Thoughts on 'Theterg y'

⇒ Is magnetostrophic turbulence as simple as it looks?

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→ initial stages ⟷ discussion, please!

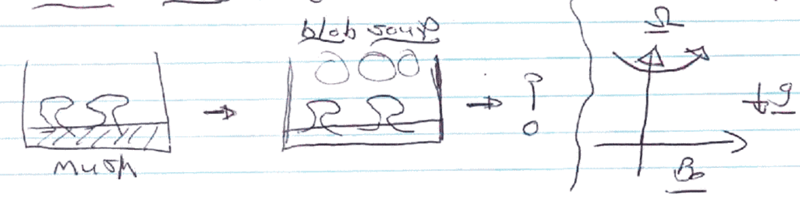
→ background: see 3 papers on Wiki

- re: RGDT (resistivity gradient driven turbulence), from Rippling Modes (a/a' FKR)
- structure similar to magnetostrophic (as is Vlasov turbulence)
- $\langle \delta T^2 \rangle$ dynamics central

→ see also: http://physics.ucsd.edu/plasmatheorygroup/ (shameless advertising)

I

→ What's Going On? (or what we think is ....)



- slugs/blobs erupt from mush at top of inner core
- slug $\begin{cases} B_0 \to \text{slices (St. Pierre)} \\ \Omega \to \text{columns} \end{cases}$ ⇒ symmetrical blob
- blobs → buoyant rise → flow → complex strain field ⇒ turbulence, cascade ...

  → wake ⟷ radiated MAC waves → 1

  ⇒ blob {fragments, radiates} during rise ⟷ turbulence

but

- Q_Θ from erupting slugs drives $\partial_z\langle\Theta\rangle$ ⇒ { $\partial_z\langle\Theta(z)\rangle$ evolves on transport time scales; local $\partial_z\langle\Theta\rangle$ → local instability? a/a' Braginskii '64 .. }

this suggests:

Turbulent blob soup { structure, collective modes }

– also blobs ⇒ shear flow

$$\underline{u}^{(s)} = -\nabla \cdot \underline{\underline{\Gamma}} \times \hat{z}/\Omega \quad , \quad \underline{\underline{\Gamma}} \sim \langle \theta^2 \rangle$$

→ structure of shearing field ? { mean / zonal } ?
→ impact on blob lifetime

→ A Few Equations

$$\boxed{\frac{\partial \theta}{\partial t} + \underline{u}\cdot\nabla\theta - \chi\nabla^2\theta = S} \qquad \begin{cases} \theta = \langle\theta\rangle + \delta\theta \\ \underline{u} = \langle \underline{u}\rangle + \delta\underline{u} \end{cases}$$

$Q_\theta, \underline{\underline{\Gamma}}, \alpha \sim \langle \delta\theta^2 \rangle_{k\omega}$

$$\boxed{\delta \underline{u}_{k\omega} = \frac{\underline{A}(k)}{d(k,\omega)}\,\delta\theta_{k,\omega}}$$

n.b.: $\langle u \rangle \sim \langle \delta\theta^2 \rangle$ ⇒ $\langle u \rangle \cdot \nabla \delta\theta \sim \delta\theta^3$

↳ $\begin{cases} g, \Omega, B_0 \\ \omega, \eta k^2 \end{cases}$ $R_m < 1$ important → η is key damping

modes: $(-i\omega + \chi k^2) = \dfrac{-\underline{A}(k)\cdot\nabla\langle\theta\rangle}{d(k,\omega)}$ ⇒ Braginskii '64... buoyancy modes

⇒ revisit with shearing, etc. ...

Quantity of Interest: $\langle \delta\theta^2 \rangle_{k,\omega}$
↓
Thetergy spectrum

II

→ Why? ⇔ What is Needed ?
→ A Statistical Theory of Thetergy / Blobs
↕

– statistically: "blob" ⇔ peak $\langle \delta\theta(1)\,\delta\theta(2)\rangle$

expect correlation structure in time:

$$\langle \theta(t)\,\theta(t+\tau)\rangle_{k} = |\theta_k|^2 e^{-i(\omega_k - \underline{k}\cdot\underline{V})\tau - \tau/\tau_{ck}}$$

mode freq; riding frozen flow; ↳ lifetime

ω_k < frequency, decay time

– apart χ, $\begin{cases} \dfrac{d\theta}{dt} = 0 \quad \rightarrow \quad \left\langle \dfrac{d\,\delta\theta^2}{dt} \right\rangle = -\dfrac{\partial \langle\theta\rangle^2}{\partial t} \\ \dfrac{\partial\langle\theta\rangle}{\partial t} = -\nabla\cdot\langle \delta\underline{u}\,\delta\theta\rangle \end{cases}$

⇒ shear + fragmentation

$$\boxed{\begin{aligned} &\frac{\partial}{\partial t}\langle \delta\theta(1)\delta\theta(2)\rangle + \langle(\underline{u}(1) - \underline{u}(2))\cdot\nabla\, \delta\theta(1)\,\delta\theta(2)\rangle + \text{diffn} \\ &= -\langle \delta\underline{u}(1)\,\delta\theta(2)\rangle\cdot\nabla\langle\theta\rangle + (1 \leftrightarrow 2) \end{aligned}}$$

i.e. $\dfrac{\partial}{\partial t}\langle \delta\theta^2 \rangle + T_{1,2}[\langle \delta\theta^2 \rangle] = P$

$T_{1,2}$ → 2 pt. evolution → buoyancy flux
P → production $\lim_{1\to 2} P = -\underline{Q}_\theta\cdot\nabla\langle\theta\rangle$ ~ entropy production

→ Fragments of the Theory

(a) $$T_{1,2}[\langle\delta\theta^2\rangle] = \frac{\partial}{\partial t}\langle\delta\theta^2\rangle + \underbrace{(\langle \underline{u}(1)\rangle - \langle \underline{u}(2)\rangle)}_{\text{shearing}}\cdot\nabla\langle\delta\theta^2\rangle$$

$$+ \langle(\delta\underline{u}(1) - \delta\underline{u}(2))\cdot\nabla_{-}\,\delta\theta^2\rangle + \text{visc.}$$

$\sim\langle\delta\theta^2\rangle$

$\sim\langle\delta\theta^3\rangle$ → closure needed .... ☹

from painful experience, expect:

$$T_{1,2} = \frac{\partial}{\partial t}\langle\delta\theta^2\rangle + (\langle\underline{u}(1)\rangle - \langle\underline{u}(2)\rangle)\cdot\nabla_{-}\langle\delta\theta^2\rangle$$

$$- \nabla_{-}\cdot\underline{\underline{D}}_{rel}\cdot\nabla_{-}\langle\delta\theta^2\rangle$$

(1): → (2): →

$$\underline{\underline{D}}_{rel} = \sum_{k,\omega}\langle\delta\theta^2\rangle_{k,\omega}\frac{\underline{A}\,\underline{A}}{|\underline{d}|^2}\tau_{c\underline{k}}\sin(\underline{k}\cdot\underline{x}_{-}) \rightarrow \underset{(1\to 2)}{0}$$

-1

⇒ for blob (lifetime)$^{-1}$: (scale ℓ)
- D/ℓ^2 relative diffusion
- $1/\tau(\ell)$ → ala' Richardson
- $\left(\frac{1}{\ell^2}\langle u\rangle'^2 D\right)^{1/3}$ → diffusion + shearing

key issues:
→ interaction locality in scale?
→ importance of shear flow

(b)
→ Production

→ not apparent that "inertial range" exists
↔ collective mode effects, $\nabla\langle\theta\rangle$, etc. ..

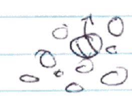
→ $P = -\underline{Q}_\theta\cdot\nabla\langle\theta\rangle = \langle\delta\underline{u}\,\delta\theta\rangle\cdot\nabla\langle\theta\rangle$

(P) → physical meaning?

- can write directly, by $\delta u \leftrightarrow \delta\theta$
- "usual" $\delta\theta = \theta^c = -R_{k\omega}\,\delta\underline{u}_{k\omega}\cdot\nabla\langle\theta\rangle$ (coherent response)

⇒ $P = \nabla\langle\theta\rangle\cdot\underline{\underline{D}}\cdot\nabla\langle\theta\rangle$
- ↝ mixing $\nabla\langle\theta\rangle$ feeds fluctuations
- ↝ analogous to F-P theory

What of wake, dynamical friction?



→ condensing a long story:

- $\delta\theta = \underset{\text{coherent}}{\theta^c} + \underset{\text{incoherent}}{\tilde{\theta}}$
- $\tilde{\theta}$ ⇒ dynamical friction → F-P drag (partial cancellation)
- θ^c screens $\tilde{\theta}$

6.

$$\therefore \quad \boxed{P = -\underline{\nabla}\langle\theta\rangle \cdot \sum_{\underline{q}\omega} \frac{\underline{A}(\underline{k})\, d(\underline{k},\omega)}{|d(\underline{k},\omega)|^2}\, \frac{\langle\tilde{\theta}^2\rangle_{\underline{k},\omega}}{|\underbrace{\epsilon(\underline{k},\omega)}_{\text{buoyancy modes}}|^2}}$$

and

$$\partial_t \langle\delta\theta^2\rangle + T_{1,2}[\langle\delta\theta^2\rangle] = P$$

$$\langle\delta\theta^2\rangle \approx \tau_L P \quad \rightarrow \quad \ldots \quad \epsilon_{EM},\ \chi_T$$

→ What do we Gain from this approach ?

- systematic statistical framework
- route to understanding wave-blob interaction ....
- lifetime ⟷ $\left\{ \begin{array}{l} \text{shearing} \\ \text{fragmentation} \end{array} \right.$
- Production ↗ diffusion, ↘ dynamical friction
  $\updownarrow$
  $\underline{\nabla}\langle\theta\rangle$

7.

III

→ Issues for Discussion

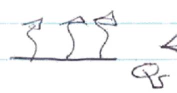
- basic picture ?

  "fragmentation" "cascade" vs. "bubble competition" "coagulation"

  i.e. large $\delta\theta$ overtake/engulf smaller ?

  ⇒ few blob state ?

- better off with macroscopics ?

  i.e.  ⟷ analogue to free convection boundary layer theory

- are collective modes important on story ?

  how best represent $\left\{ \begin{array}{l} \text{local} \\ \text{or} \\ ? \end{array} \right.$

→ structure of shear flow ?

- smooth, zonal, both ?
- modulational stability of magnetostrophic turbulence ?

→ dominant interaction: local cascade vs. non-local interaction (shearing) ?

→ structure based blob kinetics model ?