Non-Helical Large-Scale Dynamos (with zero mean kinetic helicity)

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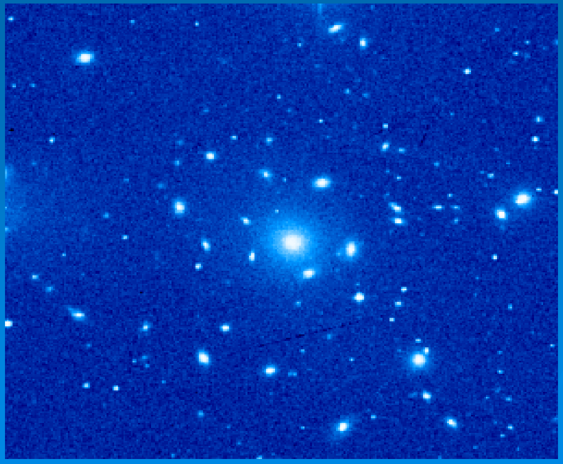


Outline

- **Numerical Simulations (non-helical turbulence in a sheared box)**
- **Effect of kinetic helicity fluctuations with zero mean on mean-field dynamo in a turbulence with large-scale shear (the tau-approach, SOCA)**
- **Shear-current effect (the tau-approach, SOCA, the renormalization approach)**
- **Generation of large-scale vorticity (vorticity dynamo) in sheared turbulence (the tau-approach, SOCA, the renormalization approach)**
- **Application to astrophysics and laboratory dynamo**

Mean-Field Dynamo

Is it possible to generate a large-scale magnetic field in a non-helical (with zero mean kinetic helicity) and non-rotating homogeneous turbulence ?

Direct Numerical Simulations (linear shear velocity)

T. A. Yousef, T. Heinemann, A.A. Schekochihin, N. Kleeorin, I. Rogachevskii, A.B. Iskakov, S.C. Cowley, J.C. McWilliams,
Phys. Rev. Lett., v.100, 184501 (2008)

1. *A white noise non-helical homogeneous and isotropic random forcing*
2. *Imposed mean linear shear flow*
3. *Sheared box (shear-periodic boundary conditions)*

$$\mathrm{Re} = \mathrm{Rm} = \mathrm{u}\, \mathrm{l}_0 / \nu = 30$$

$$L_z >> L_x = L_y \qquad L_x / l_0 = 3$$

Numerical set up

Incompressible MHD equations with background shear

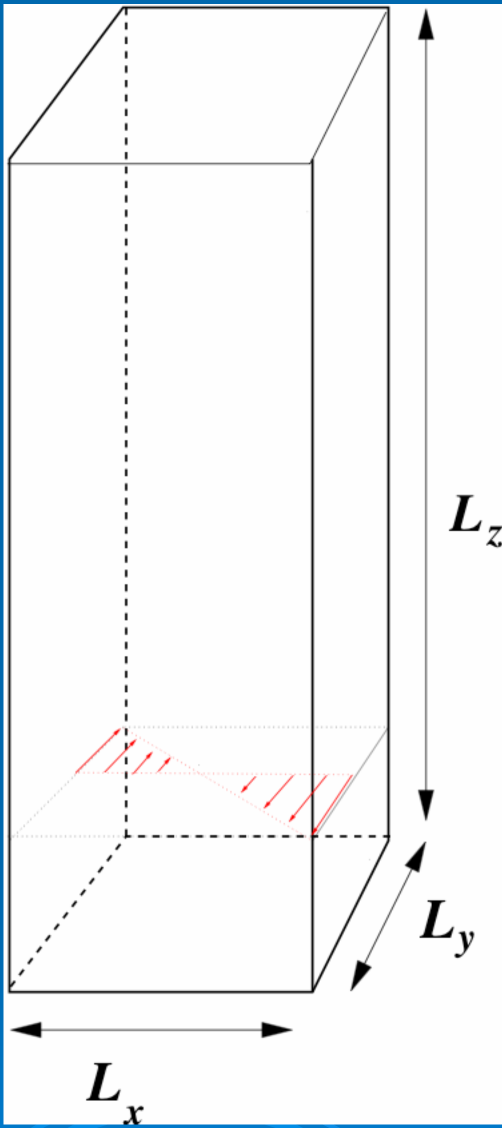
$$\frac{d\mathbf{u}}{dt} = u_x S\hat{\mathbf{y}} - \frac{\nabla p}{\rho} + \frac{\mathbf{B}\cdot\nabla\mathbf{B}}{4\pi\rho} + \nu\nabla^2\mathbf{u} + \mathbf{f},$$
$$\frac{d\mathbf{B}}{dt} = -B_x S\hat{\mathbf{y}} + \mathbf{B}\cdot\nabla\mathbf{u} + \eta\nabla^2\mathbf{B},$$

$$\nabla\cdot\mathbf{u} = 0$$

$$\mathbf{U} = -Sx\hat{\mathbf{y}}$$

$$d/dt = \partial_t - Sx\partial_y + \mathbf{u}\cdot\nabla$$

(Units: $\epsilon = \langle\mathbf{u}\cdot\mathbf{f}\rangle = 1$ $L_x = L_y = 1$)



Parameters

$$\overline{\mathbf{B}} = (\overline{B}_x(z), \overline{B}_y(z), 0)$$

Turbulence: $\epsilon = \langle\mathbf{u}\cdot\mathbf{f}\rangle = 1 \Rightarrow u_{\rm rms} \sim 1$, $l_0 \sim \frac{1}{3}$ $\Rightarrow \tau \sim \frac{1}{3}$

$$\nu = \eta = 10^{-2} \Rightarrow \mathrm{Re} \sim 30$$

$$L_B > \frac{u_{rms}}{S} = \frac{l_0}{S\tau}$$

Weak shear: $S = 2,\ 1,\ \frac{1}{2},\ \frac{1}{4},\ \frac{1}{8}$ $\Rightarrow S\tau \sim (0.04\ldots0.6)$

Linear shear velocity (DNS)

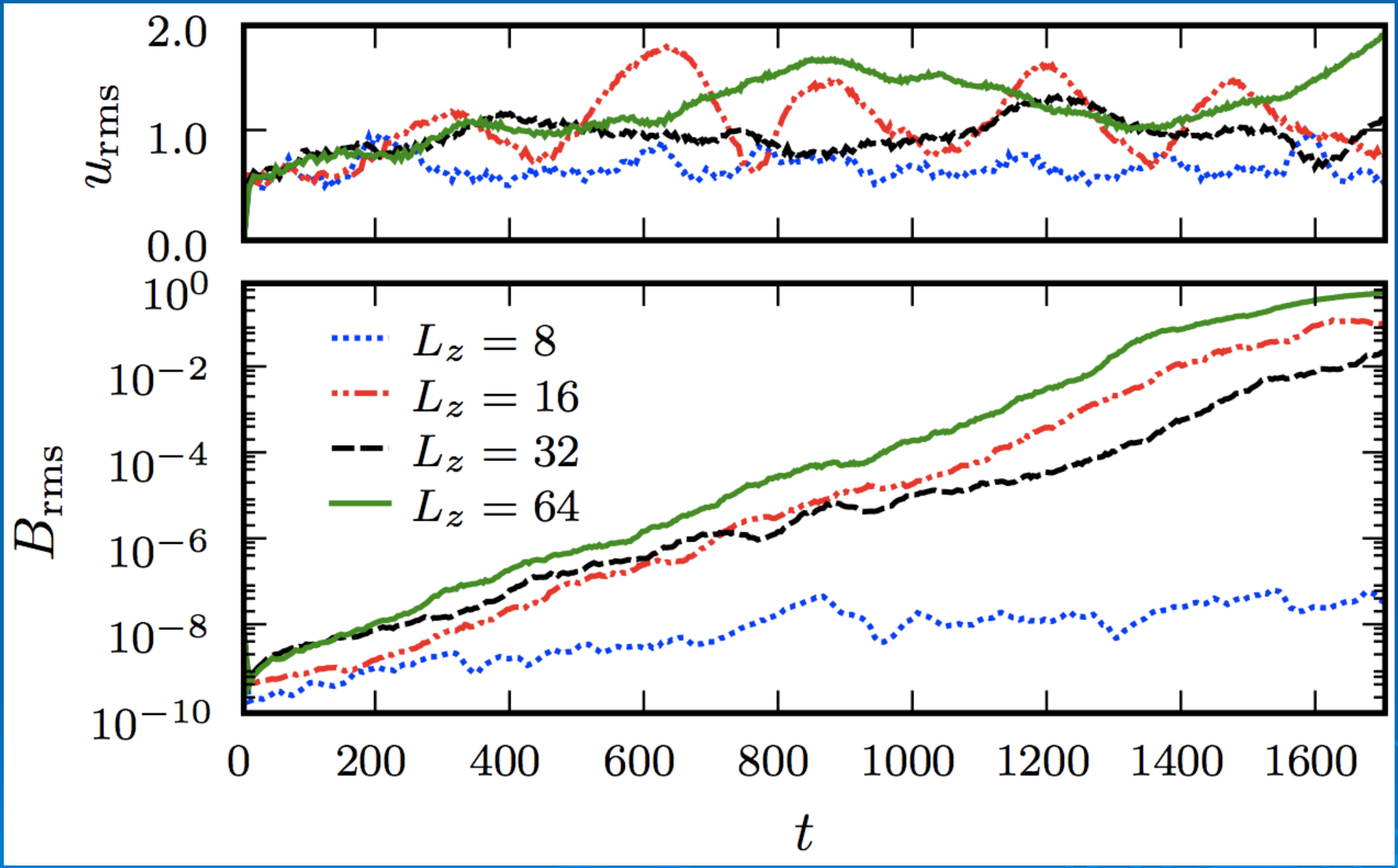
$$L_z >> L_x = L_y = 1$$

$$\overline{\mathbf{B}} = (\overline{B}_x(z), \overline{B}_y(z), 0)$$

$$L_x / l_0 = 3$$

$$l_{\overline{B}} > \frac{u_{rms}}{S}$$

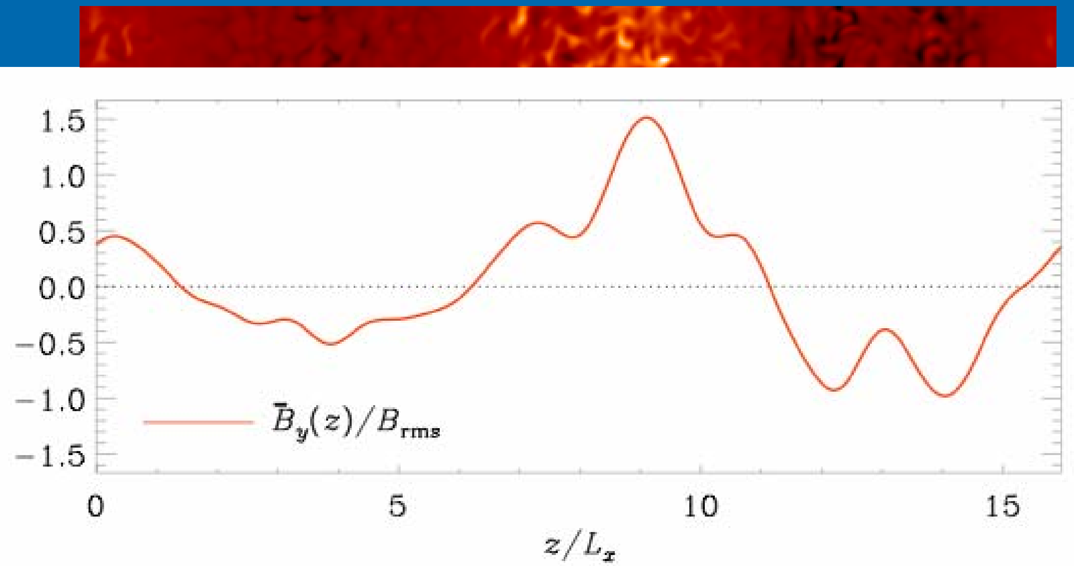
Magnetic field grows



$\mathrm{Re} = \mathrm{Rm} = u\, l_0 / \nu = 30$ $L_z >> L_x = L_y = 1$ $L_x / l_0 = 3$

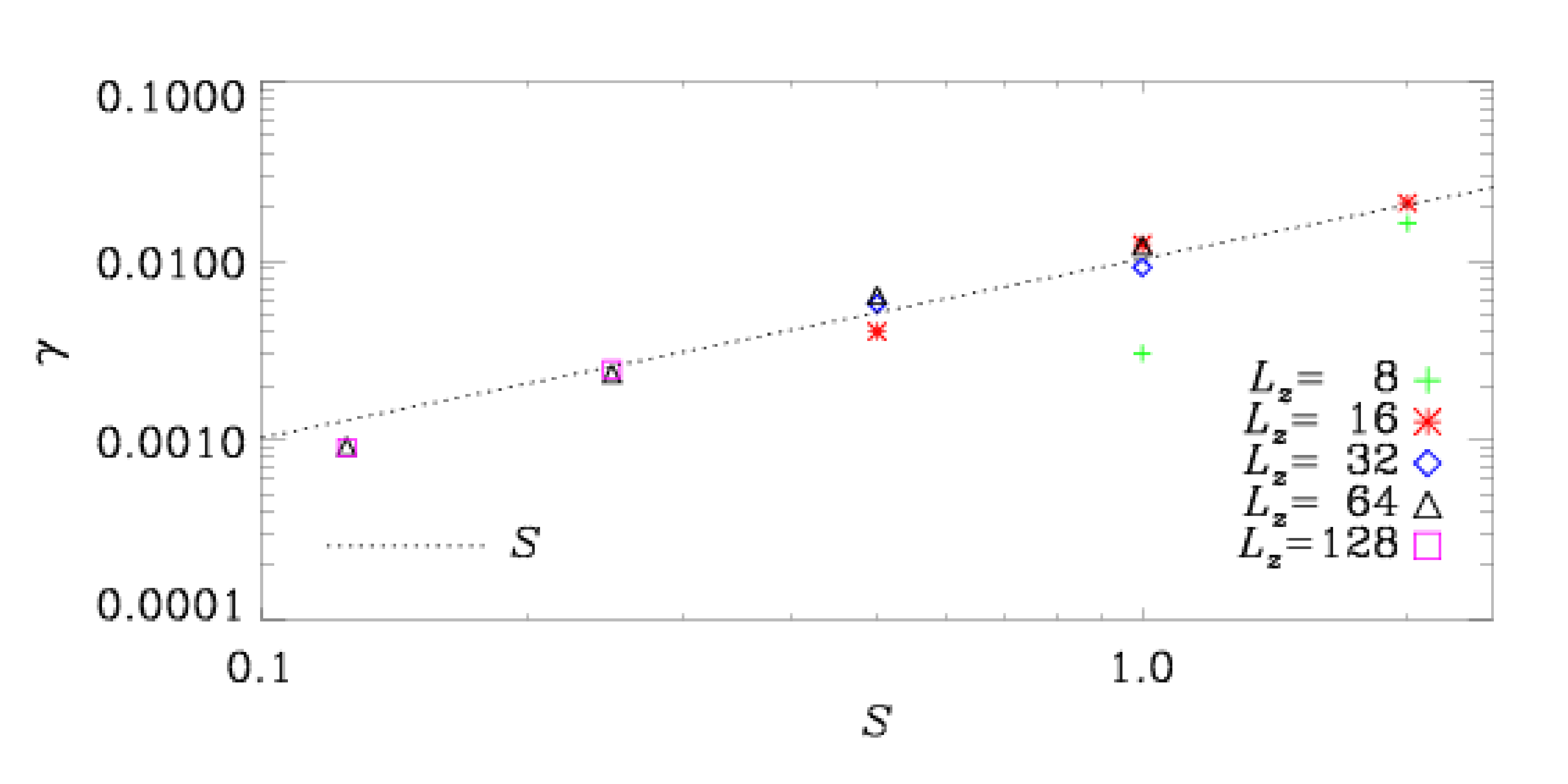
Linear shear velocity (DNS)

Generated field is large scale



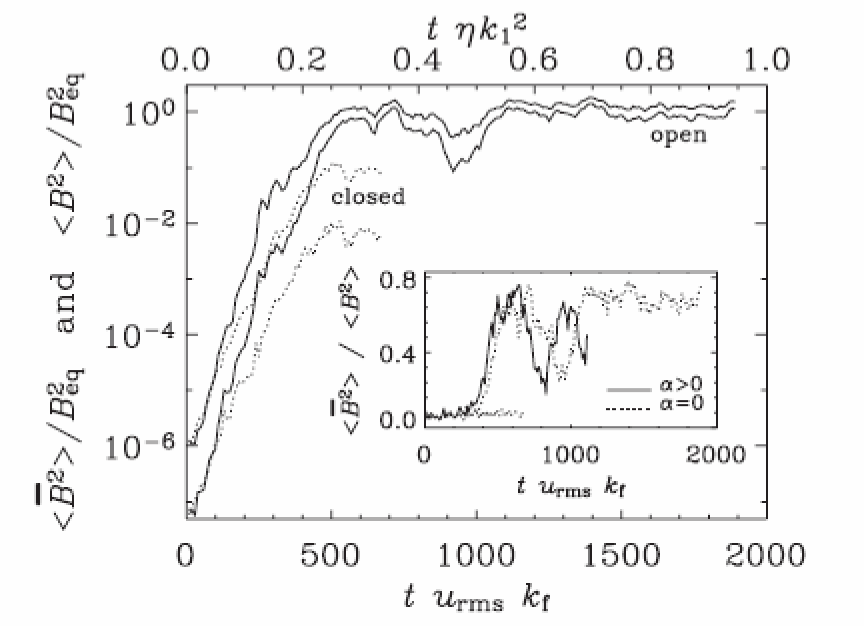
$$\overline{\mathbf{B}}(z) = \sum_{|k_z|<1} \mathbf{B}(k_x = 0, k_y = 0, k_z)\, e^{ik_z z}$$

Growth rate ∝ S



Direct Numerical Simulations (nonlinear shear velocity)

- A. Brandenburg, *Astrophys. J.* **625**, 539-547 (2005).
- A. Brandenburg, N.E.L. Haugen, P.J. Käpylä, C. Sandin, *Astron. Nachr.* ***326**, 174-185 (2005).*



1. *Non-helical constant in time sinusoidal forcing (imposed nonlinear mean velocity shear)*
2. *Open boundary conditions (non-zero flux of magnetic helicity)*

Mean-Field Dynamo

Do one need a non-zero mean kinetic helicity in order to generate a large-scale magnetic field in a homogeneous turbulent flow ?

Effect of Kinetic Helicity Fluctuations with zero mean on Mean-Field Dynamo

Kraichnan (1967)

- **Small-scale turbulence** **(forcing** F_u **)**

$$l_\nu << l_{turb} << l_0 \qquad (\tau_\nu << \tau_{turb} << \tau_0)$$

- **Kinetic helicity fluctuations** $\tilde{\alpha}$ **(forcing** F_χ **)**

$$l_0 << l << l_\chi \qquad (\tau_0 << \tau << \tau_\chi)$$

- **Mean-field effects:** $\langle \tilde{\alpha} \rangle_\alpha = 0$

$$\langle \tilde{\alpha}^2 \rangle_\alpha \neq 0$$

$$L >> l_\chi \qquad (\tau_L >> \tau_\chi)$$

Kinetic Helicity Fluctuations

R. H. Kraichnan, J. Fluid Mech. **77**, 753 (1976).

H. K. Moffatt, Magnetic Field Generation in Electrically Conducting Fluids (1978).

E. T. Vishniac and A. Brandenburg, Astrophys. J. **475**, 263 (1997).

D. D. Sokolov, Astron. Reports **41**, 68 (1997).

N. A. Silant'ev, Astron. Astrophys. 364, 339 (2000).

S. Fedotov, Phys. Rev. E **68**, 067301 (2003).

S. Fedotov, I. Bashkirtseva and L. Ryashko, Phys. Rev. E 73, 066307 (2006).

M. R. E. Proctor, Mon. Not. R. Astron. Soc.: Lett. **382**, L39 (2007).

Kinetic Helicity Fluctuations (tau-approach and SOCA)

N. Kleeorin and I. Rogachevskii, *Phys. Rev. E* ***77,*** 036307 (2008)

- **Negative contribution to turbulent magnetic diffusion**

$$\eta_T^{(\alpha)} = -\tau_\chi \left\langle \tilde{\alpha}^2 \right\rangle_\alpha$$

$$\tau_\chi = \frac{l_\chi^2}{\eta_T + \eta}$$

- **Large-scale drift velocity**

$$\mathbf{V}_{eff}^{(\alpha)} = \frac{1}{2} \tau_\chi \nabla \left\langle \tilde{\alpha}^2 \right\rangle_\alpha$$

- **Mean alpha effect due to large-scale shear and inhomogeneous kinetic helicity fluctuations, where** $\left\langle \tilde{\alpha} \right\rangle_\alpha = 0$

$$\overline{\alpha}^{(\alpha,S)} = -\frac{1}{2} \tau_\chi^2 (\mathbf{W}^S \cdot \nabla) \left\langle \tilde{\alpha}^2 \right\rangle_\alpha$$

Large-scale vorticity due to shear S:

$$\mathbf{W}^S = (0,0,S)$$

$$\mathbf{U}^S = (0,Sx,0)$$

Homogeneous Kinetic Helicity Fluctuations in Turbulence with Large-Scale Shear (tau-approach and SOCA)

N. Kleeorin and I. Rogachevskii, *Phys. Rev. E* ***77,*** 036307 (2008)

Mean-field equations: $\mathbf{B} = (B_x(z), B_y(z), 0)$

$$\frac{\partial B_x}{\partial t} = \sigma_\alpha \, S \, l_\chi^2 \, B_y'' + (\eta + \eta_{_T}) \, B_x'' \, ,$$
$$\frac{\partial B_y}{\partial t} = S \, B_x + (\eta + \eta_{_T}) \, B_y'' \, ,$$

$$\gamma_{_B} = S \, l_\chi \sqrt{-\sigma_\alpha} \, K_z - (\eta + \eta_{_T}) \, K_z^2$$

$$l_\chi = \tau_\chi \sqrt{\langle \tilde\alpha^2 \rangle^{(\alpha)}}$$

No large-scale dynamo:

$$\langle \tilde\alpha(t) \tilde\alpha(t+\tau) \rangle^{(\alpha)} \propto \exp(-\tau/\tau_\chi)$$

$$\sigma_\alpha = \int \frac{E_\alpha(k)}{[1 + \tau_\chi \, (\eta + \eta_{_T}) \, k^2]^2} \, dk > 0$$

Homogeneous Kinetic Helicity Fluctuations in Turbulence with Large-Scale Shear

Mean-field equations:

$$\frac{\partial B_x}{\partial t} = \sigma_\alpha \, S \, l_\chi^2 \, B_y'' + (\eta + \eta_T) \, B_x'' \, ,$$
$$\frac{\partial B_y}{\partial t} = S \, B_x + (\eta + \eta_T) \, B_y'' \, ,$$

$$\gamma_B = S \, l_\chi \sqrt{-\sigma_\alpha} \, K_z - (\eta + \eta_T) \, K_z^2$$

$$l_\chi = \tau_\chi \sqrt{\langle \tilde{\alpha}^2 \rangle^{(\alpha)}}$$

There can be dynamo when:

$$\langle \tilde{\alpha}(t) \tilde{\alpha}(t+\tau) \rangle^{(\alpha)} \propto \exp(-\tau/\tau_\chi) \, \cos(\omega_{\rm w} \tau)$$

$$\sigma_\alpha = \int \frac{[1 + \tau_\chi \, (\eta + \eta_T) \, k^2]^2 - (\omega_{\rm w} \tau_\chi)^2}{\{[1 + \tau_\chi \, (\eta + \eta_T) \, k^2]^2 + (\omega_{\rm w} \tau_\chi)^2\}^2} \, E_\alpha(k) \, dk$$

$$\omega_{\rm w} \tau_\chi > 1 + \tau_\chi \, (\eta + \eta_T) \, k^2, \qquad \rightarrow \qquad \sigma_\alpha < 0$$

Mean-Field Dynamo

Do one need a non-zero kinetic helicity in order to generate a large-scale magnetic field in a homogeneous turbulent flow ?

Alpha-Omega Dynamo (Mean-Field Approach)

- Induction equation for **mean magnetic field**:

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times \left(\mathbf{U} \times \mathbf{B} + \langle \mathbf{u} \times \mathbf{b} \rangle - \eta \nabla \times \mathbf{B} \right)$$

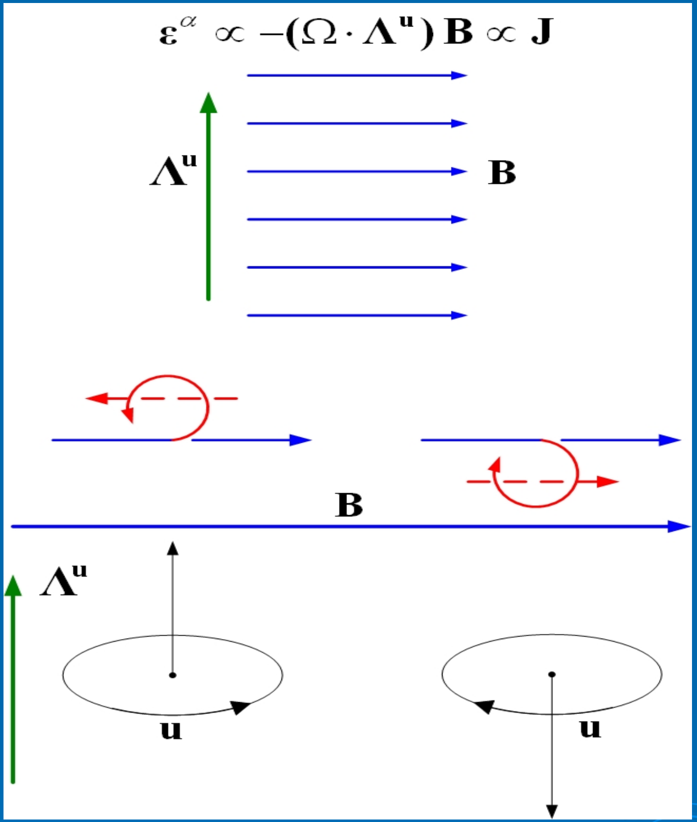
- **Electromotive force**:

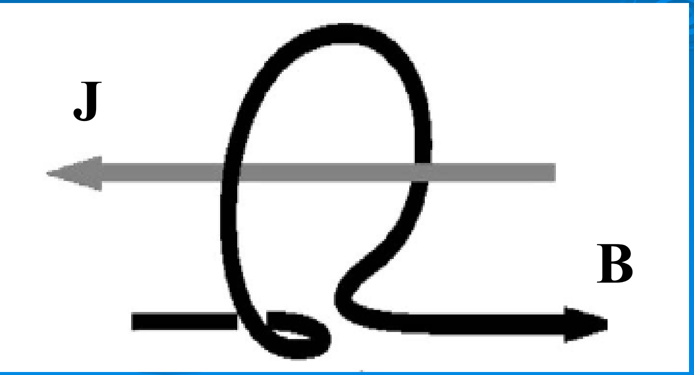
$$\boldsymbol{\varepsilon} \equiv \langle \mathbf{u} \times \mathbf{b} \rangle = \boldsymbol{\alpha} \mathbf{B} - \boldsymbol{\eta}_T \nabla \times \mathbf{B} + \ldots$$

$$\alpha \propto -\tau \langle \mathbf{u} \cdot (\nabla \times \mathbf{u}) \rangle$$

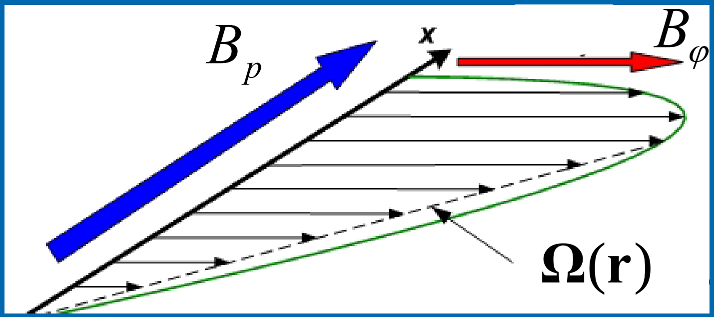
Physics of the alpha-effect

- The α -effect is related to the hydrodynamic helicity in an inhomogeneous turbulence.
- The deformations of the magnetic field lines are caused by upward and downward rotating turbulent eddies.
- The inhomogeneity of turbulence breaks a symmetry between the upward and downward eddies.
- Therefore, the total effect of the upward and downward eddies on the mean magnetic field does not vanish and it creates the mean electric current parallel to the original mean magnetic field.



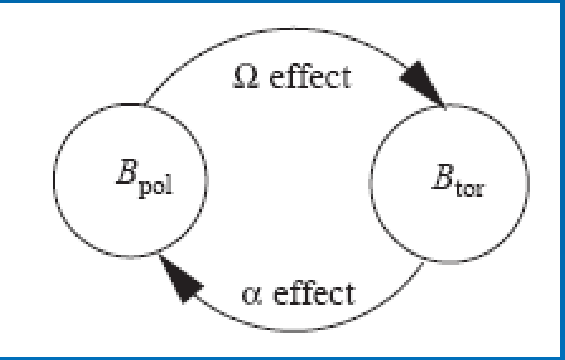


Generation of the mean magnetic field due to the $\alpha\Omega$ dynamo



Mean magnetic field:

$$\mathbf{B} = B_\varphi\, \mathbf{e}_\varphi + \nabla \times [A\, \mathbf{e}_\varphi]$$



$$\frac{\partial B_\varphi}{\partial t} = D\, [\nabla \Omega \times \nabla A]_\varphi + \Delta_s B_\varphi$$

$$\frac{\partial A}{\partial t} = \alpha\, B_\varphi + \Delta_s A$$

Dynamo number:

$$D = R_\alpha R_\Omega$$

$$\Delta_s = \Delta - \frac{1}{r^2 \sin^2 \theta}$$

Mean-Field Dynamo

Do one need alpha effect in order to generate a large-scale magnetic field in a homogeneous turbulent flow ?

Non-Helical Mean-Field Dynamos

- **Induction equation for mean magnetic field:**

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times \left(\mathbf{U} \times \mathbf{B} + \langle \mathbf{u} \times \mathbf{b} \rangle - \eta \nabla \times \mathbf{B} \right)$$

- **Electromotive force:**

$$\boldsymbol{\varepsilon} \equiv \langle \mathbf{u} \times \mathbf{b} \rangle = -\boldsymbol{\eta}_T \nabla \times \mathbf{B} - \mu \mathbf{W} \times \mathbf{J} - \boldsymbol{\kappa} \partial \mathbf{B}$$

Comparison of the alpha-effect with the "shear-current" effect

- **The α effect is caused by a uniform rotation and inhomogeneity of turbulence:**

$$\boldsymbol{\varepsilon}^{\alpha} = \alpha \, \mathbf{B} \propto - l^2 \, (\boldsymbol{\Omega} \cdot \boldsymbol{\Lambda}^u) \, \mathbf{B} \quad , \quad \text{where} \quad \boldsymbol{\Lambda}^u = \frac{\nabla \langle u^2 \rangle}{\langle u^2 \rangle}$$

- **The "shear-current" effect is related to $\mathbf{W} \times \mathbf{J}$ term and is caused by mean shear and nonuniform mean magnetic field,**

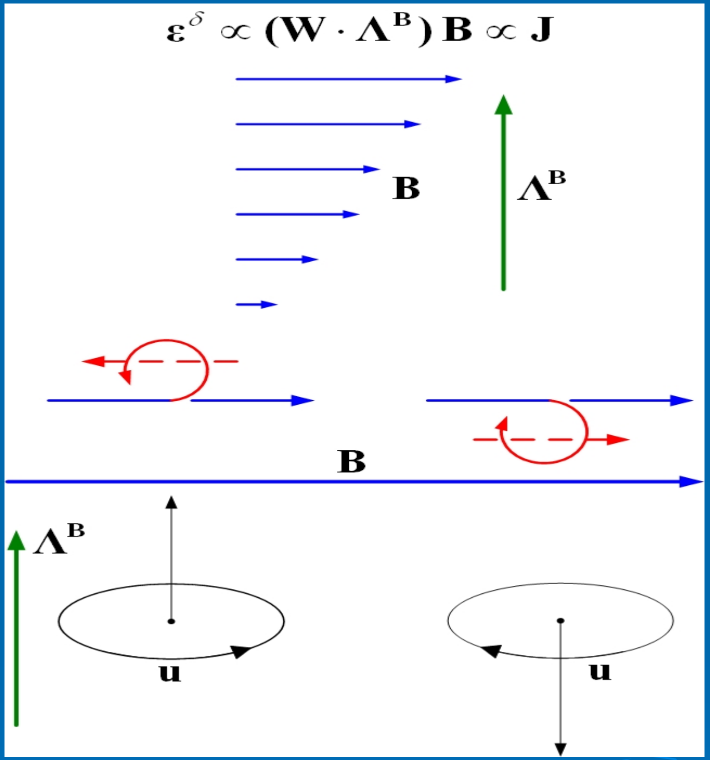
$$\boldsymbol{\varepsilon}^{\delta} \propto - l^2 \, \mathbf{W} \times \mathbf{J} \propto l^2 \, (\mathbf{W} \cdot \boldsymbol{\Lambda}^B) \, \mathbf{B} \; , \quad \text{where} \quad \boldsymbol{\Lambda}^B = \frac{\nabla \mathbf{B}^2}{\mathbf{B}^2}$$

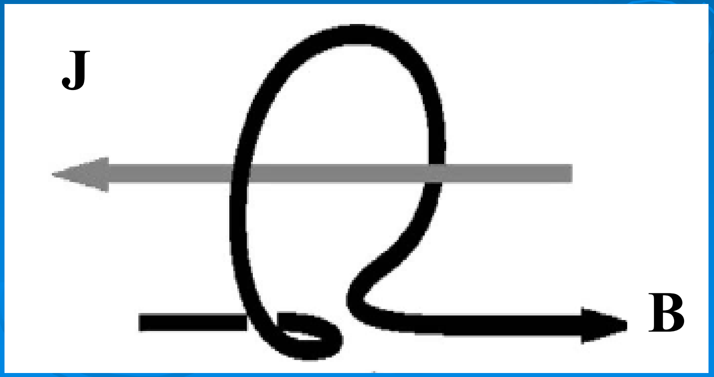
- **Therefore,** $\mathbf{W} \to \boldsymbol{\Omega}$ $\quad \boldsymbol{\Lambda}^B \to \boldsymbol{\Lambda}^u$

Physics of "shear-current" effect

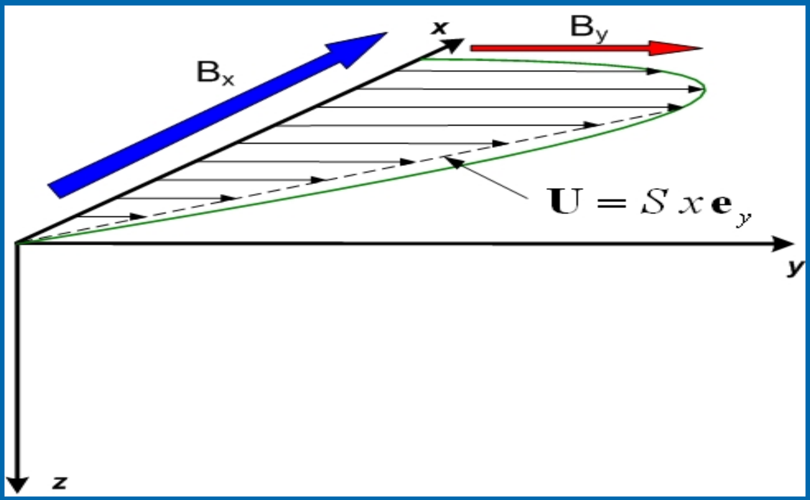
- In a turbulent flow with the mean velocity shear, the inhomogeneity of the original mean magnetic field breaks a symmetry between the influence of the upward and downward turbulent eddies on the mean magnetic field.

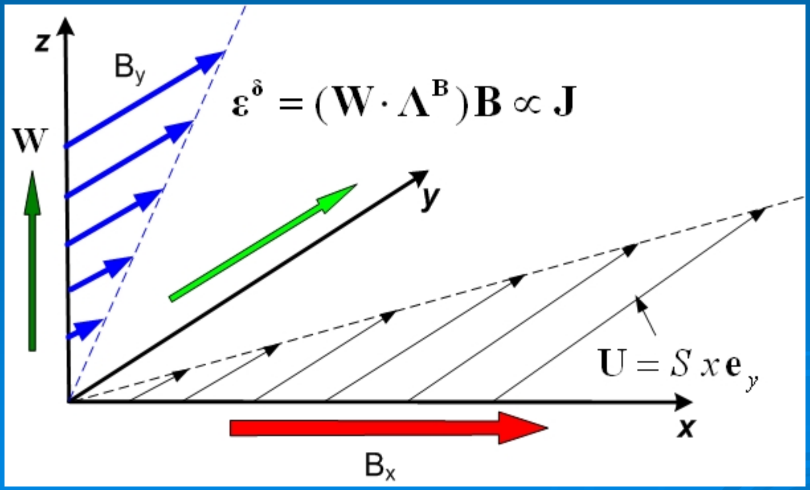
- The deformations of the magnetic field lines in the "shear-current" dynamo are caused by the upward and downward turbulent eddies which result in the mean electric current parallel to the mean magnetic field and produce the magnetic dynamo.





The effect of large-scale shear





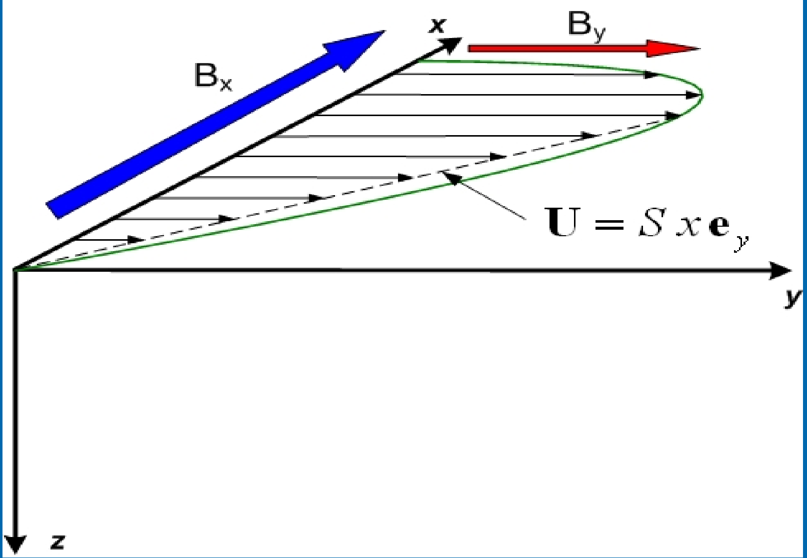
- The **large-scale shear motions** cause the **stretching** of the magnetic field B_x generating the field component B_y

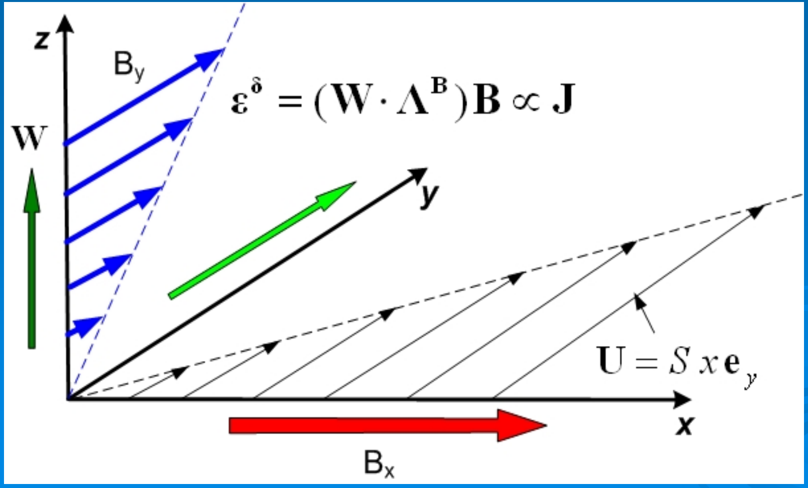
$$B_x \Rightarrow B_y$$

- The **interaction** of the non-uniform magnetic field B_y with the background vorticity $\mathbf{W}$ produces **electric current** along the field $\boldsymbol{\varepsilon}^{\delta} \propto -l^2 \mathbf{W}\times(\nabla\times\mathbf{B})$

$$B_y \Rightarrow B_x$$

Generation of the mean magnetic field due to the shear-current effect





Mean velocity shear:

$$\mathbf{U} = S\,x\,\mathbf{e_y} \qquad \mathbf{W} = S\,\mathbf{e_z}$$

$$\frac{\partial B_Y(t,z)}{\partial t} = S\,B_X + \eta_T\,B_Y''$$

$$\frac{\partial B_X(t,z)}{\partial t} = -\sigma_B\,S\,l^2 B_Y'' + \eta_T\,B_X''$$

The growth rate of B

$$\gamma = \sqrt{\sigma_B}\;S\,l\,K_Z - \eta_T K_Z^2$$

Necessary condition for the shear-current dynamo

The parameter σ_B:

$$\sigma_B \propto 1 + \frac{\tau'(k)}{\tau(k)} + c_1 \varepsilon_m + c_2 a_* > 0$$

$$a_* = \frac{2 g \tau_0 \, |\langle u_z \theta \rangle|}{\langle u^2 \rangle}$$

The growth rate of B:

$$\gamma = \sqrt{\sigma_B} \, S \, l_0 \, K_z - \eta_T \, K_z^2$$

The Kolmogorov Scaling (large Re and Rm):

$$\tau(k) \propto k^{-2/3}; \qquad \sigma_B = \frac{4}{135}(1 + 7\,\varepsilon_m + a_*) > 0; \qquad \varepsilon_m = \frac{\langle b^2 \rangle}{\langle u^2 \rangle}$$

Rogachevskii and Kleeorin (2003): there is shear-current dynamo

Small Re and Rm (random flow):

$$\tau(k) \propto \frac{1}{\nu k^2}, \frac{1}{\eta k^2}$$

$$\tau(k) \propto k^{-2}; \qquad \sigma_B \propto -1 + 5\varepsilon_m + c_2 \, a_*;$$

In an agreement with SOCA-results: for $\mathrm{Pr}_m \geq 1$ there is no dynamo for $\varepsilon_m = a_* = 0$

K.-H. Rädler and R. Stepanov, Phys. Rev. E **73**, 056311 (2006)

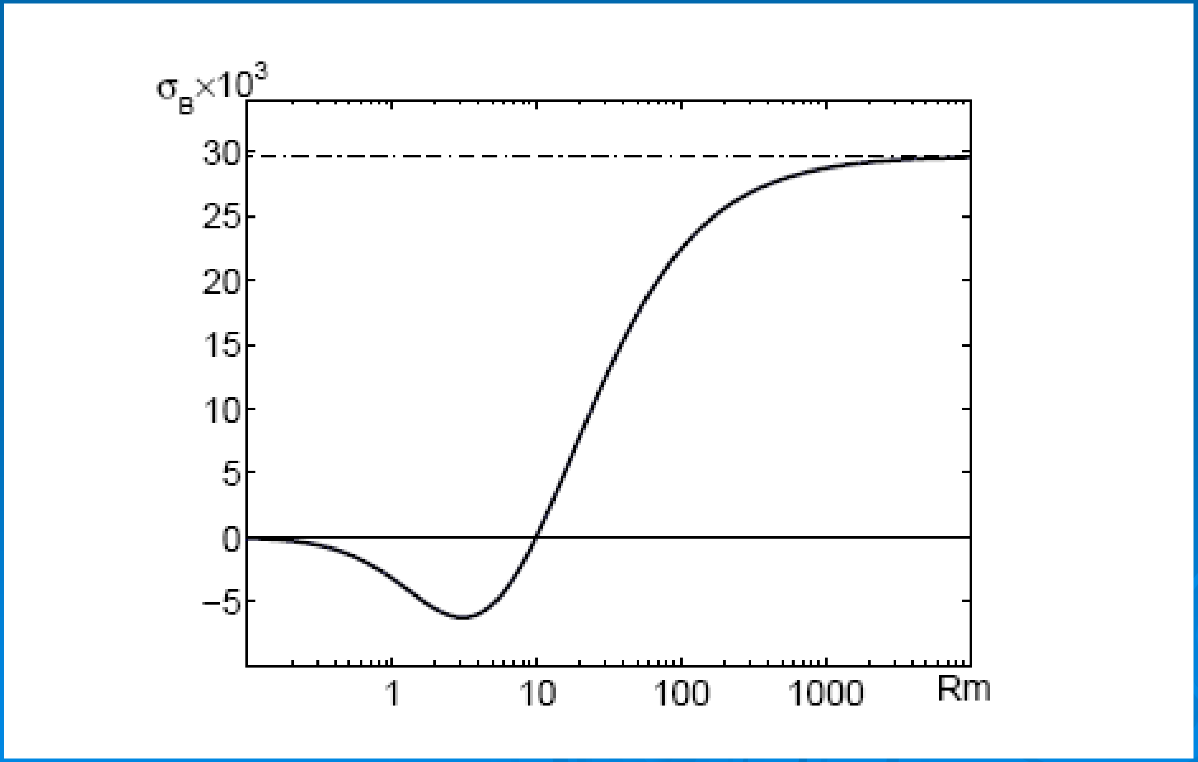
G. Rüdiger and L. L. Kitchatinov, Astron. Nachr. **327**, 298 (2006)

Necessary condition for the shear-current dynamo

Large Re and arbitrary Rm

$$\tau^{-1}(k) = \eta k^2 + \tau_0^{-1} k^{2/3}$$

$$\gamma = \sqrt{\sigma_B}\, S\, l\, K_z - \eta_T K_z^2$$

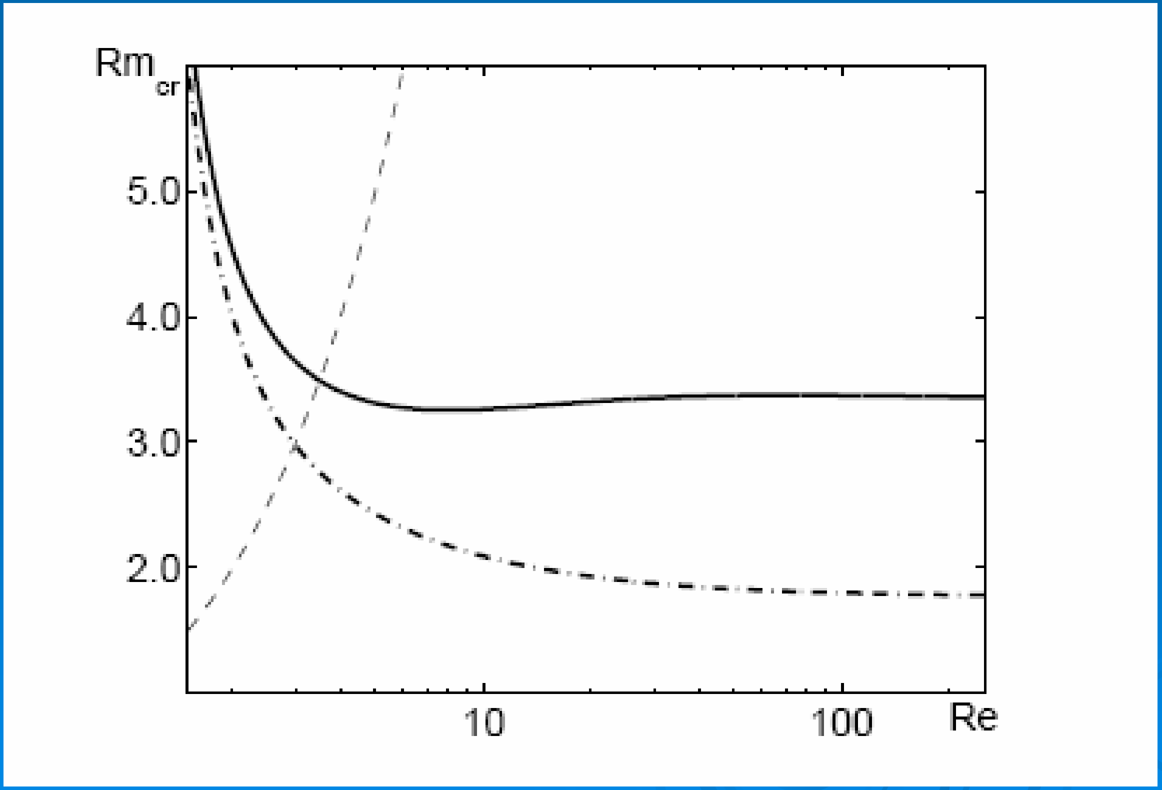


$$\mathrm{Rm}_{\mathrm{cr}} \approx 10$$

Necessary condition for the shear-current dynamo

Re > 1 and arbitrary Rm

$$\tau^{-1}(k) = (\nu + \eta) k^2 + \tau_0^{-1}$$



$$\gamma = \sqrt{\sigma_B}\, S\, l\, K_z - \eta_T K_z^2$$

$$\mathrm{Rm}_{cr} \approx 2 \text{ to } 3$$

Necessary condition for the shear-current dynamo

Kraichnan - Kazantsev model:

δ - correlated in time random velocity field

There is no shear-current dynamo

The shear-current dynamo (as well as effect of shear) requires finite correlation time of turbulent velocity field

Generation of the mean magnetic field (kinematic dynamo)

$$S_* = \frac{S\,L^2}{\eta_T}$$

$$\frac{\partial B_y}{\partial t} = -A' + B_y''$$

$$\frac{\partial A}{\partial t} = D\,B_y' + A''$$

$$\mathbf{B} = B_y\,\mathbf{e_y} - \frac{1}{S_*}A'\,\mathbf{e_x}$$

The growth rate of B:

$$\gamma = \sqrt{D}\,K_z - K_z^2$$

Critical dynamo number:

$$D = \left(\frac{l}{L}\right)^2 S_*^2\,\sigma_B > D_{cr} = \frac{\pi^2}{4}(1+2n)^2$$

$$n = 0,1,2,\ldots$$

Solution for the symmetric mode:

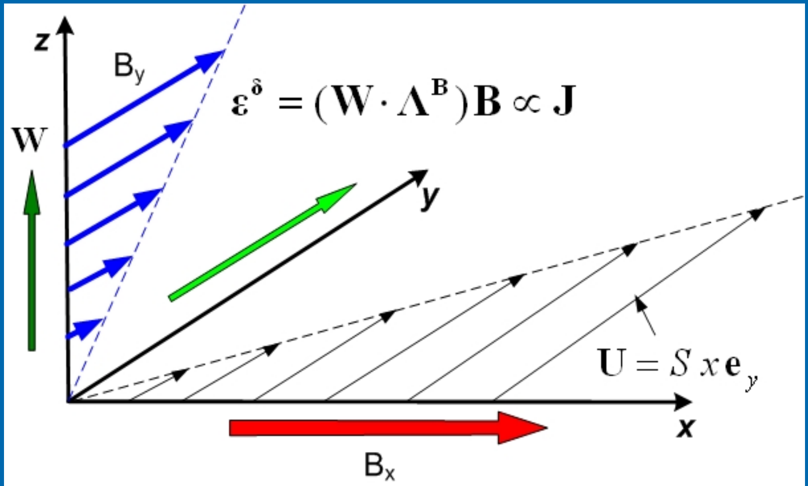
$$B_x(t,z) = \frac{l\sqrt{\sigma_B}}{L}\,K_z\,B_0\,\exp(\gamma\,t)\cos(K_z\,z)$$

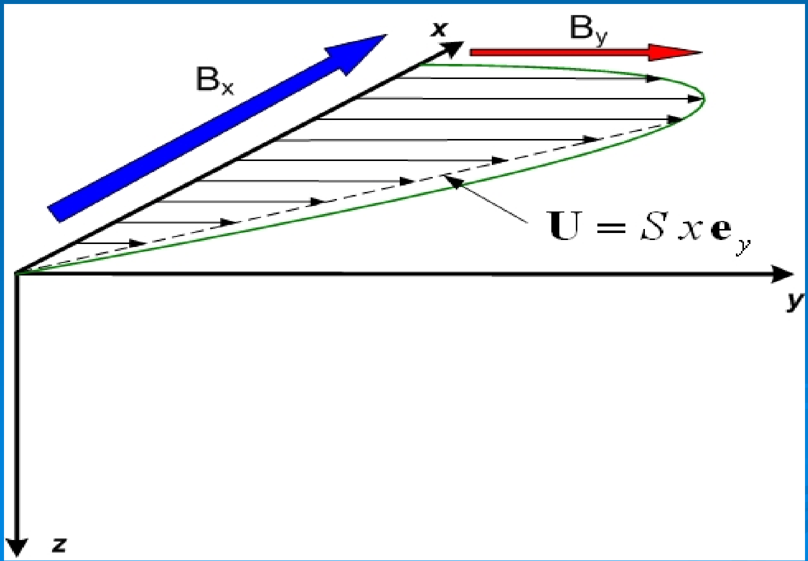
$$B_y(t,z) = B_0\,\exp(\gamma\,t)\cos(K_z\,z)$$

The magnetic scale at maximum γ :

$$L_B = \frac{4\pi}{\sqrt{D}}L$$

Anisotropic Turbulent Magnetic Diffusion





$$\frac{\partial B_X(t,z)}{\partial t} = -S\,l^2\,\sigma_B\,B_Y'' + \eta_T\,B_X''$$

$$\frac{\partial B_Y(t,z)}{\partial t} = S\,B_X + \eta_T\,B_Y''$$

$$\eta_{ij}^{T} = \begin{pmatrix} \eta_T & -S\,l^2\,\sigma_B \\ 0 & \eta_T \end{pmatrix}$$

Magnetic Field and Vorticity

- Induction equation for magnetic field:

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times \left(\mathbf{v} \times \mathbf{B} - \eta \, \nabla \times \mathbf{B} \right)$$

- Equation for vorticity:

$$\frac{\partial \mathbf{W}}{\partial t} = \nabla \times \left(\mathbf{v} \times \mathbf{W} - \nu \, \nabla \times \mathbf{W} \right)$$

$$\mathbf{W} = \nabla \times \mathbf{v}$$

Generation of the mean vorticity and magnetic field in sheared turbulence

Mean velocity shear:

$$\mathbf{U} = S\, x\, \mathbf{e_y} \qquad \mathbf{W} = S\, \mathbf{e_z}$$

The mean magnetic field

$$\frac{\partial B_X}{\partial t} = -S\, l^2\, \sigma_B\, B_Y'' + \eta_T\, B_X''$$

$$\frac{\partial B_Y}{\partial t} = S\, B_X + \eta_T\, B_Y''$$

The mean vorticity

$$\frac{\partial \tilde{W}_X}{\partial t} = S\, \tilde{W}_Y + \nu_T\, \tilde{W}_X''$$

$$\frac{\partial \tilde{W}_Y}{\partial t} = -S\, l^2\, \sigma_W\, \tilde{W}_X'' + \nu_T\, \tilde{W}_Y''$$

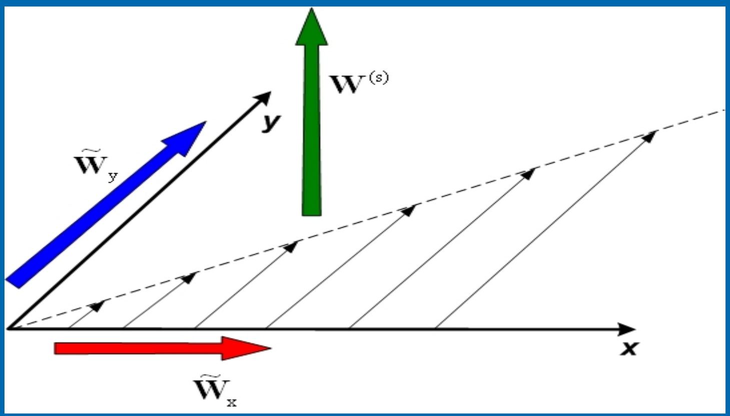
The growth rate of B

$$\gamma = \sqrt{\sigma_B}\, S\, l\, K_Z - \eta_T K_Z^2$$

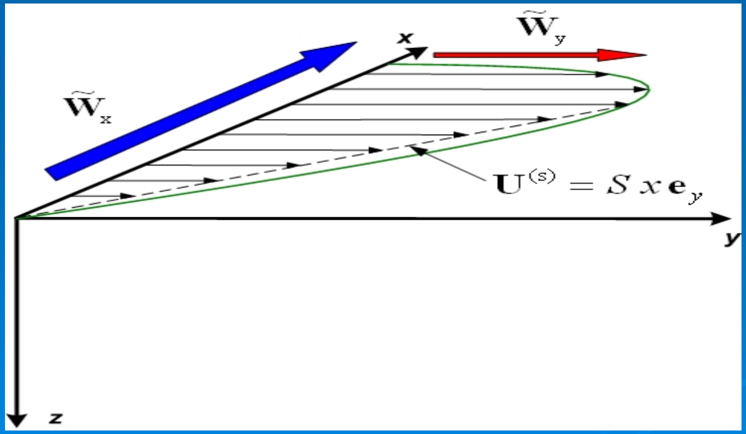
The growth rate of $\tilde{\mathbf{W}}$

$$\gamma = \sqrt{\sigma_W}\, S\, l\, K_Z - \nu_T K_Z^2$$

Generation of the mean vorticity in turbulence with mean velocity shear



$$S\,\tilde{W}_Y\,\mathbf{e}_x = (\mathbf{W}^{(s)}\cdot\nabla)\,\tilde{\mathbf{U}}_X \propto \tilde{\mathbf{W}}_Y \times \mathbf{W}^{(s)}$$



$$-l^2(\tilde{\mathbf{W}}_X''\cdot\nabla)\,\mathbf{U}^{(s)} \propto -l^2 S\,\tilde{W}_x''\,\mathbf{e}_y$$

Elperin, Kleeorin and Rogachevskii,
PRE, 68, 016311 (2003)

Mean velocity shear:

$$\mathbf{U}^{(s)} = S\,x\,\mathbf{e}_y \qquad \mathbf{W}^{(s)} = S\,\mathbf{e}_z$$

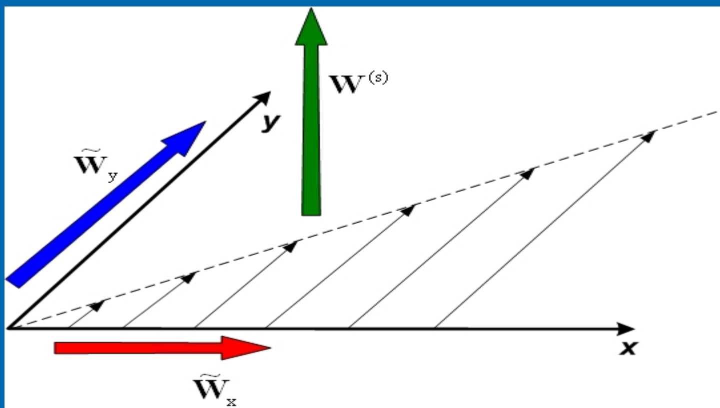
$$\frac{\partial \tilde{W}_X(t,z)}{\partial t} = S\,\tilde{W}_Y + \nu_T\,\tilde{W}_X''$$

$$\frac{\partial \tilde{W}_Y(t,z)}{\partial t} = -S\,l^2\,\sigma_W\,\tilde{W}_X'' + \nu_T\,\tilde{W}_Y''$$

The growth rate of the mean vorticity

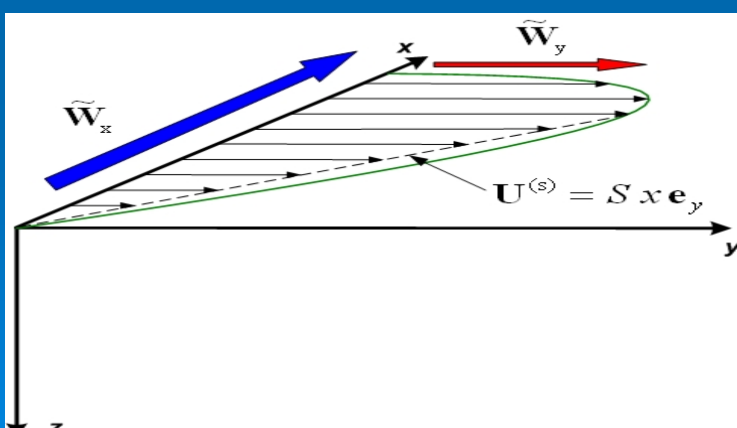
$$\gamma = \sqrt{\sigma_W}\;S\,l\,K_z - \nu_T K_z^2$$

Anisotropic Turbulent Viscosity



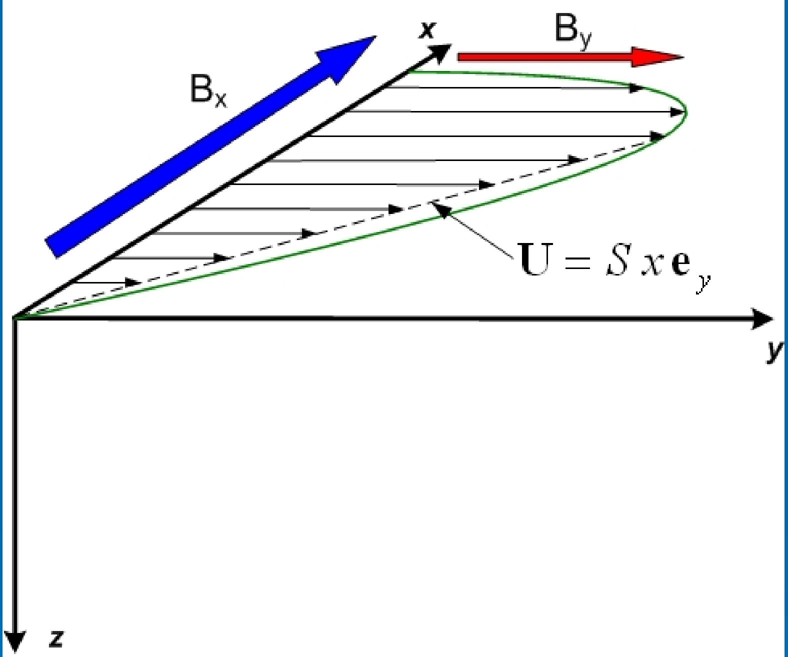
$$\frac{\partial \widetilde{W}_X(t,z)}{\partial t} = S\,\widetilde{W}_Y + \nu_T\,\widetilde{W}_X''$$

$$\frac{\partial \widetilde{W}_Y(t,z)}{\partial t} = -S\,l^2\,\sigma_W\,\widetilde{W}_X'' + \nu_T\,\widetilde{W}_Y''$$



$$\nu_{ij}^T = \begin{pmatrix} \nu_T & 0 \\ -S\,l^2\sigma_W & \nu_T \end{pmatrix}$$

The shear-current nonlinear dynamo (algebraic nonlinearity)



$$\frac{\partial B_y}{\partial t} = -A' + B_y''$$

$$\frac{\partial A}{\partial t} = D\,\sigma_N(\mathbf{B})\;B_y' + A''$$

Dynamo number:

$$D = \left(\frac{l}{L}\right)^2 S_*^2\;\sigma_B$$

Shear number:

$$S_* = \frac{S\,L^2}{\eta_T}$$

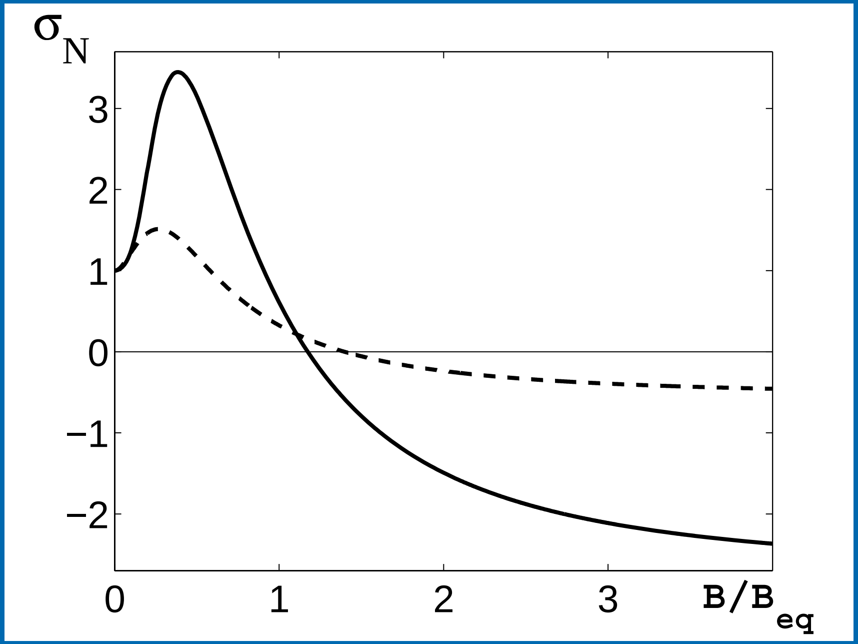
Mean magnetic field:

$$\mathbf{B} = B_y(t,z)\,\mathbf{e}_y - \frac{1}{S_*}[A'(t,z)\,\mathbf{e}_x]$$

Nonlinear shear-current effect:

$$\sigma_N(\mathbf{B}) = \frac{\sigma(\mathbf{B})}{\sigma_B}$$

Nonlinear shear-current effect



Weak magnetic field:

$$\sigma_N(\mathbf{B}) \approx 1$$

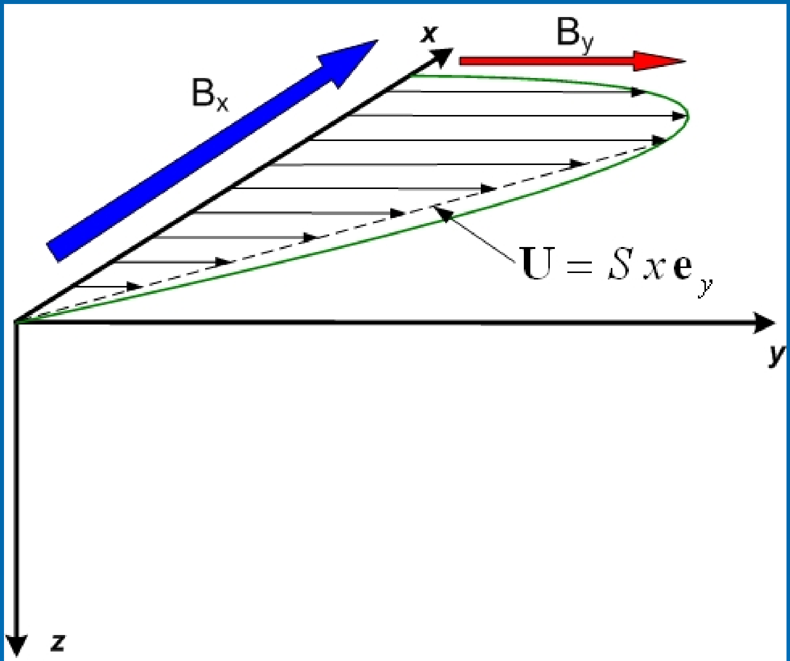
Strong mean magnetic field:

$$\sigma_N(\mathbf{B}) \approx -\frac{4}{11}\left(\frac{1+\varepsilon_m}{1+7\varepsilon_m}\right)$$

There is no quenching of the nonlinear "shear-current" effect contrary to the quenching of the nonlinear alpha effect, the nonlinear turbulent magnetic diffusion, etc.

$$\varepsilon_m = \frac{\langle \mathbf{b}^2 \rangle}{\langle \mathbf{u}^2 \rangle}$$

The shear-current nonlinear dynamo (algebraic nonlinearity)



$$\frac{\partial B_y}{\partial t} = -A' + B_y''$$

$$\frac{\partial A}{\partial t} = D\,\sigma_N(\mathbf{B})\; B_y' + A''$$

Dynamo number:

$$D = \left(\frac{l}{L}\right)^2 S_*^2\; \sigma_0$$

Shear number:

$$S_* = \frac{S\,L^2}{\eta_T}$$

Mean magnetic field:

$$\mathbf{B} = B_y(t,z)\,\mathbf{e}_y - \frac{1}{S_*}[A'(t,z)\,\mathbf{e}_x]$$

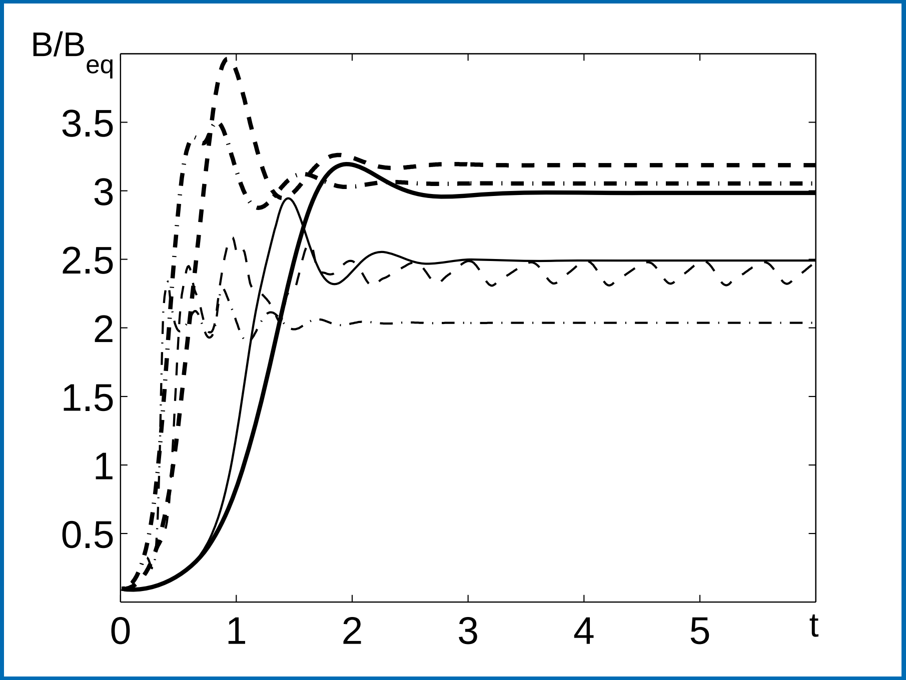
Nonlinear shear-current effect:

$$\sigma_N(\mathbf{B}) = \frac{\sigma(\mathbf{B})}{\sigma_0}$$

Nonlinear "shear-current" dynamo (algebraic nonlinearity)

$\varepsilon_m = 1\,(thick)$

$\varepsilon_m = 0\,(thin)$



$D = 50\,D_{cr}\quad (dashed - dotted)$

$D = 30\,D_{cr}\quad (dashed)$

$D = 10\,D_{cr}\quad (solid)$

Magnetic Helicity

Total magnetic helicity is conserved for very large magnetic Reynolds numbers

$$\chi_m^{\text{total}} = \mathbf{A}\cdot\mathbf{B} + \langle \mathbf{a}\cdot\mathbf{b} \rangle$$

Magnetic part of alpha effect:

$$\alpha_m(\mathbf{B}) = \Phi_N(B)\,\chi_c(\mathbf{B})$$

The nonlinear function:

$$\chi_c(\mathbf{B}) = \frac{2\langle \mathbf{a}\cdot\mathbf{b} \rangle}{9\mu\,\rho\,\eta_T}$$

Dynamics of small-scale magnetic helicity:

$$\frac{\partial \chi_c}{\partial t} + \operatorname{div}\mathbf{F} = -\left(\frac{2L}{l}\right)^2 \boldsymbol{\varepsilon}\cdot\mathbf{B} - \frac{\chi_c}{\tau_\chi}$$

$$\tau_\chi \propto \tau_0\,\mathrm{Rm}$$

Dynamics of magnetic helicity

$$\frac{\partial \chi_c}{\partial t} + \mathrm{div}\,\mathbf{F} = -\left(\frac{2L}{l}\right)^2 \boldsymbol{\varepsilon}\cdot\mathbf{B} - \frac{\chi_c}{\tau_\chi}$$

$$\chi_c(\mathbf{B}) = \frac{2\langle \mathbf{a}\cdot\mathbf{b}\rangle}{9\mu\,\rho\,\eta_T}$$

Kleeorin and Ruzmaikin (1982); Gruzinov and Diamond (1994); $\mathbf{F}=0$

Kleeorin and Rogachevskii (1999);

Kleeorin, Moss, Rogachevskii and Sokoloff (2000); $\mathbf{F}\neq 0$

Blackman and Field (2000); Vishniac and Cho (2001);

Brandenburg and Subramanian (2005); etc.

In the absence of the magnetic helicity flux,

$$\alpha_{total} = \frac{\alpha_u}{1+\mathrm{Rm}\,(\mathrm{B}/\mathrm{B}_{\mathrm{eq}})^2}$$

i.e., catastrophic quenching (Vainshtein and Cattaneo, 1992)

In the presence of the flux of magnetic helicity: $\mathbf{F}\neq 0$

$$\alpha_{total} = -\frac{\mathrm{div}\,\mathbf{F}}{(\mathrm{B}/\mathrm{B}_{\mathrm{eq}})^2}$$

$$\tau_\chi \propto \tau\,\mathrm{Rm}$$

The shear-current nonlinear dynamo (algebraic and dynamic nonlinearities)

$$\frac{\partial B_y}{\partial t} = -A' + B_y''$$

$$\frac{\partial A}{\partial t} = \alpha_m(\mathbf{B})\, B_y + D\,\sigma_N(\mathbf{B})\; B_y' + A''$$

$$\frac{\partial \chi_c}{\partial t} - \kappa_T \chi_c'' = \left(\frac{2L}{l}\right)^2 \left(A' B_y' - B_y \frac{\partial A}{\partial t}\right)$$

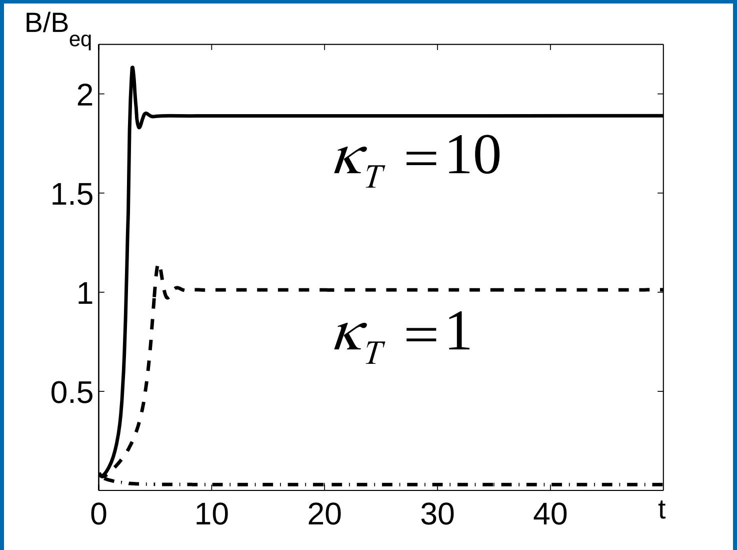
$$\alpha_m(\mathbf{B}) = \frac{1}{S_*} \chi_c(\mathbf{B})\, \Phi_N(B)$$

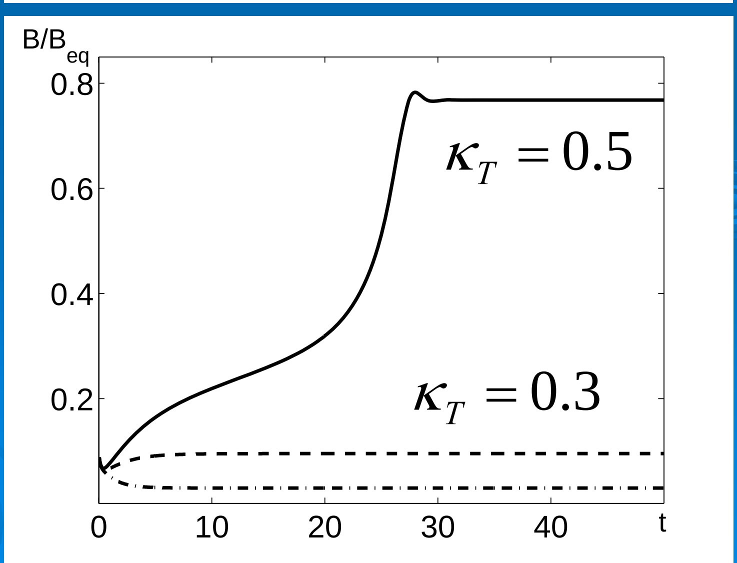
Mean magnetic field:

$$\mathbf{B} = B_y(z)\,\mathbf{e_y} - \frac{1}{S_*}[A'(z)\,\mathbf{e_x}]$$

$$D = 2\,D_{\rm cr}$$

$$L/l = 5$$





Direct Numerical Simulations (linear shear velocity)

T. A. Yousef, T. Heinemann, A.A. Schekochihin, N. Kleeorin, I. Rogachevskii, A.B. Iskakov, S.C. Cowley, J.C. McWilliams,
Phys. Rev. Lett., v.100, 184501 (2008)

1. *A white noise non-helical homogeneous and isotropic random forcing*
2. *Imposed mean linear shear flow*
3. *Sheared box (shear-periodic boundary conditions)*

Numerical set up

Incompressible MHD equations with background shear

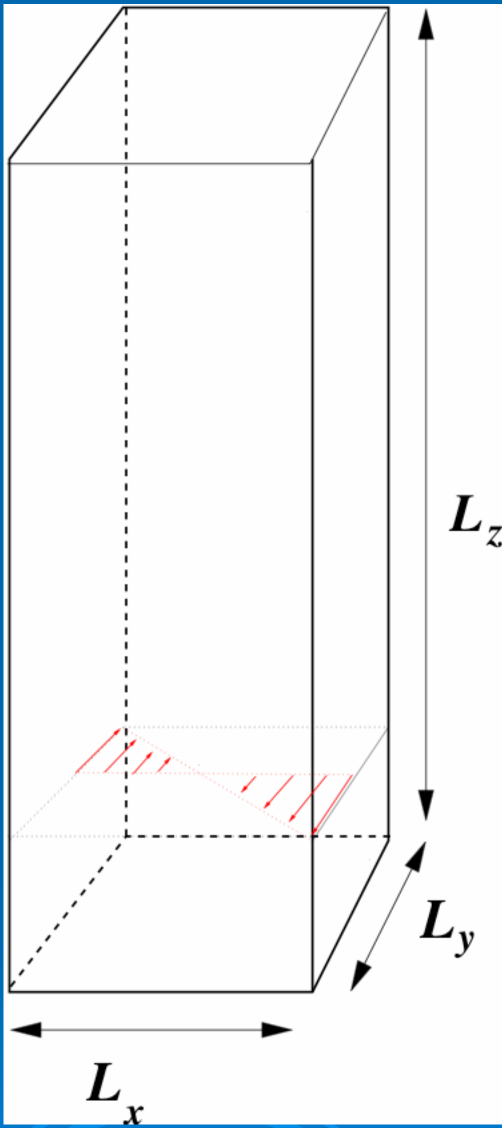
$$\frac{d\mathbf{u}}{dt} = u_x S\hat{\mathbf{y}} - \frac{\nabla p}{\rho} + \frac{\mathbf{B}\cdot\nabla\mathbf{B}}{4\pi\rho} + \nu\nabla^2\mathbf{u} + \mathbf{f},$$
$$\frac{d\mathbf{B}}{dt} = -B_x S\hat{\mathbf{y}} + \mathbf{B}\cdot\nabla\mathbf{u} + \eta\nabla^2\mathbf{B},$$

$$\nabla\cdot\mathbf{u} = 0$$

$$\mathbf{U} = -Sx\hat{\mathbf{y}}$$

$$d/dt = \partial_t - Sx\partial_y + \mathbf{u}\cdot\nabla$$

(Units: $\epsilon = \langle\mathbf{u}\cdot\mathbf{f}\rangle = 1$ $\quad L_x = L_y = 1$)



Parameters

$$\overline{\mathbf{B}} = (\overline{B}_x(z), \overline{B}_y(z), 0)$$

Turbulence: $\epsilon = \langle\mathbf{u}\cdot\mathbf{f}\rangle = 1 \Rightarrow u_{\mathrm{rms}} \sim 1$, $l_0 \sim \frac{1}{3}$ $\Rightarrow \tau \sim \frac{1}{3}$

$$\nu = \eta = 10^{-2} \Rightarrow \mathrm{Re} \sim 30$$

$$L_B > \frac{u_{rms}}{S} = \frac{l_0}{S\tau}$$

Weak shear: $S = 2,\ 1,\ \frac{1}{2},\ \frac{1}{4},\ \frac{1}{8}$ $\Rightarrow S\tau \sim (0.04\ldots0.6)$

Linear shear velocity (DNS)

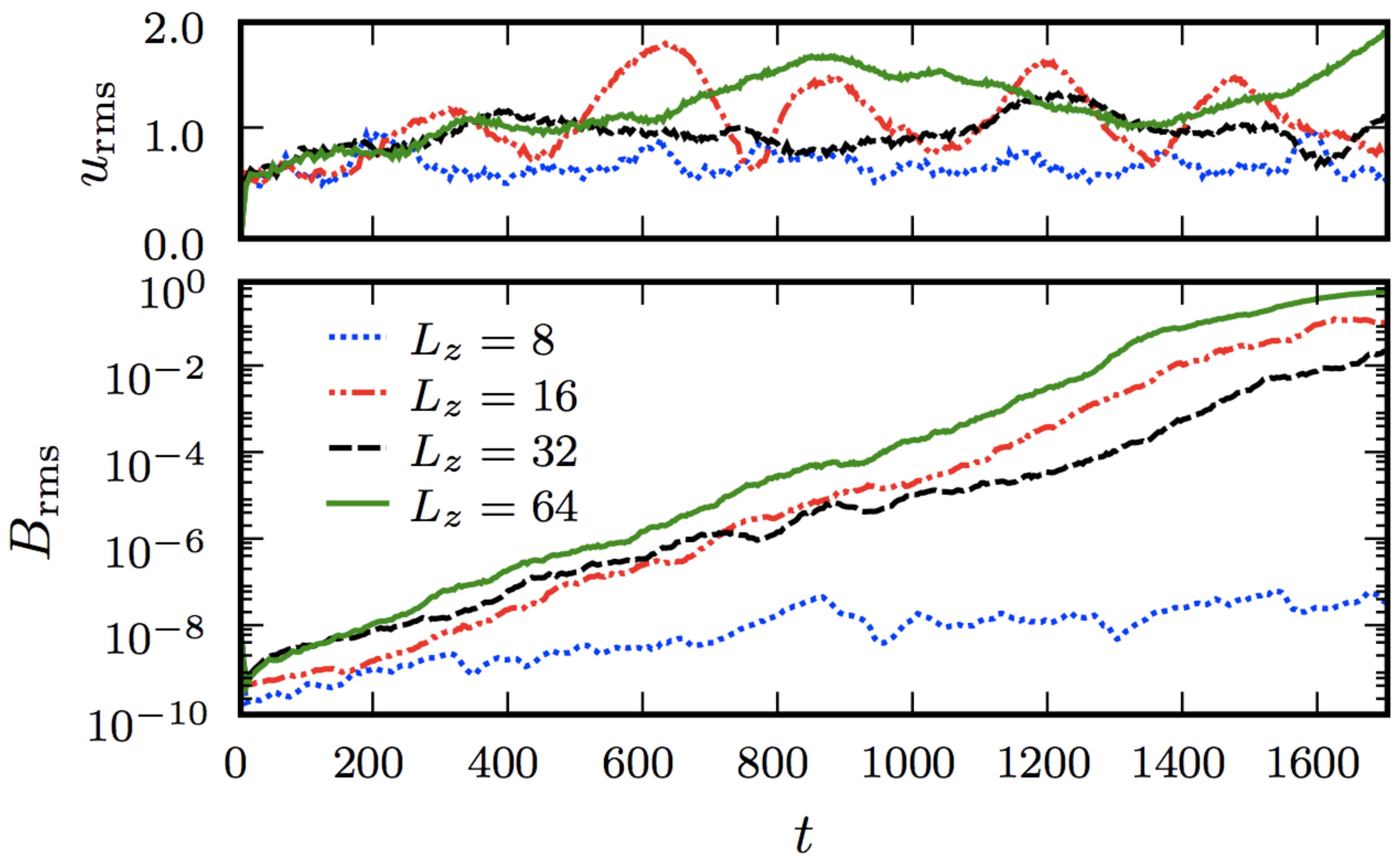
$$L_z >> L_x = L_y = 1$$

$$\overline{\mathbf{B}} = (\overline{B}_x(z), \overline{B}_y(z), 0)$$

$$L_x / l_0 = 3$$

$$l_{\overline{B}} > \frac{u_{rms}}{S}$$

Magnetic field grows



$$L_z >> L_x = L_y = 1$$

Linear shear velocity (DNS)

Linear shear velocity (DNS)

THEORY:
The growth rate of B

$$\gamma \propto S$$

$$\gamma_{\max} \propto S^2$$

$$\gamma = \sqrt{\sigma_B}\, S\, l_0\, K_Z - \eta_T K_Z^2$$

Linear shear velocity (DNS)

THEORY:

The maximum growth rate of B is at

$$\left(l_{\overline{B}}\right)_{\gamma_{\max}} \propto S^{-1}$$

$$l_{\overline{B}} > \frac{u_{rms}}{S}$$

Astrophysical clouds

- We apply the universal mechanism of generation of large-scale magnetic fields due to shear-current effect to several astrophysical objects:

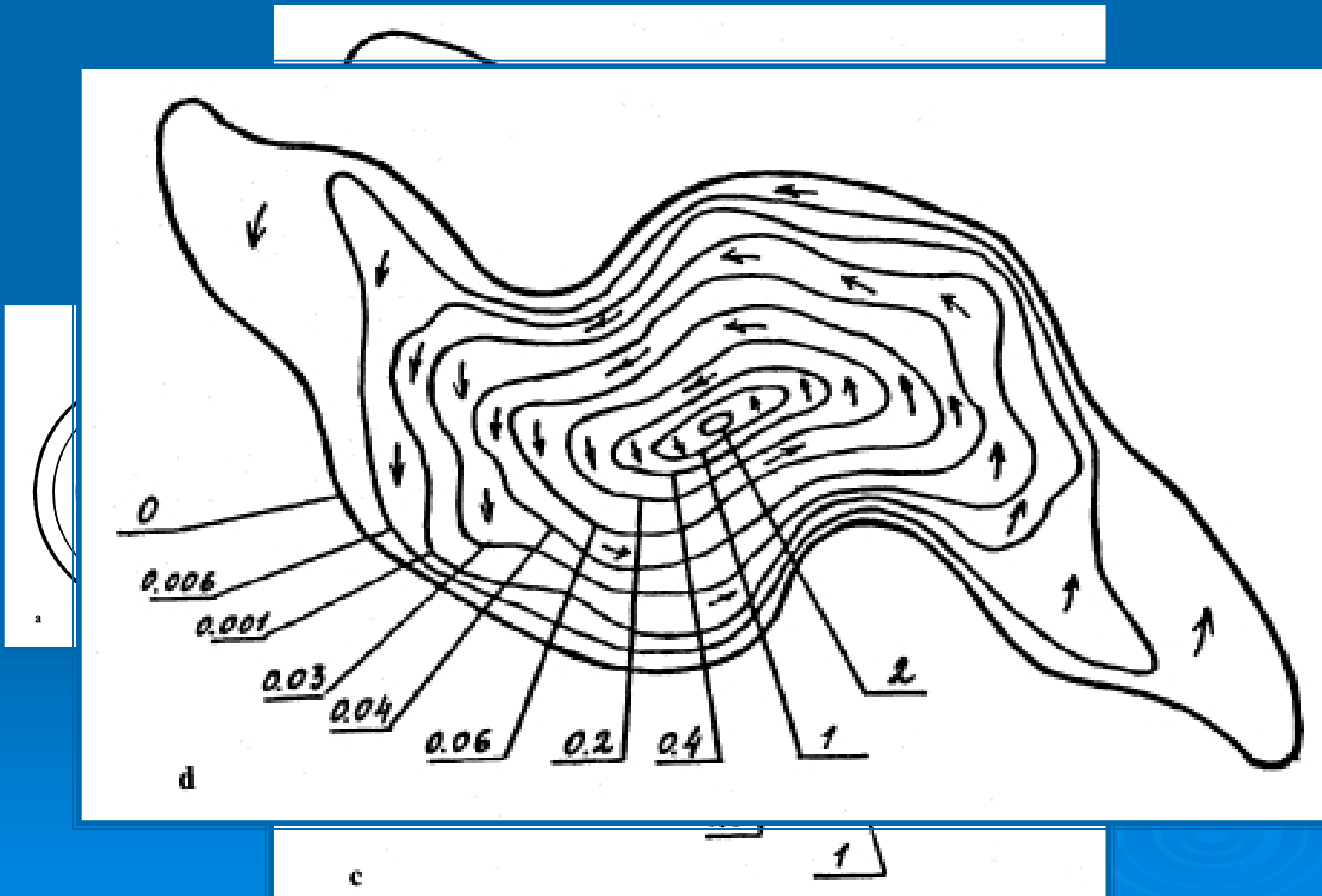
  merging protostellar clouds

  merging protogalactic clouds

  colliding giant galaxy clusters

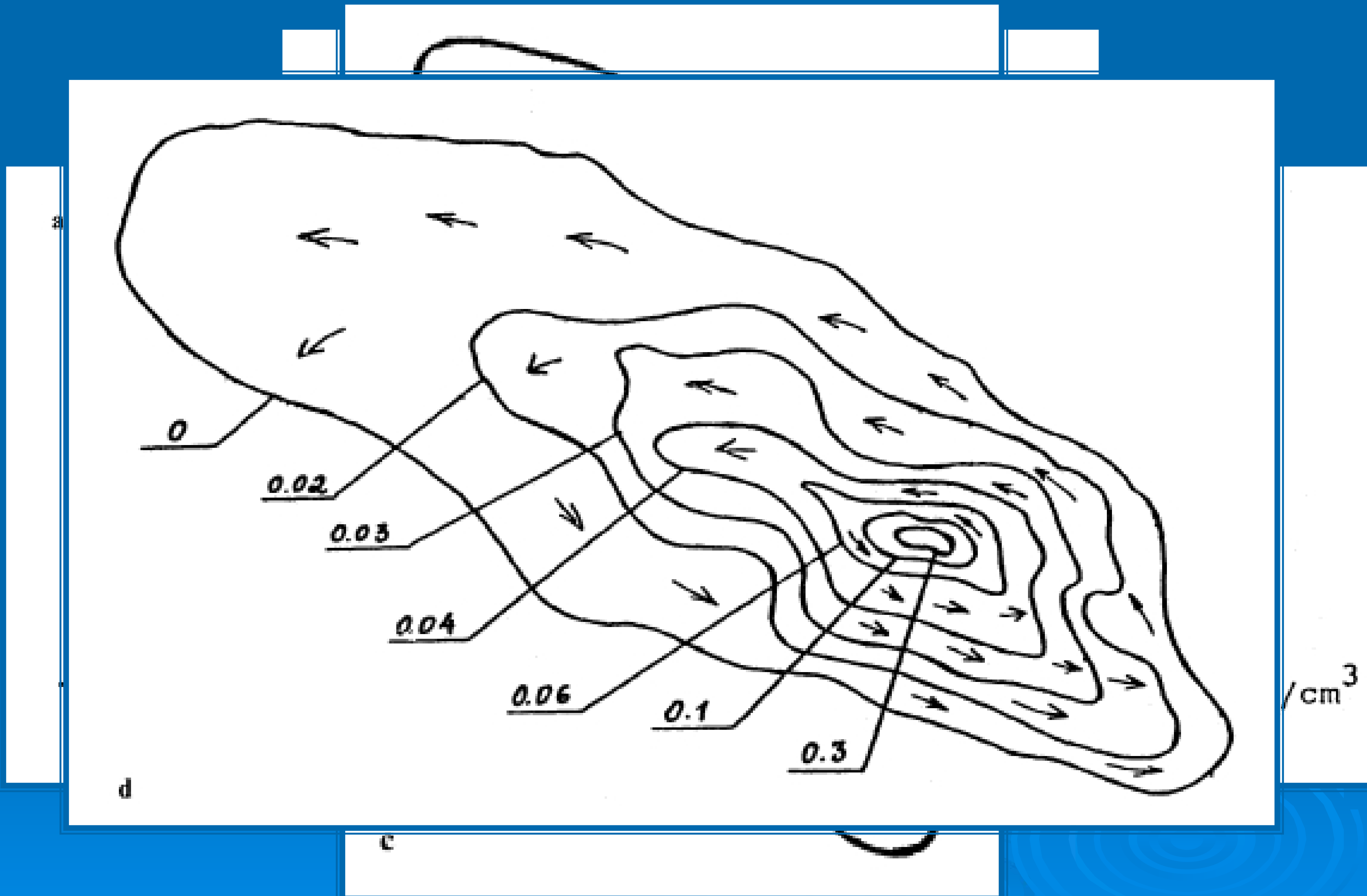
- Interactions of protostellar clouds, or colliding protogalactic clouds or giant galaxy clusters produce large-scale shear motions which are superimposed on small-scale turbulence.

Chernin (1993). Non-central collision



$$r_1 = r_2 = 10^{23}\ cm$$

Different cloud sizes, Chernin (1993)



Parameters

Parameters	Protostellar Clouds	Protogalactic Clouds	Giant Galaxy Clusters
Mass	$M < M_{SUN}$	$M < 10^{10} M_{SUN}$	$M > 10^{12} M_{SUN}$
R (pc)	$R = 3\times 10^{-2}$	$R = 3\times 10^{4}$	$R > 10^{6}$
V (cm/s)	$V = 10^{5} - 10^{6}$	$V = 10^{6} - 10^{7}$	$V = 10^{8}$
$\rho \ (g/cm^3)$	$\rho = 3\times 10^{-19}$	$\rho = 10^{-26}$	$\rho = 10^{-27}$

Parameters

Parameters	Protostellar Clouds	Protogalactic Clouds	Giant Galaxy Clusters
ΔV (cm/s)	$\Delta V = 10^5$	$\Delta V = 10^6 - 10^7$	$\Delta V = 10^7$
ΔR (cm)	$\Delta R = 10^{16} - 10^{17}$	$\Delta R = 2 \times 10^{23}$	$\Delta R = 3 \times 10^{24}$
$\eta_T \quad (cm^2/s)$	$\eta_T = 10^{19}$	$\eta_T = 10^{28}$	$\eta_T = 3 \times 10^{30}$
t_B (years)	$t_B = 10^4 - 10^5$	$t_B = 10^8 - 10^9$	$t_B = 10^{10}$
$B_{eq} \quad (\mu G)$	$B_{eq} = 10 - 75$	$B_{eq} = 0.3 - 3$	$B_{eq} = 1$

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Conclusions

- Exponential growth of large-scale magnetic field in nonhelical turbulence with superimposed linear shear is found in numerical simulations in elongated shearing boxes.

- Generation of large-scale magnetic field is caused by a "shear-current" effect which acts even in a nonrotating and nonhelical homogeneous turbulence.

- The shear-current dynamo and generation of large-scale vorticity can occur when Re>1 and Rm > 1 .

- During the growth of the mean magnetic field, the nonlinear "shear-current" effect is not quenched and it only changes its sign at some value of the mean magnetic field which can determine the level of the saturated mean magnetic field.

Conclusions

- We have taken into account the transport of magnetic helicity as dynamical nonlinearity. If the magnetic helicity flux is not small, the level of the saturated mean magnetic field is of the order of the equipartition field.

- The newly discovered shear dynamo effect potentially represents a very generic mechanism for generating large-scale magnetic fields in a broad class of astrophysical systems with spatially coherent mean flows (shear).

- Homogeneous kinetic helicity fluctuations alone with zero mean value in a sheared homogeneous turbulence cannot generally cause large-scale dynamo.

THE END

Methods and Approximations

- Second-Order Correlation Approximation (SOCA) or First-Order Smothing Approximation (FOSA)
  (a) Re << 1 (b) Rm << 1
  H. K. Moffatt (1978); F. Krause and K. H. Raedler (1980)

- Path-Integral Approach (delta-correlated in time random velocity field or short yet finite correlation time)
  R. H. Kraichnan, Phys. Fluids 11, 945 (1968)

- Tau-approaches (spectral tau-approximation, minimal tau-approximation) – third-order or high-order closure
  Re >> 1 and Rm >> 1
  A. Pouquet, U. Frisch, and J. Leorat, J. Fluid Mech. 77, 321 (1976)

- Renormalization Procedure (renormalization of viscosity, diffusion, heat conductivity and other turbulent transport coefficients) -- no separation of scales
  H. K. Moffatt, Rep. Prog. Phys. 46, 621 (1983)

Tau Approach

Equations for the correlation functions for:

- The velocity fluctuations $\left(M_{ij}^{(II)}(\mathbf{k})\right)_u = \langle u_i u_j \rangle$
- The magnetic fluctuations $\left(M_{ij}^{(II)}(\mathbf{k})\right)_b = \langle b_i b_j \rangle$
- The cross-helicity tensor $\left(M_{ij}^{(II)}(\mathbf{k})\right)_\chi = \langle b_i u_j \rangle$

The spectral τ-approximation (the third-order closure procedure)

$$\hat{D}M^{(III)}(\mathbf{k}) - \hat{D}M_0^{(III)}(\mathbf{k}) = -\frac{M^{(II)}(\mathbf{k}) - M_0^{(II)}(\mathbf{k})}{\tau_c(\mathbf{k})}$$

$$\left(M_{ij}^{(II)}(\mathbf{k})\right)_u = -\langle u_i (\mathbf{u}\cdot\nabla) u_j \rangle - \langle u_j (\mathbf{u}\cdot\nabla) u_i \rangle$$

Renormalization Procedure

- The first step is the averaging over the scale that is inside the inertial range of turbulence.
- The next stage of the renormalization procedure comprises a step-by-step increase of the scale of the averaging up to the maximum scale of turbulent motions.
- This procedure allows the derivation of equations for the turbulent transport coefficients: eddy viscosity, turbulent diffusion, turbulent heat conductivity, electromotive force coefficients, etc.
- To apply this procedure an equation invariant under the renormalization of the turbulent transport coefficients must be determined.