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Floquet codes and symmetries of topological order

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arxiv:2203.11137 (with Aasen and Hastings)

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QECC: Subspace ⊂ Subsystem ⊂ Floquet

- **Subspace codes:** code subspace $\mathrm{C} \subset L$,

  L =local Hilbert space, gs of local commuting Hamiltonians
- **Subsystem codes:** $\mathrm{C} \cong V \otimes D$, extra structure

  e.g. stabilizer codes: $1 \to A_H \to G_c \to P_D \to 1$ split extension

  $Stab(C) = A_H$, Hamiltonians with symmetries, subspace code if $D = 1d$
- **Floquet codes:** extra dynamical structure

1) honeycomb code (Hastings-Haah)

2) automorphism codes (Aasen-W.-Hastings)

3) ….

Honeycomb: volume vs area laws in monitored quantum dynamics

Automorphism: symmetries of anyon models

Manifestation of topology of spaces of Hamiltonians

Honeycomb code

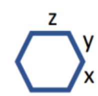
Qubits on vertices: $L = \otimes_{vertices} C^2$

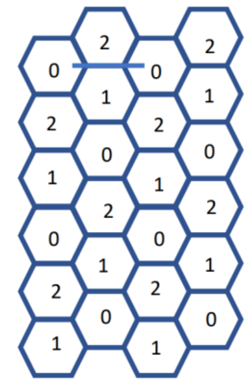
a. 3 colors of plaquettes of the honeycomb

b. 3x3 labels of each edge:

   3 round labels r=0,1, 2 and

   3 Pauli check labels xx,yy,zz (three directions)





Hastings-Haah code:

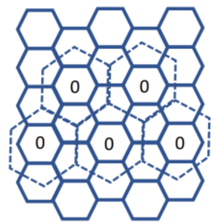
Run measurements in rounds 0,1,2,0,1,2,… and keep a log of measurements

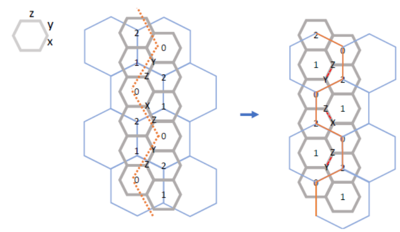
During each round r, measure the checks xx, yy, zz on the two qubits of edges with round label r.

Perfect match for Majorana quantum computing (.15 vs $\geq .8$)

Honeycomb code as dynamical toric code

- After rounds of 0,1,2,0 and wlog all measuerments=1, the states=toric code on the 0-superlattice





- Each round moves the TC subspace among corresponding superlattices
- After three rounds, the e- and m-logical operators exchanged
- $V(t) \subset L(t) \subset L, t = 0,1,2$, $V(t) \subset L(t)$ are “toric codes” for t=0,1,2 and after a cycle, the topological symmetry e-m implemented.

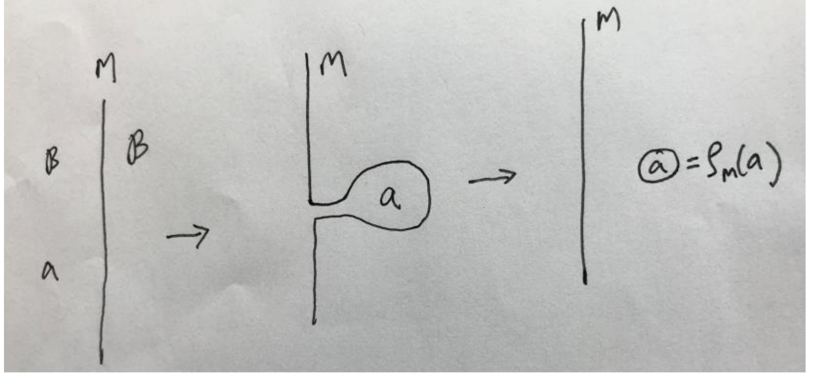
Honeycomb code = a loop of toric codes $\boldsymbol{V(t) \subset L(t) \subset L}$

that implements a symmetry when t interpolated continuously.

Floquet codes from symmetries of anyon models

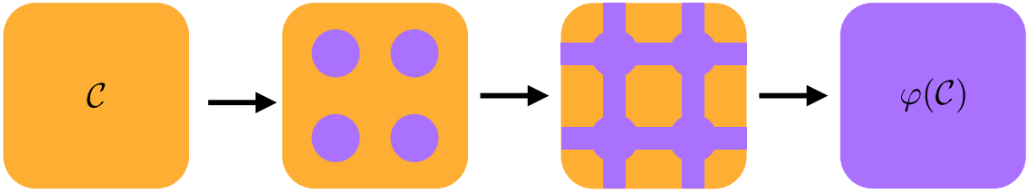
Symmetries of anyon models

- Topological symmetries of anyon models B form a group Pic(B)=classes of functors preserving anyon structures
  e.g. toric code, Pic(B)≅ Z_2 generated by e-m.
- Identified with invertible domain walls M:

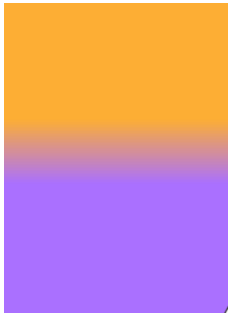


Loops of Hamiltonians and pumping domain walls

- Domain walls to loops: pump domain wall through system via Morse transitions:

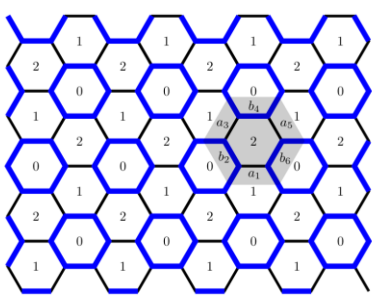


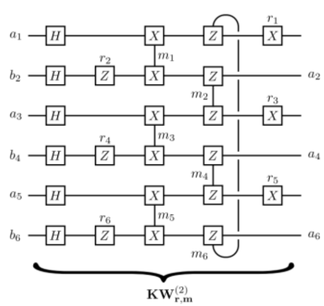
- Loops to domain walls: unrolling $R^{d-1} \times R$

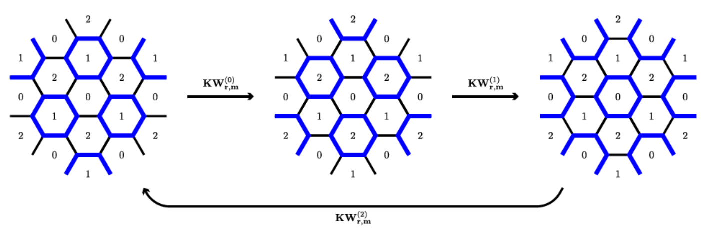


e-m automorphism code

- Symmetries $\longrightarrow$ loops of Hamiltonians $\longrightarrow$ Floquet codes
- TC: two symmetries $F_1 \longrightarrow \mathrm{TC}, F_{em} \longrightarrow$ automorphism code
- Qubits on edges: thicker=dead, thinner=active
- At round r, run the measurement based Kramers-Wannier circuits taking a_1, a_3, a_5 to a_2, a_4, a_6 of plaquettes with color r: after a cycle, “toric code” on triangular 0-superlattice (with dead qubits).

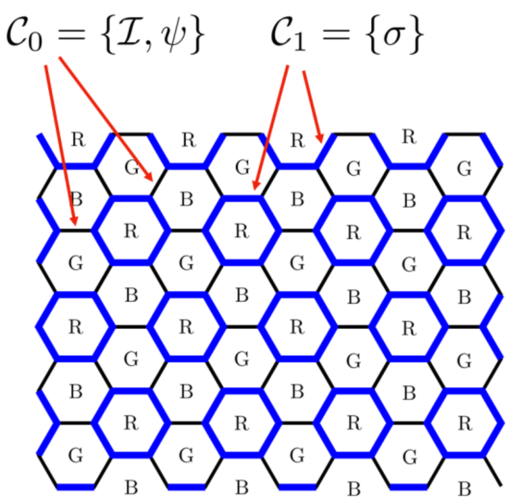


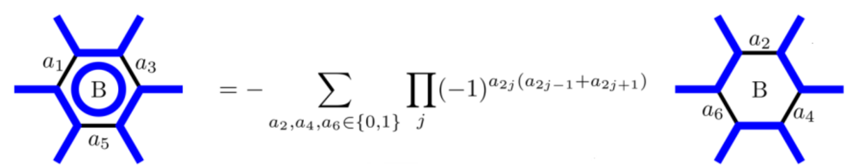




Kramers-Wannier circuit

- Identify active qubit basis as $1, \psi$ in Ising anyon model, and dead qubit with σ, then fusion a σ loop to boundaries=pumping domain wall and a version of KW duality.
- KW circuit is a measurement realization of KW duality

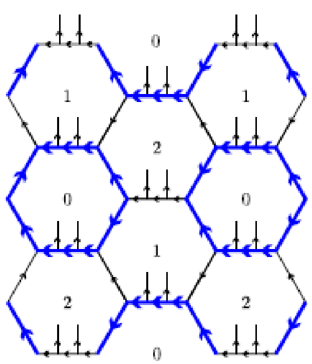


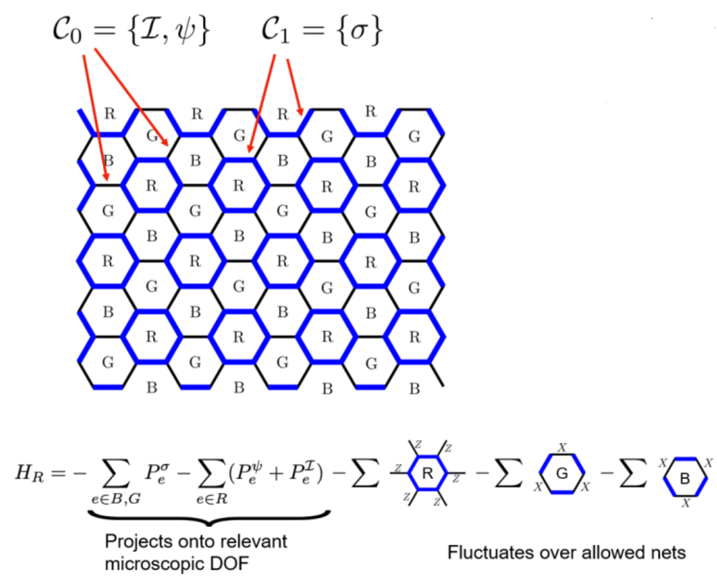


General doubled models

- Given doubled anyon model B=Z(C) and domain wall M
- $L = \bigotimes_{edges} C^{a}, a = \#\,(C \oplus M)$
- TC=$Z\left(Vec_{Z_2}\right), C \oplus M \cong Ising$ as fusion category
- a) Extended Levin-Wen model

  b) Generalized Kitaev model





Manifestation of Topology

T=topological order, e.g. anyon model in 2D

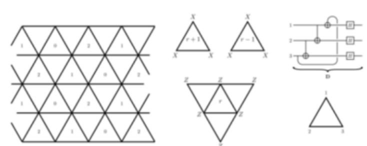
S_T =space of gapped physical Hamiltonians

that realize T modulo invertible ones

e.g.

A non-trivial loop of toric code

An explicit loop:



A Homotopy Conjecture

B=an anyon model, e.g. toric code

Pic(B)≅**categorical** symmetry 2-group of B

B**Pic(B)**=classifying space of **Pic(B)**

Conjecture:

$S_B \sim$ B**Pic(B)**, homotopy equivalent

Weak:

$$\pi_n(S_B) \cong \pi_n(B\boldsymbol{Pic}(\boldsymbol{B}))$$

Classification of topological codes (~order)

- Spaces of Hamiltonians of topological order are invariants of the topological order:

  Any invariants derived from such a space are invariants of topological order.

- “Good” complete invariants of topological order?