Magnetic Field Amplification in MRI-driven turbulence

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Outline.

1. Introduction to MRI
2. Shearing Box & Numerical Methods
3. Current studies of MRI.
4. Other instabilities in accretion disks.
5. Summary

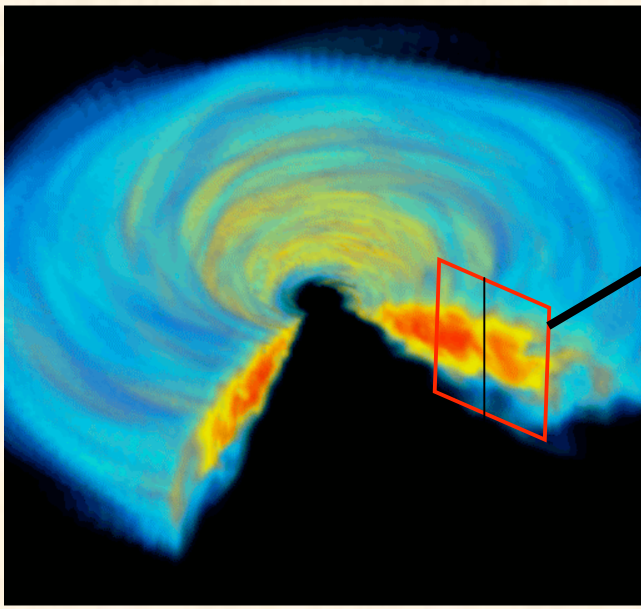
Will try to make connections to dynamo theory throughout.

Also, see talks from recent workshop on the MRI organized by M. Pessah at IAS: http://www.sns.ias.edu/mri-2008

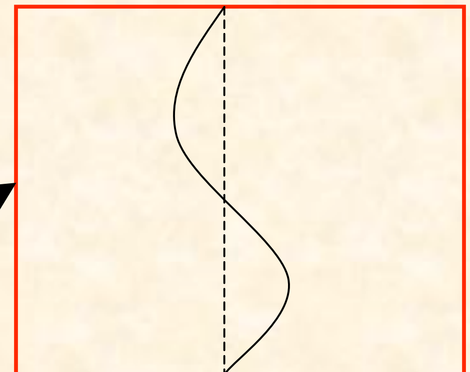
The MRI in accretion disks.

Weakly magnetized Keplerian rotation profiles are subject to local, linear instability.

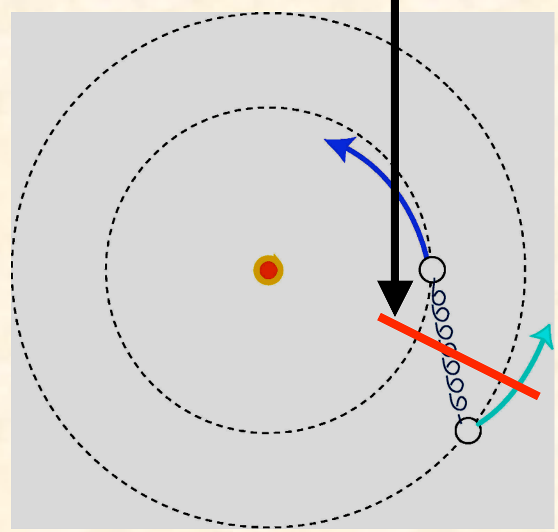
Mechanism:



Radial perturbations to vertical B



Side view
(r-z plane)



Top view
(r-ϕ plane)

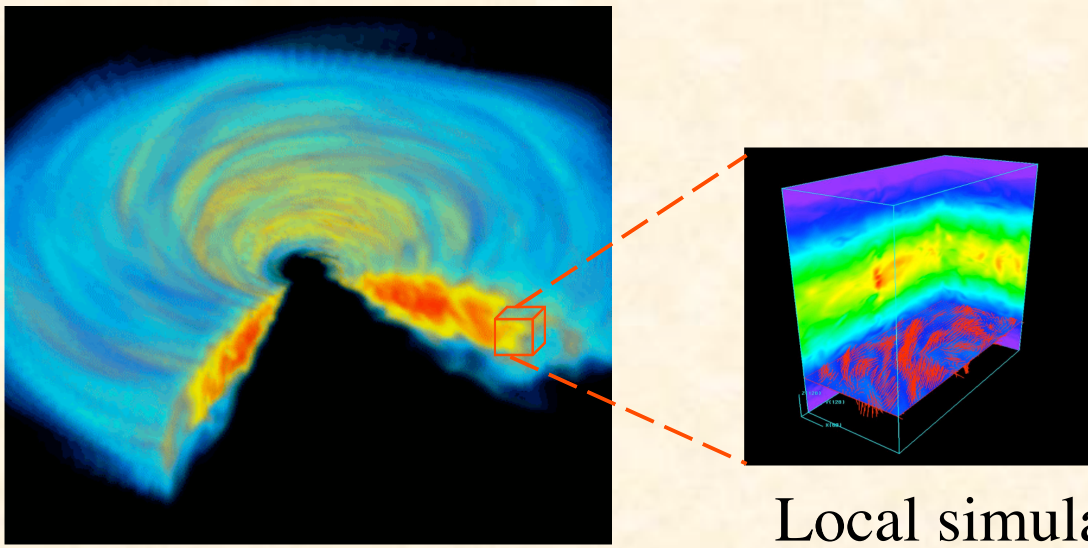
Instability when

$$(\mathbf{k} \cdot \mathbf{V_A}) \leq \sqrt{3}\Omega$$

Numerical methods for MHD required to study nonlinear regime. Many codes are used:

- ZEUS (Stone & Norman 1992a; b)
  - See David Clarke's ZEUS-3D webpage for Falle (2002) tests..
- Athena (Gardiner & Stone 2005; 2008; Stone et al. 2008)
  - Higher-order Godunov scheme
  - Both nested and adaptive meshes
  - Download from http://www.astro.princeton.edu/~jstone
- Other codes in use:
  - Pencil code
  - Nirvana
  - RAMSES
  - PLUTO

Numerical methods for MRI: Shearing-box source terms.



Global simulation

Local simulation

- Shearing box → Expand MHD equations in a frame orbiting at the local angular velocity Ω_0.
- Introduces Coriolis force and tidal gravity as *source terms*

Introduces Coriolis force and tidal gravity as *source terms*

$$\frac{\partial q}{\partial t}+\frac{\partial F}{\partial x}+\frac{\partial G}{\partial y}+\frac{\partial H}{\partial z}=S$$

$$S_{\rho v_x}=3\Omega_0^2 x\rho+2\Omega_0\rho v_y$$

$$S_{\rho v_y}=-2\Omega_0\rho v_x$$

$$S_E=3\Omega_0^2 x\rho v_x$$

Important ingredients of algorithm:

- Update *momentum fluctuation* equations with Crank-Nicholson
- Update tidal gravity term as an effective potential
- Total Energy (including tidal potential) is *conserved*.
- Remap emfs at x-faces to conserve total magnetic flux.

Numerical methods for MRI: FARGO.

Azimuthal velocity can be decomposed into fluctuations and background Keplerian shear.

$$V_y = V_{orb} + \delta V_y$$

V_{orb} is a stationary flow.

Suggest an operator split approach:

- Normal shearing sheet algorithm using for MHD equations written using δV_y.
- Background shear taken into account via linear advection equations using V_{orb}

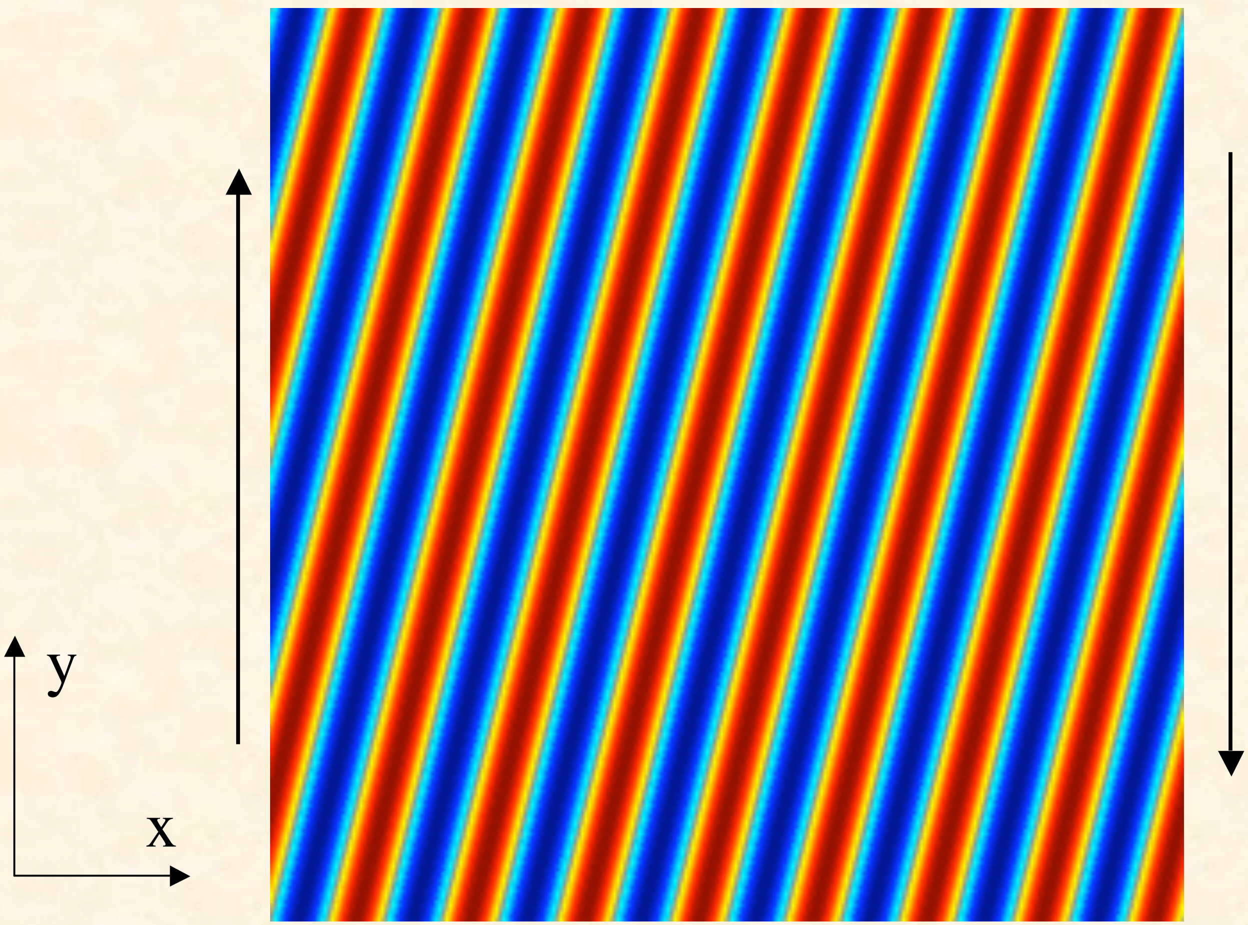
Advantage: easy to design an algorithm for linear advection in which the CFL condition does not apply

- Timestep with FARGO can be very large (set by CFL condition on $\delta V_y + C_f$)

Resulting method is fully 2nd order accurate, and seems to be *more* accurate than shearing sheet without FARGO.

FARGO can be used for both shearing-sheet and global disk models.

A test of FARGO: shear amplification of a planar MHD wave.



Vx

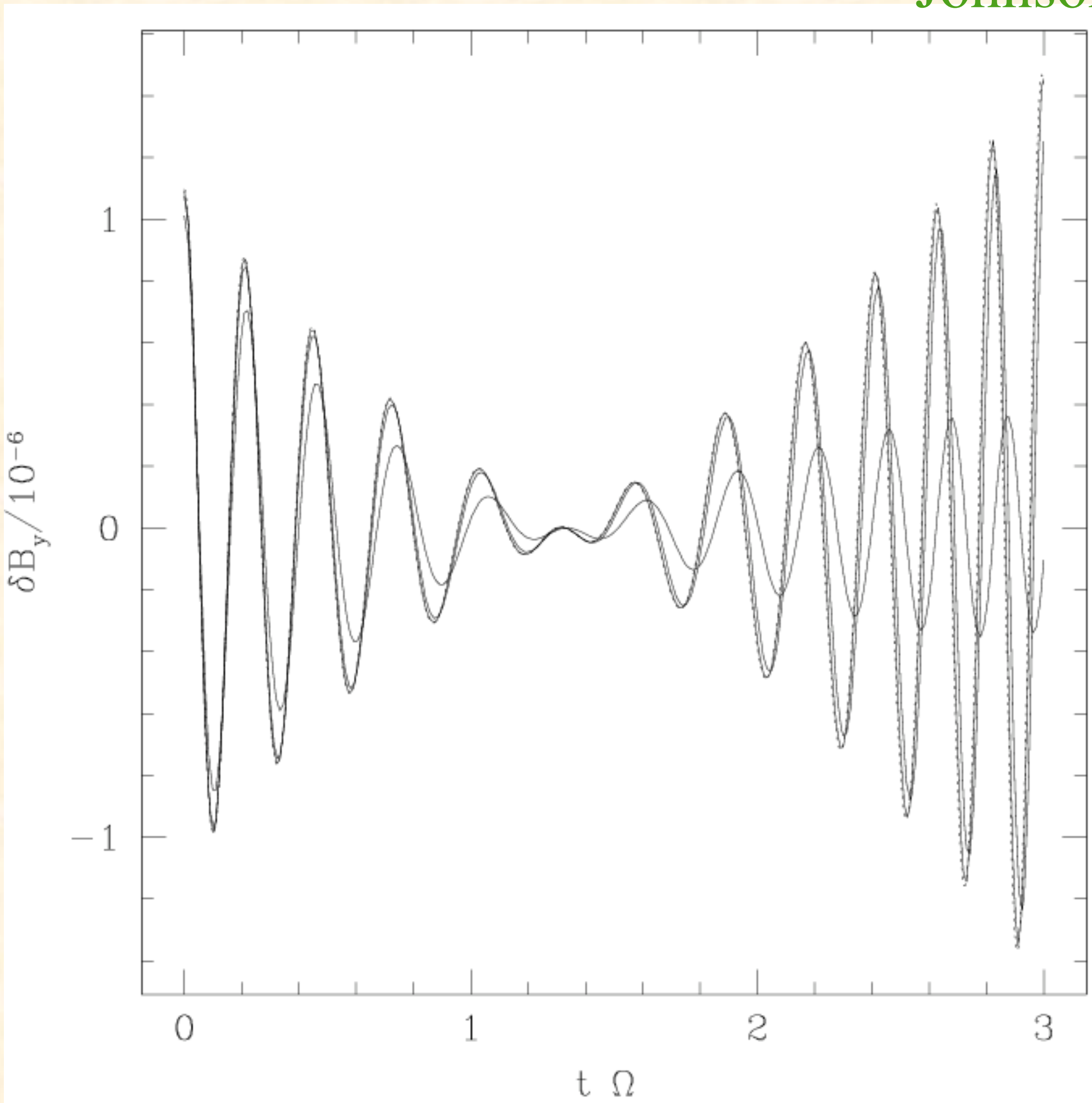
256^2 grid

$(k_x/k_y)_0 = 4$

Uses the local shearing box approximation in the x-y (r-ϕ) plane

Evolution of wave amplitude.

Johnson, Guan, & Gammie (2008)



Athena with FARGO

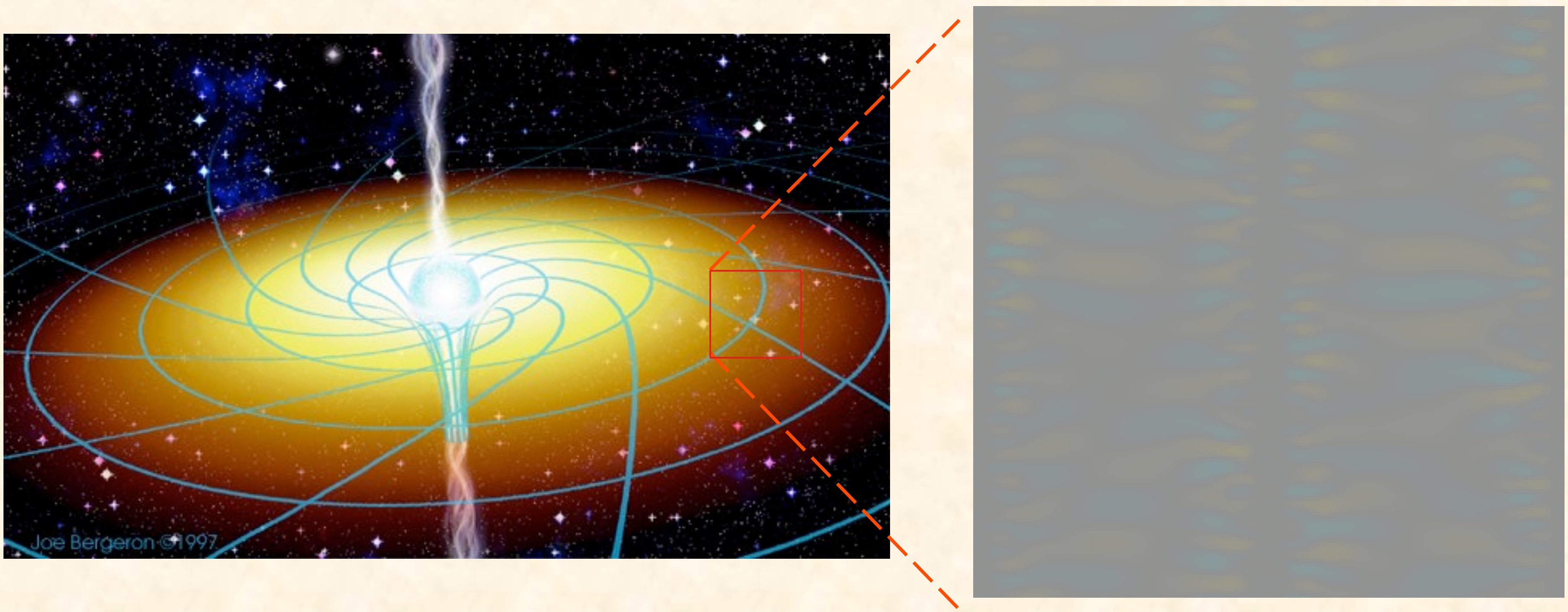
Resolution 8, 16, 32 points per λ.

Dotted line analytic solution.

Shearing box captures the essential dynamics of orbital flow. Still need global simulations to study structure and evolution of disks.

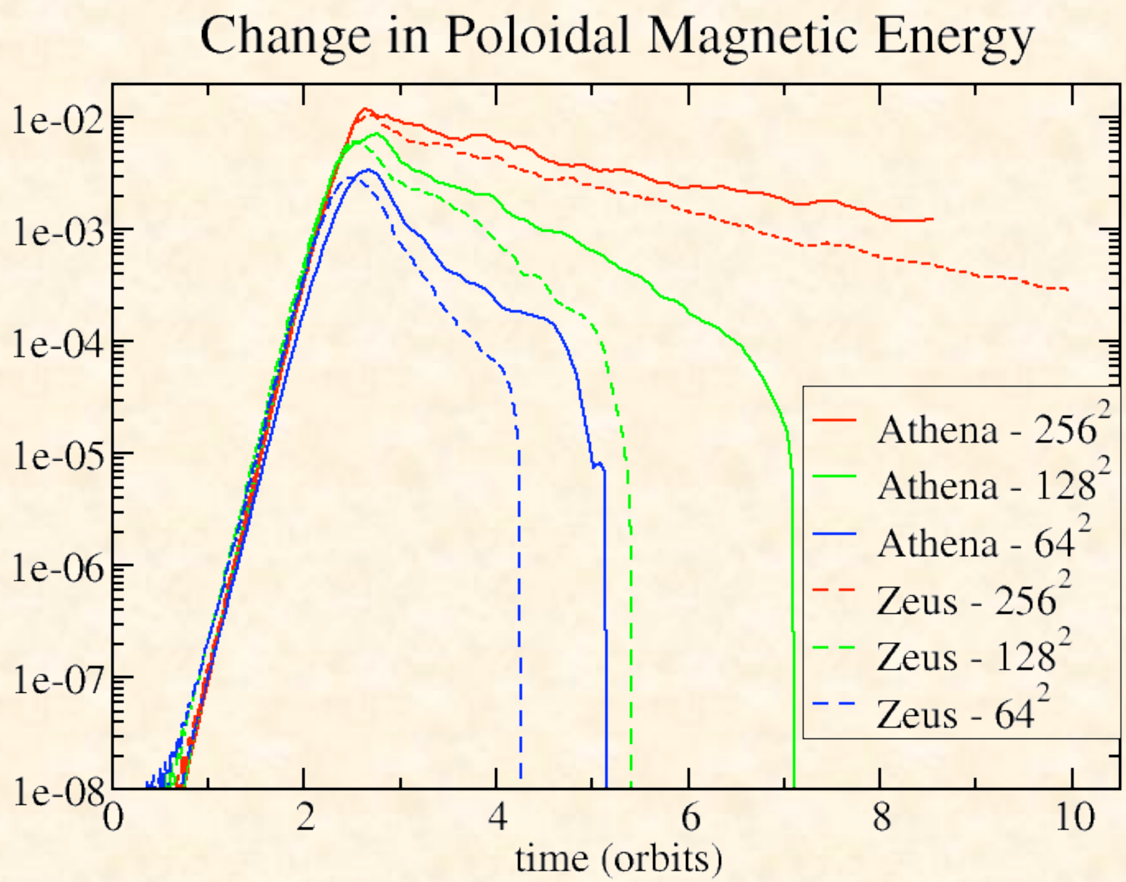
Example of nonlinear saturation of MRI.

Start from a vertical field with zero net flux: $B_z = B_0 \sin(2\pi x)$
Sustained turbulence not possible in 2D… (anti-dynamo theorem)



Animation of angular velocity fluctuations: $\delta V_\phi = V_\phi - V_{Kep}$

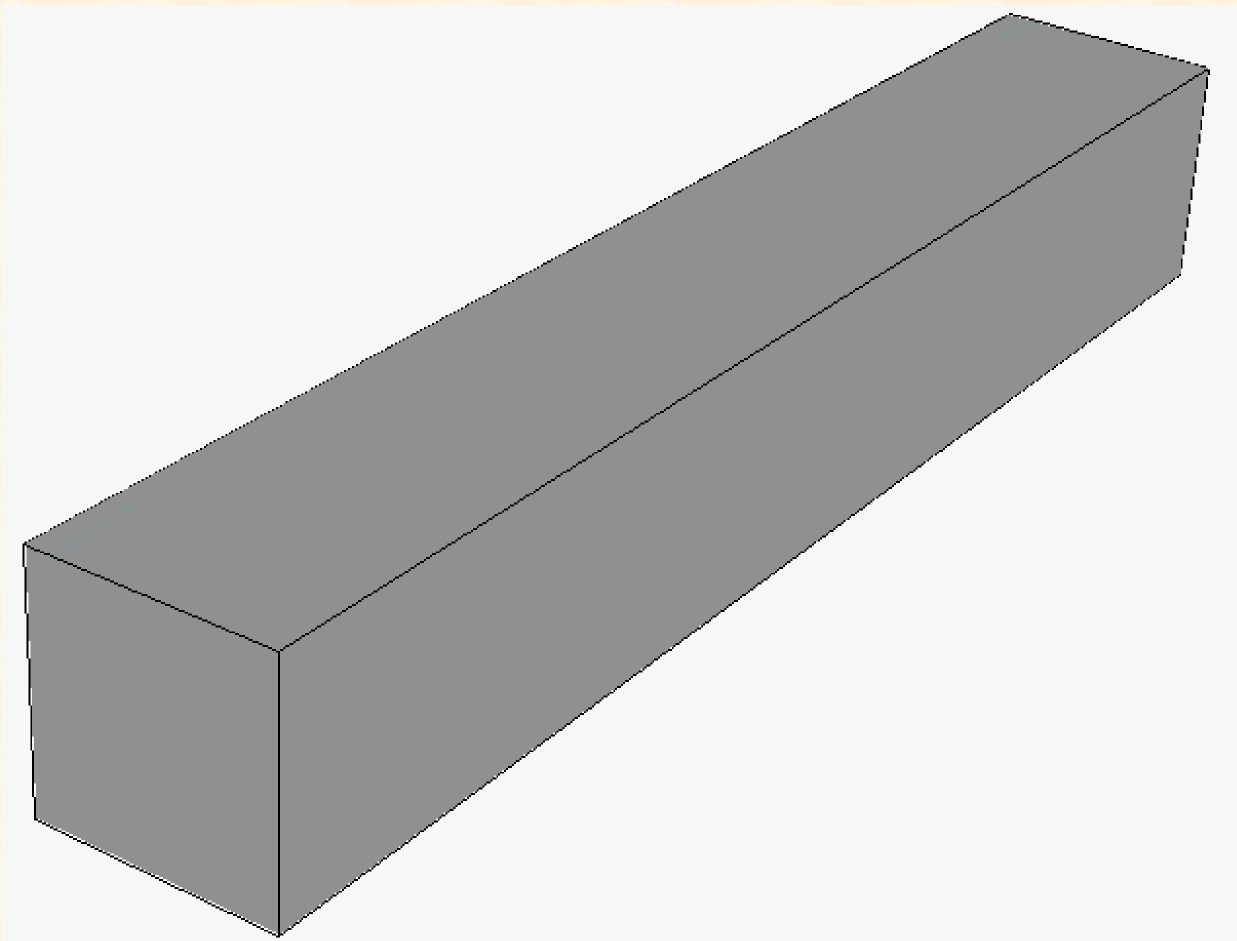
Decay of Magnetic Energy in 2D MRI with no-net-flux is a good code test.



Numerical dissipation is ~1.5 times smaller with Athena compared to ZEUS.

3D MRI

Animation of angular velocity fluctuations: $\delta V_{\phi} = V_{\phi} - V_{Kep}$
Initial Field Geometry is Uniform B_y

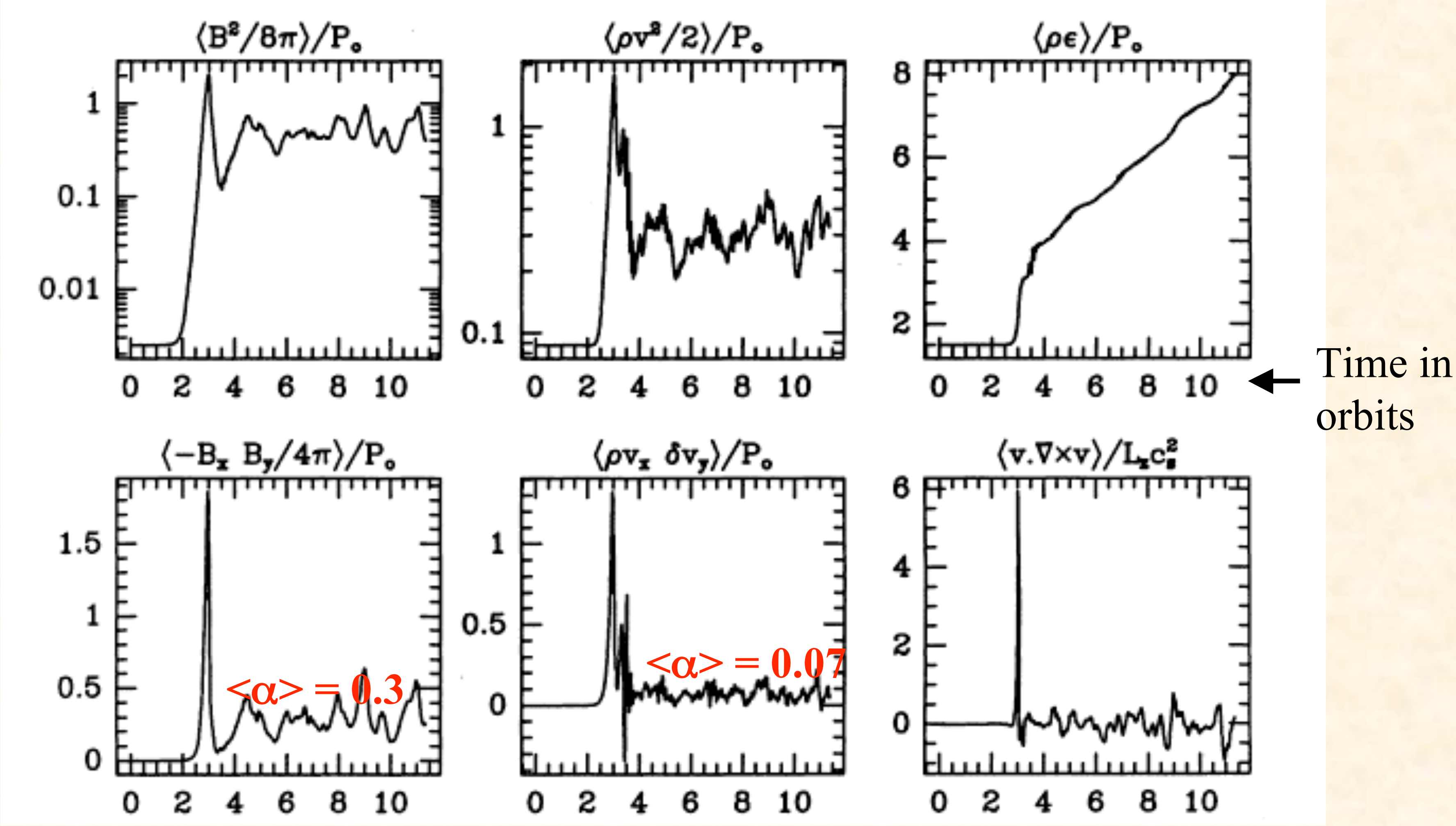


128 x 256 x 128 Grid
β_{min}= 100, orbits 4-20

In 3D, sustained turbulence

Primary importance of MRI is total stress in saturated state.

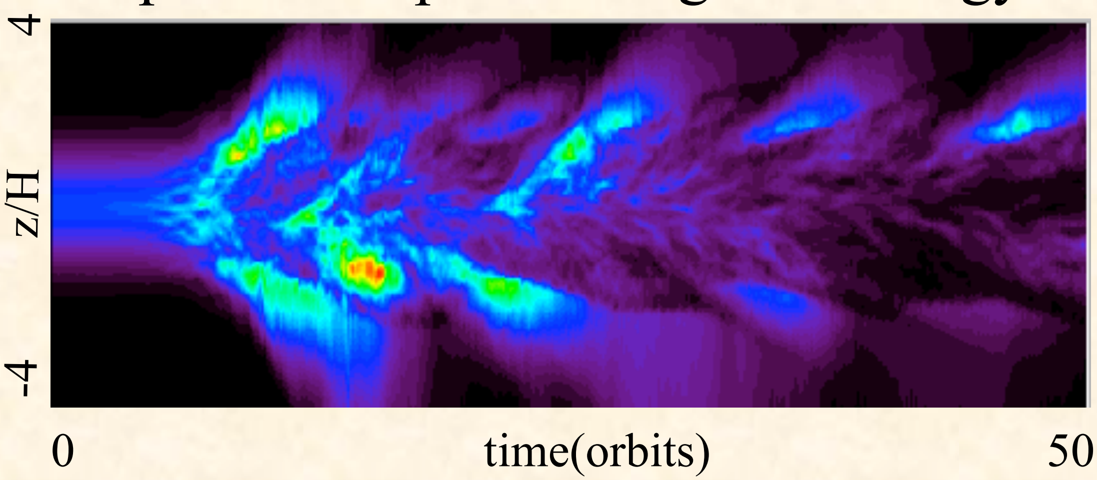
Time-evolution of volume-averaged quantities:



Also note: Sustained amplification of B indicates dynamo action.
Maxwell stress with even a small net flux gives a large α

In vertically stratified disks, MRI generates magnetized corona.

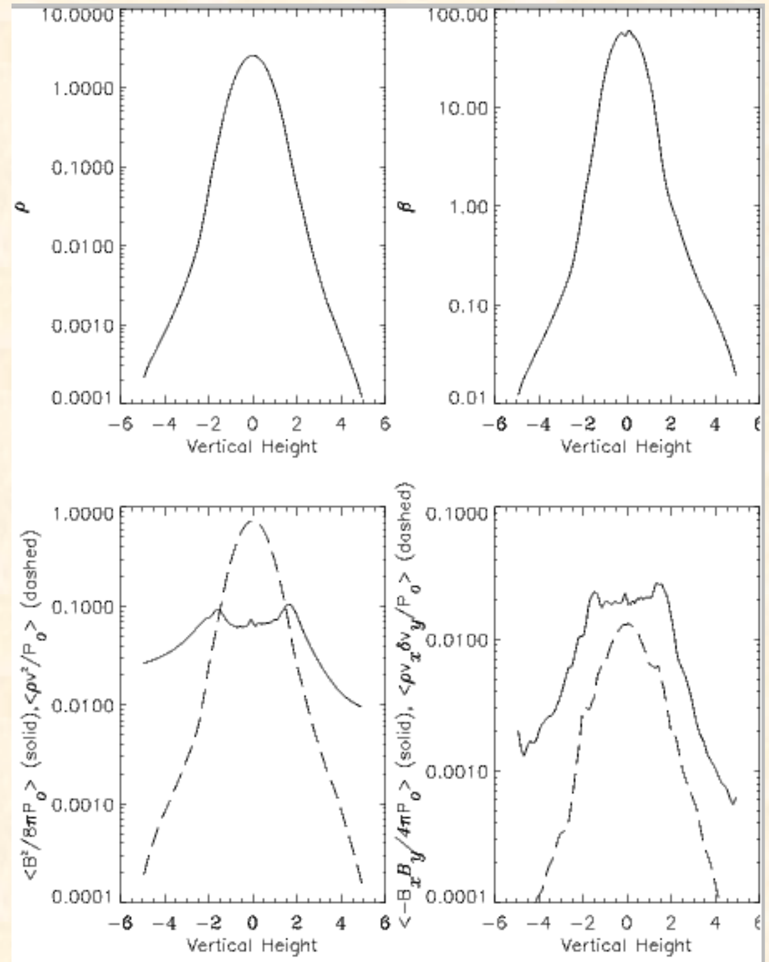
“spacetime” plot of magnetic energy



Clearly shows how buoyant field rises into corona.

$$F(z,t) = \frac{\int\int f(x,y,z,t)dxdy}{\int\int dxdy}.$$

Vertical profiles of t-averaged quantities.

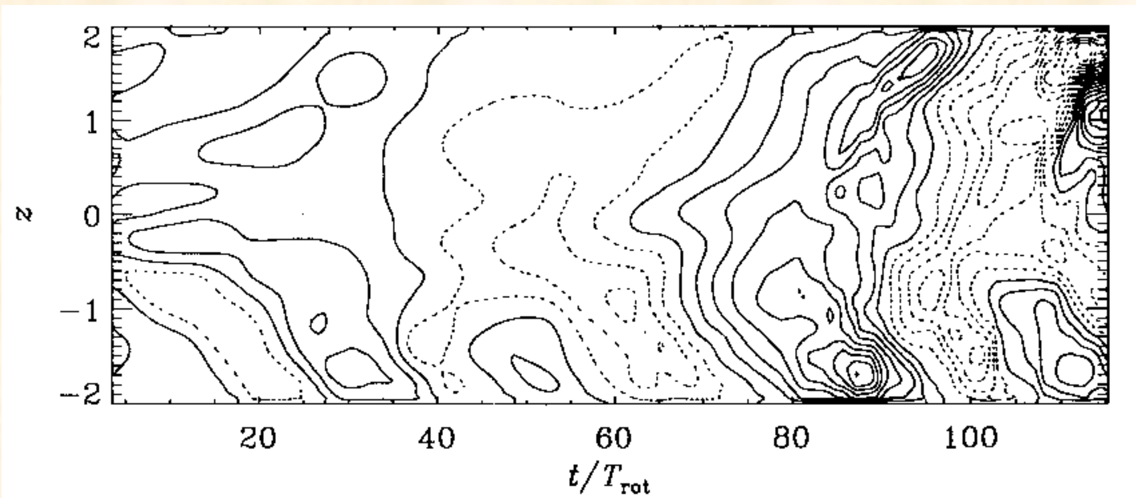


Miller & Stone 1999

Is there a mean field dynamo in a stratified disk?

Brandenburg et al. (1995)

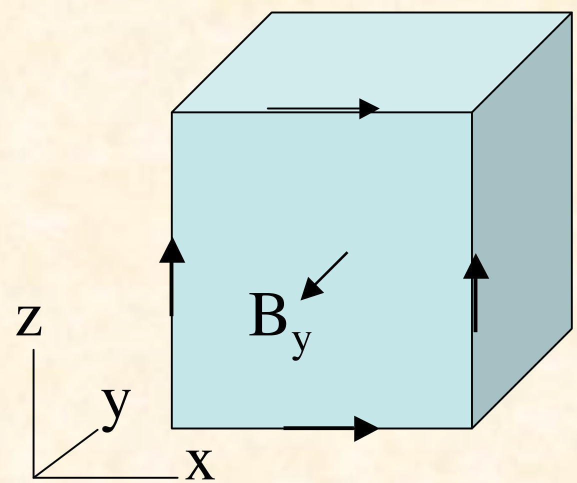
Oscillating net $<B_y>$ seen in stratified disks



But net B_y must be generated by currents at vertical boundaries.

Different BCs, different results.

More results in Axel's talk.



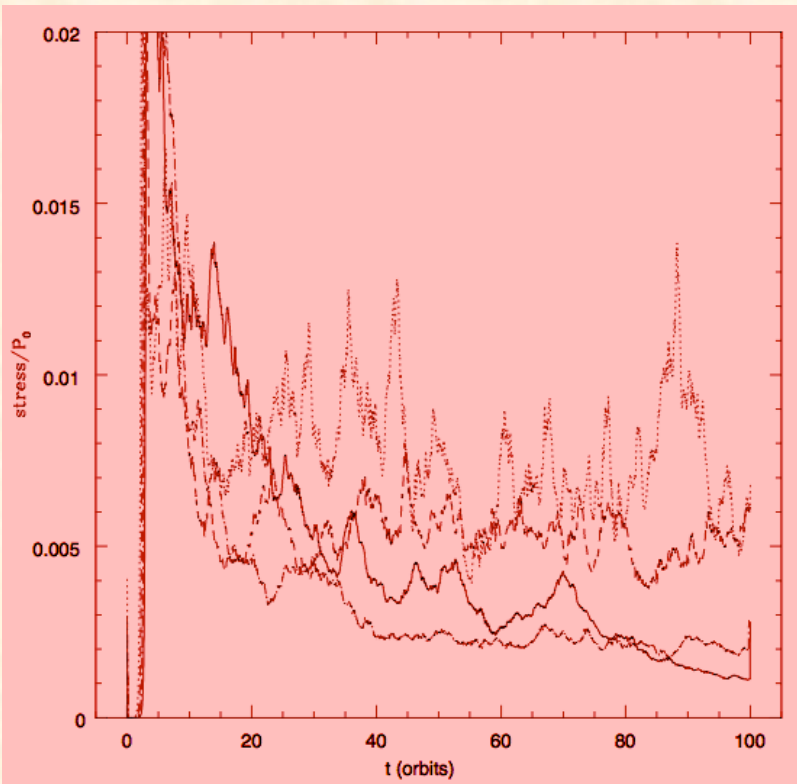
E_z cancel with shearing periodic BCs

Some Issues Currently Being Studied.

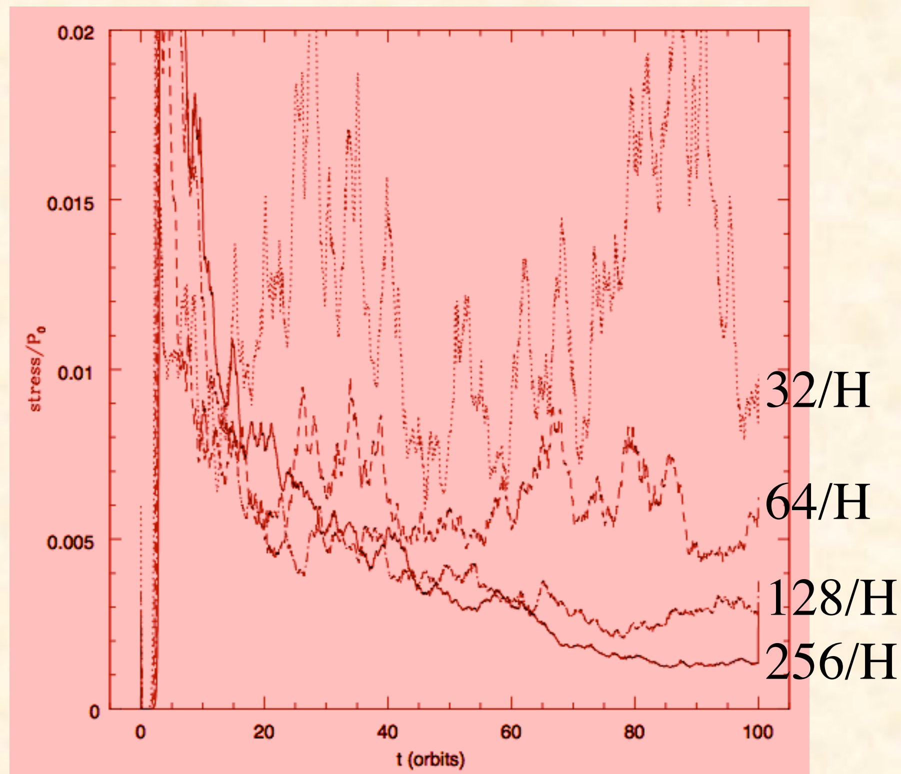
- Convergence with zero-net-flux
- Effect of finite $P_m = \nu/\eta$
- Effect of bigger radial box size
- Measurement of turbulent Pm

Current studies: *convergence with no net-flux.*

Fromang & Papaloizou (2007) have shown than with zero net-flux, the saturation amplitude of the MRI *decreases* with *increasing* resolution.



Athena -- no FARGO

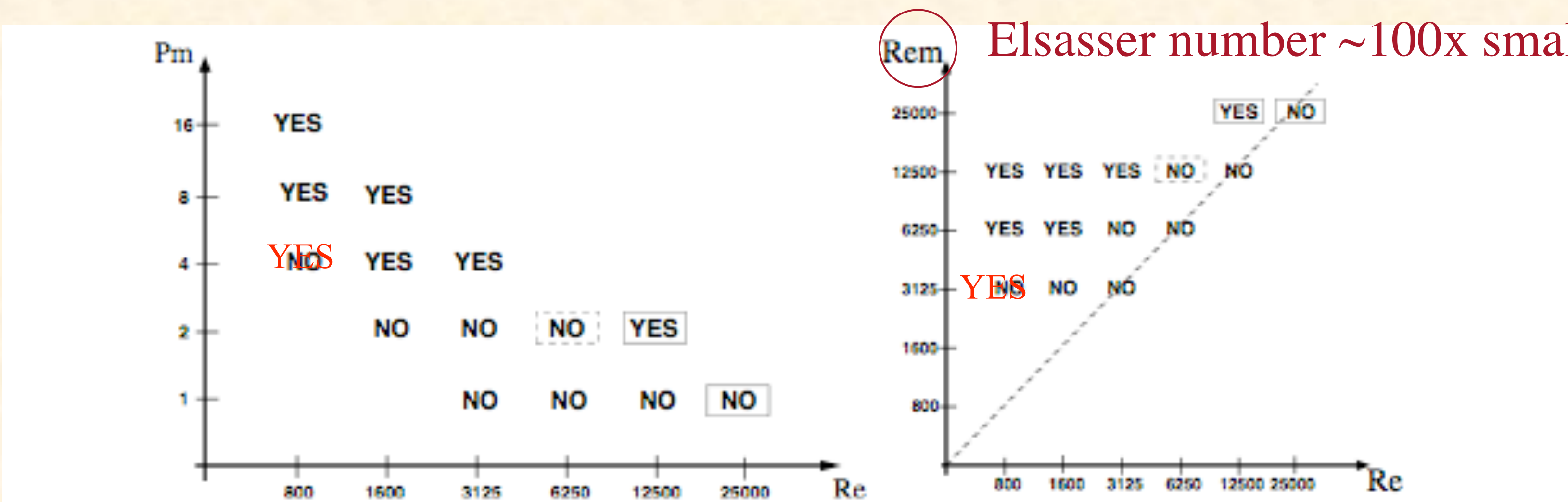


Athena -- FARGO

Is zero net-flux shearing box relevant to real disks?

Current studies: *effect of finite dissipation.*

Fromang et al. (2007) have found that MRI-driven turbulence dies away in the shearing box with no net field at low magnetic Prandtl number Pr = ν/η, and low Reynolds number Re = cH/ν.



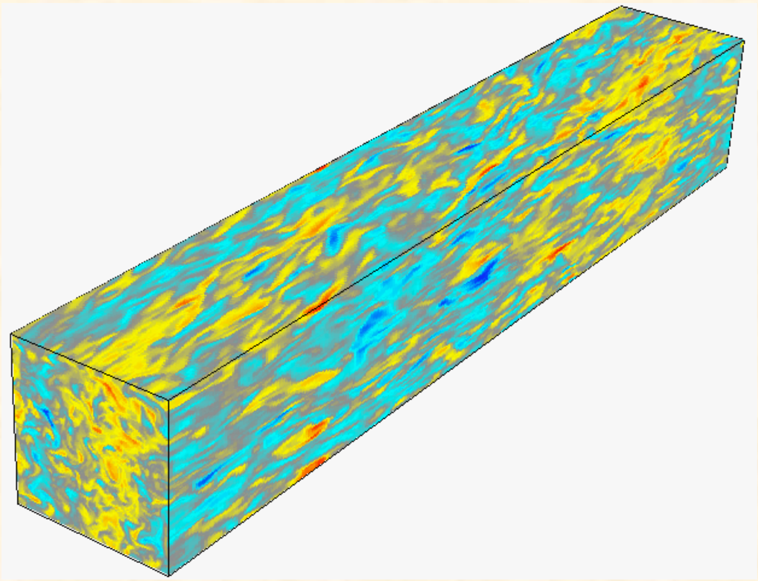
Elsasser number ~100x smaller

YES = sustained turbulence
NO = turbulence eventually dies away… WHY?

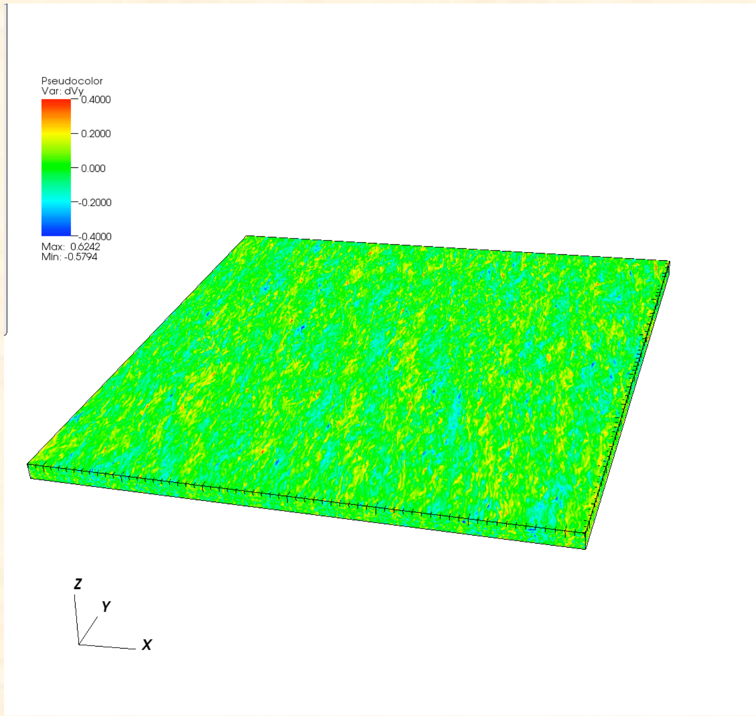
Exact YES/NO boundary seems to depend on boxsize.
Saturation amplitude can change by ~50% with Godunov schemes.

Current studies: *effect of increasing boxsize.*

Almost all 3D simulations of the MRI to date use very narrow domain in the radial (x) direction.



Thin box: H x 4H x H

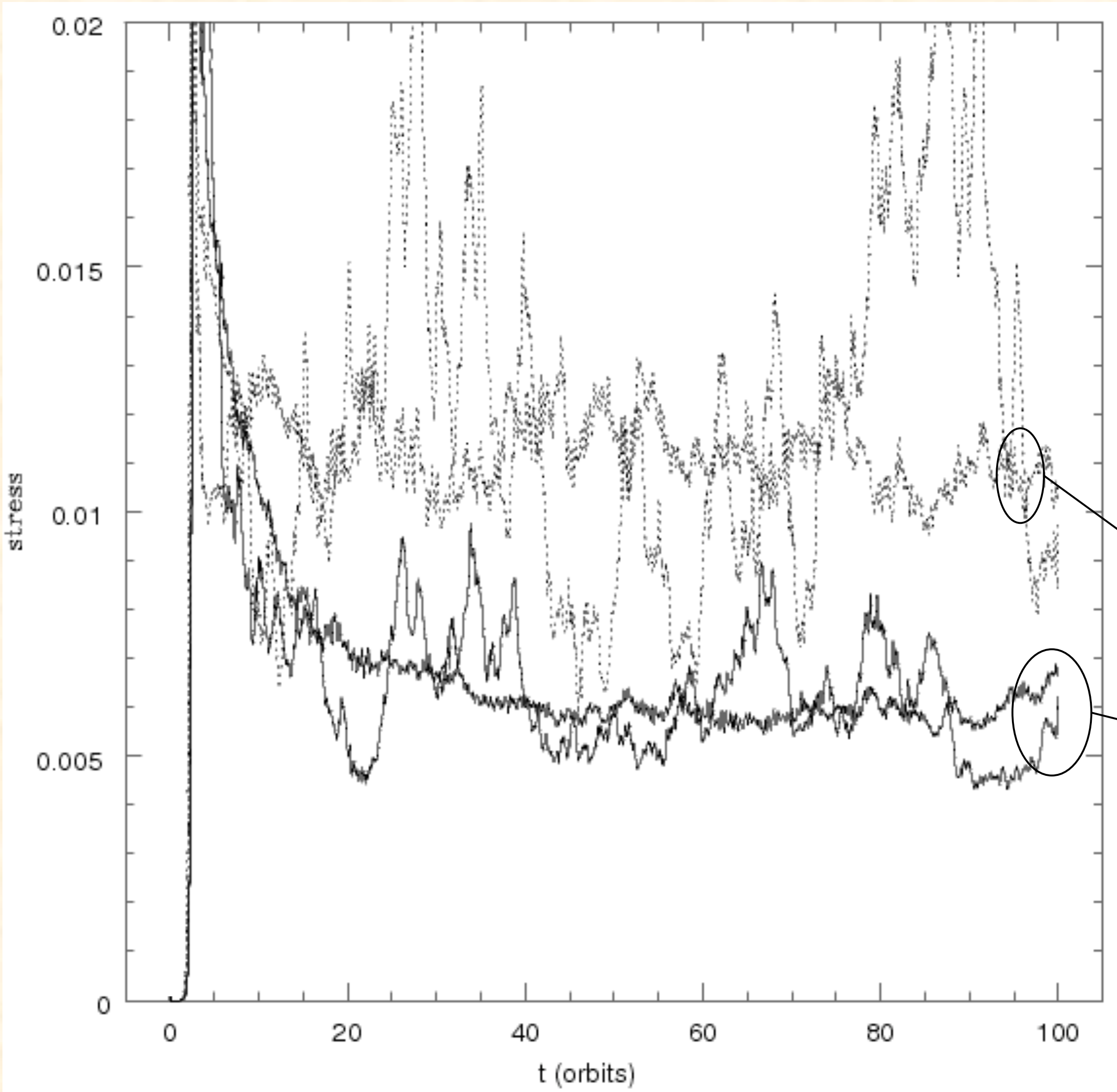


Wide box: 32H x 32H x H

Is the saturation level the same in wide boxes?

(Requires FARGO algorithm for circumventing CFL condition on orbital velocity.)

Preliminary results: no-net-flux.



$B_z = B_0 \sin(2\pi x)$
$\beta = 2P_0/B_0^2 = 400$

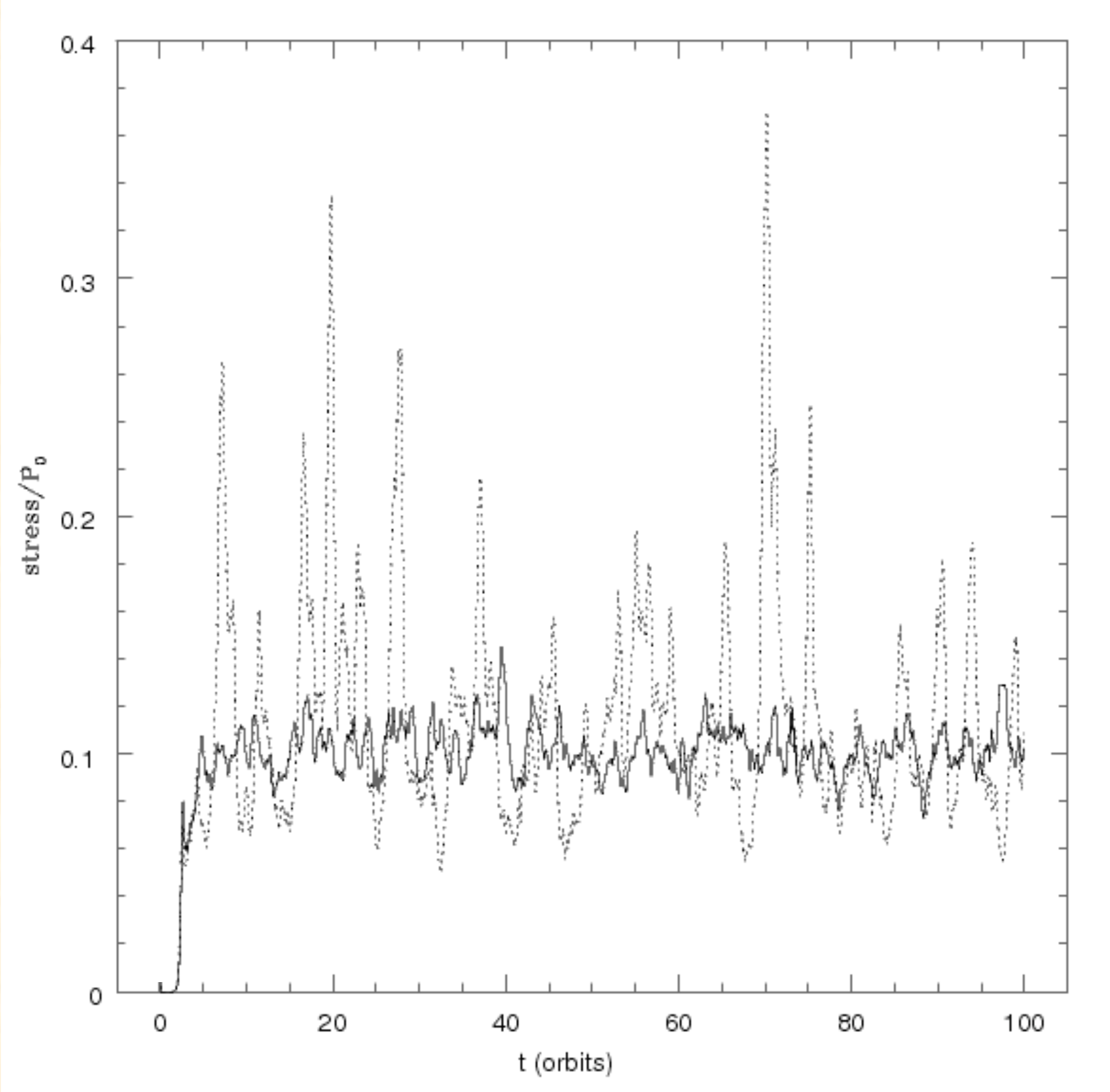
Domain size either:
H x 4H x H
or 8H x 8H x H

resolution 32/H

resolution 64/H

Less time-variability due to better statistics, additional parasitic modes

Preliminary results: net flux.



$B_z = B_0$
$\beta = 2P_0/B_0^2 = 1600$

Domain size either:
H x 8H x H
or 8H x 8H x H

Resolution 64/H

See Bodo et al (2008) for extensive analysis of this problem.

Volume-averages do ***not*** differ significantly

We need to understand much more than just the MRI.

Other MHD instabilities that can be important in disks:

1. *Parker instability.* Produces vertical flux of magnetic energy.
2. *Magneto-viscous instability (MVI).* Important in hot, diffuse plasmas (AGN disks, coronae)
3. *Magneto-thermal instability (MTI).* Important in diffuse, thermally stratified plasmas
4. *Photon bubble instability.* May be important in radiation dominated disk atmospheres.
5. *RT and KH instabilities*. May be important in star-disk interaction region.

MTI & MVI: Instabilities in the weakly collisional regime.

If mean-free-path of particles in a plasma is long compared to gyro-radius, anisotropic transport coefficients can be important (Braginskii 1965)

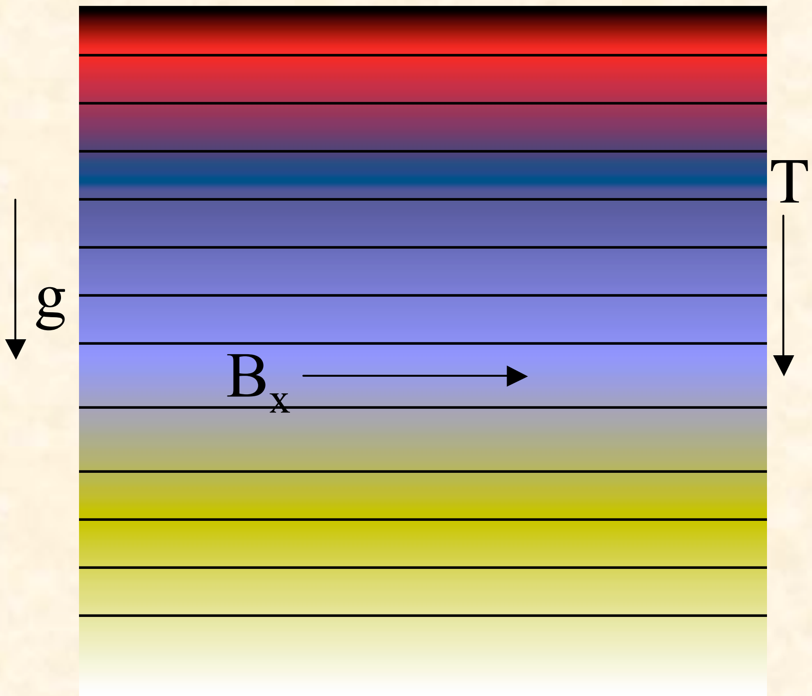
These effects can produce *qualitative* changes to the dynamics:

1. with anisotropic heat conduction, the convective instability criterion becomes $dT/dz < 0$ (Balbus 2000) (magneto-thermal instability) (Parrish & Stone 2005; 2007)
2. Hot accretion flows around compact objects may be nearly collisionless; growth and saturation of MRI can be different in kinetic MHD. (Sharma et al. 2006)
3. In fact, a new shear instability (magneto-viscous instability) exists with anisotropic viscosity (Islam & Balbus 2008)

Magneto-thermal instability (MTI).

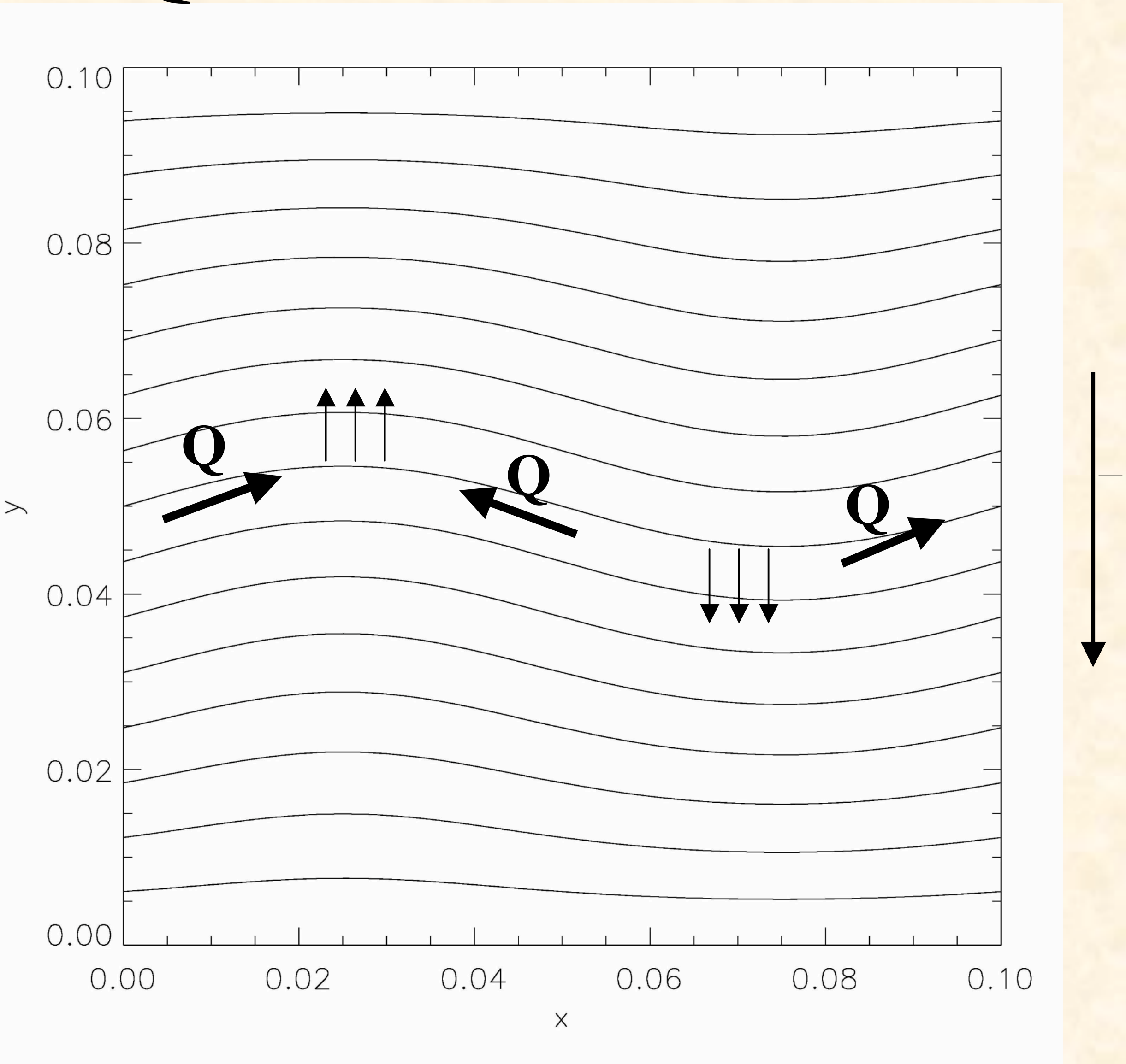
with Ian Parrish

Temperature Profile



- Convectively Stable (dS/dz > 0)
- MTI unstable (dT/dz < 0)
- Ideal MHD + Anisotropic Heat Conduction
- g = constant
- BC's: adiabatic at y-boundary, periodic in x

Qualitative Mechanism



Field Line Plot of Single Mode Perturbation

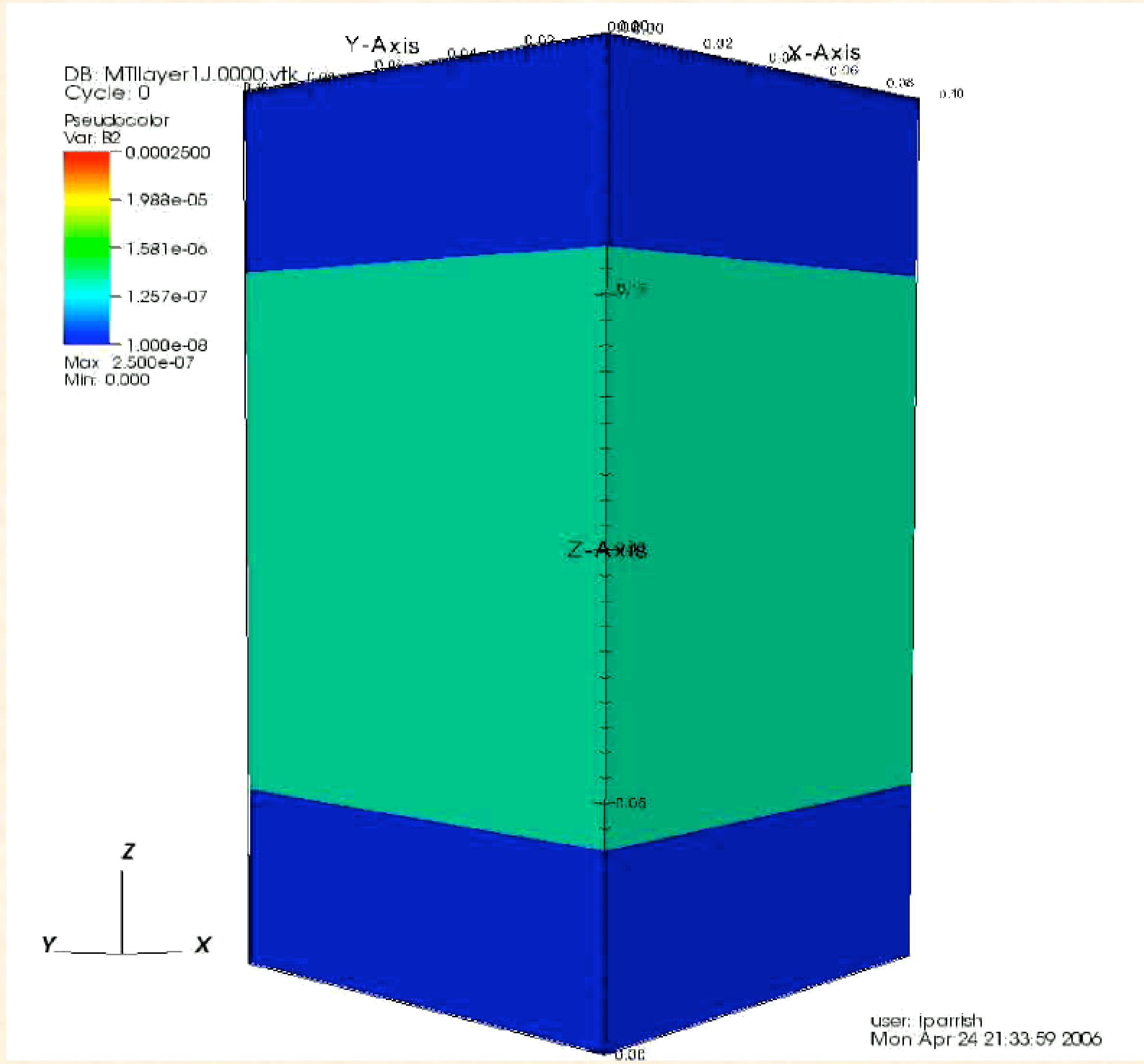
Convection in stratified atmosphere is vigorous, and sustained in 3D (Parrish & Stone 2007)

Vertical equilibrium identical to 2D runs with stable layers

B = ($B_o$sin(z),$B_o$cos(z),0)

128x128x256 grid

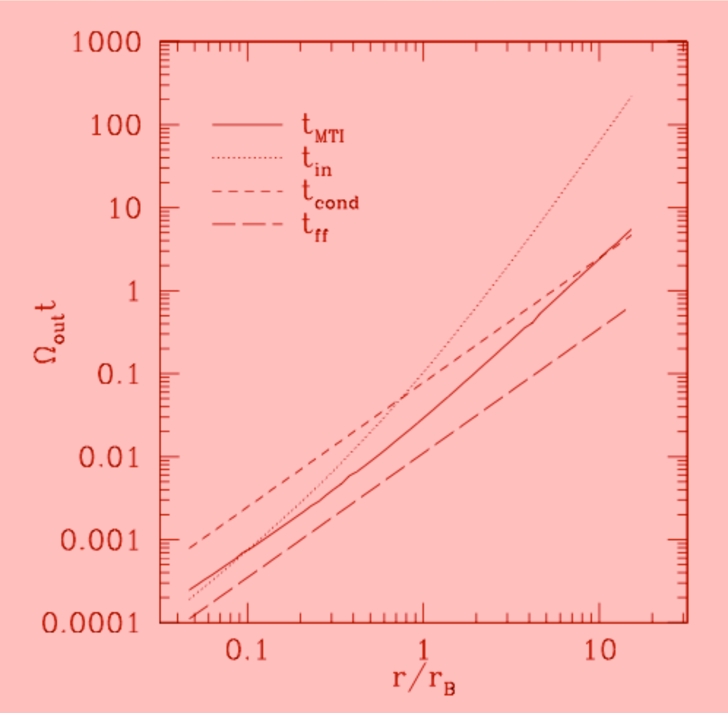
Animation of B^2

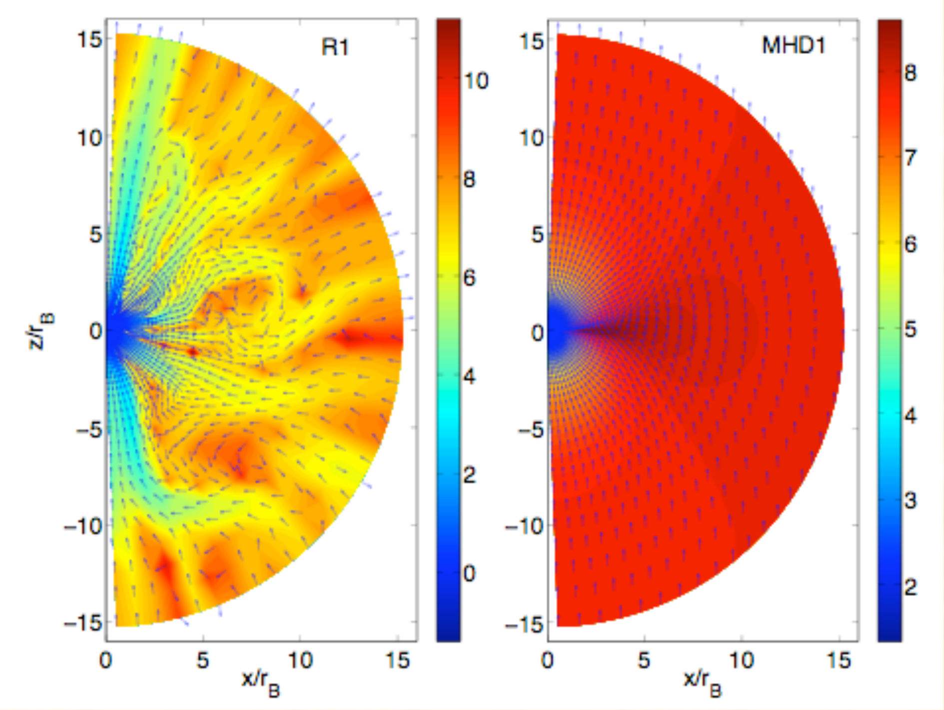


MTI in Bondi accretion flow.

Sharma et al., 2008

Growth time of MTI is shorter than infall time in Bondi flow.





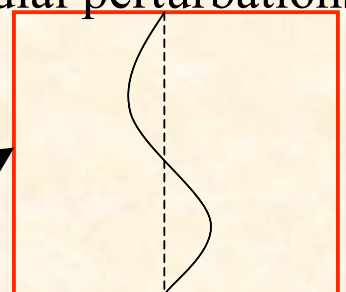
This leads to large amplification of field in convective turbulence, but little change to time-averaged ρ(r) and T(r)

Magneto-viscous instability.

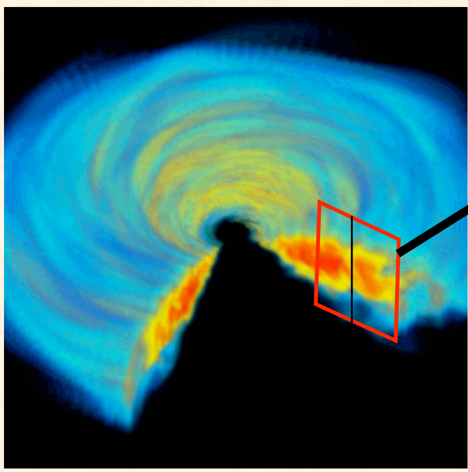
In a weakly colisional plasma, viscous transport is only along magnetic field lines (Braginskii 1964). Relevant to inner regions of AGN disks.

This leads to the magneto-viscous instability in disks (Islam & Balbus 2008). Mechanism is identical to MRI, except viscosity (rather than Maxwell stress) transports angular momentum!

Radial perturbations to vertical B

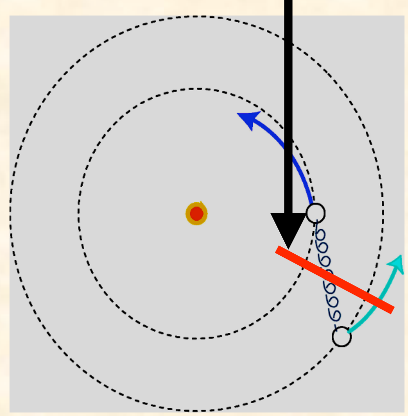


Side view
(r-z plane)



Instability when

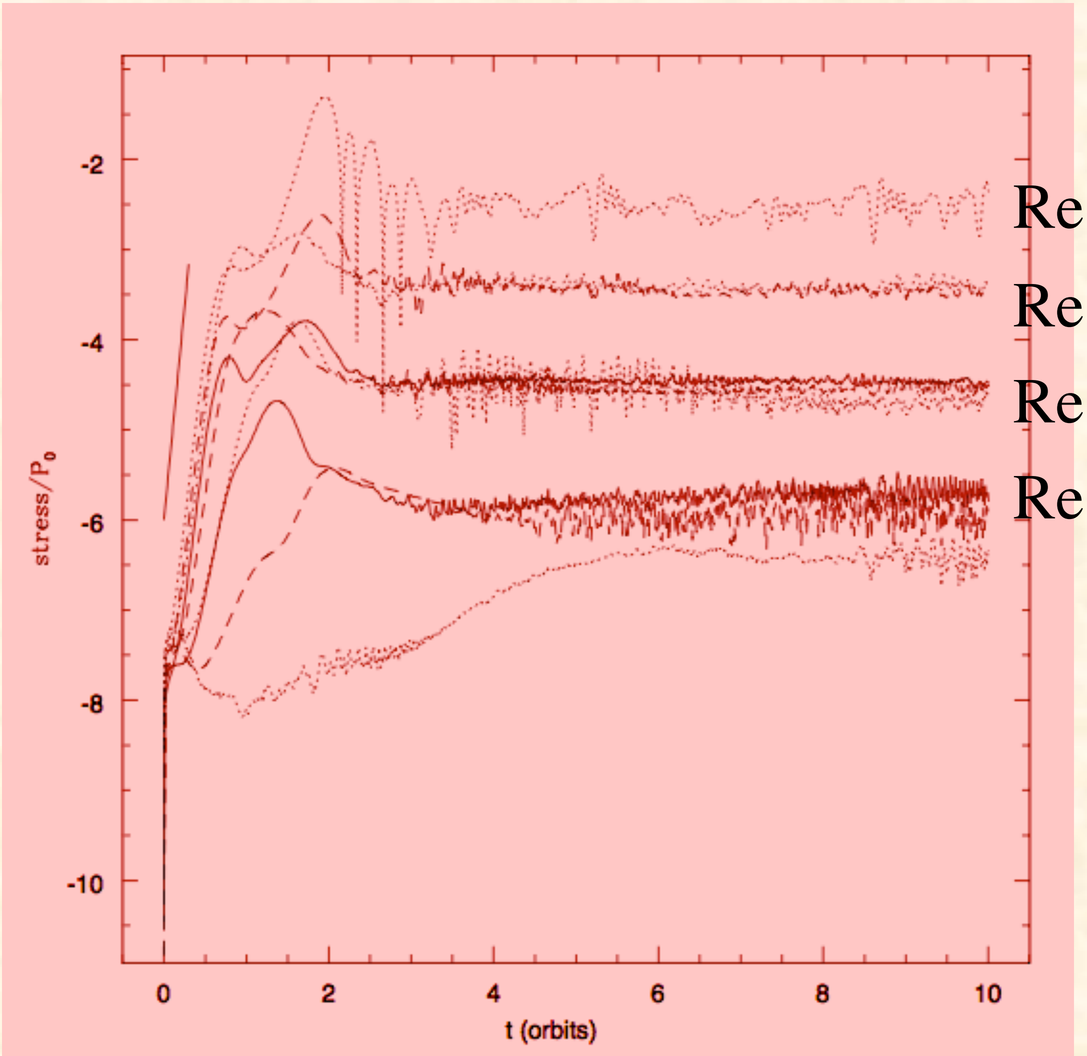
$$(\mathbf{k} \cdot \mathbf{V_A}) \leq \sqrt{3}\Omega$$



Will the saturation of the MRI be different when the MVI is also present?

In nonlinear regime, pure MVI produces turbulence and significant Reynolds stress.

$\beta = 10^{10}$; field is too weak for MRI



Re = $C_s^2/\Omega\nu$ = 10

Re = 10^2

Re = 10^3

Re = 10^4

H x 4H x H box

Numerical resolution:

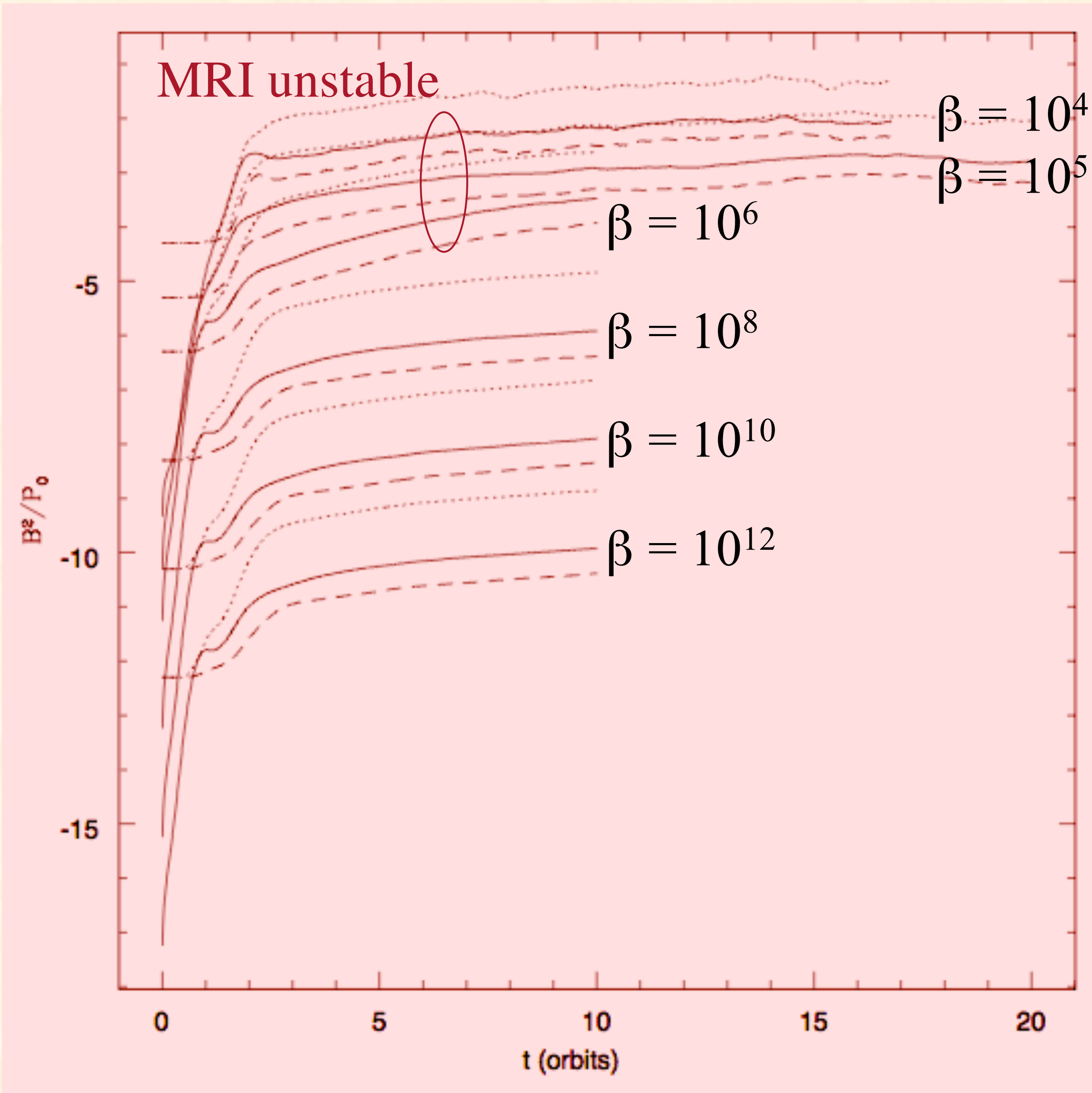
dotted = 32/H

dashed = 64/H

solid = 128/H

With explicit resistivity, saturation independent of P_m

In nonlinear regime, MVI+MRI produces significant amplification of magnetic field.



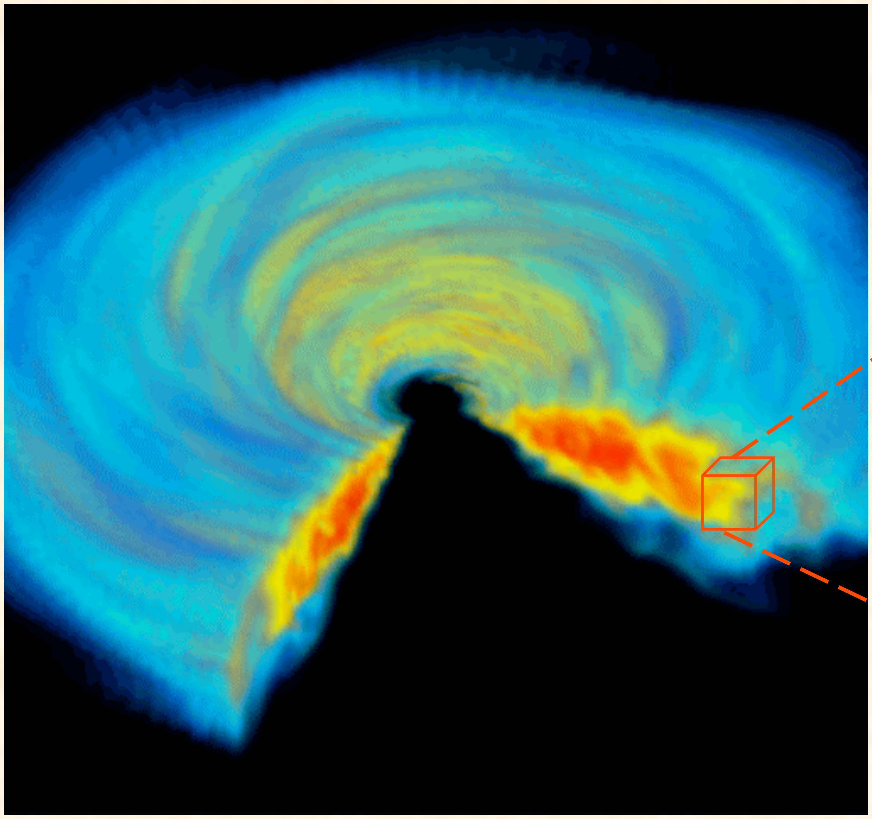
MVI + MRI provides whole new context to study dynamo action in AGN and protogalactic disks.

There is still so much to do to apply the MRI to different astrophysical systems.

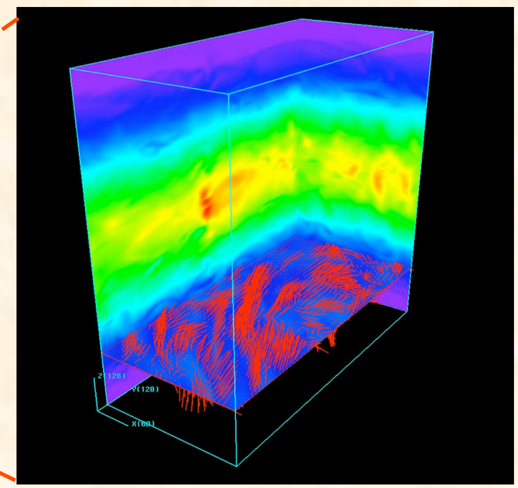
- Disks around BHs and NSs
  - –Radiation MHD (Turner et al., Hirose et al.)
  - –Anisotropic viscosity and conductivity (Sharma et al., Islam & Balbus)
  - –Self-gravity (Fromang & Balbus)
- Disks around WDs
  - –Radiative cooling
  - –Temperature dependent resistivity
  - –Modeling boundary layer
- Disks around YSOs
  - –Non-ideal MHD (Hall, Ohmic, AD) (Salmeron & Wardle, et al.)
  - –Disk-stellar magnetosphere interaction
  - –Disk-planet interaction (Winters et al., Nelson & Papaloizou)
  - –Effect of dust (Johansan et al., Turner & Sano)

Moreover, *global* simulations will be needed to understand each of these systems

e.g., global GR MHD studies of black hole accretions disks have been reported by various groups.



Global simulation



Local simulation

Hawley, Balbus, & Stone 2001; Miller & Stone 1999

Summary

1. MRI studies in the shearing box are being informed by dynamo theory (esp. no net-flux case).

   For example, microscopic diffusivities affect saturation.

2. Modeling astrophysical disks require more physics than the MRI (e.g. radiation, Hall MHD, etc.).

   Much work remains to understand the interplay of the MRI with this physics.

3. Global simulations have been reported for geometrically-thick disks.

   Geometrically-thin disks await systematic study with global simulations.