THE ROLE OF TURBULENCE IN LIQUID METAL DYNAMO EXPERIMENTS

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Sodium Dynamo Experiments on Two Continents: History and Current Successes

Dynamo Theory

Kavli Institute for Theoretical Physics UCSB

10th JUNE 2008

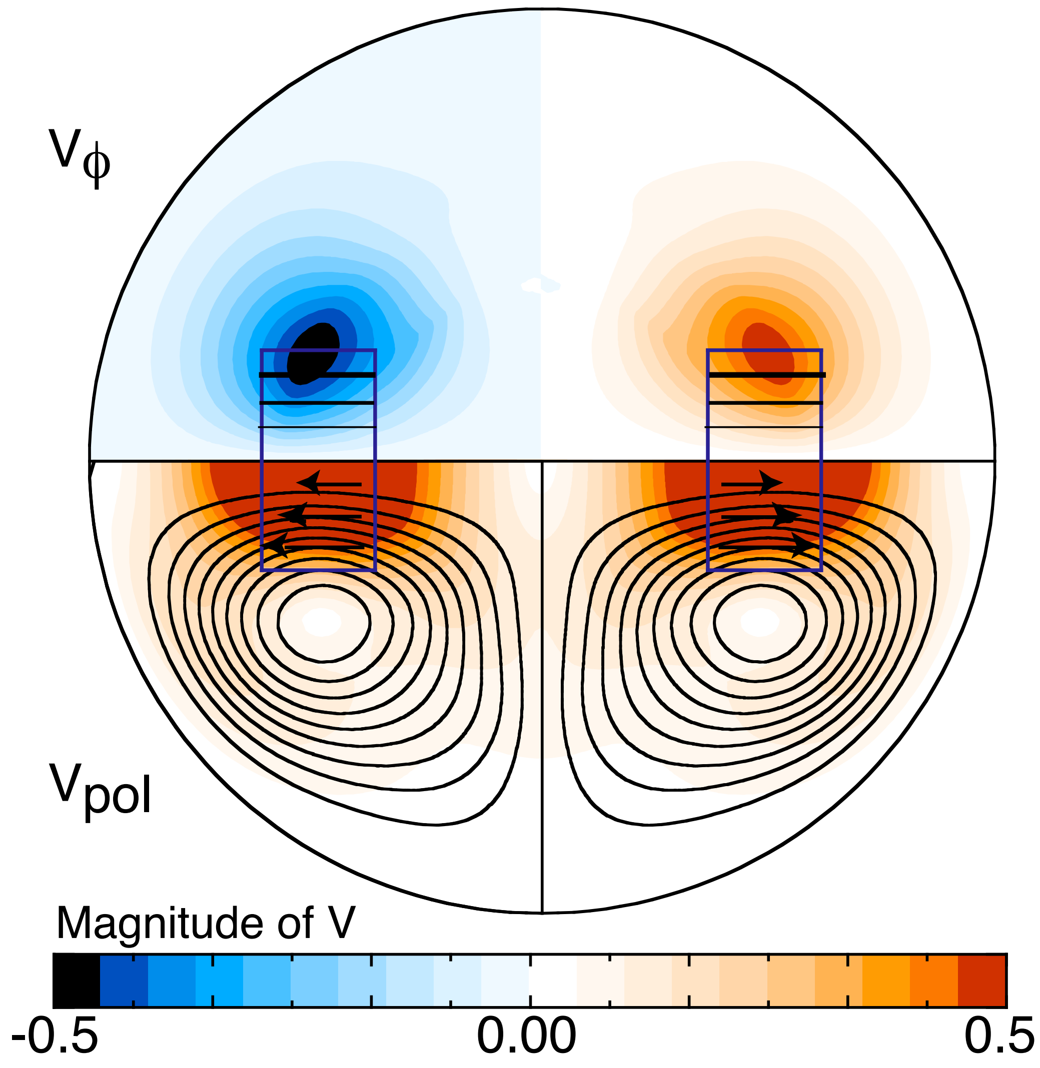
Do Mean-Field Currents Exist in Weakly Magnetized, Turbulent MHD Flows?

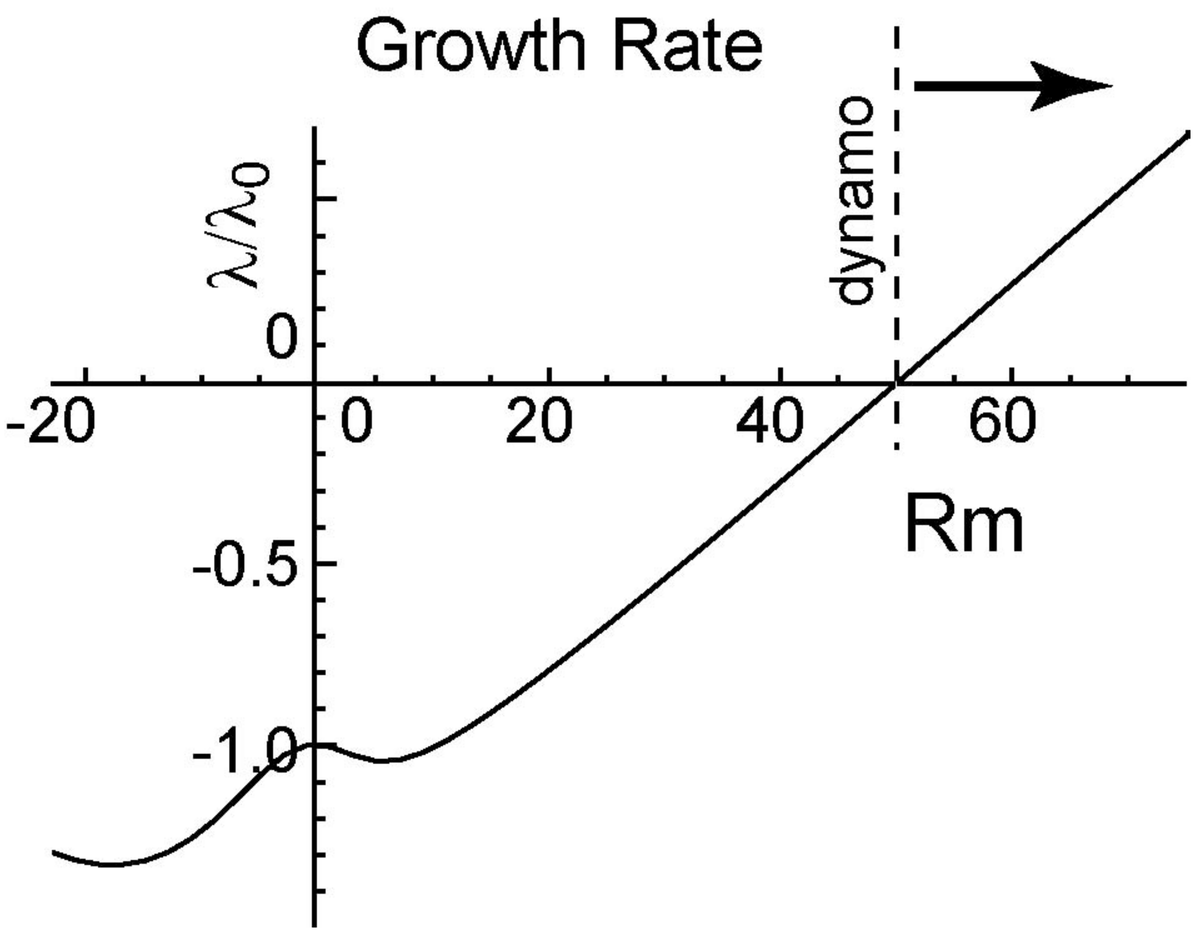
- Not previously observed
- Essential component of the Standard Model of the self-excited dynamo
- understand role of MFED in liquid metal dynamo experiments (helpful, or detrimental?)
- long standing prediction

Outline

- Introduction
  - overview of the Madison experiment
  - role of turbulence in dynamo onset
- Observation of fluctuation driven currents
- Future plans
  - turbulence reduction and flow control
  - A Plasma based MRI and Dynamo Experiment

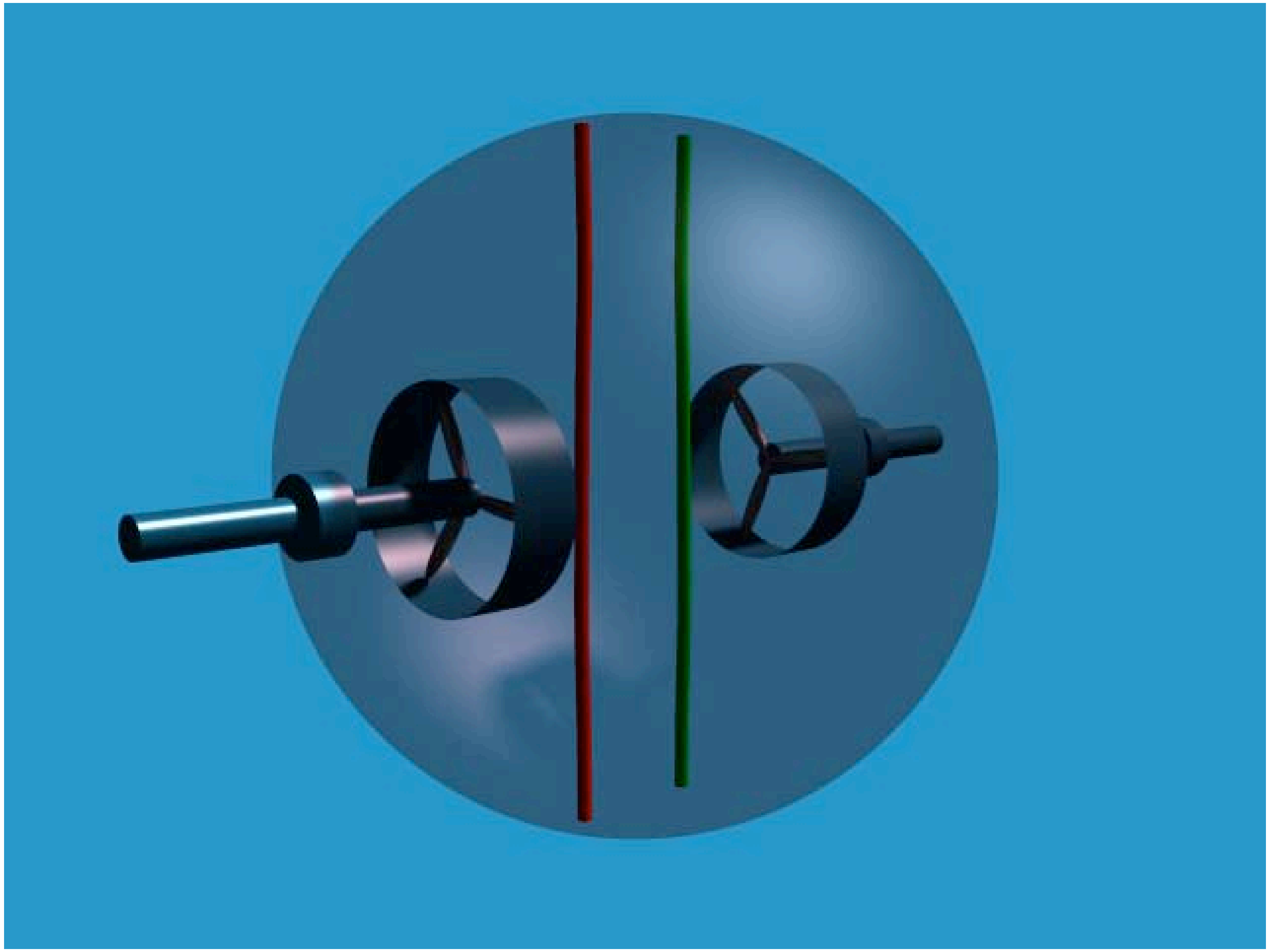
This simplest possible self-exciting flow: a two vortex flow with Rm_{crit}~50





Dudley and James, *Time-dependent kinematic dynamos with stationary flows*, Proc. Roy. Soc. Lond. A. **425** 407 (1989).

Dynamo is of the stretch-twist-fold type: field line stretching, geometric reinforcement, and reconnection leads to dynamo



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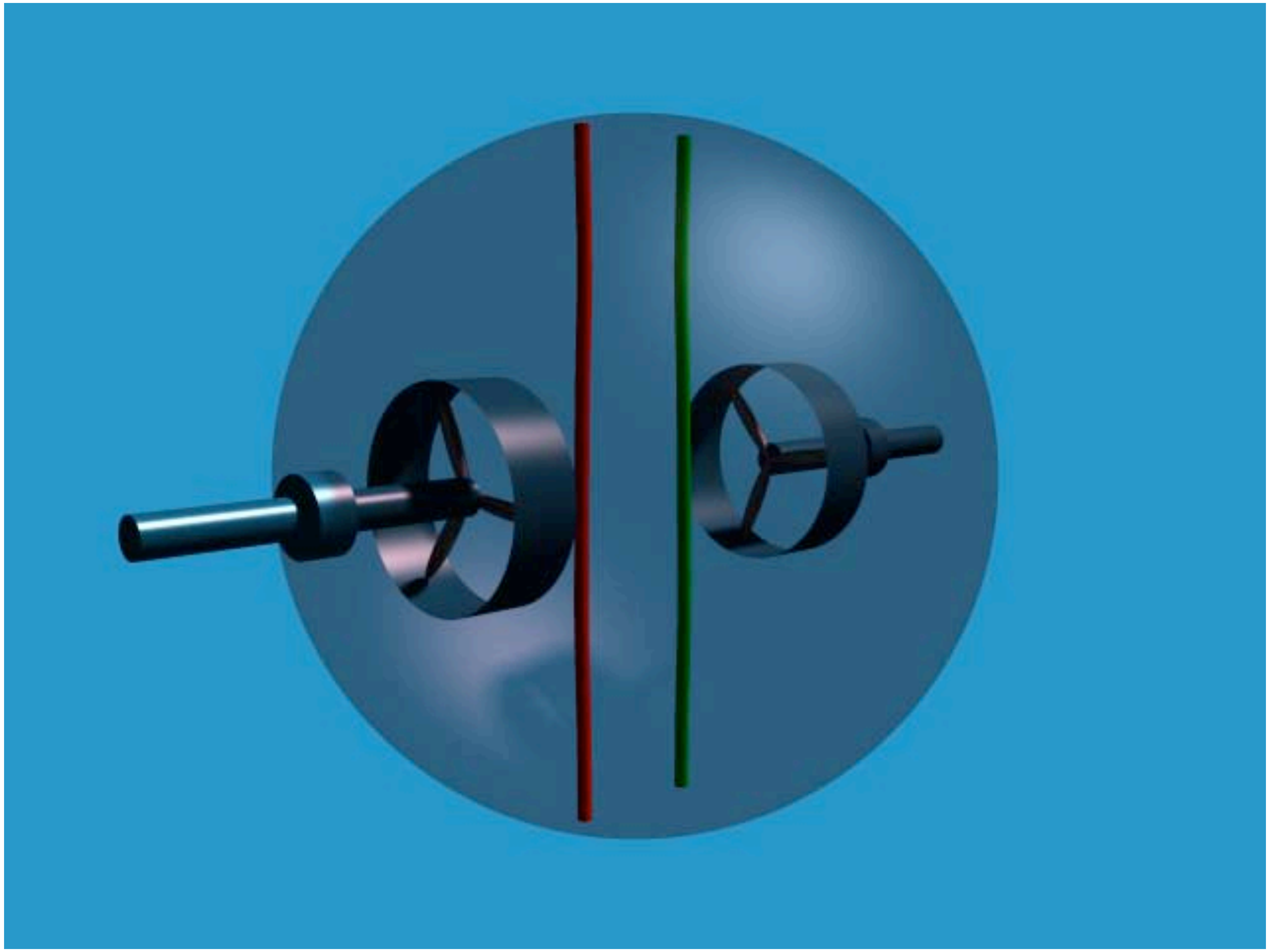
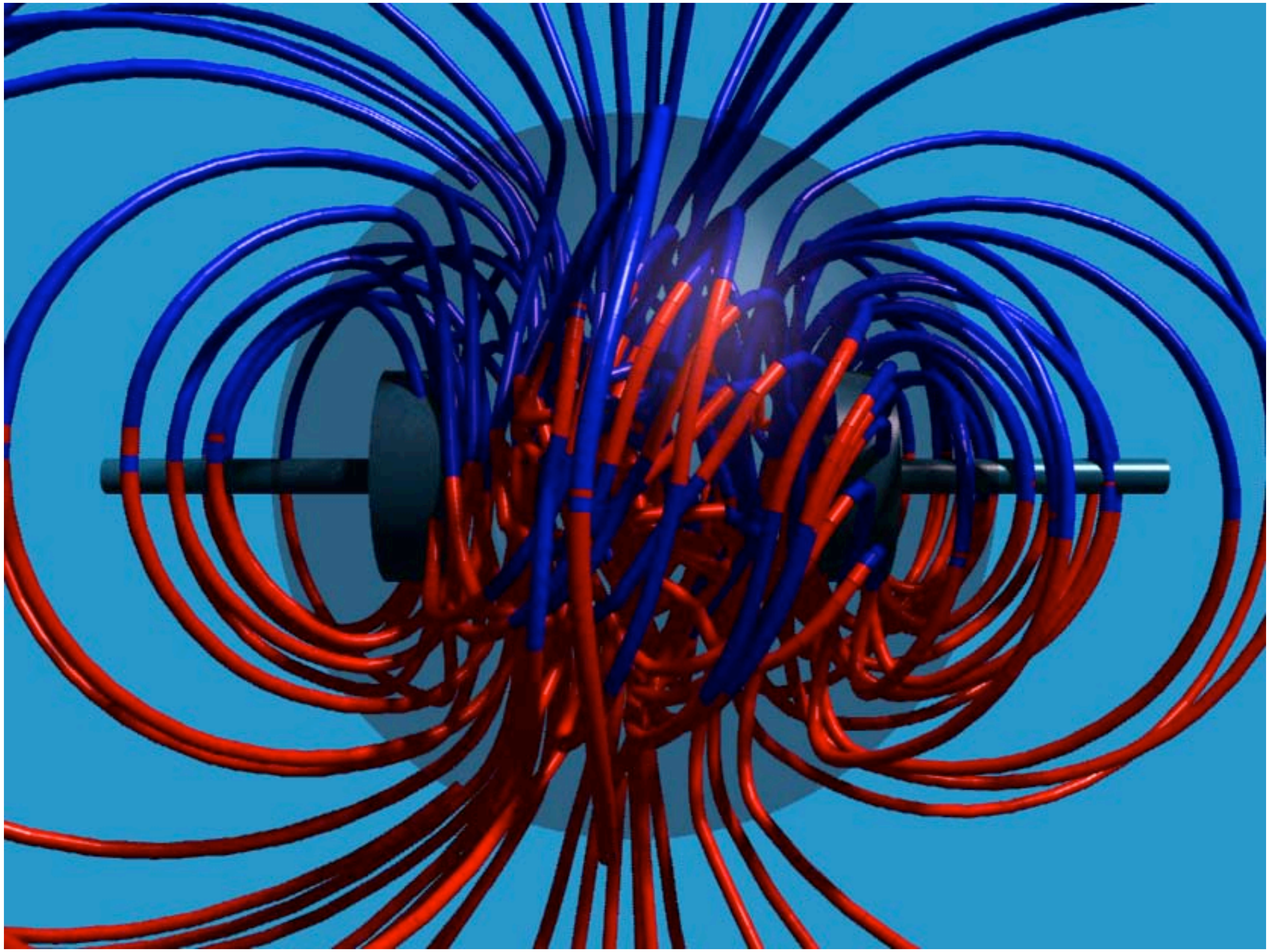


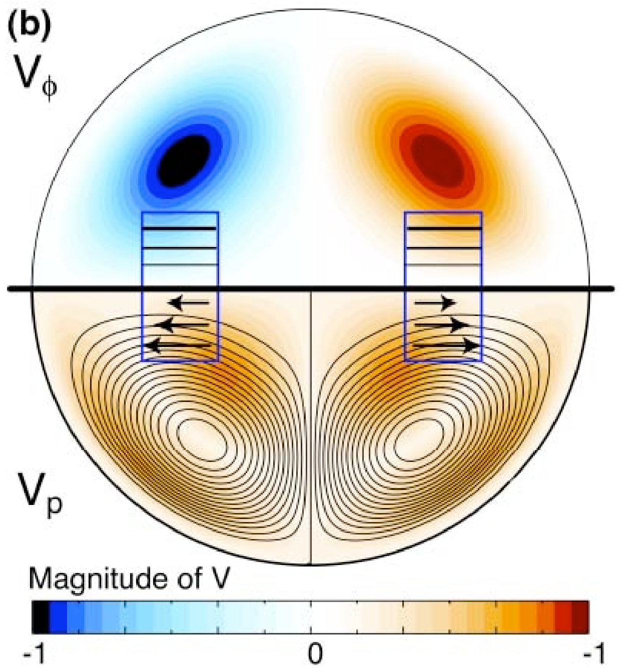
The saturated magnetic eigenmode (from a full 3D, non-linear MHD computation) is an equatorial diplole



Bayliss, Nornberg, Terry and Forest, *Numerical simulations of current generation and dynamo excitation in a mechanically-forced, turbulent flow, Phys. Rev. E,* (2006)

For liquid metals, Re>>Rm

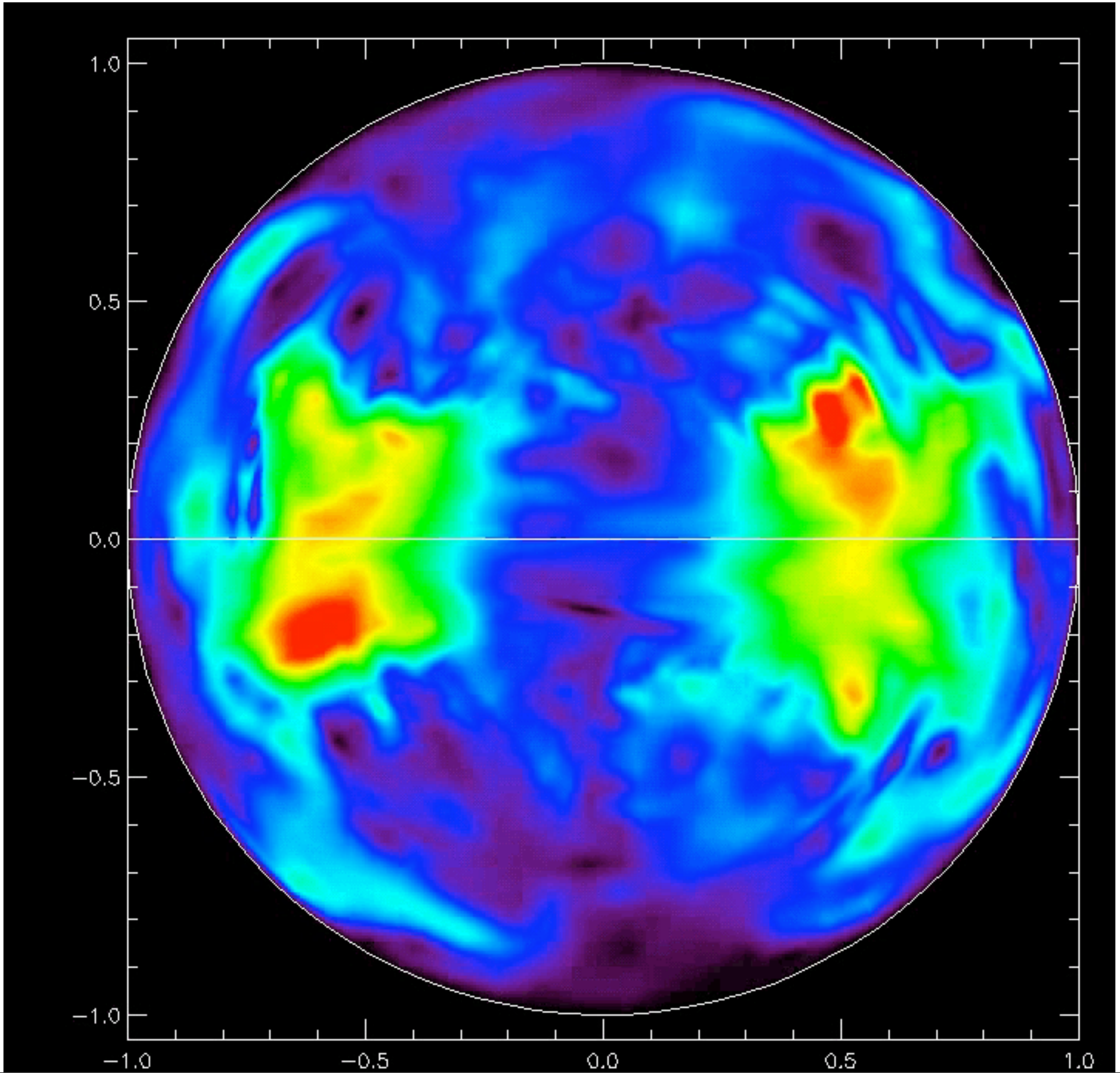
- Direct Numerical Simulations of MHD equations with mechanical forcing
- Re=2200; turbulence for Re>450



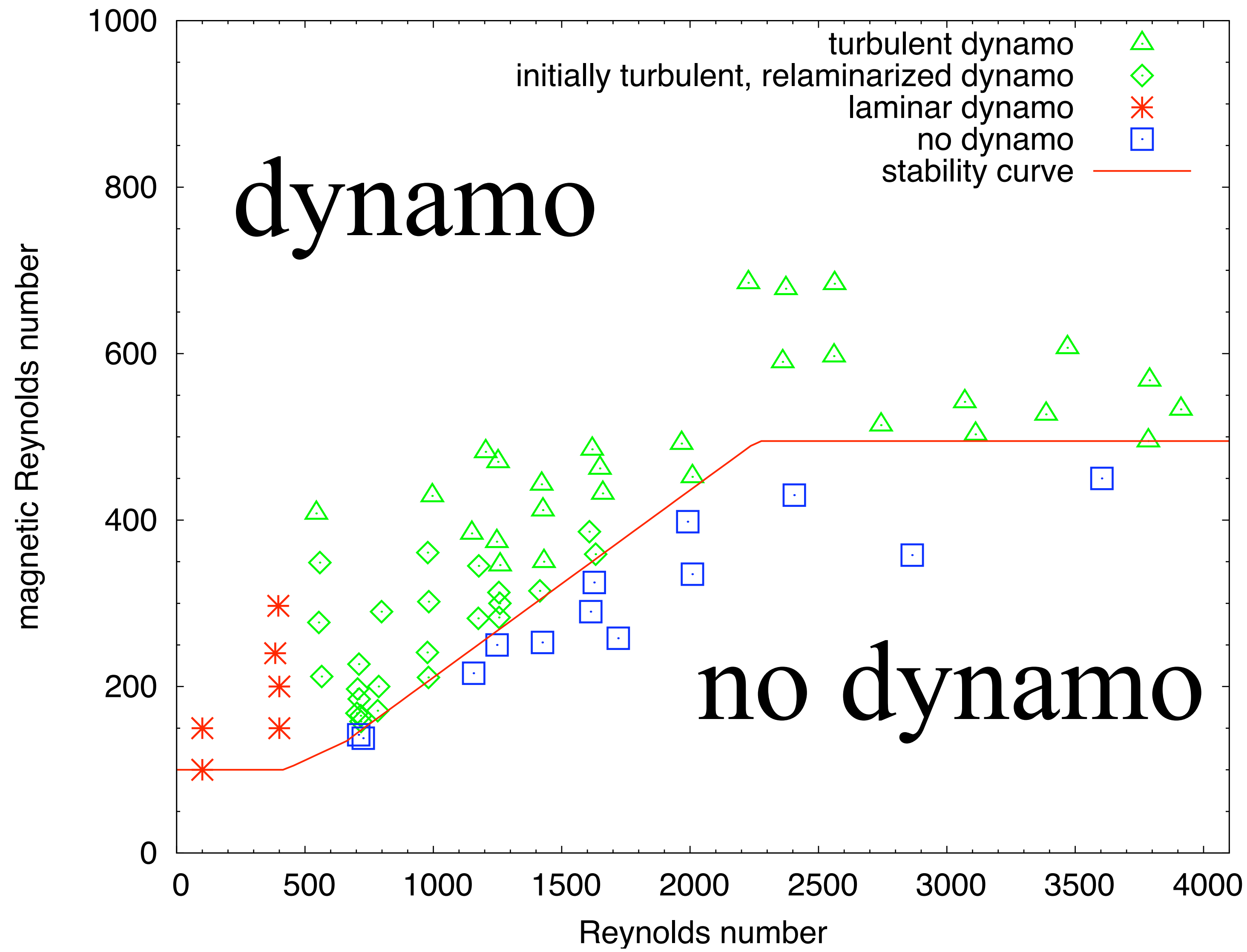
$$F_\phi(\rho, z) = \rho^2 \sin(\pi \rho b)$$

$$F_z(\rho, z) = -\epsilon \sin(\pi \rho c)$$

$$0.25a < |z| < 0.55a,\ \rho < 0.3a$$



Turbulence, in the two-vortex dynamo, increases Rm_{crit} by factor of 5



- Recent, fully resolved MHD simulations (no hyperviscosity, no LES) extended to Re~5000
- proper boundary conditions and mechanical forcing term

In a homogenous isotropic turbulence, flow fluctuations should reduce the conductivity

- Mean field theory predicts:

$$(1 + \mu_0 \sigma \beta)\mathbf{J} = \sigma \left(\mathbf{E} + \mathbf{V} \times \mathbf{B} + \alpha \mathbf{B}\right) \ \text{with } \beta = \frac{1}{3}\tilde{v}\tau_{corr}$$

- Define a factor that can be determined empirically

$$\tilde{v} = C(Pm, L, \ell_v) \langle V \rangle$$

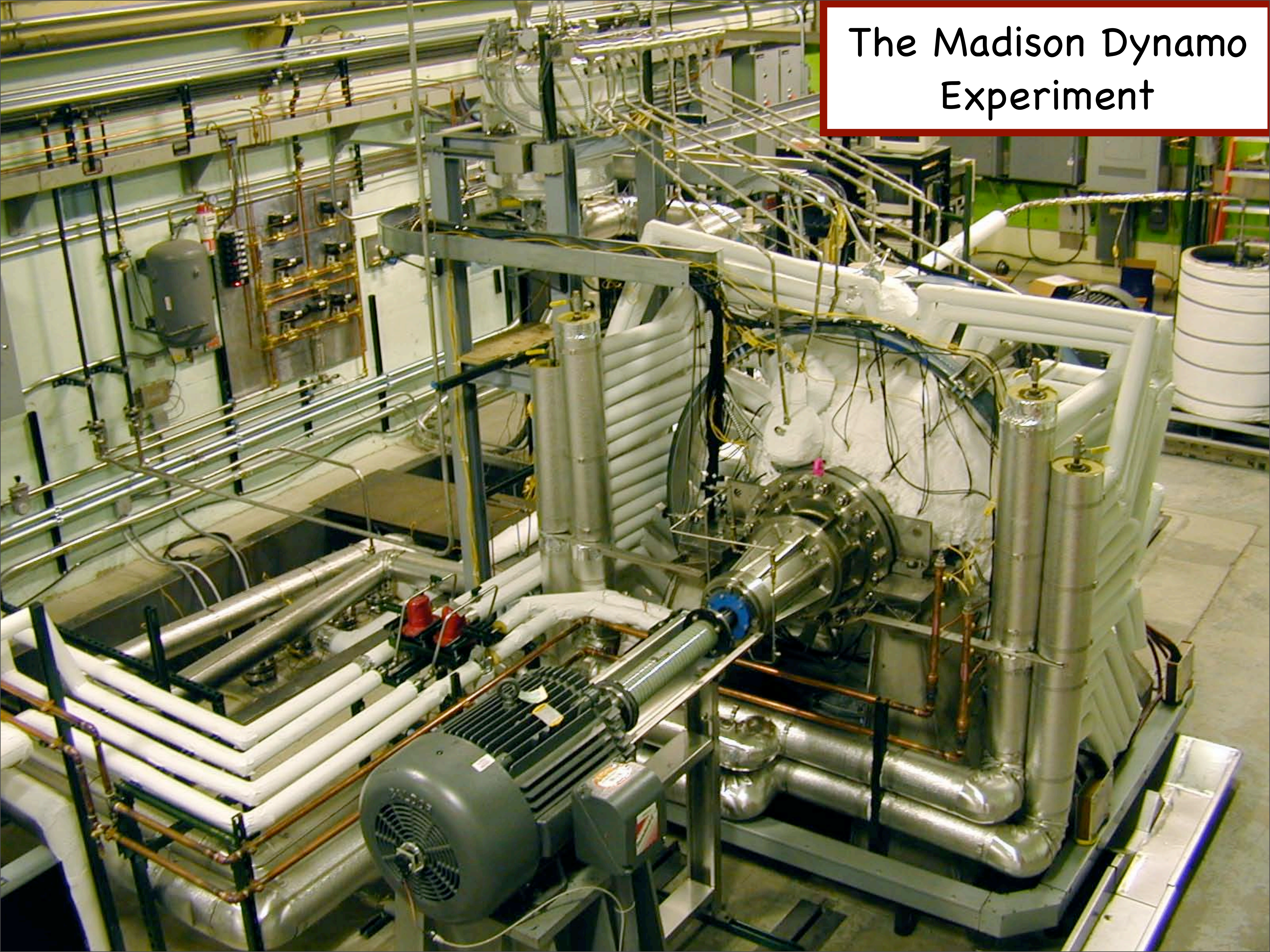
- Flow fluctuations should reduce effective conductivity:

$$\sigma_T = \frac{\sigma}{1 + C Rm \ell_v / 2L}$$

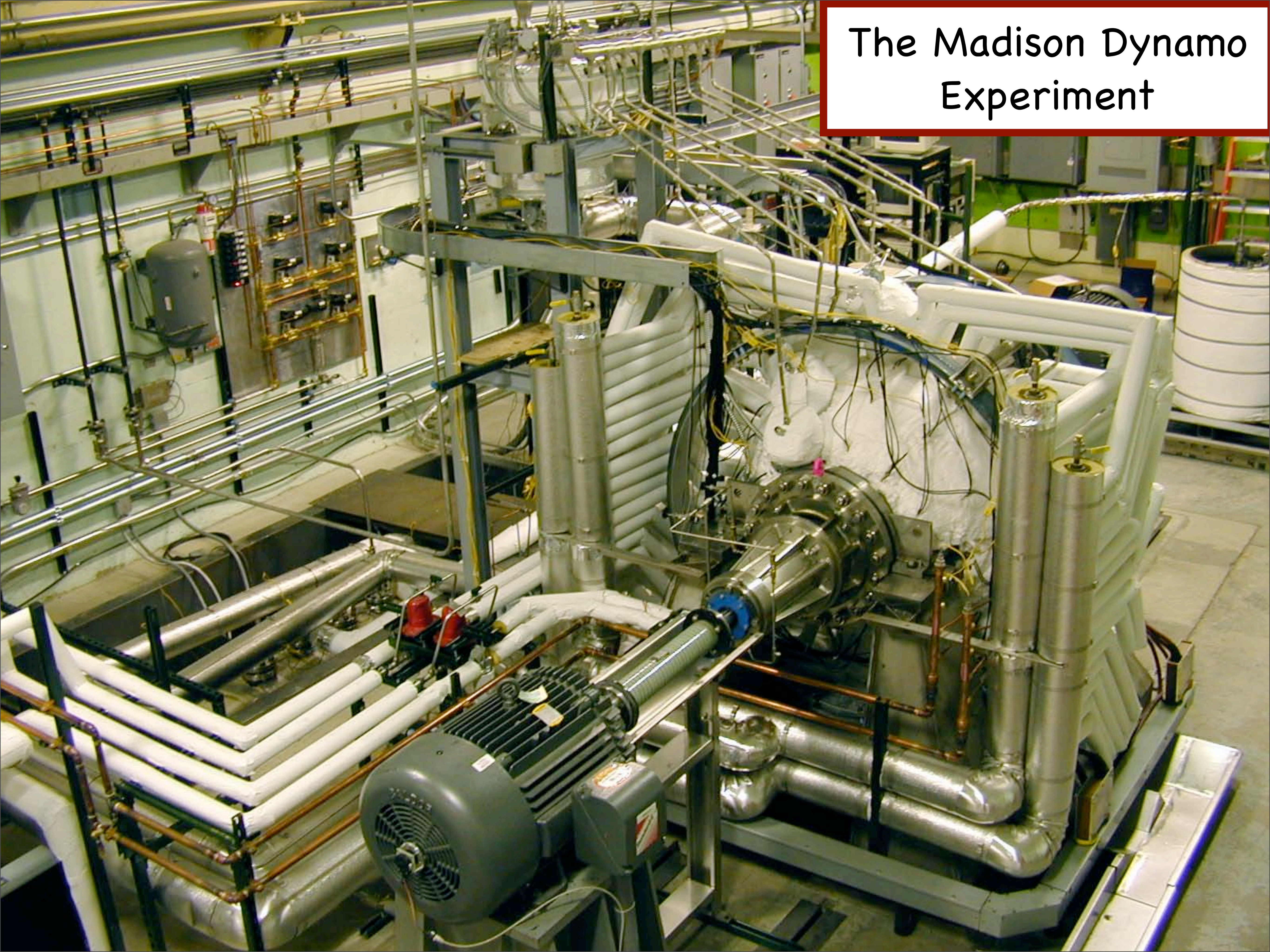
- Requiring $\mu_0 \sigma_T V_0 L > Rm_{crit}$ the condition for dynamo onset is:

$$Rm = \mu_0 \sigma V_0 L > \frac{Rm_{crit}}{1 - C \ell_v Rm_{crit} / 3L}$$

The Madison Dynamo Experiment



The Madison Dynamo Experiment

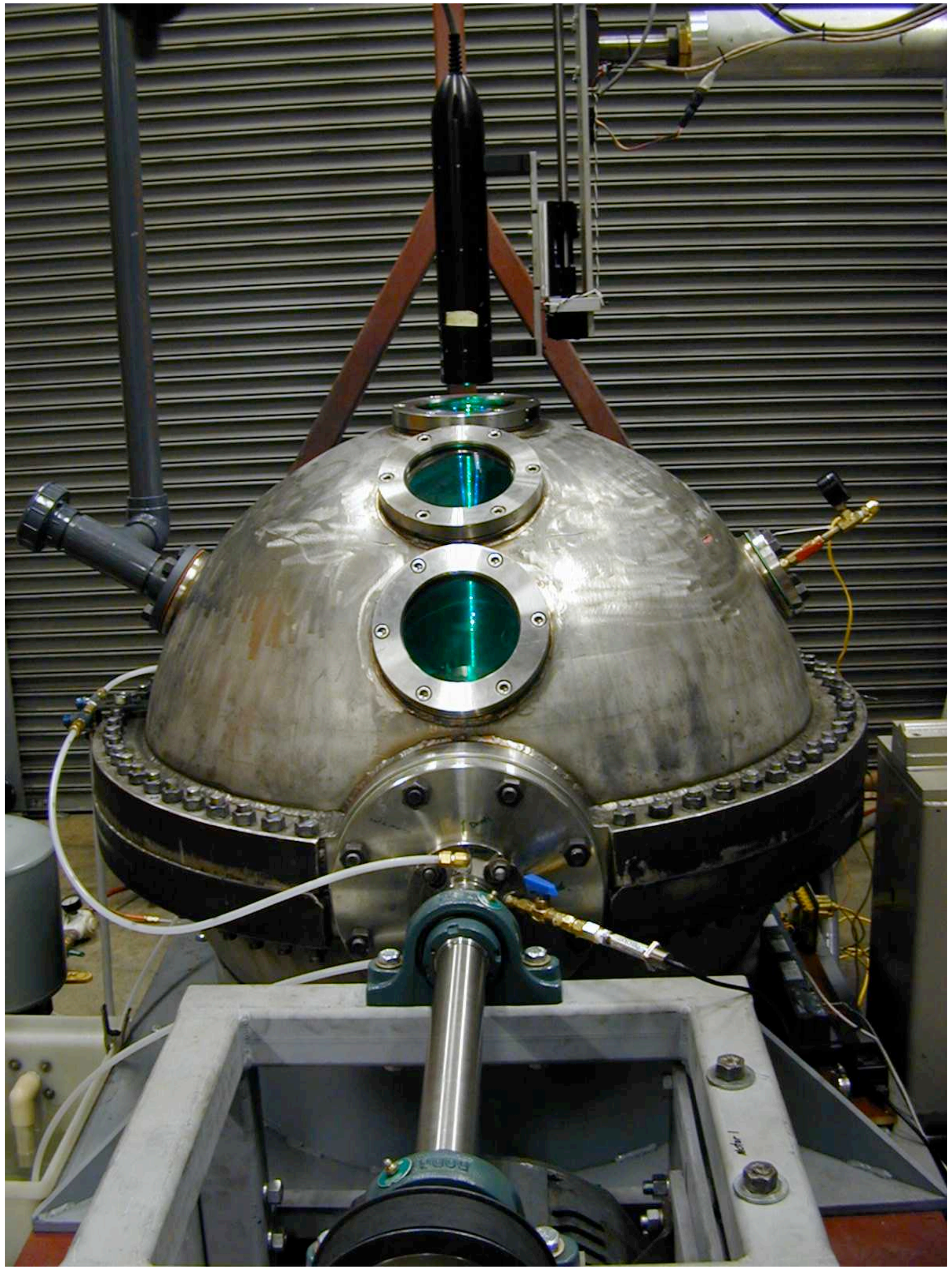


Dimensionally identical water experiment was used to demonstrate feasibility

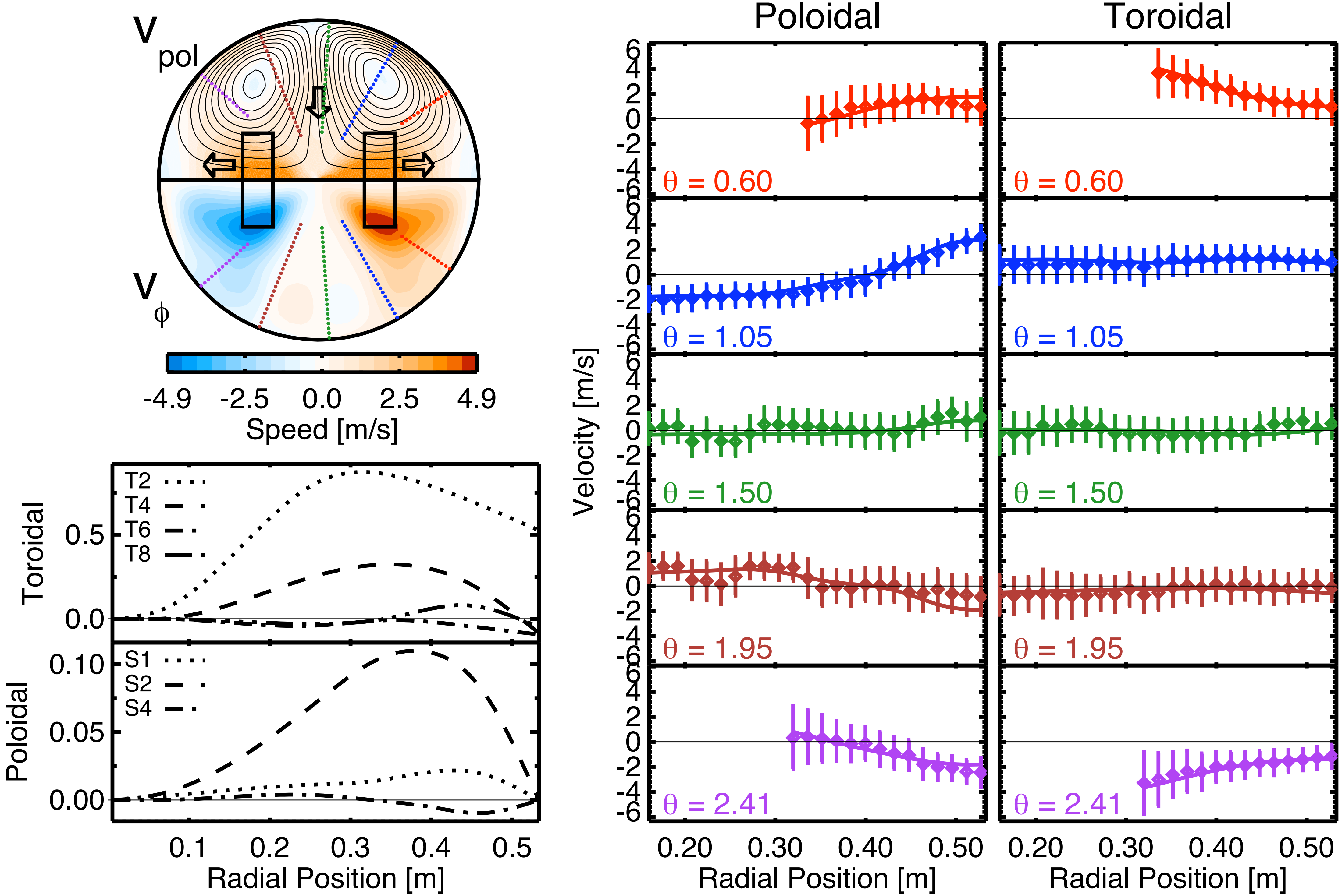
- Laser Doppler velocimetry is used to measure vector velocity field
- Measured flows are used as input to MHD calculation
- Full scale, half power

	Sodium	Water
Temperature	$110^\circ C$	$50^\circ C$
viscosity	$0.65\times10^{-6}\ \mathrm{m}^2\,\mathrm{sec}^{-1}$	$0.65\times10^{-6}\ \mathrm{m}^2\,\mathrm{sec}^{-1}$
mass density	$0.925\ \mathrm{gm\ cm}^{-3}$	$0.988\ \mathrm{gm\ cm}^{-3}$
resistivity	$10^{-7}\ \Omega\mathrm{m}$	

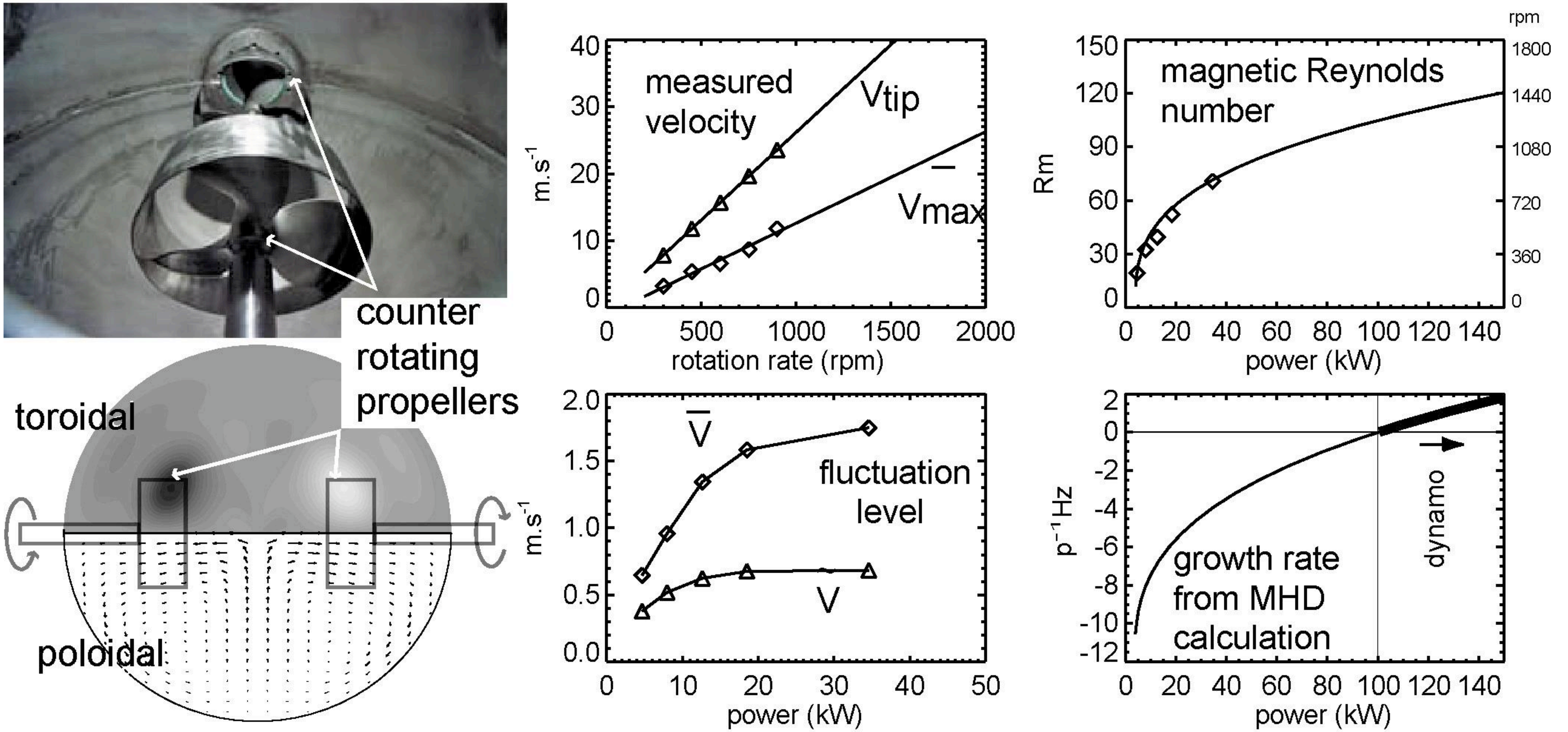
$$\longrightarrow \quad Rm = \frac{\mu_0 a V}{\eta} = 4\pi\, a(m)\, V(m/s)$$



LDV measurements provide data for a reconstruction of the mean velocity field



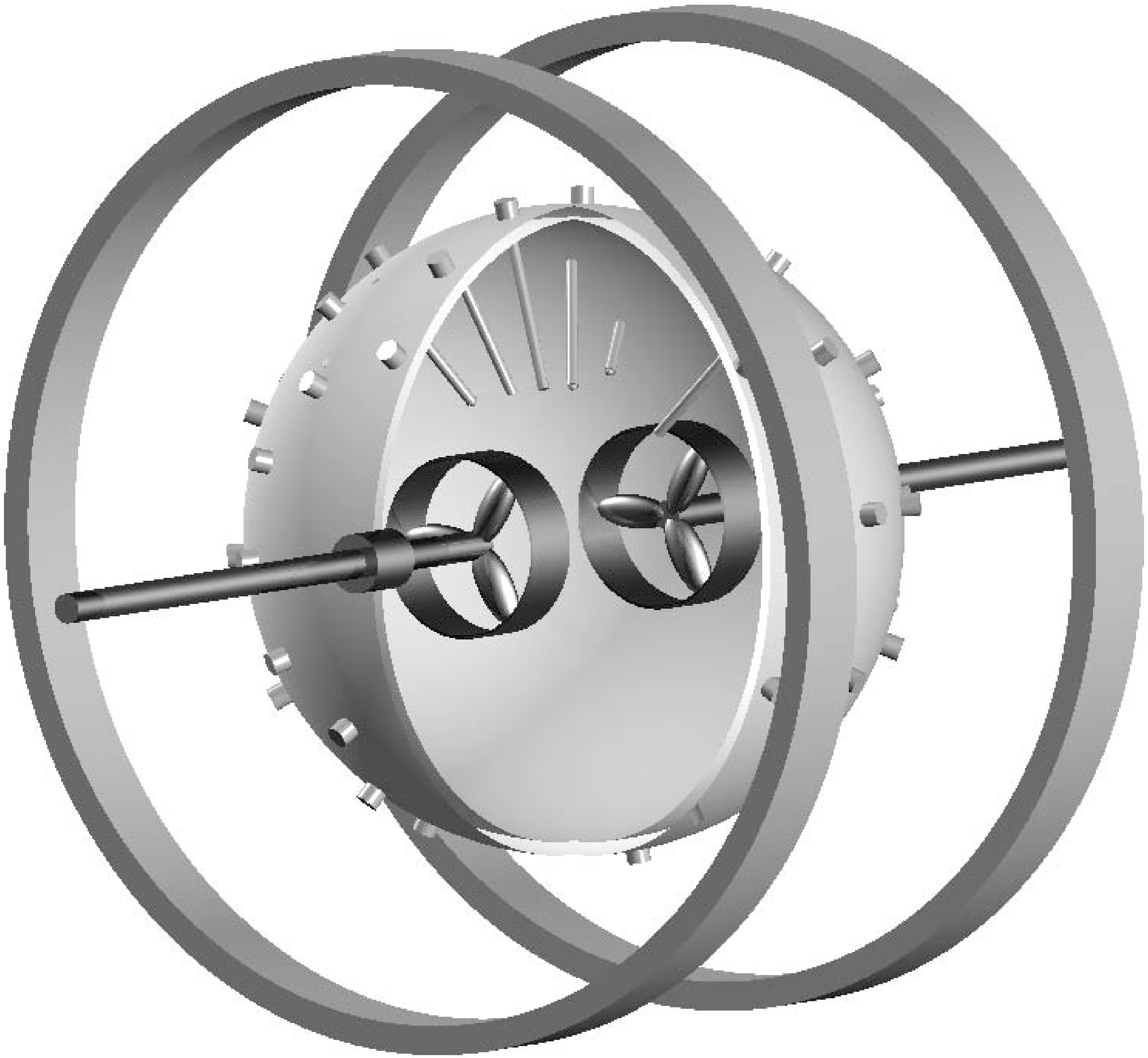
Velocity fields can be generated in water which lead to dynamo action



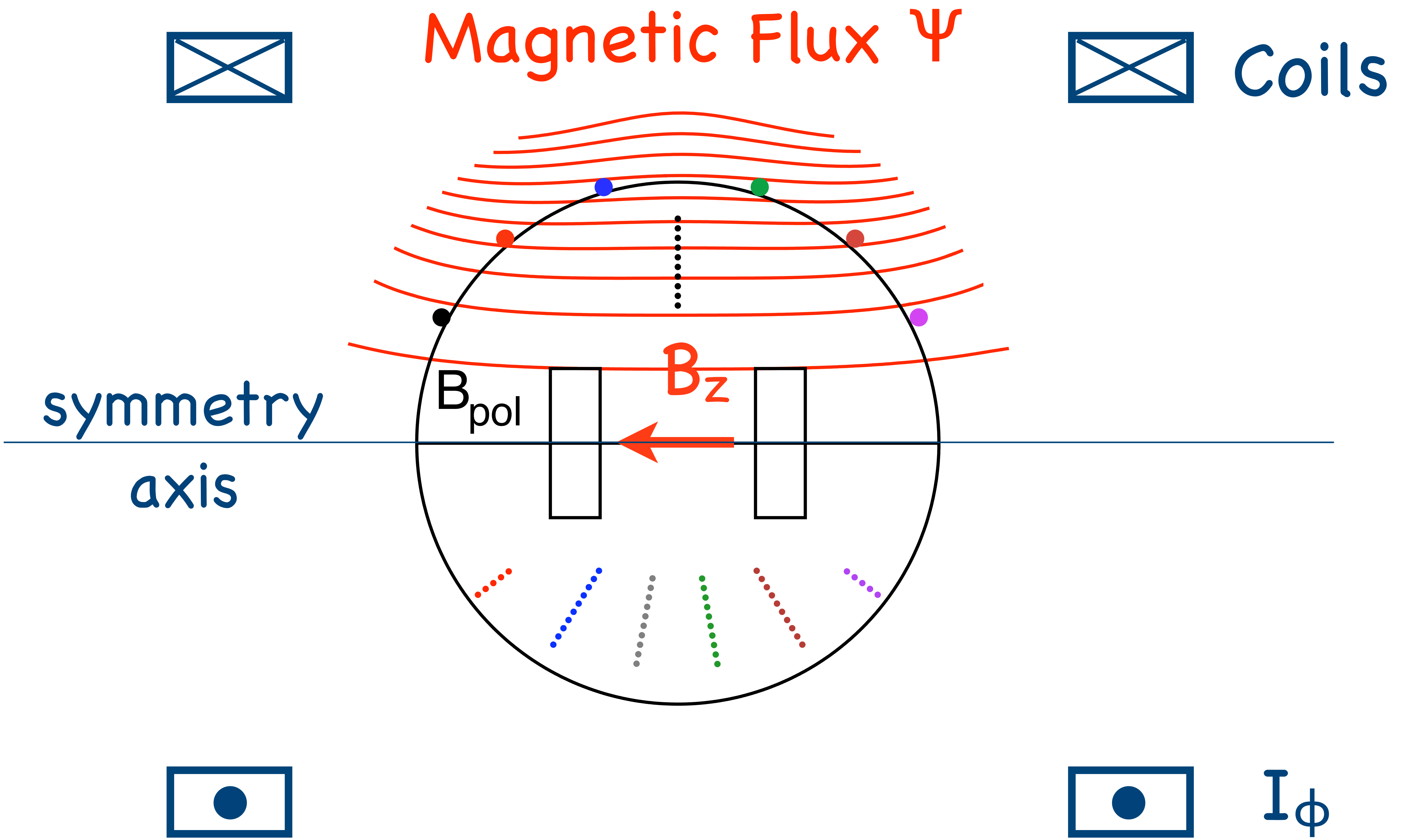
$a=0.5$ m, $\sigma=10^7$ mhos

Magnetic field is measured both internally and externally; external magnetic fields can be applied to probe experiment

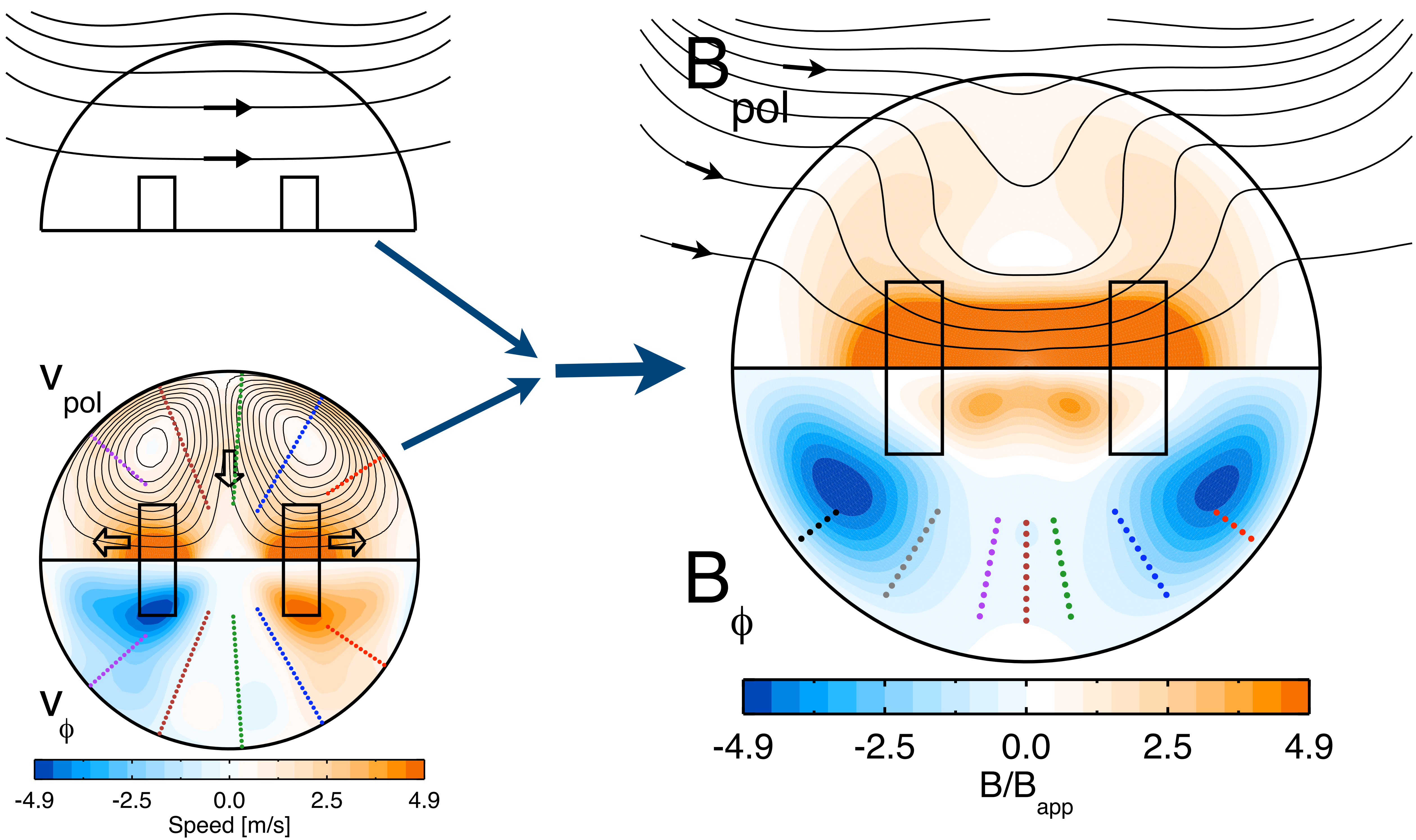
- $B_z \leq 100$ gauss
- Measure
  - surface probes
    - $B_r(a,\theta,\phi)$
    - Y_{lm} for $l \leq 6$, $|m| \leq 4$
  - Internal Probes
    - $B_\phi(r,\theta_p)$, 6 arrays
    - $B_z(r,\theta=\pi/2)$



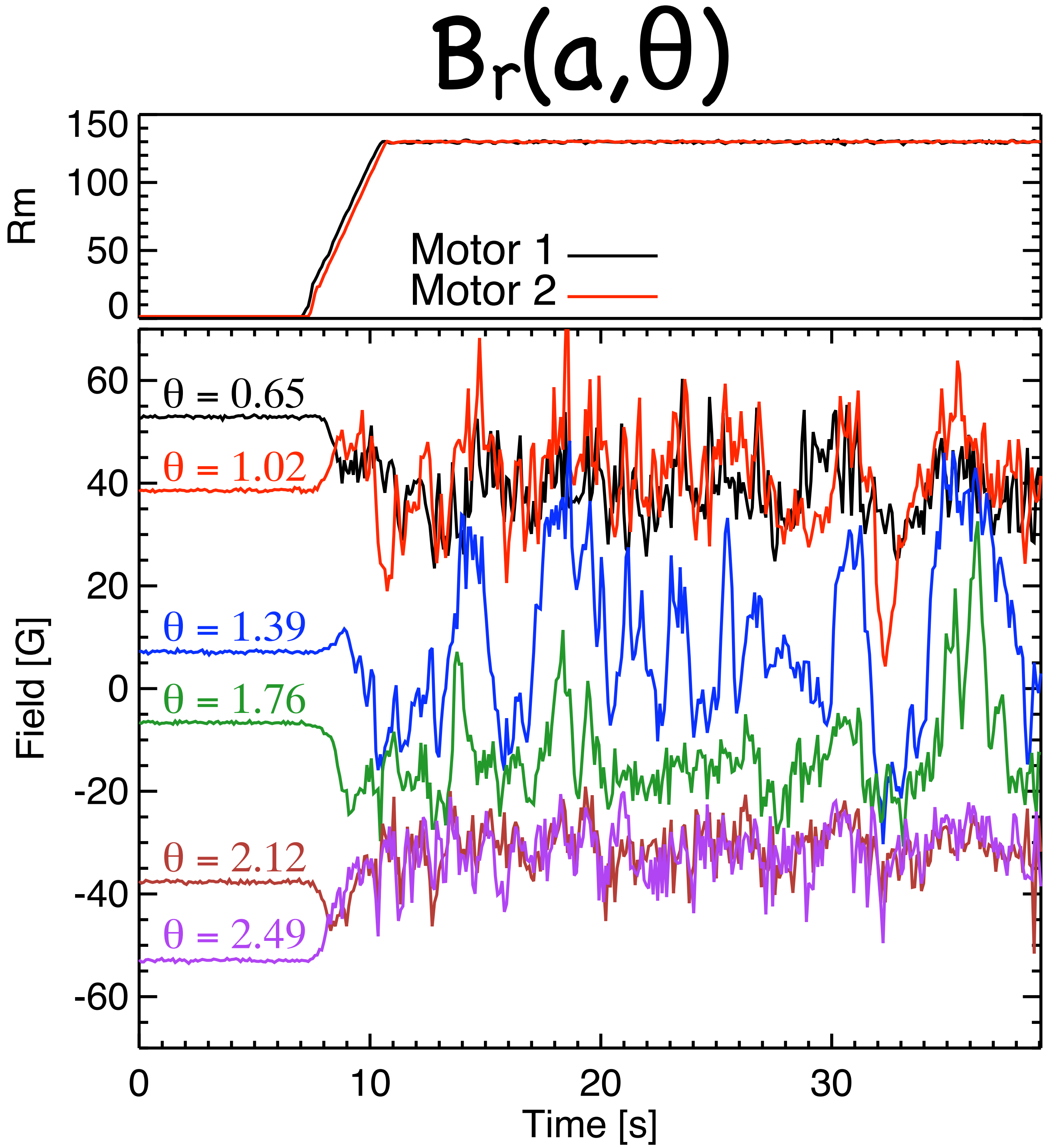
Experiment: apply axisymmetric poloidal seed field to sphere and measure induced magnetic fields

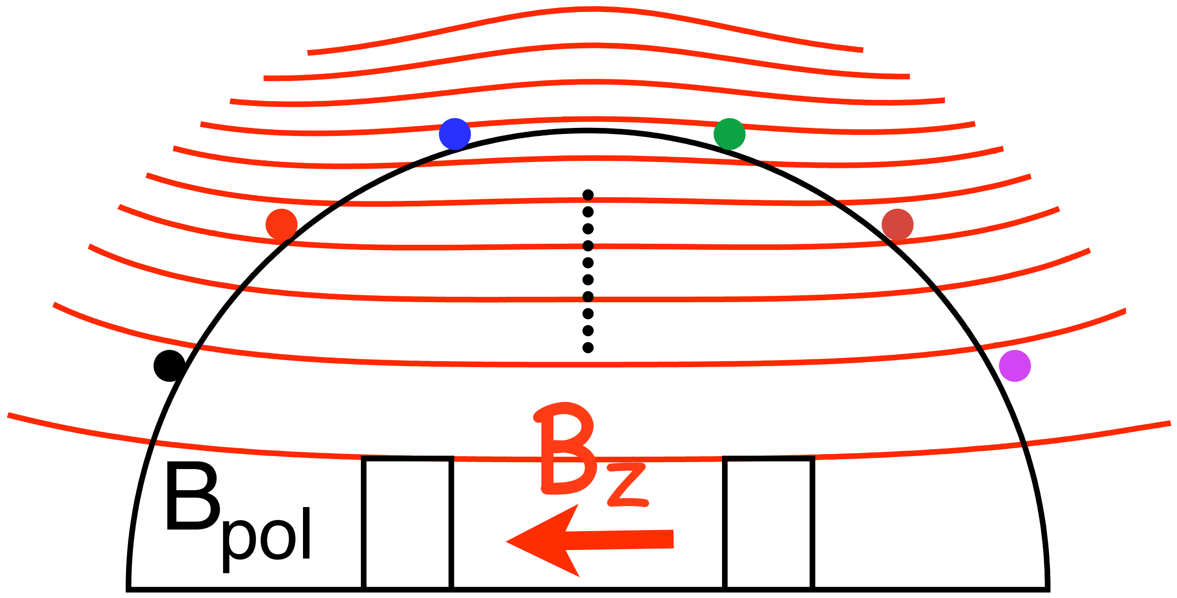


Predicted total magnetic fields

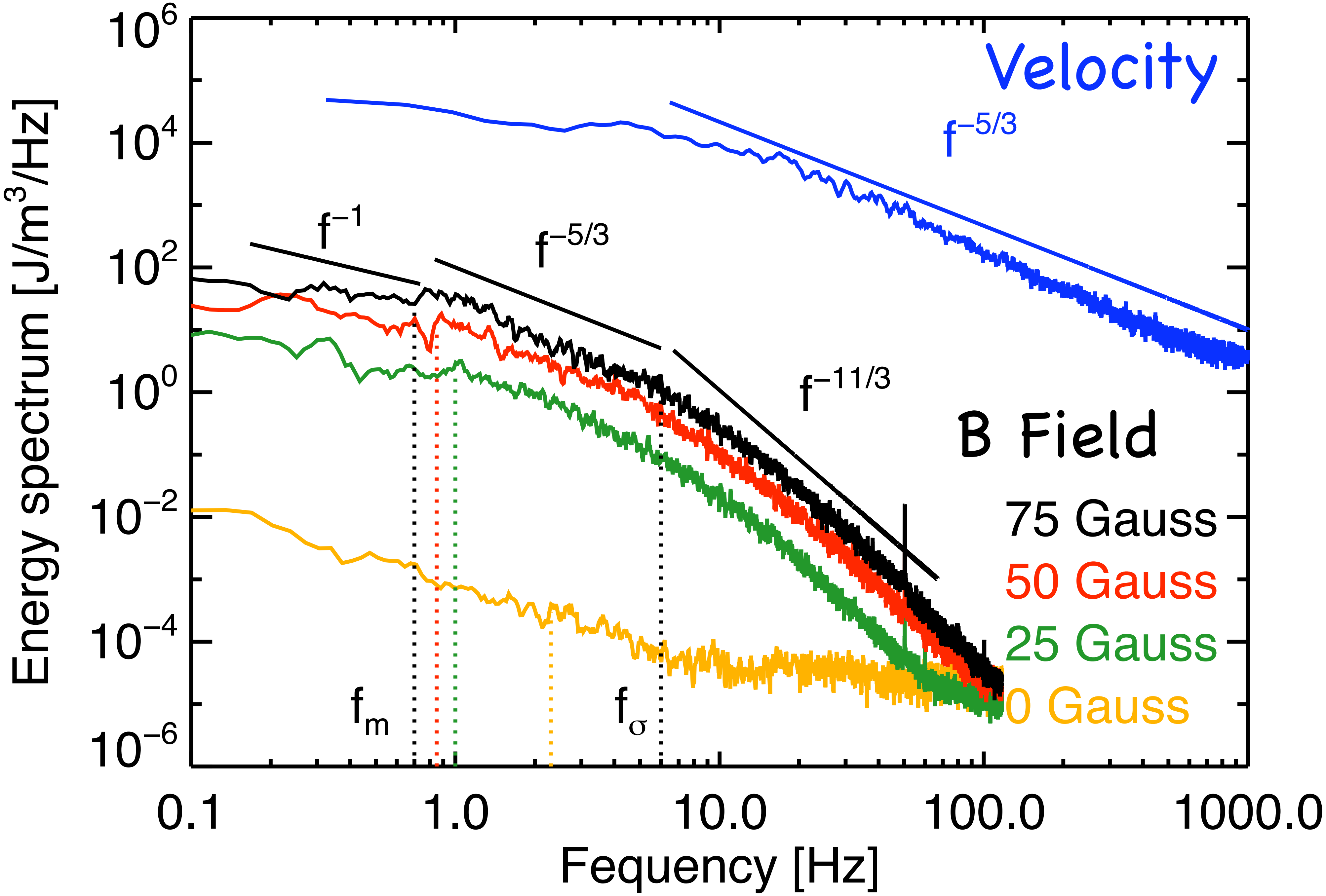


Large scale (mean) and small scale (turbulent) magnetic fields are generated by liquid sodium flows





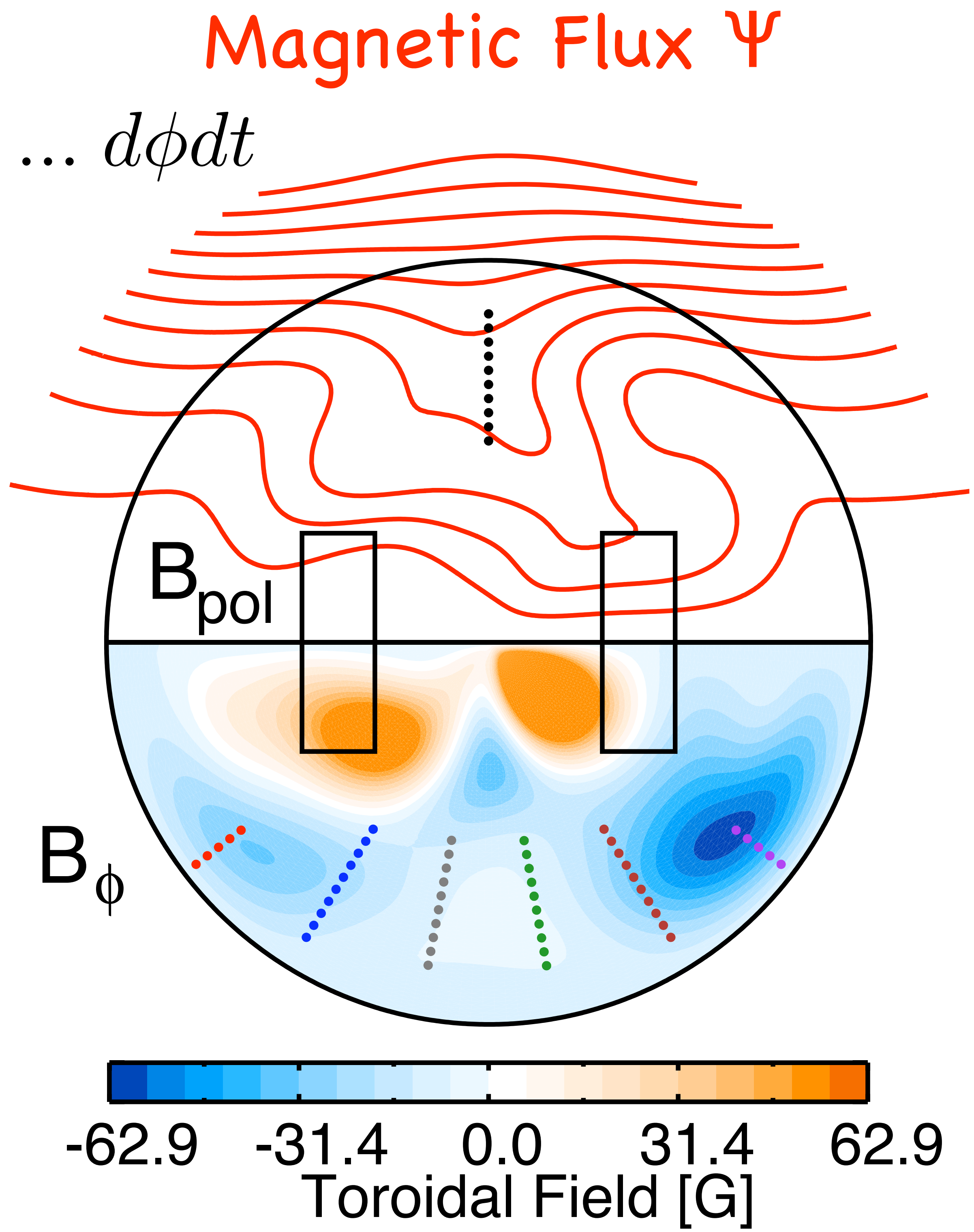
Spectra are turbulent: the turublent magnetic energy is much smaller than the kinetic energy



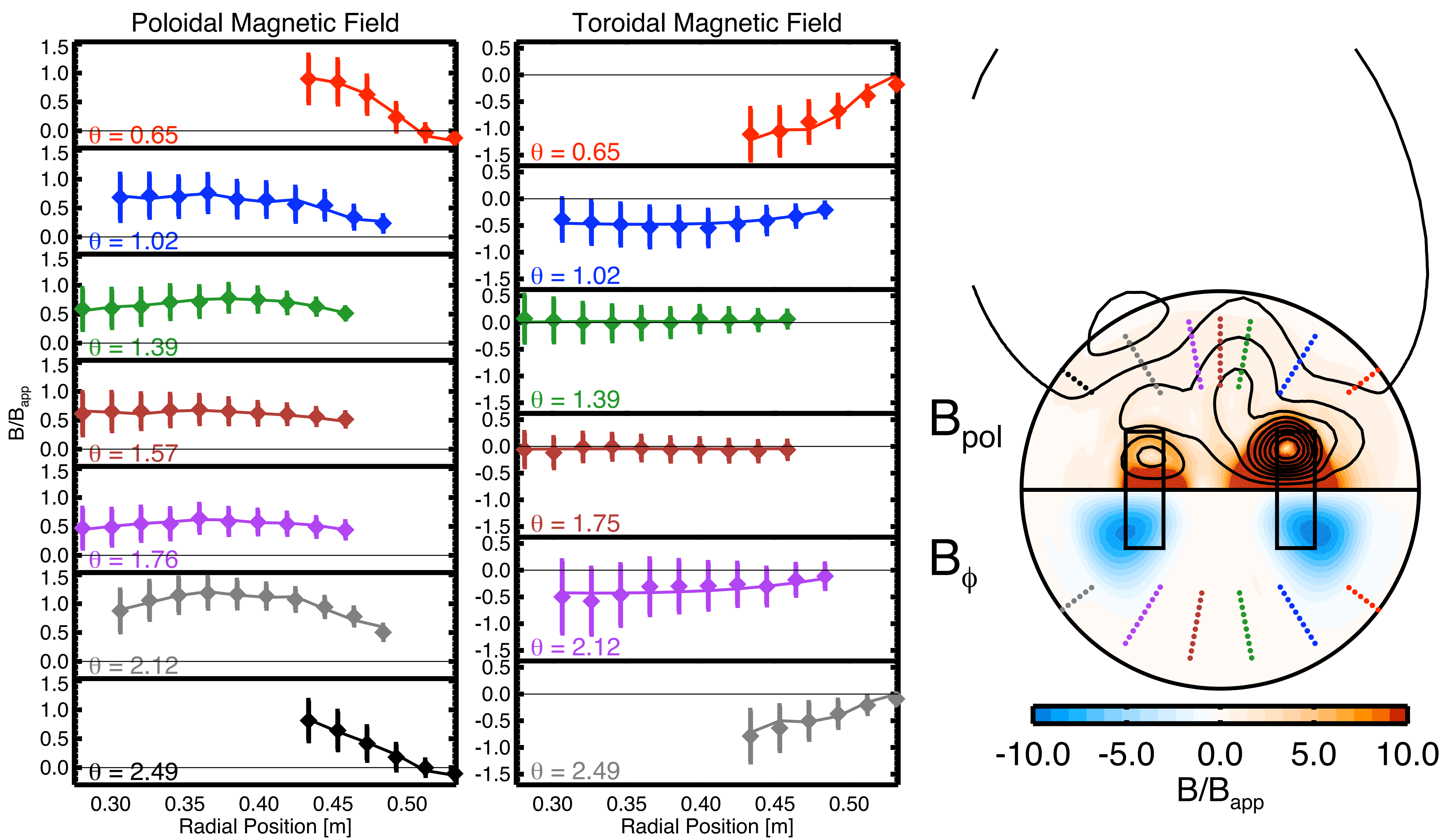
Nornberg, Spence, Bayliss, Kendrick, and Forest, *Measurements of the magnetic field induced by a turbulent flow of liquid metal*, Phys. Plasmas **13** 055901 (2006).

The time-averaged, axisymmetric part of the magnetic field shows poloidal flux expulsion and a strong Ω effect

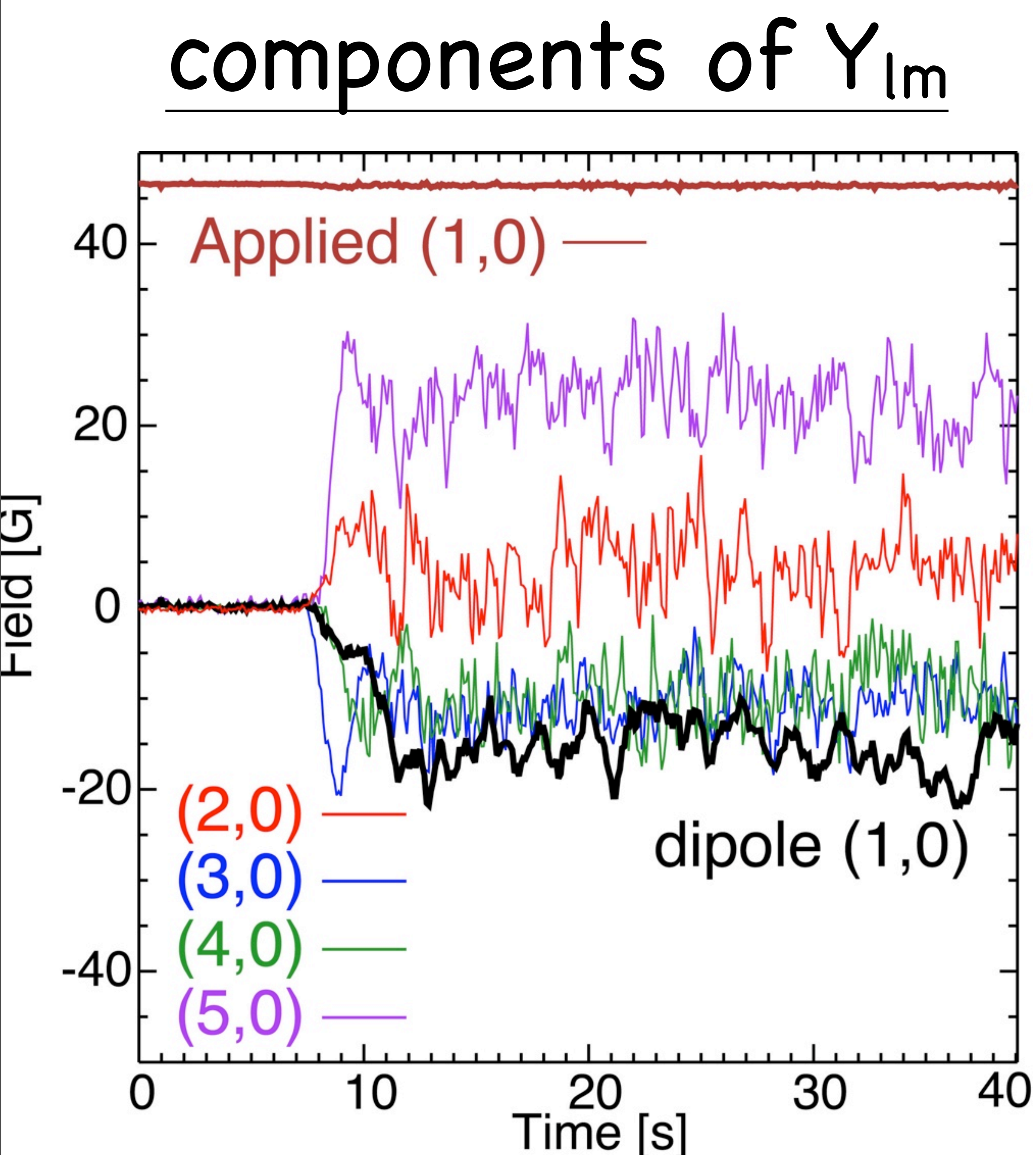
$$\langle \ldots \rangle \equiv \frac{1}{2\pi T}\int^{T}\int_{0}^{2\pi} \ldots \, d\phi dt$$

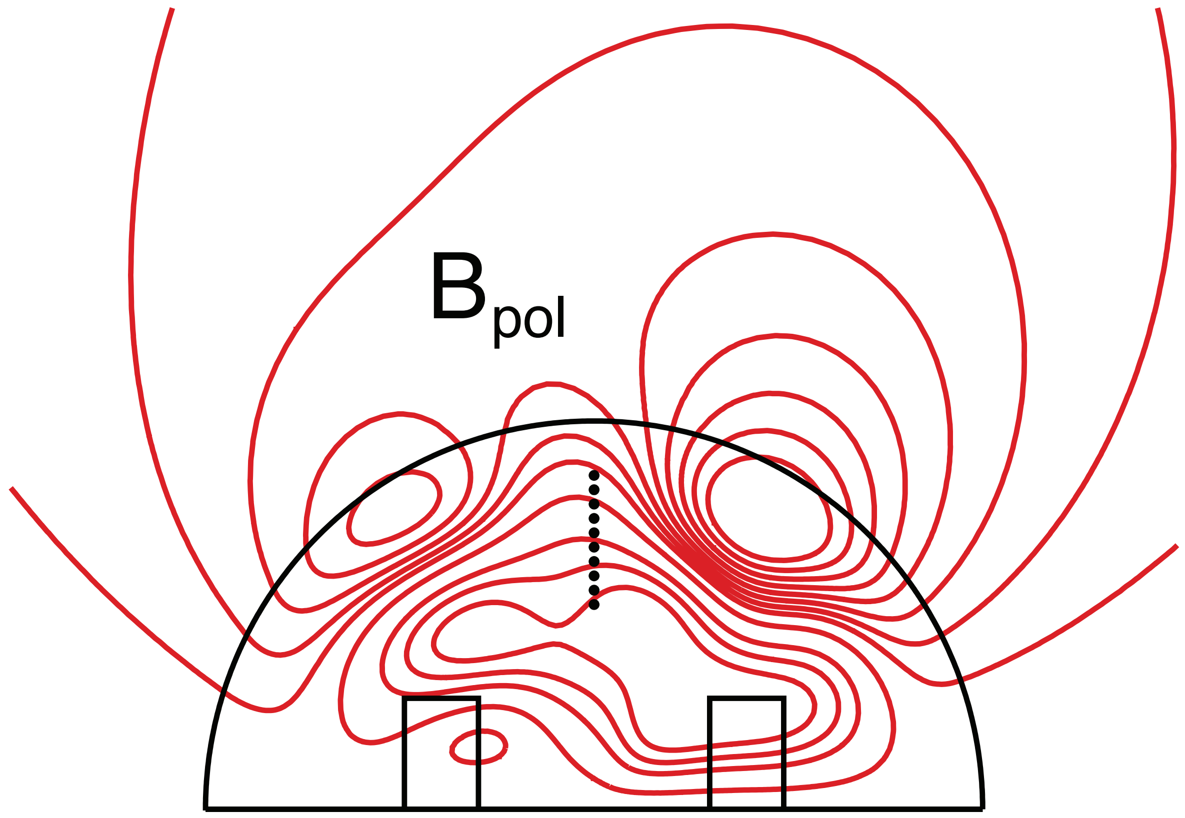


Magnetic field is reconstructed from magnetic field measurements at discrete positions



The mean induced magnetic field has a dipole moment





Impossible to reconstruct with axisymmetric flows!

Spence, Nornberg, Jacobson, Kendrick, and Forest, *Observation of a turbulence-induced large-scale magnetic field*, Phys. Rev. Lett. **96** 055002 (2006).

Theorem: For a stationary, axisymmetric flow and magnetic field, no dipole moment can exist for the current distribution inside the experiment (even with externally applied fields)

Use cylindrical coordinates (s, Z, ϕ) and stream functions for velocity and magnetic fields:

$$\vec{v} = \nabla\Phi \times \nabla\phi + v_\phi \hat{\phi} \quad (1)$$

$$\vec{B} = \nabla\Psi \times \nabla\phi + B_\phi \hat{\phi} \quad (2)$$

The dipole moment $\mu_z = \int s J_\phi d^3x$ is generated by toroidal currents:

$$J_\phi = \sigma \vec{v} \times \vec{B} \cdot \hat{\phi} \quad (3)$$

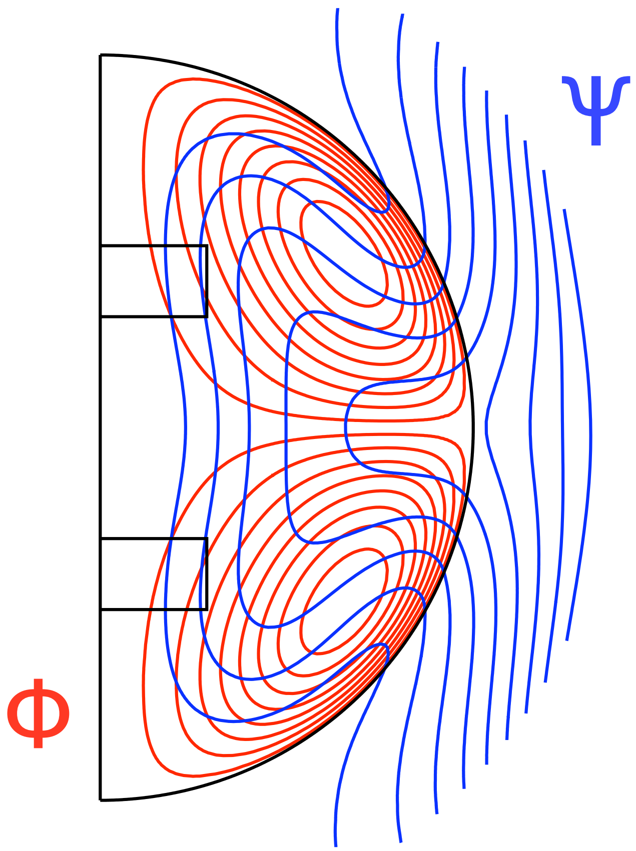
$$= \sigma \frac{|\nabla\Phi \times \nabla\Psi|}{s^2} \quad (4)$$

Switching to flux coordinates (Ψ, ℓ) where $d^3x = \frac{d\ell d\Psi}{B_p}$, the dipole becomes

$$\mu_z = \sigma \int \int |\nabla\Phi \times \nabla\Psi| \frac{d\ell d\Psi}{s B_p} \quad (5)$$

$$= \sigma \int d\Psi \int \frac{\partial\Phi}{\partial\ell} d\ell \equiv 0 \quad (6)$$

Proof continued



Integrating Φ along open poloidal flux contours gives

$$\int_a^b \frac{\partial \Phi}{\partial \ell} d\ell = \Phi(b) - \Phi(a) = 0$$

since vessel boundary had $\Phi = const.$ Closed poloidal flux contours give

$$\oint \frac{\partial \Phi}{\partial \ell} d\ell \equiv 0$$

Therefore, $\mu_z = 0$ for axisymmetric flows. QED

- Conclusion: symmetry breaking fluctuations must be responsible for observed dipole
  - consistent with an α-effect and the self-generated toroidal field: $J_\varphi = \sigma \alpha B_\varphi$

Question: Does a simple Ohm's law make sense?

$$\langle \boldsymbol{J} \rangle = \sigma \left(\langle \boldsymbol{E} \rangle + \langle \boldsymbol{V} \rangle \times \langle \boldsymbol{B} \rangle + \left\langle \widetilde{\boldsymbol{v}} \times \widetilde{\boldsymbol{b}} \right\rangle \right)$$

$\langle \ldots \rangle \equiv \frac{1}{2\pi T} \int^T \int_0^{2\pi} \ldots \, d\phi dt$

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Measured in Sodium Experiment

$$\langle \boldsymbol{J} \rangle = \sigma \left(\langle \boldsymbol{E} \rangle + \langle \boldsymbol{V} \rangle \times \langle \boldsymbol{B} \rangle + \left\langle \widetilde{\boldsymbol{v}} \times \widetilde{\boldsymbol{b}} \right\rangle \right)$$

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Measured by LDV

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Measured by LDV

Fluctuation Driven Currents

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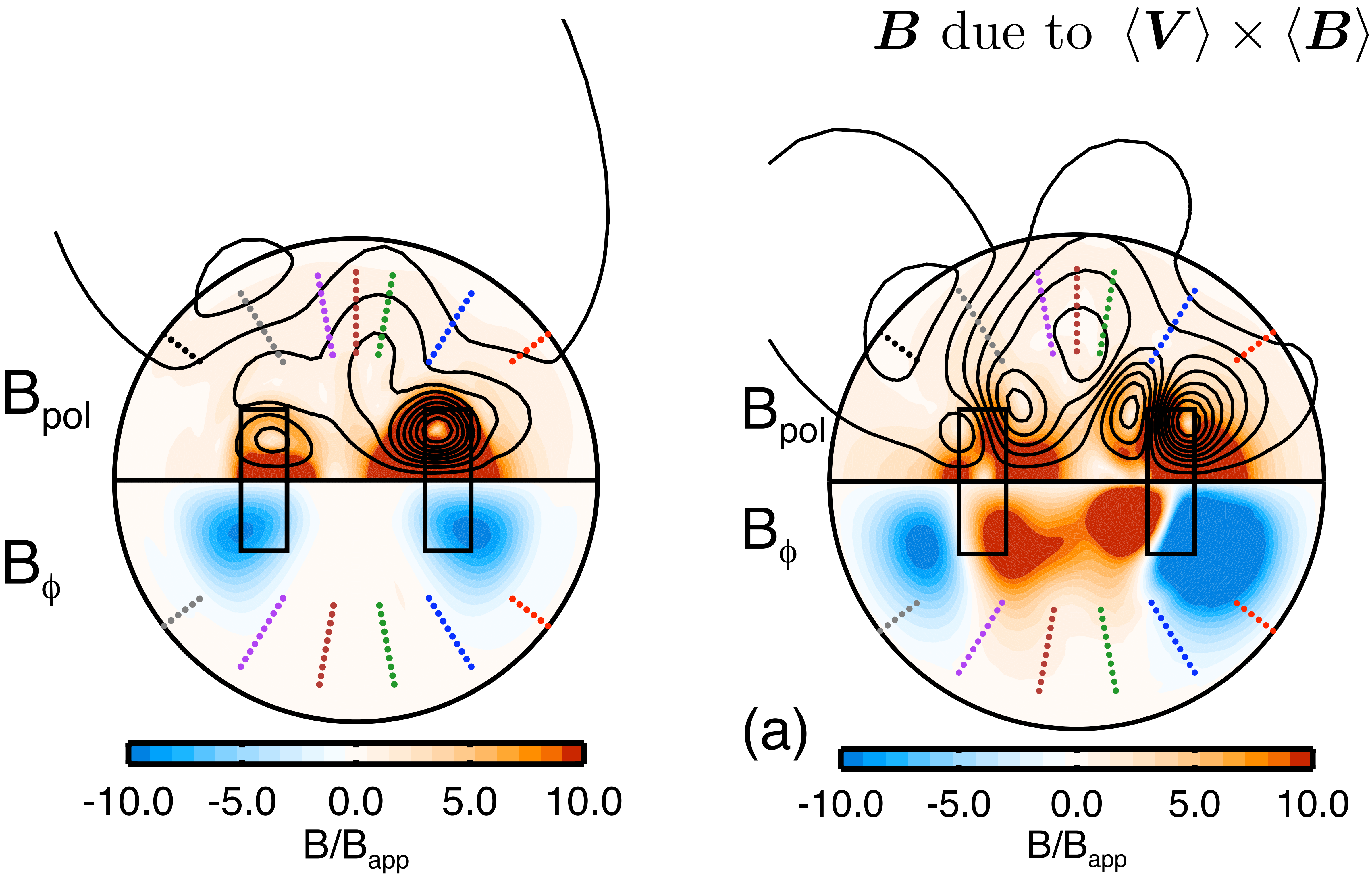
Measured by LDV

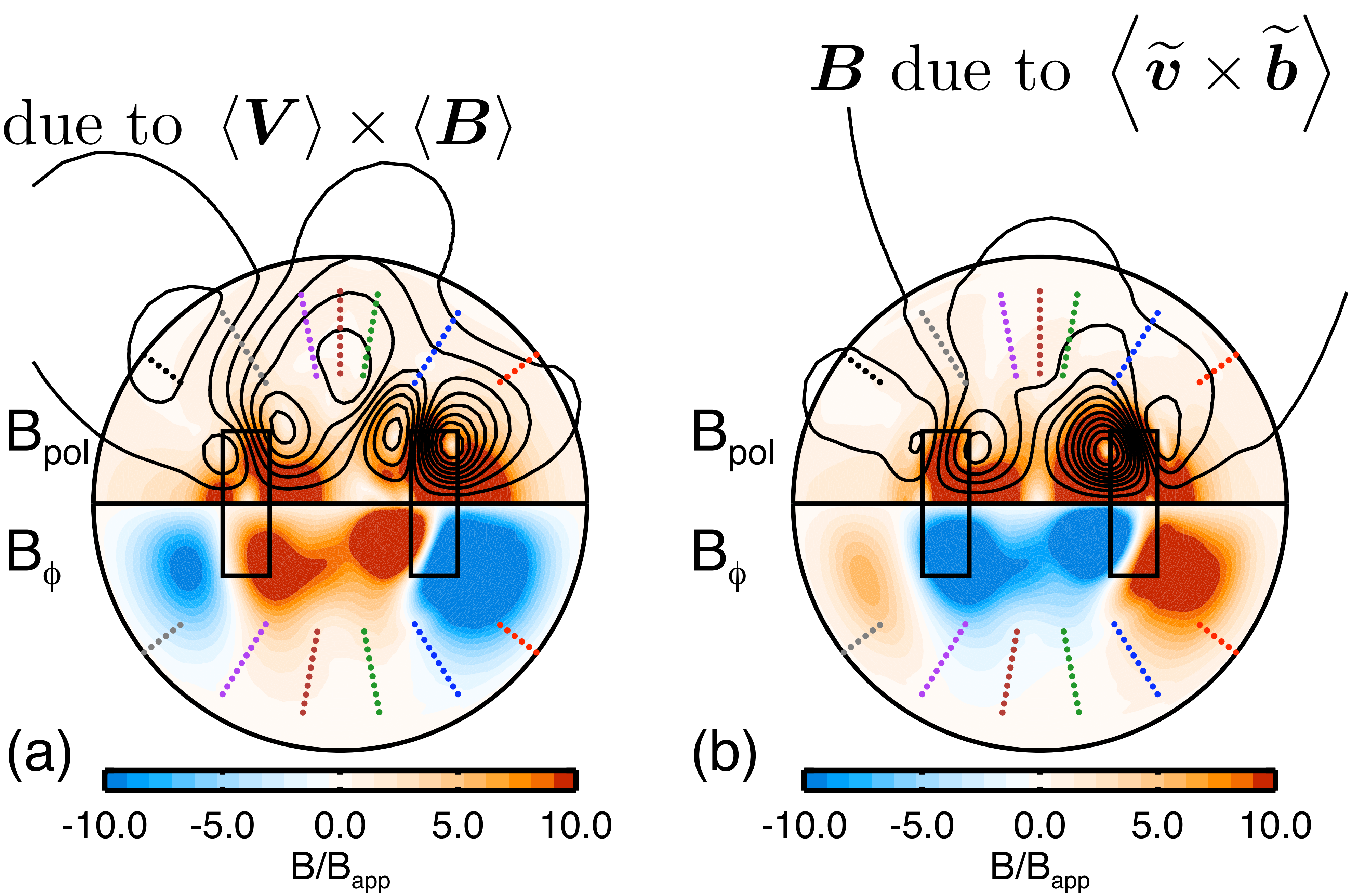
Fluctuation Driven Currents

$$\langle \ldots \rangle \equiv \frac{1}{2\pi T} \int^{T} \int_{0}^{2\pi} \ldots \, d\phi dt$$

<V>x<B> does not account for measured field: turbulence must be generating current

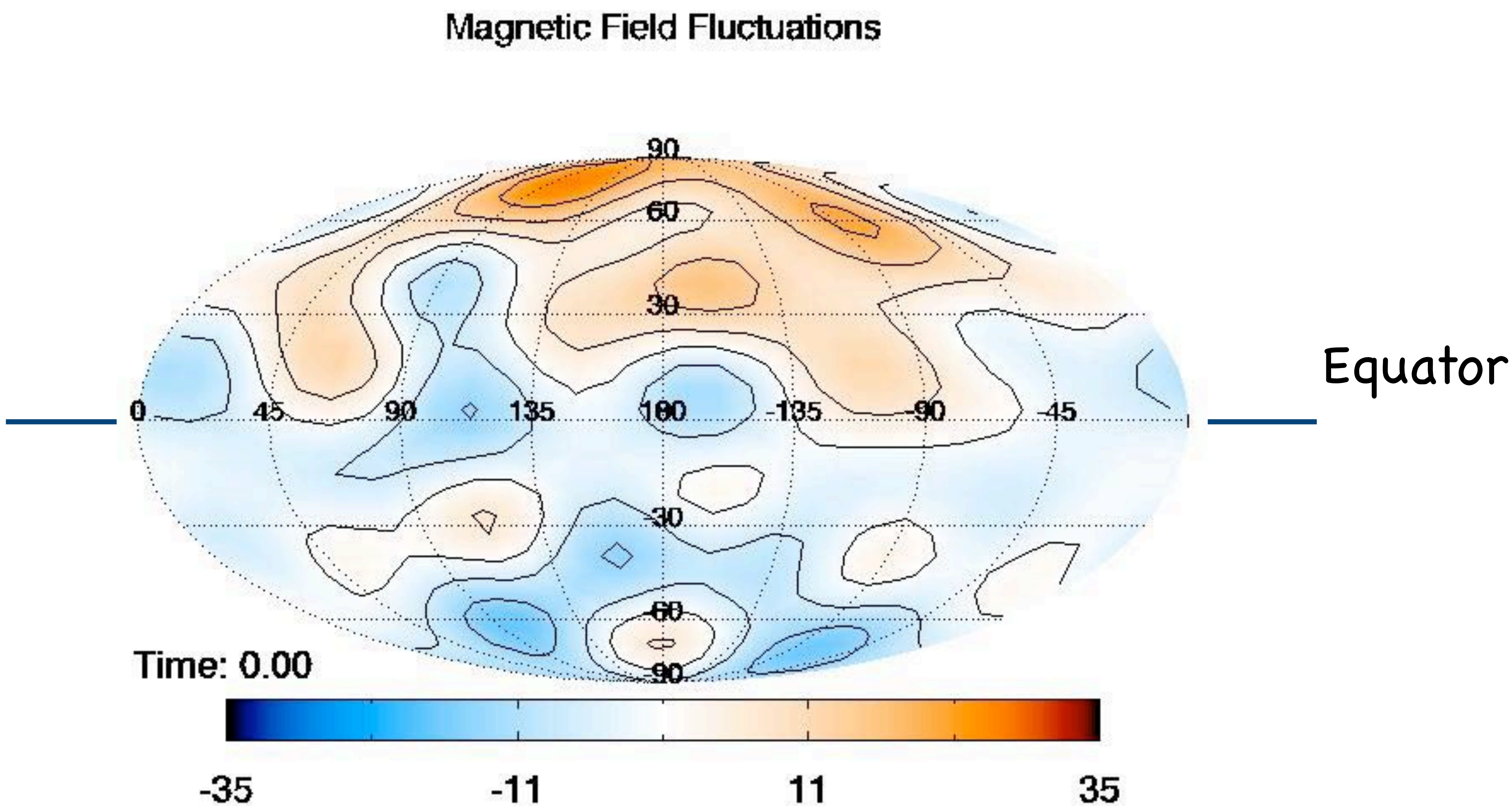


Field can be separated into mean-flow, mean-field driven currents and fluctuation generated currents



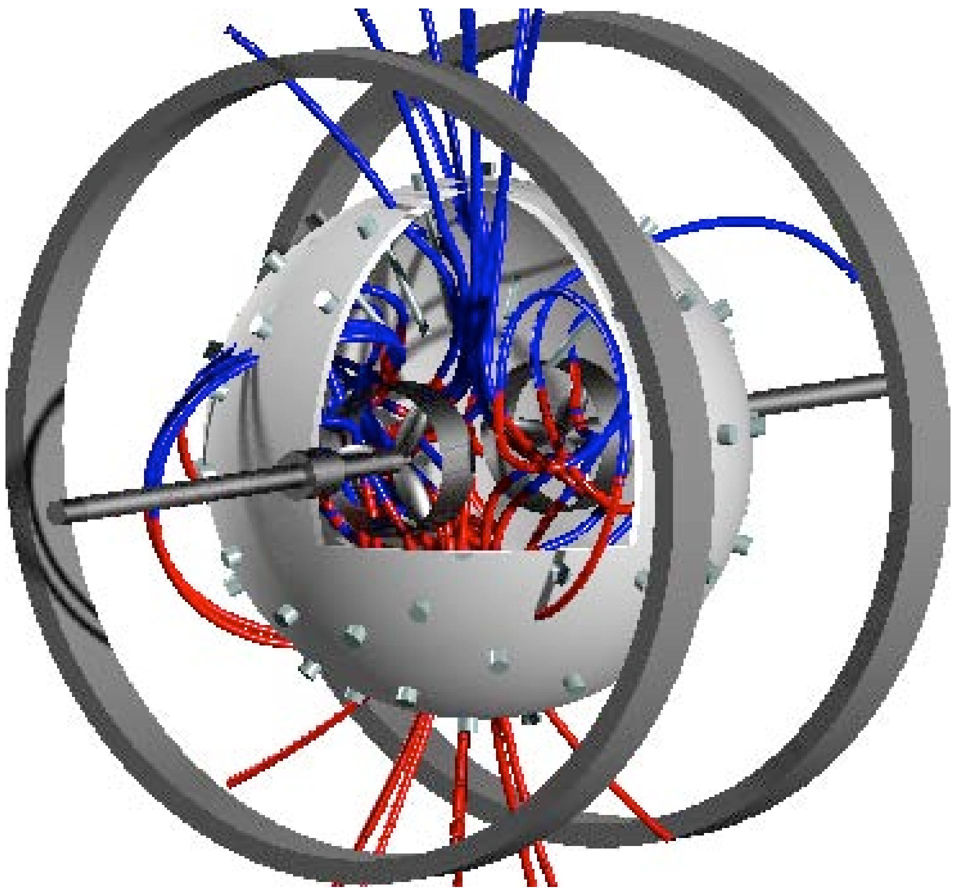
Spence, Nornberg, Jacobson, Parada, Kendrick, and Forest, *Turbulent Diamagnetism in Flowing Liquid Sodium*, Phys. Rev. Lett. **98** 164503 (2007).

Intermittent equatorial dipole is observed on surface of sphere

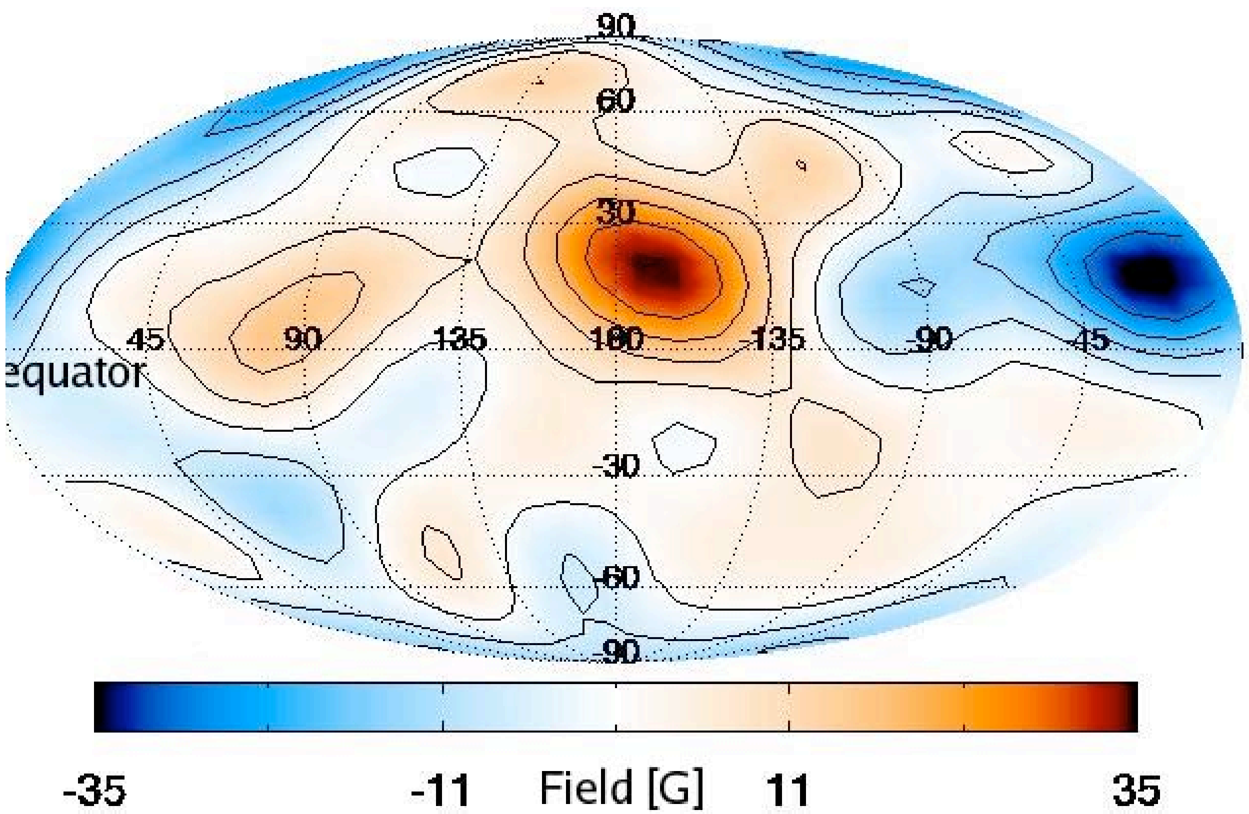


Excited eigenmode has structure similar to that predicted for the mean-flow, self-generated dynamo

Predicted



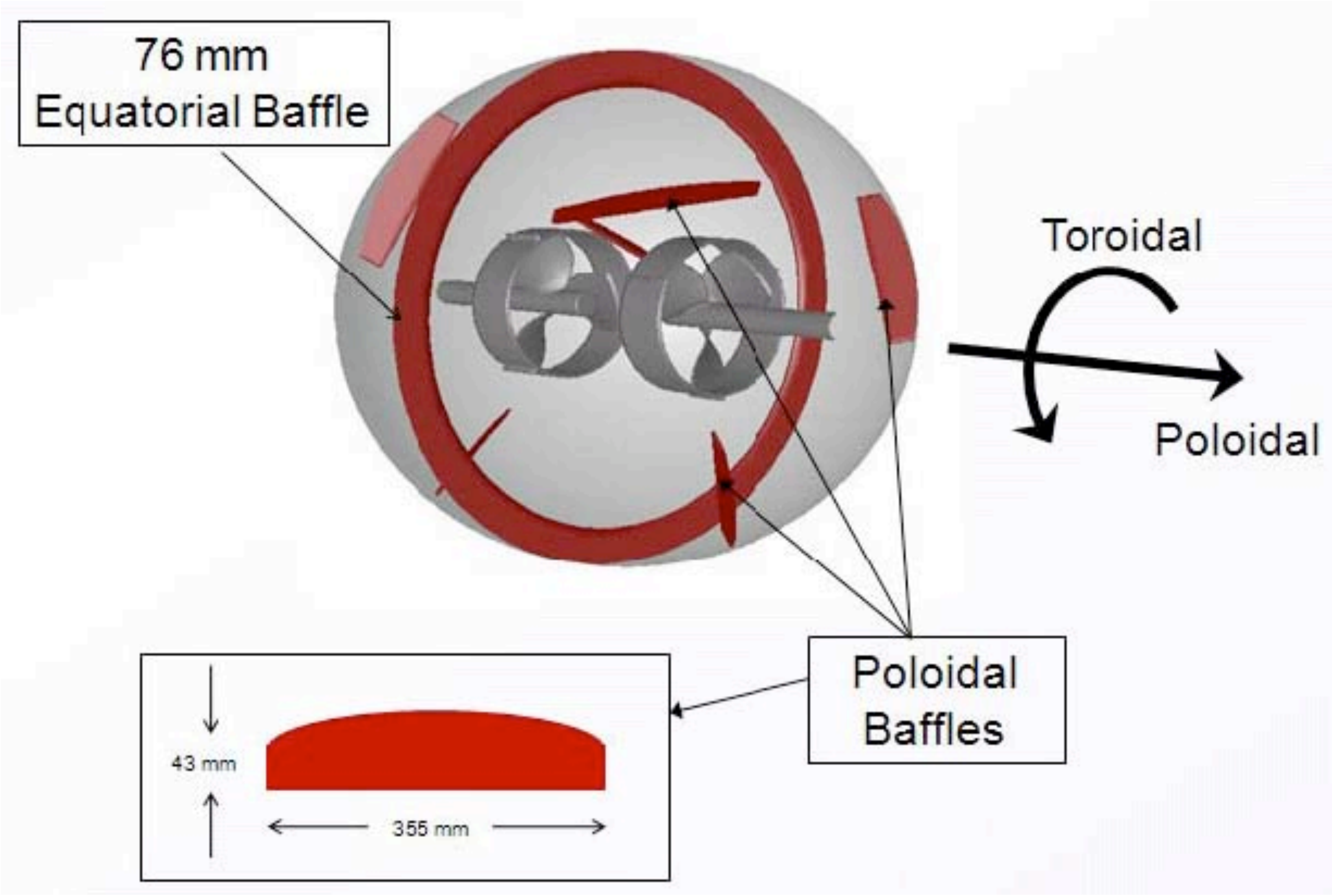
Observed



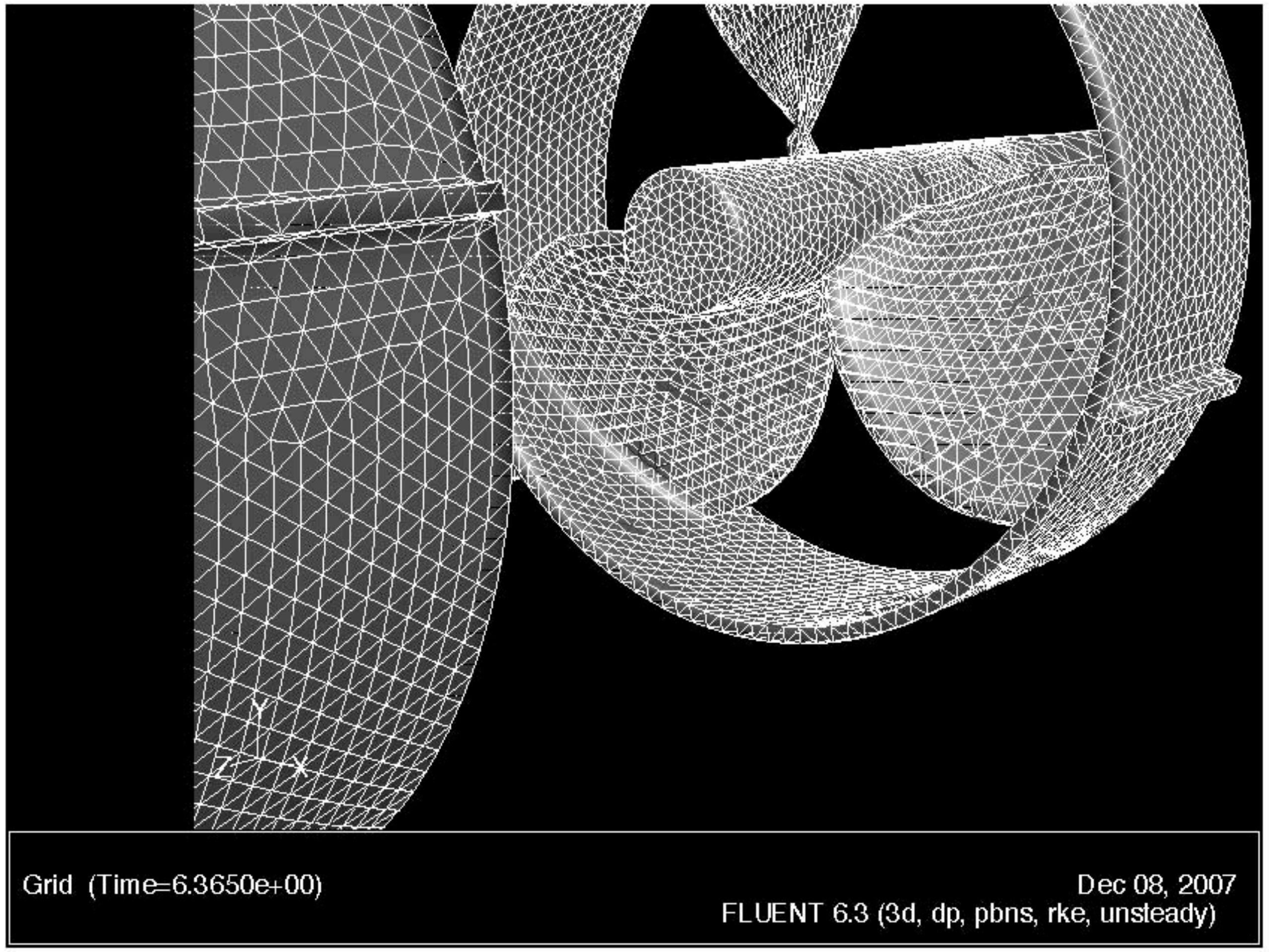
Nornberg, Spence, Jacobson, Kendrick, and Forest, *Intermittent magnetic field excitation by a turbulent flow of liquid sodium,* Phys. Rev. Lett. 97 044503 (2006).

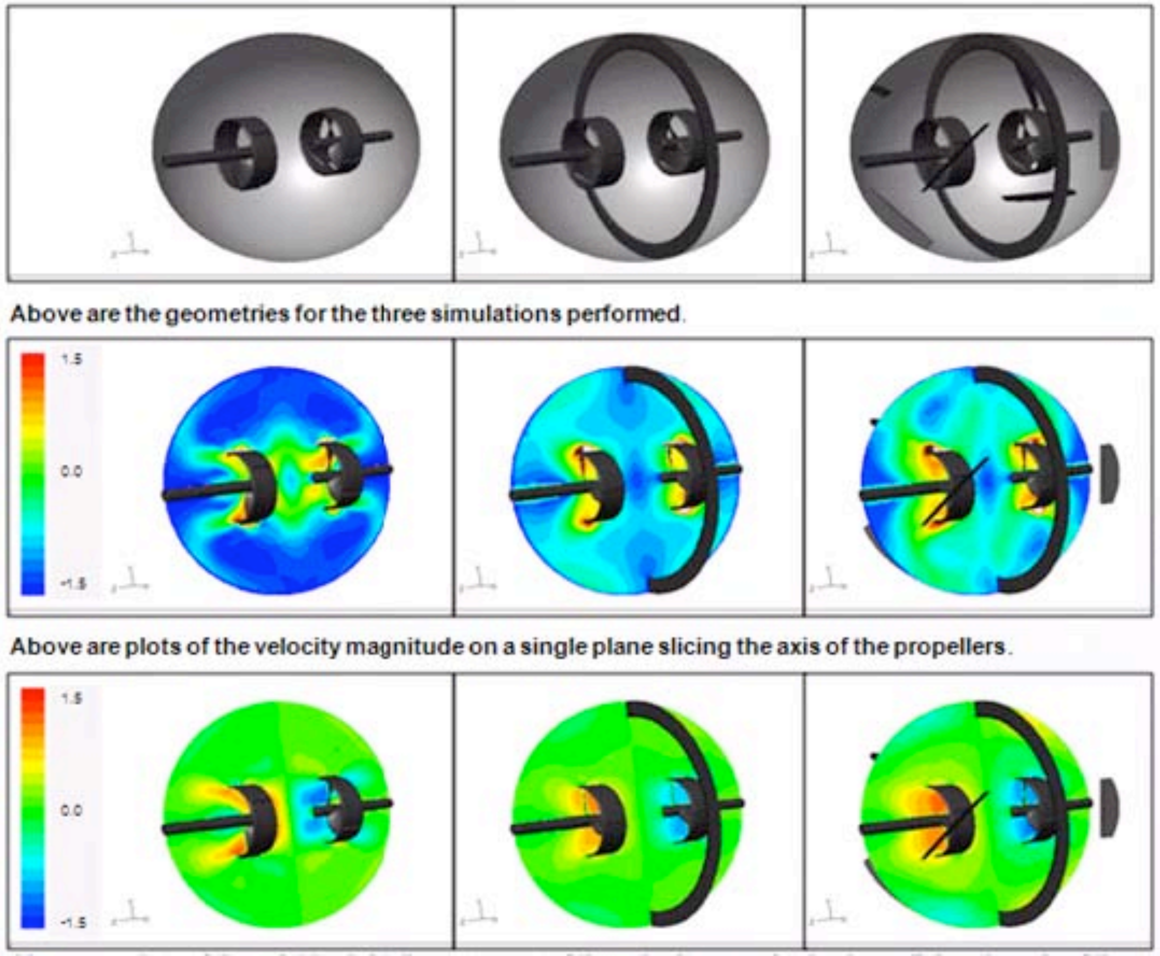
Future Plans for Madison Dynamo Experiment

- Adding internal baffles for flow control and turbulence reduction

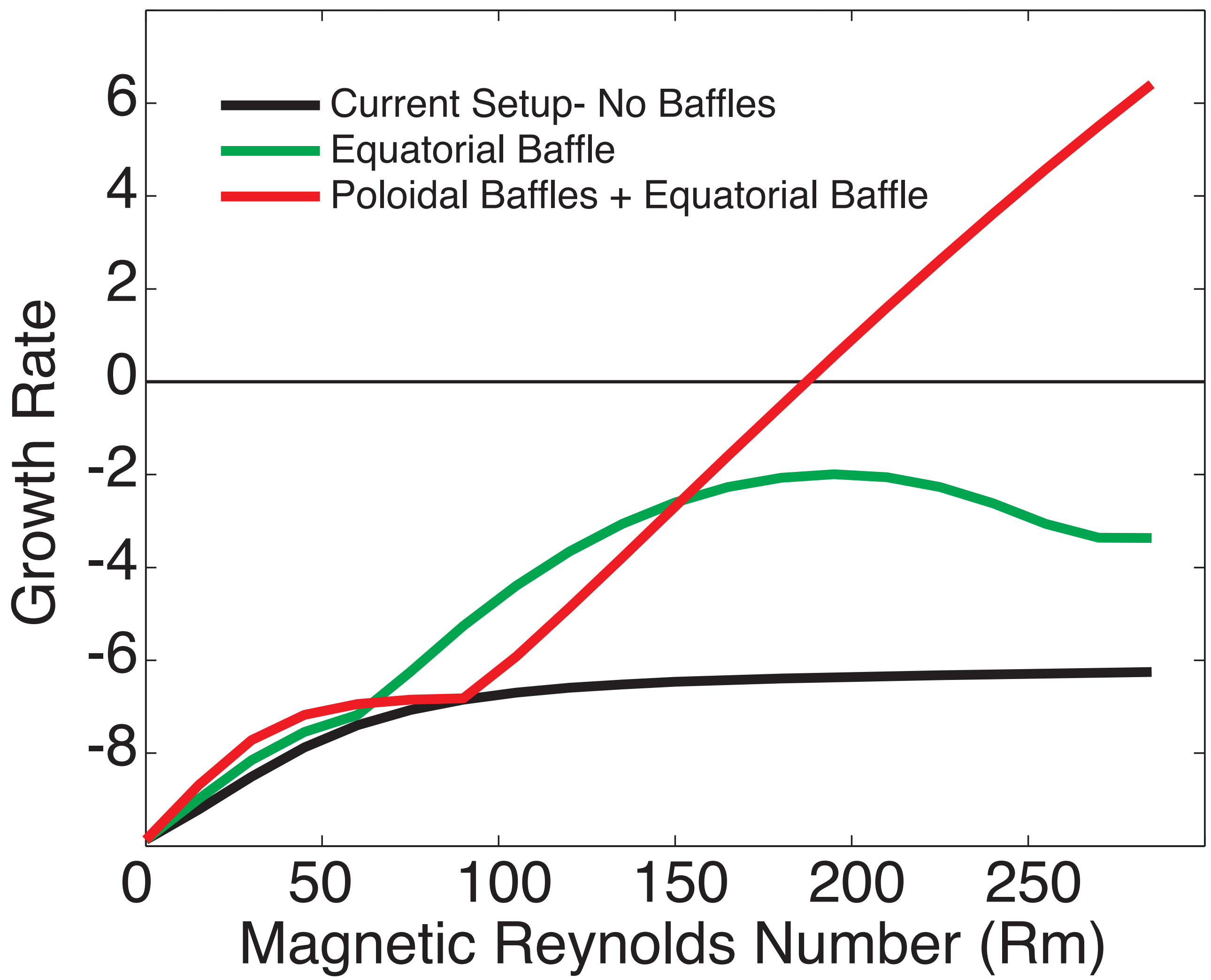


CFD (FLUENT) has been used to study baffles and further optimize flow





CFD predicts lower fluctuation levels and better optimized pitch of propellors



case	turbulent energy
no baffles	$0.71\ m^2/s^2$
equatorial baffle	$0.42\ m^2/s^2$
poloidal vane	$0.10\ m^2/s^2$

Summary

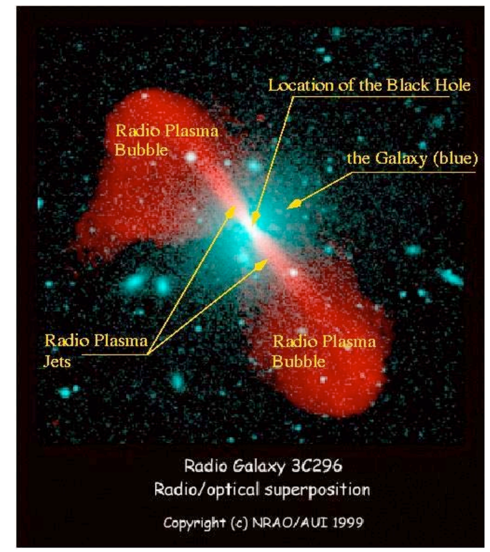
- Mean field electrical currents are observed in a turbulent flow of liquid sodium
- turbulence can be beneficial or detrimental to dynamo onset (detrimental for MDE flows)
- A plasma-based dynamo experiment can access astrophysically important dynamo processes

Big Questions in Astophysics have a Common Theme Related to Magnetic Field Generation from Plasma Flow

SOLAR MAGNETIC FIELD

-Dynamic and well measured
-weak large scale strong smale scale
Rm = 10^7
Pm = 10^{-3}

ACCRETION DISKS

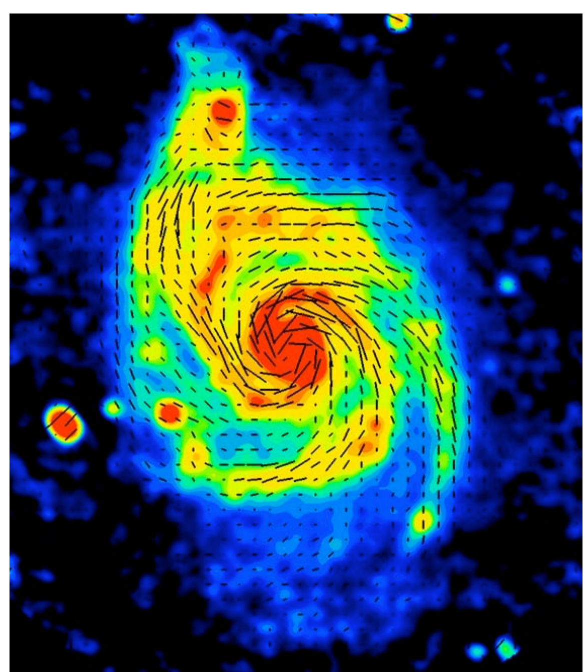


-Collisionless close to hole
-Galaxy is ejecting plasma and magnetic field into the surrounding IGM
-Rm = 10^{19}
-Pm = 10^5

GALACTIC MAGNETIC FIELD

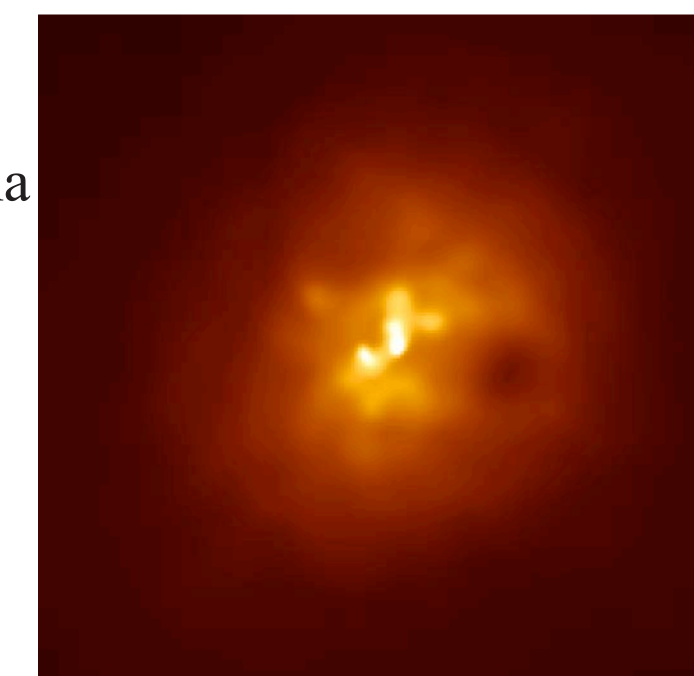
-M51 Spiral Galaxy
-Polarization of 6cm emission Indicates direction of B field in the hot plasma between the stars.
-Rm = 10^{14} (?)
-Pm = 10^5

Large scale coherent field



GALAXY CLUSTERS

Xray image of Abel 2597 from Chandra
-Collisionless plasma (Te=10 keV); mean Free path size of a galaxy.
-Turbulent.
-Magnetized: $\beta \sim 3$
-Rm = 10^{29}
-Pm = 10^4



Poorly Understood, Fundamental Plasma and MHD Processes Can Benefit from Experimental Studies

- **Large Scale Dynamo**: What is the size, structure and dynamics of the mean magnetic field created by high magnetic Reynolds number flows—particularly rotating flows? At low Pm, does turbulence suppress the Large Scale Dynamo? Is helical turbulence necessary for a turbulent LSD?
- **Small Scale Dynamo**: How do random turbulent (high Rm) flows create random and turbulent magnetic fields—what is the structure of these fields?
- **Plasma Turbulence**: What is the nature of plasma turbulence when magnetic fields and velocity fields are in near equipartition? How is energy dissipated? How are heat, momentum and current transported in stochastic magnetic fields that have little large scale structure?
- **Magnetorotational Instability**: How does angular momentum get transported by magnetic instabilities? Can the MRI be a dynamo?
- **Explosive Reconnection Driven by Plasma Flow**: How does plasma flow generate magnetic energy which can accumulate and ultimately be released in explosive instabilities?
- **Plasma Instabilities**: Do plasma instabilities beyond MHD such as the firehose, mirror, or energetic particle driven exist in collisionless, turbulent plasma flows? How do these instabilities saturate? Do they change the macroscopic dynamics?

Important Dimensionless Numbers

Cowling	C	$\frac{\frac{B^2}{2\mu_0}}{\frac{1}{2}\rho U^2}$
Magnetic Reynolds	Rm	$\mu_0 \sigma U L$
Reynolds	Re	$\frac{UL}{\nu}$
Magnetic Prandtl	Pm	$\mu_0 \sigma \nu$

Minimum requirements for experimentally addressing each Plasma Process

Plasma Process	Rm_{crit}	Re	C	$\frac{\lambda}{L}$	β
large scale dynamo					
laminar	$\gtrsim 100$	< 100	$\ll 1$	-	-
with turbulence	$\gtrsim 500$	> 1000	$\ll 1$	-	-
small scale dynamo	$\gtrsim 500$	$\gtrsim 1000$	$\ll 1$	?	?
MHD turbulence	$\gtrsim Re$	$\gtrsim 1000$	~ 1	-	-
MRI					
with mean field	$\gtrsim 10$	—	$\lesssim 1$	?	?
without mean field	$\gtrsim 15000$	—	$\ll 1$	?	?
B field stretching	$\gtrsim 100$	< 100	~ 1	-	-
Plasma Instabilities	$\gtrsim Re$	$\gtrsim 1000$	$\lesssim 1$	$\gtrsim 1$	$\gg 1$

Large, High Te, fast flowing plasmas

Low B, fast flowing plasmas

Minimum requirements for experimentally addressing each Plasma Process

$\tau_\sigma = \mu_0 \sigma a^2$

Plasma Process	Rm_{crit}	Re	C	$\frac{\lambda}{L}$	β
large scale dynamo					
laminar	$\gtrsim 100$	< 100	$\ll 1$	-	-
with turbulence	$\gtrsim 500$	> 1000	$\ll 1$	-	-
small scale dynamo	$\gtrsim 500$	$\gtrsim 1000$	$\ll 1$	?	?
MHD turbulence	$\gtrsim Re$	$\gtrsim 1000$	~ 1	-	-
MRI					
with mean field	$\gtrsim 10$	—	$\lesssim 1$	?	?
without mean field	$\gtrsim 15000$	—	$\ll 1$	?	?
B field stretching	$\gtrsim 100$	< 100	~ 1	-	-
Plasma Instabilities	$\gtrsim Re$	$\gtrsim 1000$	$\lesssim 1$	$\gtrsim 1$	$\gg 1$

Large, High Te, fast flowing plasmas

Low B, fast flowing plasmas

Liquid Metal Experiments are limited: the next frontier for experimental dynamo studies should be plasma based

- Liquid metals have advantage that confinement is free and conductivity is independent of confinement, BUT:
  - ➡ Unfortunate Power Scaling Limitation: $P_{mech} \sim Rm^3 / L$
  - ➡ Prandtl Number is always very small: $Rm \ll Re$
- Plasmas have the potential for
  - Variable Pm
  - $Rm \gg 100$
  - intrinsically include "plasma effects" important for astrophysics (compressibility, collisionality)
  - broader class of available diagnostics

Dynamo and MRI Process

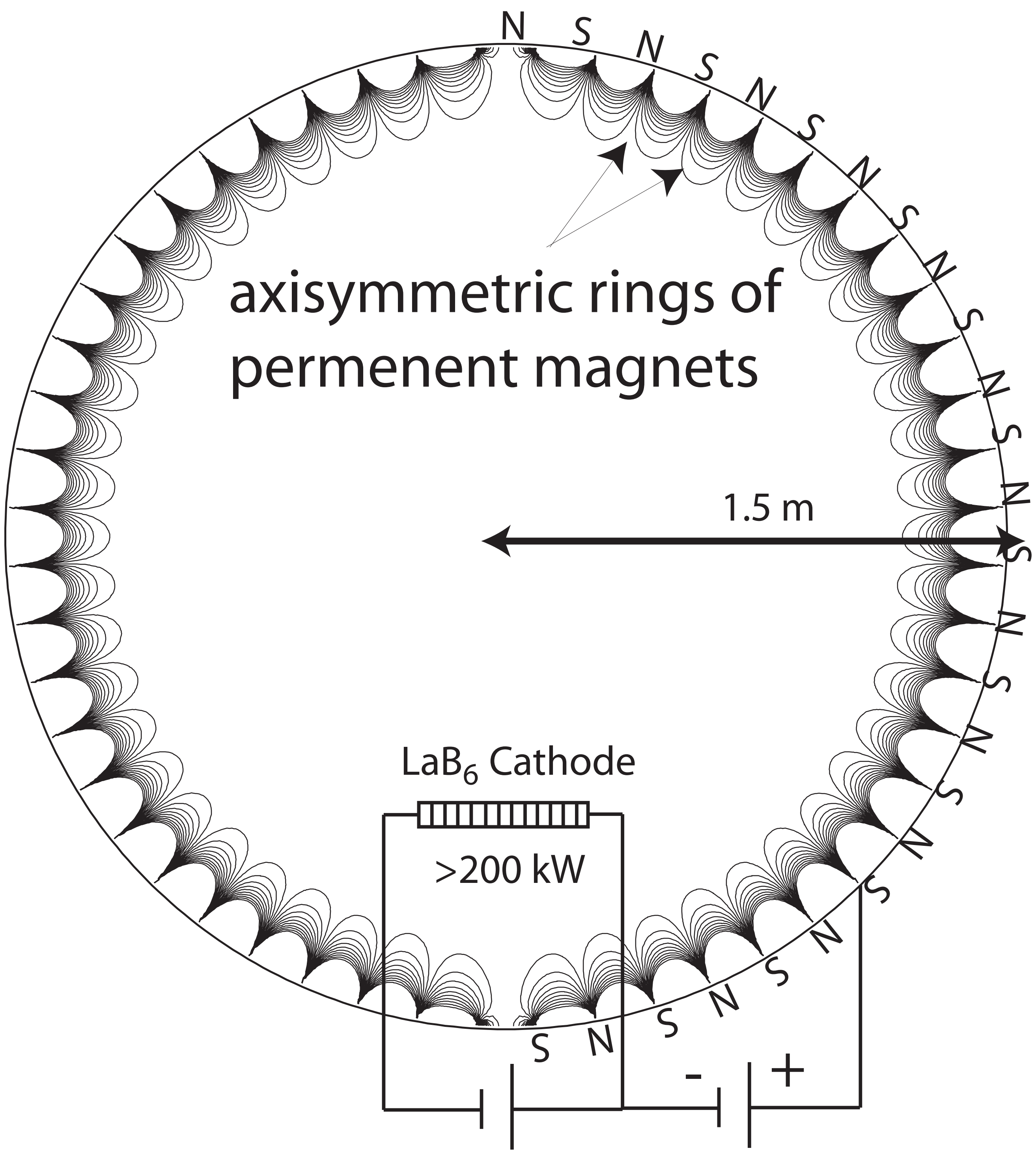
1. Begin with small magnetic field ($C \ll 1$)
2. Stir until $Rm > Rm_{crit}$
3. Magnetic field spontaneously created

Challenge: to create a large, highly conducting, unmagnetized, fast flowing laboratory plasma for study

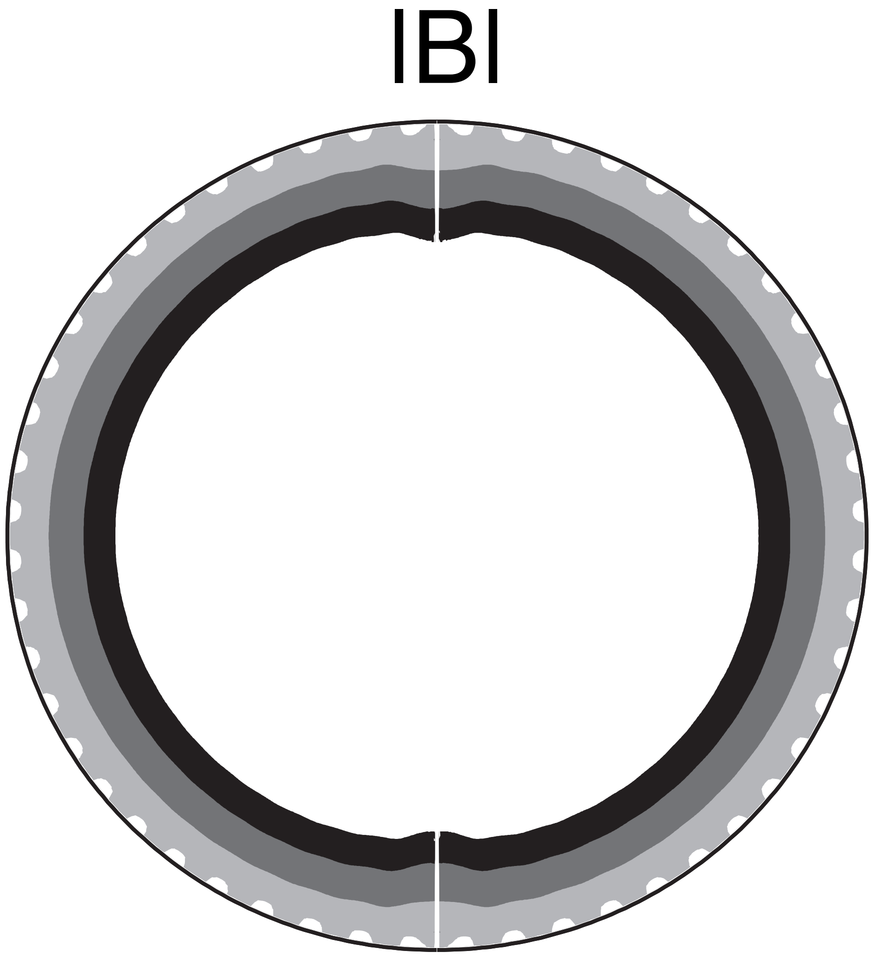
- -difficult to stir a plasma
- -need some confinement for plasma to be hot

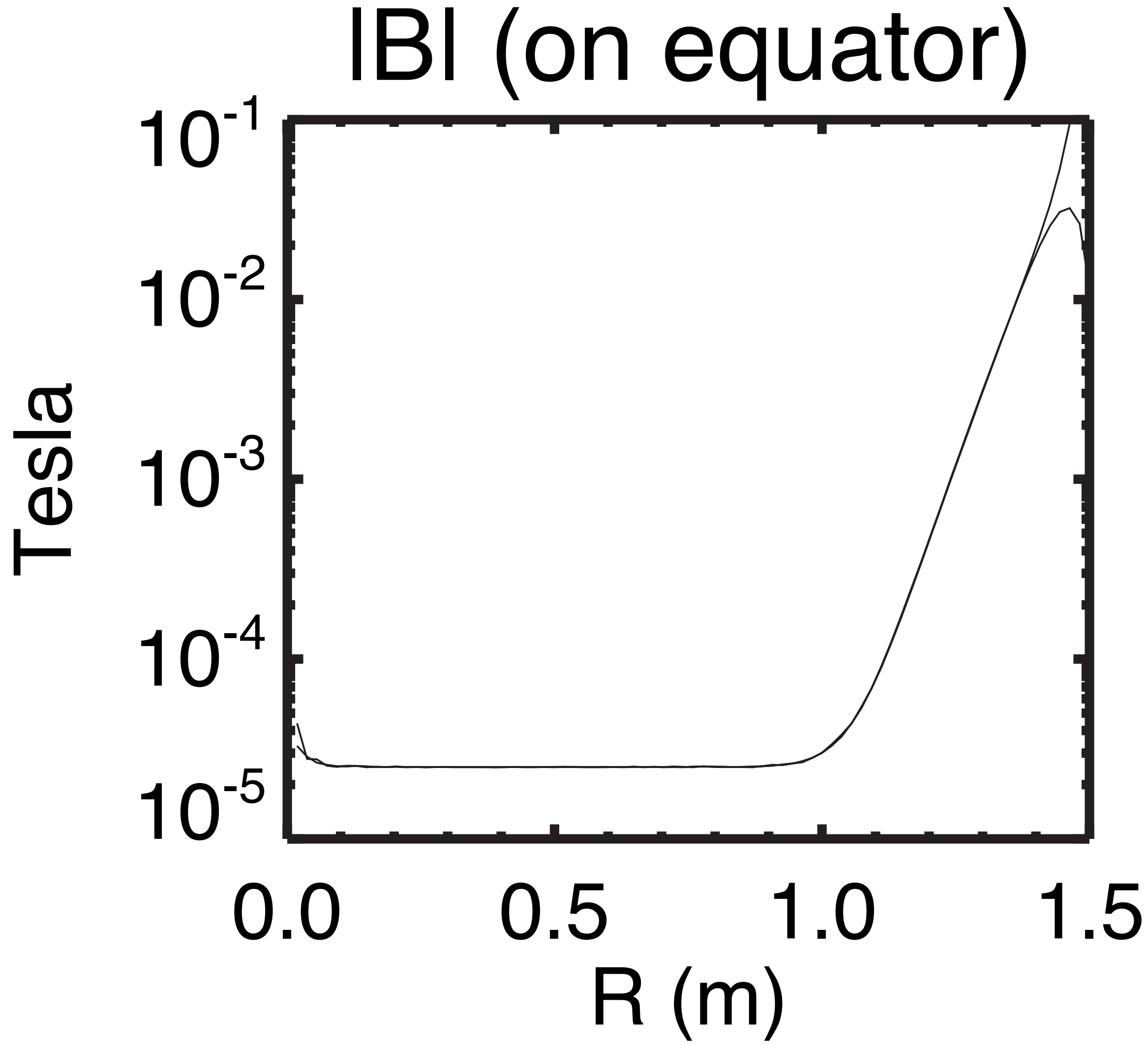
Plasma Dynamo Facility is needed to study high Rm, high C plasmas

- Axisymmetric Ring Cusp
- edge confinement provided by 1.5 T, NdFeB Magnets
- high power plasma source using LaB_6
  - 200 kW, DC power supplies
  - similar to LAPD, CDX technology
- Challenges
  - cooling of magnets
  - insulators



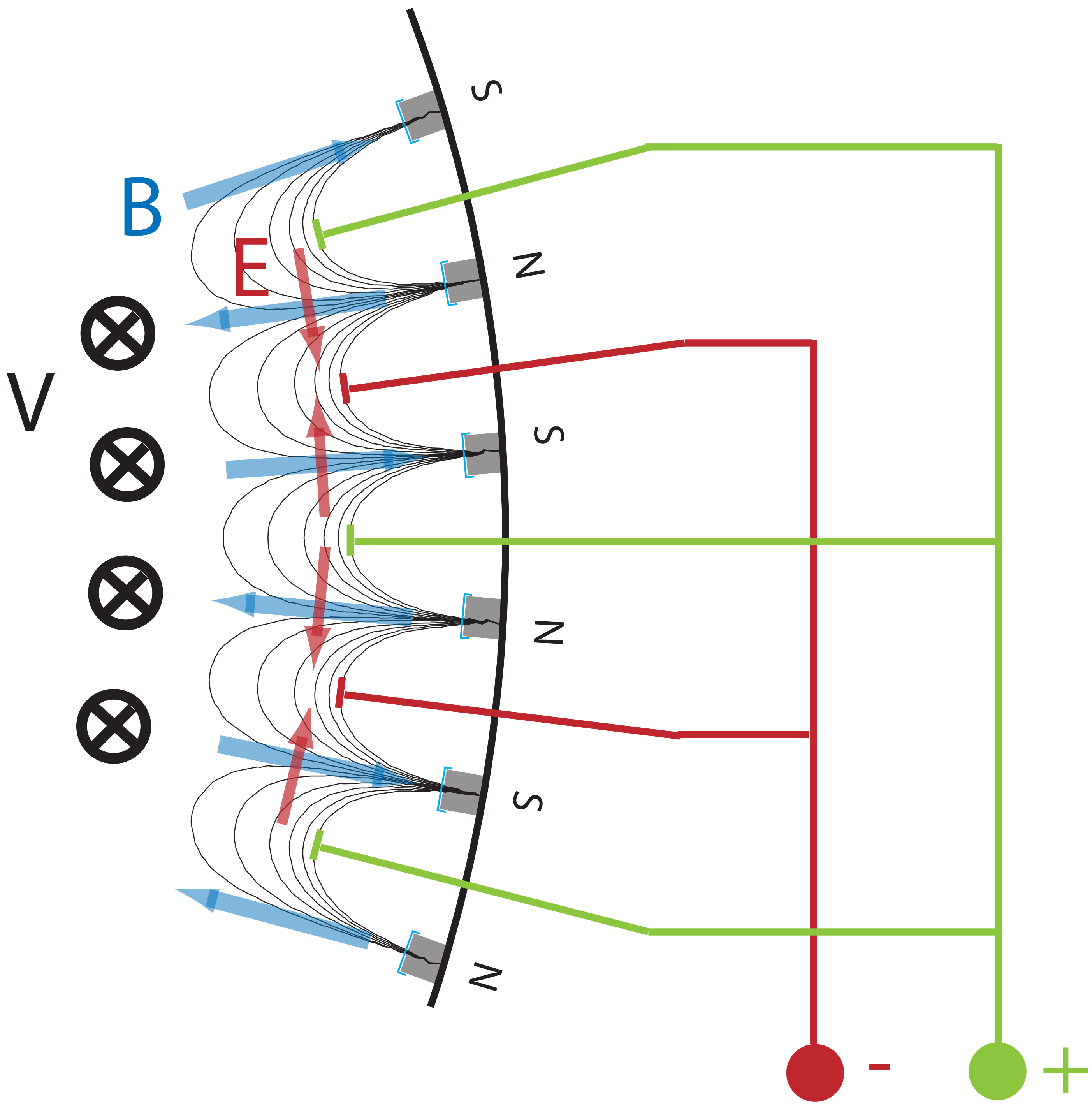
Large, Magnetic Field Free Volume Plasma





Magnetic field provides confinement similar to wall in fluid experiments

Multipole Magnetic Field can be used to drive flow at edge



Arbitrary Vφ (r = a, θ)

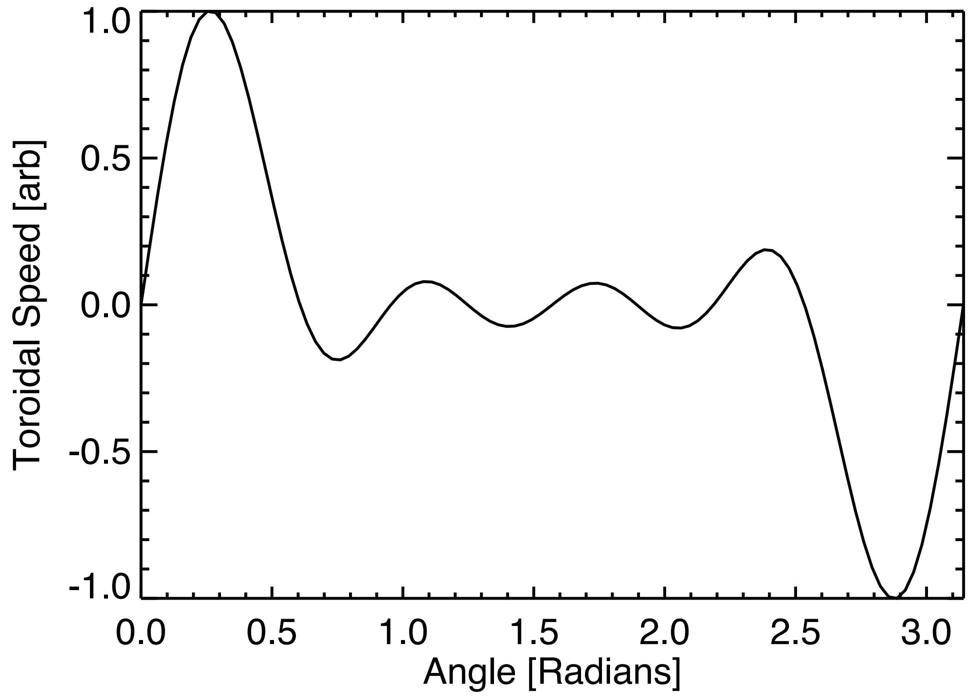
Formulary of Key Dimensionless Parameters

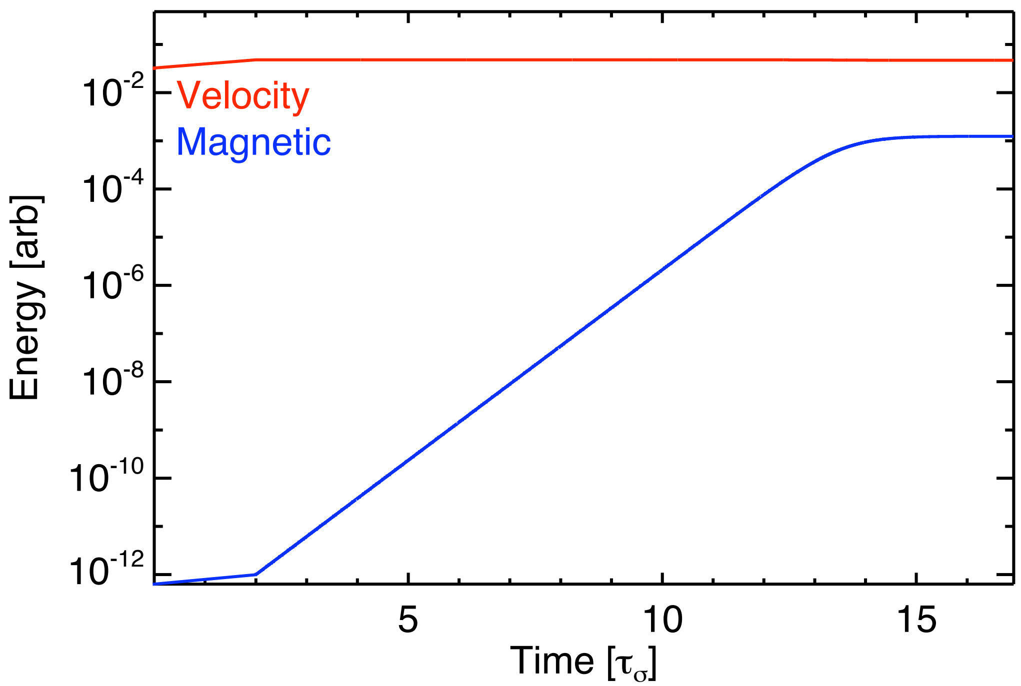
Magnetic Reynolds Number	Rm	$\mu_0 \sigma U L$	1.5	$\frac{T_{\rm e,eV}^{3/2} U_{\rm km/s} L_{\rm m}}{Z}$
Reynolds Number	Re	$\frac{UL}{\nu}$	8	$\frac{a_{\rm m} U_{km/s} \mu^2 n_{18}}{T_{\rm i,eV}^{5/2}}$
Magnetic Prandtl Number	Pm	$\mu_0 \sigma \nu$	0.18	$\frac{T_{\rm e,eV}^{3/2} T_{i,eV}^{5/2}}{\mu^2 n_{18}}$
Cowling Number	C	$\frac{\frac{B^2}{2\mu_0}}{\frac{1}{2}\rho U^2}$	4.75	$\frac{B_{\rm G}^2}{\mu n_{18} U_{km/s}^2}$
Lundquist Number	Lu	$Rm \times C^{1/2}$	3.26	$\frac{T_{\rm e,eV}^{3/2} B_{\rm G} L_{\rm m}}{Z\sqrt{\mu n_{18}}}$
Magnetization		$\frac{\rho_e}{L}$	0.0238	$\frac{T_{\rm e,eV}^{1/2}}{B_{\rm G} L_{\rm m}}$
Ion Collisionality		$\frac{\lambda_{mfp}}{L}$	0.012	$\frac{T_{\rm i,eV}^2}{n_{18} L_{\rm m}}$
Plasma Pressure	β	$\frac{2\mu_0 n T}{B^2}$	40	$\frac{n_{18} T_{\rm e,eV}}{B_G^2}$

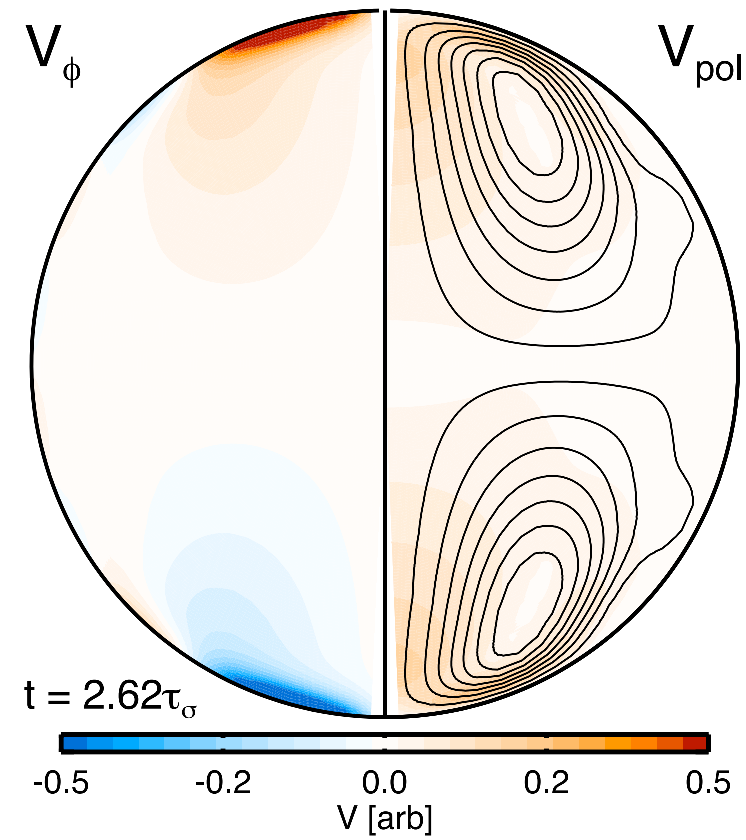
Plasma Parameters

plasma radius	a	1.5	m
density	n	10^{17}—10^{19}	m^{-3}
electron temperature	T_e	2—20	eV
ion temperature	T_i	0.5—2	eV
peak flow speed	U_{max}	0—20	km/s
ion species	H, He, Ne, Ar	1, 4, 20, 40	amu
magnetic field	$r < 1.2$ m	<0.1	gauss
magnetic field	at cusp	$>10^4$	gauss
current diffusion time	$\mu_0 \sigma a^2$	50	msec
pulse length	τ_{pulse}	5	sec
heating power	P	< 0.5	MW
	Rm_{max}	> 1000	
	Re	24—3.8×10^6	
	Pm	3×10^{-4}—56	
	C	10^{-4}	
	β	10^4	

Two Vortex Plasma Dynamo Flow can be driven at boundary (spherical Von Karman Flow)







- Plasma Rm=300, Re=100
  - Te=10 eV
  - U=10 km/s,
  - $n=10^{18}\ m^{-3}$
  - Hydrogen

Small Scale Dynamo at Pm>1

- Rm=1000
- Re=400
- Plasma
    - Te = 13 eV
    - Ti = 1 eV
    - deuterium
    - U = 15 km/s
    - $n = 10^{18}\ m^{-3}$