CONTROL VOLUME METHOD FOR (GEO)DYNAMO SIMULATIONS

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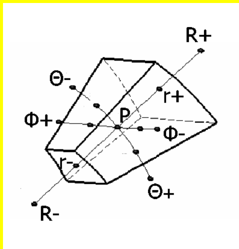
Introduction

Numerical modelling of self-consistent dynamos has made noticeable progress in the last decade due to the progress in computer technology. Almost all of physical models are based on the same numerical method, namely the spectral method. However, they are not able to run in an Earth-like parameter regime because of the considerable spatial resolution that is required. At some resolution, grid methods could be more efficient on parallel computer architectures. The control volume method is one of the local methods which would be available for dynamo simulations, and which achieves a given accuracy at high resolutions.
It is an other numerical method available for numerical modelling of a self-consistent dynamo, it was used to investigate the miscellaneous dynamo models for various input parameters and geometric configurations and was successfully tested on the so-called numerical dynamo benchmark for Case 0 and Case 2. The computations had high demands on the computer time. The forward integration of the equations was possible only with a very small time step. Results indicate that our code based on control volume method is effective on large parallel systems (consisting of a few hundreds of processors) and to expect that it will be much more effective than codes based on the spectral methods on very large parallel systems (consisting of a few thousands of processors), especially at the study of turbulence. We present the test on benchmark solution, on the efficiency of our numerical code and of parallelization and a small review of investigated models.

Control volume method

The basic strategy of the control volume method - to express the differential equations in conservative form, integrate them over the control volumes and convert every such integral into the sum of fluxes over the boundary faces by means of Gauss' theorem. It is advantageous to employ a different grid for each component of vector fields (and an additional grid for the scalar field).



Governing equations

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{V} \times \mathbf{B}) + q^{-1}\nabla^2\mathbf{B}$$

$$P_r^{-1}E\left(\frac{\partial \mathbf{V}}{\partial t} + (\mathbf{V}\cdot\nabla)\,\mathbf{V}\right) = -\nabla P + \mathbf{F} + E\nabla^2\mathbf{V}$$

$$\frac{\partial T}{\partial t} + (\mathbf{V}\cdot\nabla)\,T = \nabla^2 T + G(r)$$

$$\nabla\cdot\mathbf{B} = 0, \qquad \nabla\cdot\mathbf{V} = 0$$

where

$$\mathbf{F} = -\mathbf{1}_z \times \mathbf{V} + R_a T r \mathbf{1}_r + (\nabla\times\mathbf{B})\times\mathbf{B}$$

and

$$EI\frac{\partial \omega}{\partial t} = P_r r_i \oint_S \tau_{r\phi}\Big|_{r=r_i} \sin\theta \, \mathrm{d}S$$

The equations are scaled with the outer radius of the shell L, which makes the dimensionless radius $r_{\mathrm{CMB}} = 1$; the inner core radius r_{ICB} is usually put equal to 0.35, which is the value valid for the Earth.

Benchmark tests

$G(r) = 0$, $E = 2.1125 \times 10^{-4}$ (10^{-3}) and $P_r = 1$

Case 0 – Non-magnetic convection, inner core co-rotating with the outer boundary

$R_a = 76.92$ (100) and $q = 1$

K_r	K_θ	K_φ	E_K	T	V_φ	ω
35	35	64	61.986	0.4386	-9.958	0.2638
45	45	64	60.825	0.4316	-9.978	0.1123
55	55	64	60.563	0.4313	-10.029	0.0886
65	65	64	60.450	0.4312	-10.055	0.0984
85	85	64	60.282	0.4311	-10.082	0.1338
105	105	64	60.179	0.4310	-10.095	0.1556
45	45	96	59.994	0.4294	-10.121	0.0853
85	85	96	59.414	0.4291	-10.175	0.1187
Standard solution			**58.348**	**0.4281**	**-10.157**	**0.1824**

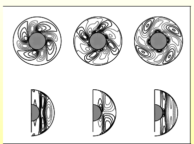
where K_r, K_θ, K_φ are numbers of grid points, E_K is the mean kinetic energy, T is the local temperature and V_φ local velocity and ω is the drift velocity. The bottom line corresponds to the suggested standard solution.

Case 2 – Dynamo with a conducting and freely rotating inner core

$R_a = 85$ (110) and $q = 5$

K_r	K_θ	K_φ	E_k	E_m	E_m^{ic}	ω	ω_{ic}
65	65	120	45.082	870.69	849.34	-4.1811	-2.9337
85	85	160	43.691	857.27	834.09	-3.9731	-2.7993
Standard solution			**42.388**	**845.60**	**822.67**	**-3.8027**	**-2.6595**

where E_k is the mean kinetic energy, E_m the mean magnetic energy in the shell, E_m^{ic} the mean magnetic energy in the core, ω is the drift velocity and ω_{ic} angular frequency of differential rotation of the inner core.

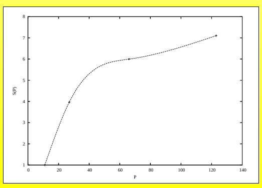


V_r, V_θ, V_φ: 85 × 85 × 160

B_r, B_θ, B_φ: 85 × 85 × 160

Test on the efficiency of parallelization

Number of processors	Time steps / 1 hour	Time steps / processor
3 × 3 + 2	3 412	379
5 × 5 + 2	13 526	541
8 × 8 + 2	20 461	319
11 × 11 + 2	24 246	200



Speed-up, $S(P)$, in dependence on number of processors, P

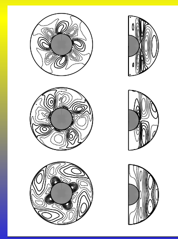
Non-uniform stratification

$$\mathbf{G(r)} = \frac{9rr_{ICB}^2 - 12r + 6r^2r_{ICB}^2 + 60r^2 - 2r_{ICB}^2 - 8 + r_{ICB}^4 - 12r_{ICB}r^2 - 6rr_{ICB}^3 - 18r_{ICB}r}{r(r_{ICB}^2 - 4)}$$

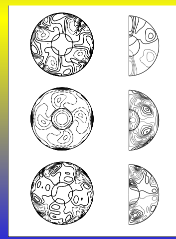
$R_a = 550$ and $P_r = 1$

E	q	E_k	E_m	E_m^{ic}	ω	ω_{ic}
10^{-1}	8	805.3	3619	3365	-1.6541	-0.9337
10^{-2}	5	773.6	3403	2722	-1.5342	-0.7962
10^{-3}	5	1459	16049	14922	-1.5913	-0.8527
10^{-4}	2	2318	44042	40950	-1.6443	-0.9116

$E = 10^{-4}$



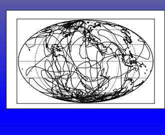
V_r, V_θ, V_φ

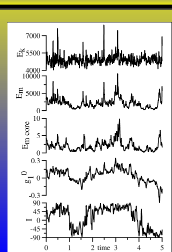


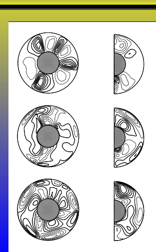
B_r, B_θ, B_φ

Reversals & Thin shell

G(r) = 0, $E = 3 \times 10^{-3}$,
$P_r = 1$, $R_a = 10^3$ and $q = 2$



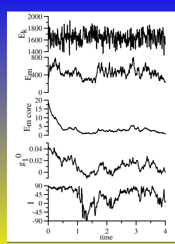


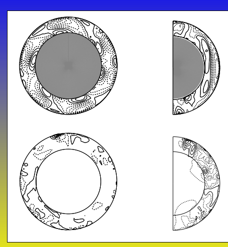


V_r, V_θ, V_φ

B_r, B_θ, B_φ

G(r) = 3, $E = 3 \times 10^{-4}$,
$P_r = 10^{-1}$, $R_a = 10^3$, $q = 8$
and $r_{\mathrm{ICB}} = 0.67$





V_φ (top) and B_φ (bottom)

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