Dynamo action with wave motion

Andreas Tilgner,   University of Göttingen

Well known mechanisms for generating magnetic fields:

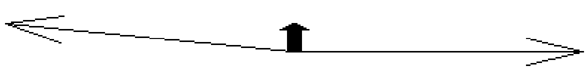
- α - effect
- Lagrangian chaos

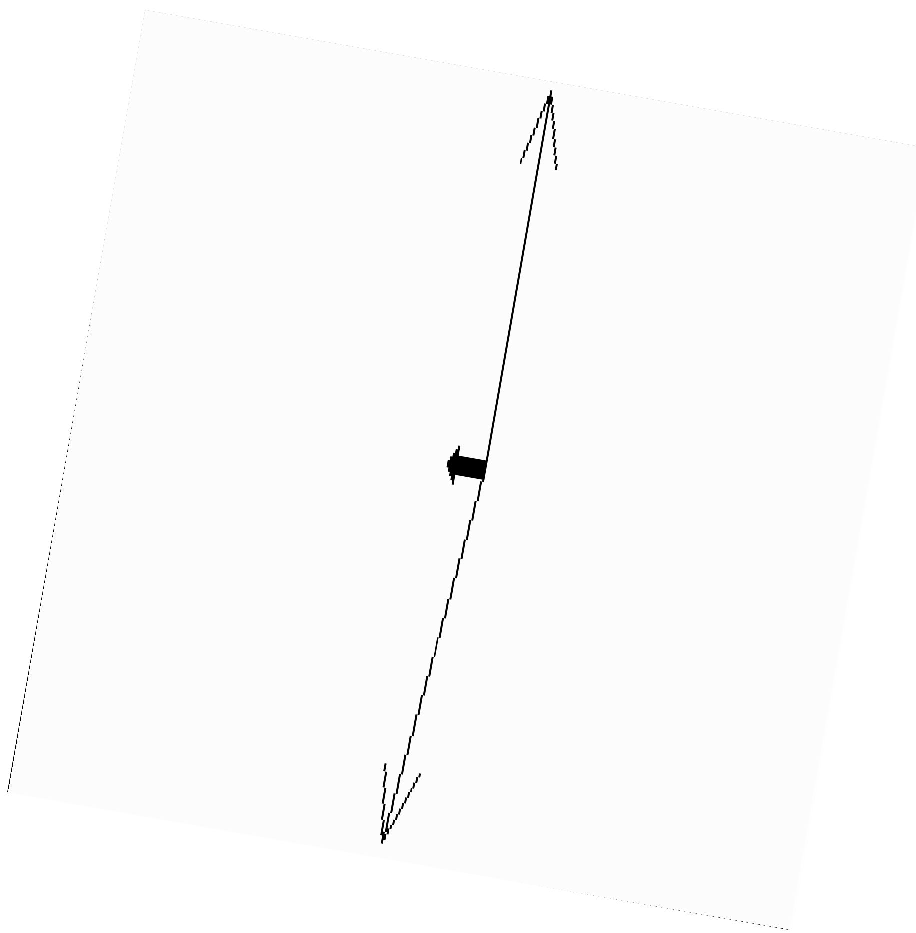
Here:

Mixing of eigenstates of a non-normal operator

Problems with non-normal operators:

Transition to turbulence in shear flows => transient growth

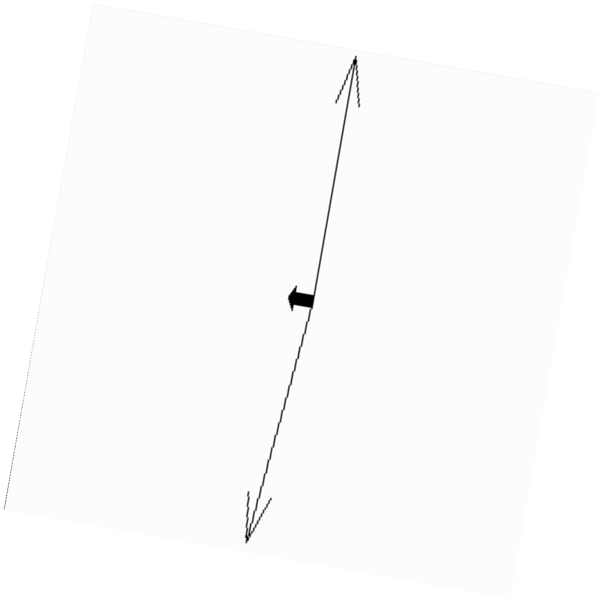




Transient growth → sustained growth

• transition to turbulence : non-linear terms

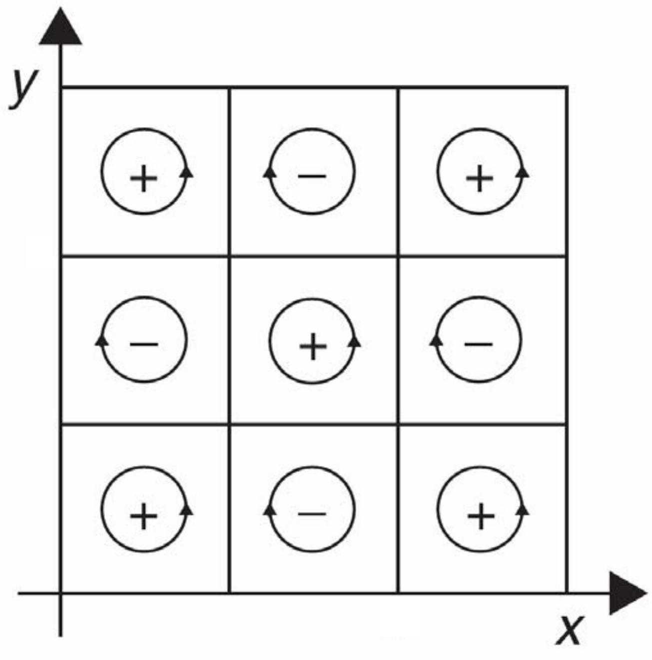
• kinematic dynamo : time dependent eigenstates



• slow time dependence of eigenvectors : no effect
• fast time dependence of eigenvectors : averaging, decay
• intermediate time dependence: growth is possible, even though all eigenvectors decay

Periodic 2D dynamo with drift

$$\boldsymbol{v}_I = \begin{pmatrix} \sqrt{2}\sin[2\pi(x - v_d t)]\cos(2\pi y) \\ -\sqrt{2}\cos[2\pi(x - v_d t)]\sin(2\pi y) \\ 2W\sin[2\pi(x - v_d t)]\sin(2\pi y) \end{pmatrix}$$



$\longrightarrow \; v_d$

Propagating wave:

- Eigenfunctions at different times differ by a translation
- The scalar product between eigenfunctions is independent of time
- Propagation is equivalent to a “rotation” in function space

Roberts flow:

Many orthogonal eigenvectors because of symmetries

Non-orthogonal eigenvectors only within one symmetry class

Uniformly drifting velocity pattern is equivalent to a stationary flow in a co-moving frame:

$$\boldsymbol{v}(\boldsymbol{r}) = \boldsymbol{v}_I(\boldsymbol{r}, t = 0) - v_d \hat{\boldsymbol{x}}$$

$$\frac{\partial}{\partial t}\boldsymbol{B} + \boldsymbol{\nabla} \times (\boldsymbol{B} \times \boldsymbol{v}) = \frac{1}{\mathrm{Rm}} \nabla^2 \boldsymbol{B}, \qquad \boldsymbol{\nabla} \cdot \boldsymbol{B} = 0$$

Solved with periodic boundary conditions in the box

$$0 \leq x,\ y \leq 1,\ 0 \leq z \leq 2$$

Compute solutions of the form:

$$\boldsymbol{B}(\boldsymbol{r}, t) = \boldsymbol{B}_0(x, y) e^{\mu t} e^{i\pi z}$$

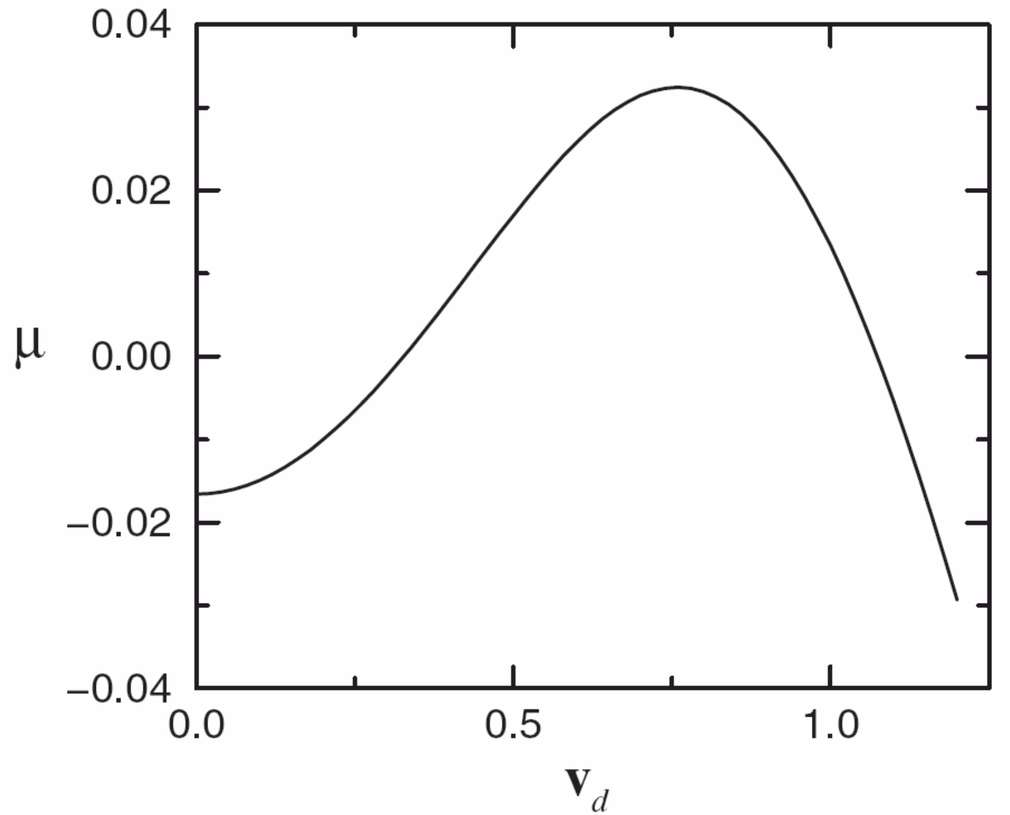


FIG. 1. Growth rate μ of the eigenmode with the largest growth rate as a function of v_d for $W = 0.25$ and Rm = 20.

Slow drift : Negligible effect.

Fast drift : Distortions of magnetic field lines which occurred during one half period are reverted during the next half period.

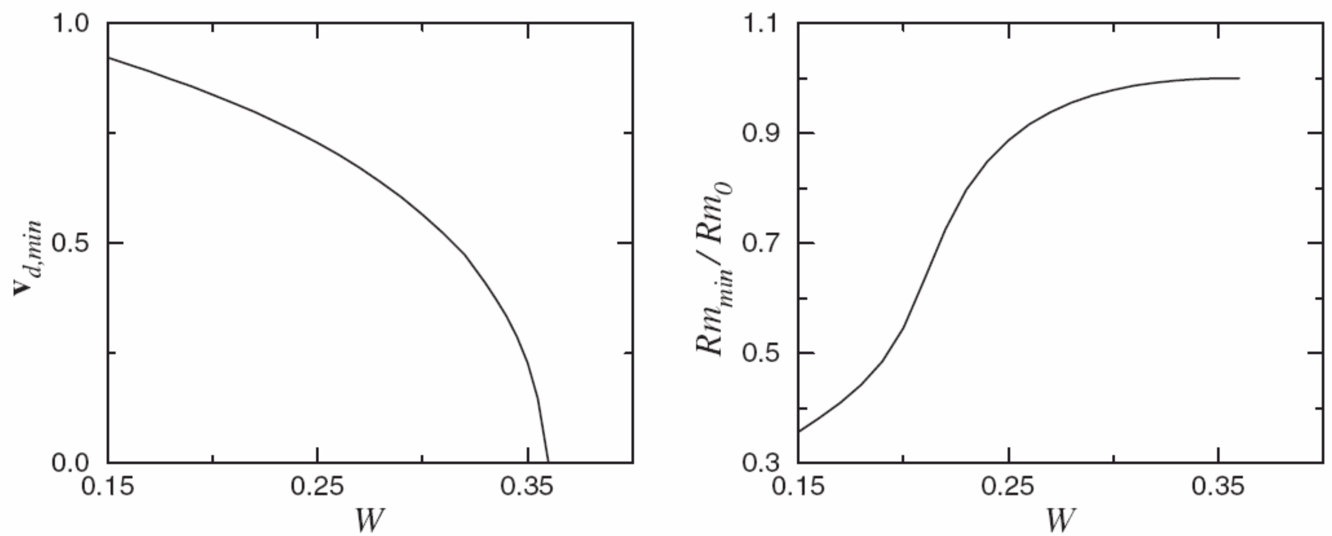


FIG. 2. Left panel: The optimal value of the drift velocity $v_{d,\mathrm{min}}$ as a function of W. Right panel: The critical magnetic Reynolds number, $\mathrm{Rm}_{\mathrm{min}}$, when the drift velocity has its optimal value, divided by the critical magnetic Reynolds number of the same flow with zero drift velocity, Rm_0, as a function of W.

Propagating waves in convective dynamos

28 U.R. Christensen et al. / Physics of the Earth and Planetary Interiors 128 (2001) 25–34

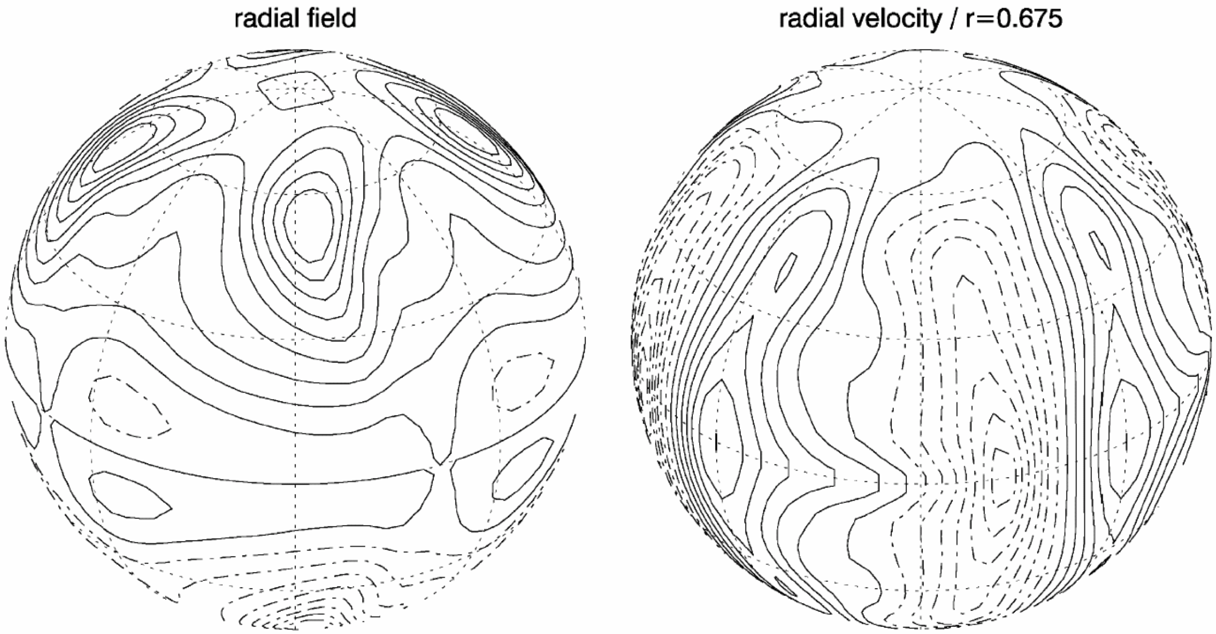


Fig. 1. Case 1: left, contours of B_r on outer boundary in steps of 0.25; right, contours of u_r at mid-depth in the shell, interval 2. Positive and zero contours solid lines, negative contours dash-dotted.

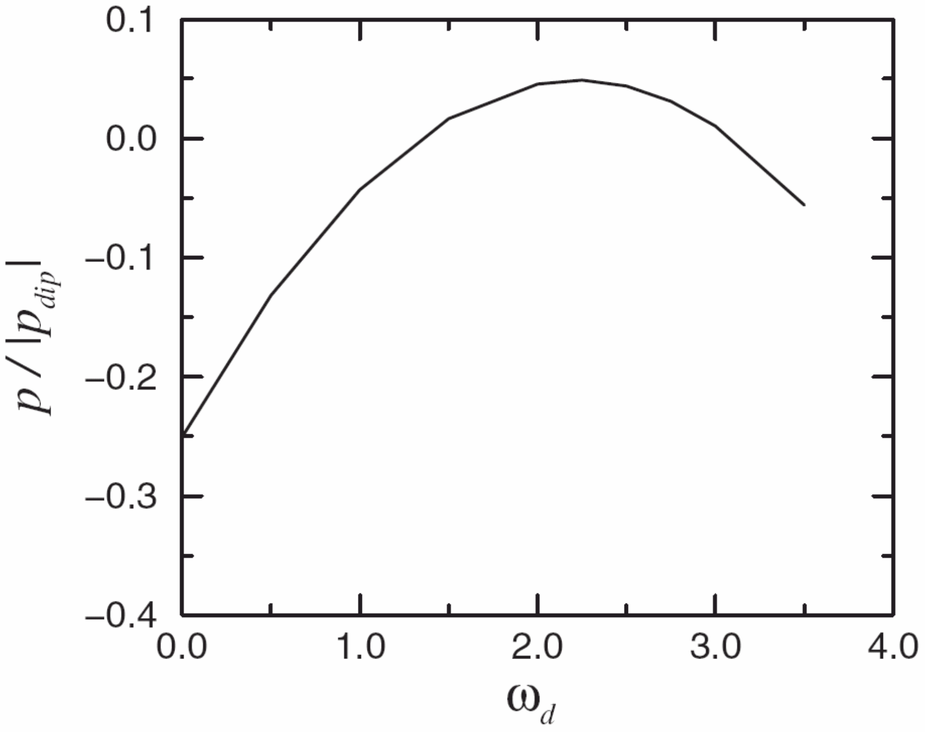
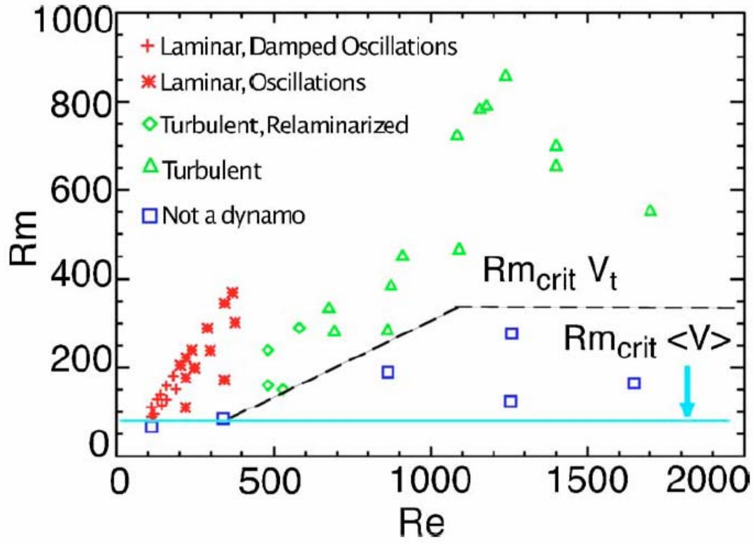


FIG. 3. Growth rate p, normalized by the free dipole decay rate, $|p_{\mathrm{dip}}|$, as a function of drift frequency ω_d for the velocity pattern taken from the benchmark dynamo.

Vary the drift frequency of the velocity pattern

Find two kinematic dynamos, one of which satisfies the full dynamo problem



Bayliss et al.
PRE (2007)

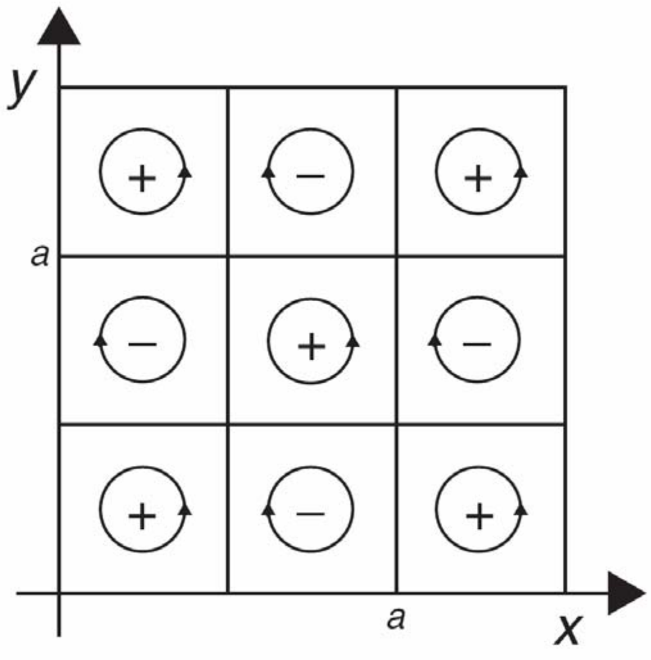
FIG. 7. (Color online) Re–Rm phase diagram. A number of simulations whose hydrodynamic and final saturated states are documented in Fig. 5. Rm_{crit} for the mean flow $\langle \mathbf{V} \rangle$ is essentially independent of Re, whereas the effective dynamo threshold grows with Re. The dashed line shows the qualitative behavior of the dynamo threshold in turbulent flows (V_T).

Higher Re introduces small scales, increases Rm,crit (Tilgner, NJP (2007))

Periodic 2D dynamos as toy models for turbulent dynamos

$$\boldsymbol{v} = \sum_n c_n \boldsymbol{v}_n$$

$$\boldsymbol{v}_n = \begin{pmatrix} \sqrt{2}\sin\left(\frac{2\pi}{a_n}x\right)\cos\left(\frac{2\pi}{a_n}y\right) \\ -\sqrt{2}\cos\left(\frac{2\pi}{a_n}x\right)\sin\left(\frac{2\pi}{a_n}y\right) \\ 2\sin\left(\frac{2\pi}{a_n}x\right)\sin\left(\frac{2\pi}{a_n}y\right) \end{pmatrix}$$



Periodicity length along z:
d = 1

Sketch of $\boldsymbol{v}_1$

Magnetic field generated by eddy of size a:

Different structure at scales larger or smaller than a!

Example: Helicity (negative at large scale, positive at small scale for right handed eddy)

Without small scales:

Helicity at lengthscale a positive

With small scales (smaller than a):

If the small scales generate enough magnetic field

=> positive helicity at scale a

=> wrong structure for eddy of size a to produce magnetic field

Conclusion

- An alternative view of magnetic field production: mixing of non-normal eigenmodes.

- Examples of time dependent dynamos, for which no snapshot is a dynamo.

- Clearest demonstration for uniform drift (wave propagation), qualitatively the same must happen for more complicated time dependencies.

- Mean field MHD: The term responsible for the drift effect is of the same order as other terms neglected in FOSA.

- Adjustment of time dependence is part of the saturation process.

References

New Journal of Physics 9, 270 (2007)

Physical Review Letters 100, 128501 (2008)