Dynamical Regimes in Planetary Dynamos

Chris Jones

Binod Sreenivasan

University of Leeds, UK

Small E, low Pm numerical dynamo

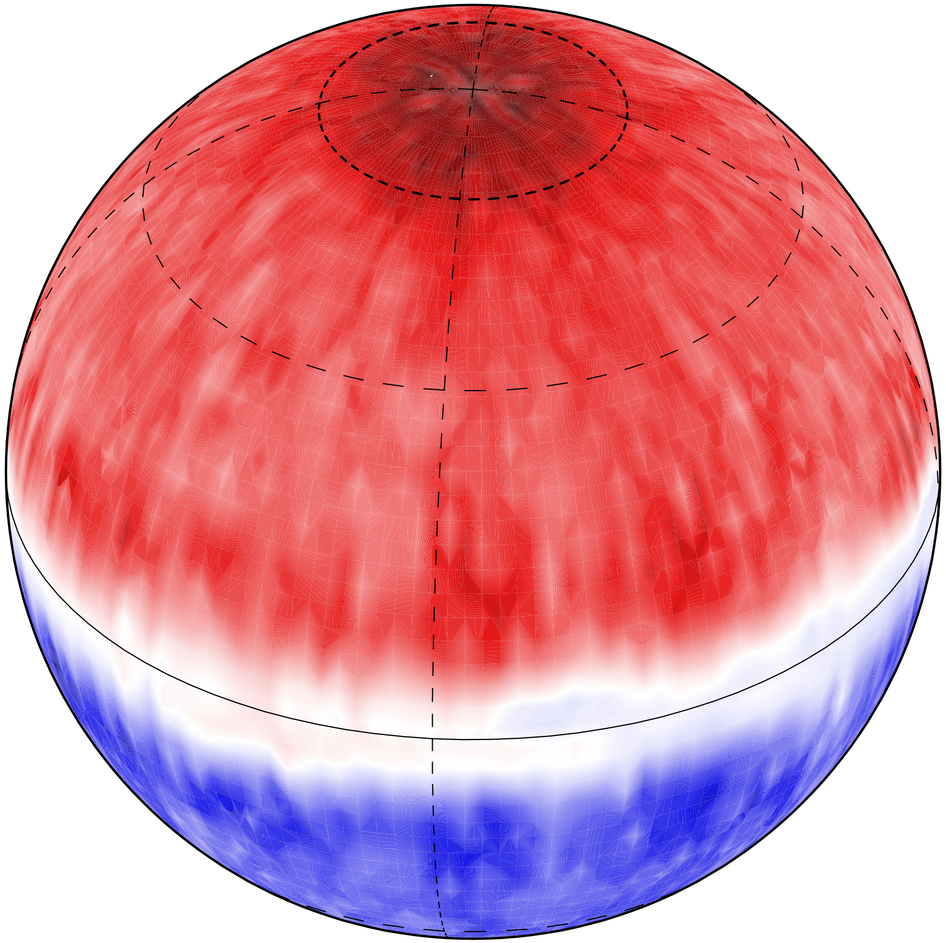
$E = \frac{\nu}{\Omega d^2} = 3 \times 10^{-6}$, $Pr = \nu/\kappa = 1$, $Pm = \nu/\eta = 0.1$ $R \approx 50 R_{crit}$

Field is strongly dipolar: $R_m \approx 125$, Elsasser number $\Lambda = B^2/\rho\mu\Omega\eta \approx 0.5$. Models typically produce dipole dominated fields if inertia is not dominant. Local Rossby number $U/\ell\Omega = (d/\ell) R_m E Pm^{-1} \ll 1$

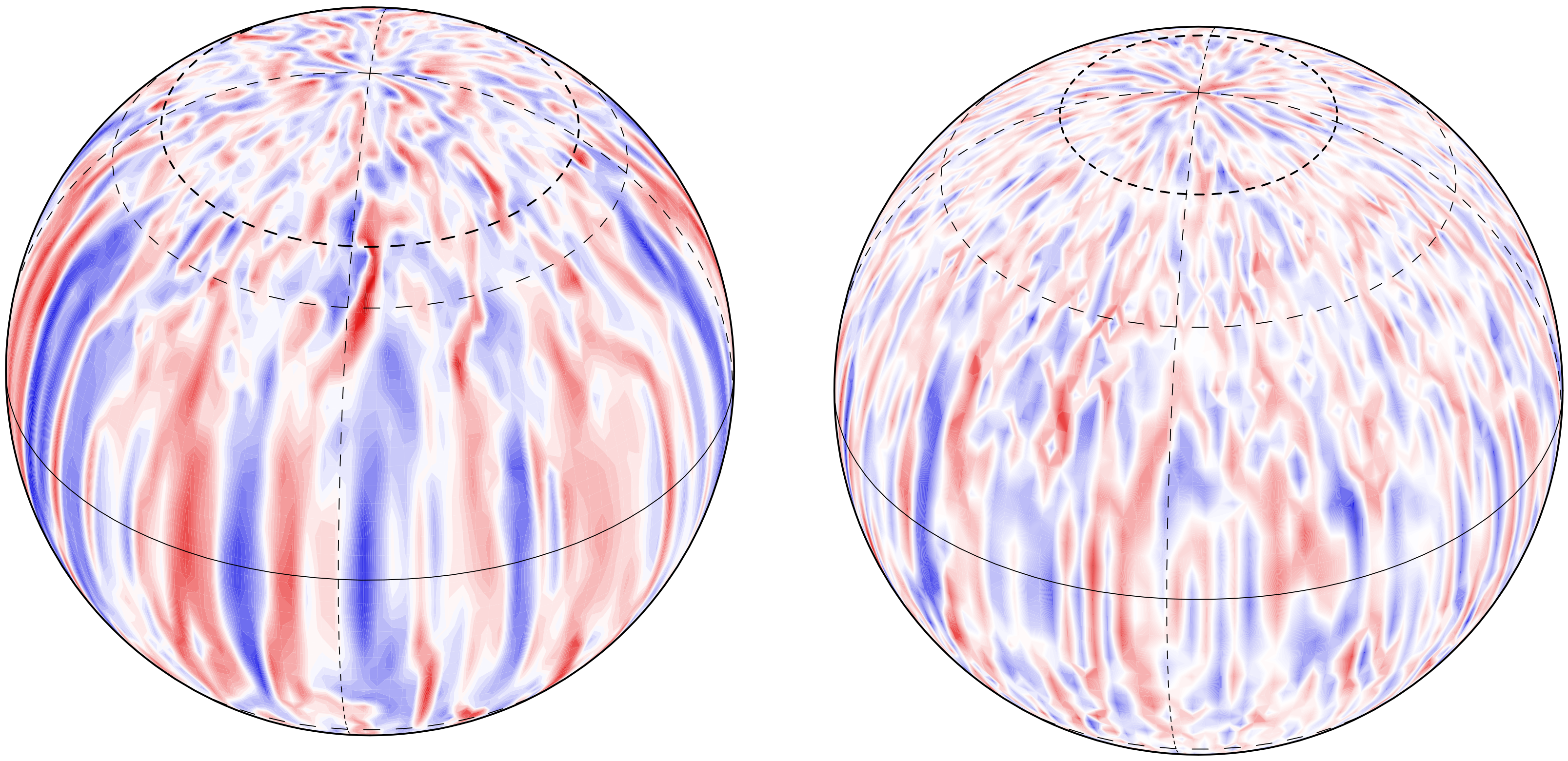
Columnar structure is evident in the dynamo simulations, particularly near the tangent cylinder, and this seems to be related to the dipole dominance. These dynamos won't reverse.

For this run, Kinetic and Magnetic energies similar:

K.E. $\approx$ M.E., and also ohmic dissipation similar to viscous dissipation.



Field at the Core Mantle Boundary. Dipole dominant! Note the weaker field near the pole, inside the tangent cylinder.

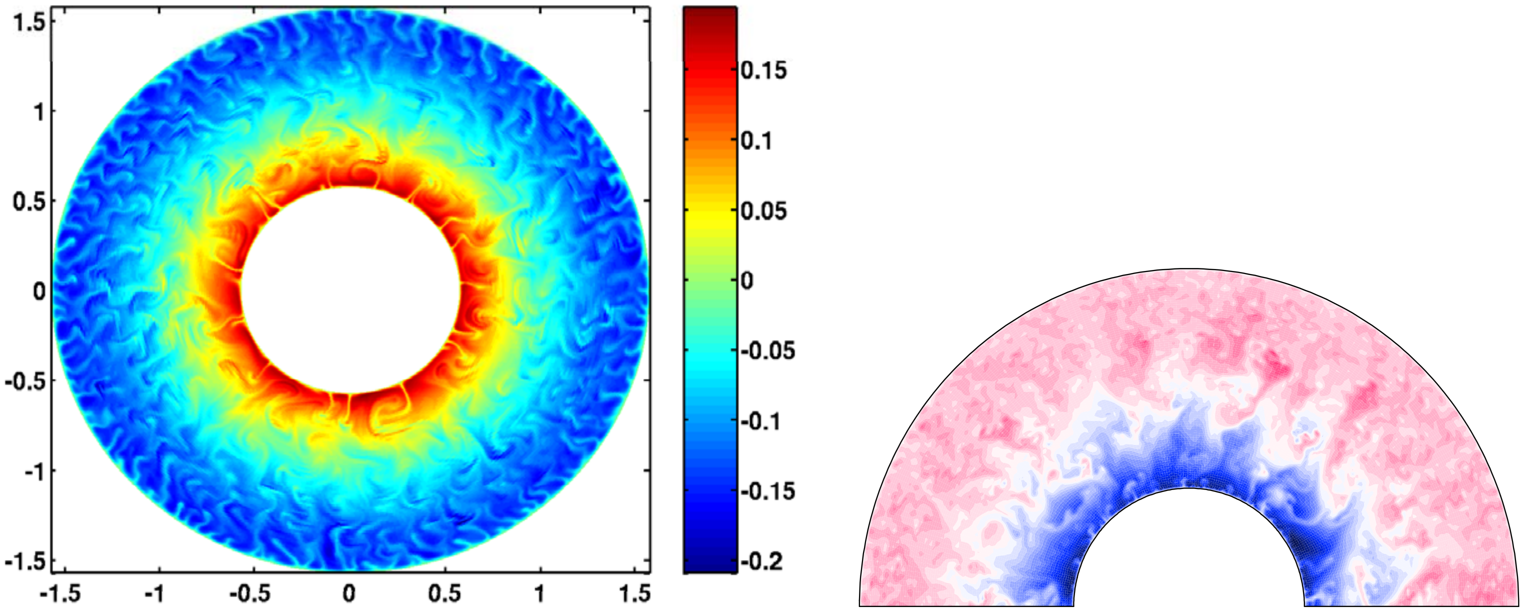


(a) (b)

(a) Contours of u_r at $r = r_i + 0.5d$ (b) Contours of u_r at $r = r_i + 0.8d$

Snapshot. Flow is strongly time-dependent, individual rolls having lifetime over only of order a turnover time.

Magnetic field has remarkably little effect on structure of flow

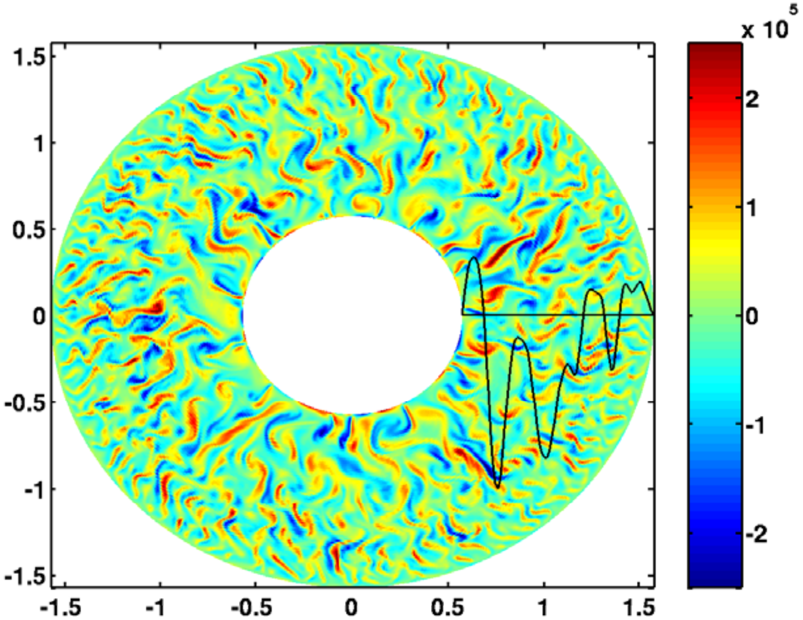


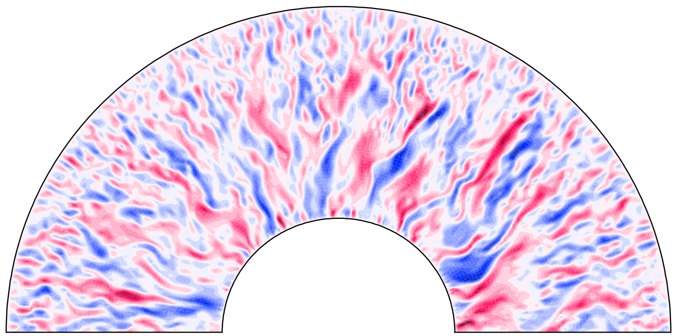
(a) (b)

(a) Non-magnetic temperature snapshot in equatorial plane.

$P = 7.0$, $E = 6.5 \times 10^{-6}$, $Ra = 42.7 R_c$
(Gillet and Jones, 2006)

(b) Temperature snapshot from the dynamo simulation





(a) (b)

(a) Non-magnetic vorticity snapshot.

(b) u_r in equatorial plane from dynamo simulation

Parameter regimes: simulations and geodynamo

Simulations generally have $100 < R_m < 500$, $R_m \sim 500$ plausible geodynamo value. With $\eta \sim 2\ \mathrm{m}^2\mathrm{s}^{-1}$, typical velocity $U_* \sim 5 \times 10^{-4}\ \mathrm{ms}^{-1}$, consistent with secular variation.

Typical simulation has

$$Pr = \frac{\nu}{\kappa} \sim 1, \quad Pm = \frac{\nu}{\eta} \sim 1, \quad E = \frac{\nu}{\Omega d^2} \sim 10^{-5}$$

Then

$$\Omega = E^{-1} Pm \frac{\eta}{d^2} \sim 5 \times 10^{-8} s^{-1}, \quad \text{should be} \quad 7 \times 10^{-5} s^{-1}$$

Need scaling laws to extrapolate to realistic parameter regime.

Typical Velocity: Inertial theory

$$\mathbf{u} \cdot \nabla \mathbf{u} + 2\boldsymbol{\Omega} \times \mathbf{u} = -\nabla p/\rho + \mathbf{j} \times \mathbf{B}/\rho + g\alpha T'\hat{\mathbf{r}} + [\nu \nabla^2 \mathbf{u}]$$

$$\mathbf{u} \cdot \nabla \omega - 2(\Omega \cdot \nabla)\mathbf{u} = \nabla \times g\alpha T'\hat{\mathbf{r}} + \frac{1}{\rho}\nabla \times (\mathbf{j} \times \mathbf{B}), \quad \text{vorticity eqn}$$

We assume a balance $\dfrac{U_*^2}{\ell^2} \sim \dfrac{\Omega U_*}{d} \sim \dfrac{g\alpha T'}{\ell}$

Here U_* is typical convective velocity. $|\omega| \sim U_*/\ell$

$d = r_{cmb} - r_{icb}$, ℓ is length scale perpendicular to z, the roll axis.

Lorentz force ignored (?).

$$\ell \sim \left(\frac{U_* d}{\Omega}\right)^{1/2} \sim \left(\frac{5 \times 10^{-4} \times 2 \times 10^6}{7 \times 10^{-5}}\right)^{1/2} \sim 4km$$

ℓ is Rhines length, balance of inertia and Coriolis. On longer length scales, inertia $<<$ Coriolis. Can such short length scales be relevant to the geodynamo?

Convective heat flux per square metre $F_{conv} \sim \rho c_p U_* T'$

Eliminate T' to get

$$\frac{U_*}{\Omega d} = Ro \sim \left(\frac{g\alpha F_{conv}}{\rho c_p \Omega^3 d^2}\right)^{2/5} = (Ra_Q)^{2/5}$$

For compositional convection, $Ra_Q = gF_m/\rho\Omega^3 d^2$, F_m being light material mass flux, Kg m^{-2} s^{-1}.

Fitting data from dynamo simulations, Christensen & Aubert 2006 obtained

$$Ro = 0.85 Ra_Q^{0.41}$$

very close to inertial scaling.

Taking typical core velocity as 15 km/year from the secular variation gives $Ro = U_*/\Omega d = 2.9 \times 10^{-6}$, giving

$$Ra_Q \sim 2 \times 10^{-14} \rightarrow F_{conv} \sim 3.9 \times 10^{-3}\mathrm{Wm}^{-2} \rightarrow Q_{conv} \sim 0.6\mathrm{TW}$$

with usual estimates for c_p etc. Lowish, but right ball park! Heat and composition fluxes vary across the core.

Some length scales in the core

Linear onset length $L_{crit} = d \left(\frac{E(1+Pr)}{Pr} \right)^{1/3}$,

Core value ~ 0.8 km

Rhines length, inertia $\sim$ Coriolis, $L_R = \left(\frac{U_* d}{\Omega} \right)^{1/2}$,

~ 4 km in core

Minimum length for dynamo action, $L_{dyn} = 10\eta/U_*$,

~ 40 km in core

Magnetic length $L_B = \left[\int \mathbf{B}^2 \, dv / \int |\nabla \times \mathbf{B}|^2 \, dv \right]^{1/2}$,

~ 70 km in core

All fairly similar in numerical simulations!

Magnetic field controlling roll size?

In simulations, little sign of magnetic field influencing roll-width ℓ

But what if it does at lower EPm^{-1}? A simple model is to take ℓ constant, rather than decreasing as $Ra_Q^{1/5}$ as predicted by inertial theory.

Then $\dfrac{\Omega U_*}{d} \sim \dfrac{g\alpha T'}{\ell}, \quad F_{conv} \sim \rho c_p U_* T'$

giving

$Ro \sim Ra_Q^{1/2}$ rather than the $Ro \sim Ra_Q^{2/5}$ law of inertial theory and the simulations.

Not hugely different, but will affect the heat flux arguments, because of the large extrapolation range.

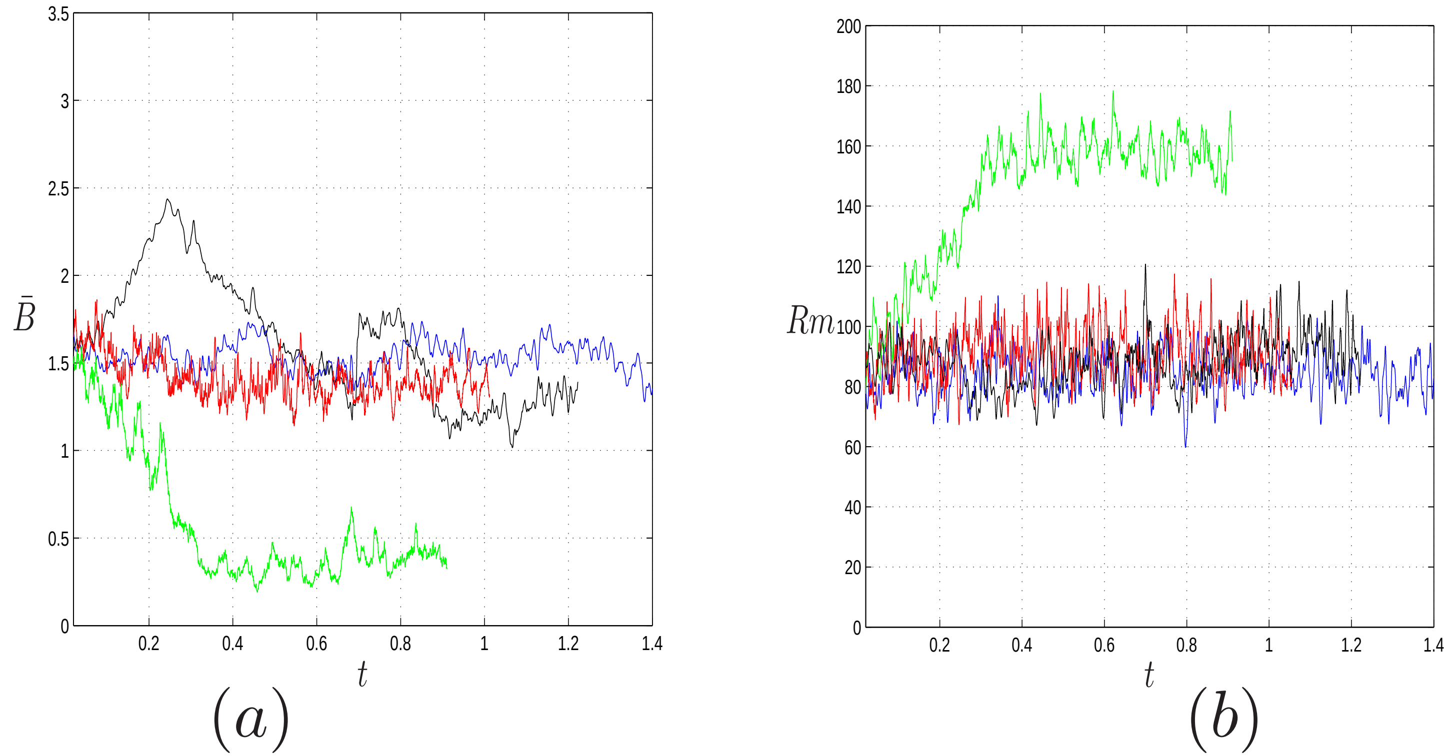
Reversals and inertia in dynamo models

At fixed E, if Pr and Pm increased, dynamo enters 'inertia-free' regime (Sreenivasan & Jones, 2006). Very dipole dominant, non-reversing regime.

In inertial regime, field strength lower and morphology more complex, reversals occur.

Olson & Christensen (2006) note known reversing dynamos lie on boundary between the two regimes. Also suggest boundary at $d\,Ro/\ell \sim 0.1$.

Requires very short length scales, $\sim 0.1 km$, to be important in the dynamo process. Locally low R_m motion is strongly constrained by rotation and magnetic field (Braginsky and Meytlis, 1990).



(a) Mean magnetic field for (i) $Pr = Pm = 5$ (Black)

(ii) $Pr = Pm = 1$ (Blue) (iii) $Pr = Pm = 0.5$ (Red)

(iv) $Pr = Pm = 0.2$ (Green) $E = 10^{-4}$

(b) Magnetic Reynolds number

Note the sharp distinction between the non-inertial solutions, independent of Pr and the green inertial solution.

Scaling for magnetic field

Energy input from buoyancy per unit volume balances ohmic and viscous dissipation. If f_{ohm} is fraction of total dissipation that is ohmic

$$f_{ohm} \frac{g\alpha F}{c_p} \sim \frac{\eta |\nabla \times \mathbf{B}|^2}{\mu}$$

To get magnetic energy, need a relation between the field and the current i.e. the magnetic length scale.

$L_B = \left[\int \mathbf{B}^2 \, dv / \int |\nabla \times \mathbf{B}|^2 \, dv \right]^{1/2}$. Note L_B^2/η is time for magnetic energy to dissipate due to ohmic heating.

Assume dissipation mainly ohmic, not viscous, $f_{ohm} \approx 1$.

Christensen and Tilgner 2004 suggest $L_B \sim dR_m^{-1/2}$, from simulations and lab experiments. Kinematic flux expulsion models also give this scaling.

Assuming this relation for L_B is correct,

$$B \sim \mu^{1/2} d^{1/2} \left(\frac{g \alpha F}{c_p} \right)^{1/2} \frac{1}{U_*^{1/2}}$$

Now we can eliminate U_* using our favourite scaling, With the $Ro \sim {Ra_Q}^{1/2}$ velocity scaling,

$$B \sim \mu^{1/2} d^{1/2} \Omega^{1/4} \rho^{1/4} \left(\frac{g\alpha F}{c_p} \right)^{1/4}$$

and with the $Ro \sim {Ra_Q}^{2/5}$ velocity scaling

$$B \sim 0.45 \mu^{1/2} d^{2/5} \Omega^{1/10} \rho^{1/5} \left(\frac{g\alpha F}{c_p} \right)^{3/10}$$

Both predict remarkably weak dependence of the field on rotation rate.

Of course, these formulae can only be valid in the rapidly rotating limit.

Alignments

—

Why not balance buoyancy and Lorentz directly?

$$g\alpha T' U_* \sim \mathbf{u} \cdot \mathbf{j} \times \mathbf{B} \sim \frac{U_* B^2 R_m^{1/2}}{\mu\rho}$$

In terms of heat flux

$$B^2 \sim \mu \frac{g\alpha F}{c_p} \frac{R_m^{-1/2}}{U_*}$$

Factor $R_m^{-1/2}$ too small.

$$\mathbf{u} \cdot \mathbf{j} \times \mathbf{B} << |\mathbf{u}||\mathbf{j}||\mathbf{B}|$$

Alignment of field and flow!