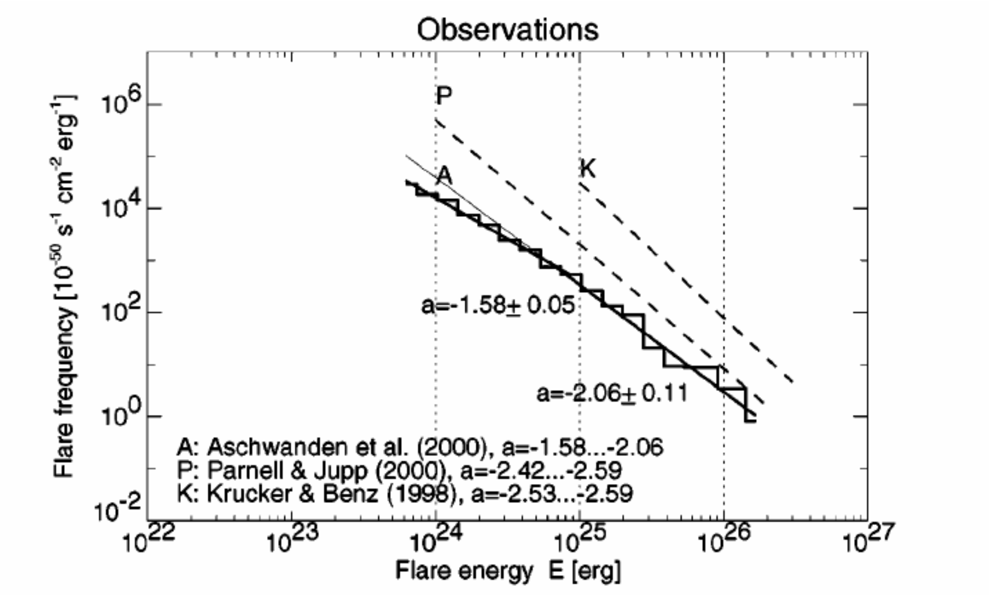
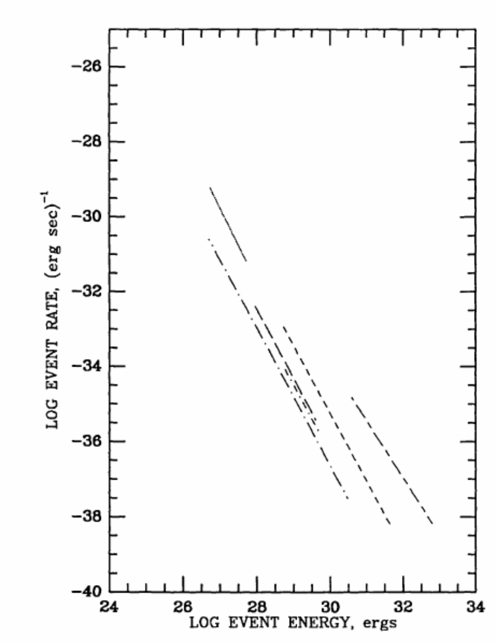
Self-organized braiding of magnetic fields

Mitchell Berger University of Exeter

- Why is the corona heated to 1 – 2 million degrees?
- What causes flares, μflares, and νflares? Why do they have a power law energy distribution?

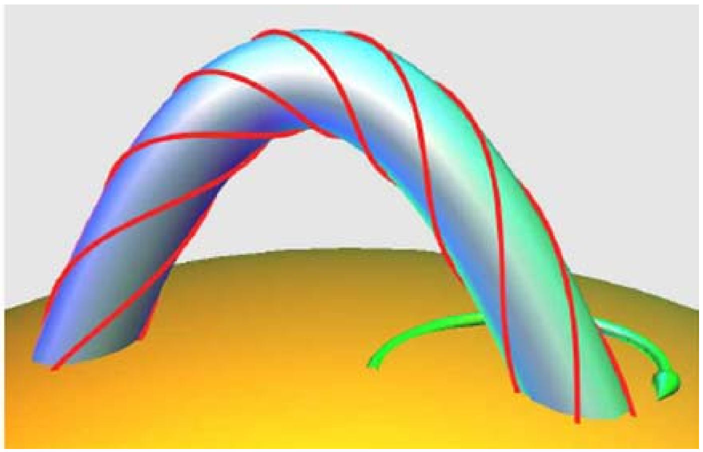




Hudson 1993

Sturrock-Uchida 1981

- Random twisting of one tube



Energy is quadratic in twist, but mean square twist grows only linearly in time.

Power = dE/dt independent of saturation time.

Parker 1983

- Braiding of many tubes

412 E. N. PARKER: LOW

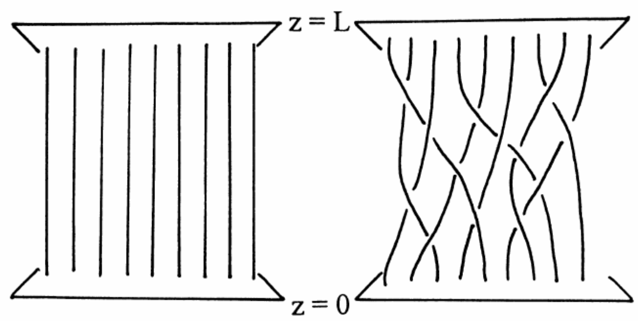


Fig. 1. A sketch of the arbitrary interlace field created by the arbitrary stream function ψ throughout $0 < z < L$.

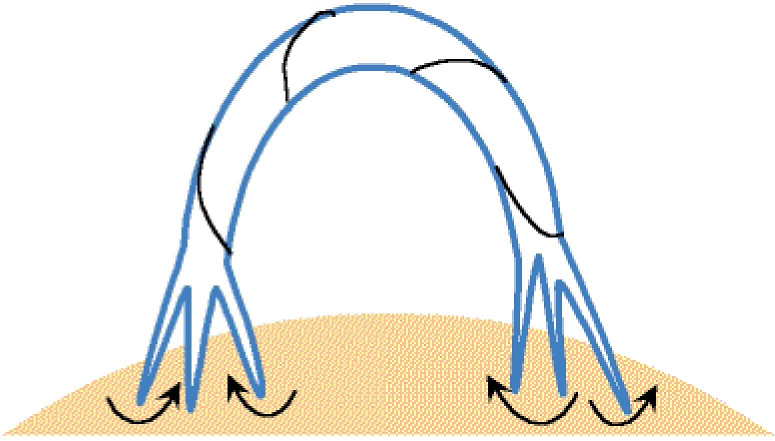
Energy grows as t^2 .

Power grows linearly with saturation time.

Twisting is faster, but braiding is more efficient!

Flux Tube Splitting

More recent models add interactions with small low lying loops Ruzmaikin & Berger 98, Schriver 01



Flux tube endpoints constantly split up and gather together again, but in new combinations. This locks positive twist away from negative twist. Berger 94

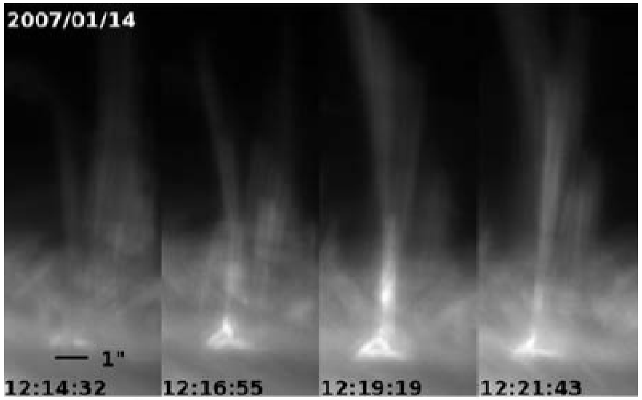
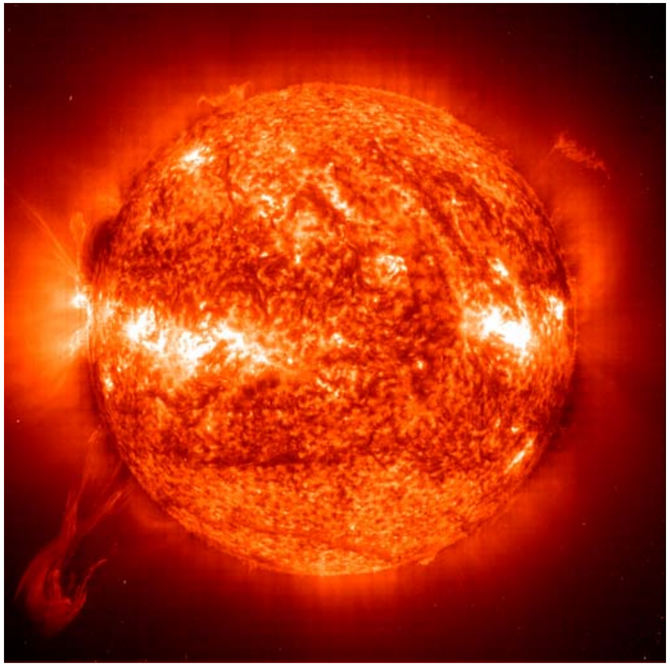


Fig. 2. Time evolution of typical Ca jets observed in Ca II H broadband filter of Hinode/SOT. Times are shown in UT.

Shibata et al 2007 Hinode “Anemone jets”

Topological Dissipation

Given the boundary flux distribution and the topology of the magnetic field lines, there is a set of possible fields. Does this set contain a smooth equilibrium? If not, the field cannot relax without developing a current sheet (discontinuity in the field). This triggers reconnection – e.g. flares and Tokomak disruptions.



Yohkoh Image

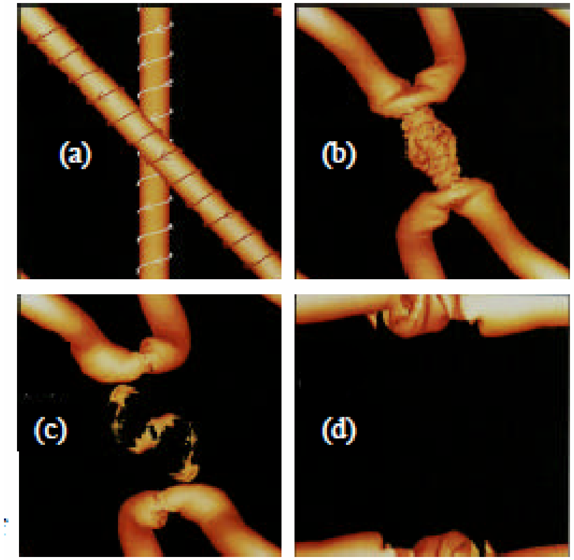
Parker's topological dissipation scenario:

1. Corona evolves quasi-statically due to footpoint motions (Alfvén travel time 10-100 secs for loop, photospheric motion timescale ~2000 seconds)
2. Smooth equilibria scarce or nonexistent for non-trivial topologies – current sheets must form
3. Slow burn while stresses buildup
4. Eventually something triggers fast reconnection

Dahlburg, Klimchuck & Antiochos 2005: trigger when flux becomes mis-aligned through more than 30^o or so – "secondary instability".

Reconnection

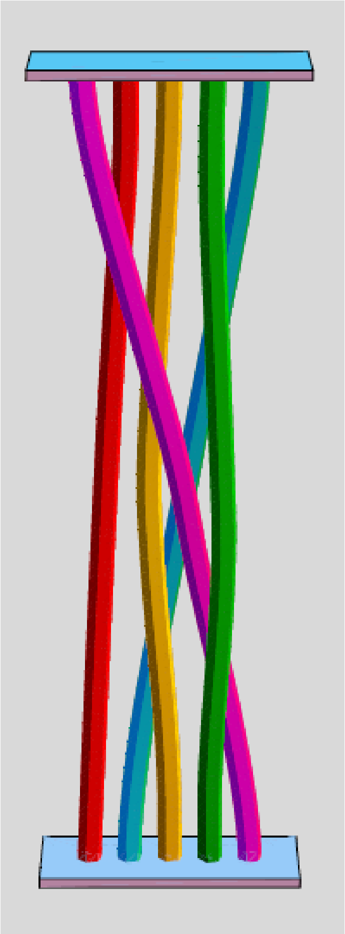
When neighbouring tubes are misaligned by ~ 30 degrees, reconnection removes a crossing, releasing magnetic energy into heat – a nanoflare.



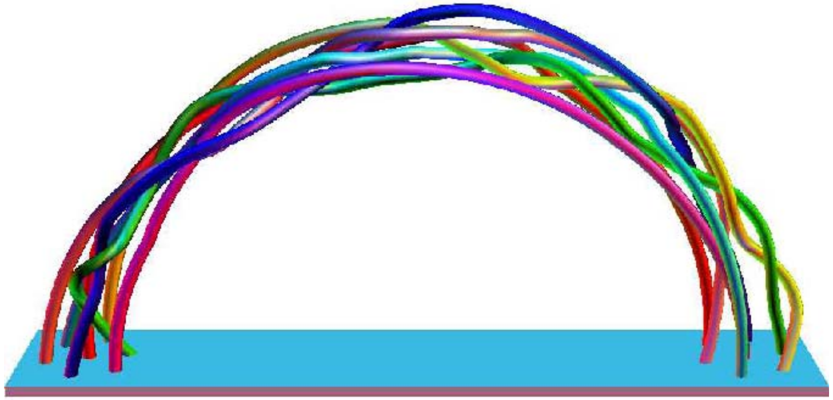
Linton, Dahlburg and Antiochos 2001

The Parker Conjecture

Start with a uniform vertical field between two planes. Braid the field lines by smooth motions at the boundary planes. Now fix the endpoints.
Conjecture: when the field is *sufficiently* braided, no smooth equilibria exist. Parker 79, 94, Ng & Bhattacharjee 98



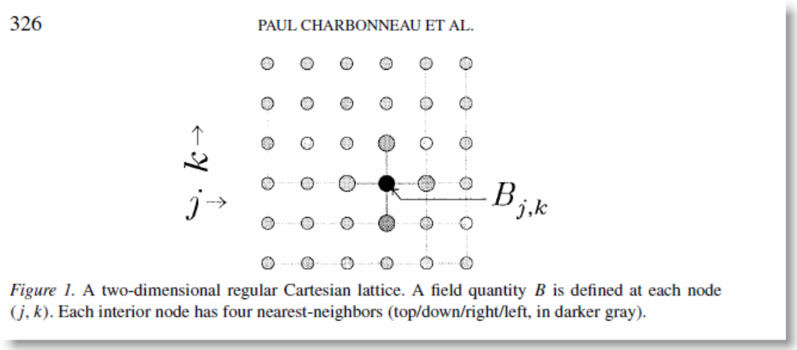
Application: the interior structure of coronal loops



Avalanche Models

Ed Lu

- Introduced by Lu and Hamilton (1991)
- Bits of energy are added randomly to nodes on a grid. When field energy or gradients exceed a threshold, a node shares its excess with its neighbours.

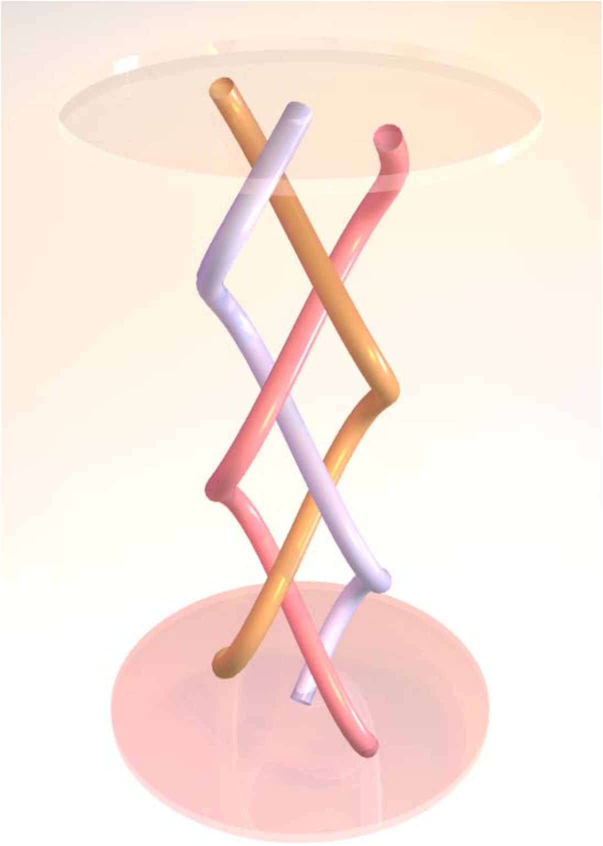


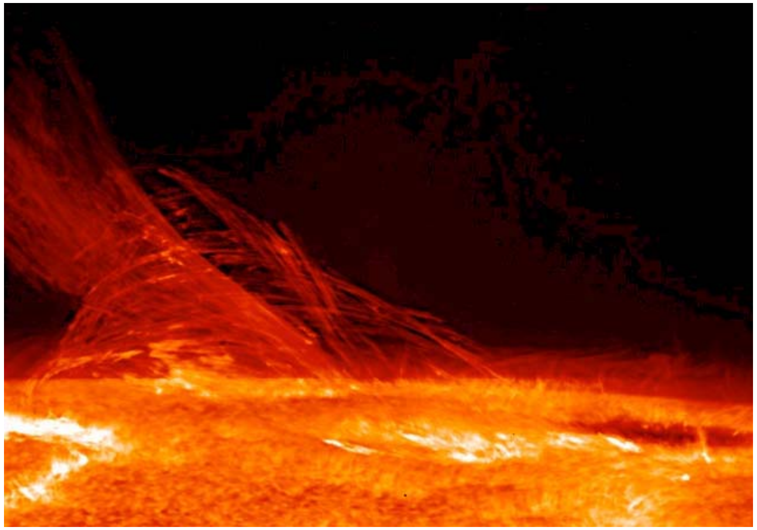
326 PAUL CHARBONNEAU ET AL.

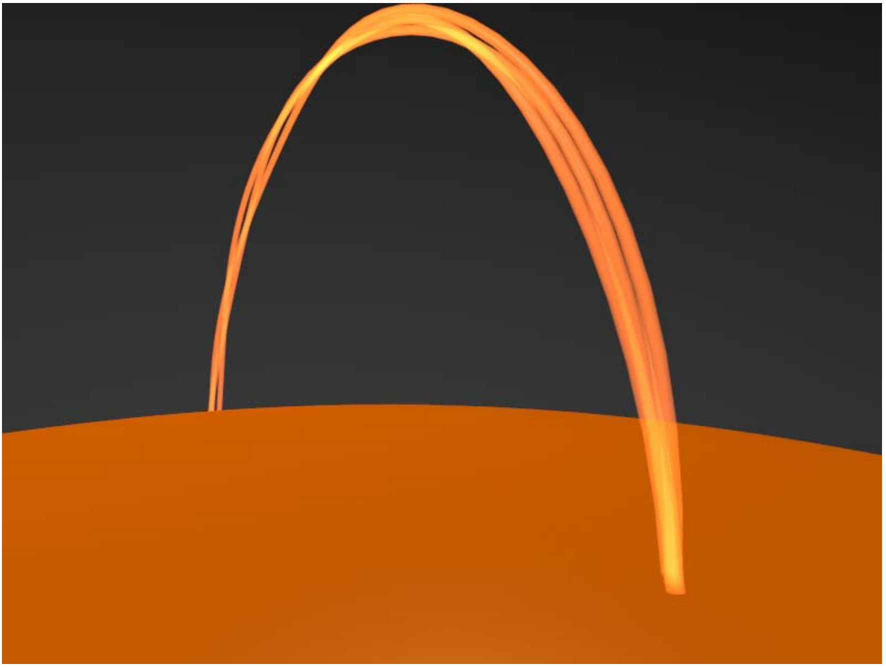
Figure 1. A two-dimensional regular Cartesian lattice. A field quantity B is defined at each node (j, k). Each interior node has four nearest-neighbors (top/down/right/left, in darker gray).

This triggers more events. Eventually a *self-organized critical state* emerges with structure on all length scales. Total avalanche energies, peak power output, and duration of avalanches all follow power laws.

Will we be able to see braids on the sun?







Analysis of braid structures

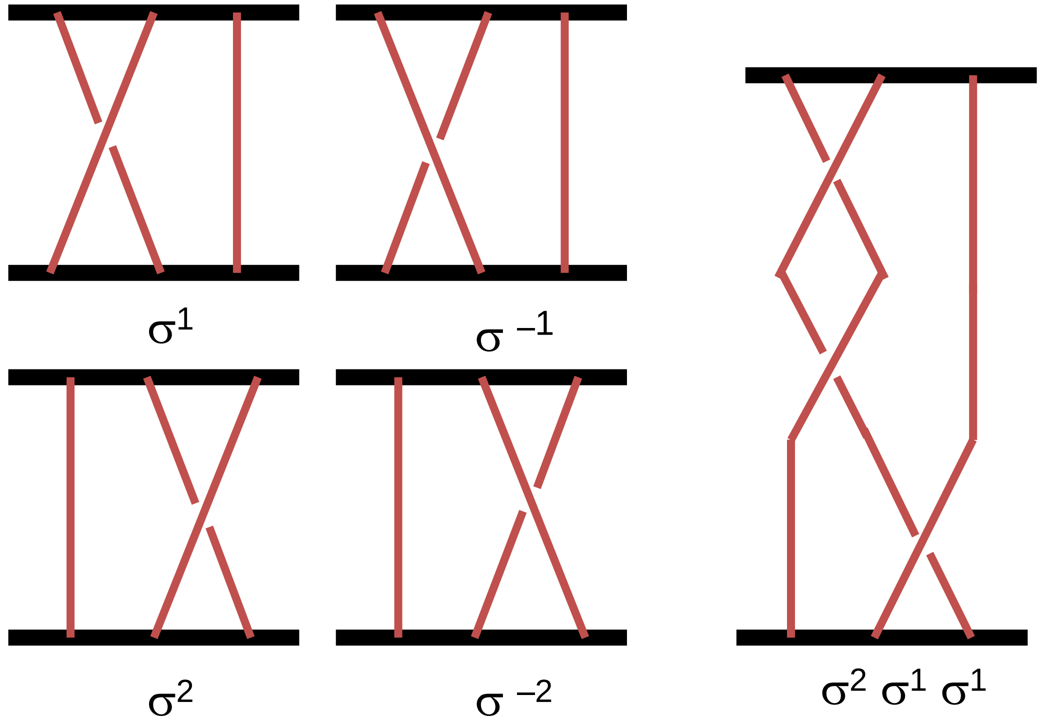
A braided field will evolve depending on the nature of its sources (e.g. foot point motion) and sinks (e.g. reconnection). Reconnection can also increase braiding (Schriver 07)!

Understanding the braid structure can also impact

-- MHD turbulence with dominant magnetic component (e.g. Oughton & Prandi 2000)

-- particle transport and acceleration (e.g. Jokipii & Parker 1969, Geiseler & Kirk 1999)

The braid group (for 3 strings)



A braid word is a sequence of σ symbols, subject to the relation $\sigma^1 \sigma^2 \sigma^1 = \sigma^2 \sigma^1 \sigma^2$

A reconnection removes one of the σ crossings – will this trigger more cancellation of crossings?

Example: $\sigma^1 \sigma^2 \sigma^2 \sigma^{-1} \sigma^{-2} \sigma^{-2} \sigma^1$

→ $\sigma^1 \sigma^2 \sigma^2 \quad \sigma^{-2} \sigma^{-2} \sigma^1$

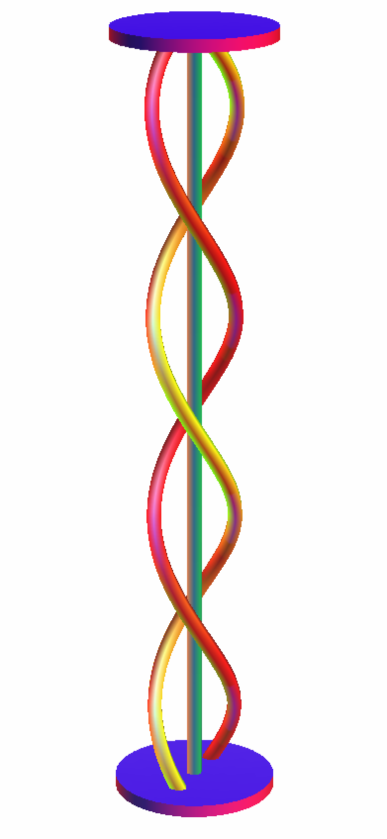
→ $\sigma^1 \sigma^1$

Ignoring the relation $\sigma^1 \sigma^2 \sigma^1 = \sigma^2 \sigma^1 \sigma^2$, the probability of one extra cancellation (of two σ crossings) is ¼ . Including the relation makes it 1 in 3.

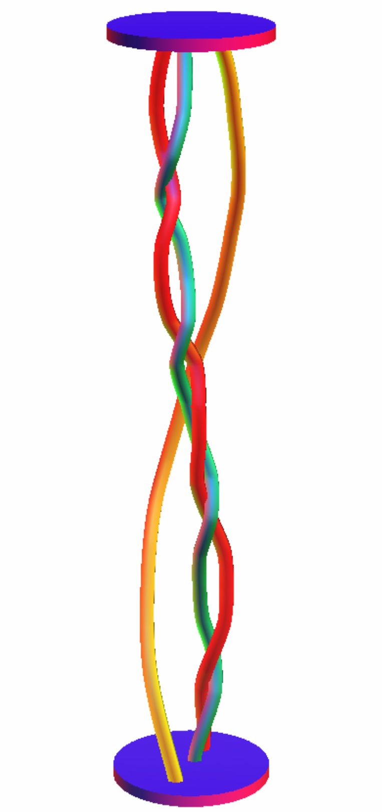
The probability of *n* cancellations is 3^{-n} .

This leads to an exponential distribution of flare energies – not good!

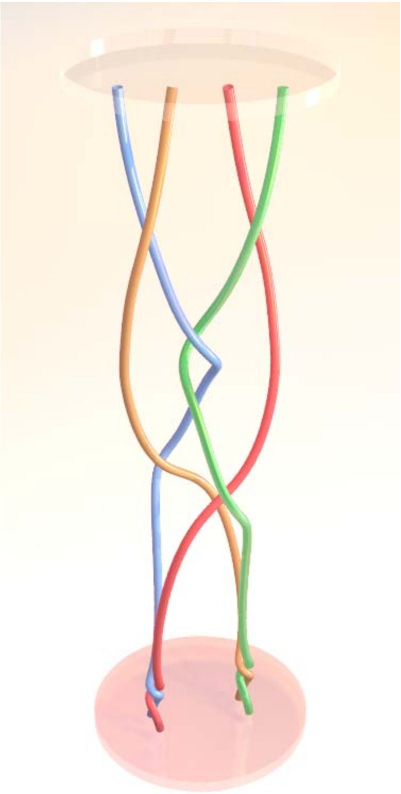
Classification of Braids



Periodic
(uniform twist)

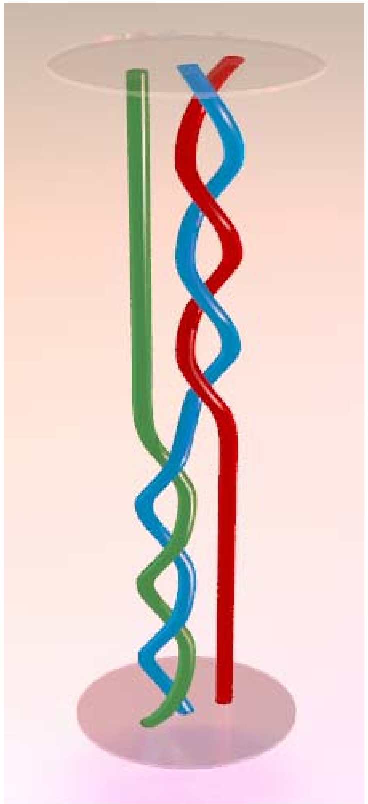


reducible



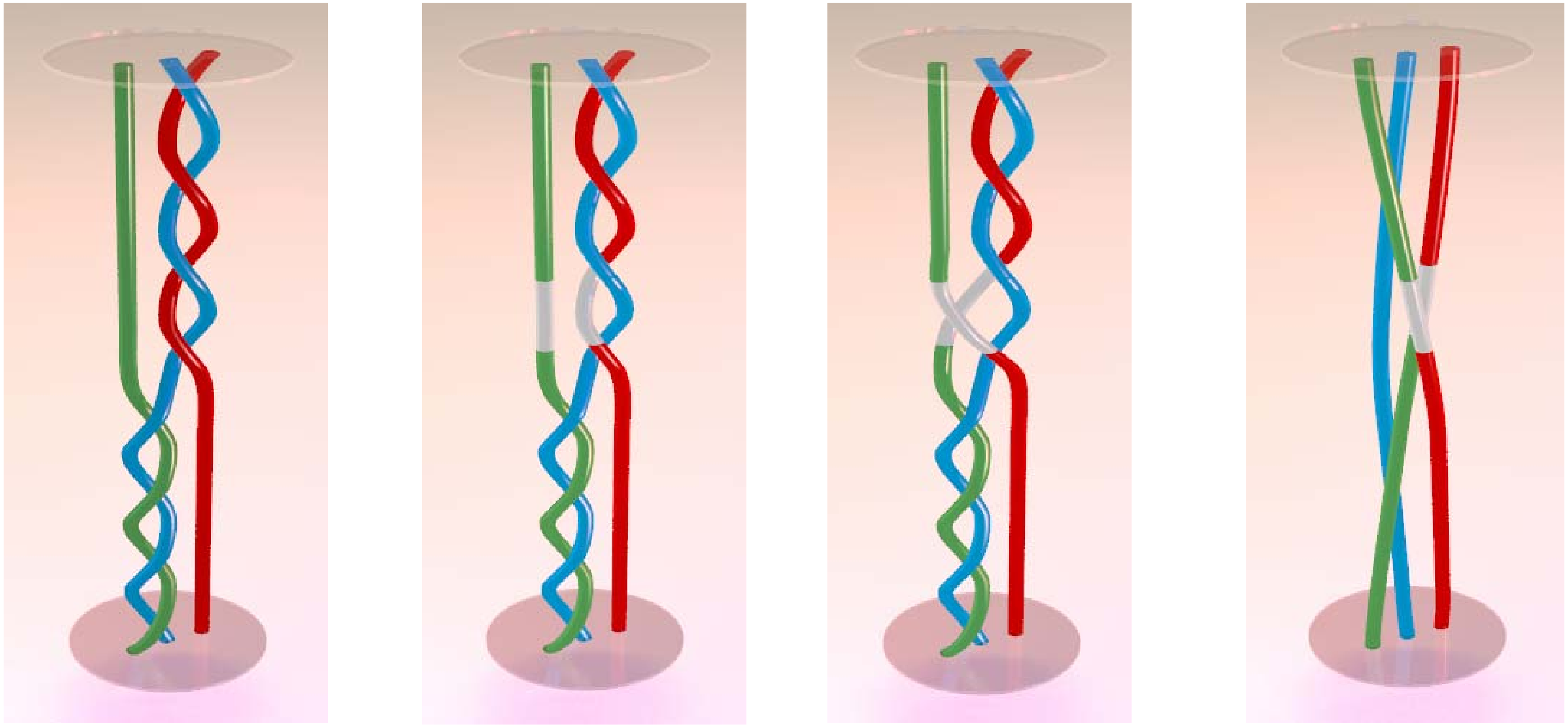
Pseudo-Anosov
(anything else)

Braids with some amount of coherence

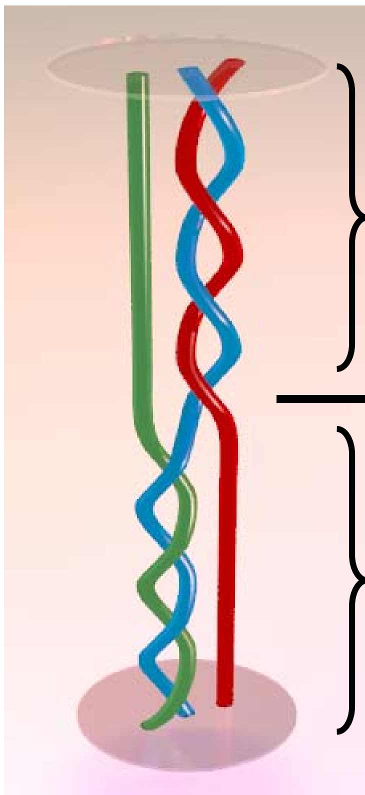


Two reducible braids stuck on top of each other. This can be generated by a rotational motion at one boundary of one pair of tubes, and at the other boundary by a different pair. Or – anti-clockwise motion at the lower boundary followed by clockwise.

Reconnection in a coherent braid can release a large amount of energy…



A simple model for producing coherent braids



coherent section (twist 2)

barrier

coherent section (twist 2)

Suppose at each time step one new "coherent section" is created at the boundaries (photosphere). The size (twist) of the new section is *T*. The probability of creating *T* is *p(T).*

Also at each time step one barrier is removed by reconnection. The neighbouring sections merge.

What is the steady state distribution *n(T)* *of coherent sections with twist T?*

At each time step, *n(T)* changes by *Nδn(T)* (*N*=total number):

$$N\delta n(T) = p(T) - 2n(T) + \int_{-\infty}^{\infty} n(W)n(T-W)\mathrm{d}W.$$

In a steady state,

$$p(T) - 2n(T) + \sqrt{2\pi}(n * n)(T) = 0,$$

where f*g is the Fourier convolution. Take the transform:

$$\sqrt{2\pi}\ \tilde{n}^2(k) - 2\ \tilde{n}(k) + \tilde{p}(k) = 0.$$

Thus

$$\tilde{n}(k) = \frac{1}{\sqrt{2\pi}}\left(1 \pm \sqrt{1 - \sqrt{2\pi}\tilde{p}(k)}\right).$$

(Take negative square root for good behaviour at infinity).

Examples

Input as a Poisson process:

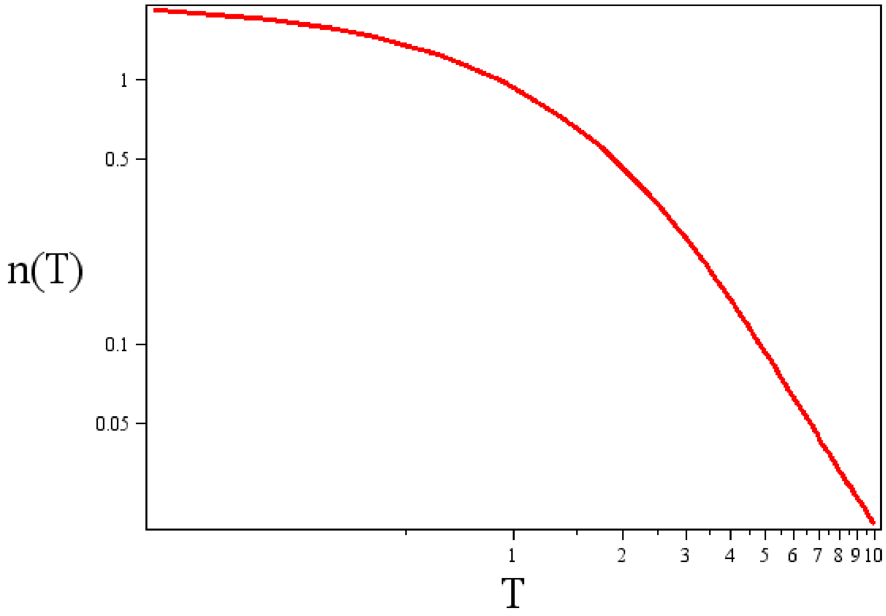
$$p(T) = \frac{1}{2} e^{-|T|}$$

$$p(k) = 1 - \sqrt{\frac{k^2}{1 + k^2}}$$

Solution:

n(T) = ½ (L$_{-1}$(T) - I$_1$ (T))

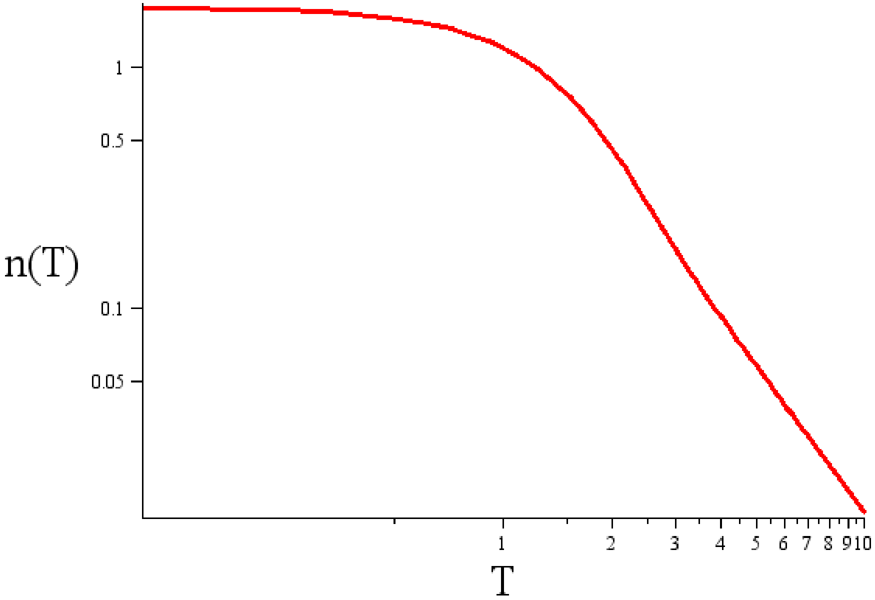
where L_{-1} is a Struve *L* function, and I_1 is a Bessel *I* function.



Power law with slope -2.

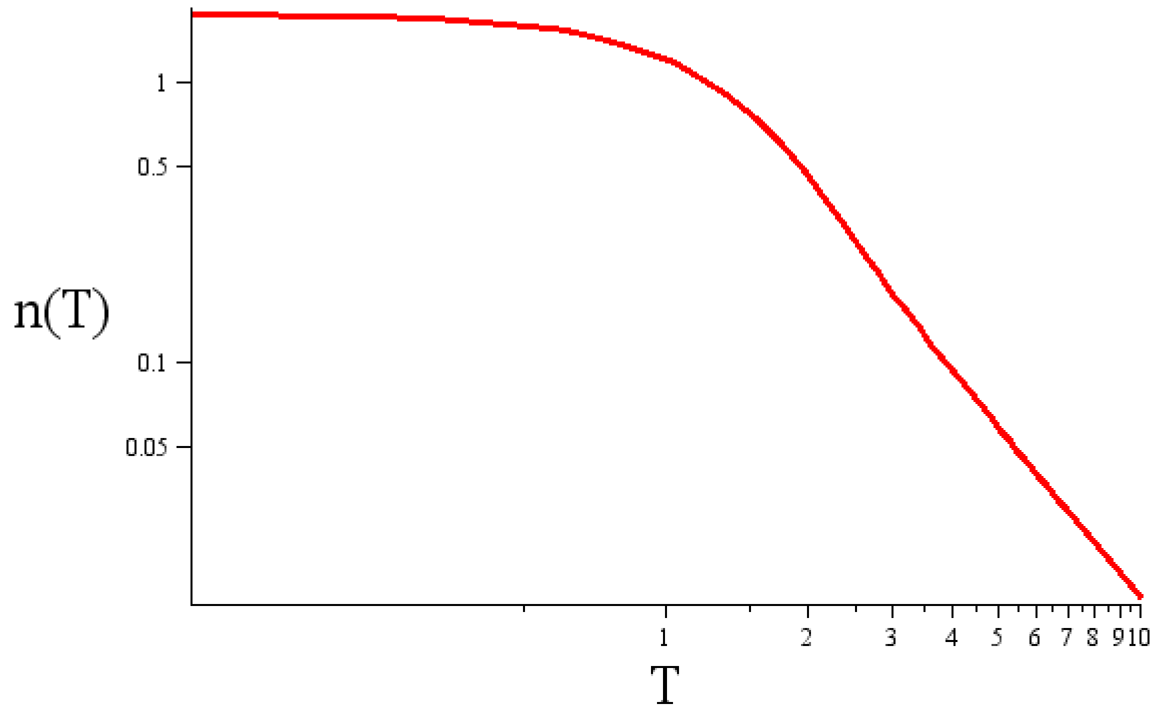
Input as a Normal Distribution:

$$p(T) = \frac{1}{\sqrt{2 \cdot \pi}} e^{-\frac{T^2}{2}}$$



Again a power law with slope -2.

Input $p(T) = (1+T^2)^{-1}$



Power law (-2) input --- slope -3/2 output

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Conclusions

- Braided coronal loops may be self-organized along their length, with power law distributions in size and released energy.
- Braid structure in self-organized state forgets size of input perturbation.
- Self-organized braids are less random , therefore more compact and less obvious in observations.
- Both twist and random walks at photosphere will contribute to braid structure.