Discussion on Alignment and Cross-Helicity

Annick Pouquet, NCAR

- 1 - The growth of global and local v.B correlations and the quenching of nonlinearities in NS and MHD
- 2 - Different energy spectra when v.B correlations are strong
- 3 - Equipartition- and correlation- defects have steeper spectra
- 4 - Third-order flux scaling laws and compatibility relations

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pouquet@ucar.edu

[1] Quadratic invariants ($\nu = \eta = 0$) with direct cascades

* Energy: $Q_1 = E^T = 1/2 < v^2 + B^2 >$
* Cross helicity: Q_2 = **H^C**= < **v.B** > *(Woltjer, 1958)*

One normalized correlation coefficient:

$R_2(\mathbf{x}) = Q_2 \,/\, Q_1 = \mathbf{v.B} \,/\, [v^2+b^2] = H^c(\mathbf{x}) \,/\, (2)\, E^T(\mathbf{x})$

Mid 80's: **Growth** of the correlation coefficients, *viewed as a long-term process*

(Dobrowolny et al.; Frisch etc.; Montgomery and Matthaeus etc.)

(selective decay, dynamic alignment)

[1] Quadratic invariants ($\nu = \eta = 0$) with direct cascades

* Energy: $Q_1 = E^T = 1/2 < v^2 + B^2 >$ and cross helicity: $Q_2 = H^C = < v.B >$ *(Woltjer, 1958)*

Normalized correlation coefficients:

$R_2(\mathbf{x}) = Q_2 \; / \; Q_1 \; = \mathbf{v.B} \; / \; [v^2+b^2] = H^c(\mathbf{x}) \,/\, (2)\, E^T(\mathbf{x})$

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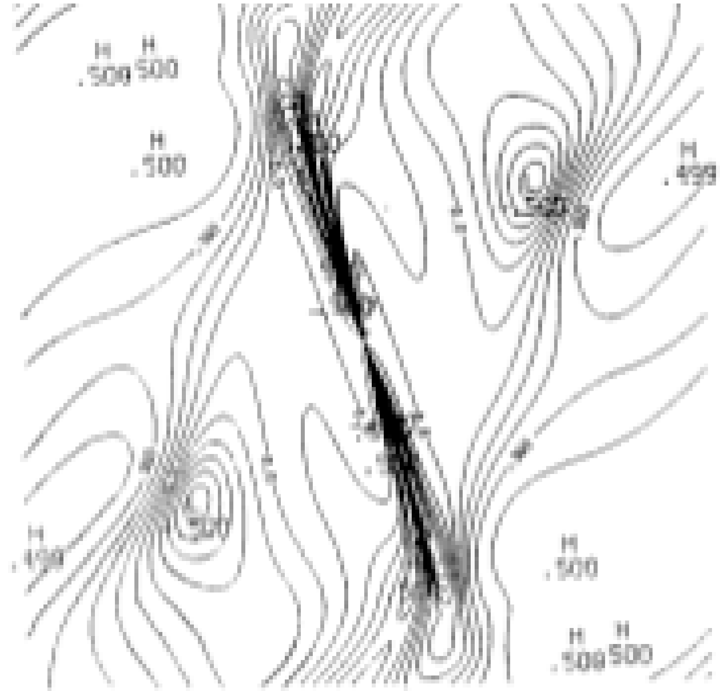
(selective decay, dynamic alignment)

Point-wise growth on a ``fast'' timescale:

$\mathbf{D_t\ (v.B) = B\ .\ \nabla[-p + v^2/2]}$ *(Matthaeus et al., PRL **100**, 2008)*

Batchelor analogy between vorticity ω and induction: v.ω follows the same equation

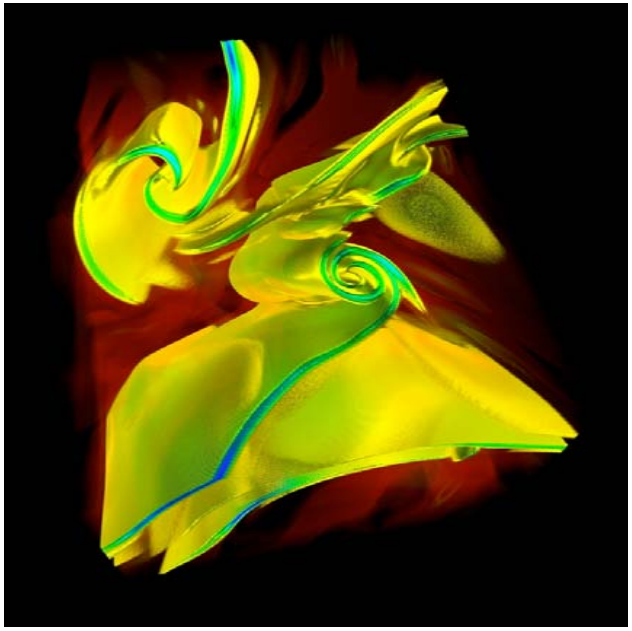
Two-dimensional case

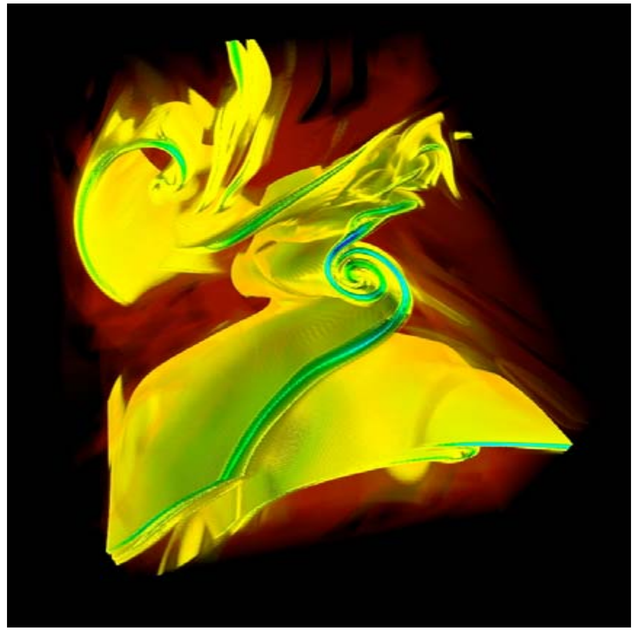


Zoom on current structure in Orszag-Tang vortex: *Meneguzzi et al., JCP **123** (1996)*

Global view, random run: *Matthaeus et al. PRL **100**, 085003 (2008)*

Kinetic helicity: Moffatt, …





1536^3 decay run

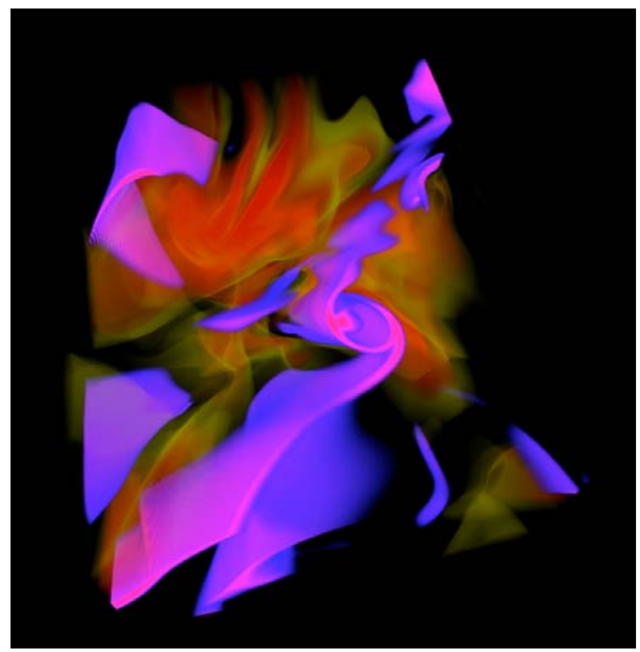
Zoom on a structure

early phase

Vorticity & current,

and cos(v,B)
when strong

3D run: *Mininni et al.,, Phys. Rev. Lett.* ***97****, 244503 (2006);*
see also PRL ***99****, 254502 (2007).*



*cisl.****ucar****.edu/hss/dasg/software/vapor*

[2] Energy spectra in the presence of v-B correlations

Say that $E^T(k) \sim \mathbf{k}^{-\alpha}$

Note on inertial index α:

Kolmogorov (K41) or Alfvénic (IK) or weak turbulence (WT); k can be $k_\perp$

(intermittency corrections are not considered here)

Elsässer variables $\mathbf{z}^\pm = \mathbf{v} \pm \mathbf{b}$ with spectra $E^\pm(k) = [\, E^T(k) \pm H^c(k) \,]$

$E^+(k) \sim k^{-p}$ and $E^-(k) \sim k^{-m}$

with **$m + p = 2\,\alpha$** *(e.g., m+p=3 for the IK case)*

EDQNM closure, phenomenologies, numerical simulations, and more recently weak turbulence theory

Mid '80s: *Frisch etc.*

~ 15 years later, for weak turbulence: *Galtier et al., Goldreich and Sridhar*

[2] Energy spectra in the presence of v-B correlations

Elsässer **$z^{\pm}=v\pm b$**

$E^{+}(k) \sim k^{-p}$ $E^{-}(k) \sim k^{-m}$

2D MHD direct numerical simulations, decay runs

cos θ

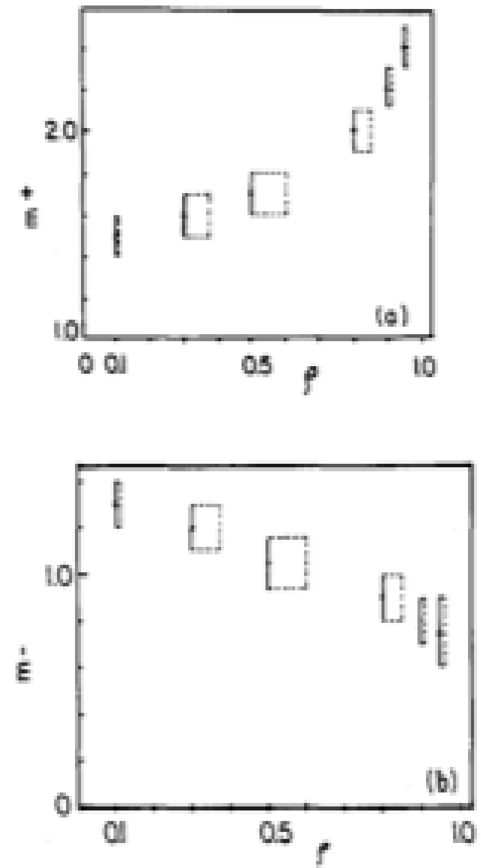


FIG. 12. Inertial range spectral exponents m^+ and m^- versus the initial degree of correlation.

Pouquet et al., 1988

[3] Residual energy and cross-helicity spectra

With $E^T(k) \sim k^{-\alpha}$, what is the spectral density $H^c(k)$?

$H^c(k)$ is steeper than $E^T(k)$:

$H^c(k) \sim k^{-2}$ when $E^T(k) \sim k^{-3/2}$ *(low global v-B correlation)*

[EDQNM, *Grappin et al., A&A **105**, 1982)]*

Defect of Alfvénicity as measured either in the (v, B) or in the (z^+, z^-) variables:

Differential Δ_1 in (v, B) energies, assumed small : $E^R = | v^2 - B^2 |$
Differential Δ_2 in (+, -) energies, assumed small : $H^c = | E^+ - E^- |$

$$\boldsymbol{\Delta(k) = \sim k^{-r}} \sim [\tau_A/\tau_{NL}]^2 E^T(k)$$

hence: **$r = 2\alpha - 1$** *(r=2 for α =3/2:* $E^R(k) \sim k^{-2}$ and $H^c(k) \sim k^{-2}$*)*

(Pouquet et al. 1976; Heyvaerts and Priest 1983; Müller and Grappin 2005)

[4] Two **coupled** scaling laws

``exact but not rigorous''

Politano+AP , Geophys.Res. Lett. ***25****, 273 (1998)*

5 hypotheses

Structure function for **F** : $\boldsymbol{\delta F}(\boldsymbol{l}) = \mathbf{F}(\mathbf{x}+\boldsymbol{l}) - \mathbf{F}(\mathbf{x})$ (longitudinal $\delta F_L(r) = \boldsymbol{\delta F} \cdot \boldsymbol{l} / |\boldsymbol{l}|$).
Energy and cross-helicity invariance lead to flux relationships:

$$< \delta v_L[\delta v_i^2+\delta b_i^2] > - 2 < \delta b_L \, \delta \mathbf{v}.\delta \mathbf{b} > \quad = - (4/d) \, \boldsymbol{\varepsilon}^{\mathbf{T}} \, l$$

$$- <\delta b_L[\delta v_i^2+\delta b_i^2] > + 2 < \delta v_L \, \delta \mathbf{v}.\delta \mathbf{b} > \quad = - (4/d) \, \boldsymbol{\varepsilon}^{\mathbf{c}} \, l$$

with $\varepsilon^T = - d_t E^T = -d_t(E^V + E^M)$ and $\varepsilon^c = - d_t H^C = -d_t <v.b>$ (*d is the space dimension)*

- Either $\delta v \sim \delta b \sim l^{1/3}$, or else v-B correlations must play a role

[4] *and …*

Boldyrev et al., 2006

The role of v-B correlations can be modelized as:

$< \delta v_L(l)\, \delta b_i^2\, (l)\, \theta\, (l)> + \ldots = -(4/3)\, l$

where $\theta = (\mathbf{v},\mathbf{B})$

$\delta v \sim \delta b \sim l^{1/4} \sim \theta$

Hence, there is compatibility between the -3/2 spectral law and the exact flux scaling laws

[4] *and yet another compatibility remark*

Differential in angle: $\Delta a = | \cos\theta - 1 | \sim \theta^2$

$H^c(k) \sim k^{-2}$ hence $kH^c(k) \sim k^{-1} \sim l \sim v_l\, b_l\, \theta^2_l$

With $v_l \sim b_l \sim l^{1/4}$, then θ must scale as $\sim l^{1/4}$ as well in order to satisfy $H^c(k) \sim k^{-2}$.

In other words, the spectrum $H^c(k) \sim k^{-2}$ and the $\theta \sim l^{1/4}$ scaling are also compatible

Thanks!

Also possibly for the discussion: alpha effect with cross helicity (Yoshizawa, PoF B2, 1589, 1990)