Dynamic Alignment in Driven MHD Turbulence

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Energy Transfer

$$\partial_t \mathbf{z}^{\pm} \mp \mathbf{V}_A \cdot \nabla \mathbf{z}^{\pm} + \mathbf{z}^{\mp} \cdot \nabla \mathbf{z}^{\pm} = -\nabla P, \quad \mathbf{z}^{\pm} = \mathbf{v} \pm \mathbf{b}$$

- **Iroshnikov (1964) Kraichnan (1965)**

  Isotropic. Weak interactions $\tau_l \sim lV_A / \delta v_l^2$. $E(k) \propto k^{-3/2}$.

- **Goldreich & Sridhar (1995)**

  Anisotropic. Critical balance $l_{||} / l_{\perp} \sim V_A / \delta v_{\perp}$. $E(k_{\perp}) \propto k_{\perp}^{-5/3}$.

[Muller & Biskamp (2000); Haugen, Brandenburg & Dobler (2004); Cho & Vishniac (2000); Cho, Lazarian & Vishniac (2002)] [Maron & Goldreich (2001); Muller et al. (2003)]

- **Dynamic Alignment (Boldyrev 2005; 2006):**

  Magnetic field and velocity fluctuations align within a scale dependent angle θ_λ

  3-d anisotropic and $E(k_{\perp}) \propto k_{\perp}^{-3/2}$

[Maron & Goldreich (2001); Muller, Biskamp & Grappin (2003); Muller & Grappin (2005); Mason, Cattaneo & Boldyrev (2006); Mason, Cattaneo & Boldyrev (2008)]

Theory of Polarization Alignment

Decaying MHD turbulence

• **Evolves towards the perfectly aligned configuration $\mathbf{b} = \pm\mathbf{v}$**

[Dobrowolny et al. (1980); Grappin et al. (1982); Pouquet et al. (1986)]

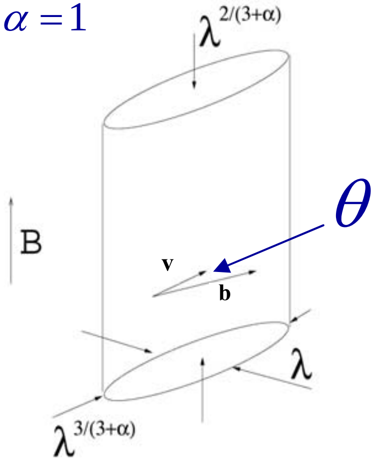
Driven MHD turbulence

• **Energy cascade must be maintained by the nonlinear terms**
• **Magnetic field and velocity fluctuations align within a scale dependent angle θ_λ**
• **If $\theta_\lambda \propto \lambda^{\alpha/(3+\alpha)}$ then constant energy flux implies**

$$E(k_\perp) \propto k_\perp^{-(5+\alpha)/(3+\alpha)}$$

• **Conservation of cross helicity: minimizing total alignment angle yields $\alpha = 1$**

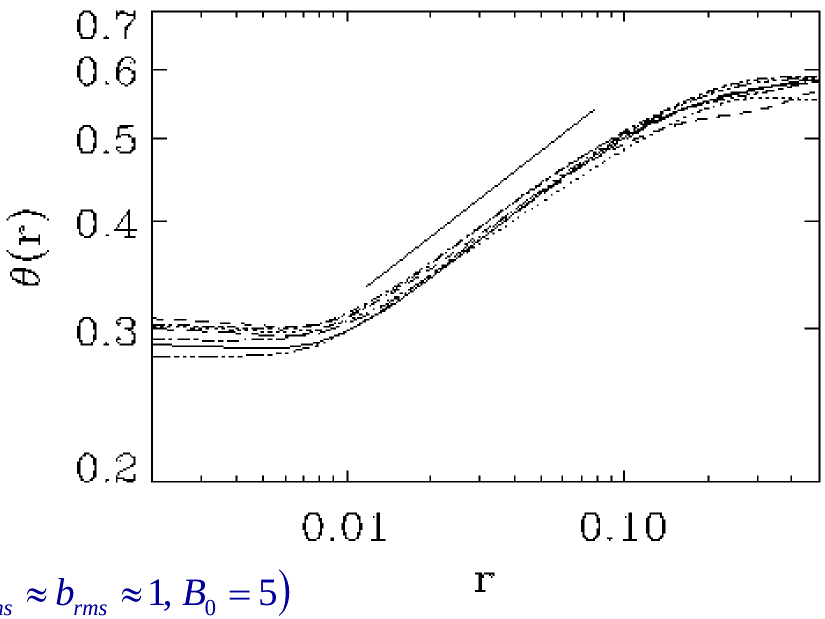
$$\theta_\lambda \sim \lambda^{1/4},\ E(k_\perp) \propto k_\perp^{-3/2}$$



• **Boldyrev (2005, 2006)**

Measuring Alignment

$$\partial_t \mathbf{u} + (\mathbf{u}.\nabla)\mathbf{u} = -\nabla p + \mathbf{j}\times\mathbf{B} + \nu\nabla^2\mathbf{u} + \mathbf{f}_u$$
$$\partial_t \mathbf{B} = \nabla\times(\mathbf{u}\times\mathbf{B}) + \eta\nabla^2 B + \mathbf{f}_B$$
$$\nabla\cdot\mathbf{u} = 0, \quad \nabla\cdot\mathbf{B} = 0$$



$1{:}1{:}\, B_{0,}\ 512^3, \quad P_m = \frac{\nu}{\eta} = 1,\ R_e = \frac{uL}{\nu} \approx 2200\ \ (u_{rms} \approx b_{rms} \approx 1,\ B_0 = 5)$

- **Cascade mainly in field-perpendicular plane:**

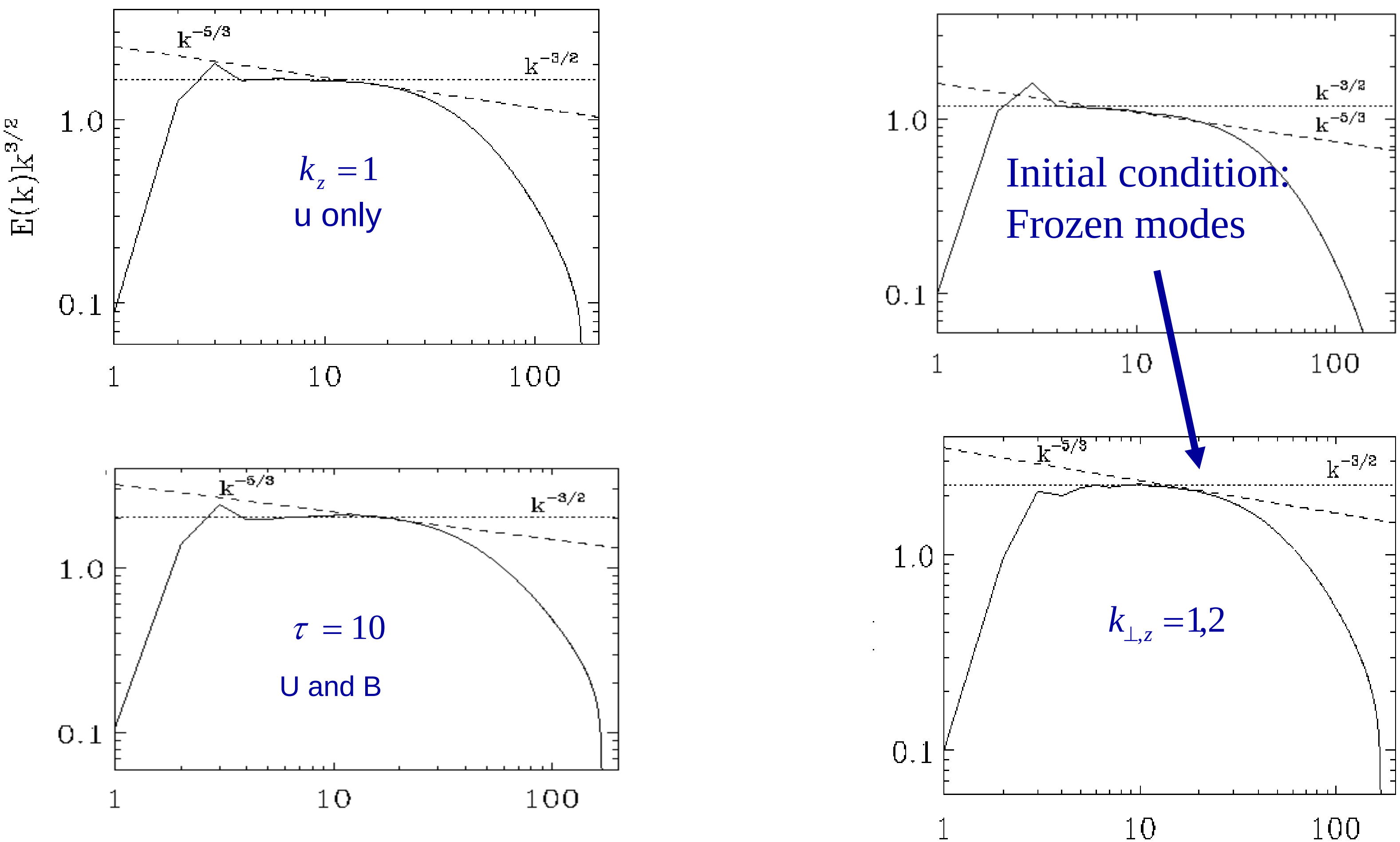
$$\delta\mathbf{v}_\lambda \equiv \mathbf{v}(\mathbf{x}+\boldsymbol{\lambda}) - \mathbf{v}(\mathbf{x}), \qquad \delta\tilde{\mathbf{v}}_\lambda = \delta\mathbf{v}_\lambda - (\delta\mathbf{v}_\lambda\cdot\mathbf{n})\mathbf{n}, \quad \mathbf{n} = \frac{\mathbf{B}(x)}{|\mathbf{B}(x)|}$$

- **Measure alignment between rms fluctuations**

$$\frac{\left\langle \left|\delta\tilde{\mathbf{v}}_\lambda \times \delta\tilde{\mathbf{b}}_\lambda\right| \right\rangle}{\left\langle \left|\delta\tilde{\mathbf{v}}_\lambda\right| \left|\delta\tilde{\mathbf{b}}_\lambda\right| \right\rangle} \approx \sin(\theta_\lambda) \approx \theta_\lambda \propto \lambda^{1/4}$$

- **Mason, Cattaneo & Boldyrev (2006)**

Energy Spectrum



- $k_{\perp} = \sqrt{k_x^2 + k_y^2}, \quad E(k_{\perp}) = \langle |\mathbf{v}(k_{\perp})|^2 \rangle k_{\perp} + \langle |\mathbf{b}(k_{\perp})|^2 \rangle k_{\perp}$
- **Mason, Cattaneo & Boldyrev (2008); Perez & Boldyrev (2008)**

Kolmogorov's 4/5-law

- **Hydrodynamic turbulence** $\left\langle \delta v_L^3(\mathbf{r}) \right\rangle = -\frac{4}{5}\varepsilon r \;\rightarrow\; E(k) \propto k^{-5/3}$,

where $\delta v_L(\mathbf{r}) \equiv \left[\mathbf{v}(\mathbf{x}+\mathbf{r}) - \mathbf{v}(\mathbf{x})\right]\cdot \mathbf{r}$

- **Isotropic MHD turbulence (Politano & Pouquet, 1998)** $\boldsymbol{\delta z}^{\pm} = \boldsymbol{\delta v} \pm \boldsymbol{\delta b}$

$$\left\langle \delta z_L^-\left(\delta\mathbf{z}^+\right)^2 \right\rangle = -\frac{4}{3}\varepsilon^+ r \quad \text{and} \quad \left\langle \delta z_L^+\left(\delta\mathbf{z}^-\right)^2 \right\rangle = -\frac{4}{3}\varepsilon^- r$$

- **For a strong guiding field**

$$\left\langle \delta z_L^-\left(\delta\mathbf{z}^+\right)^2 \right\rangle = -2\varepsilon^+ r_\perp \quad \text{and} \quad \left\langle \delta z_L^+\left(\delta\mathbf{z}^-\right)^2 \right\rangle = -2\varepsilon^- r_\perp$$

~> Consistent with $E(k_\perp) \propto k_\perp^{-3/2}$ **?**

- ***Check the theory of dynamic alignment***

$$\left\langle \delta z_L^-\left(\delta\mathbf{z}^+\right)^2 \right\rangle = \theta_r \delta v_r^3 \sim r$$

$$\delta z^+ \sim \delta v, \;\; \delta z_L^- = \delta z^- \cos(\phi) \approx \delta z^- \approx \delta v \sin(\theta) \approx \delta v\, \theta,$$