Angular momentum transport and dynamos in stably stratified domains

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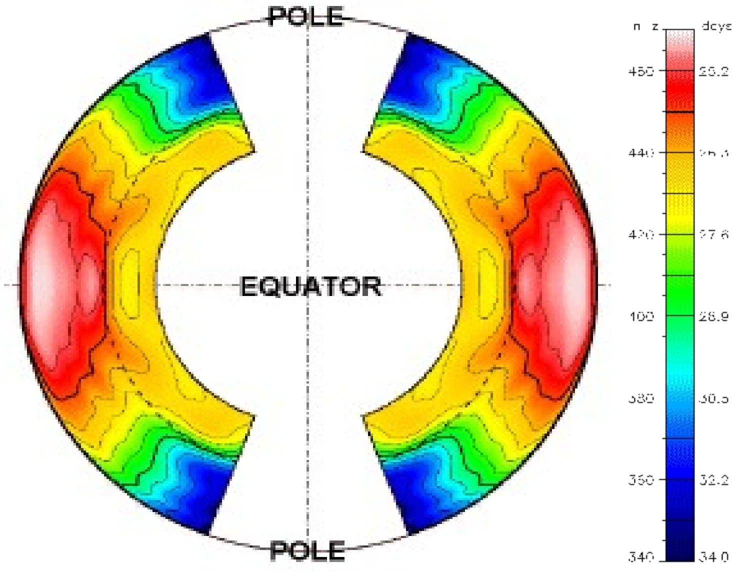
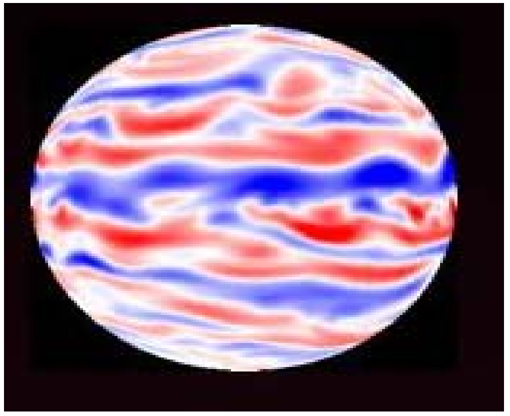
Outline

- The role of magnetic fields in stably stratified domains
- Shallow water and shallow water MHD
- A turbulent shallow water dynamo
- A simpler shallow water dynamo

Stable layer dynamics

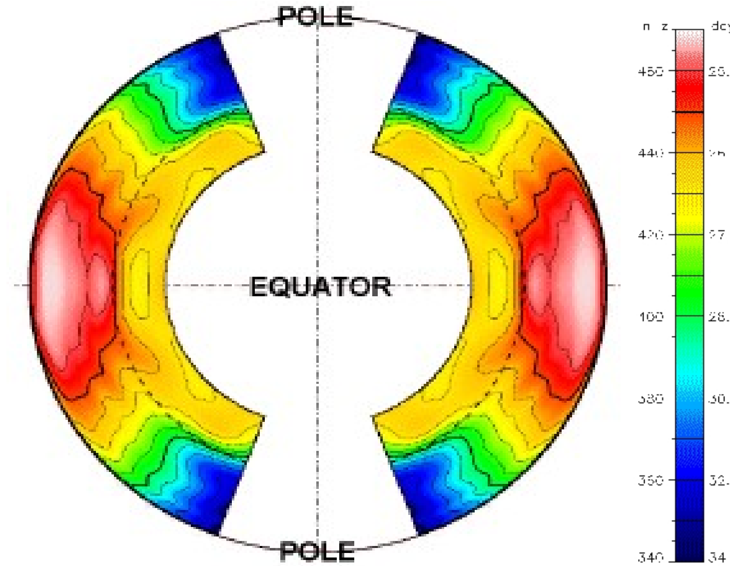
- In many astrophysical situations interesting dynamics takes place in stably stratified regions of the astrophysical body.
- The solar tachocline
- The outer layers of some planets

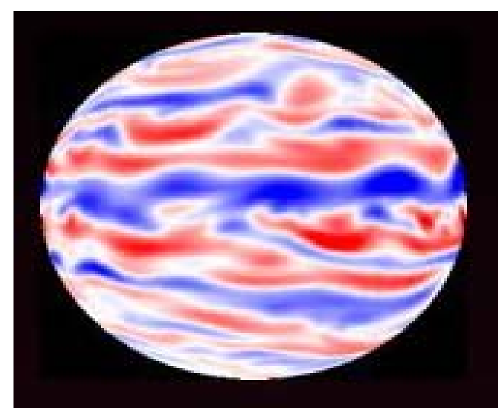




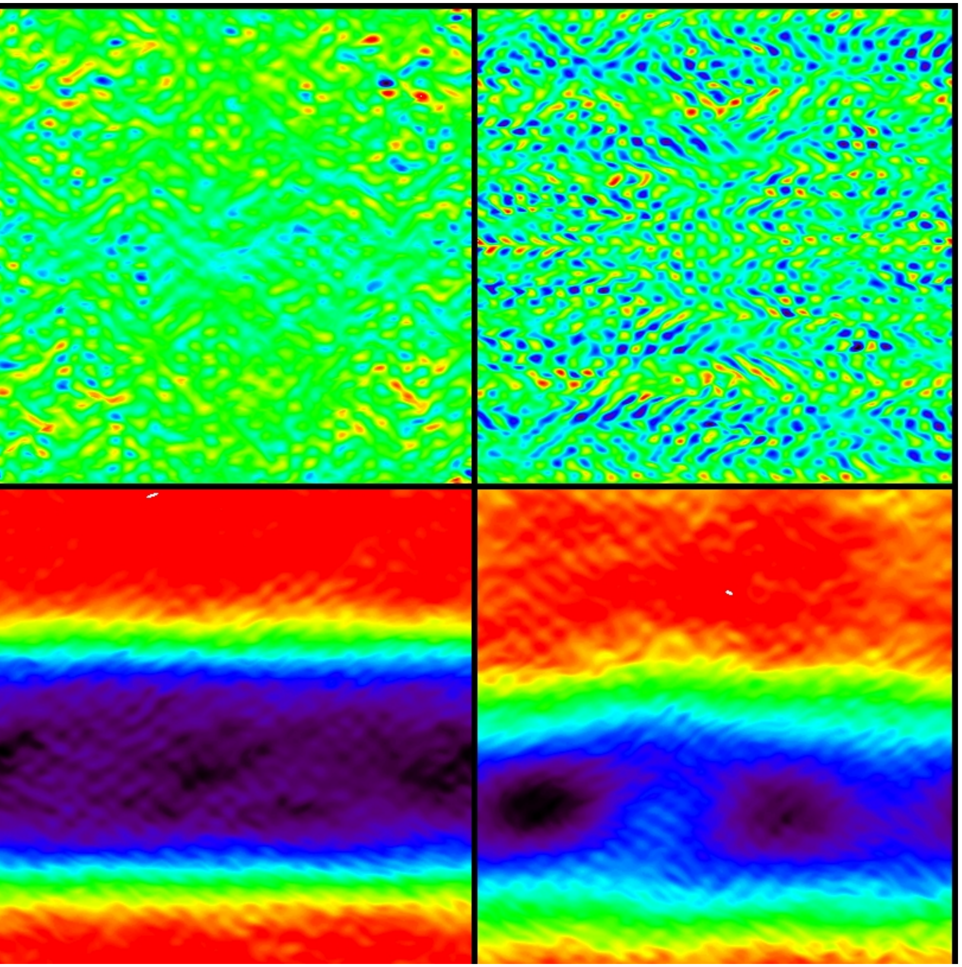
Angular momentum transport

- Angular momentum transport in stably stratified domains is subtle.
- Nonlinear interactions of Reynolds stresses can lead to generation of strong zonal flows (jets e.g. Earth's atmosphere, Jupiter?)
- Interesting form of turbulence where interactions of mean flows and dispersive waves are important (see e.g. Diamond et al 2005)





Angular momentum transport and magnetic fields



- Tachocline is ionised and presumably magnetised
- Outer layers of extra-solar planets close to parent stars are hot enough to be ionised (see e.g. Cho et al 2008)
- Magnetic fields can play a significant part in modifying the transport properties in these domains

Tobias, Diamond & Hughes (2007 ApJ)

But can magnetic field be sustained in these stably stratified systems – if so how?

Shallow water dynamos

- Either field is transported into stable region or it is generated there.
- Stable stratification → nearly 2d dynamics
- 2D dynamics → no dynamo. $\mathbf{B} = (B_x, B_y, 0), \mathbf{u} = (u, v, 0)$
- In atmospheric literature: shallow water system used extensively
- For MHD: Shallow water → Shallow water MHD (Gilman 2000)

$$\partial_t \mathbf{u} + \mathbf{u}.\nabla\mathbf{u} + f\mathbf{k}\times\mathbf{u} = -c^2\nabla\tilde{h} + \mathbf{B}.\nabla\mathbf{B} + \nu\nabla^2\mathbf{u}$$

$$\partial_t \mathbf{B} + \mathbf{u}.\nabla\mathbf{B} = \mathbf{B}.\nabla\mathbf{u} + \eta\nabla^2\mathbf{B}$$

$$\partial_t \tilde{h} + \nabla.((1+\tilde{h})\mathbf{u}) = 0$$

$$\nabla.((1+\tilde{h})\mathbf{B}) = 0 \qquad c^2 = gH$$

- System used extensively to examine large-scale instabilities of strong fields and differential instabilities (see e.g. Gilman & Cally 2007)
- These large-scale joint instabilities can be sustained (Miesch 2007)

Shallow water dynamos

- Do traditional turbulent dynamos work in such a system (see e.g. Lillo et al 2005)?
- How does the field amplification occur
- How does saturation occur?
- Stable layers are often forced from above or below by convection, vortical flow stresses or shear flow stresses.
- These lead to (nearly 2d) shallow water turbulence
- Set

$$u = \nabla \times (\psi \hat{z}) + \nabla_H \chi$$

- And

$$\omega = -\nabla^2 \psi; \qquad \delta = \nabla^2 \chi$$

vorticity **divergence**

Shallow water forcing

to get (ignoring magnetic field see e.g. Lorenz 1980; Yuan & Hamilton 2006):

$$\partial \omega / \partial t + J(\psi, \omega) + \nabla.(\omega \nabla \chi) + f\delta = D_\omega + F_\omega$$

$$\partial \delta / \partial t + J(\psi, \delta) - \nabla.(\omega \nabla \psi) +$$

$$\nabla^2 \left[(|\nabla \psi|^2 + |\nabla \chi|^2)/2 + J(\psi, \chi) + gh - f\psi \right] = D_\delta + F_\delta$$

$$\partial h / \partial t + J(\psi, h) + \nabla.(h \nabla \chi) + H\delta = F_H$$

In principle can force in any equation:

Small-scale vortical forcing, small-scale compressional forcing, forcing of upper layer.

Relax to a stable/unstable shear.

Shallow water: non-dimensionalisation

- Shallow water dynamics is usually examined in unforced situations – run-down from given flow field U_0
- With forcing F_0 included new set of non-dimensional parameters

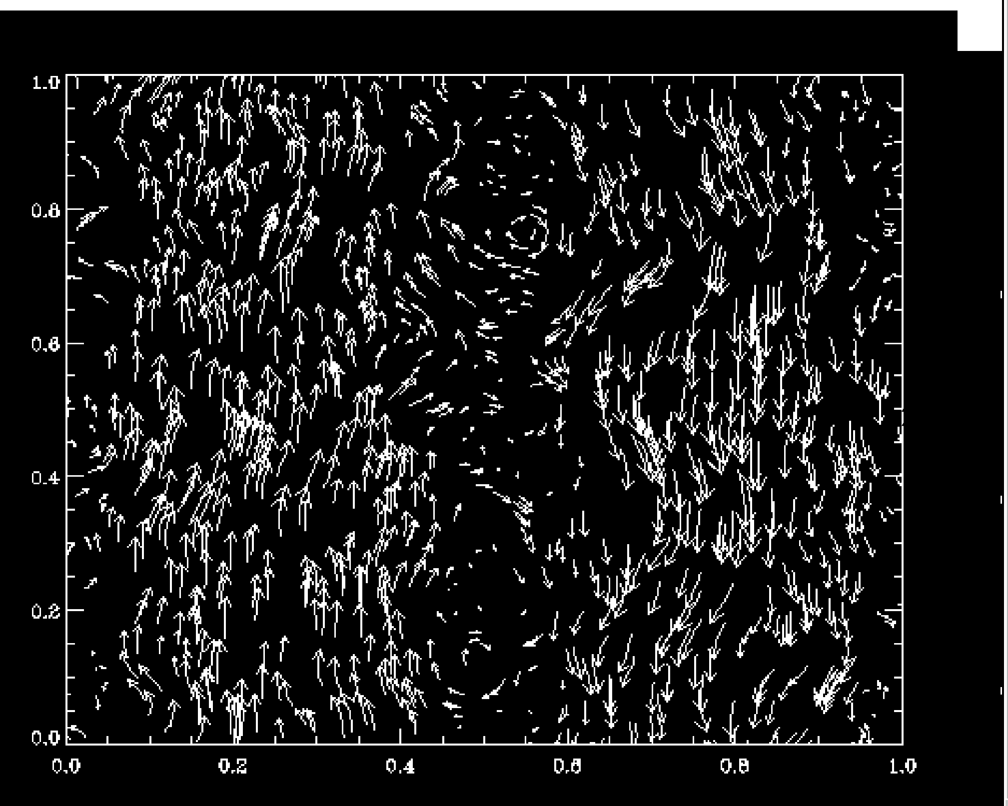
$$Fr = \left({gH}/{F_0 L} \right)^{-1/2} \qquad Ro = {(F_0 L)^{1/2}}/{fL}$$

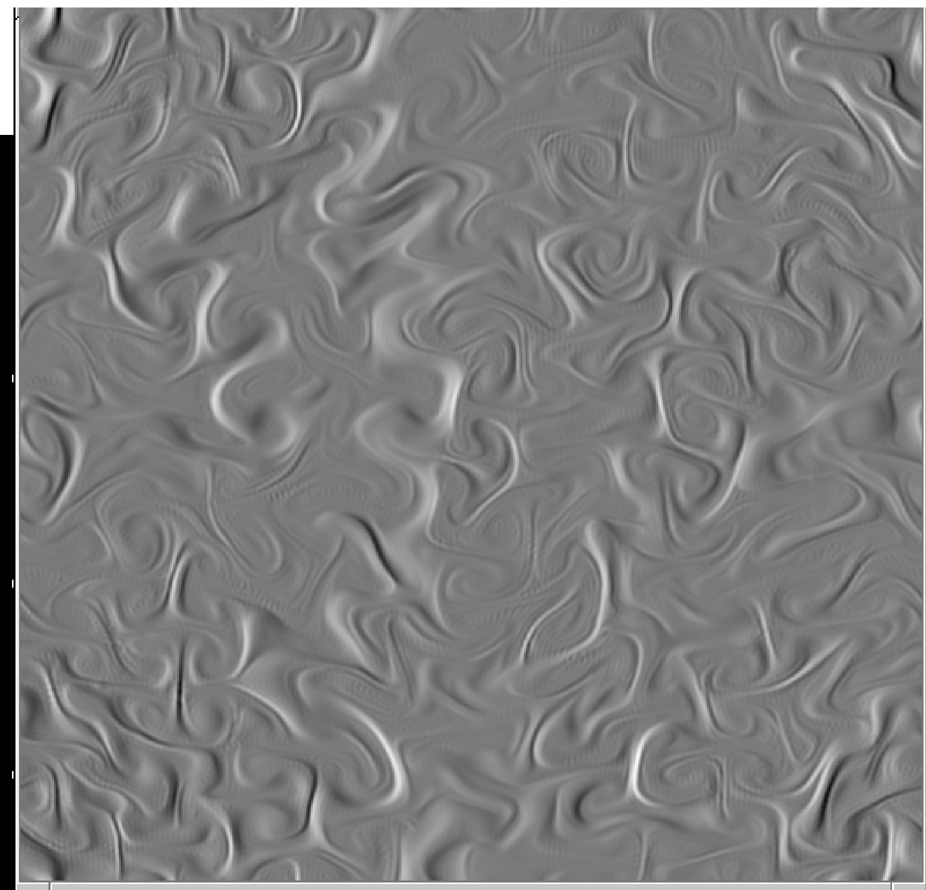
$$\mathrm{Re} = {(F_0 L)^{1/2} L}/{\nu} \qquad Rm = Pm\,\mathrm{Re}$$

$$Bu = {gh}/{f^2 L^2} = {Ro^2}/{Fr^2}$$

The turbulent system

- Force with small-scale vortices.
- System inverse cascades (depending on Bu)
- Turbulent flow is dynamo
- Field grows, saturates and the flow can forward cascade!





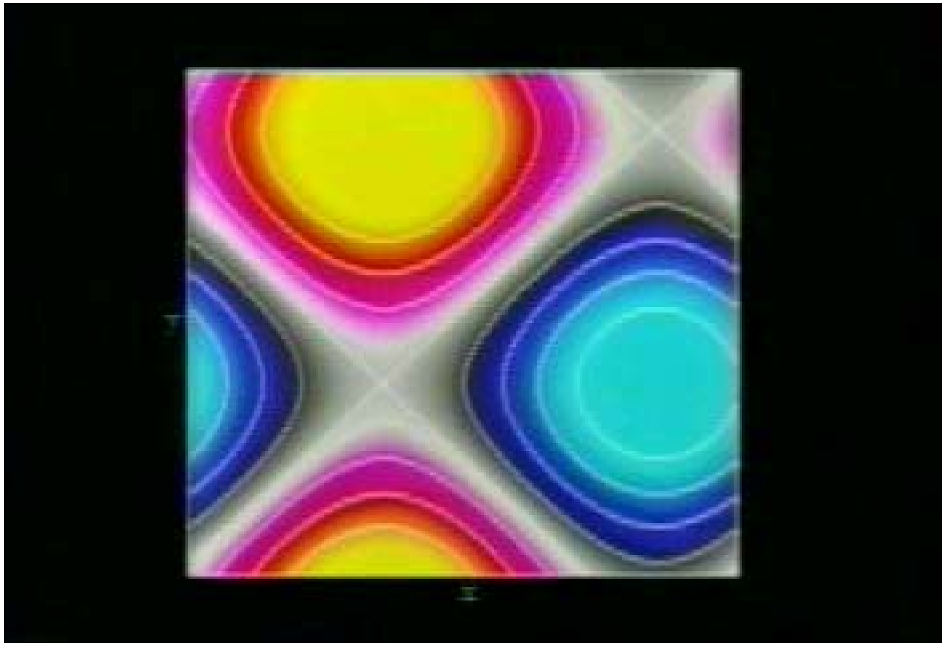
Making a better dynamo

These dynamos are fairly hard to excite
These dynamos are very hard to analyse
So need a simpler/better dynamo to understand what is going on
Try to get the best dynamo possible

The Galloway Proctor Flow

$$u = \nabla \times (\psi \hat{z}) + \psi \hat{z}$$

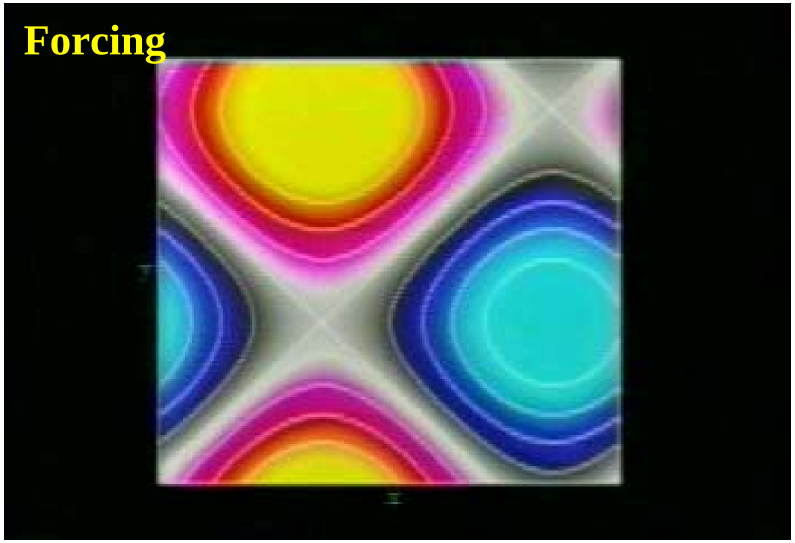
Great dynamo, but vertical velocity is prescribed and w not small



Making a better dynamo

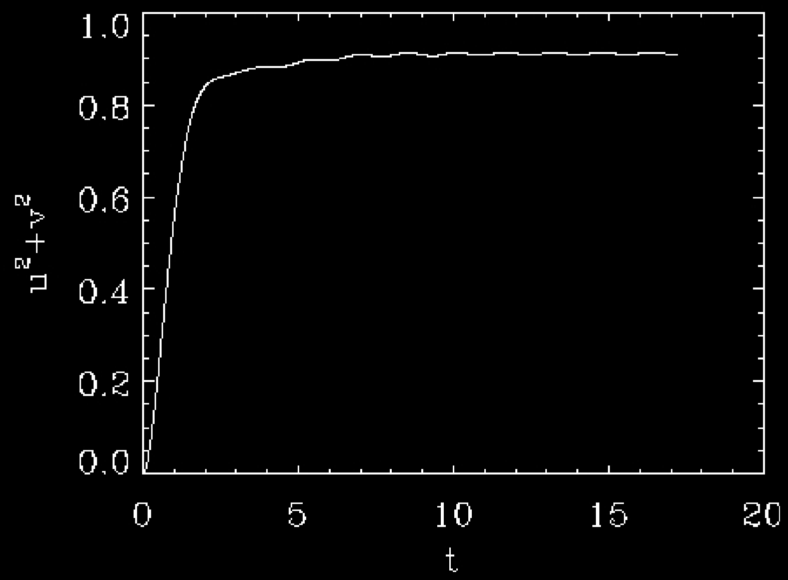
Would like similar dynamics (at least in the plane)
But can't prescribe w
Select forcing to drive vorticity such that if h were constant would get GP flow in plane.
Let δ (and therefore h) evolve naturally

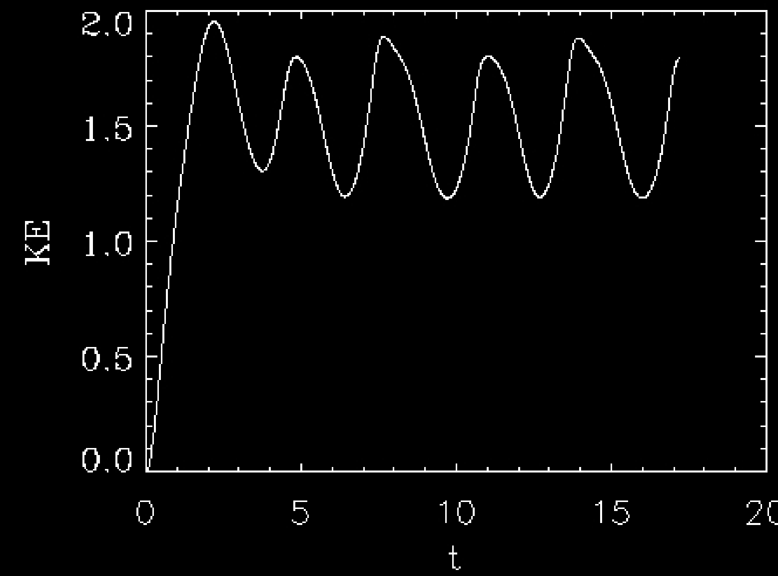
The "Shallow"ay Proctor flow



(with apologies to Dave G)

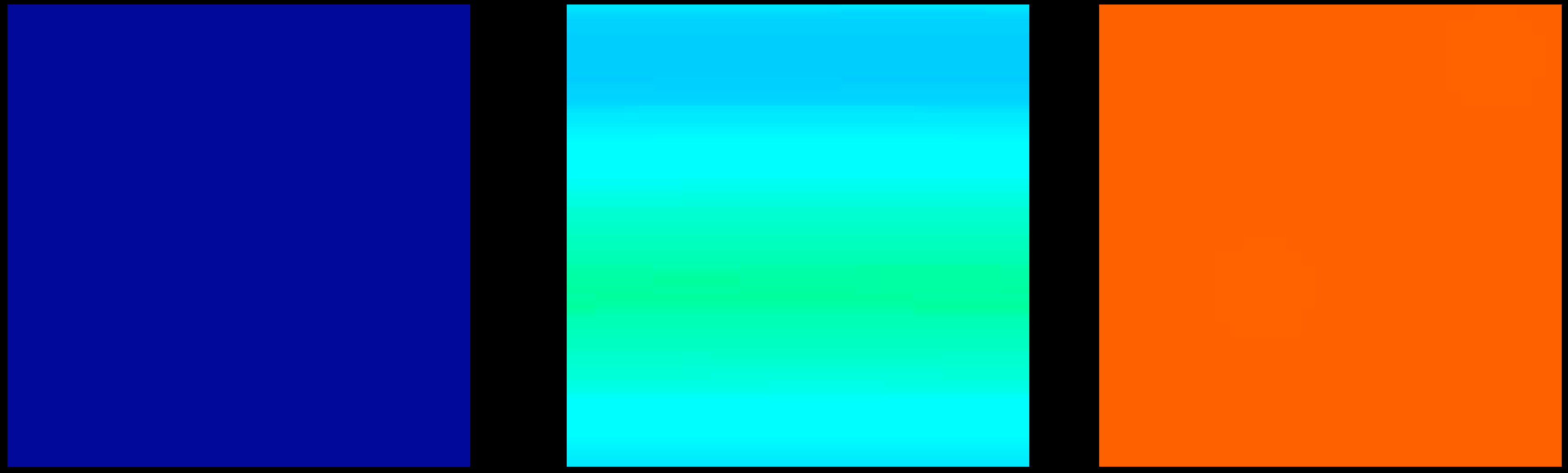
Evolution of both components depend on f and g (and ν) (or Fr, Ro, Re)

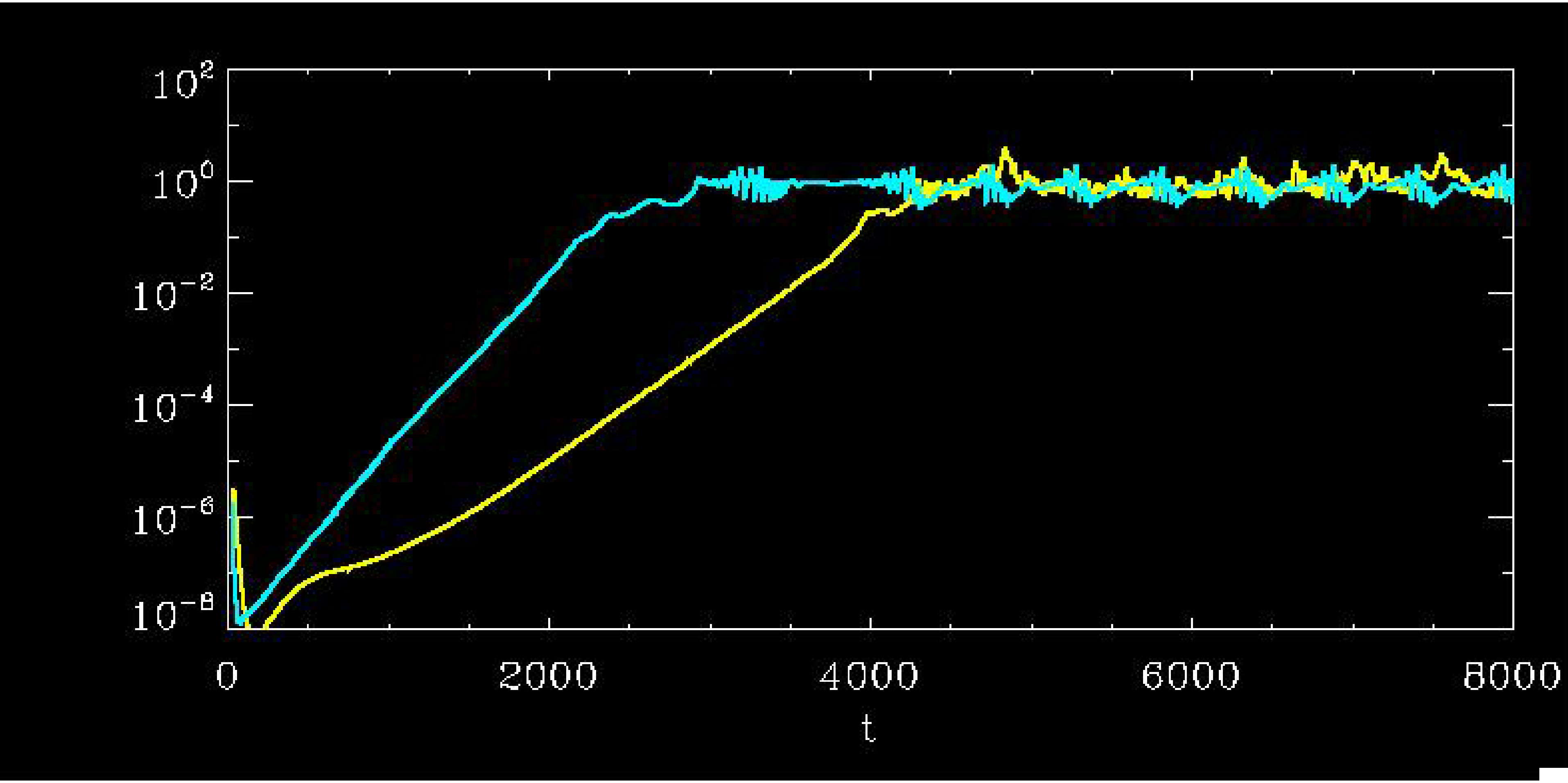




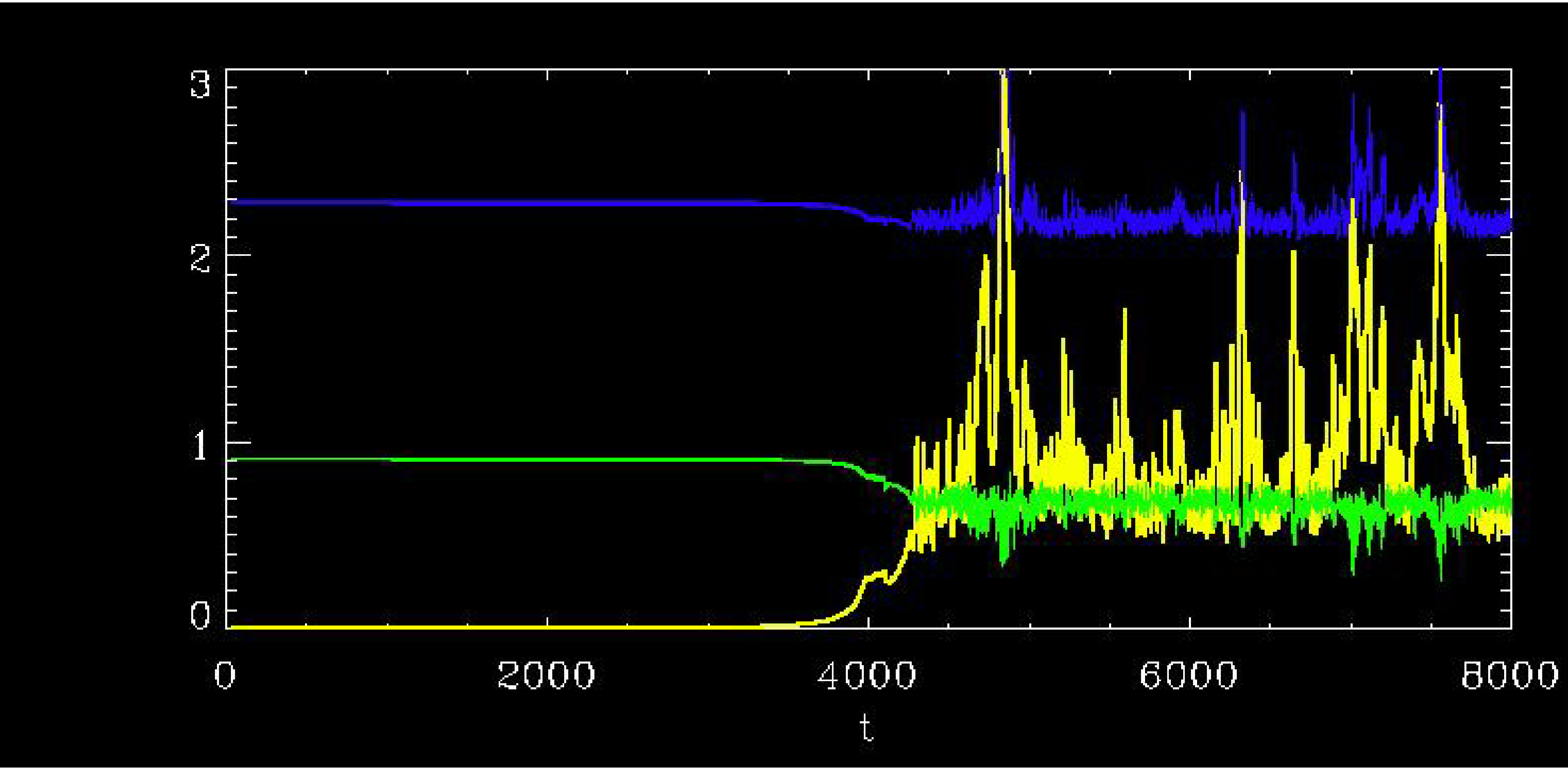
Tobias 2008b

h **ω** **δ**



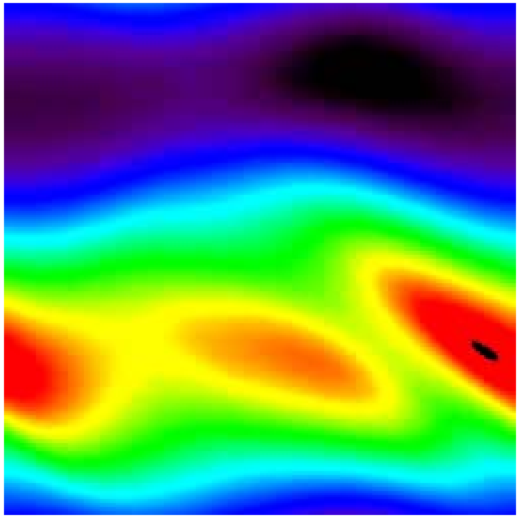


10^2
10^0
10^{-2}
10^{-4}
10^{-6}
10^{-8}
0
2000
4000
6000
8000
t

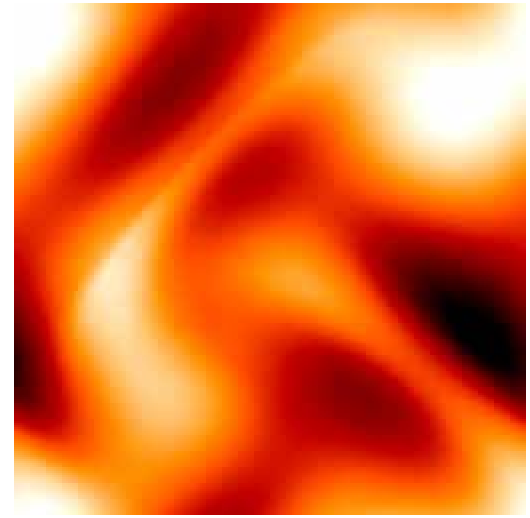


3
2
1
0
0
2000
4000
6000
8000
t

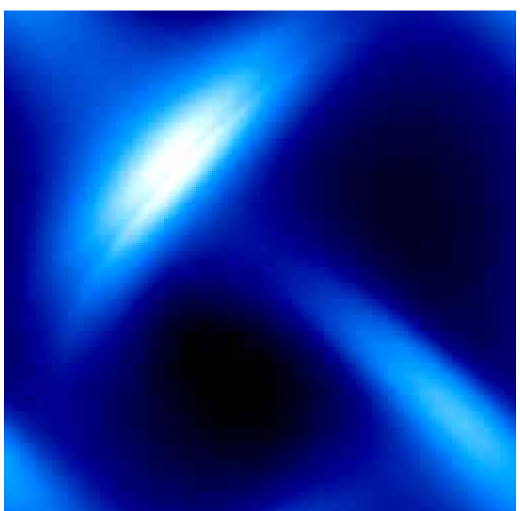
ω



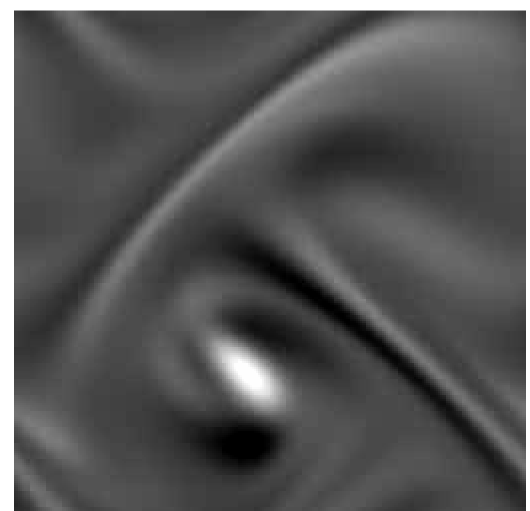
δ



h



B_x



Saturation

- Saturation occurs via
  - Back-reaction on divergence
  - Back-reaction on vorticity
  - Making flow more 2-dimensional
- Which mechanism is more important depends on Bu, Re and Rm
- May also cause forward cascade for smaller-scale forcing

Conclusions & Future work

- Stably stratified domains can have interesting dynamics, inverse cascades, zonal flows
- This can be mediated by presence of magnetic field
- Turbulent dynamo can operate in shallow water – saturation mechanism difficult to interpret
- Simple model with vortical forcing can drive vertical flows which lead to dynamo action
- These saturate in a number of ways, but lead to magnetic fields close to equipartition.
- May lead to large-scale fields and interact with shear flows.