Can we predict alpha?

- theory that may work (sheared turbulence)?

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Shear flows

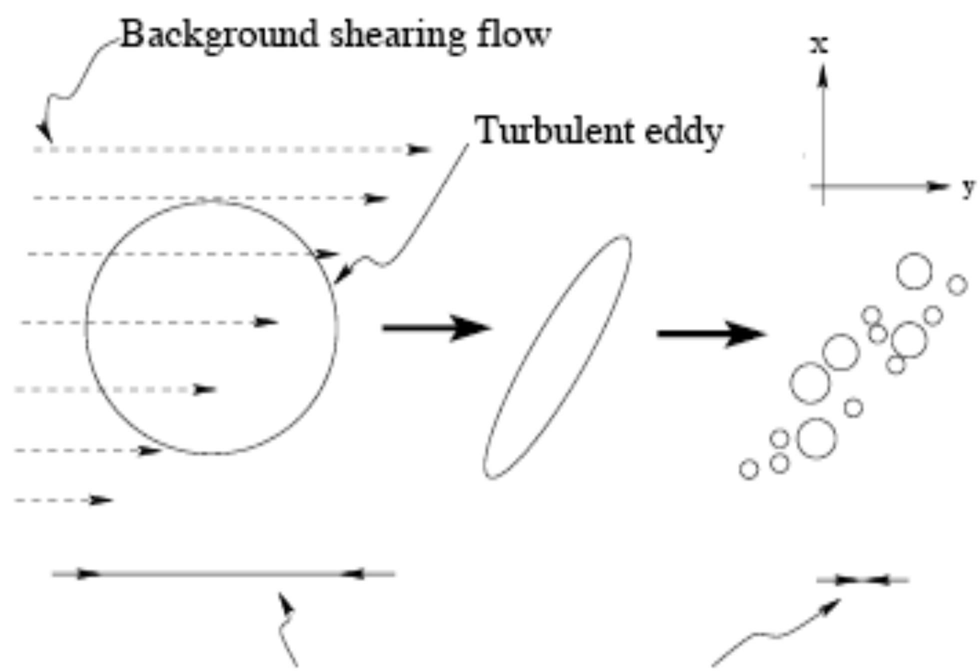
1. Source of turbulence
2. Direct interaction with mean fields (e.g. shearing of poloidal magnetic fields)
3. Stable shear flows: weak turbulence by quenching of turbulence level and turbulent transport

-> Exclude 1 and 2 from now on!

Turbulence + Shear flows

• Shear distortion and disruption of turbulent eddies ($\mathbf{v}$) $\Rightarrow$ enhanced dissipation

→ Reduce turbulent transport and turbulence level [Burrell 97, Hahm 94, Diamond and Biglari 91, Shats et al 02, Kim et al 02-08]



Typical distance an eddy can transport a passvie scalar field

cf: Waves

- Quenching transport without necessarily reducing turbulence level

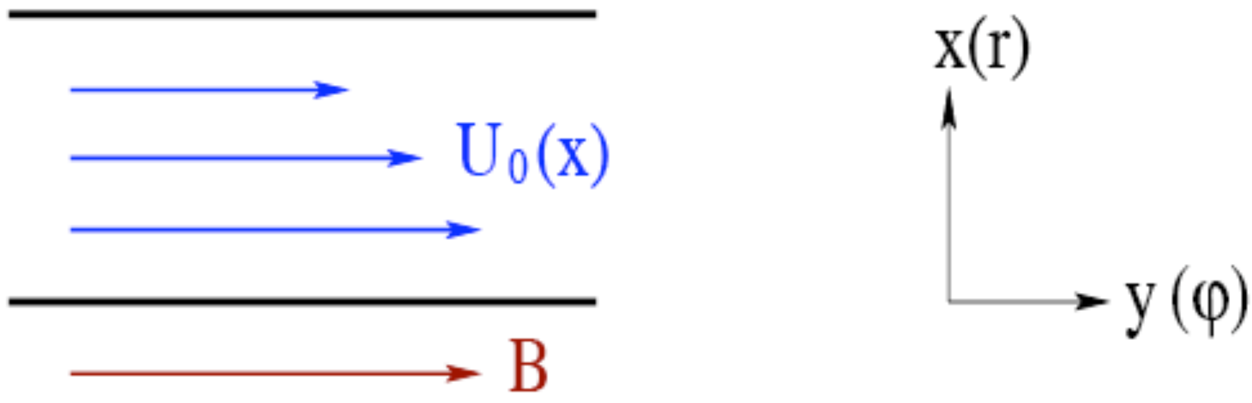
Example: $\langle av \rangle = a_{rms}\, v_{rms} \cos\phi$

*Shear flows: can reduce a_{rms}, v_{rms}, $\cos\phi$

*Waves: $\cos\phi \sim 0$

Turbulent transport in 2D MHD

[Kim and Dubrulle, PoP (2001); Kim, PRL (2006); Newton and Kim, in preparation (2008); Douglas, Kim, and Thyagaraja, PoP (2008)]



$$(\partial_t + \mathbf{u}\cdot\nabla)\omega = -(\mathbf{b}\cdot\nabla)\nabla^2 a + \nu\nabla^2\omega + F$$
$$(\partial_t + \mathbf{u}\cdot\nabla)a = \eta\nabla^2 a$$

where F is the small–scale fluid forcing. ν and η are viscosity and Ohmic diffusivity. $\mathbf{b} = \nabla\times a\hat{z} = (\partial_y a, -\partial_x a, 0)$; $\omega = -\nabla_\perp^2\phi$;
$\omega\hat{z} = \nabla_\perp\times\mathbf{u} = (\partial_x u_y - \partial_y u_x)\hat{z}$.

Evolution of mean–fields:

- For $\langle a\rangle = A(x)$ $[\langle \mathbf{b}\rangle = \hat{y}B(x)]$ and $\langle \mathbf{u}\rangle = \hat{y}U(x)$

$$\begin{aligned} \partial A &= \eta\,\partial_{xx}A - \partial_x\langle u'_x a'\rangle \\ \partial_t U &= \nu\,\partial_{xx}U - \partial_x\langle u'_x u'_y - b'_x b'_y\rangle - \partial_y\langle \Pi\rangle \end{aligned}$$

where $\Pi \equiv p + b'^2/2$

- Turbulent magnetic diffusivity η_T and viscosity ν_T:

$$\langle u'_x a'\rangle = -\eta_T \partial A$$

$$\langle u'_x u'_y - b'_x b'_y\rangle = -\nu_T \partial_x U$$

$\Rightarrow$

$$\begin{aligned} \partial_t A &= \partial_x[(\eta + \eta_T)\,\partial_x A] \\ \partial_t U &= \partial_x[(\nu + \nu_T)\partial_x U] - \partial_y\langle \Pi\rangle \end{aligned}$$

Theoretical prediction

[Kim and Dubrulle, PoP, **8** 813 (2001); Kim, PRL, **96**, 084504 (2006)]

For $\langle \mathbf{B} \rangle = B_0 \hat{y}$ (constant), $\langle \mathbf{u} \rangle = -x\mathcal{A}\hat{y}$

Weak B_0:

$$\nu_T \propto -\frac{1}{\mathcal{A}^2} < 0, \qquad \eta_T \propto \frac{1}{\mathcal{A}^2}$$

where ν_T and η_T are turbulent viscosity, magnetic and diffusivities.

Strong B_0:

$$\nu_T \propto \frac{1}{B_0^2} (> 0) \,, \qquad \eta_T \propto \frac{1}{B_0^2 \mathcal{A}^{2/3}}.$$

- Damping of Ω due to Maxwell stress [Alfenization]
- B_0 and $\mathcal{A}$ both reduce turbulent transport

Numerical Experiments

[Newton and Kim, in preparation (2008)]

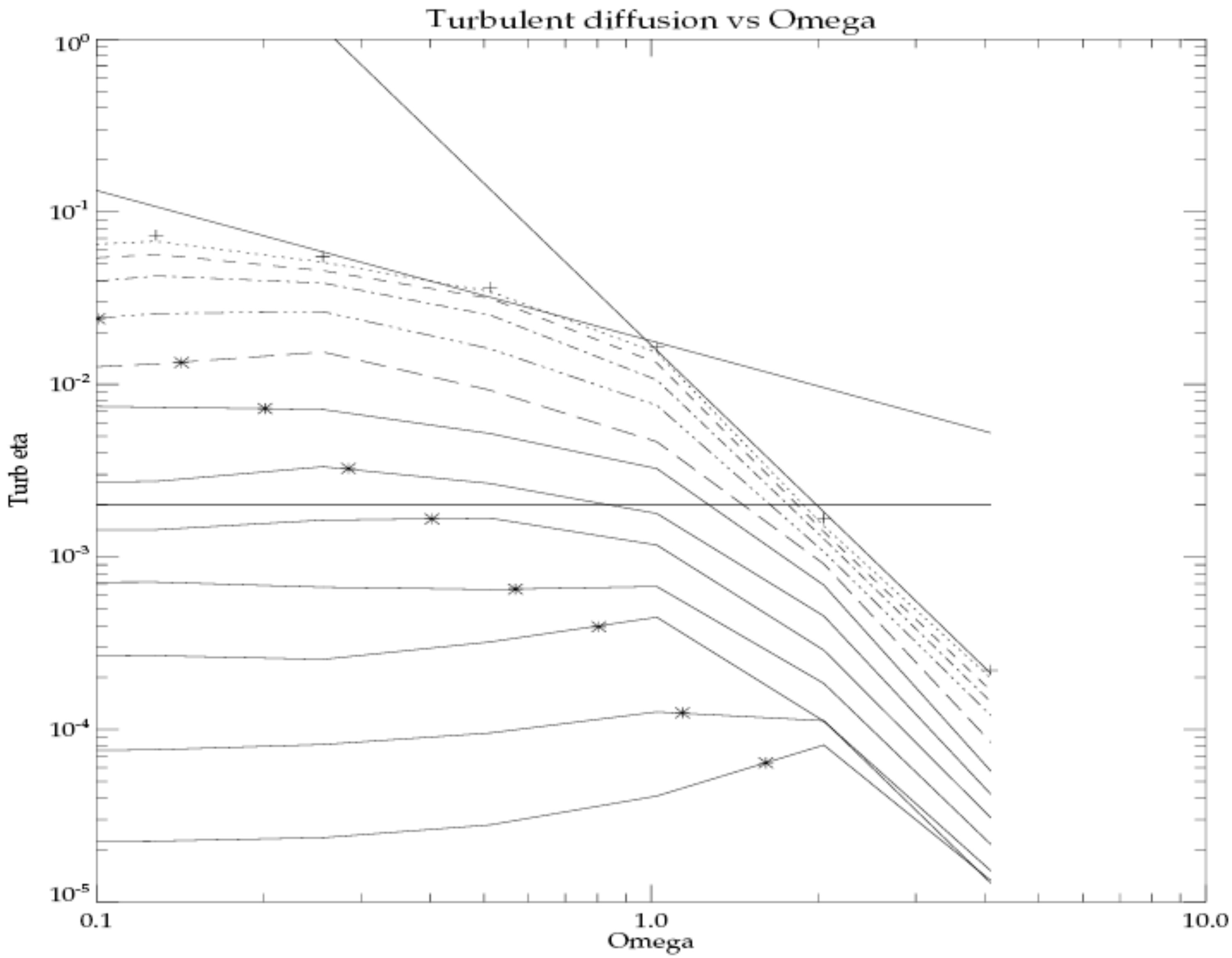
- Start with an initial large-scale magnetic field $\langle a \rangle = A \sin x$ ($\langle \mathbf{b} \rangle = \hat{y} B_0 \cos x$)
- Impose a large-scale shear flow $\langle \mathbf{u} \rangle = \hat{y} U_0 \sin x$ in parallel with $\mathbf{B}_0$

$\Rightarrow$ Dissipation rate of A: η_T?

$\Rightarrow$ Effects of shear flow and magnetic field B_0 on η_T?

NOTE: For weak B_0 and $U_0 = 0$, turbulence driven by the forcing F dissipates $\langle A \rangle$ rapidly at turbulent rate ($\eta_T = vl$)

Turbulent diffusion η_T vs shear

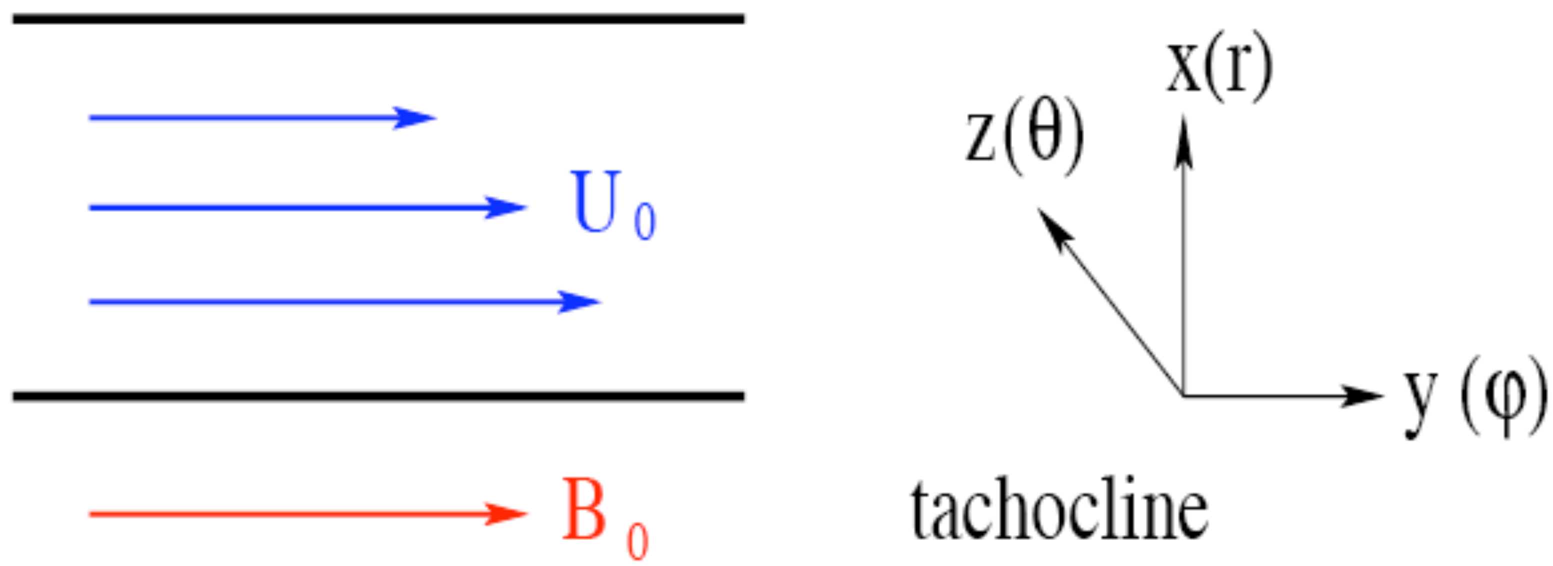


Dynamo quenching by shear flow

[Leprovost and Kim, Phys. Rev. Lett. 100, 144502 (2008)]

$$\begin{aligned} \partial_t \mathbf{V} + \mathbf{V} \cdot \nabla \mathbf{V} &= -\nabla p + \mathbf{B} \cdot \nabla \mathbf{B} + \nu \Delta \mathbf{V} + \mathbf{f} \\ \partial_t \mathbf{B} + \mathbf{V} \cdot \nabla \mathbf{B} &= \mathbf{B} \cdot \nabla \mathbf{V} + \eta \nabla^2 \mathbf{B} \\ \nabla \cdot \mathbf{B} = 0; &\qquad \nabla \cdot \mathbf{V} = 0 \end{aligned}$$

where $\mathbf{B} = \langle \mathbf{B} \rangle + \mathbf{b}$ and $\mathbf{V} = \langle \mathbf{V} \rangle + \mathbf{v}$; $\langle \mathbf{V} \rangle = U_0 = -x\mathcal{A}\hat{\mathbf{y}}$ $(\mathcal{A} \sim \Delta\Omega)$ and a constant large-scale magnetic field $\langle \mathbf{B} \rangle = B_0\hat{\mathbf{y}}$ parallel with $\langle \mathbf{V} \rangle$.


U0
B0
x(r)
z(θ)
y (φ)
tachocline

Strategy

- Time dependent Fourier transform for shearing effect with $k_x(t) = k_x(0) + k_y \mathcal{A} t$ (Goldreich and Lynden-bell 64)

$$\mathbf{v}(\mathbf{x}, t) = \int d^3k \tilde{\mathbf{v}}(\mathbf{k}, t) \exp\{i(k_x(t)x + k_y y + k_z z)\}$$

- Quasi-linear analysis [validity: Lee et al 90, Leconte et al 06; Newton and Kim 08]

- Solve for fluctuations for given forcing

$$\langle \tilde{f}_i(\mathbf{k_1}, t_1) \tilde{f}_j(\mathbf{k_2}, t_2) \rangle = \tau_f \, (2\pi)^3 \delta(\mathbf{k_1} + \mathbf{k_2}) \, \delta(t_1 - t_2) \phi_{ij}(\mathbf{k_2})$$

where

$$\phi_{ij}(\mathbf{k}) = E(k) \left(\delta_{ij} - \frac{k_i k_j}{k^2} \right) + i \epsilon_{ijp} k_p H(k)$$

$$\Rightarrow \mathcal{E}_y = \langle v_z b_x - v_x b_z \rangle = \alpha_{yy} B_y = \alpha B_0 \, , \langle v_x v_y \rangle = \nu^T \mathcal{A}$$

Strong shear limit

$$\gamma = \frac{B_0 k_y}{\mathcal{A}} \ll 1\,, \quad \xi = \frac{\nu k_y^2}{\mathcal{A}} \ll 1$$

A. Turbulence amplitude

$$\langle v_x^2 \rangle \sim \xi \langle v_0^2 \rangle, \qquad \frac{\langle v_x^2 \rangle}{\langle v_z^2 \rangle} \sim \xi^{\frac{1}{3}} \ll 1$$

Here, $e_0 \equiv \langle \mathbf{v}_0^2 \rangle \sim \tau_f \int dk E(k) \frac{k_H^2}{\nu k^2}$ is the energy driven by a forcing in the absence of magnetic fields and shear flow.

- Shear flow ($\mathbf{U}_0 = x\mathcal{A}\hat{y}$) quenches turbulence, leading to weak turbulence
- Shear flow induces anisotropic turbulence which is much weaker in shear (x) direction: $\langle v_x^2 \rangle \ll \langle v_z^2 \rangle$

B. Turbulent transport

$$\nu_T = \xi^2[1 + \gamma^2 \xi^{-2/3}]\nu_{T0} \propto \mathcal{A}^{-2}$$

$$\alpha = \xi^{5/3}[1 - \gamma^2 \xi^{-2/3}]\alpha_0 \propto \mathcal{A}^{-5/3}$$

Here, $\nu_{T0} = e_0/\nu k^2$ and $\alpha_0 = -h_0/\nu k^2$; h_0 is flow helicity driven by forcing when $\mathcal{A} = 0$ and $B = 0$: $h_0 \equiv \langle \mathbf{v}_0 \cdot \nabla \times \mathbf{v}_0 \rangle \sim \tau_f \int dk H(k) \frac{k^2}{\nu}$

- Shear flow ($\mathbf{U}_0 = x\mathcal{A}\hat{y}$) quenches transport ($\nu_T \sim \tau_c v^2$ with $\tau_c = \mathcal{A}^{-1}$)

- Shear flow induces anisotropic transport which is much weaker in shear (x) direction: $\nu_T^{xx} \ll \nu_T^{zz}$

- Quenching of alpha effect ($\propto \mathcal{A}^{-5/3}$) and momentum transport ($\propto \mathcal{A}^{-2}$) by flow shear $\mathcal{A}$

- Further reduction in alpha effect due to magnetic fields for $B_0 < \mathcal{A}/(kR_m^{1/3})$

cf: Strong magnetic field: $\gamma \gg 1$: $\nu_T, \alpha \propto B^{-2}$

*Numerical confirmation by D.MITRA et al(2008)

*Q: Important in the solar tachocline?

*Q: alpha quenching in turbulence through multi-scale interactions (e.g. localized shear)?

Other related works

- Anisotropic turbulence and momentum transport [Kim, A&A **441**, 764 (2005); Leprovost and Kim, A&A **456**, 617 (2006)]
- Stratified magnetized sheared turbulence [Kim and Leprovost, A&A **465**, 633 (2007); **468,** 1025 (2007)]: Negative eddy viscosity
- 2D Magnetzed Rossby wave turbulence [Leprovost and Kim, ApJ **154**, 1166 (2006)]
- 3D Rotating sheared turbulence [Leprovost and Kim, A&AL, **463** L9-L12 (2007)]: Negative eddy viscosity, non-diffusive momentum flux