Numerical models of small-scale dynamo action in compressible convection

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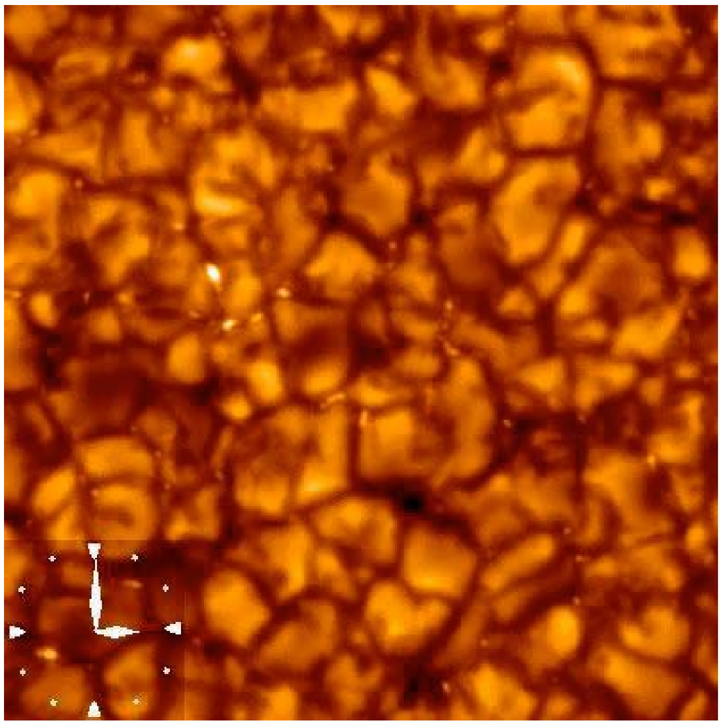
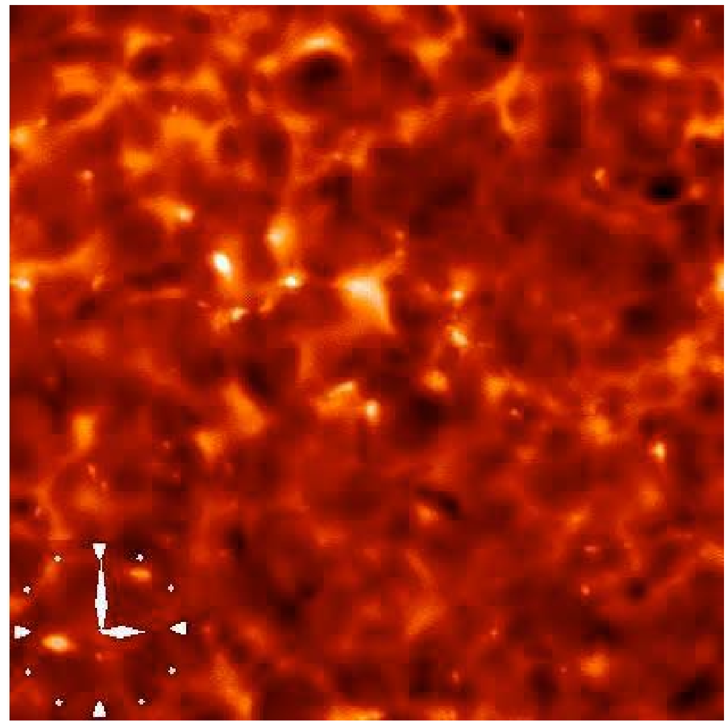
Observations:

Granular boundaries at the quiet solar surface are associated with a network of mixed polarity magnetic flux - show up in G-Band images as localised bright points (Image taken from Hinode's website)





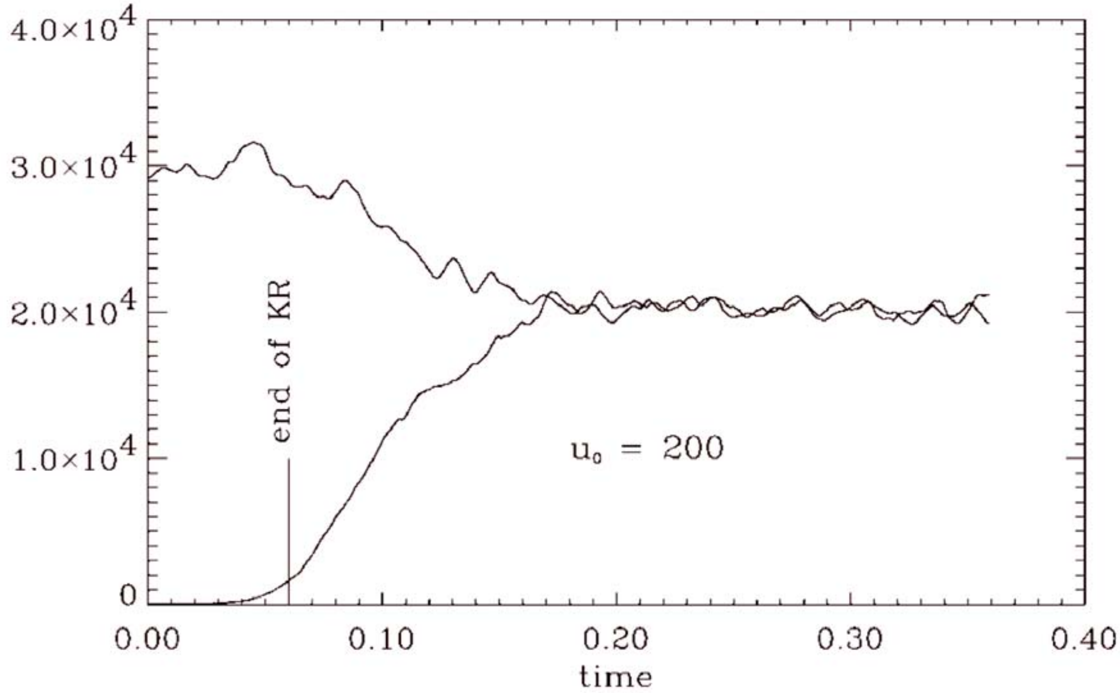
G-Band (430nm) Ca II H (397nm)

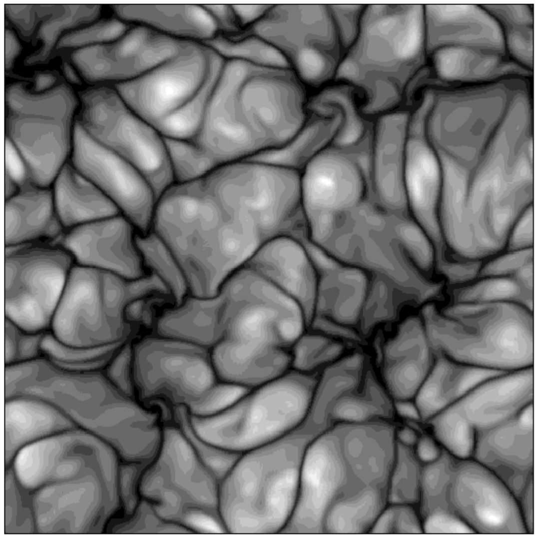
Cattaneo (1999):

Dynamo action in Boussinesq convection: (Rm=1000, Re=200)

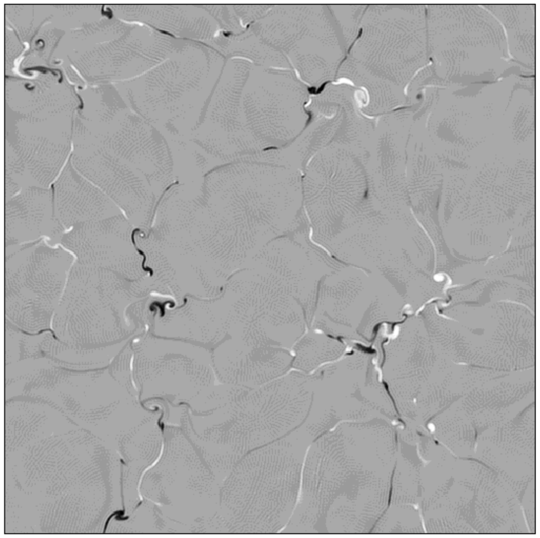
Right: Kinetic energy + 5x magnetic energy vs time

Below: Horzontal cuts through the computational domain

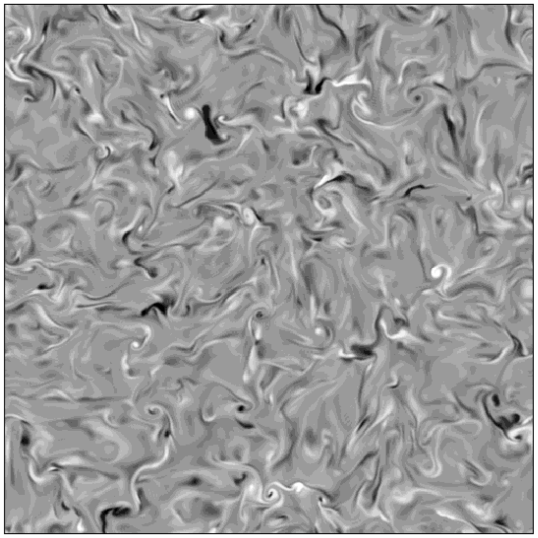




T (surface)



Bz (surface)



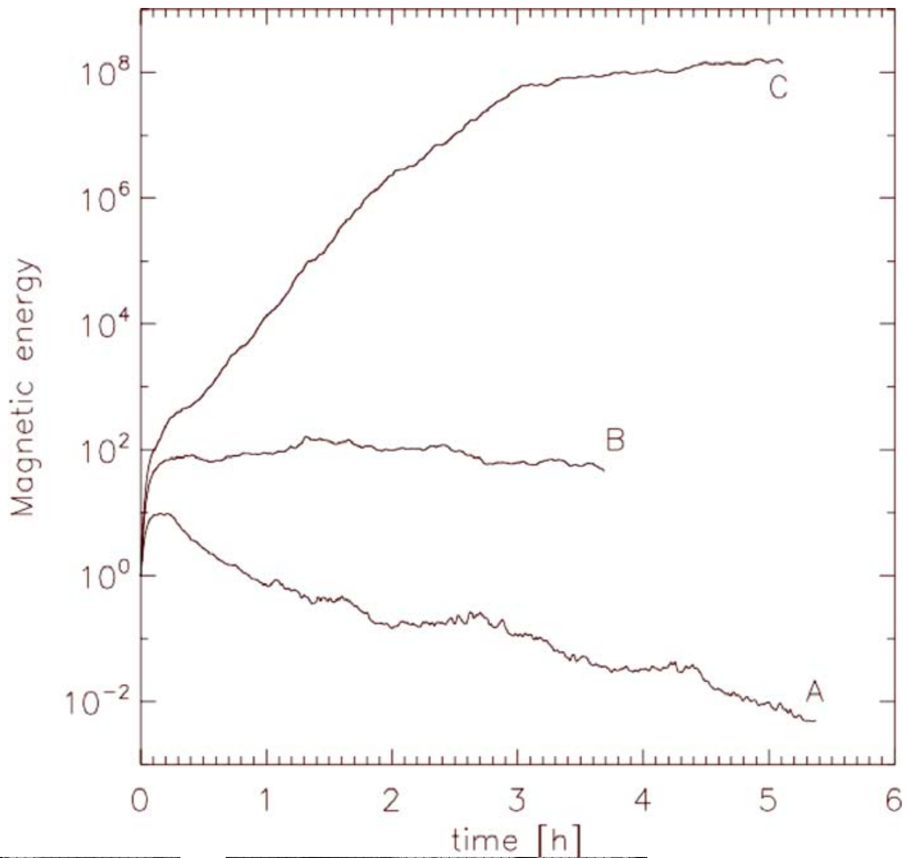
Bz (midlayer)

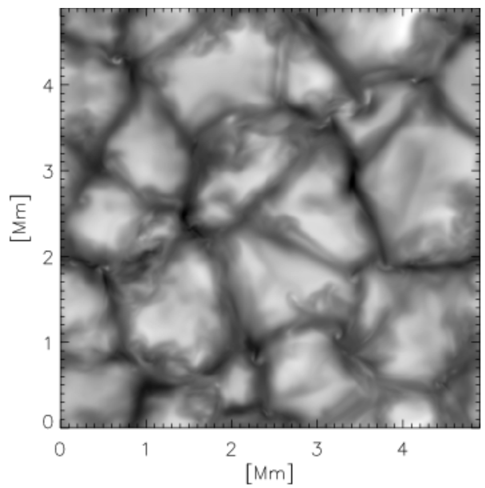
Vögler & Schüssler (2007):

LES simulation of dynamo action in radiative **compressible** convection

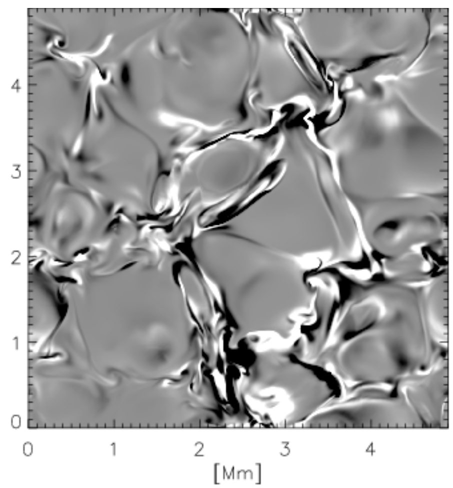
Right: Results from 3 runs (A: Rm=300; B: Rm=1300; C:Rm=2600), showing magnetic energy as a function of time.

Below: A surface snapshot from run C (taken from Schüssler & Vögler 2008).

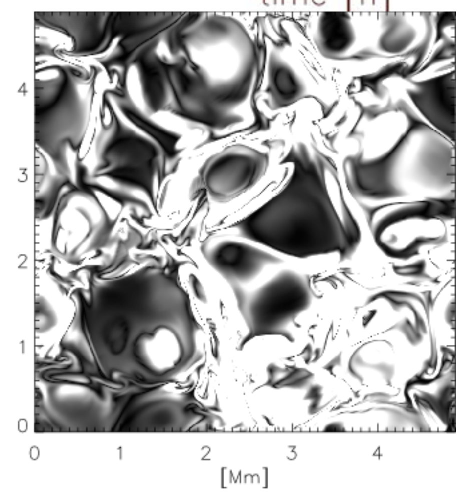




“Continuum”



Bz



Horizontal field

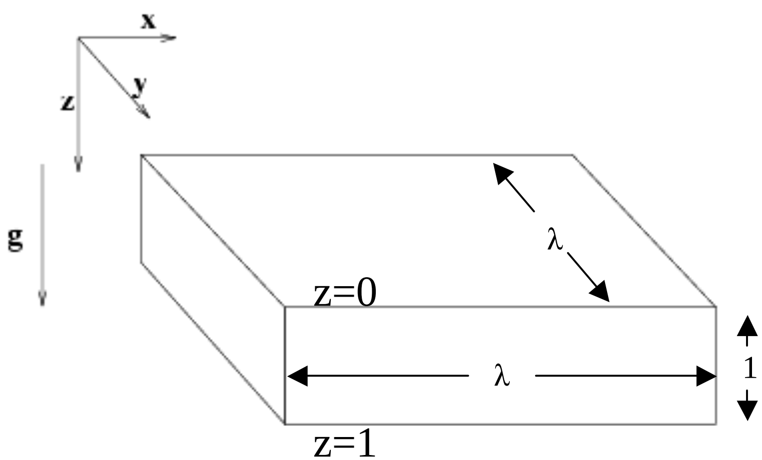
Model setup: Non-dimensional equations

$$\frac{\partial \rho}{\partial t} = -\nabla \cdot (\rho \mathbf{u}) \qquad P = \rho T$$

$$\frac{\partial}{\partial t}(\rho \mathbf{u}) = -\nabla \left(P + B^2/2\right) + \theta(m+1)\rho\hat{\mathbf{z}} + \nabla \cdot (\mathbf{BB} - \rho\mathbf{uu} + \sigma\kappa\tau)$$

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{u} \times \mathbf{B} - \zeta_o \kappa \nabla \times \mathbf{B}) \qquad \nabla \cdot \mathbf{B} = 0$$

$$\frac{\partial T}{\partial t} = -(\mathbf{u} \cdot \nabla) T - (\gamma - 1) T \nabla \cdot \mathbf{u} + \frac{\gamma\kappa}{\rho}\nabla^2 T + \frac{\kappa(\gamma-1)}{\rho}(\sigma\tau^2/2 + \zeta_o|\nabla \times \mathbf{B}|^2)$$



Initially: Fully-developed hydrodynamic convection - density and temperature vary by an order of magnitude across the layer.

$$\mathbf{B} = \epsilon \cos(2\pi x/\lambda)\cos(2\pi y/\lambda)\hat{\mathbf{z}}$$

A horizontally-periodic Cartesian domain (λ typically 4 or 8)

Upper and lower boundaries**:** Impermeable, stress-free, vertical field, fixed T

Model setup (cont.)

Numerical method (Direct numerical simulation)

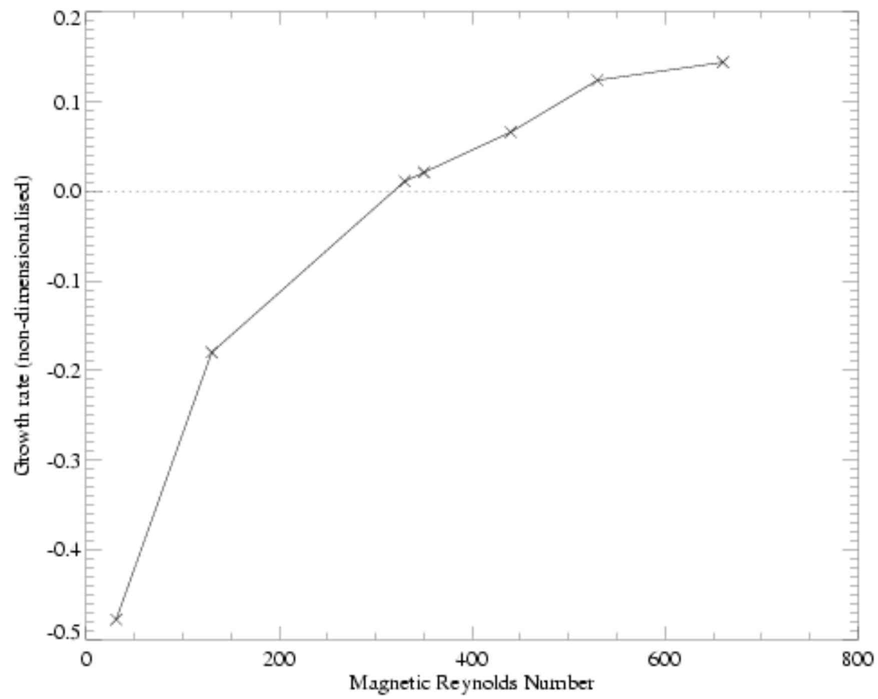
- Mixed finite-difference/pseudo-spectral scheme
- Horizontal derivatives evaluated in Fourier space
- Fourth order finite differences (either upwinded or centred, as appropriate) are used to calculate vertical derivatives
- Typical computational meshes use 256/512 points in each horizontal direction and > 100 points vertically
- Code parallelised using MPI

Key Parameters: (Photospheric estimates given in brackets)

Rayleigh number:	$Ra = 4 \times 10^5 \sim 300 Ra_{crit}$	(10^{16})
Reynolds number:	$Re \sim 150$	(10^{12})
Mag. Reynolds number:	$Rm \sim 60 - 660$	(10^6)
Prandtl number:	$\sigma = 1$	(10^{-7})
Mag. Prandtl number:	$Pm \sim 0.4 - 4.4$	(10^{-6})

Convective dynamo action

Right: The Rm dependence of the kinematic growth rate of the convectively-driven dynamo



- Critical magnetic Reynolds number is approximately 300
- Growth rate appears to be converging at large Rm, but this may be an indication that numerical diffusion is becoming increasingly important in this parameter regime

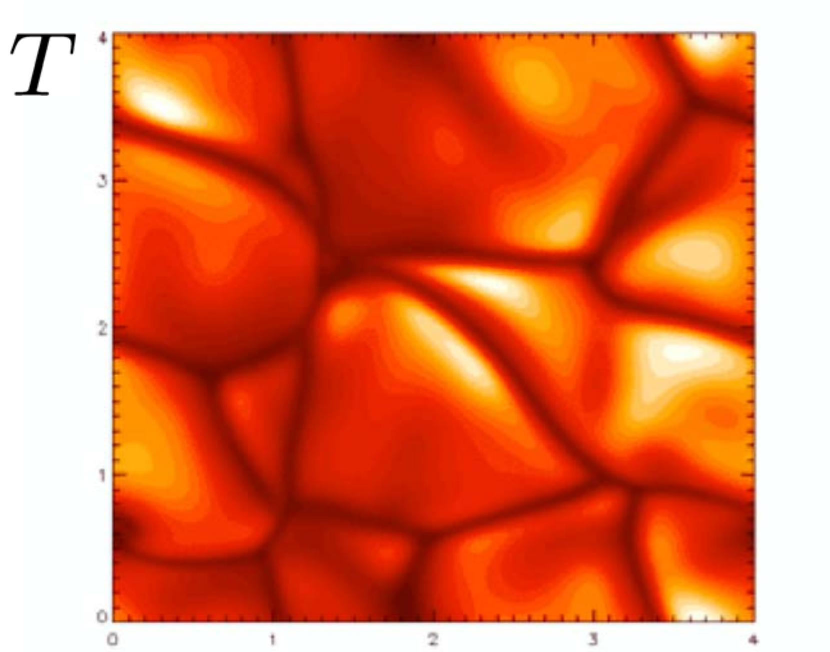
Magnetic Prandtl number: There is some debate regarding the viability of small-scale dynamos at low magnetic Prandtl number (e.g. Boldyrev & Cattaneo 2004; Schekochihin et al. 2005) -- impossible to resolve this debate using DNS at present (with current computational facilities)

For this set of parameters: $Pm \sim 2$ when $Rm = Rm_c \sim 300$

Convective dynamo action

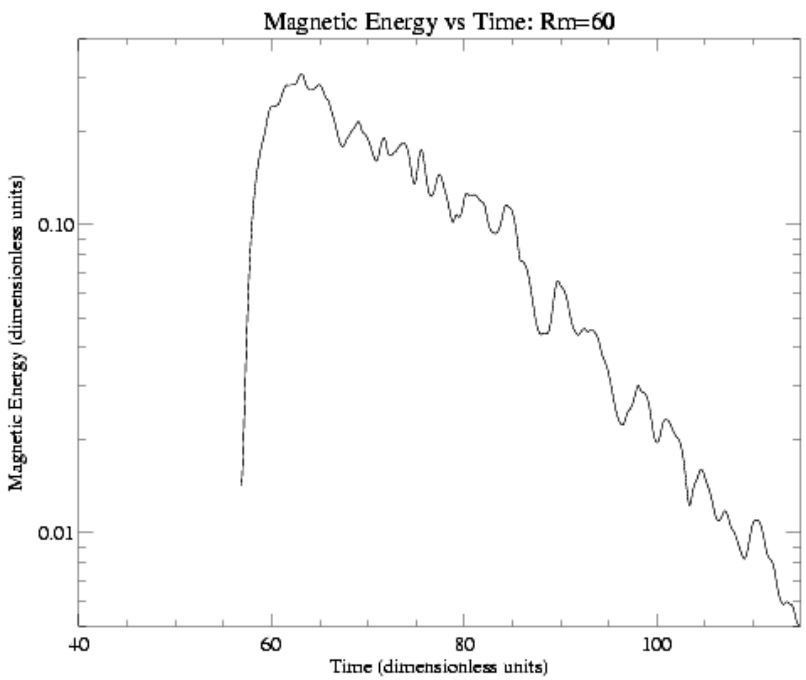
Right: The initial state – a fully developed non-magnetic convective state

Not really "turbulent" (Reynolds number is too small), but highly time-dependent.



By varying the (constant) magnetic diffusivity, different magnetic Reynolds numbers can be investigated

Right: Rm=60 – too small for dynamo action → magnetic energy decays exponentially



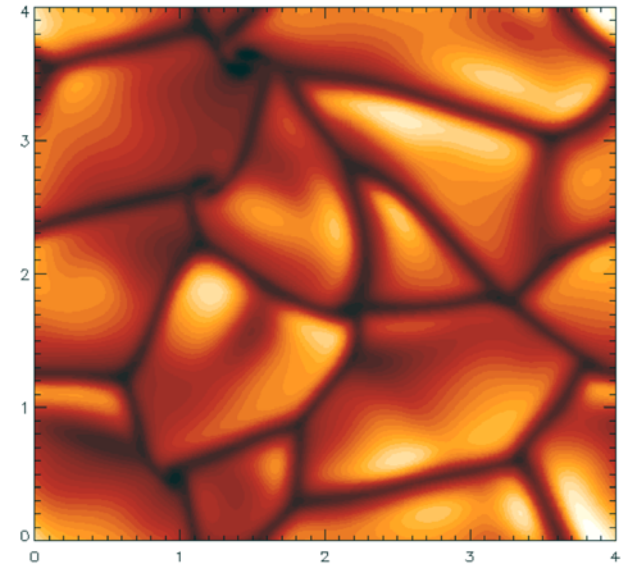
Convective dynamo action (cont.)

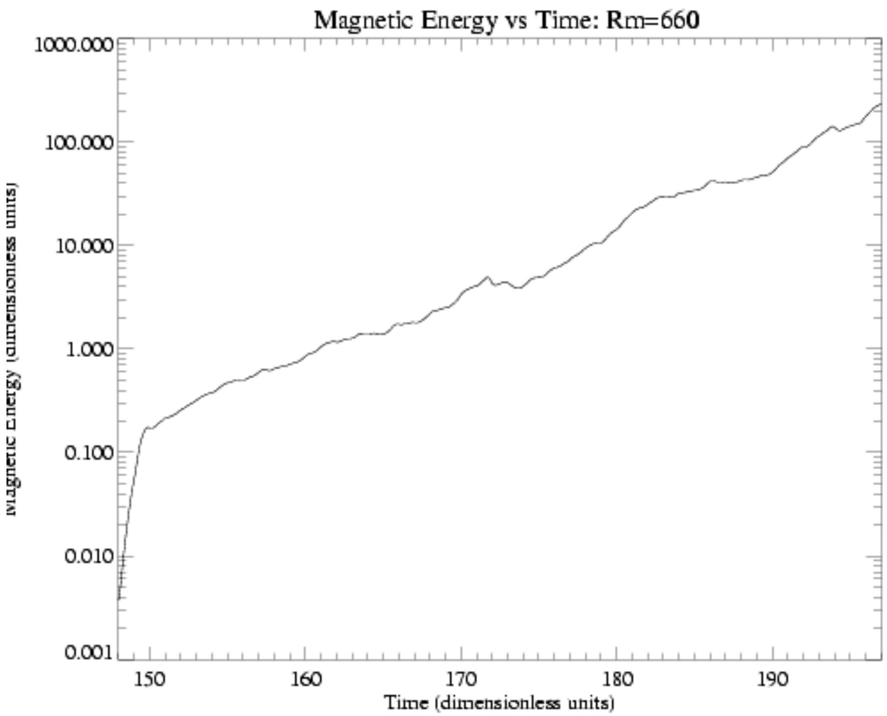
A kinematic dynamo:

$$\lambda = 4 \quad Rm \sim 660 \quad Re \sim 150$$

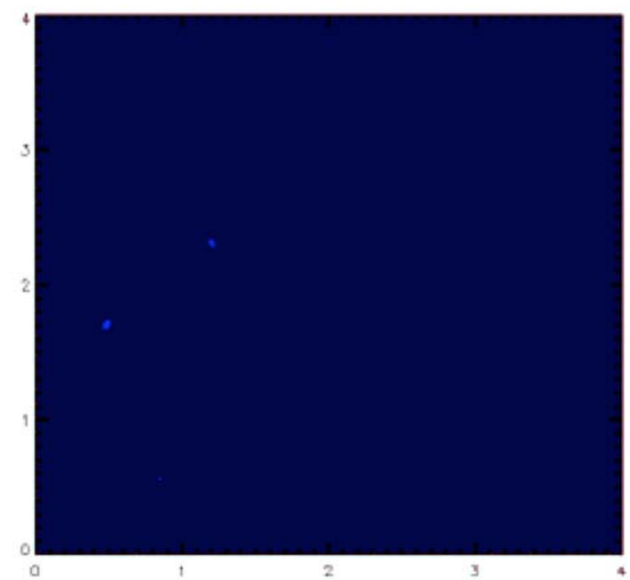
Numerical resolution: $256 \times 256 \times 160$

T





B^2



Convective dynamo action (cont.)

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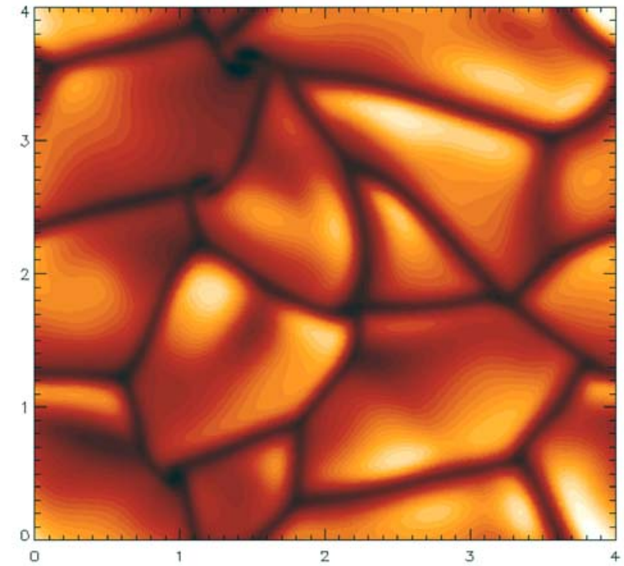
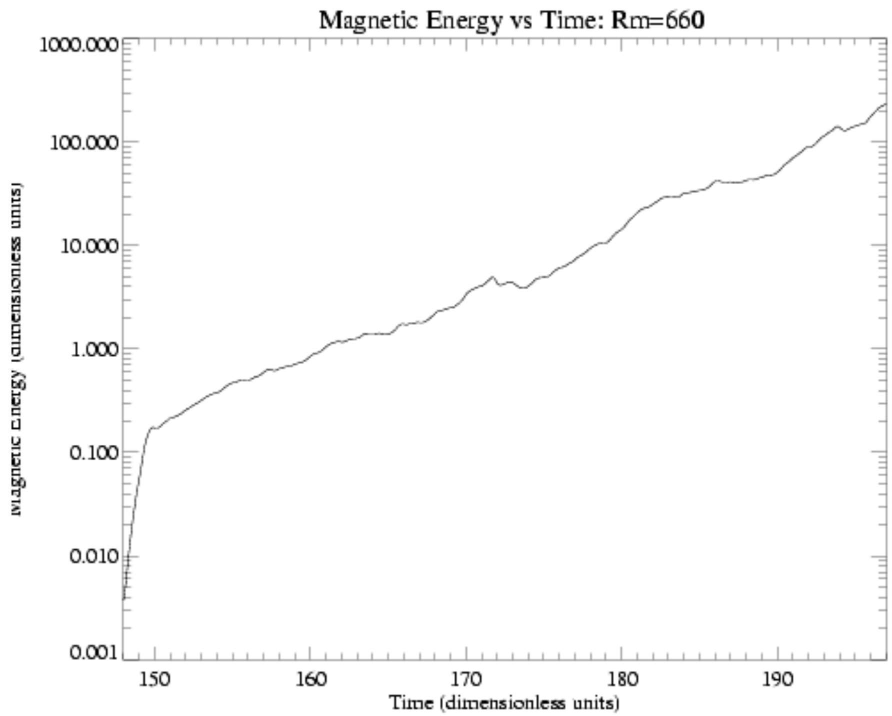
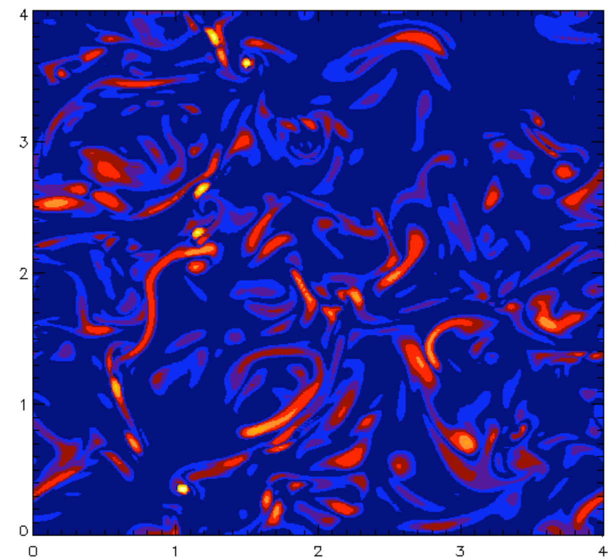




B^2

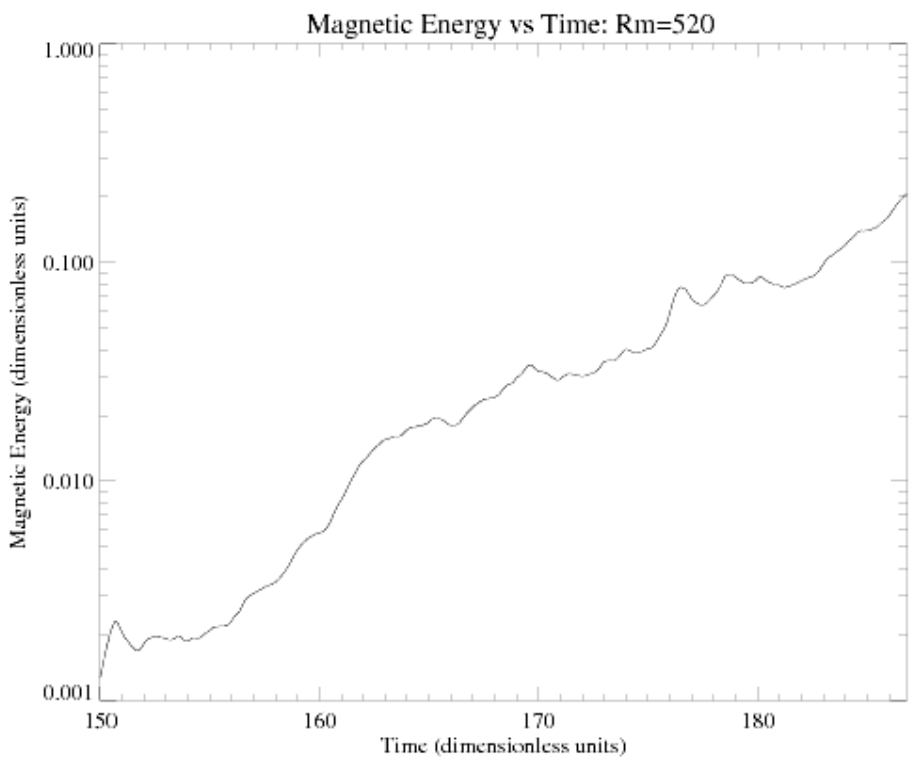


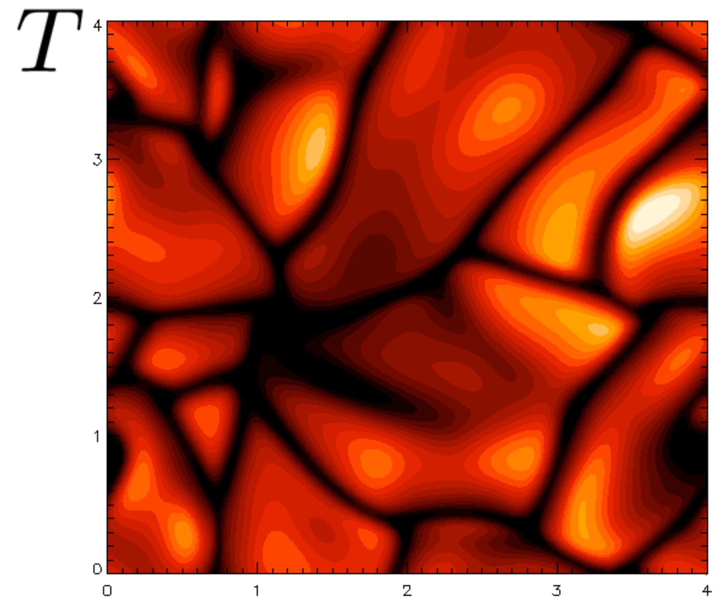
Convective dynamo action (cont.)

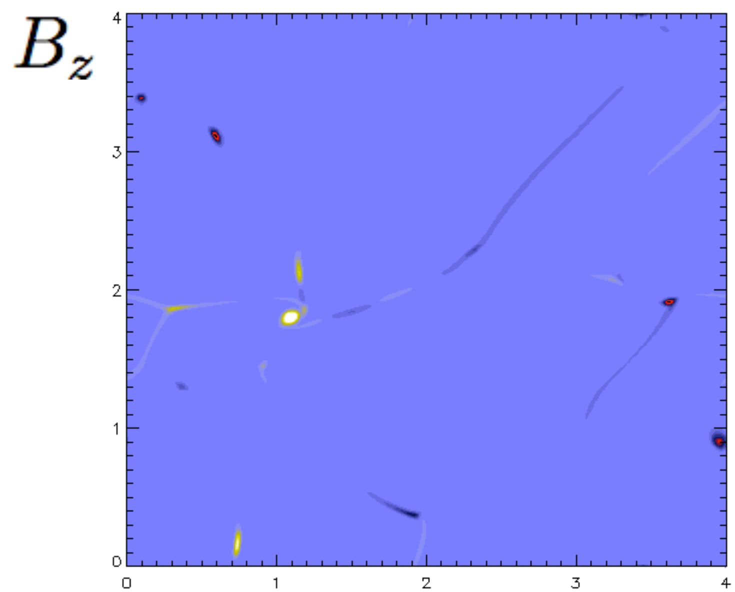
A nonlinear dynamo:

$$\lambda = 4 \qquad Rm \sim 520 \quad Re \sim 150$$

Numerical resolution: $512 \times 512 \times 160$





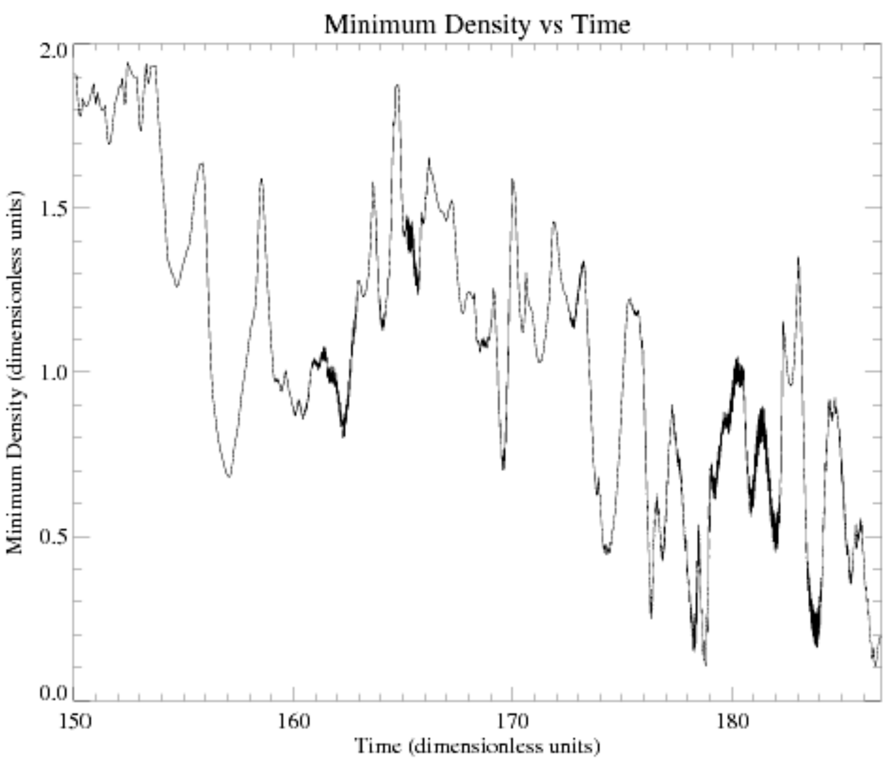


Convective dynamo action (cont.)

Evacuation of magnetic elements:

As magnetic concentrations form, the resulting high magnetic pressure tends to lead to the partial evacuation of these regions

Below left: A plot of the minimum density against time for this nonlinear case.



Implications for numerics:

Alfvén speed, $V_A \sim \frac{B_0}{\sqrt{\rho}}$

Coefficient of thermal diffusion $\sim \frac{\kappa}{\rho}$

Both of these increase rapidly

The time-scales associated with thermal diffusion and alfvénic disturbances therefore become very small → critical time-step for the stability of the (explicit) numerical scheme becomes very small.

Summary (and suggestions)

- All simulations are in the high Pm regime. Using DNS, not possible to resolve necessary scales with available computing resources - using LES, what is Pm?
    - This issue will **not** be resolved by numerical approaches in the near future - could a simpler model be considered?
- Convective dynamos do work in the high Pm regime, although the partial evacuation of the resulting magnetic regions leads to numerical difficulties….
    - Anelastic approach may be a good compromise (although this will underestimate the peak fields that can be produced)
    - Dynamo problem may be well suited to AMR-type approaches - not investigated yet, but would allow us to focus the necessary resolution upon the magnetic structures…..