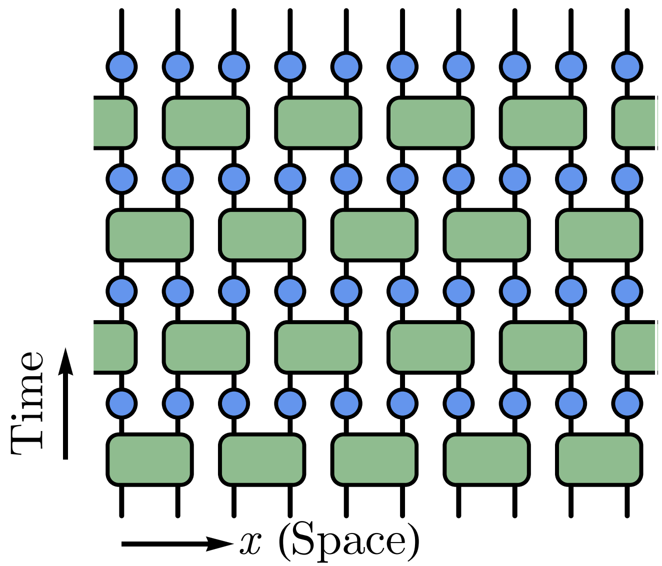
Learning global charges from local measurements

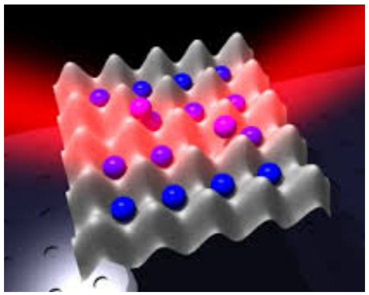
Romain Vasseur
(UMass Amherst)
KITP conference: NISQ systems: advances & applications



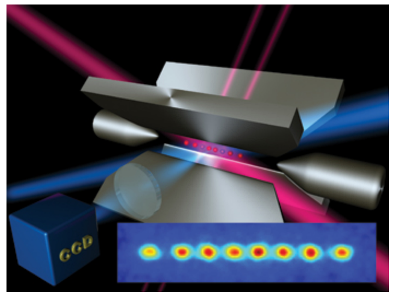
Agrawal, Zabalo, Chen, Wilson, Potter, Pixley, Gopalakrishnan, RV, PRX '22 (in press)
Barratt, Agrawal, Gopalakrishnan, Huse, RV, Potter, PRL '22
Barratt, Agrawal, Potter, Gopalakrishnan, RV, 2206.12429

Non-equilibrium quantum systems and NISQ era

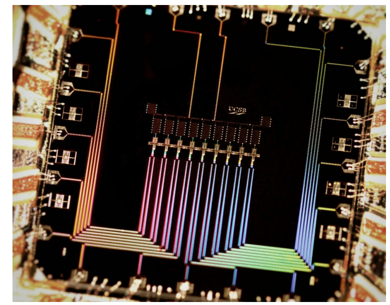
Ultracold atoms



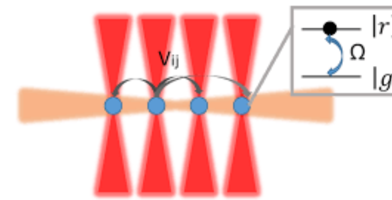
Trapped Ions



Superconducting Qubits



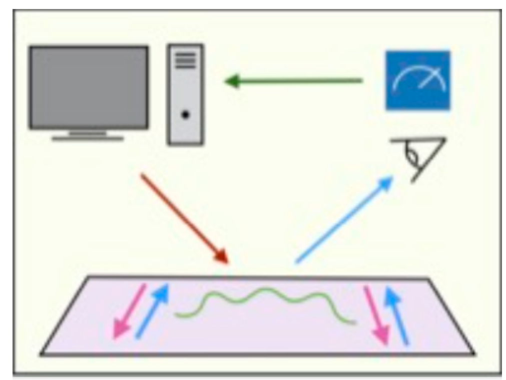
Rydberg Arrays



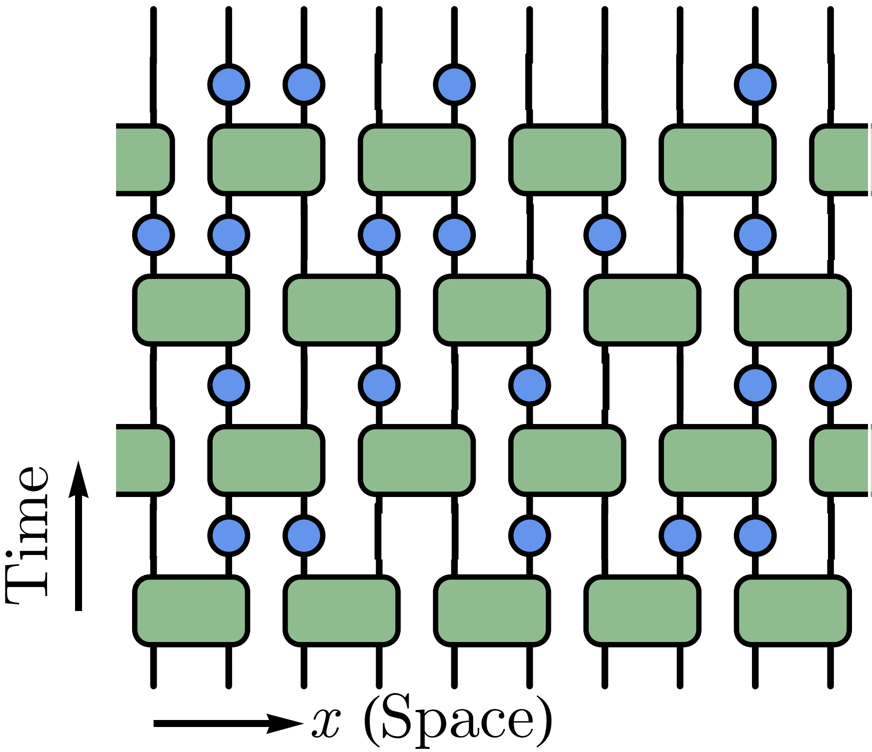
Many fundamental questions:

"Interactive" many-body dynamics

Role of the environment & observer?



Measurement-induced transitions



Skinner, Ruhman & Nahum ’19
Li, Chen, Fisher ’19

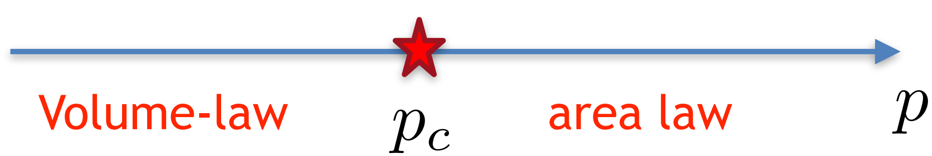
Projective measurement:

$$|\psi\rangle \to |\psi_m\rangle = \hat{P}_m |\psi\rangle$$

Born probability: $p_m = \langle\psi_m|\psi_m\rangle$

Reviews: Potter & Vasseur 2111.08018
Fisher, Khemani, Nahum & Vijay 2207.14280

Competition between scrambling/chaotic dynamics and disentangling measurements



QEC perspective & post selection problem

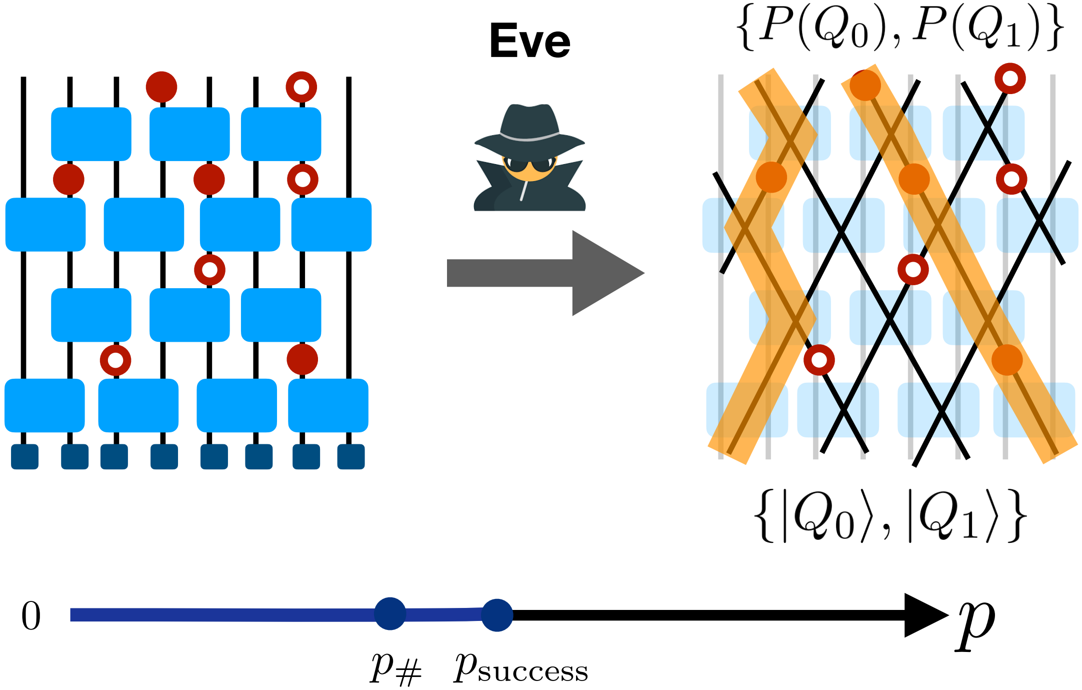
- In volume-law phase, quantum dynamics effectively hides quantum information from measurements (emergent code)

Choi, Bao, Qi & Altman ’20, Gullans & Huse ‘20

- In area-law/pure phase, measurements efficiently extract information about the state

- Current experimental realizations limited to small systems (naively requires post-selection, decoding problem possible for Clifford gates only)

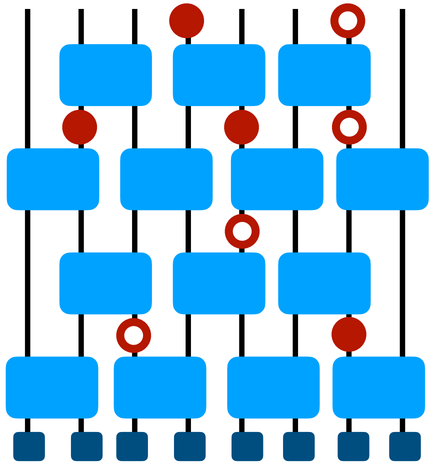
Noel et al ’21, Koh et al ‘22



Beyond postselection:
Ippoliti & Khemani ’21 (space time duality), Dehghani et al ’22 (Machine Learning), Buchhold et al (feedback) ’22, Y. Li et al (XEB) ’22

This talk: predict a simpler observable, global charge! Fundamentally limited by “charge-sharpening” transition

Let’s play some game!



$$|\psi_0\rangle = \sum_Q |Q\rangle$$

$$|\psi_0\rangle = |Q_0\rangle \; , |Q_1\rangle$$

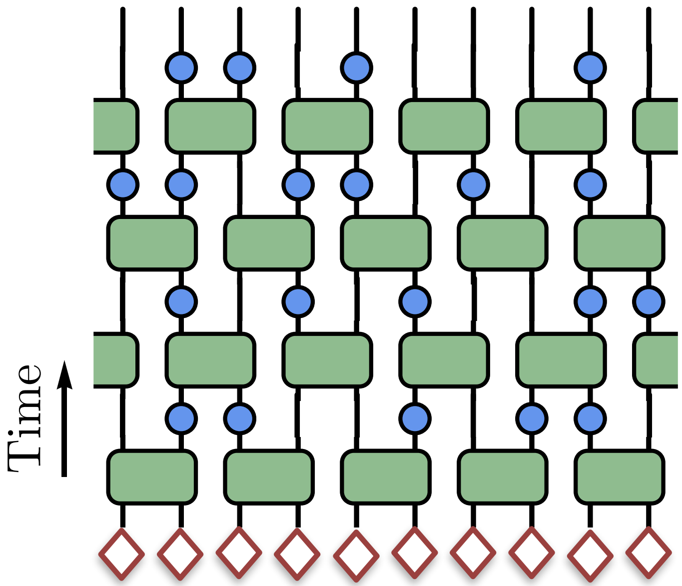
Eve

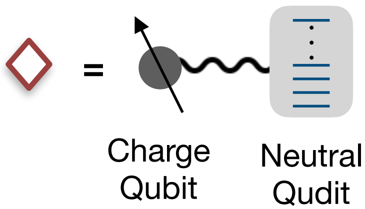
1. Collect random local charge measurements for time t=O(L) and rate p.
2. Guess global charge Q of the state
3. Measure global charge in one shot, did Eve get it right?

Fundamentally limited by “charge-sharpening” transition: Eve can only learn the charge above a threshold $p_{\#}$

U(1) Symmetric circuits

Model:

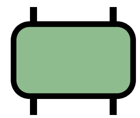




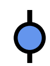
$$\mathcal{H}_i = \mathbb{C}^2 \otimes \mathbb{C}^{d^2}$$

(Focus on large-d limit)

Charge conserving gates and measurements:



$$= \begin{pmatrix} U^0_{d^2\times d^2} & & \\ & U^1_{2d^2\times 2d^2} & \\ & & U^2_{d^2\times d^2} \end{pmatrix}$$

 = measure qubit in charge-basis, qudit in whatever basis

Motivation(s):

- Scrambling of hydrodynamic modes
- Interplay of symmetry and entanglement growth
- How much information about charge can an observer extract from local measurements?

Independent measurements: $N_{\#} \sim L^2$

Unitary version:
Khemani, Huse, Vishwanath PRX ‘18, Rakovsky, Pollmann, Von Keyserlingk, PRX ’18, PRL ’19,
MRC version: Agrawal, Zabalo, Chen, Wilson, Potter, Pixley, Gopalakrishnan, RV

New “charge-sharpening” transition

Luttinger liquid-like correlations!

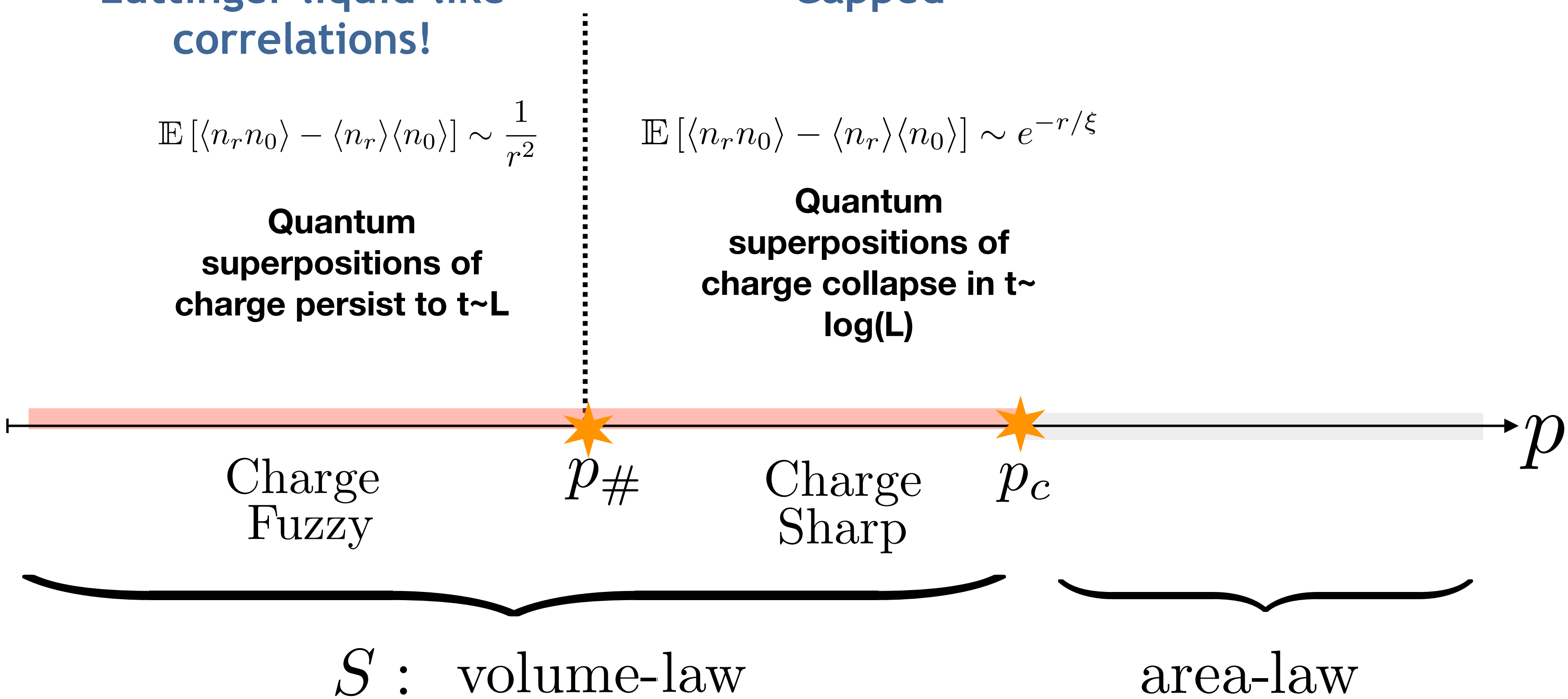
$$\mathbb{E}\left[\langle n_r n_0\rangle - \langle n_r\rangle\langle n_0\rangle\right] \sim \frac{1}{r^2}$$

Quantum superpositions of charge persist to t~L

Gapped

$$\mathbb{E}\left[\langle n_r n_0\rangle - \langle n_r\rangle\langle n_0\rangle\right] \sim e^{-r/\xi}$$

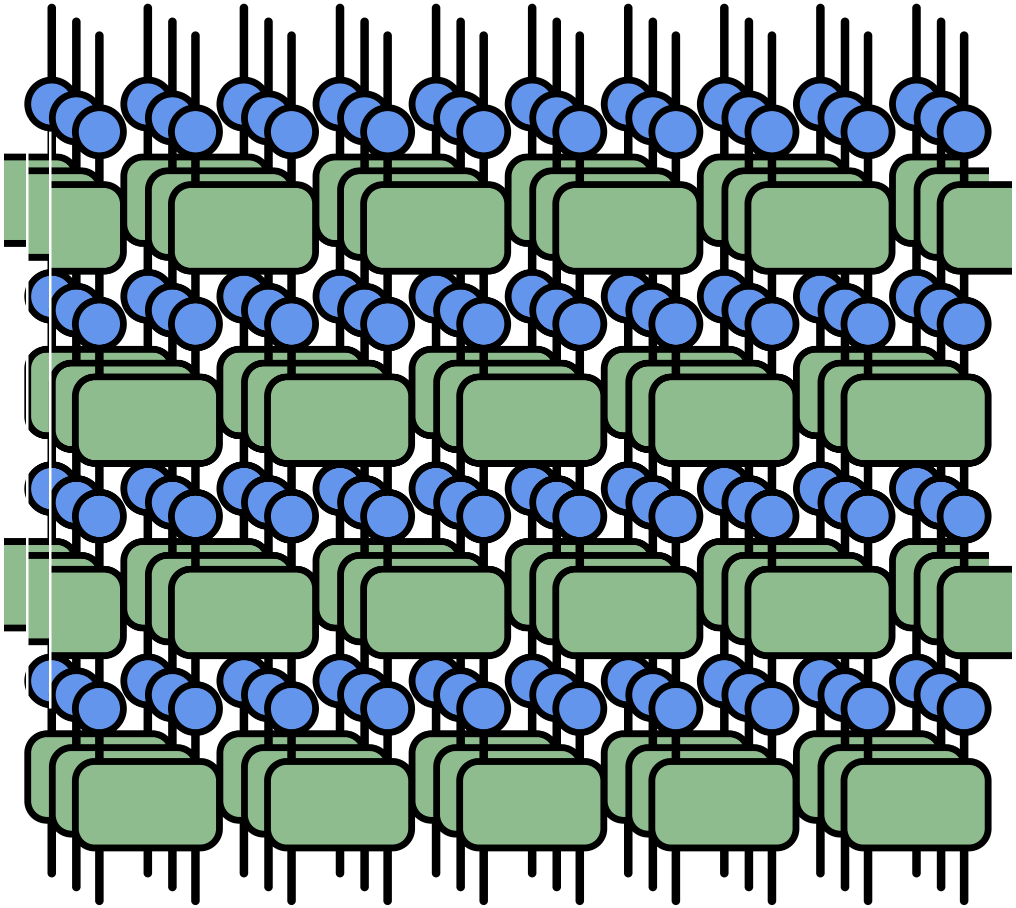
Quantum superpositions of charge collapse in t~ log(L)



Agrawal, Zabalo, Chen, Wilson, Potter, Pixley, Gopalakrishnan, RV, PRX ’22 (in press)

See also: Transitions inside volume law phase with discrete symmetry: Bao, Choi & Altman ’21
Free fermion transitions, see Sebastian’s talk

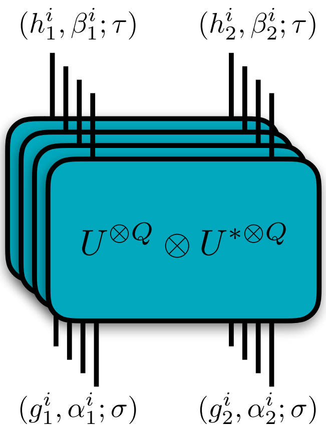
Replica stat mech model and effective field theory



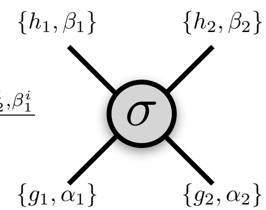
$$\rho^{\otimes Q}$$

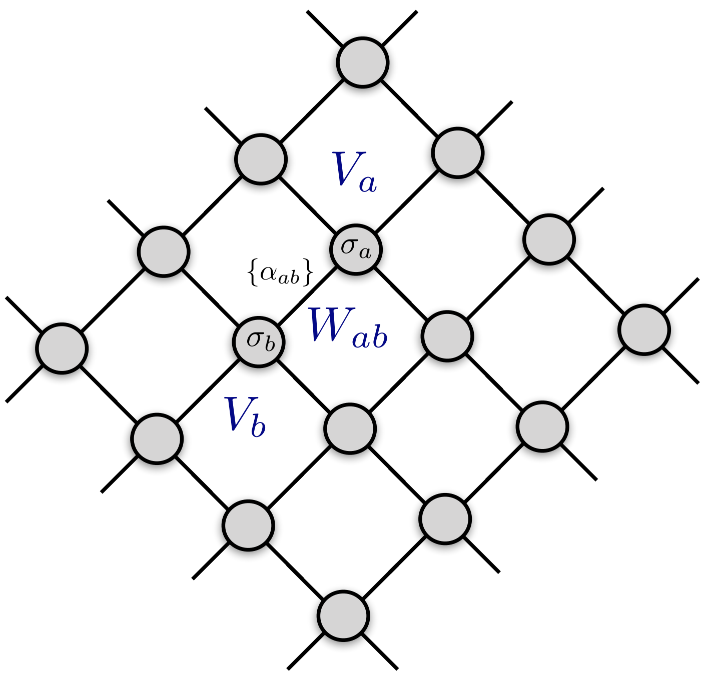
Step 1: **replicate**

Replica stat mech model and effective field theory



$$\simeq \delta_{\sigma,\tau} \prod_{i=1}^{Q} \frac{1}{d^2} \frac{\delta_{\alpha_1^i,\beta_1^i}\delta_{\alpha_2^i,\beta_2^i} + \delta_{\alpha_1^i,\beta_2^i}\delta_{\alpha_2^i,\beta_1^i}}{2}$$



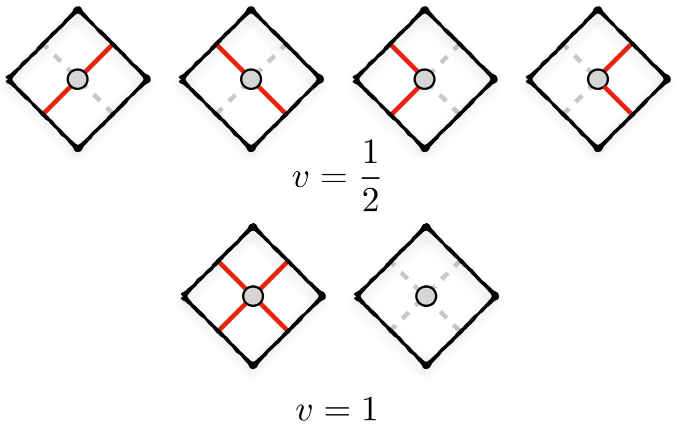


Step 2: **average over circuits**

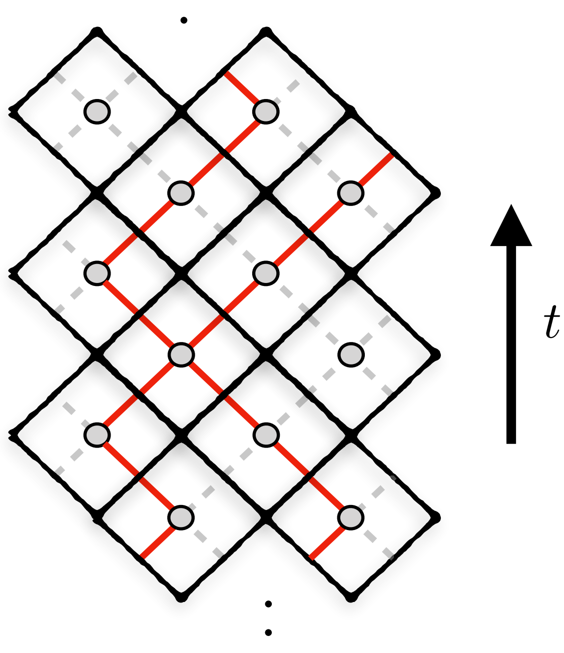
Permutation "spins"
+ charges dof's

RV, A.C. Potter, Y-Z. You and A.W.W. Ludwig '18
Zhou & Nahum '18
Bao, Choi & Altman '20
C-M. Jiang, RV, Y-Z. You and A.W.W. Ludwig '20

Replica stat mech model and effective field theory



(Disordered) 6-vertex model aka symmetric exclusion process

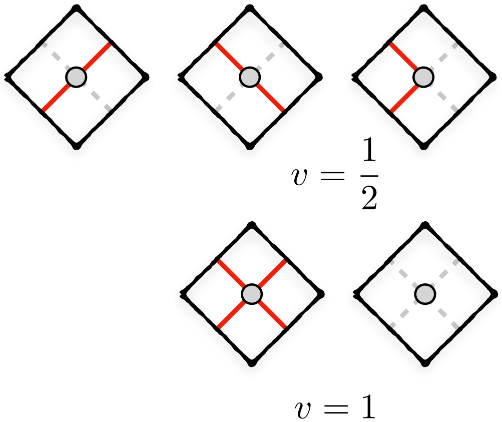


Step 3: **Effective charge dynamics**

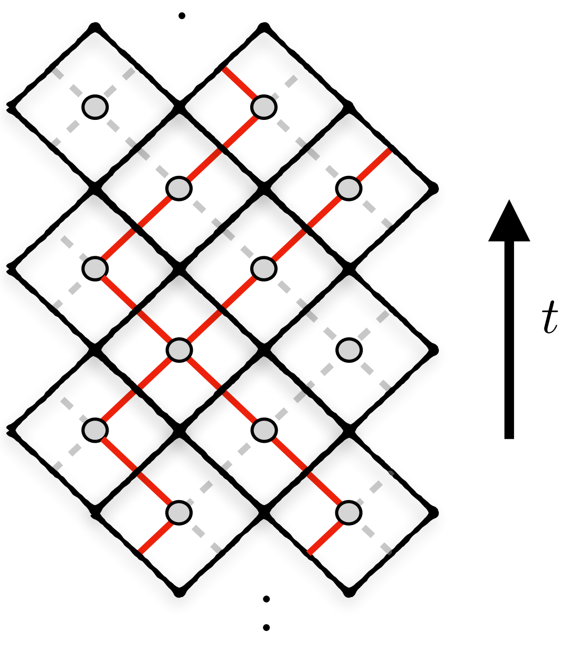
Volume-law regime:

- Average over circuits leads to permutation "spins"
- Gapped excitations
- Integrate out => effective field theory for charge DOF only
- Can do this analytically at large d, including replica-limit

Replica stat mech model and effective field theory



(Disordered) 6-vertex model aka symmetric exclusion process



Step 3: **Effective charge dynamics**

Replicated disordered SEP problem:

- **Charge Fuzzy**: Strong fluctuations between replicas
- **Charge Sharp**: inter-replica charge fluctuations pinned to 0

Controlled theory:
1/d corrections RG irrelevant!

Replica stat mech model and effective field theory

Hamiltonian limit (weak measurements):

$$H_U = -J \sum_{a,\langle i,j \rangle} \vec{S}_{a,i} \cdot \vec{S}_{a,j}$$

Gapless, z=2 “magnons” => charge diffusion

$$H_M = \gamma \sum_{a,b;i} S^z_{a,i} \Pi_{ab} S^z_{b,i}$$

$$\Pi_{ab} = \delta_{ab} - \frac{1}{Q}$$

projector onto inter-replica fluctuations
(replica-symmetric mode unaffected by measurements)

Step 4: Hamiltonian limit

Continuous time and weak measurements

Replica stat mech model and effective field theory

Goldstone action (linearize about ordered state)

$$\mathcal{L} = iS\delta\bar{\theta}\partial_\tau\bar{\phi} + \frac{S^2 J}{2}\left(\nabla\delta\bar{\theta}^2 + \nabla\bar{\phi}^2\right) + \frac{S^2}{2}\left(\frac{1}{\gamma}\left(\partial_\tau \Pi\phi\right)^2 + J\left(\nabla\Pi\phi\right)^2\right)$$

Replica-average part:
Heisenberg FM, gapless z=2

Inter-replica fluctuations
(Q-1) independent z=1 modes

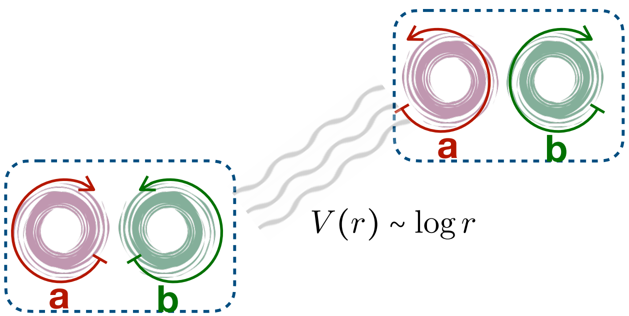
$$\rho_s \sim \sqrt{\frac{J}{\gamma}}$$

unitary dynamics

measurement strength

“Superfluid stiffness”

Inter-replica vortices



KT* transition $\rho_s^c = \frac{1}{2}\rho_s^{\mathrm{BKT}}$

Step 5: Effective field theory

Main prediction: charge-sharpening transition

~ **Kosterlitz-Thouless universality class** with

“superfluid stiffness” $\rho_s \sim \frac{1}{\sqrt{p}}$

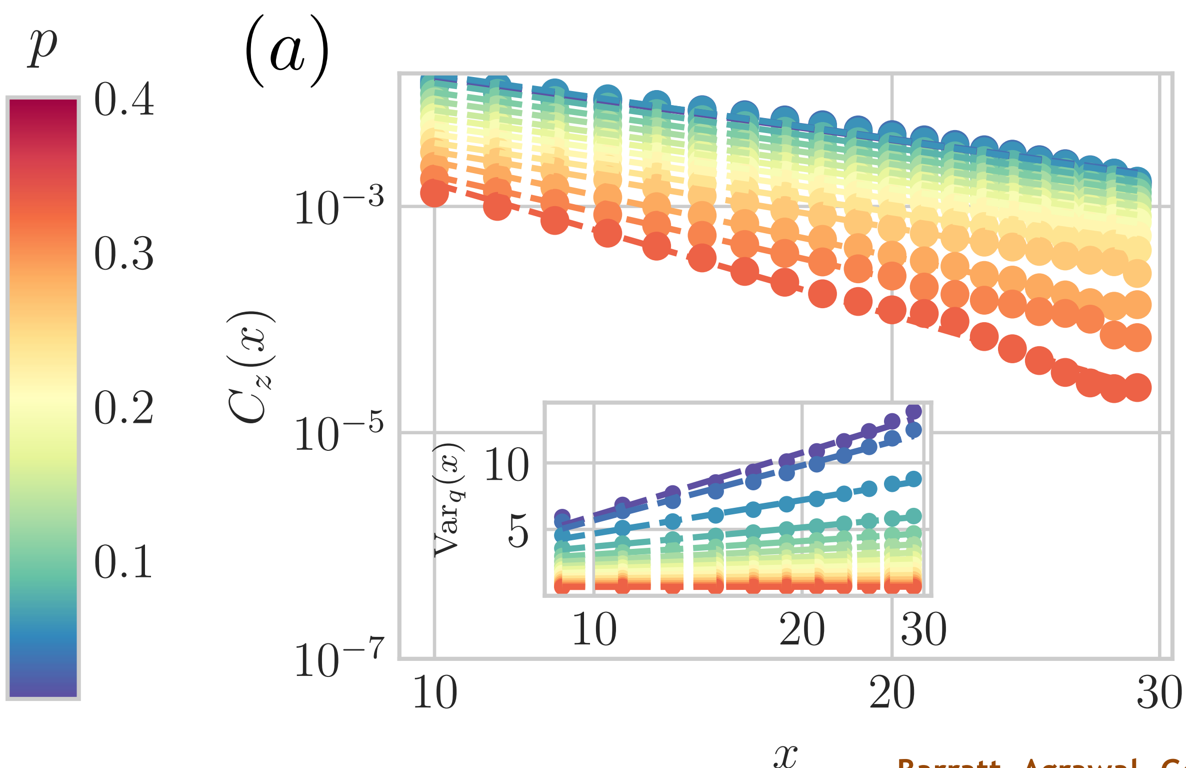
Numerics and critical properties

TEBD on stat-mech model is efficient, even though original model is in highly-entangled phase!

Charge Fluctuations $C_z(x) = \mathbb{E}_{U,M}\left[\langle S_0^z S_x^z\rangle - \langle S_0^z\rangle\langle S_x^z\rangle\right]$

Effective field theory predictions:

$$C_z(x) \sim \begin{cases} \rho_s (a/x)^{-2} & p \le p_\# \\ e^{-x/\xi} & p > p_\# \end{cases} + \dots$$



In Fuzzy phase:

$$\delta Q_m^2 \sim \rho_s \log L$$

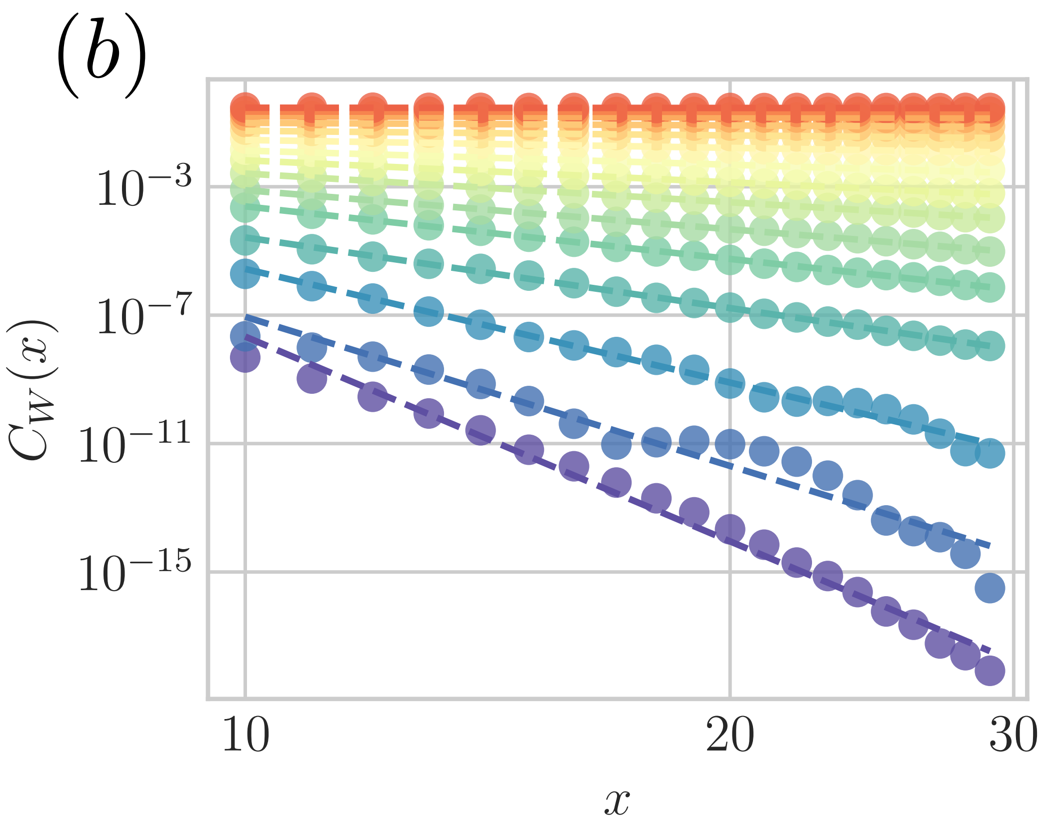
Barratt, Agrawal, Gopalakrishnan, Huse, RV, Potter, PRL ‘22

Continuously varying exponents

String Correlators:

$$W_{[0,x]} = \prod_{0 \leq i < x} \sigma_i^z$$

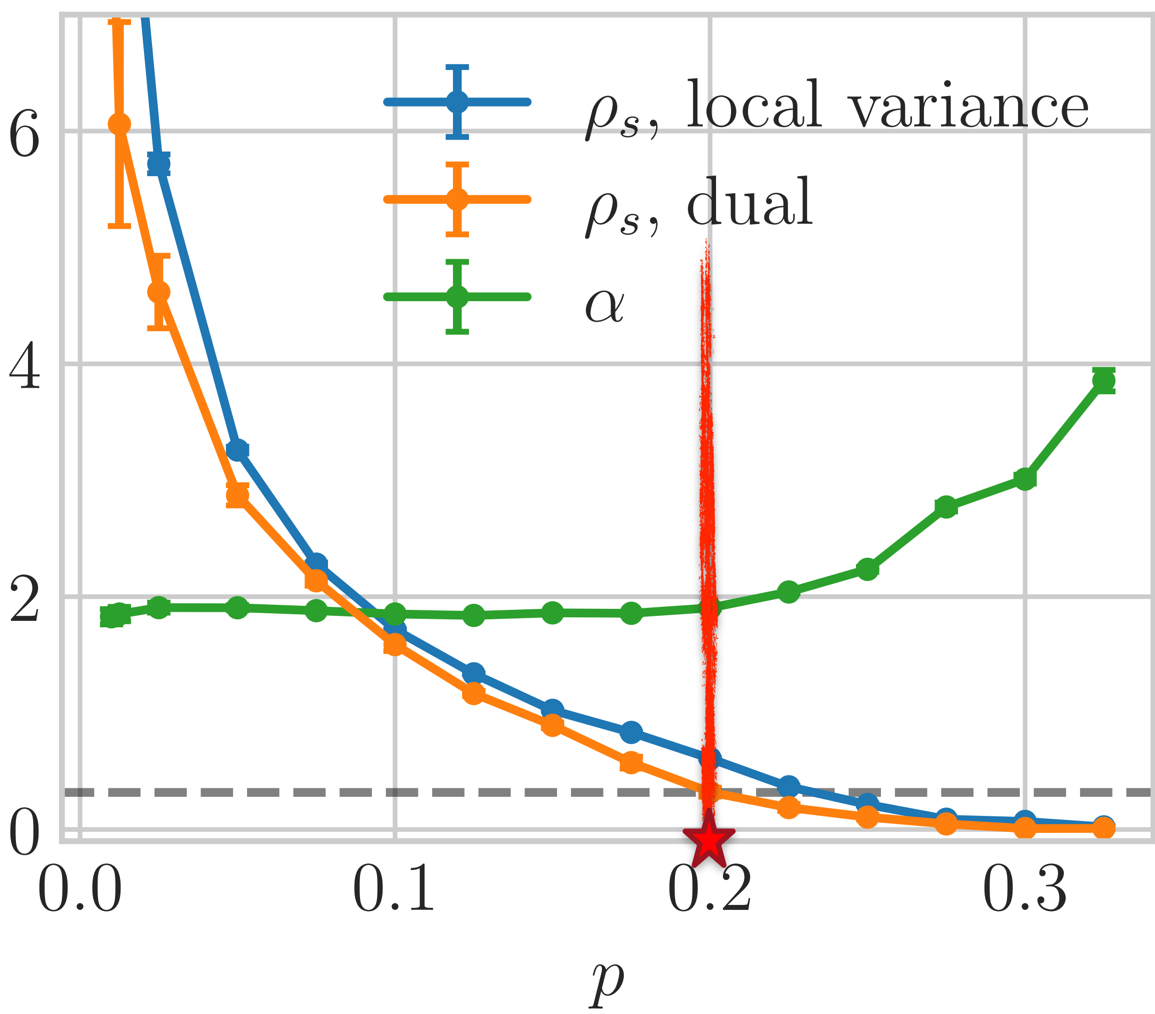
$$C_W(x) = \mathbb{E}_{U,M}\left[\langle W_{[0,x]}\rangle^2\right]$$



$$C_W(x) \approx \begin{cases} |x|^{-2\pi\rho_s} & p \leq p_\# \\ \text{constant} & p > p_\# \end{cases}$$

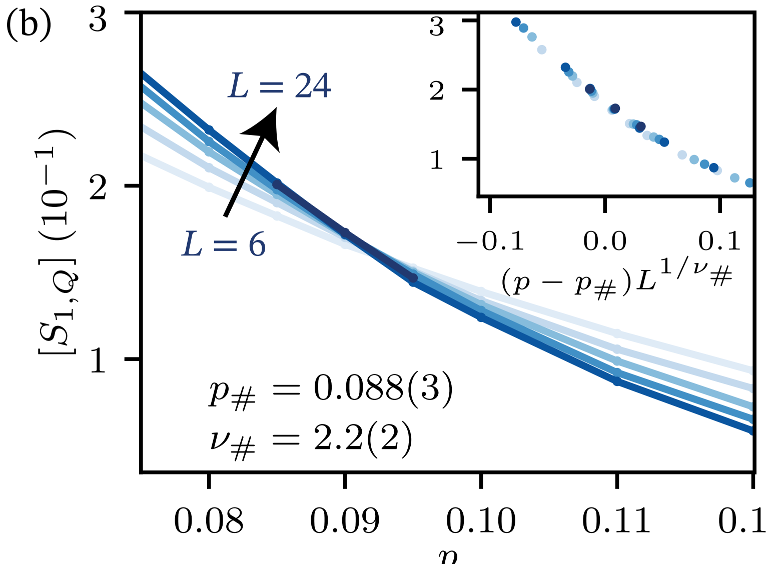
Barratt, Agrawal, Gopalakrishnan, Huse, RV, Potter, PRL '22

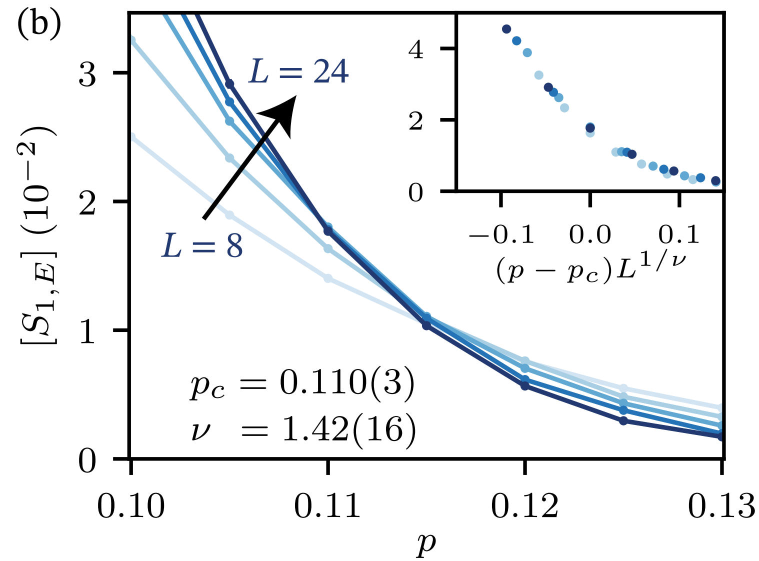
Numerics and critical properties



Barratt, Agrawal, Gopalakrishnan, Huse, RV, Potter, PRL '22

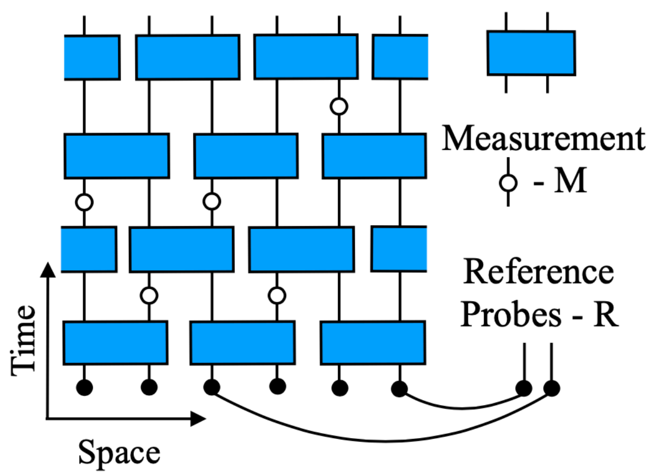
Qubit numerics

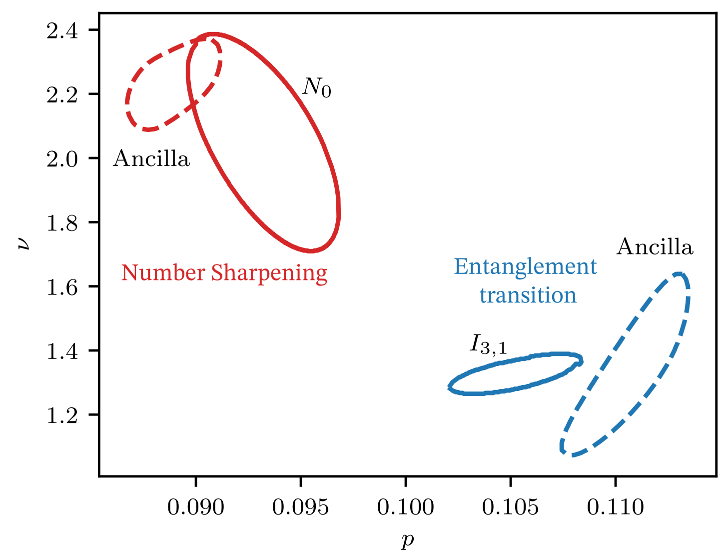




$$|\Psi\rangle = |\psi_{\mathcal{Q}}\rangle|0\rangle + |\psi_{\mathcal{Q}-1}\rangle|1\rangle$$

$$|\Psi\rangle = (1/\sqrt{2})(|\psi_1\rangle|1\rangle + |\psi_0\rangle|0\rangle)$$



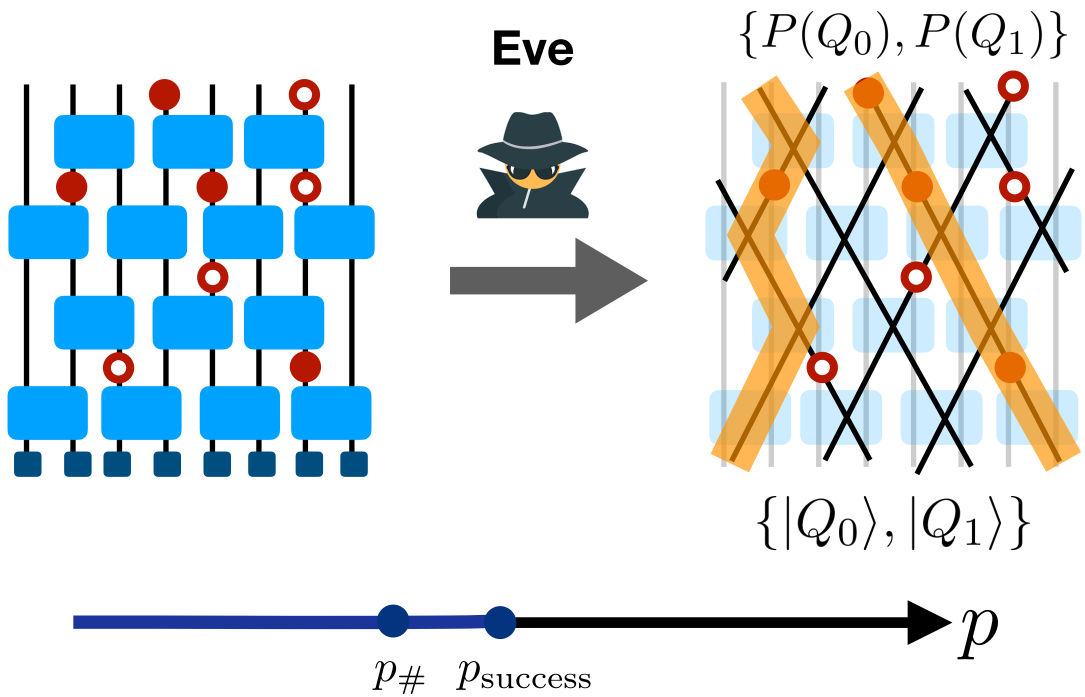


Using Ancilla probe of Gullans & Huse ’19

Eavesdropping problem

Barratt, Agrawal, Potter, Gopalakrishnan, RV, 2206.12429

Transitions in the learnability of global charges from local measurements



Can Eve predict global charge from local measurements?

With unlimited resources, yes above the sharpening transition $p > p_\#$

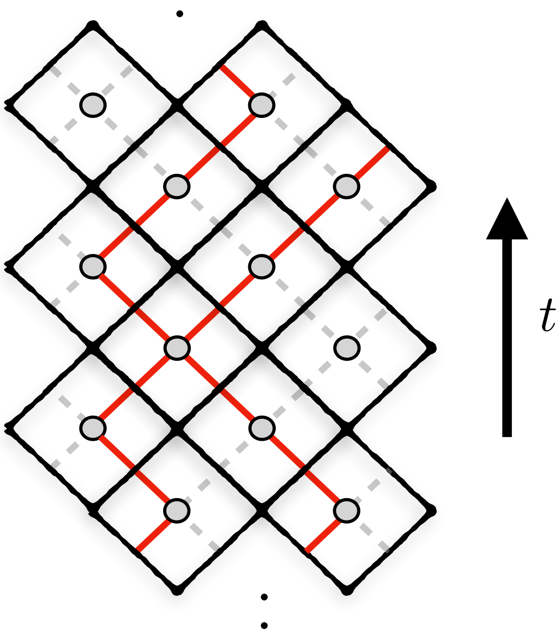
What about with less knowledge (no knowledge of the gates) and limited resources?

Eavesdropping problem

Barratt, Agrawal, Potter, Gopalakrishnan, RV, 2206.12429

- Optimal classifier without knowledge of circuit: maximize

$$p(Q|\{m\}) = \frac{\mathbb{E}_U p(\{m\}|U,Q)}{\sum_{Q'} \mathbb{E}_U p(\{m\}|U,Q')}$$



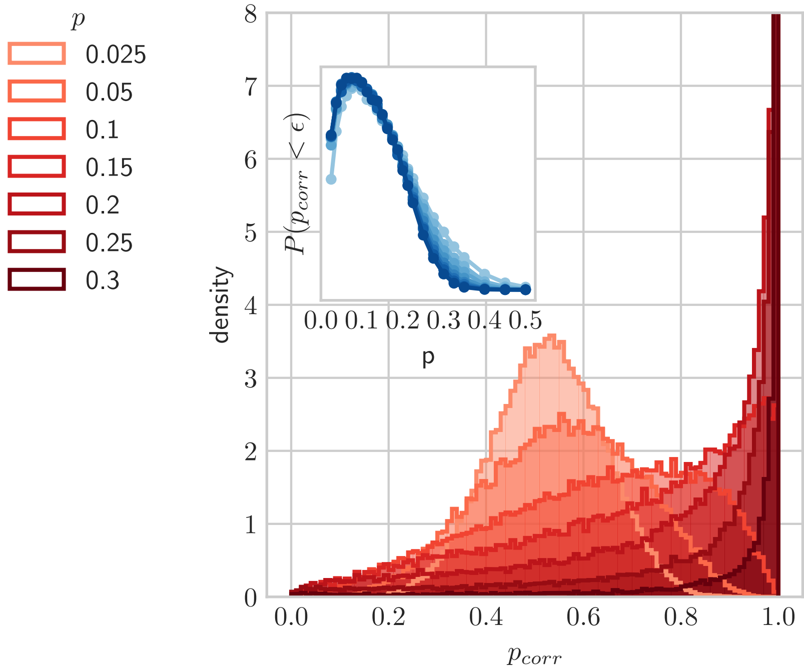
- Marginalized Born probabilities can be computed from disordered SEP problem

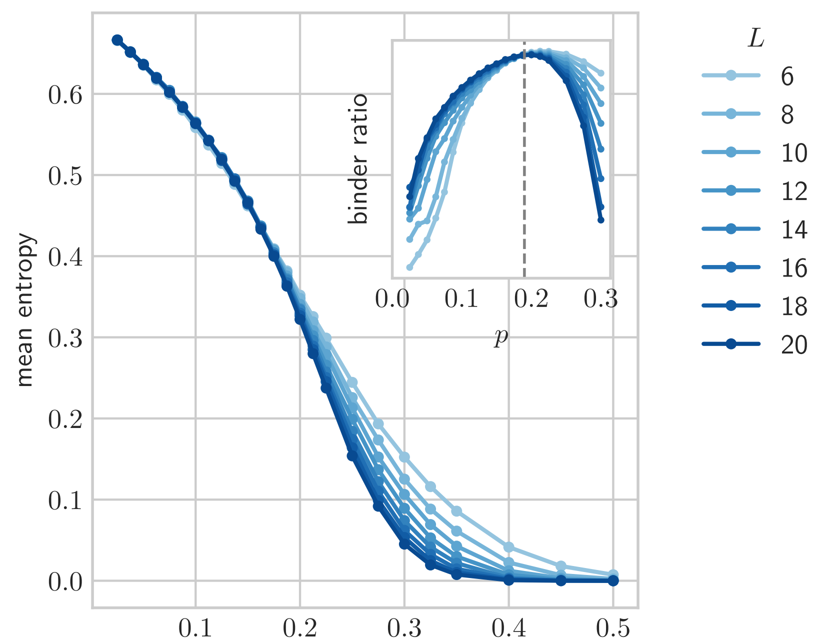
- Can be computed efficiently using MPS / TEBD

$$\mathbb{E}_U p(\{m\}|U,Q) = (1|\prod_{t=1}^{t_f} T(\vec{m}_t)|Q).$$

Eavesdropping problem

Barratt, Agrawal, Potter, Gopalakrishnan, RV, 2206.12429





- Optimal classifier/decoder: count charge random walks constrained by measurement outcomes. Determine probabilities $P(Q_i|\{m\})$
- Eve can learn the charge with polynomial resources! (Can be solved efficiently with TEBD)

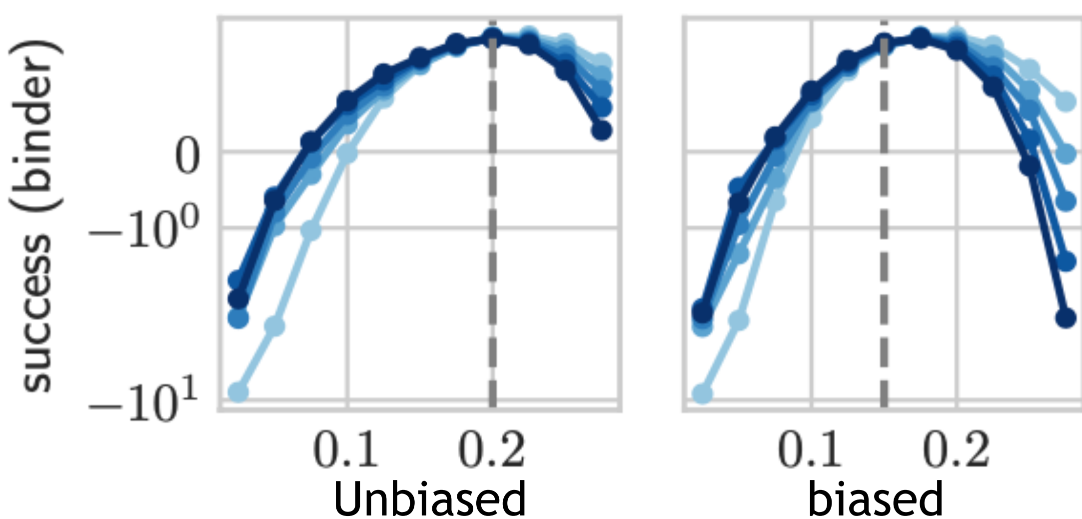
Eavesdropping problem

Barratt, Agrawal, Potter, Gopalakrishnan, RV, 2206.12429

- Optimal classifier without knowledge of circuit: maximize

$$p(Q|\{m\}) = \frac{\mathbb{E}_U p(\{m\}|U,Q)}{\sum_{Q'} \mathbb{E}_U p(\{m\}|U,Q')}$$

- If Eve knows the circuit, we can do better while keeping a simple classical classifier:



Use biased random walks using hopping probability:

$$|\langle 01| U |10\rangle|^2$$

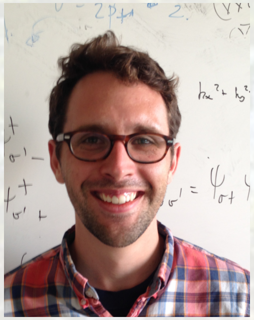
Conclusion

- New type of measurement-induced criticality
- Transition in ability of measurements to reveal a conserved charge inside the scrambled, entangling phase!
- Stat mech mappings allow for efficient TEBD + Effective Field theory approaches
- Higher d? Efficient decoders in general case? Non-Abelian symmetries? Competing measurements? Experiments?

UMass Amherst

Thank you!

Slides credit thanks:
Utkarsh & Drew



A.C. Potter
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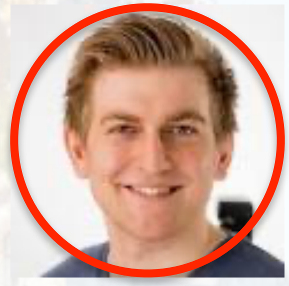

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