Quenching and irreversibility in 2D “wavy” MHD

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Dynamo ‘08

Q: How ***robust*** is η_T – quenching?

- A: Exactly as robust as the Zel'dovich theorem:

$$\eta_T = \eta_c \frac{\langle b^2 \rangle}{\langle B \rangle^2}$$

- Result that follows from
  - stationarity
  - eddy transport is diffusive
  - periodic boundary conditions
  - spectral convergence of **hb^2i** (not passive!)

- Implicit assumption: irreversibility is due to *molecular collisions*

“Wavy” MHD turbulence

- MHD + **additional body forces:** { Coriolis force, buoyancy }
- Large-scale eddies ⇨ dispersive waves: { Rossby waves, internal waves }

“Wavy” MHD = MHD + **dispersive waves**

c.f. K. Moffatt (1970, 1972), Vainshtein & Zel’dovich (1972), A. Soward (1975)…

- Simple example: MHD turbulence on a beta-plane

$$\left(\partial_t + \boldsymbol{v}\cdot\boldsymbol{\nabla}\right)\nabla^2\psi + \beta v_y = \left(B_0\partial_x + \boldsymbol{b}\cdot\boldsymbol{\nabla}\right)\nabla^2\tilde{A} + \nu\nabla^2\nabla^2\psi,$$

$$\left(\partial_t + \boldsymbol{v}\cdot\boldsymbol{\nabla}\right)\tilde{A} + v_y B_0 = \eta\nabla^2\tilde{A}.$$

- Dispersive (non-dispersive) on large (small) scales

“Wavy” MHD turbulence

- ***Attractive feature***: When the wave-slope is small, wave turbulence theory is applicable.

- Origin of irreversibility is ray chaos induced by overlapping **three-wave resonances**; present even in the *absence* of collisions (Rm ! 1)

- Dual asymptotics: Rm >> 1, wave-slope << 1. Hence, *rigorous* (though limited) theory with no unconstrained parameters / hidden microphysics (e.g. τ of EDQNM)

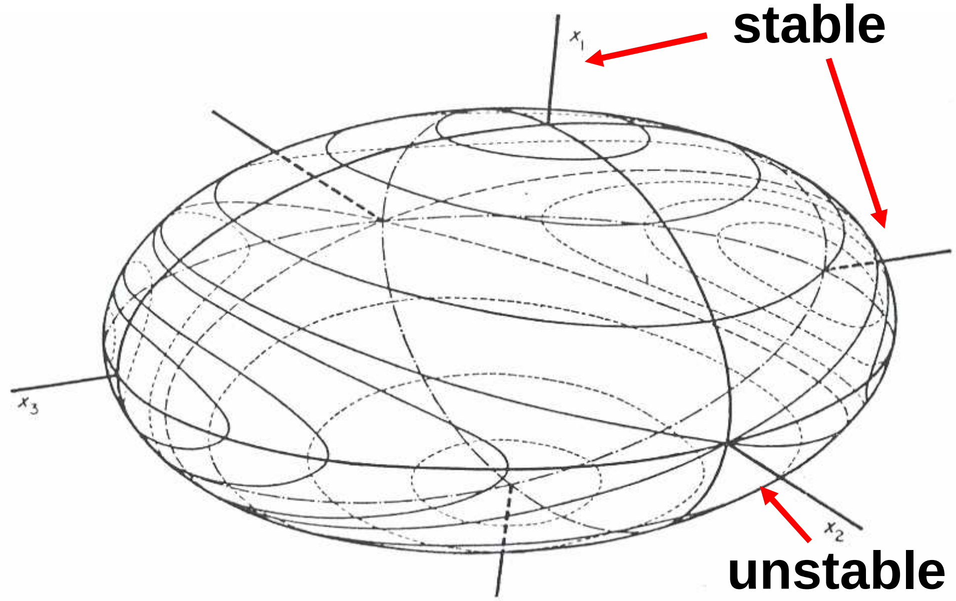
- Analytical result provides useful (and falsifiable!) test of theory of η_T –quenching in a regime where physics of irreversibility is unambiguous

Wave turbulence theory

- describes the slow transfer of energy among a triad of waves satisfying the resonance conditions:

$$\begin{aligned} \boldsymbol{k} + \boldsymbol{k}' + \boldsymbol{k}'' &= \boldsymbol{0} \\ \omega + \omega' + \omega'' &= 0 \end{aligned}$$

- analogous to free asymmetric top ($I_3 > I_2 > I_1$)



- for *ensemble* of triads, origin of irreversibility is ray chaos via overlapping resonances

- Energy transfer can be modeled as a random walk when

$$\tau_{coherence} << \tau_{transfer}$$

Wave turbulence theory: validity

- Wave turbulence theory requires a **broad spectrum** of **dispersive**, **weakly interacting** waves

- broad enough for triad to remain coherent during interaction

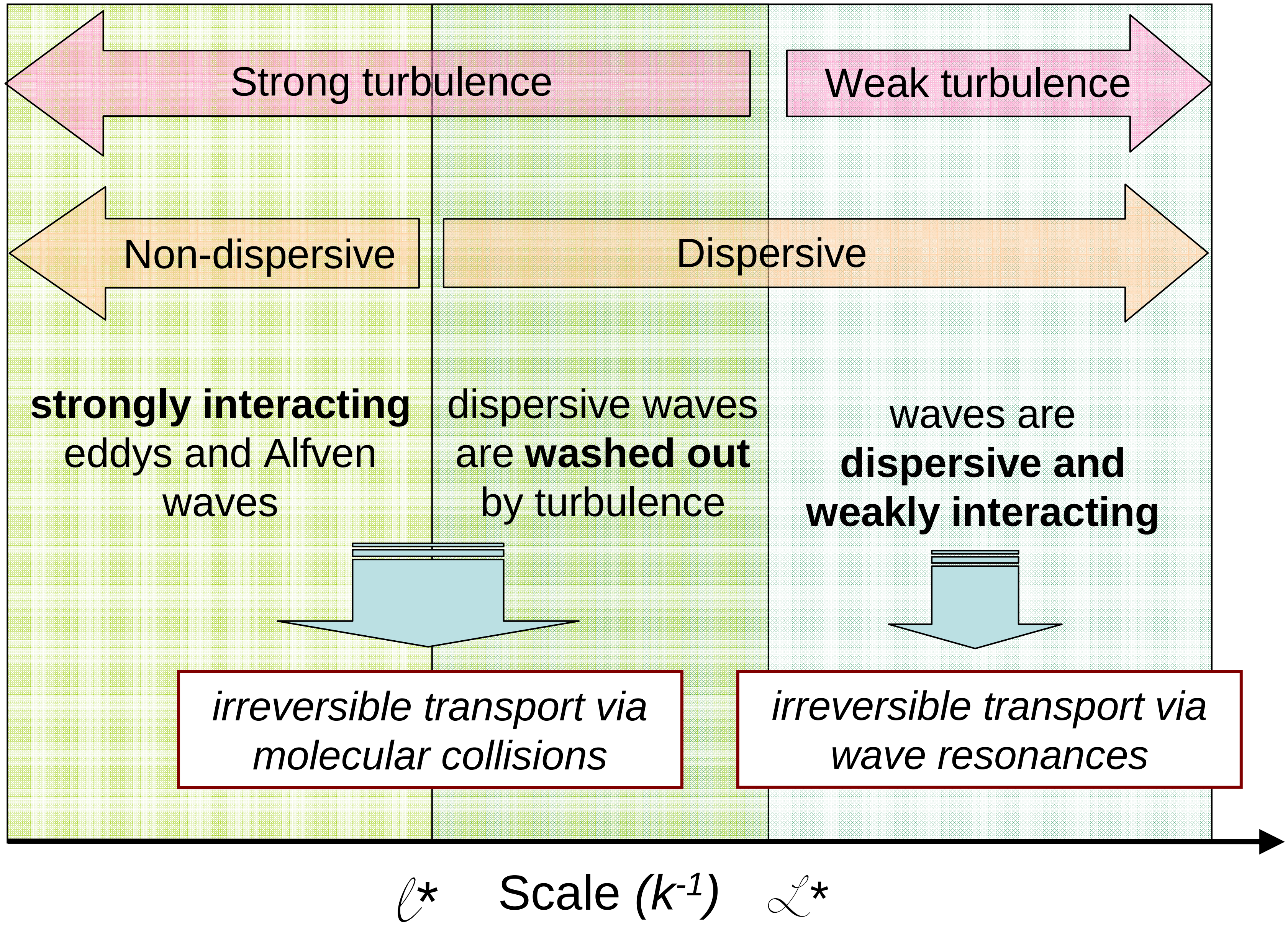
- resonance manifold is empty for non-dispersive waves

- turbulent decorrelation doesn't wash out wave interactions:

$$\frac{\text{decorrelation rate}}{\text{wave mismatch}} \approx \frac{k\tilde{V}}{\omega_{\boldsymbol{k}}} \approx \text{wave slope} \ll 1$$

- these are *equal* at the cross-over scale *L**

Spectral regimes



The flux of *A* due to wave resonances

$$\delta\Gamma_A = \langle v_z \delta A \rangle + \langle A \delta v_z \rangle$$

$$= \langle \epsilon \mathbf{v} \cdot \nabla \delta A \rangle - \eta \langle \epsilon \nabla^2 \delta A \rangle$$

Response to wave interactions

Flux due to: (wave-wave resonances) (molecular collisions)

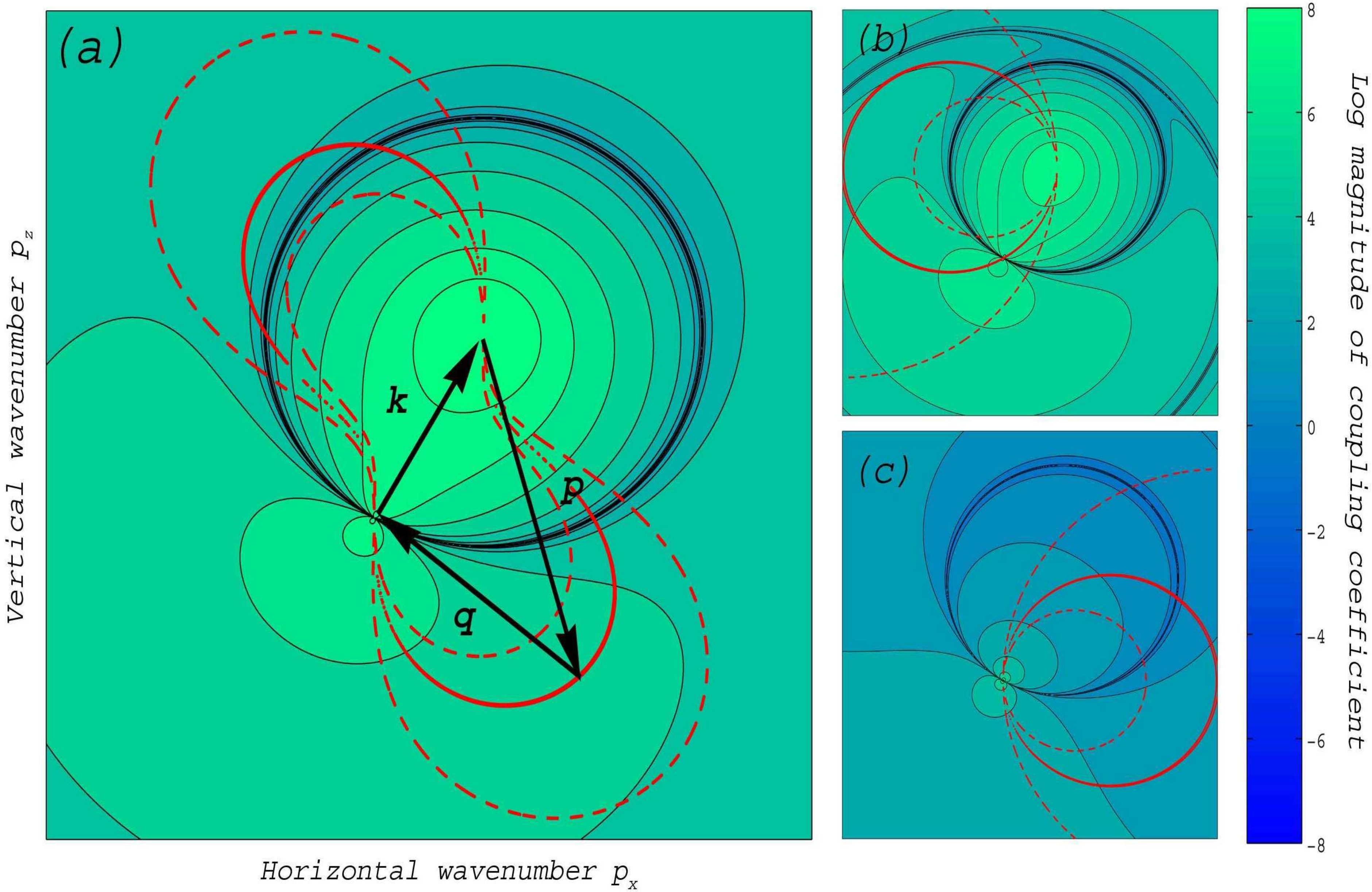
- Expand δA in powers of the wave-slope $\sigma = k\varepsilon < 1$
- $O(\sigma^2)$ still tied to η (QLT gives usual quenching result)
- First nonzero contribution to resonant flux is $O(\sigma^4)$

$$\eta_{\mathrm{ww}}^{(4)} = \frac{\pi}{8} \sum_{\Delta} g_{\mathbf{k}',\mathbf{k}''} (\mathcal{C}^+ \theta^+ - \mathcal{C}^- \theta^-) |\sigma_{\mathbf{k}'\omega'}|^2 |\sigma_{\mathbf{k}''\omega''}|^2$$

Response time: $\theta^{§} = \mathrm{Re}\, i\, (\omega' + \omega'' \, § \, \omega_{k'+k''} + i\, 0^+)^{-1}$

$= \delta\, (\omega' + \omega'' \, § \, \omega_{k'+k''})$

Analysis of the result: β -plane MHD



Summary and discussion

- ***In a nutshell***: ray chaos, induced by overlapping dispersive-wave resonances, will drive a diffusive flux of magnetic potential in that is ***independent*** of magnetic Reynolds number

- **Issues for discussion:**
  - What is microphysics of τ in EDQNM calculations? What is origin of irreversibility?
  - What is the nature of the relationship between spectral transfer of energy and spatial transport of magnetic potential?
  - *Shameless speculation*: wave resonances drive helicity transport and/or dynamo action?