Hydrodynamic instabilities in the solar Tachocline

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Model

Layer in the slow Tachocline. Rotating frame, layer located at latitude λ.

Cartesian model, $r \rightarrow z$, $\theta \rightarrow y$, $\phi \rightarrow x$.

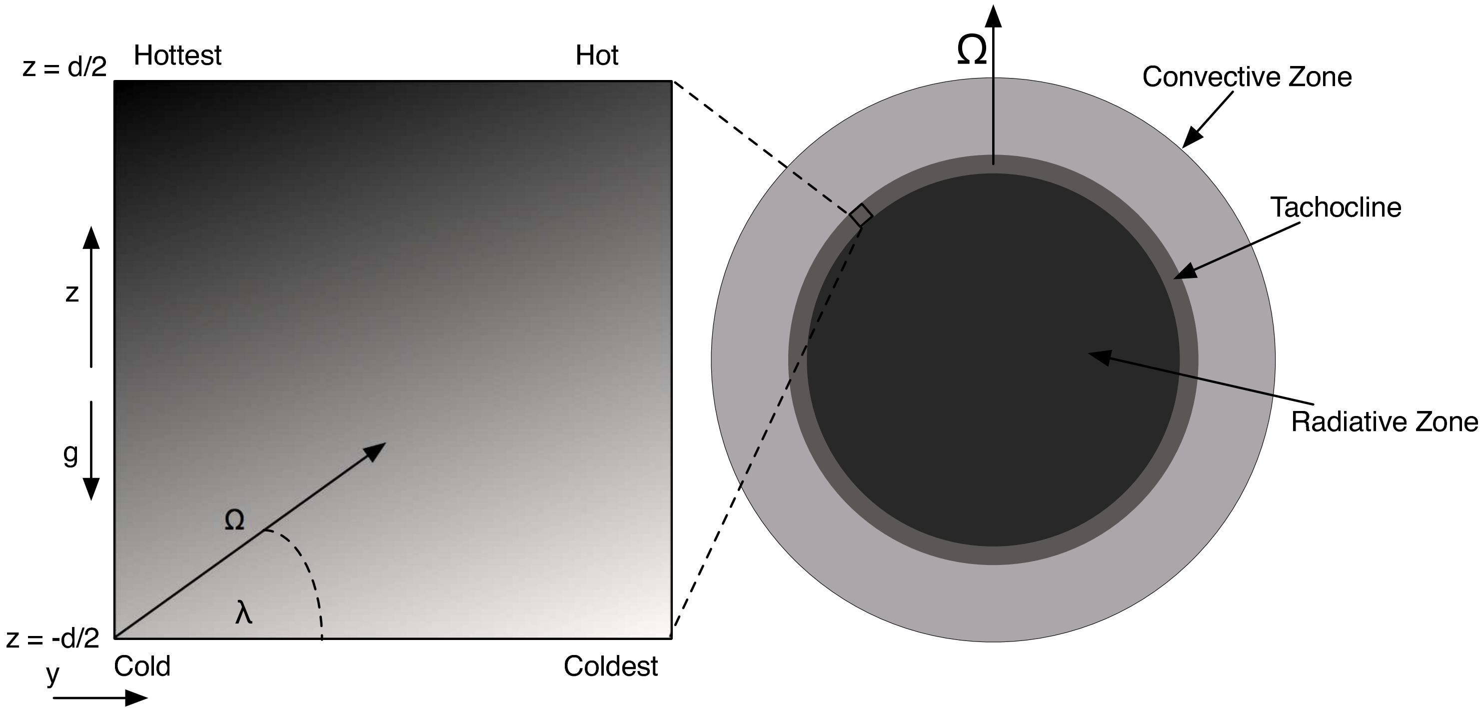
Layer is stably stratified, thickness d. Tachocline shear velocity

$$\mathbf{u} = (V'z, 0, 0)$$

so the superadiabatic temperature is

$$T_0 = \beta z - \frac{2\Omega U' \sin\lambda}{g\alpha} y$$

Examine stability: expect nonaxisymmetric baroclinic modes (Eady modes) and axisymmetric GSF (Goldreich-Schubert-Fricke) modes.



Stably stratified layer, with latitudinal superadiabatic temperature gradient maintaining a thermal wind, an eastward flow sheared in the z direction.

Linearised Equations

$$\frac{D\omega_z}{Dt} - Ro\frac{\partial u_z}{\partial y} - \sin\lambda\frac{\partial u_z}{\partial z} - \cos\lambda\frac{\partial u_z}{\partial y} = E\nabla^2\omega_z$$

$$\frac{D}{Dt}\nabla^2 u_z + \sin\lambda\frac{\partial \omega_z}{\partial z} + \cos\lambda\frac{\partial \omega_z}{\partial y} = \nabla_H^2\theta + E\nabla^4 u_z$$

$$\nabla_H^2\Big(\frac{D\theta}{Dt} + RiRo^2 u_z - EPr^{-1}\nabla^2\theta\Big) = \sin\lambda Ro\Big(\frac{\partial\omega_z}{\partial x} - \frac{\partial^2 u_z}{\partial y\partial z}\Big)$$

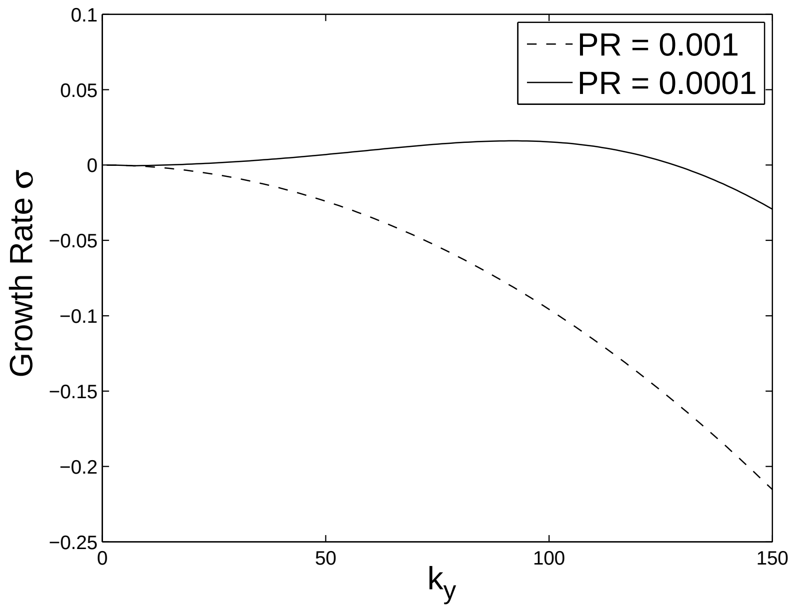
$$\text{where} \qquad \frac{D}{Dt} \equiv \frac{\partial}{\partial t} + zRo\frac{\partial}{\partial x}.$$

$$E = \frac{\nu}{2\Omega d^2}, \quad Ro = \frac{U'}{2\Omega}, \quad Ri = \frac{g\alpha\beta}{(U')^2}, \quad Pr = \frac{\nu}{\kappa},$$

Ri small favours instability, but Ri large in tachocline.

$Ro \sim 1$, E and Pr both small.

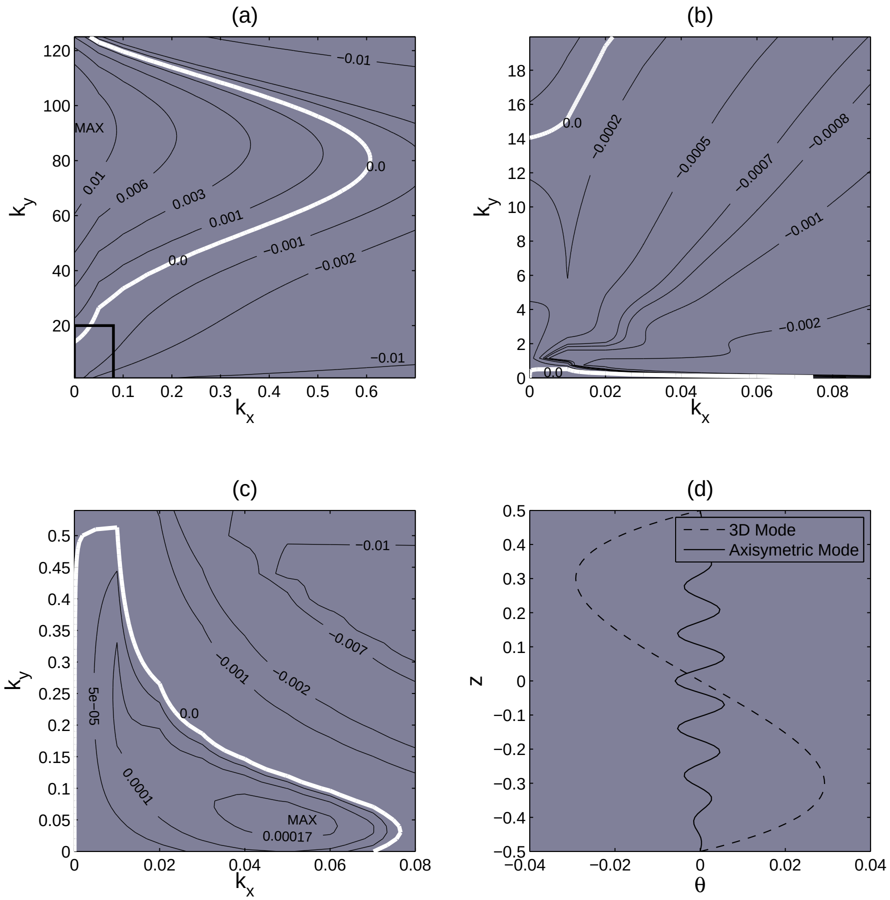
GSF instability



The growth-rate of the axisymmetric mode as a function of wavenumber k_y at $Pr = 10^{-3}$ and $Pr = 10^{-4}$ for $Ri = 2000$, $Ro = 1$, $E = 10^{-5}$, $\lambda = 90°$.

The axisymmetric mode is stable for all wavenumbers at $Pr = 10^{-3}$, but growing modes occur for $Pr = 10^{-4}$, with fastest growth-rate at $k_y \approx 92$. $RiPr$ is key parameter.

Baroclinic Eady Modes



GSF modes have fastest growth rate when axisymmetric, $k_x = 0$. Eady modes have both k_x and k_y small but non-zero. There are always unstable Eady modes, but growth rate is slow. The GSF modes have very oscillatory eigenfunctions, Eady modes simpler.

In spherical geometry, $m = 1$ is least possible, so unstable Eady modes may not fit in.

Asymptotic theory

In the relevant limit $E \to 0$, $Ri \to \infty$, $Pr \to 0$ with $RiPr$ fixed, both types of mode can be found by analytical methods. Excellent agreement with numerics.

GSF modes have $k_y \sim E^{-1/3}$, are unstable if $RiPr < 1/4$, and fluid moves in the wedge $\pi/2 - \tan^{-1} 2Ro < \gamma < \pi/2$ in the $y - z$ plane. (Unstable inertial-gravity waves). Growth rate can be $O(\Omega)$, though don't know yet how much angular momentum these short wavelength instabilities transport.

Eady modes have wavelengths $Ri^{1/2}$ times Tachocline thickness, (in both eastward and latitudinal direction), growth rate $O(\Omega Ri^{-1})$.

Cross-over point where GSF and Eady modes have same growth rate always close to $RiPr = 1/4$.

Conclusions

Eady modes and GSF modes can work on timescales which are fast enough to be significant in the slow Tachocline.

Eady modes don't rely on thermal diffusion, and are large scale modes, but they need $Ri < (R/d)^2$ to work.

GSF modes need Pr small enough for $RiPr < 1/4$. When they happen, they grow quickly.

Simplified geometry makes nonlinear studies feasible, and this work is in progress.