

A strong field dynamo in the tachocline

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Strong field dynamos

Dynamos with \mathbf{U} and \mathbf{B} comparable (in Alfvénic scaled units) over a large fraction of the flow domain.

Example: Archontis dynamo

$$\mathbf{F} = v(\sin z, \sin x, \sin y)$$

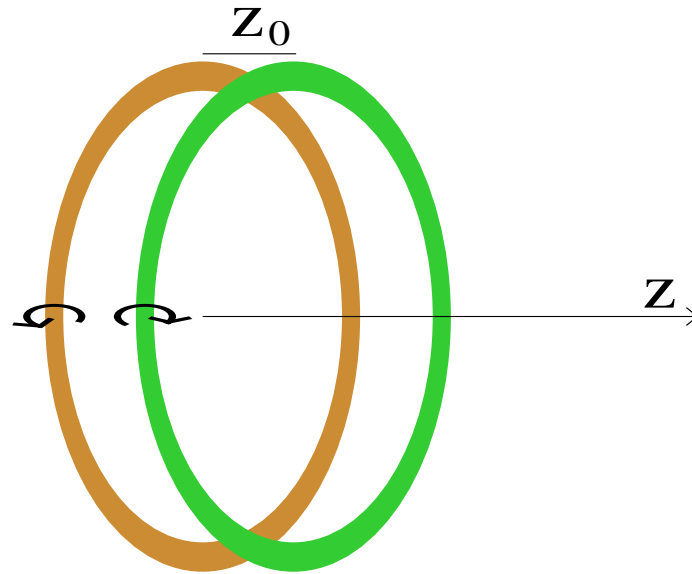
$$\rightarrow \mathbf{U}/\mathbf{B} / 0.5(\sin z, \sin x, \sin y) + \text{few \% terms}$$

A scaling argument: dynamos to order

Suppose we have any steady solution \mathbf{B}_0 to the induction equation when solved with a velocity field \mathbf{U}_0 and a magnetic diffusivity η_0 . We can now generate an equilibrium solution to the whole dynamo problem (including the momentum equation) for $\eta = \varepsilon \eta_0$. This is $\mathbf{U}_1 = \varepsilon \mathbf{U}_0 + \lambda \mathbf{B}_0$, $\mathbf{B}_1 = \mathbf{B}_0$, \mathbf{F} = whatever is necessary to satisfy the momentum equation. This dynamo has the property that \mathbf{U} tends to $\lambda \mathbf{B}$ as the diffusivity tends to zero. If the flow is incompressible, λ can be spatially varying but must be constant on each field line. Note the stability of the resulting object is uncertain. Friedlander and Vishik have shown that the ideal MHD case is neutrally stable.

Strong-field Gailitis dynamo

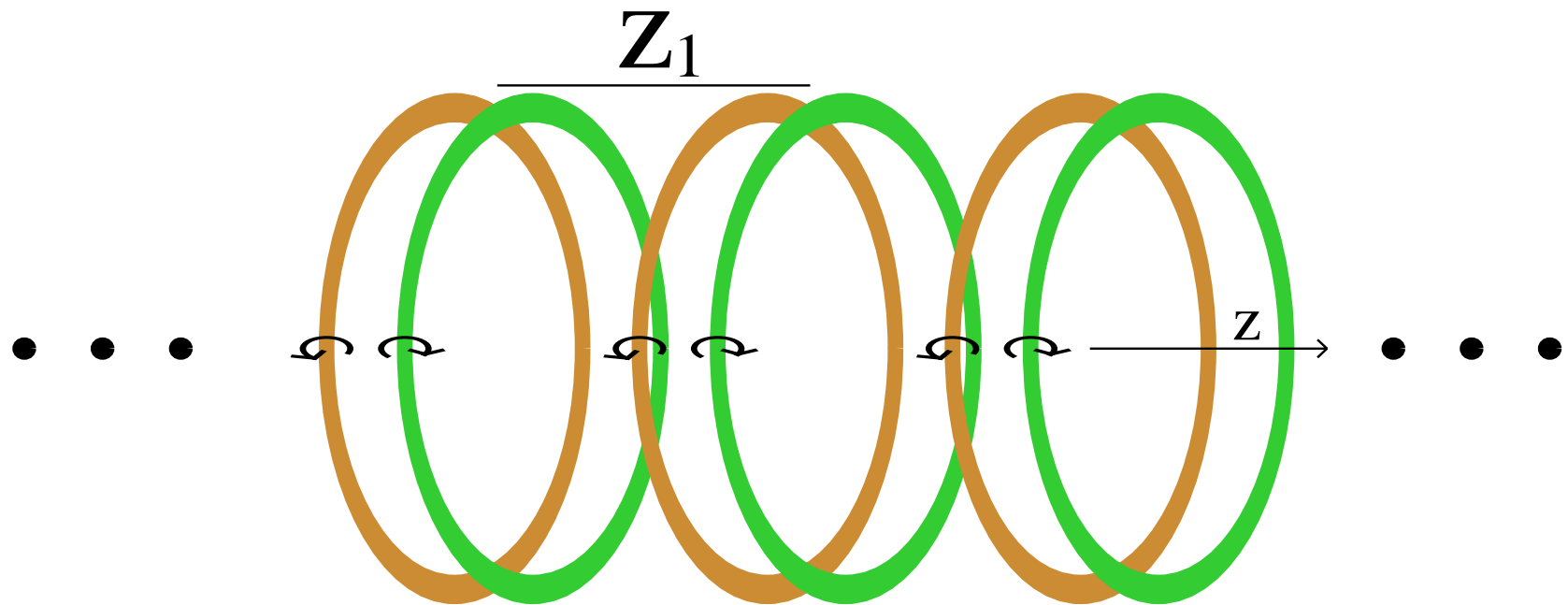
Gailitis's 1970 kinematic dynamo consists of two axisymmetric rings rotating in opposite directions in their meridional plane. Cowling's theorem tells us no axisymmetric dynamo is possible, but a non-axisymmetric field where the field from one ring acts as a seed field for the other can be shown to work.



Let c be the distance out from the Z -axis to the centre of the cross-section of each ring, a be the radius of the cross-section, and Z_0 be the separation as shown. Then Gailitis's theory gives the critical magnetic Reynolds number for kinematic dynamo action as $(c^2/a^2F(Z_0/c))$, where F is an integral.

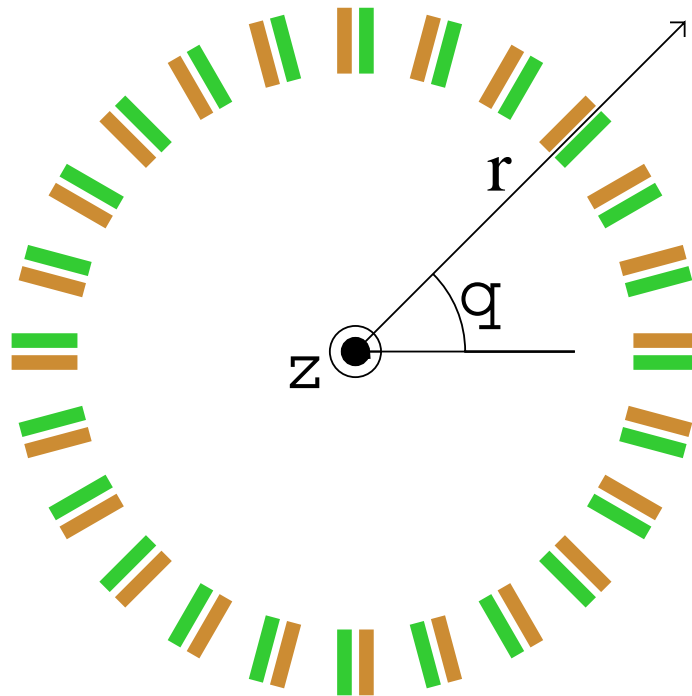
Here we are assuming an $e^{i\varphi}$ dependence for the magnetic field, where φ is the angle around the z -axis. The field is predominantly from L to R at the back of the rings and from R to L at the front (say).

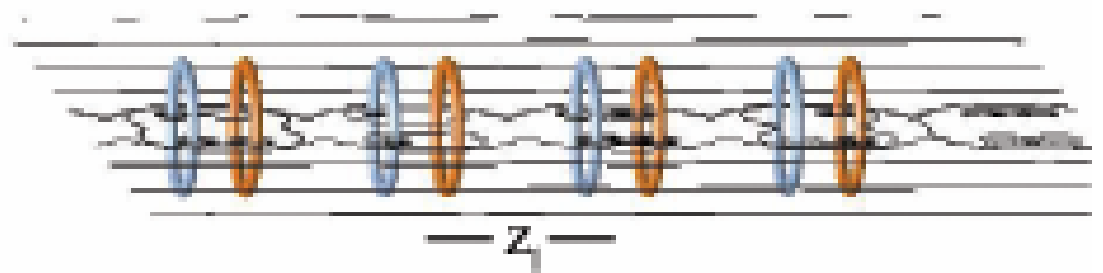
This can be generalised to a line of such pairs (which we call Gailitis building blocks)



Critical R_m is now an infinite sum of integrals similar to the pair case

...then closed round into a ring rather like a tokamak.....





(b)



(c)

Figure 1. The geometries of the three dynamos considered in this paper: (a) a single pair of Gailitis rings; (b) a small segment of a line of such pairs and some of the magnetic field lines; and (c) an indicative plot of the geometry when the line is bent to form a circle. It is the third of these which, when embedded in a circulating tachocline, can reproduce the observed properties of the solar cycle.

...then scaled up into a $U=B$ dynamo as specified earlier

- The set of rings can then be parked at the tachocline (a strong shear layer at the base of the Sun's convection zone)
- Superimposing a meridional flow (which is thought for other reasons to be a feature of the tachocline) and letting it have a circulation time of 22 years, the $e^{i\phi}$ is wafted around to give fields of different polarities to be picked up by the convection zone and carried quickly to the surface every 11 years-a new theory for the solar cycle! Inferred velocity of around 1m/s is reasonable.

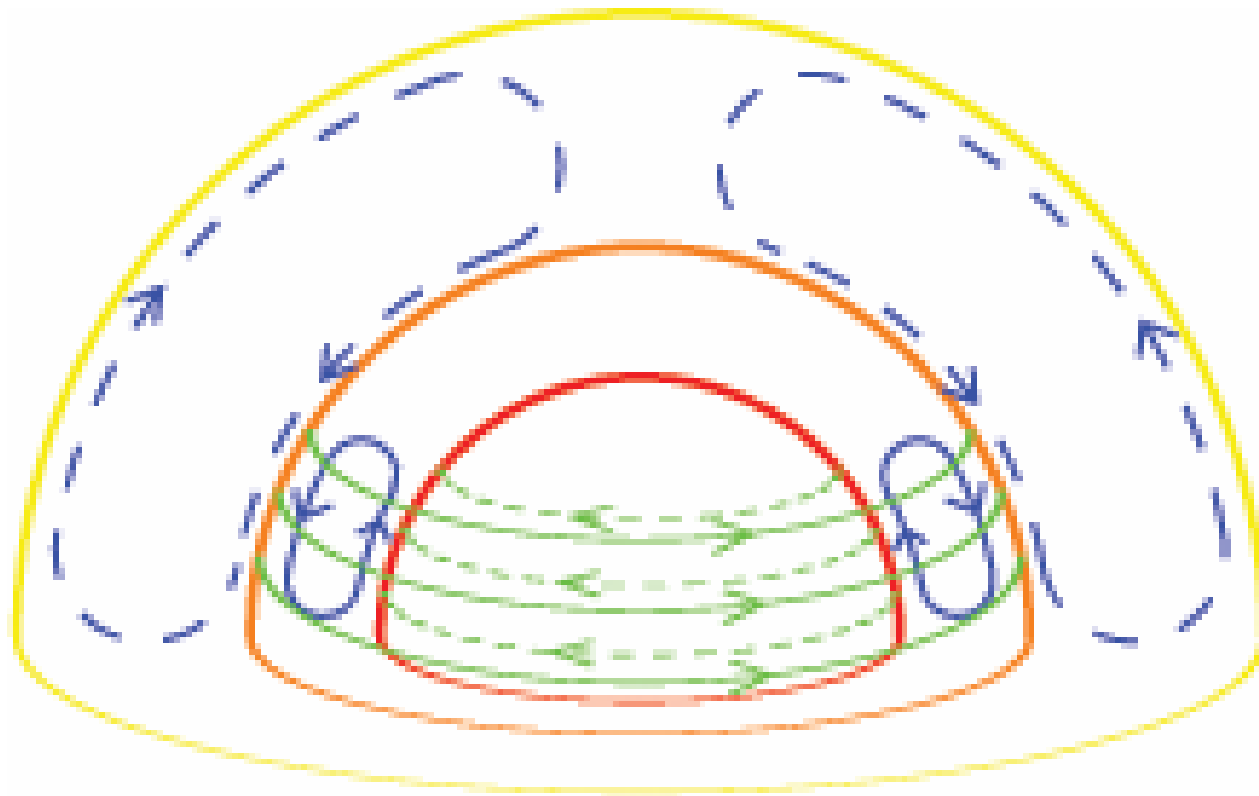


Figure 2. The engine room of the solar dynamo: a schematic illustration of the large-scale features of the proposed dynamo mechanism. The Sun is here divided into a radiative interior (below the red curve), a tachocline (which lies between the red and orange curves), and a convection zone (which lies between the tachocline and the solar surface which is shown in yellow). For illustrative purpose the thickness of the tachocline has been vastly exaggerated. The assumed meridional velocity field is shown using the blue curves and the large scale component of the magnetic field is shown in green. Note that the field has opposite directions at the top and bottom of the tachocline. This field is advected by the meridional circulation and thus presents oppositely directed poloidal field at the base of the convection zone over the course of a 22-year magnetic cycle.

- Surface reacts almost instantaneously to BC presented by tachocline to lower boundary of convection zone (timescale is around 1 month)
- Field strength is limited by quenching of differential rotation---estimates give predicted field strengths of around 1T in tachocline. This agrees well with estimates based on how field evolves up to surface via magnetic buoyancy
- Explains Hale polarity laws, equatorwards progression of butterfly diagram, and most or all other aspects of the solar cycle
- No attempt so far to couple hemispheres via interactions near the equator: slight asymmetries could explain could explain Gnedyshev-Ohl rule on odd/even cycles
- In this model fields in the photosphere/corona/solar wind are lost as waste products from what is happening deep down

- Uncertainties:
- Depends on interactions with differential rotation within the tachocline: latter is not at all understood
- Needs $u=B$ in top/bottom of tachocline, $u=-B$ in bottom/top (because differential rotation apparently does not vary with solar cycle)
- $U=B$ dynamos so far only produced in periodic geometries (or infinite for Gailitis); effects of boundary conditions must modify things at least locally
- Convection zone aspects: picture as proclaimed so far (rising twists to give tilts, magnetic buoyancy, etc. etc) to be largely taken over lock stock and barrel---perhaps peaceful coexistence is possible!