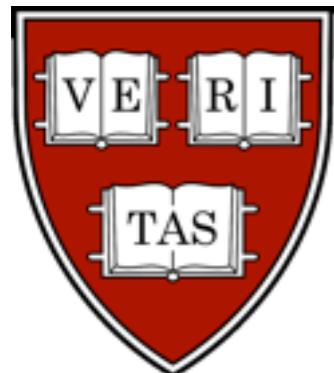
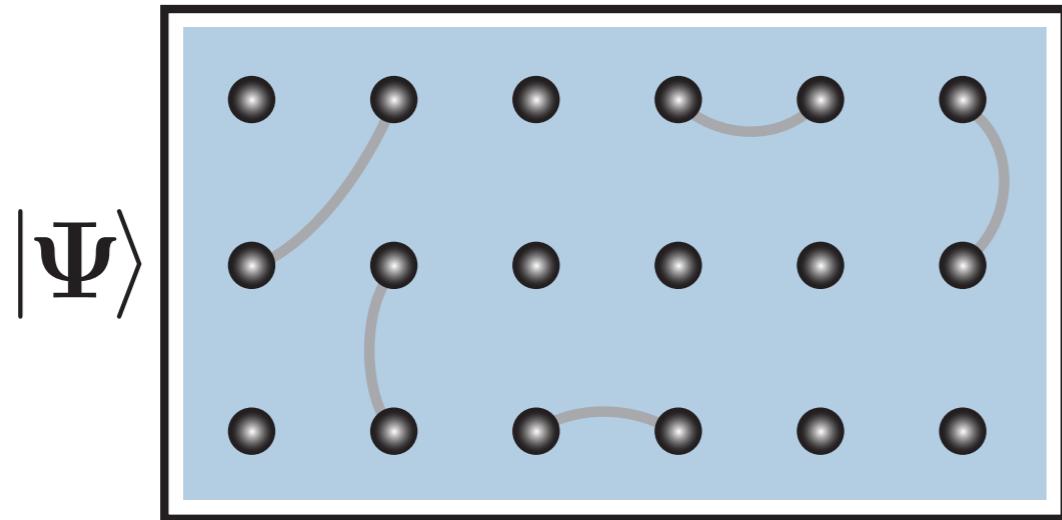


Probing entanglement and correlations dynamics in many-body localized system

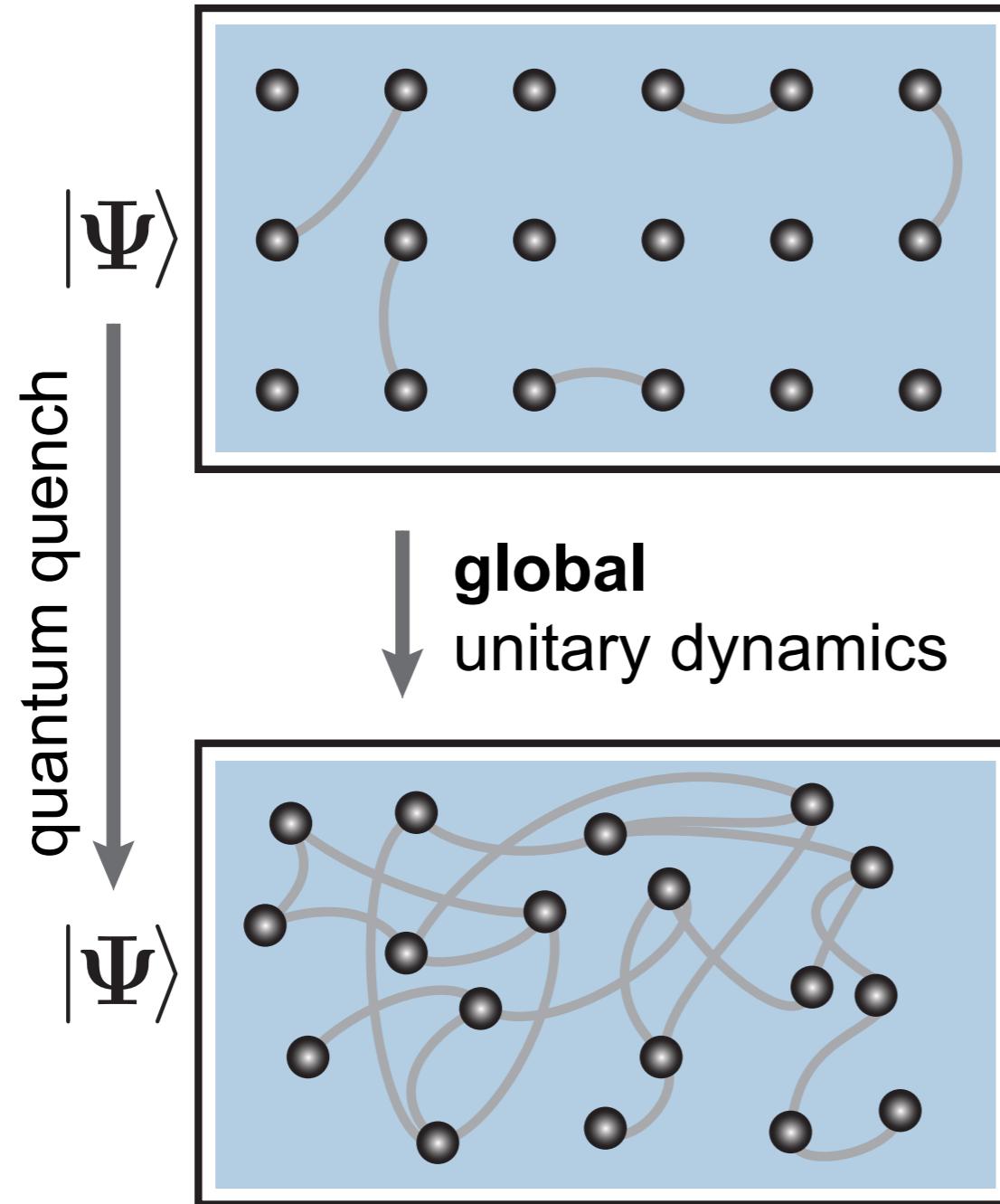
Alexander Lukin
Harvard University



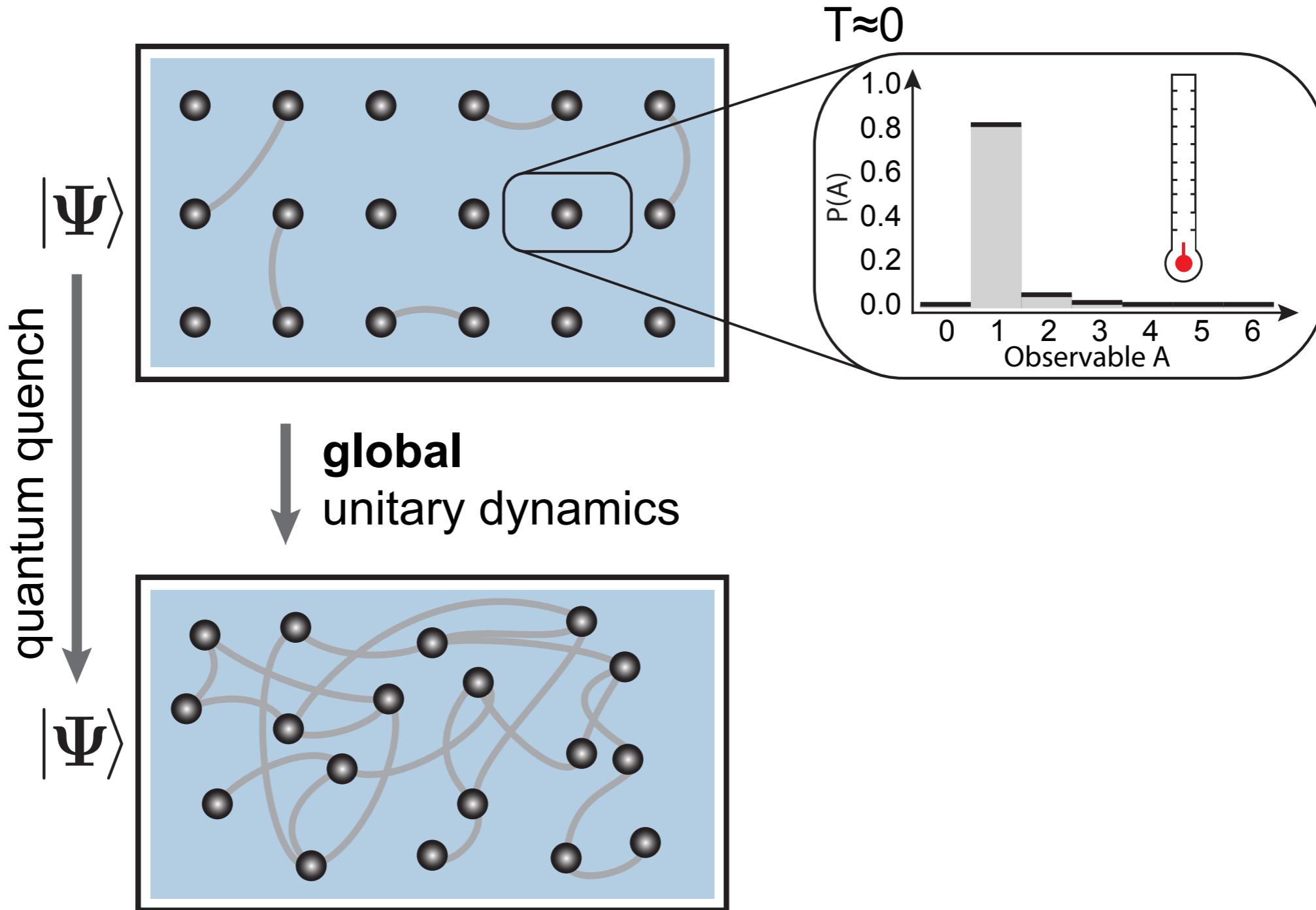
Thermalization in quantum system



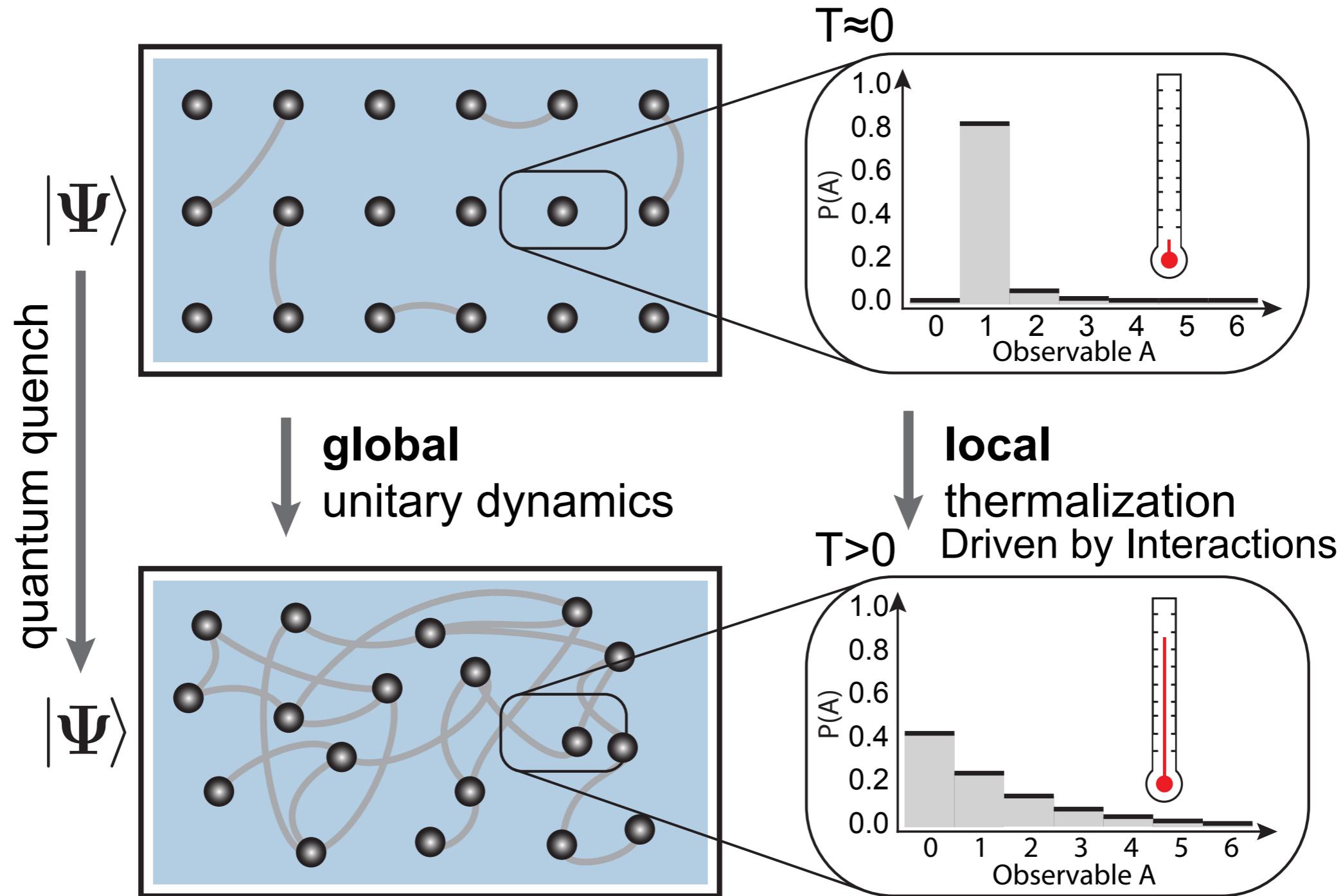
Thermalization in quantum system



Thermalization in quantum system

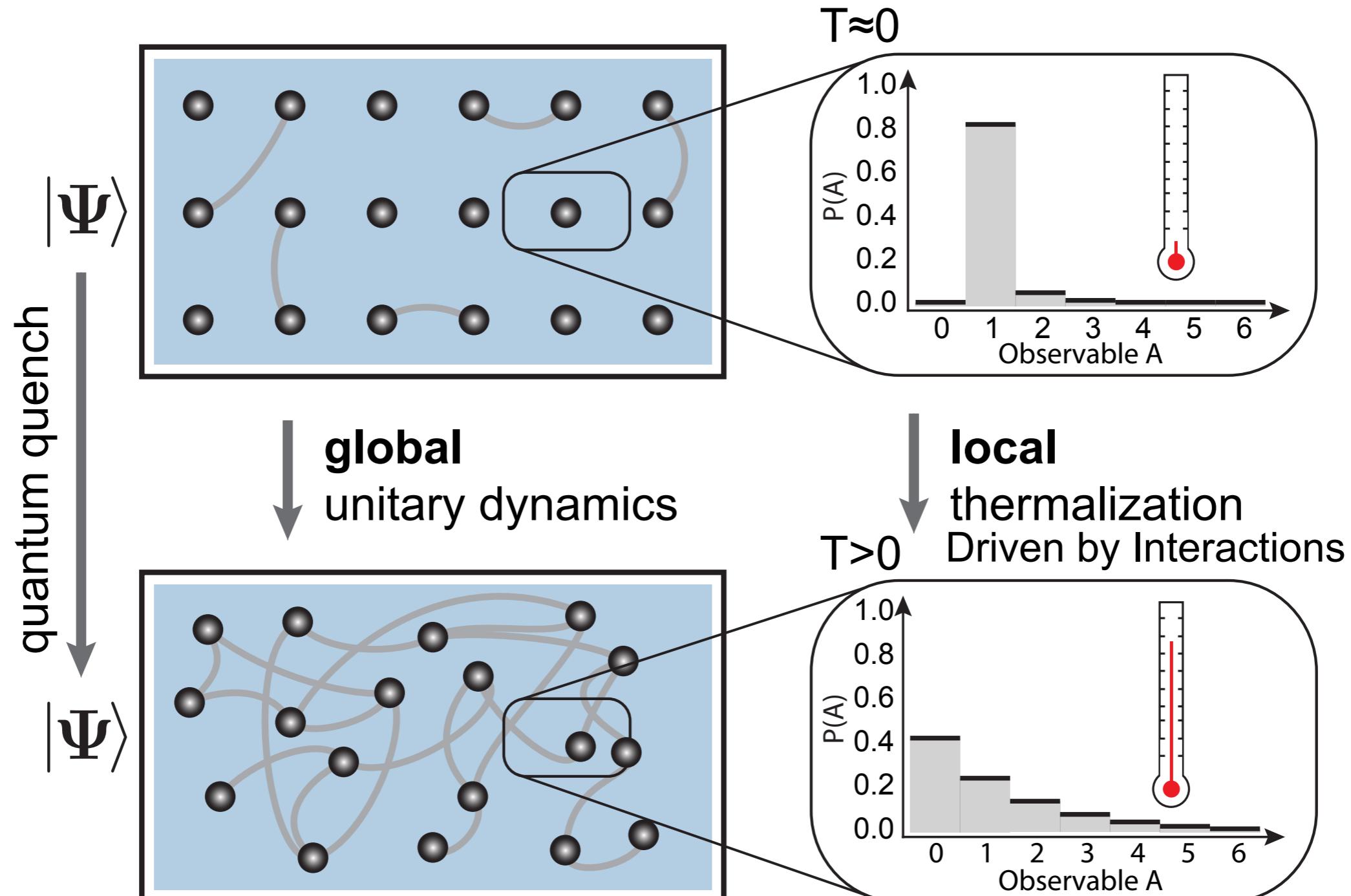


Thermalization in quantum system



- J.M Deutch, PRA 43,2046 (1991)
M. Srednicki et. al., PRE 50,888 (1994)
M, Rigol et. al., Nature 452, 854 (2008)
Langen et. al., Science 348, 207 (2014)
Neill et. al. , Nat Phys 12, 1037 (2016)
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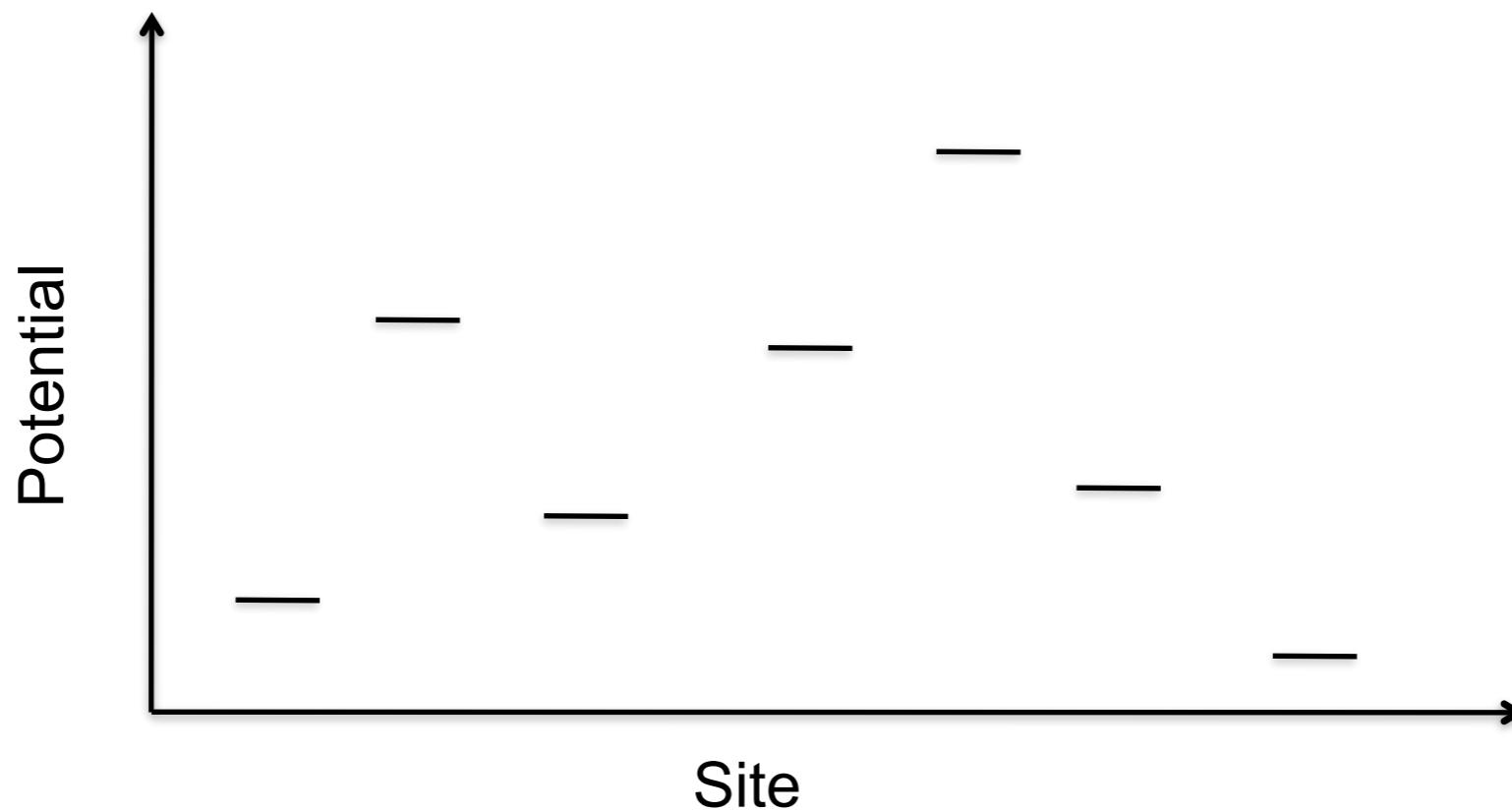
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Single-particle Anderson localization

$$\hat{\mathcal{H}} = -J \sum_i \left(\hat{a}_i^\dagger \hat{a}_{i+1} + h.c. \right) + \sum_i W_i \hat{n}_i$$

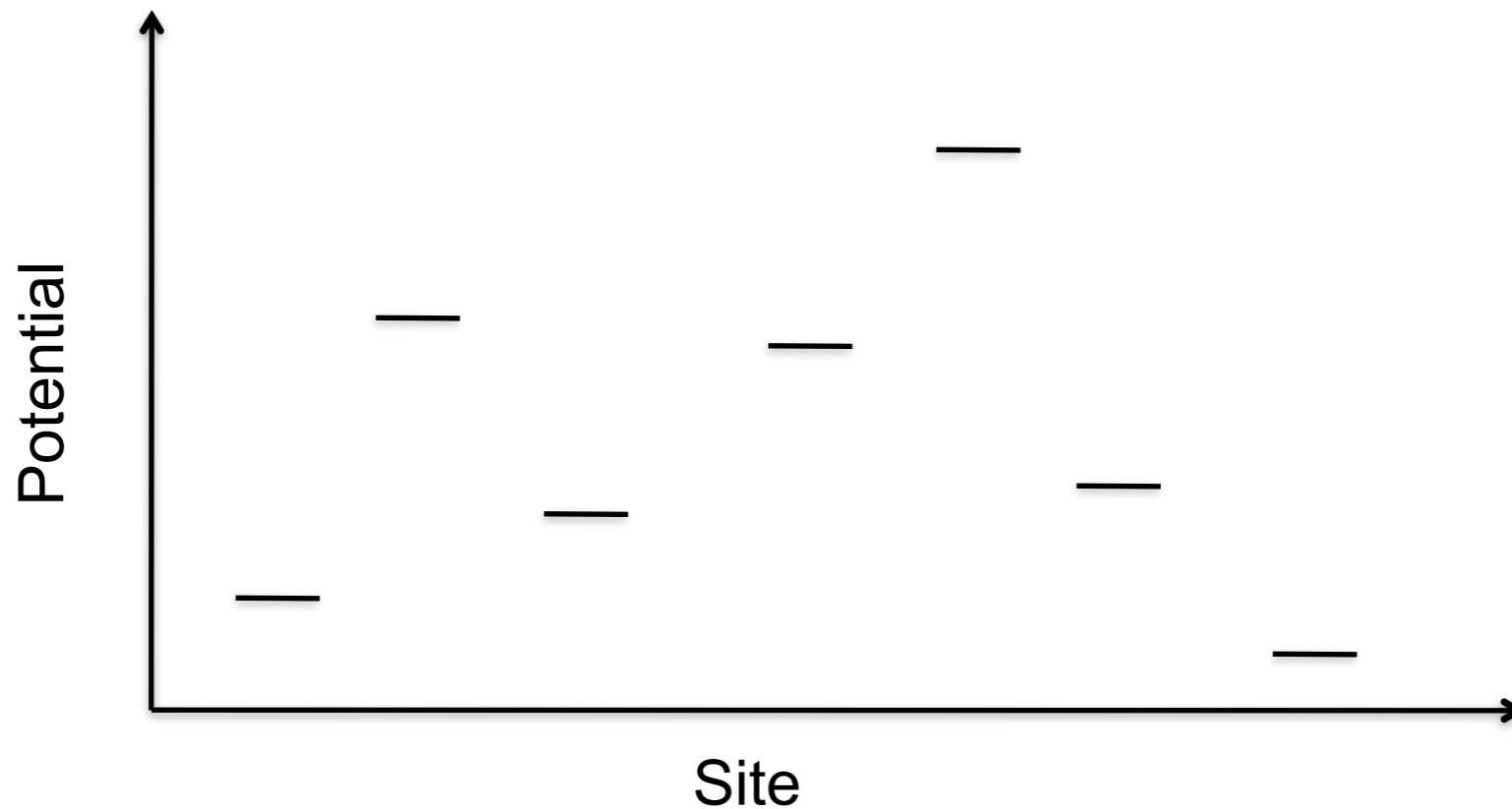
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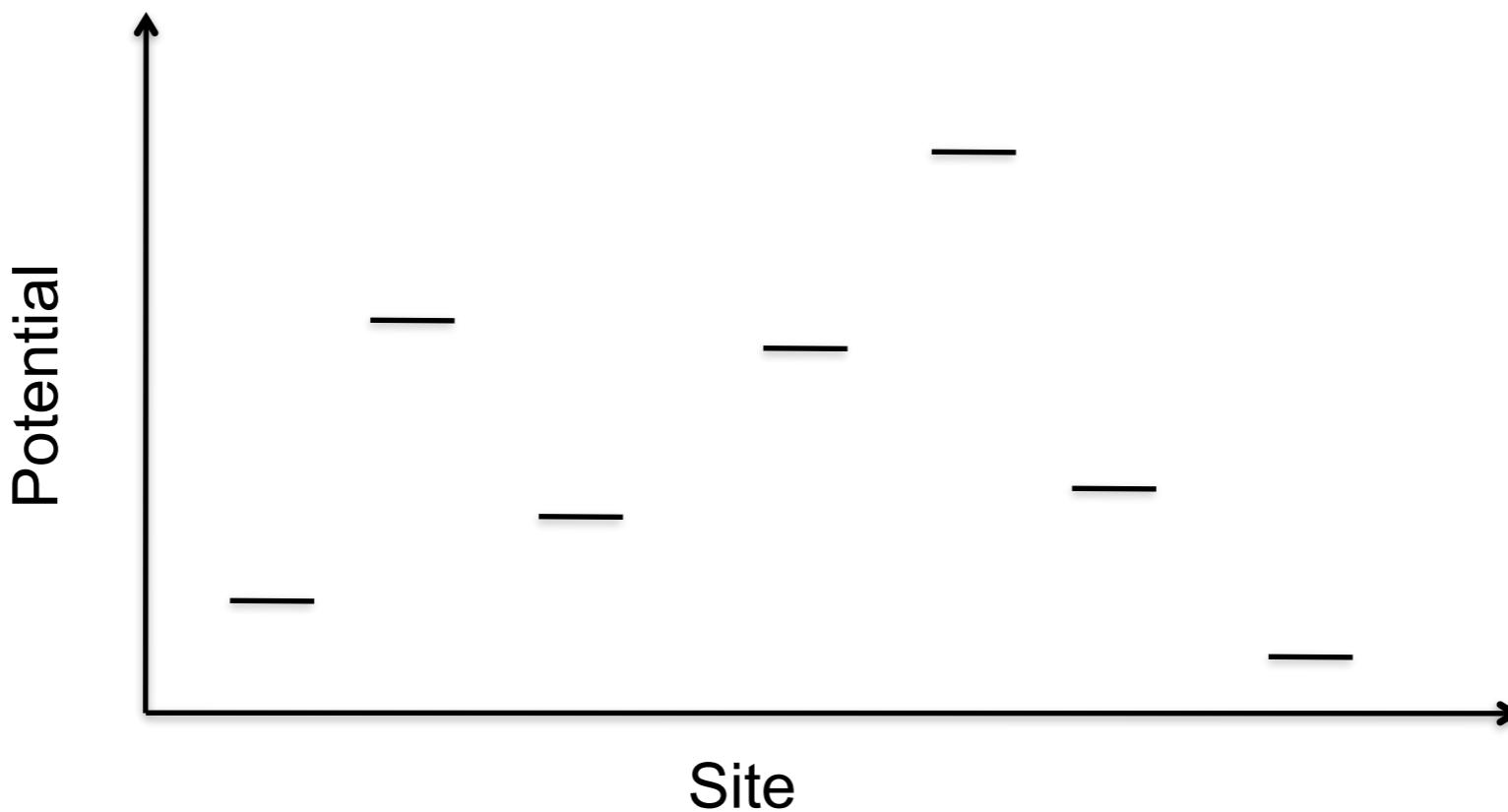
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Off-resonant hopping fails to hybridize sites at long-distances

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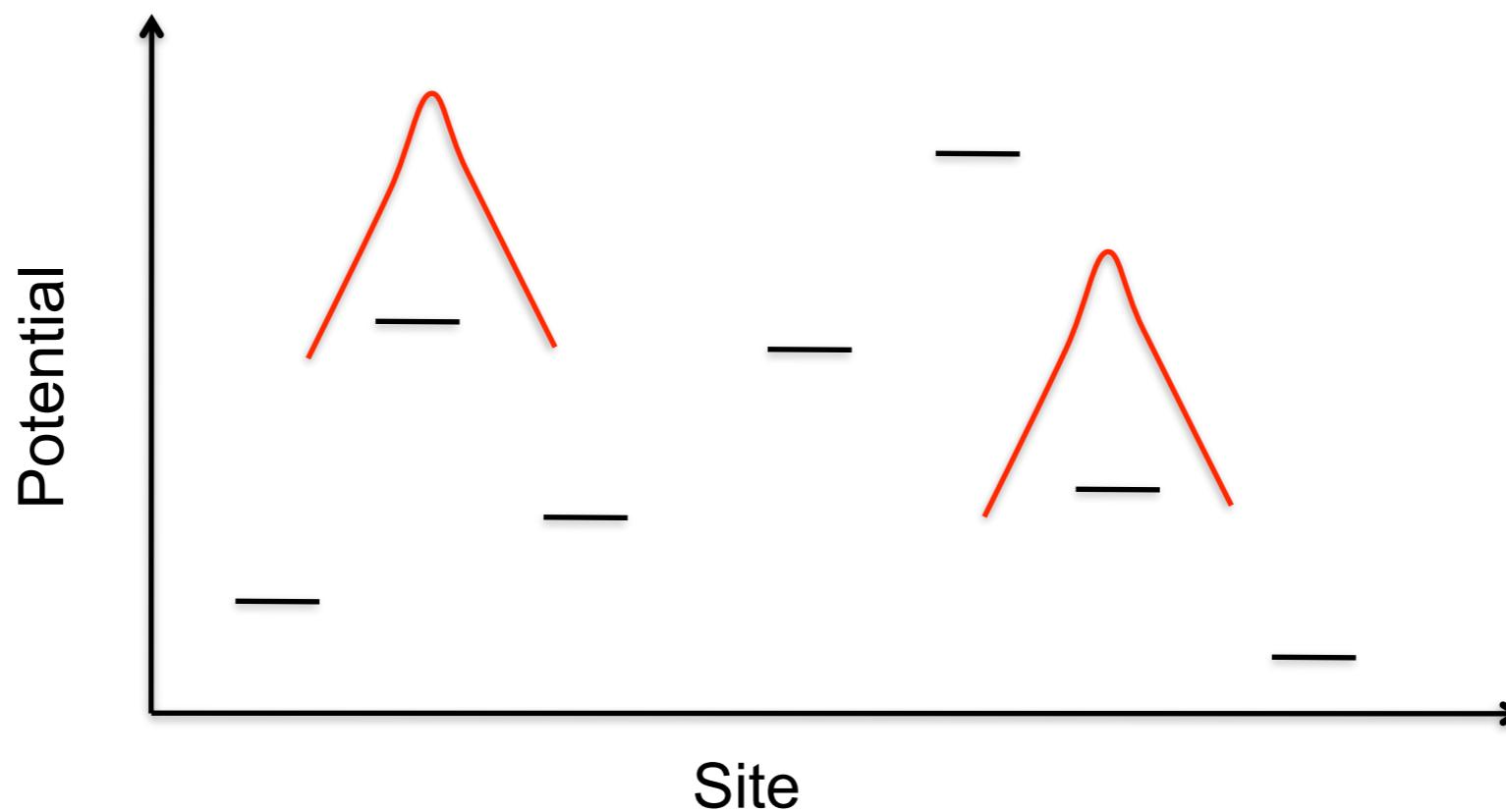


Off-resonant hopping fails to hybridize sites at long-distances

Localized $|\phi(r)|^2 \sim e^{-r/\xi}$

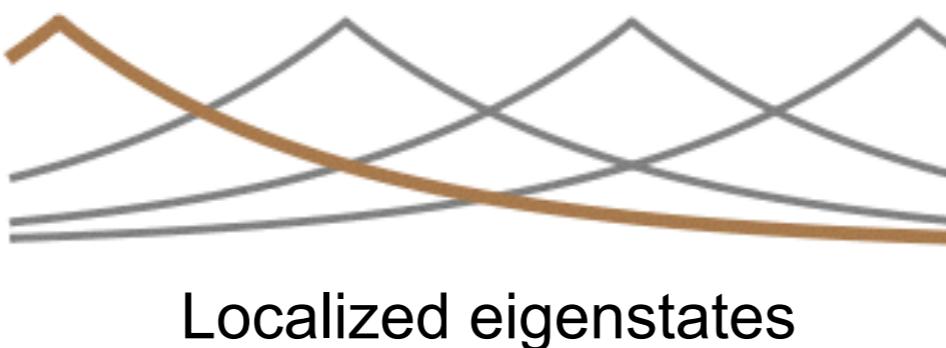
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Many-body localization (MBL)

$$\hat{\mathcal{H}} = -J \sum_i \left(\hat{a}_i^\dagger \hat{a}_{i+1} + h.c. \right) + \sum_i W_i \hat{n}_i + \frac{U}{2} \sum_i \hat{n}_i (\hat{n}_i - 1)$$

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Extended eigenstates



Localized eigenstates

Many-body localization (MBL)

$$\hat{\mathcal{H}} = -J \sum_i \left(\hat{a}_i^\dagger \hat{a}_{i+1} + h.c. \right) + \sum_i W_i \hat{n}_i + \frac{U}{2} \sum_i \hat{n}_i (\hat{n}_i - 1)$$



Localized eigenstates

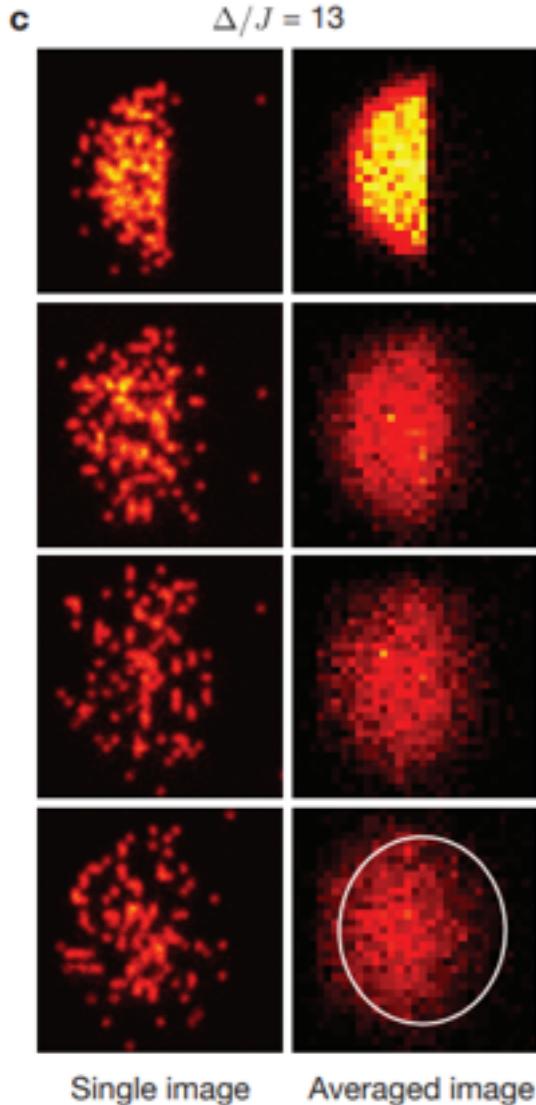
Fleishman, Anderson (1980)
Basko Aleiner, Altshuler (2006)
Huse and Oganesyan (2007)
Imbrie (2014)

Many-body localization (MBL)

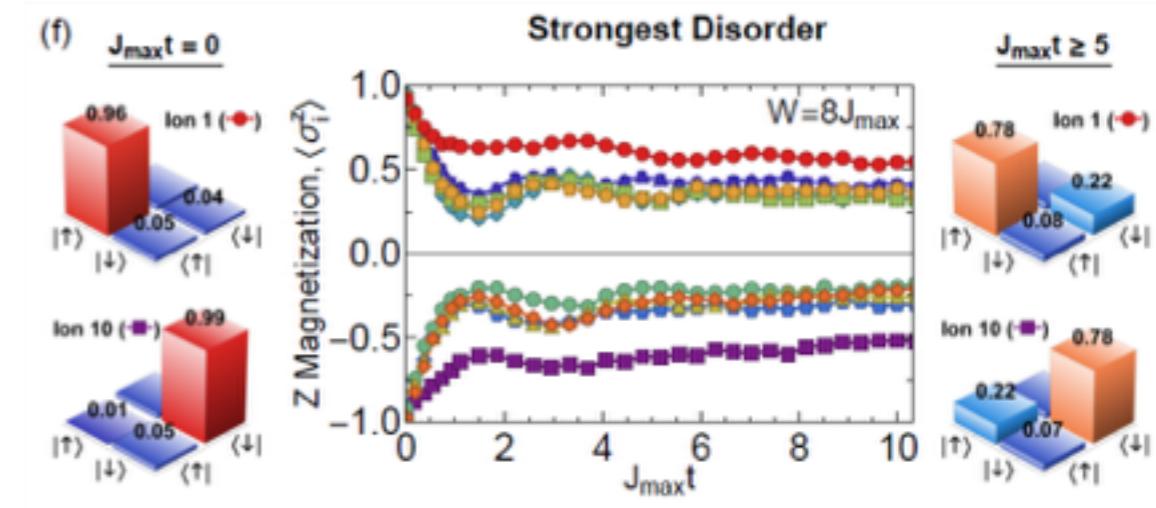
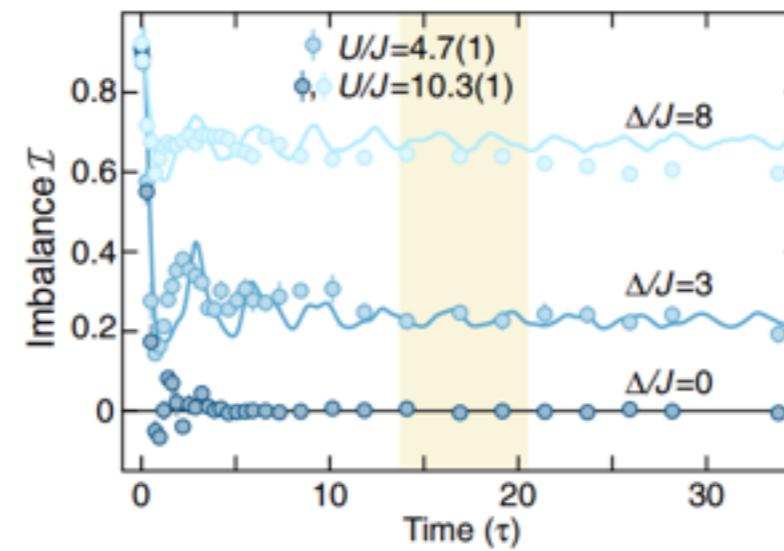
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Localized eigenstates



Breakdown of ergodicity:
Michael Schreiber, et. al. (2015)
Jacob Smith, et. al. (2016)
J. Choi, et. al. (2016)



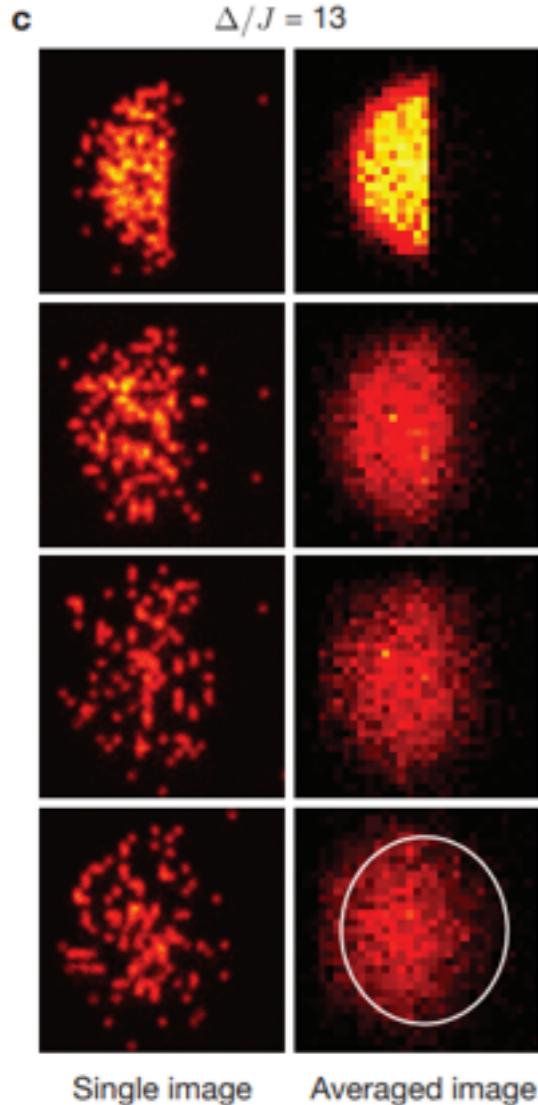
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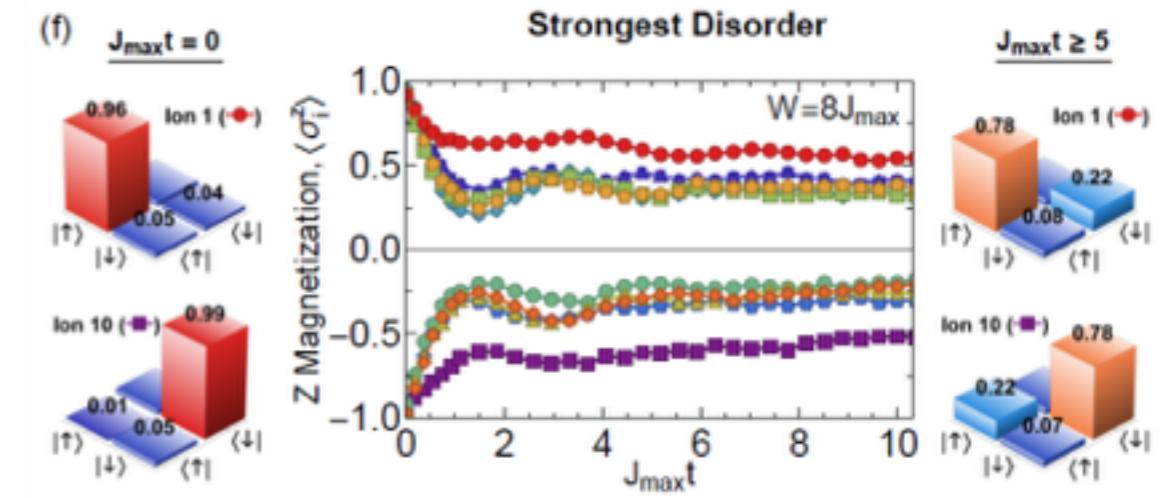
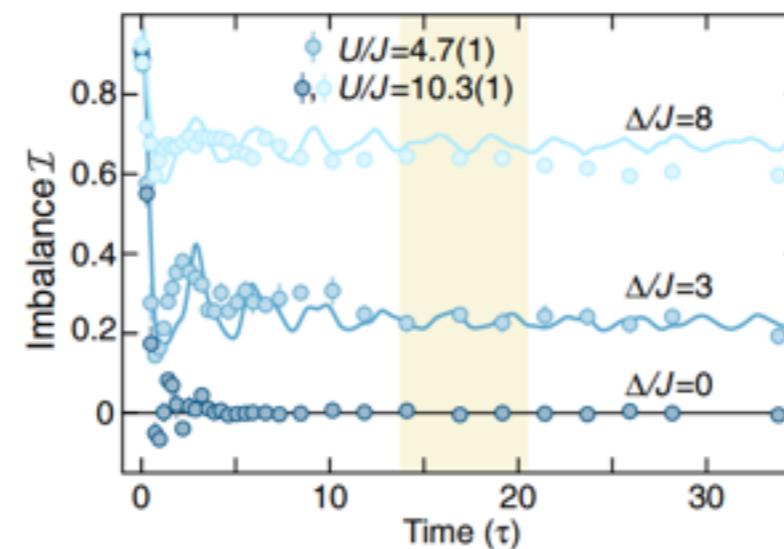
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Localized eigenstates



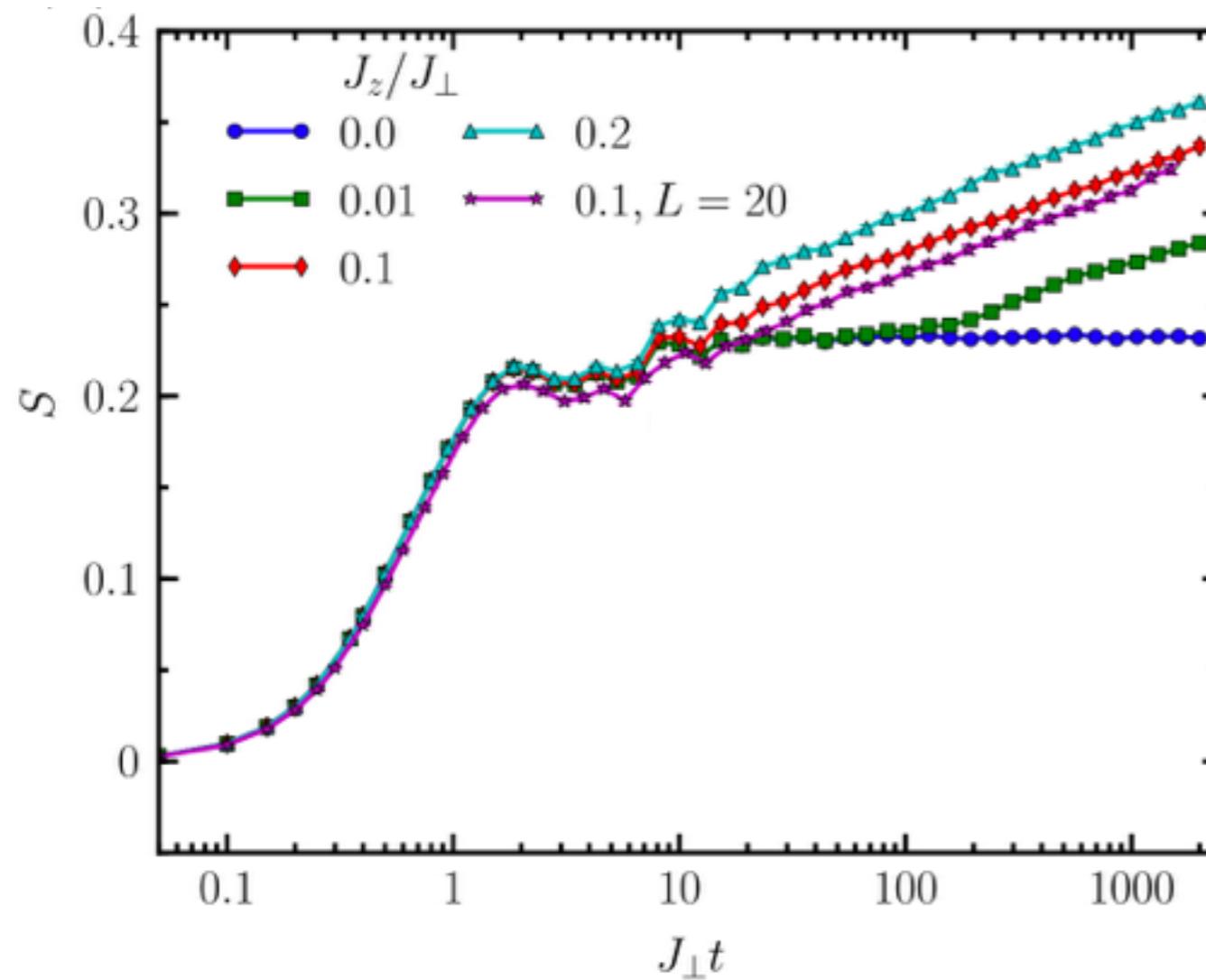
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dynamics limited by decoherence

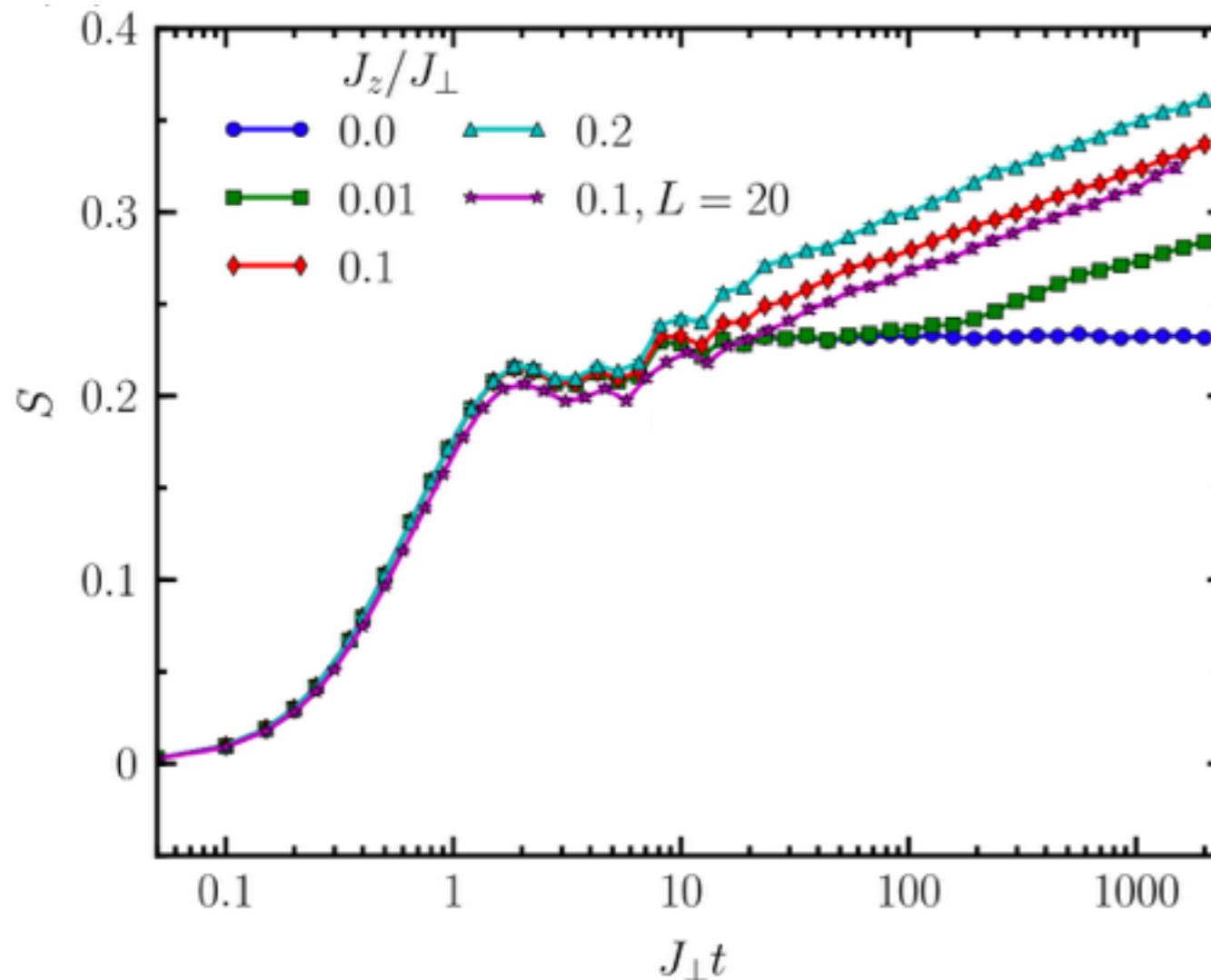
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Hidden quantum many-body dynamics



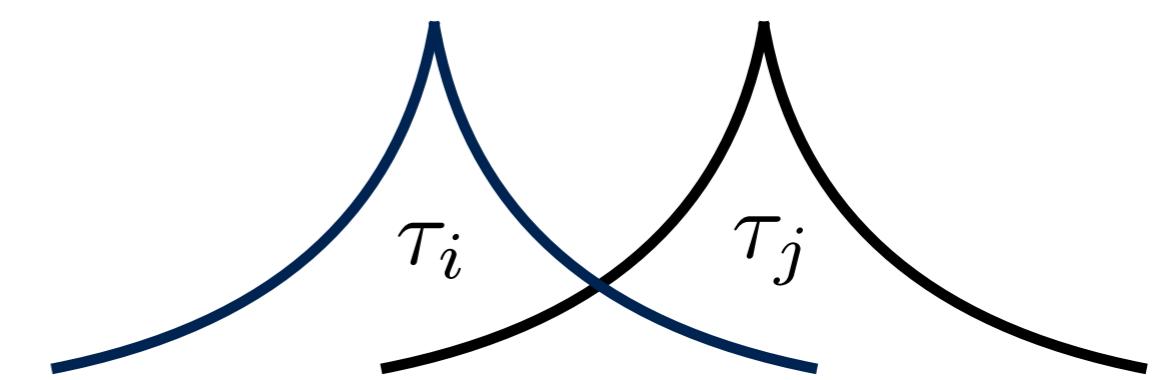
M. Znidaric et al, PRB 77, 064426 (2008)
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Hidden quantum many-body dynamics



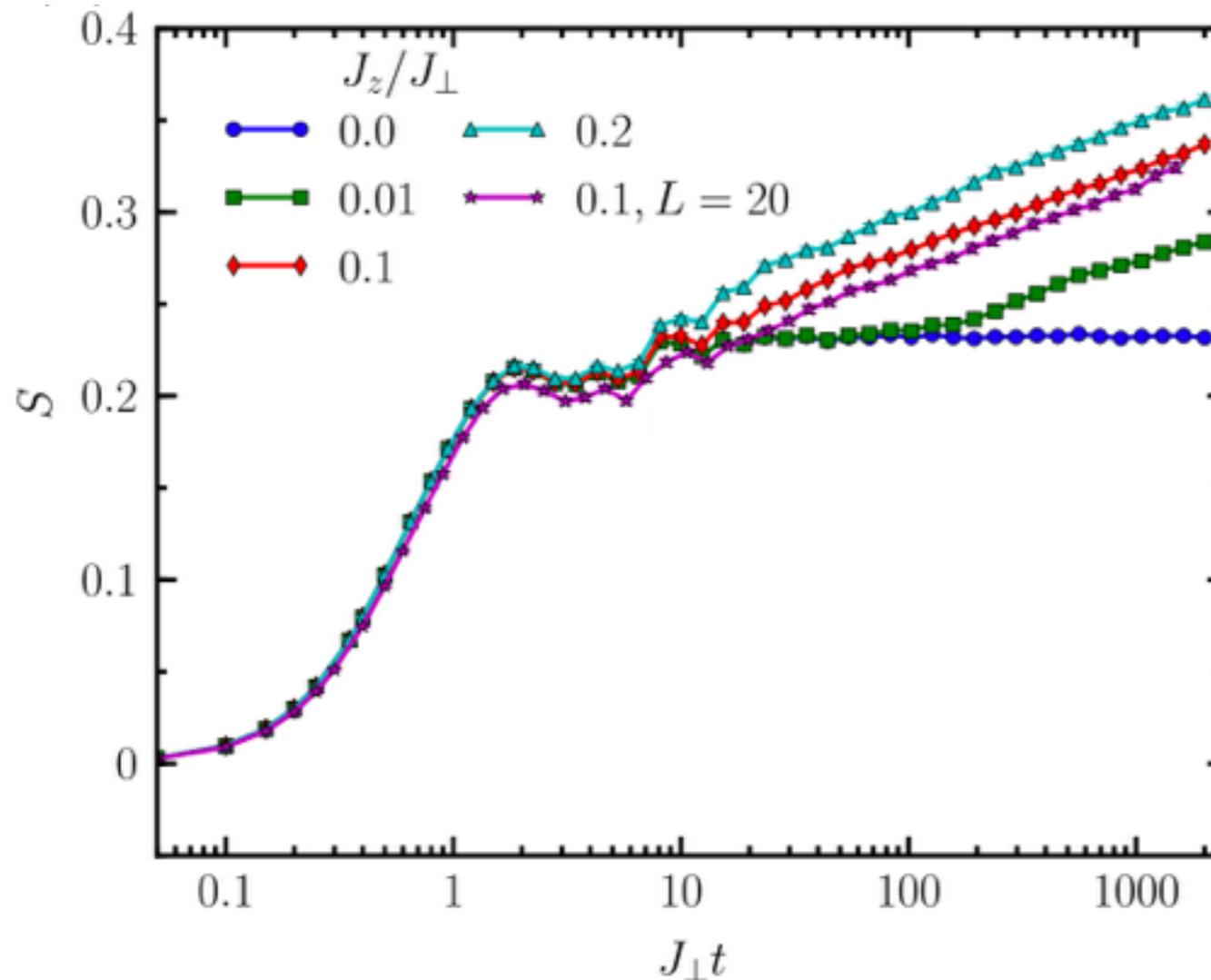
M. Znidaric et al, PRB 77, 064426 (2008)
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concerned operators
with exponentially localized support

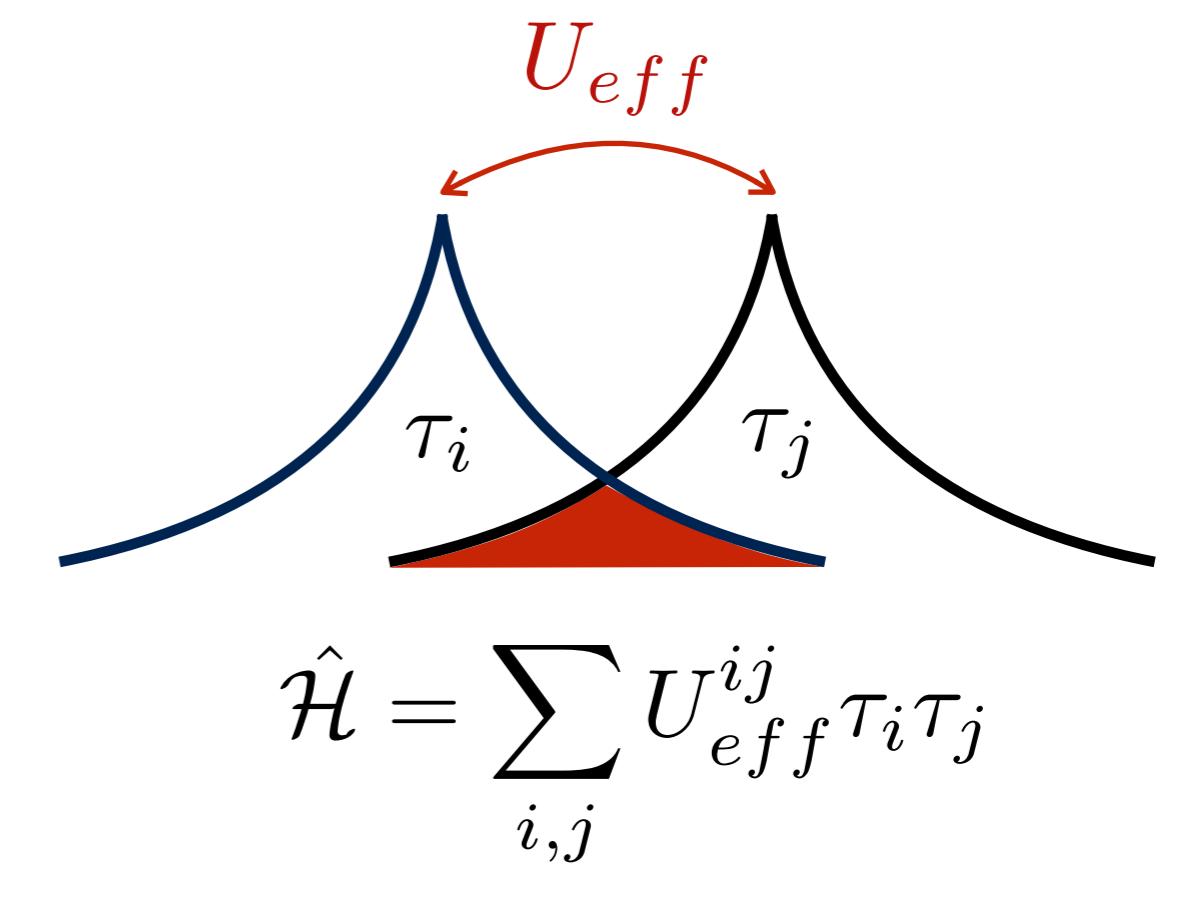


M. Serbyn, et. al., PRL 110, 260601 (2013)
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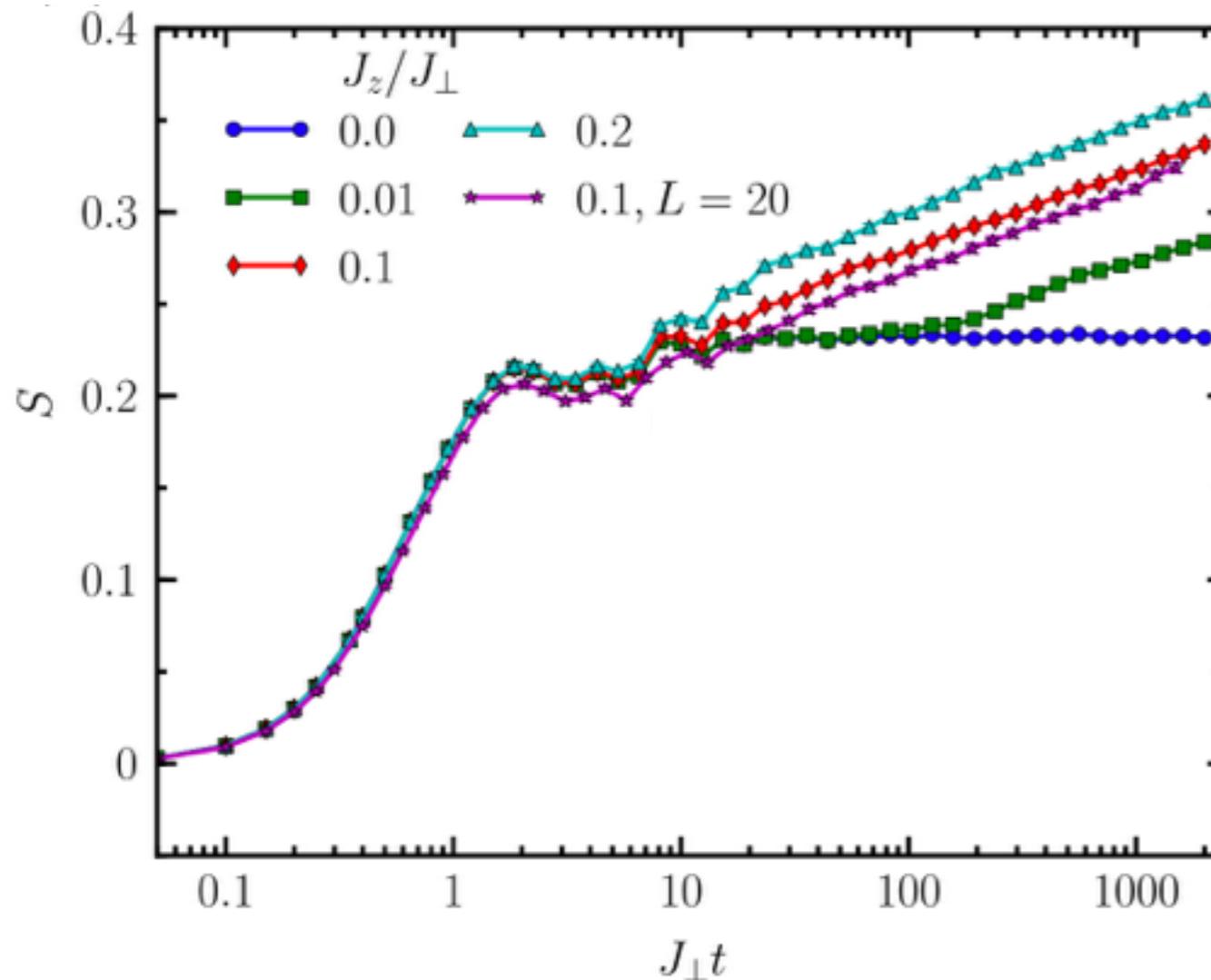


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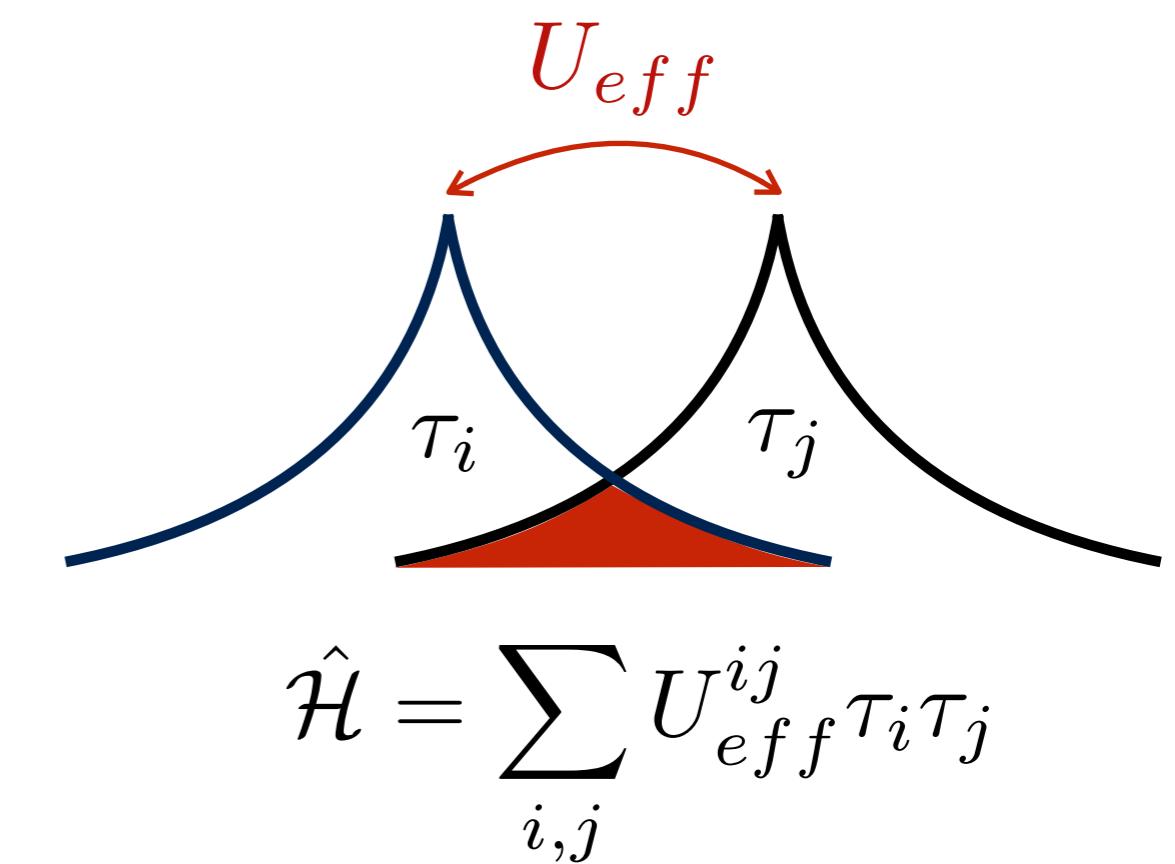


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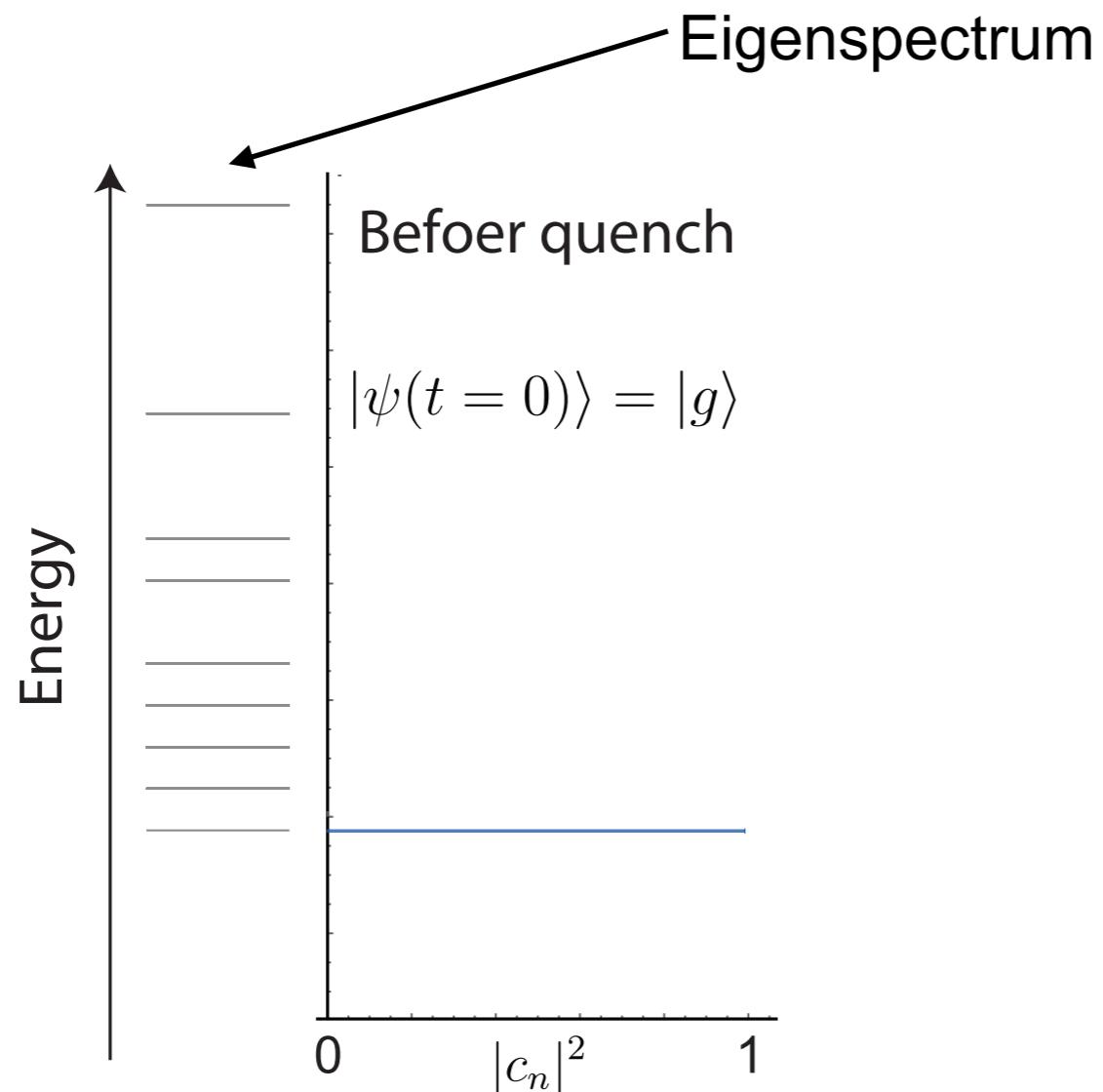
local interactions + tunneling + disorder
↓
non-local quantum dynamics hidden from local probes

Quantum quenches

Ergodic to MBL:
transition in excited eigenstates

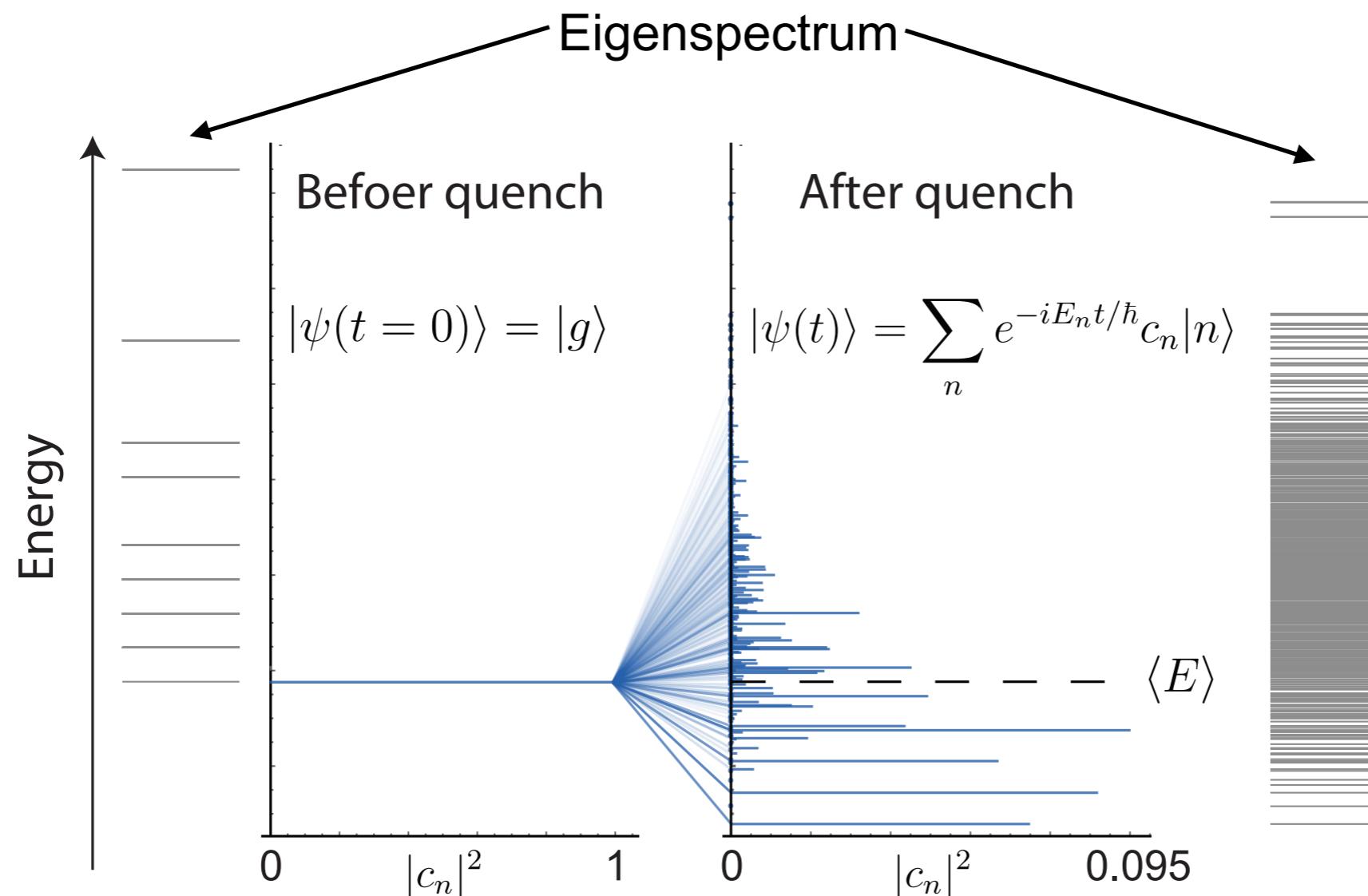
Quantum quenches

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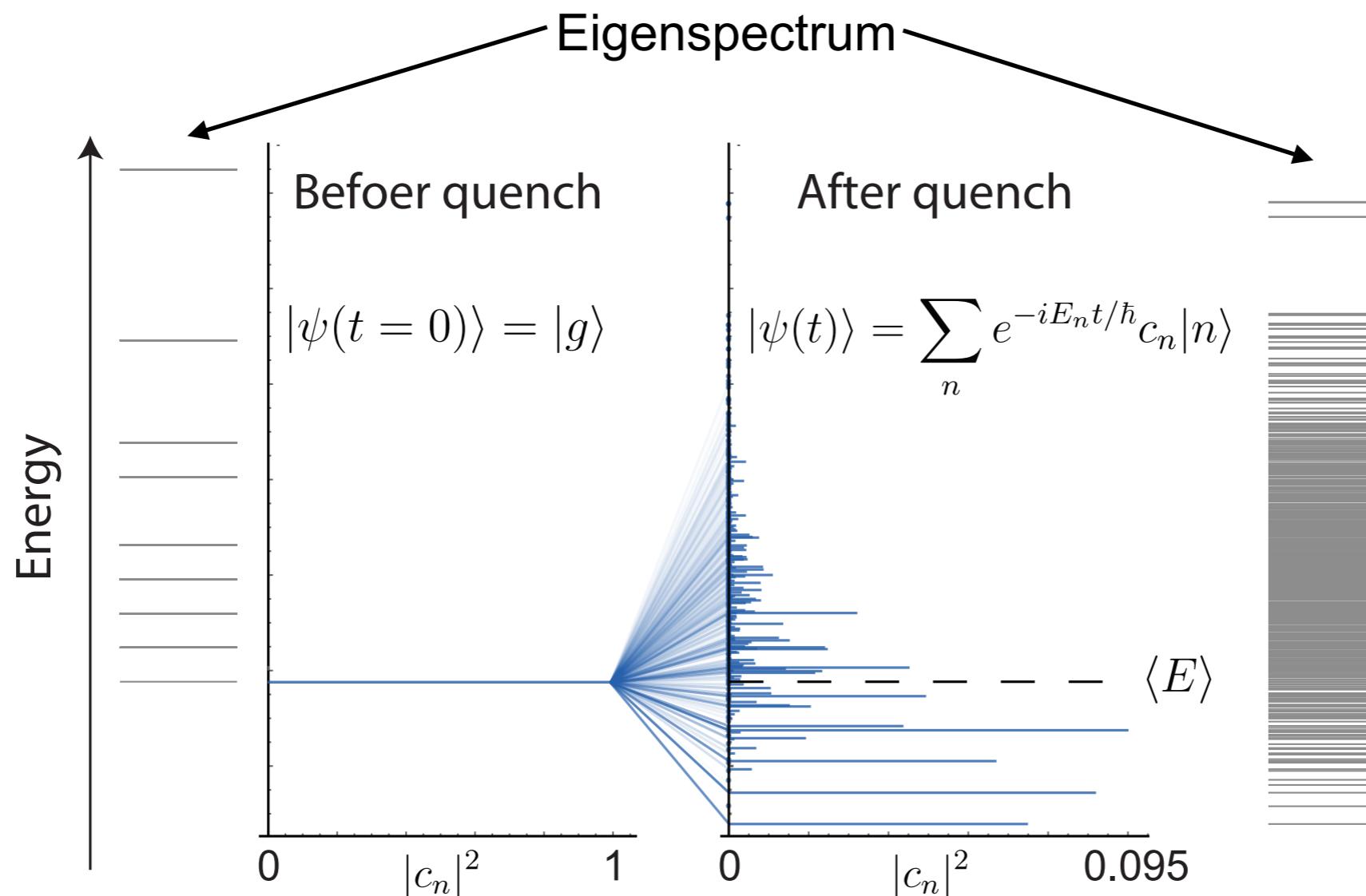
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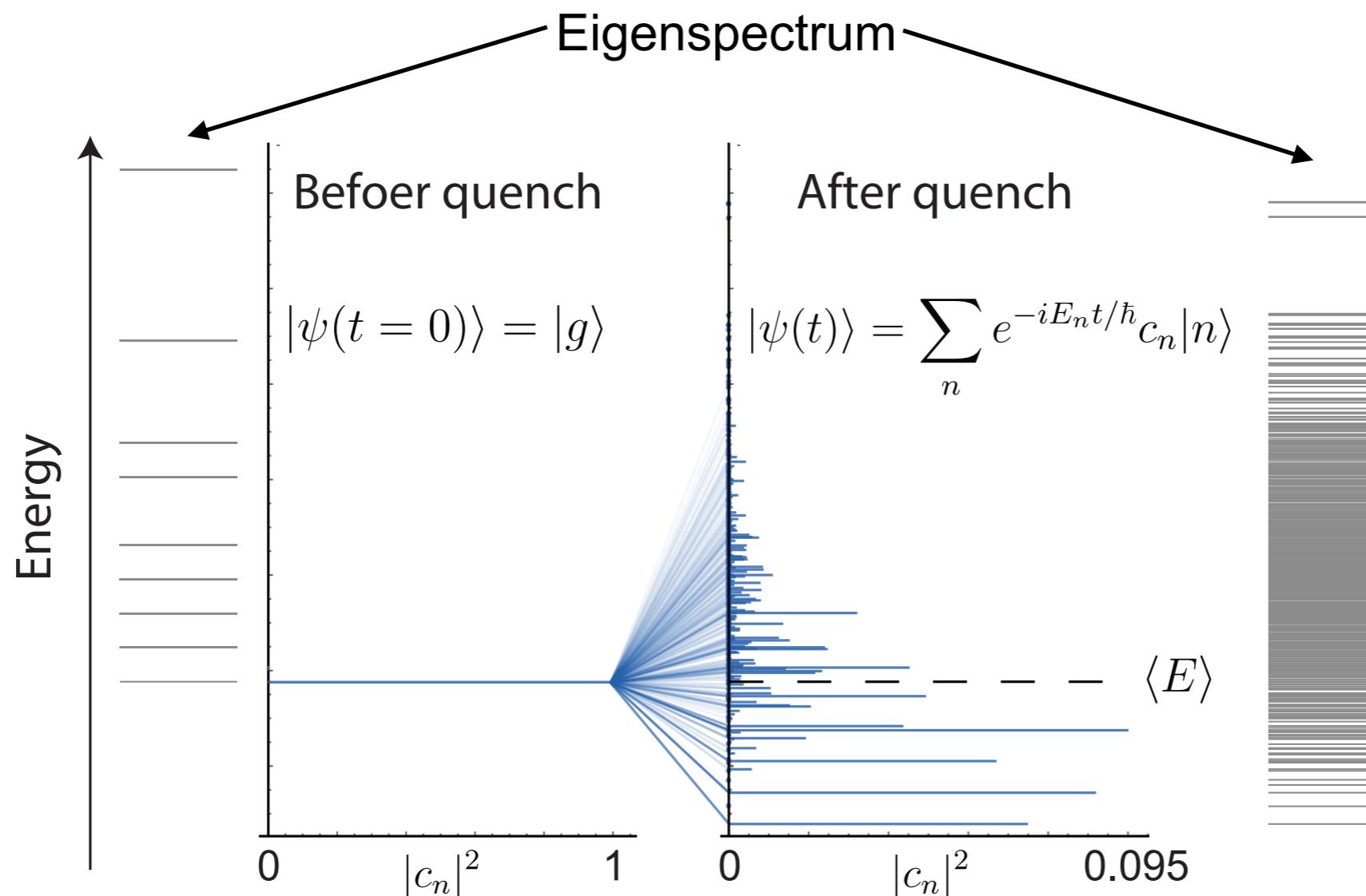
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$$\langle O(t) \rangle = \sum_{m,n} c_m^* c_n O_{mn} e^{i(E_m - E_n)t/\hbar}$$

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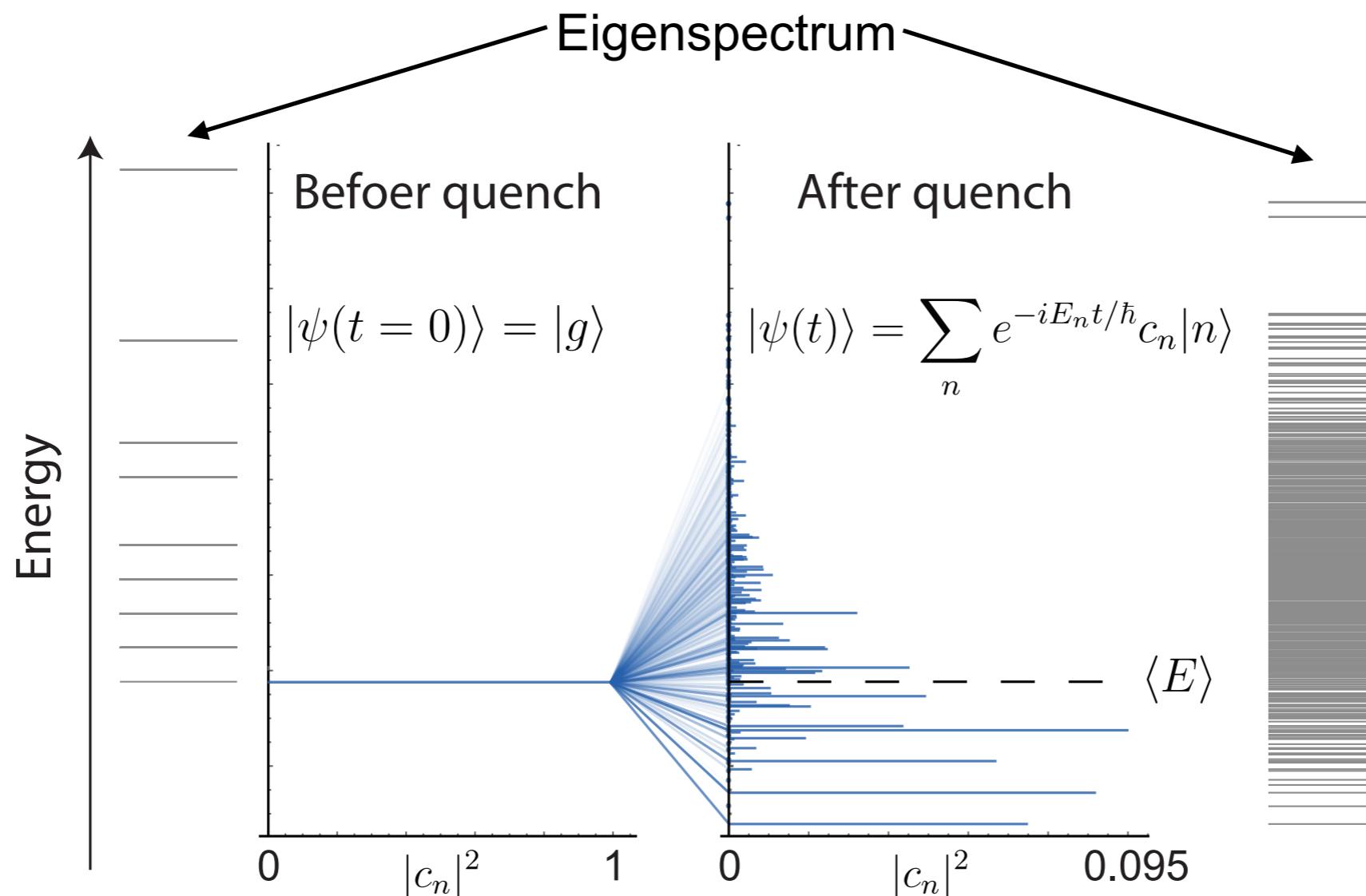
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Long coherent evolution required!

Outlook: key features of MBL

- Experimental setup

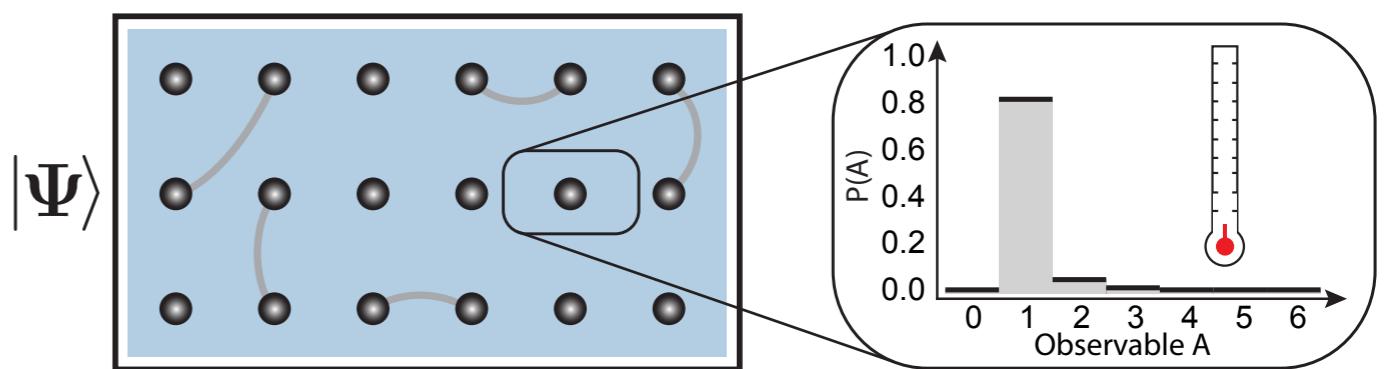


Outlook: key features of MBL

- Experimental setup



- Breakdown of thermalization

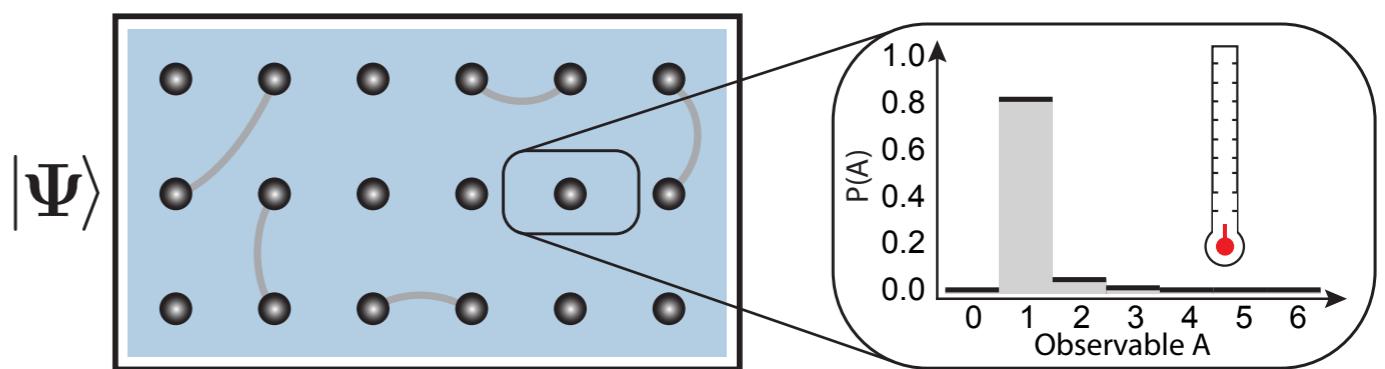


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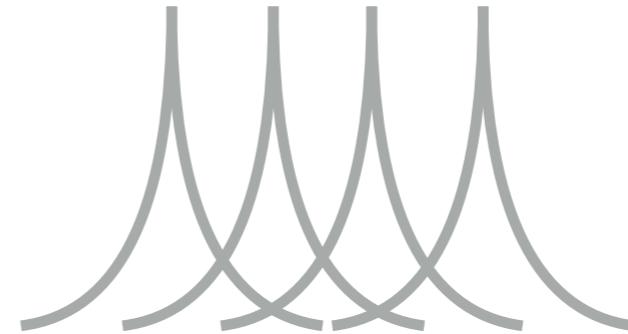
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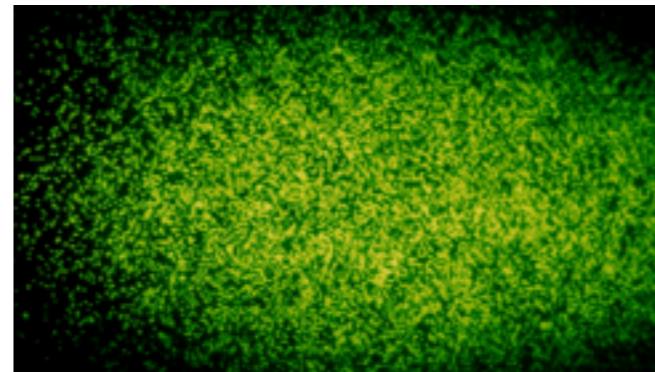


- Spatial localization

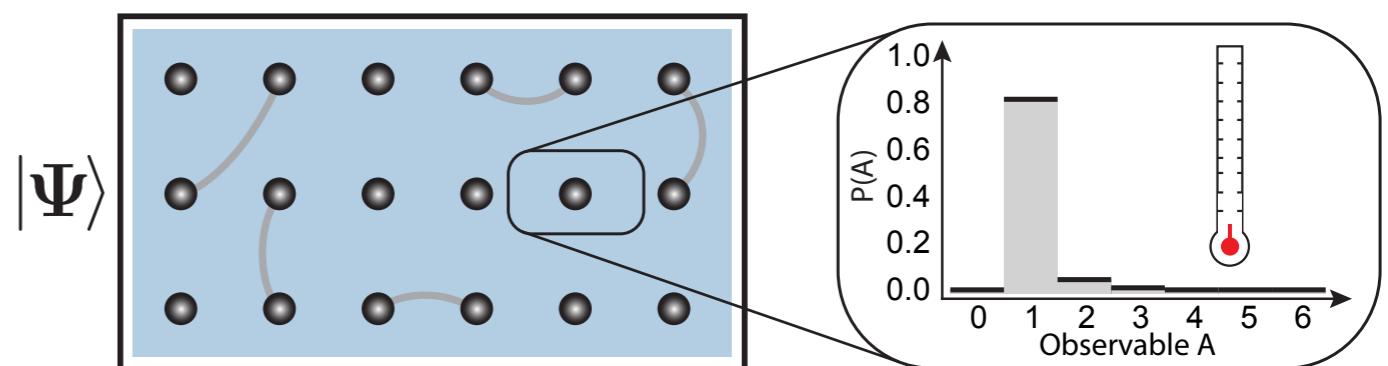


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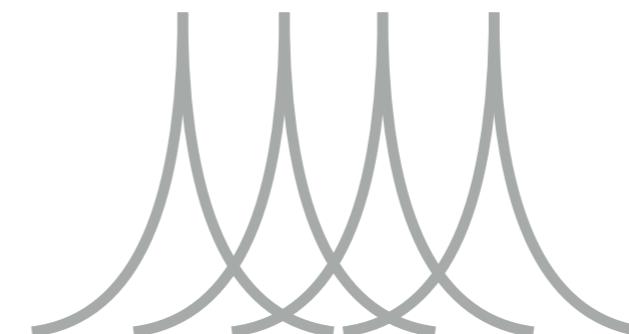
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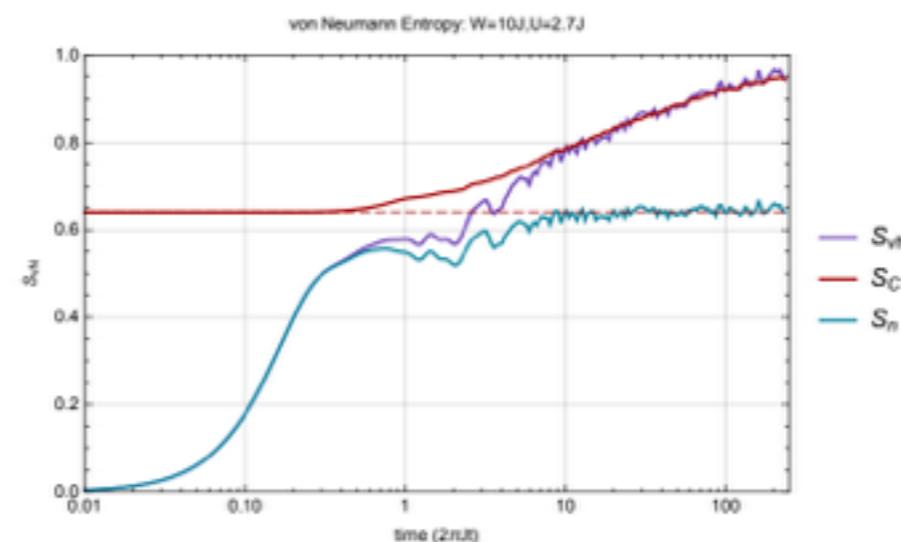
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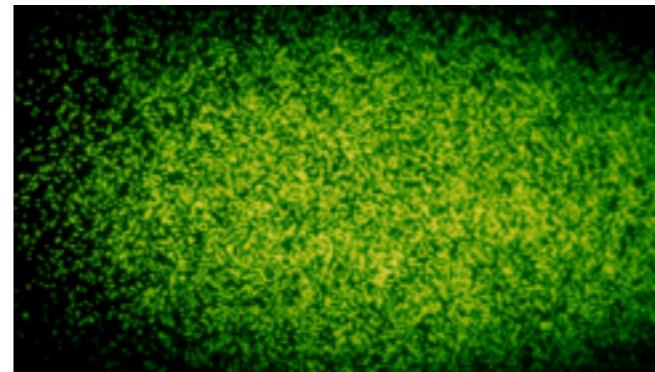


- Entanglement growth

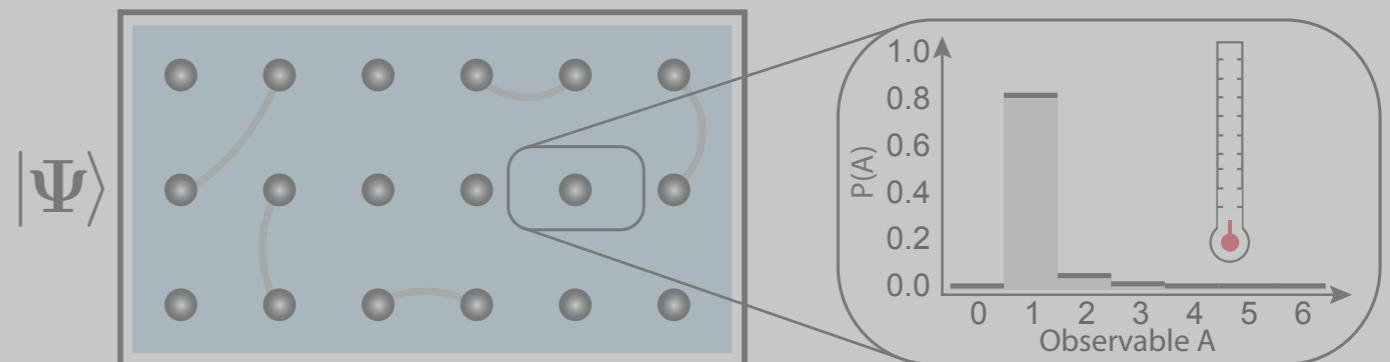


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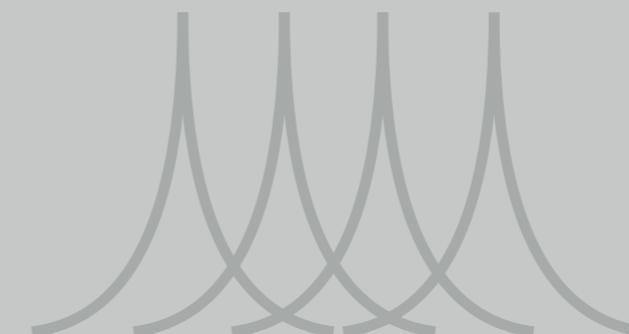
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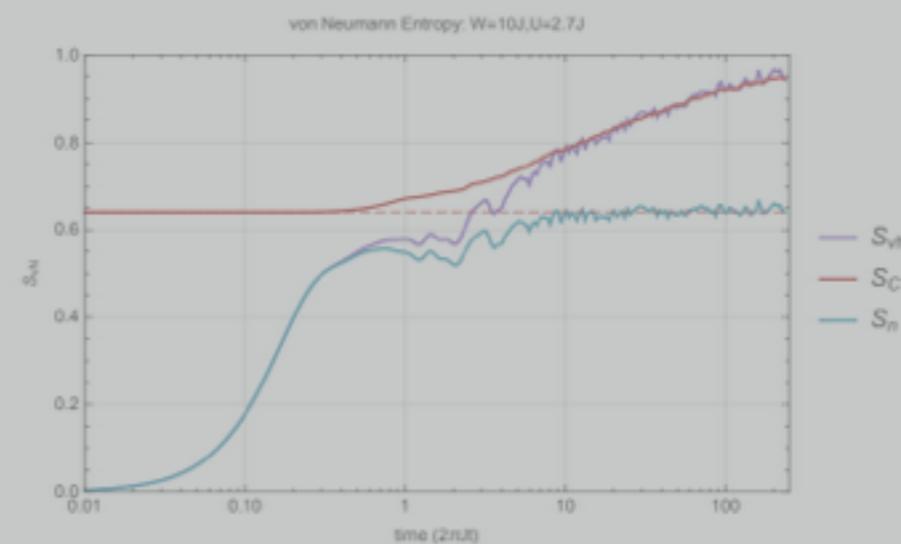
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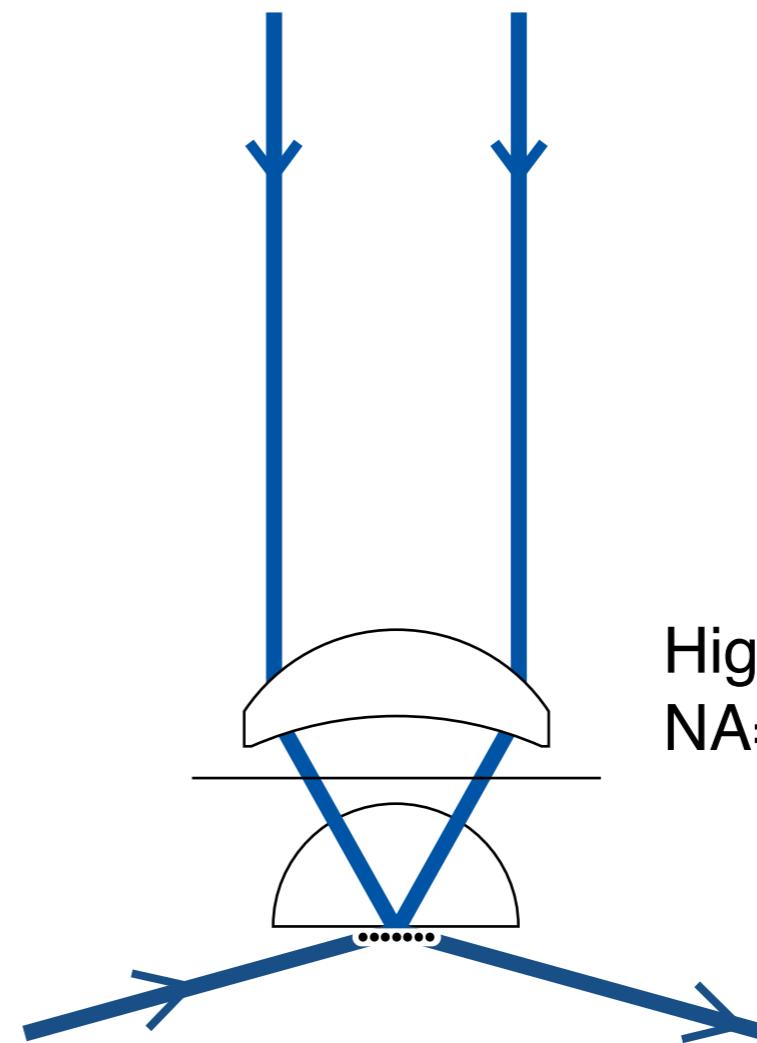
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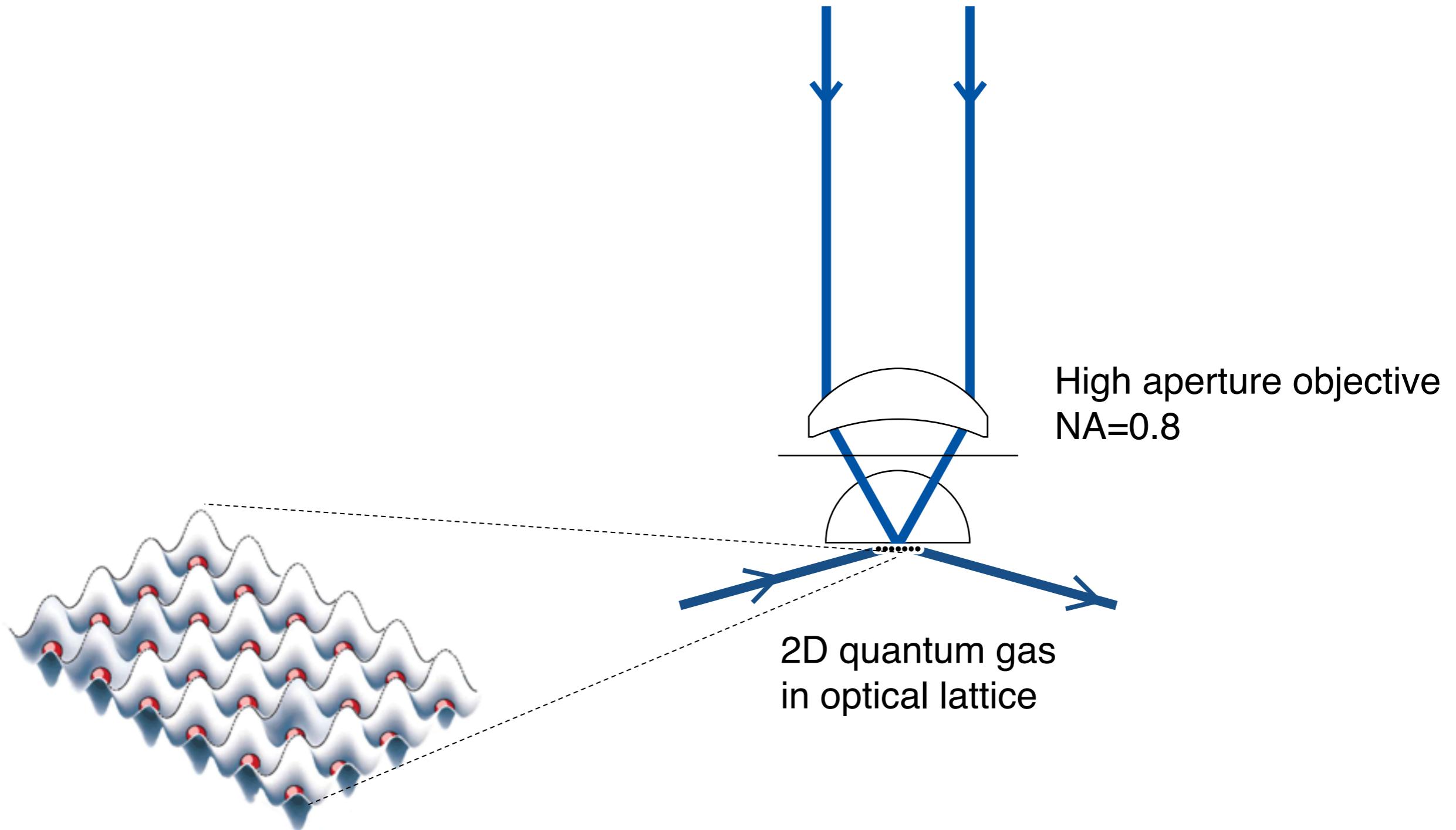
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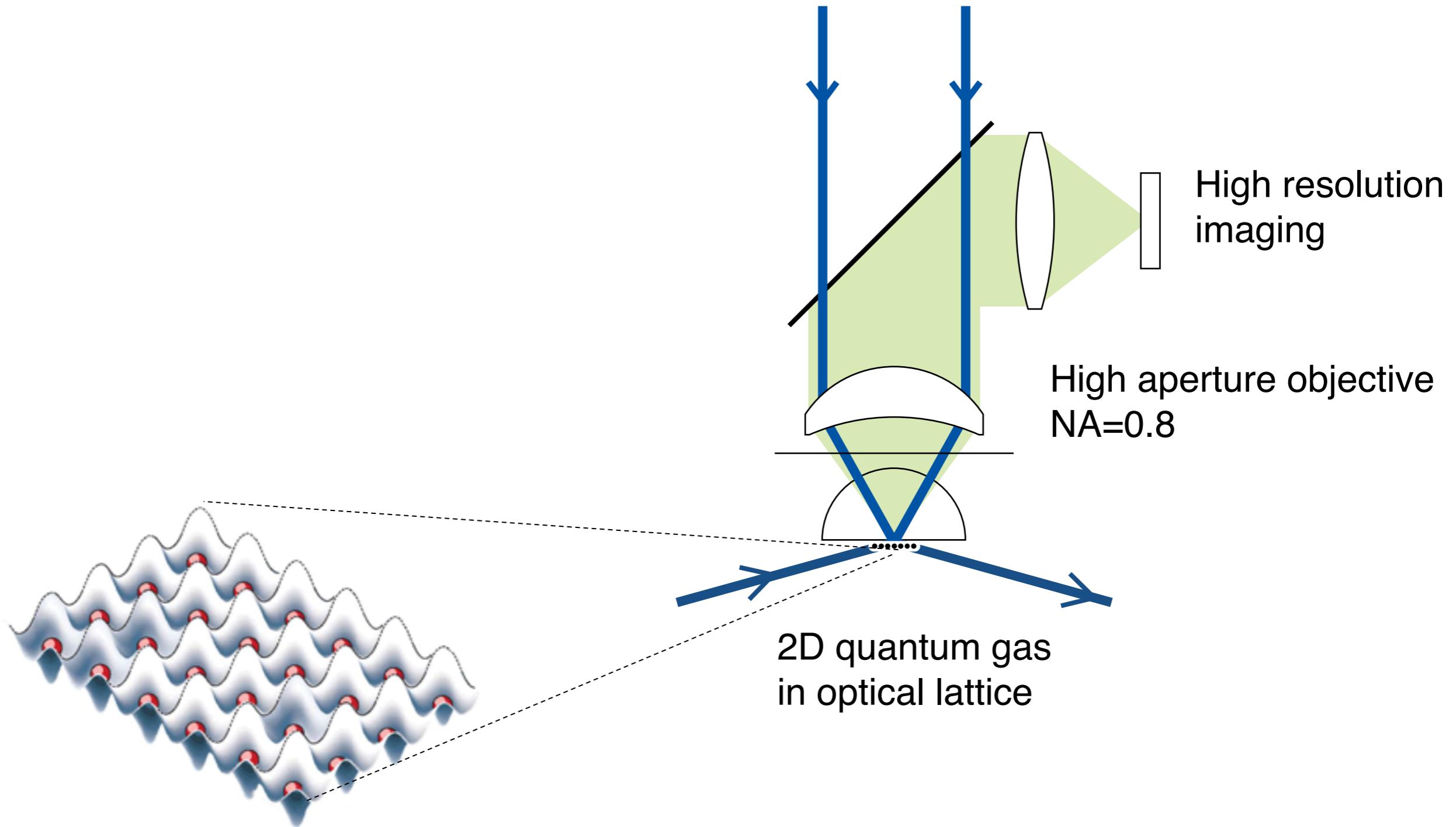
Quantum gas microscope



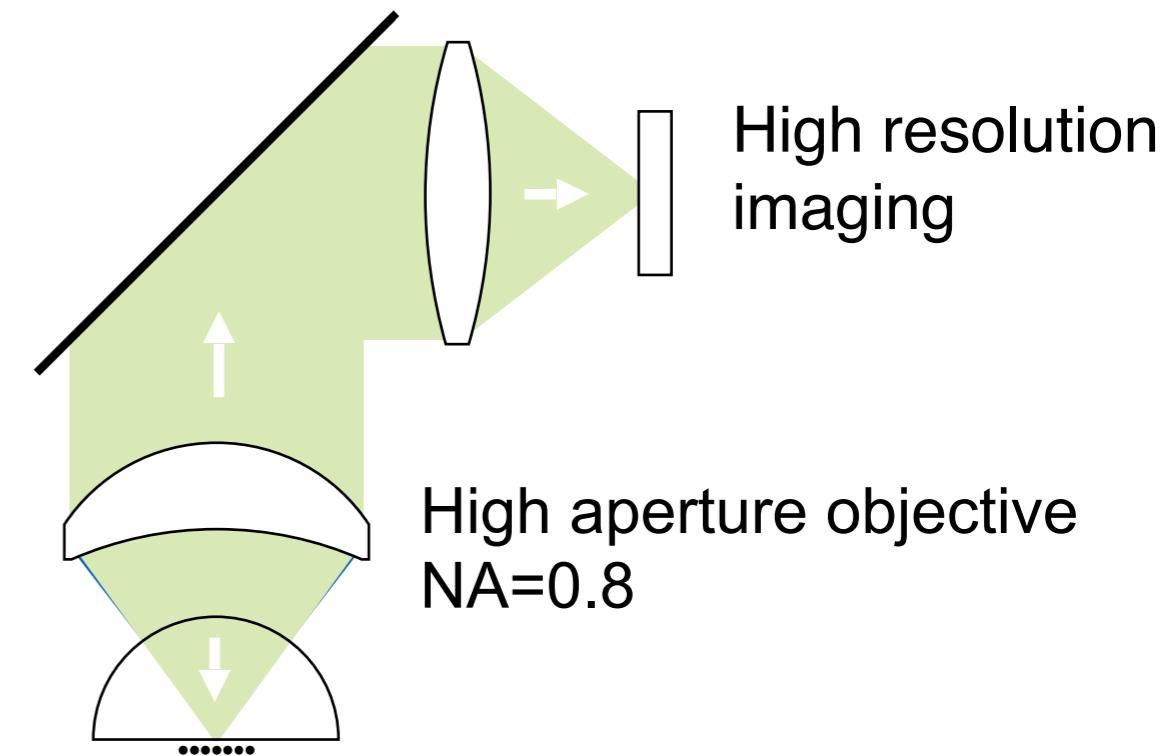
Quantum gas microscope



Quantum gas microscope

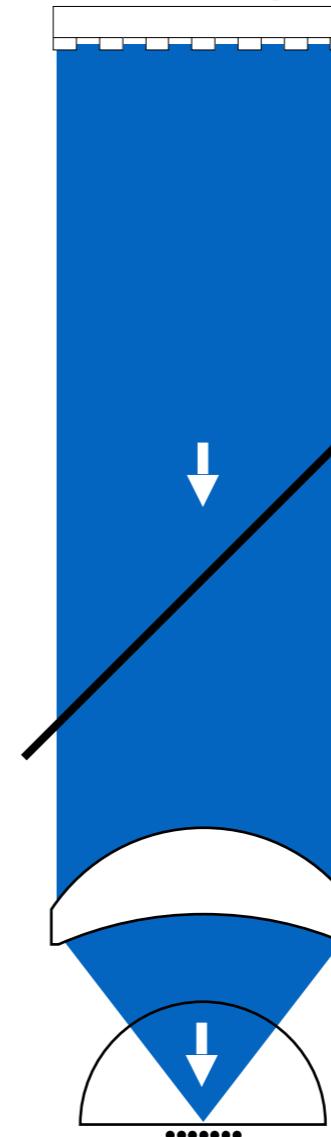


Fourier plane DMD



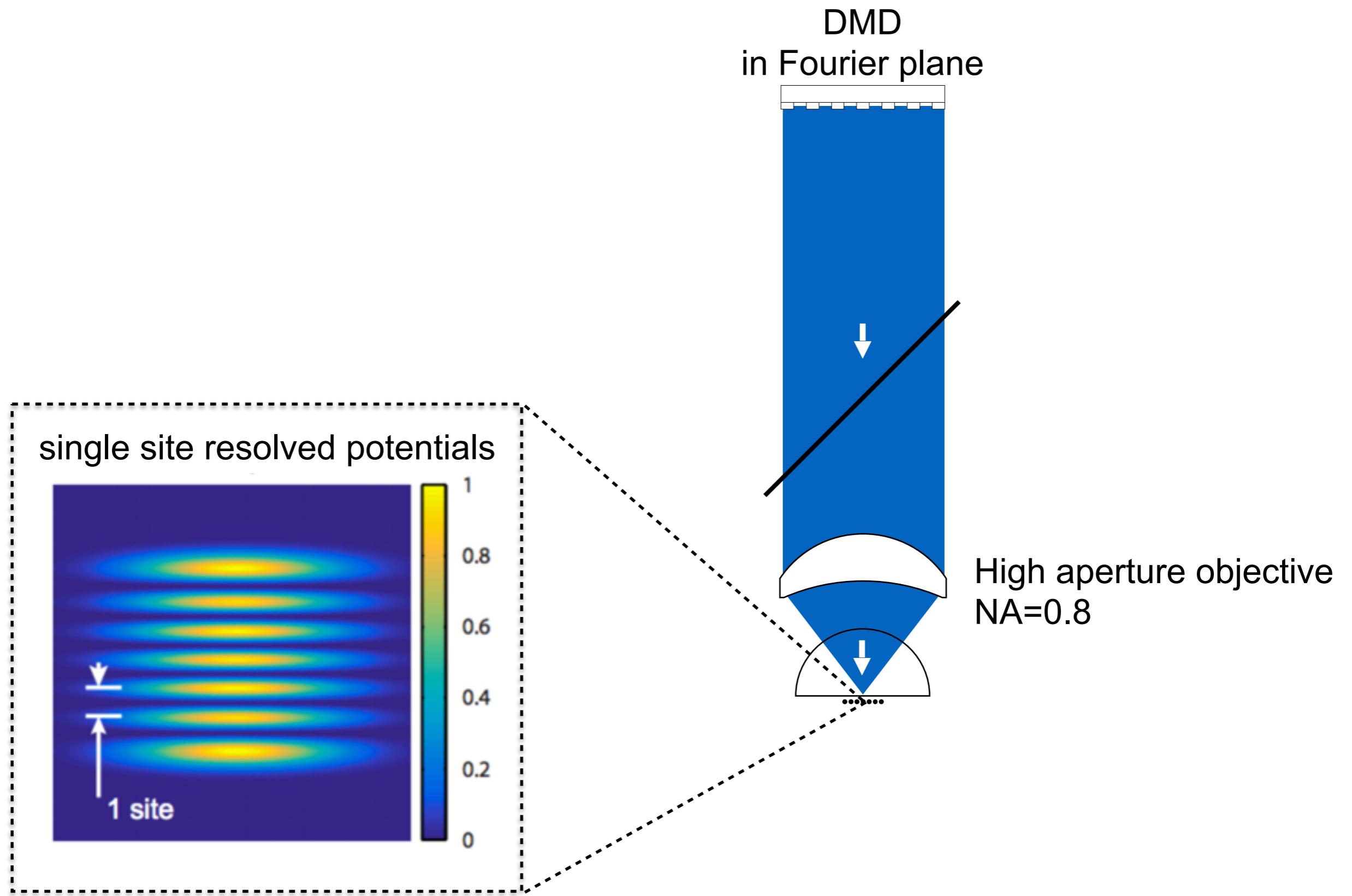
Fourier plane DMD

DMD
in Fourier plane

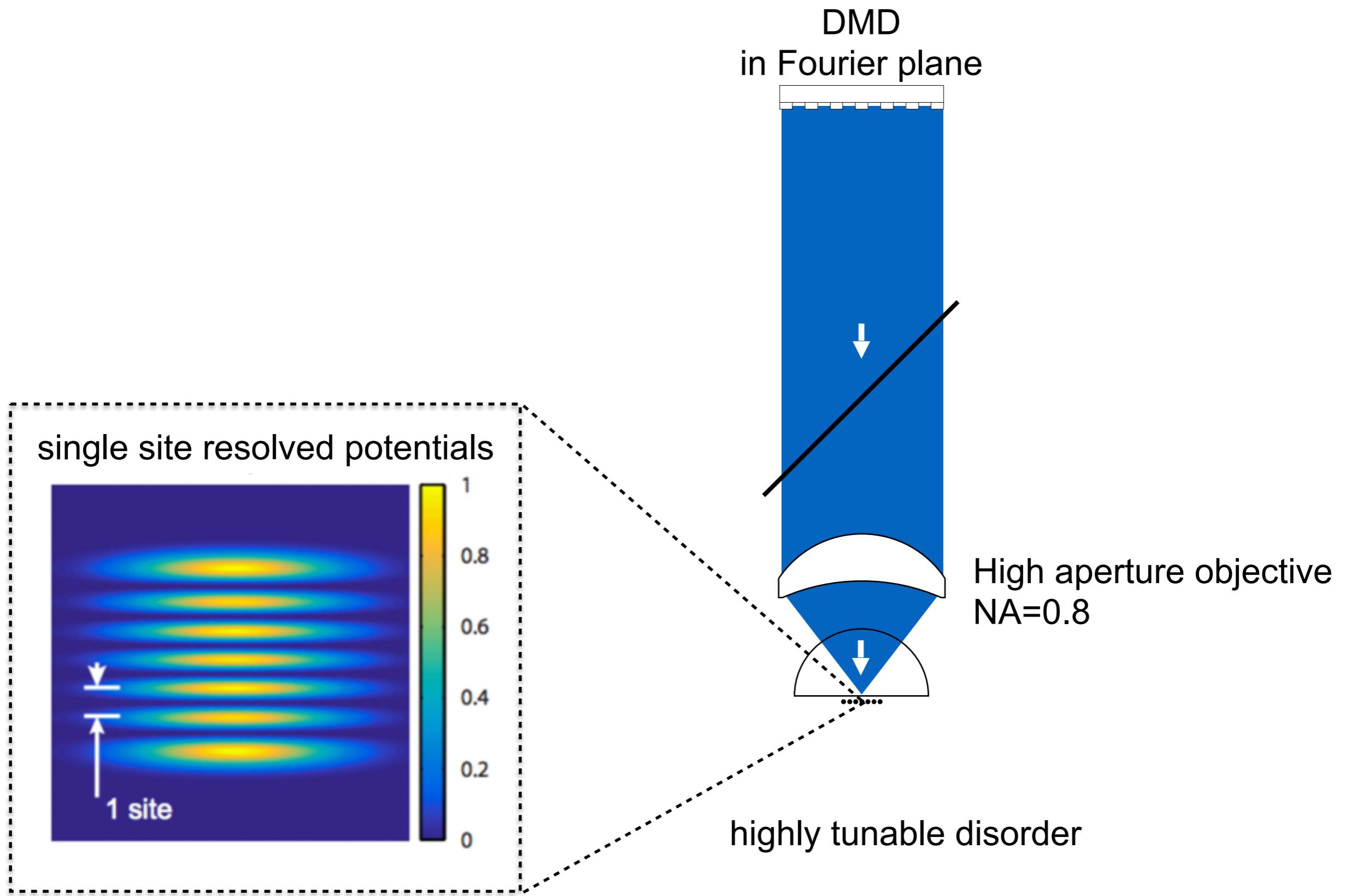


High aperture objective
NA=0.8

Fourier plane DMD

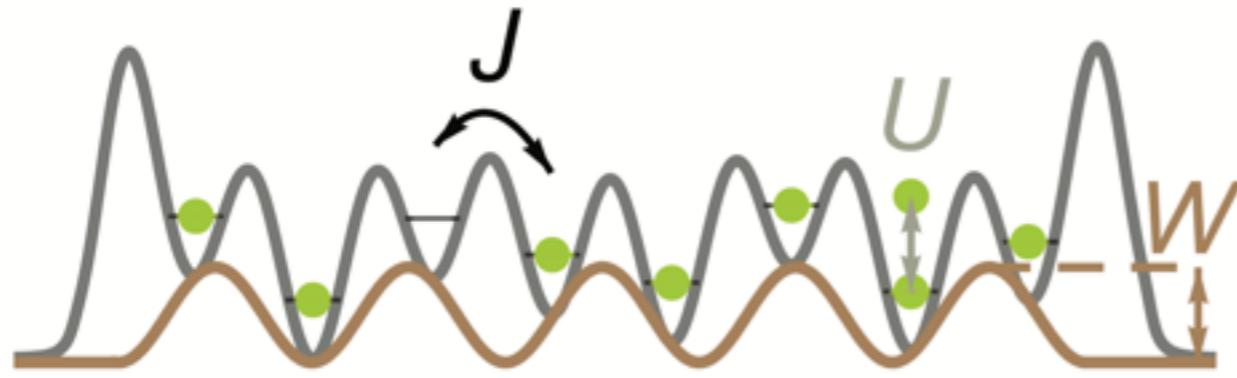


Fourier plane DMD



Experimental system

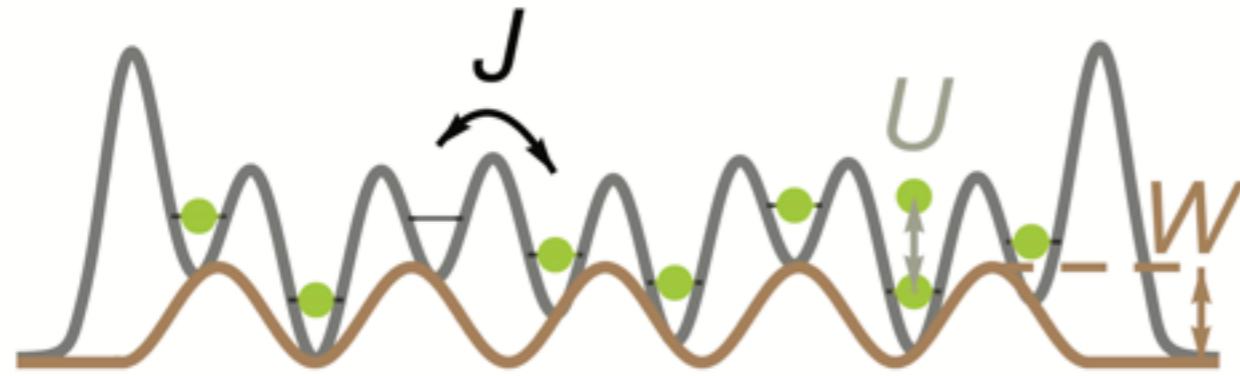
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interacting Bosonic Aubry-André model

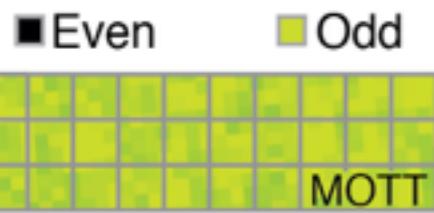
Experimental system

$$\hat{\mathcal{H}} = -J \sum_i (\hat{a}_i^\dagger \hat{a}_{i+1} + h.c.) + \frac{U}{2} \sum_i \hat{n}_i (\hat{n}_i - 1) + W \sum_i \cos(2\pi\beta i + \phi) \hat{n}_i$$



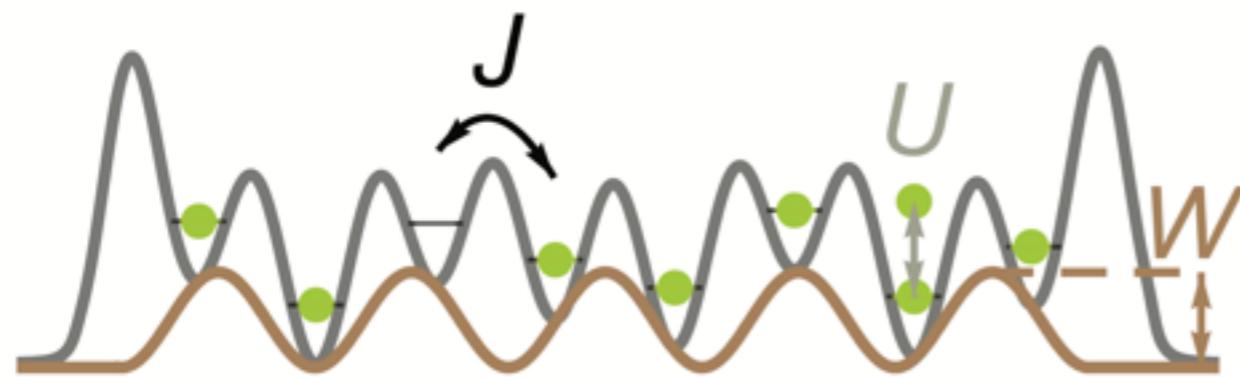
interacting Bosonic Aubry-André model

Site-Resolved Readout

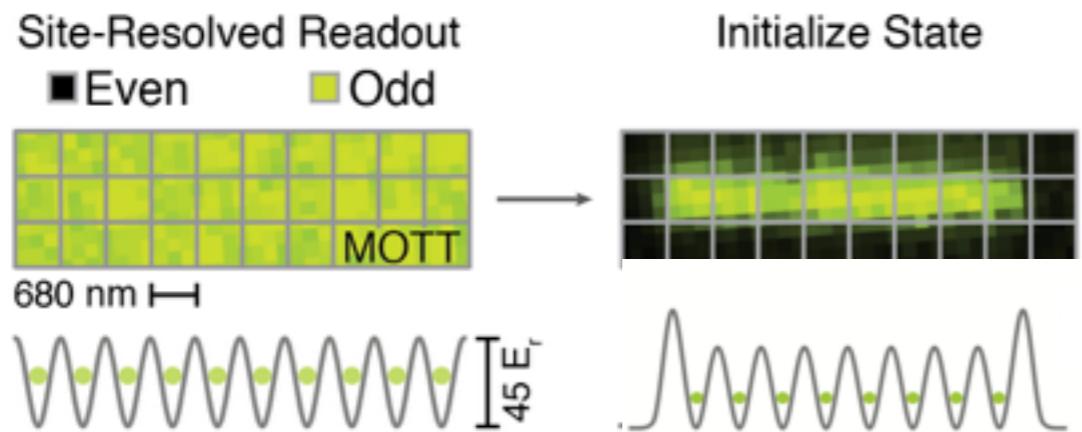


Experimental system

$$\hat{\mathcal{H}} = -J \sum_i (\hat{a}_i^\dagger \hat{a}_{i+1} + h.c.) + \frac{U}{2} \sum_i \hat{n}_i (\hat{n}_i - 1) + W \sum_i \cos(2\pi\beta i + \phi) \hat{n}_i$$

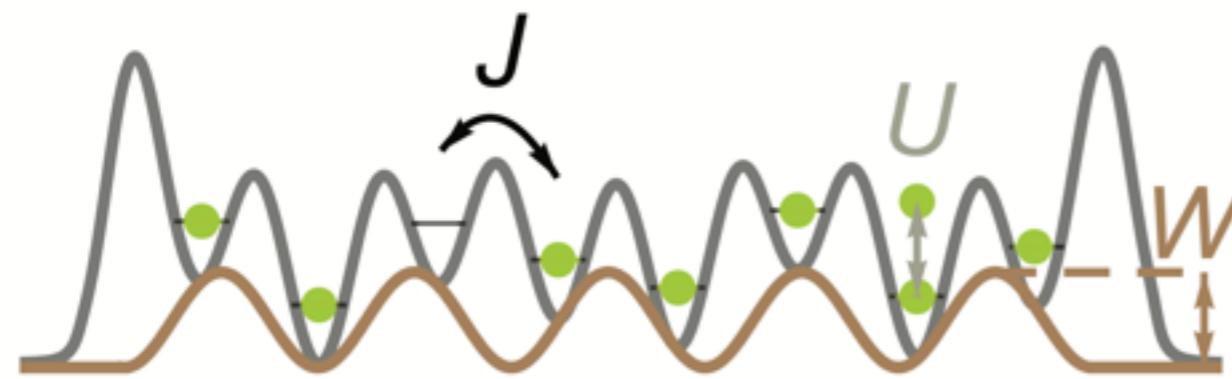


interacting Bosonic Aubry-André model

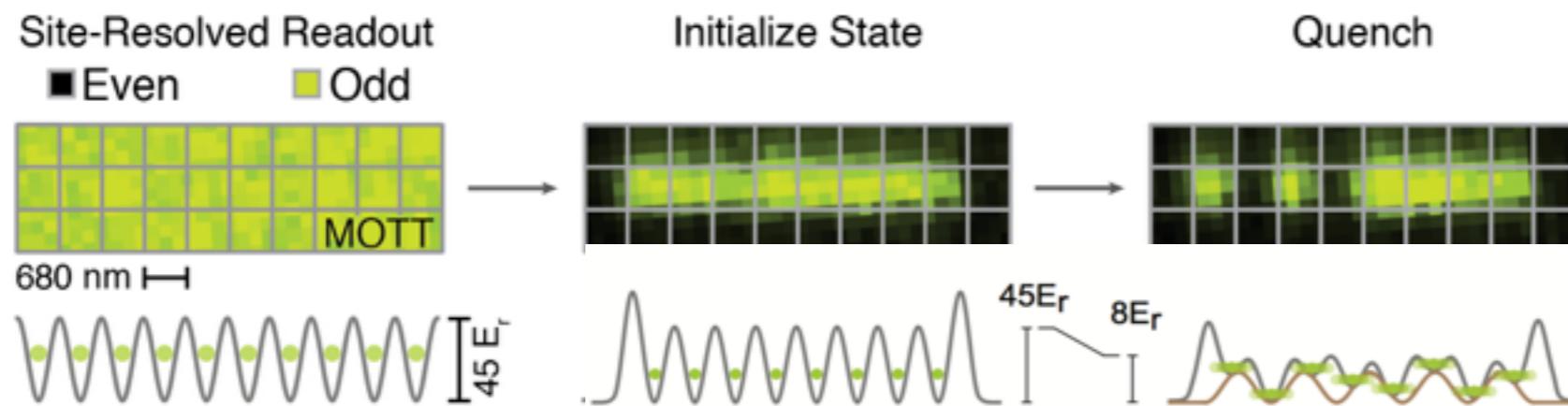


Experimental system

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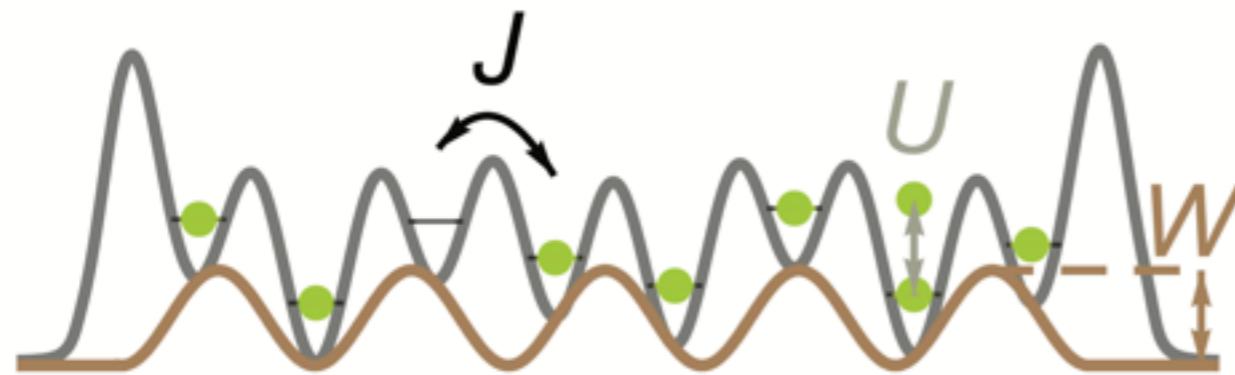


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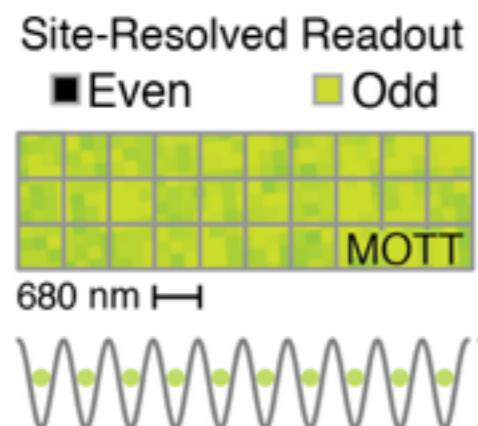


Experimental system

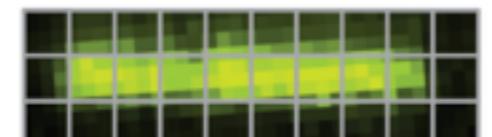
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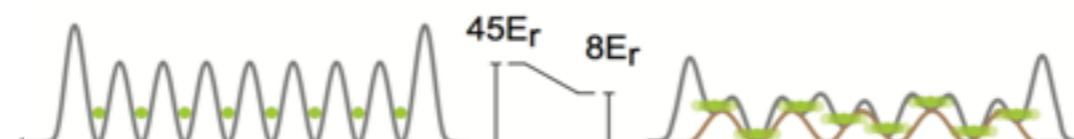
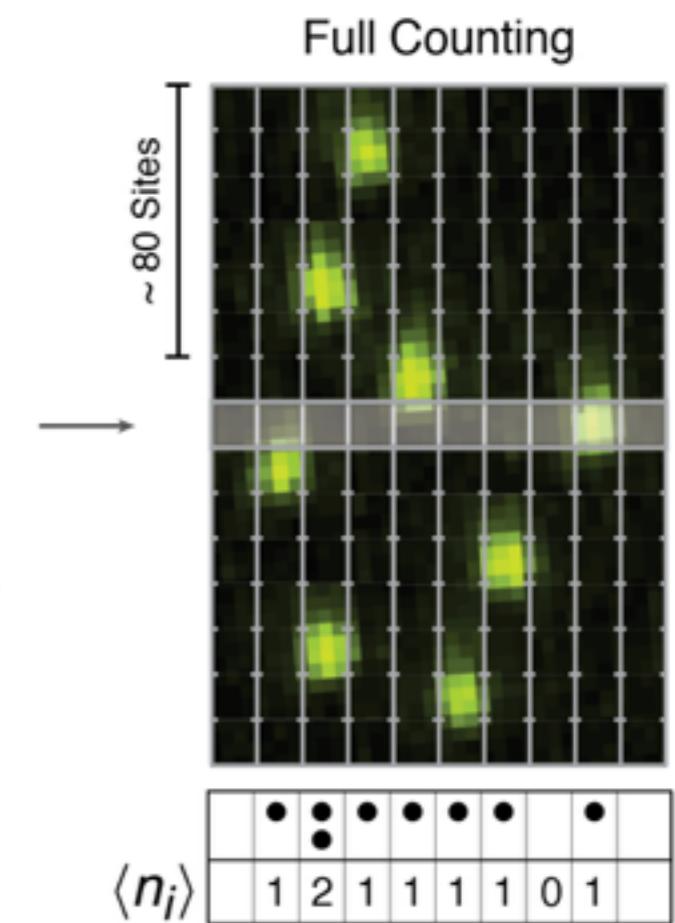
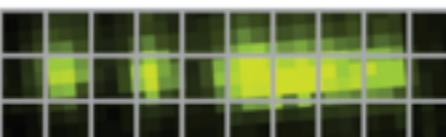
interacting Bosonic Aubry-André model



Initialize State

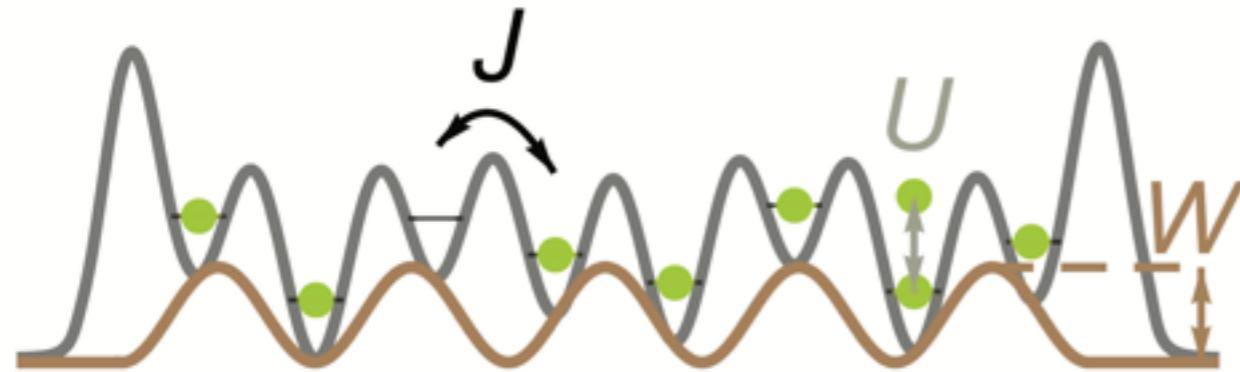


Quench

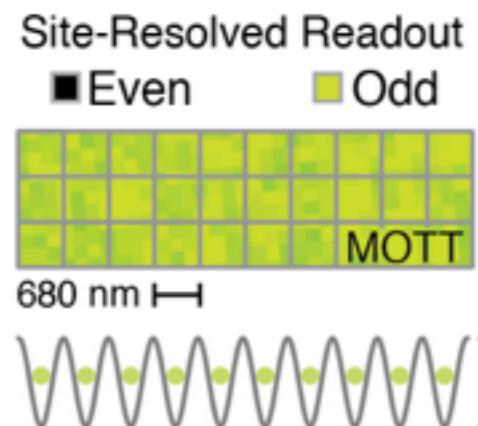


Experimental system

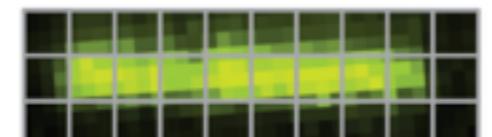
$$\hat{\mathcal{H}} = -J \sum_i (\hat{a}_i^\dagger \hat{a}_{i+1} + h.c.) + \frac{U}{2} \sum_i \hat{n}_i (\hat{n}_i - 1) + W \sum_i \cos(2\pi\beta i + \phi) \hat{n}_i$$



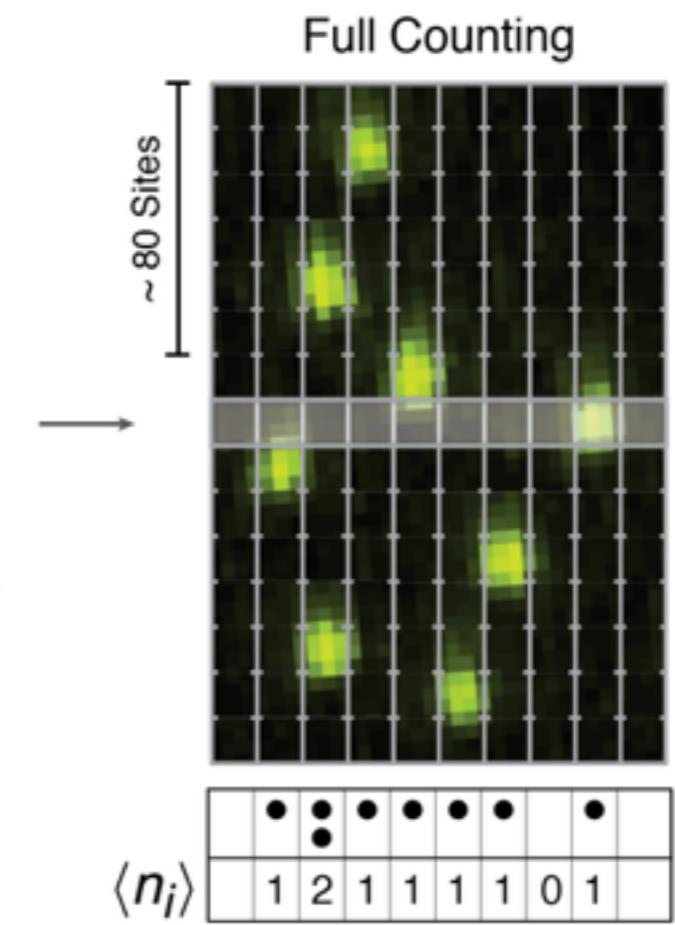
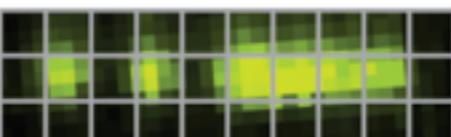
interacting Bosonic Aubry-André model



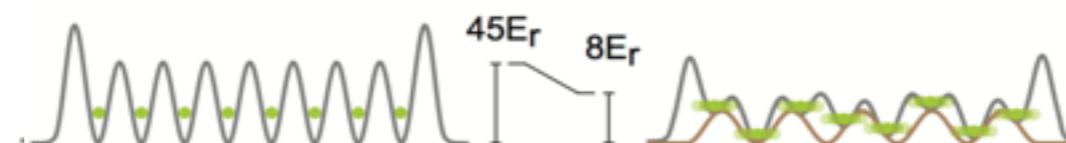
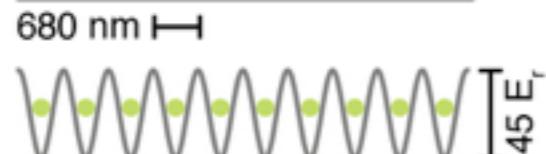
Initialize State



Quench

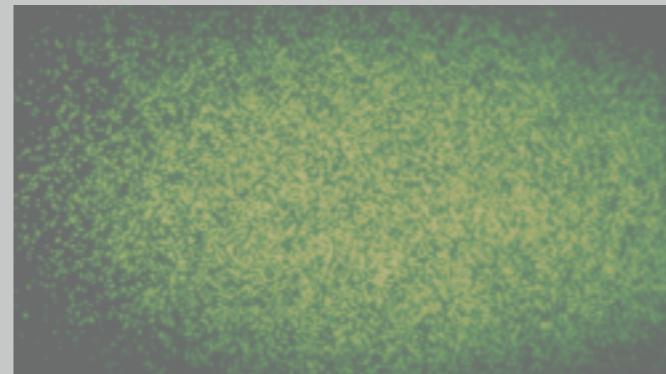


Fully coherent evolution of the system

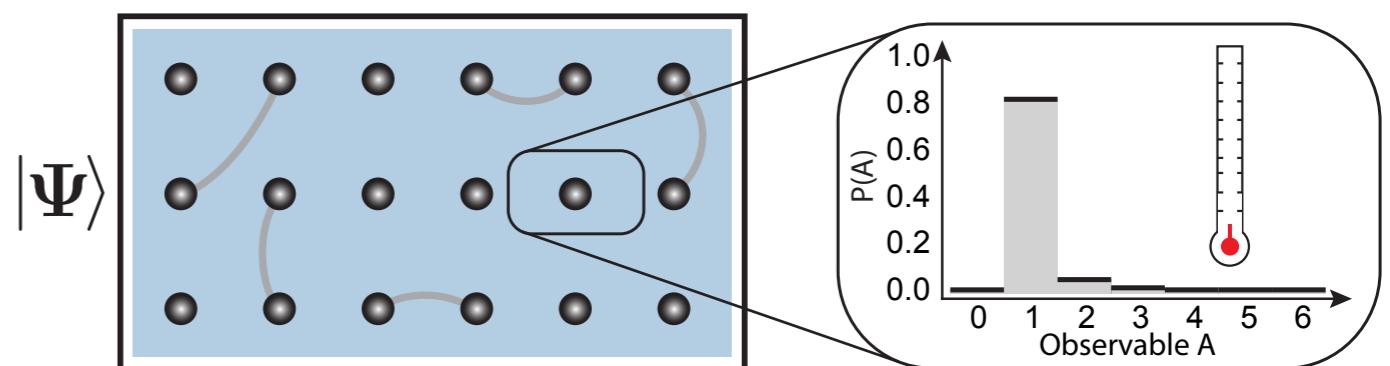


Key features of MBL

- Experimental setup



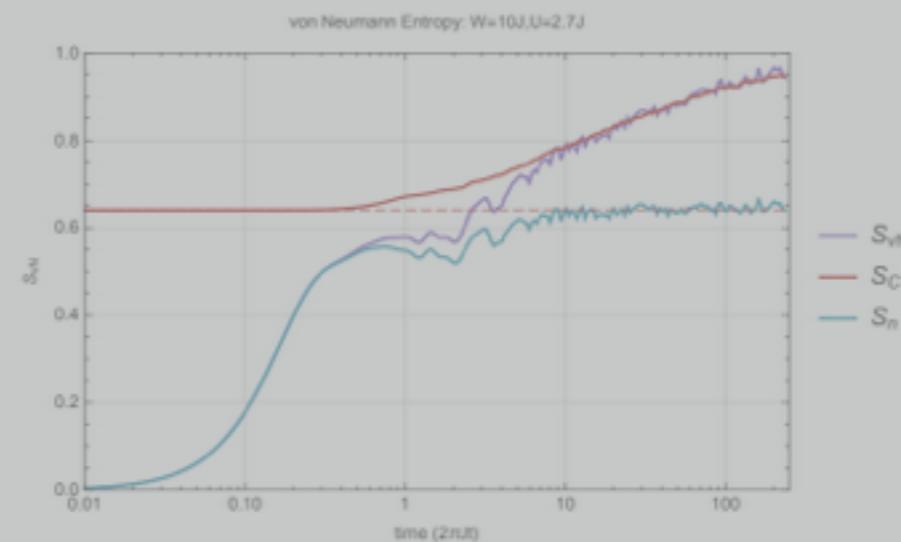
- Breakdown of thermalization



- Spatial localization



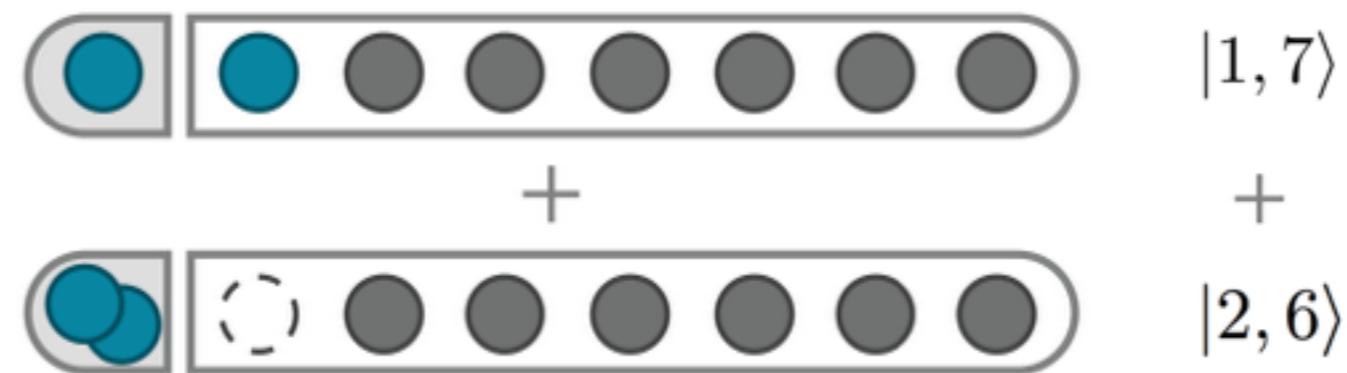
- Entanglement growth



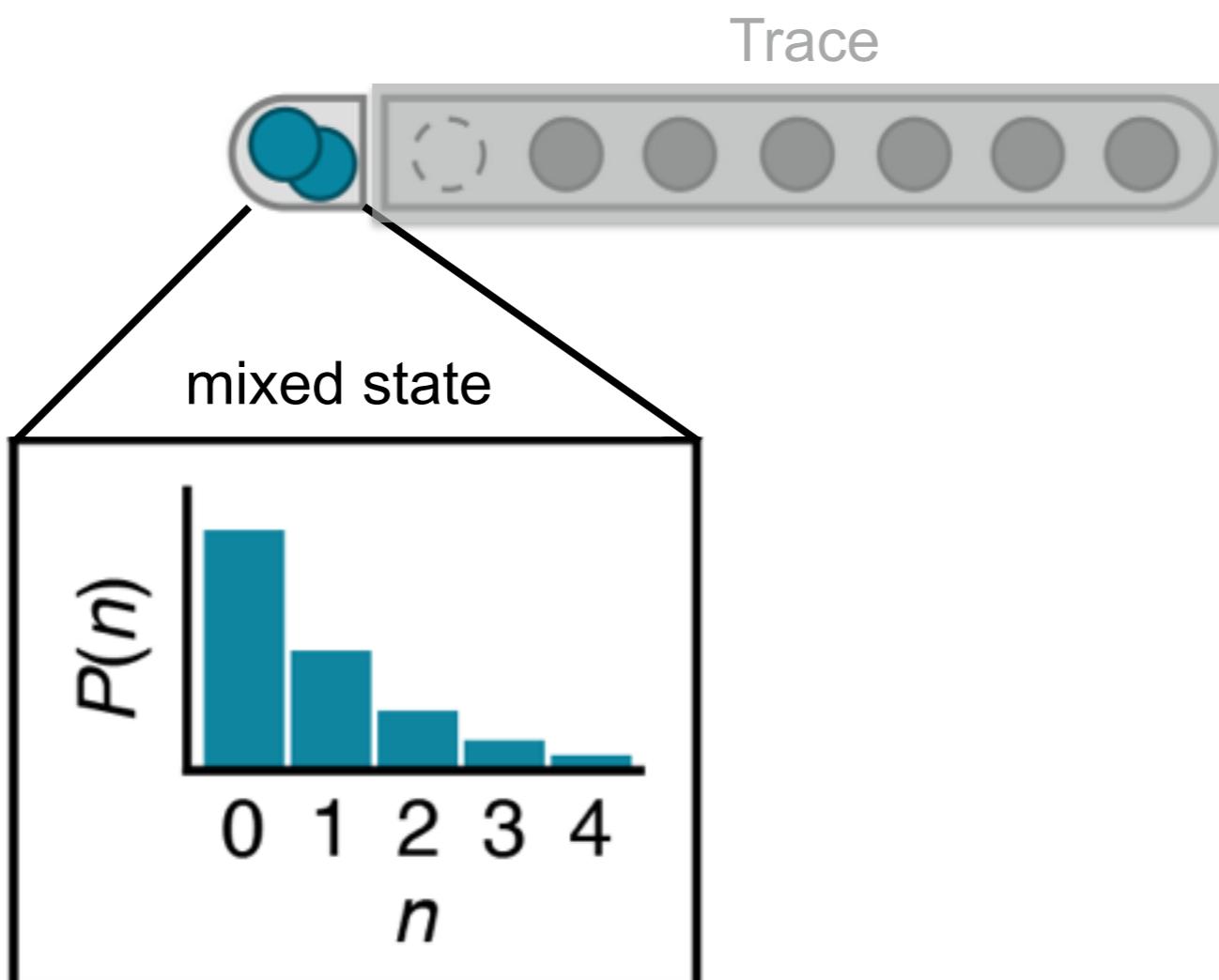
Single site entropy



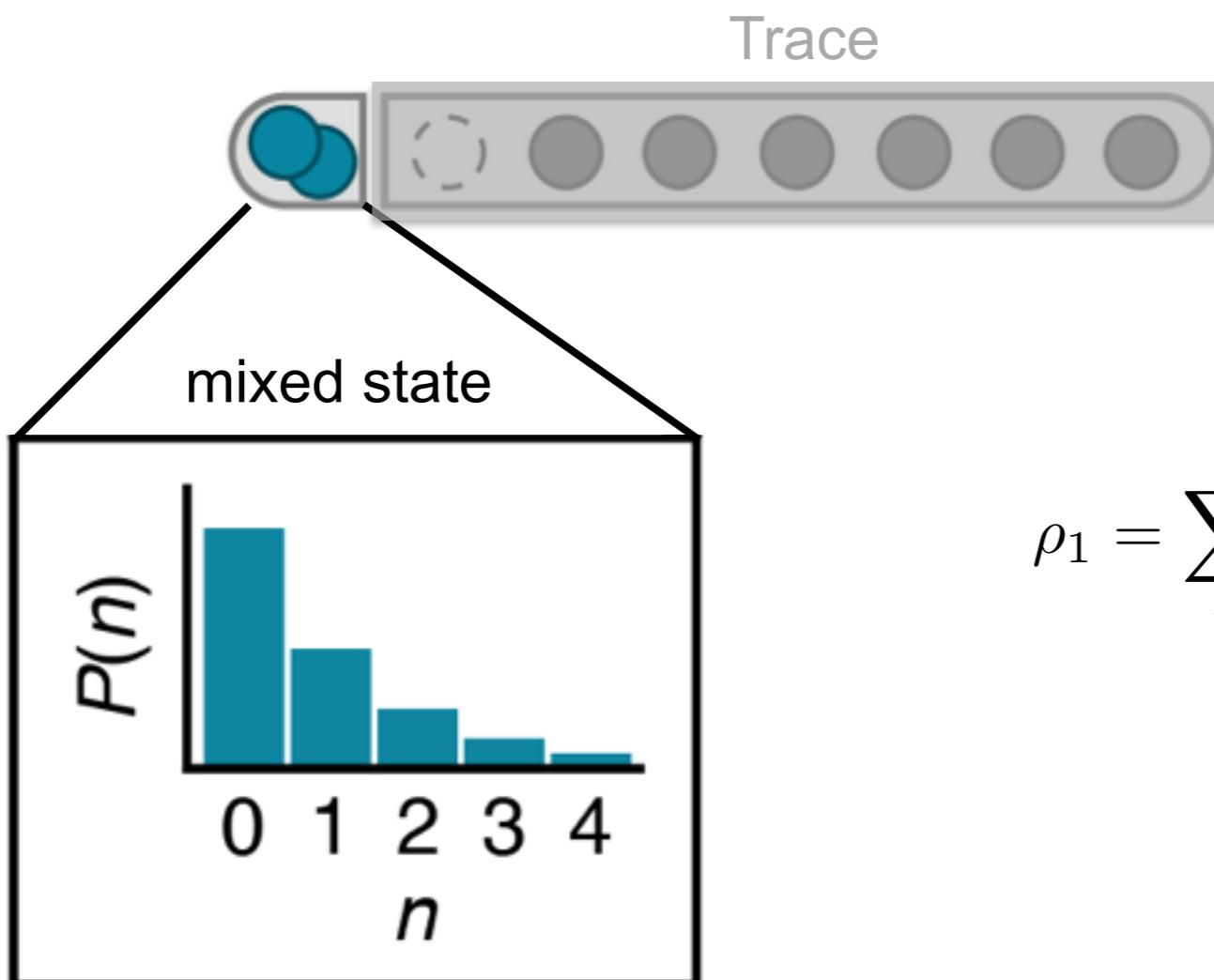
Single site entropy



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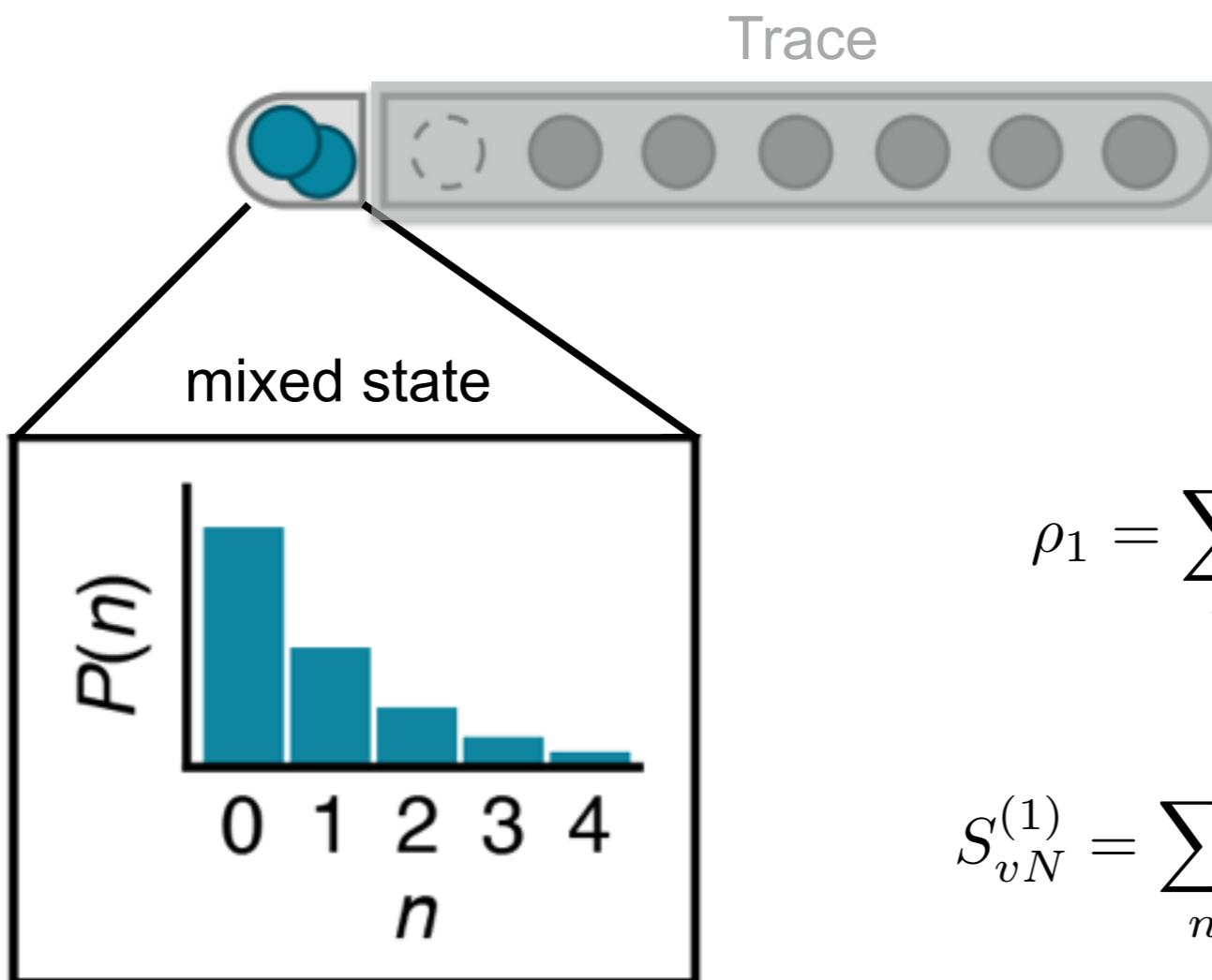


Single site entropy



$$\rho_1 = \sum_n P(n) |n\rangle \langle n|$$

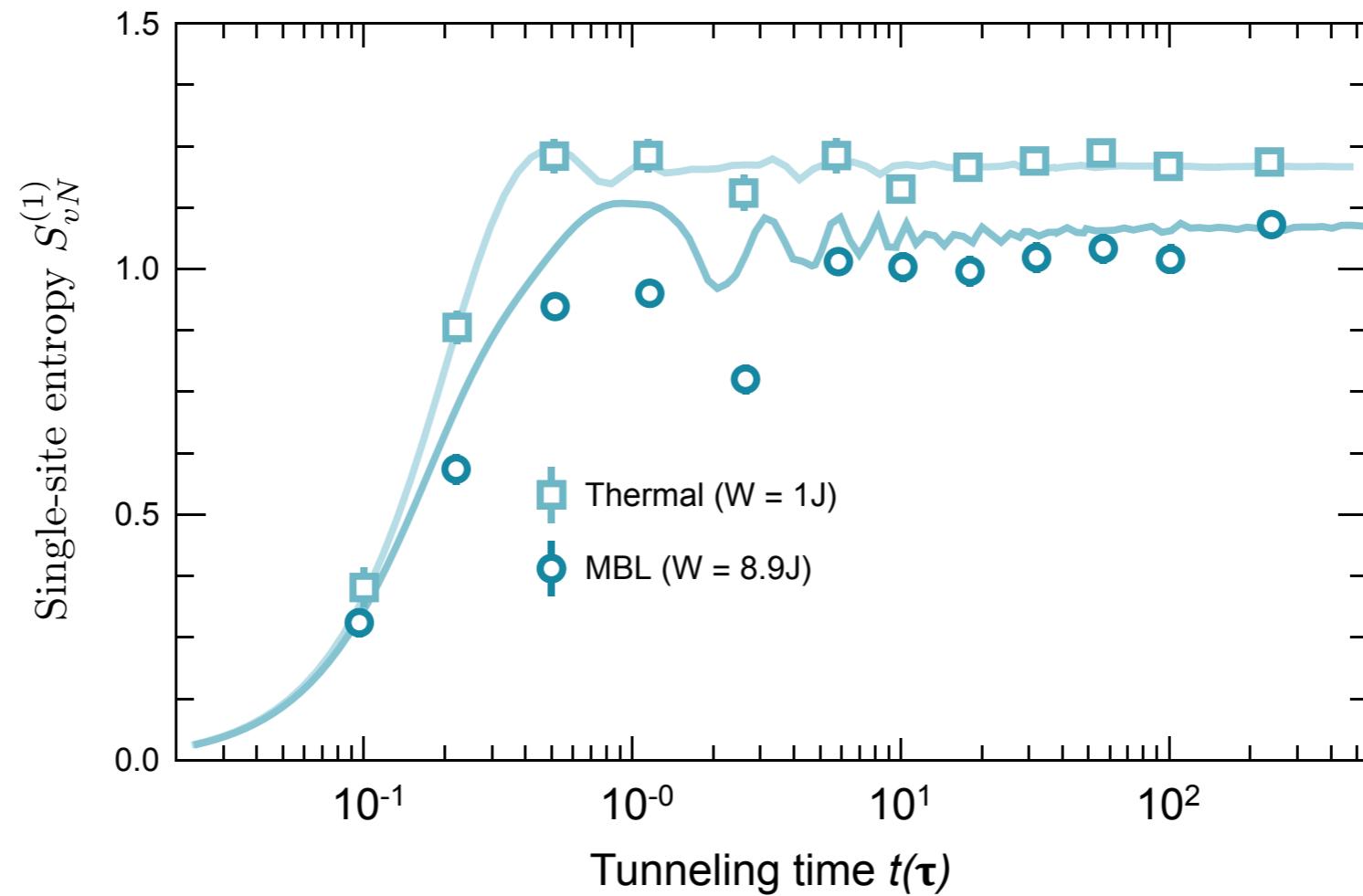
Single site entropy



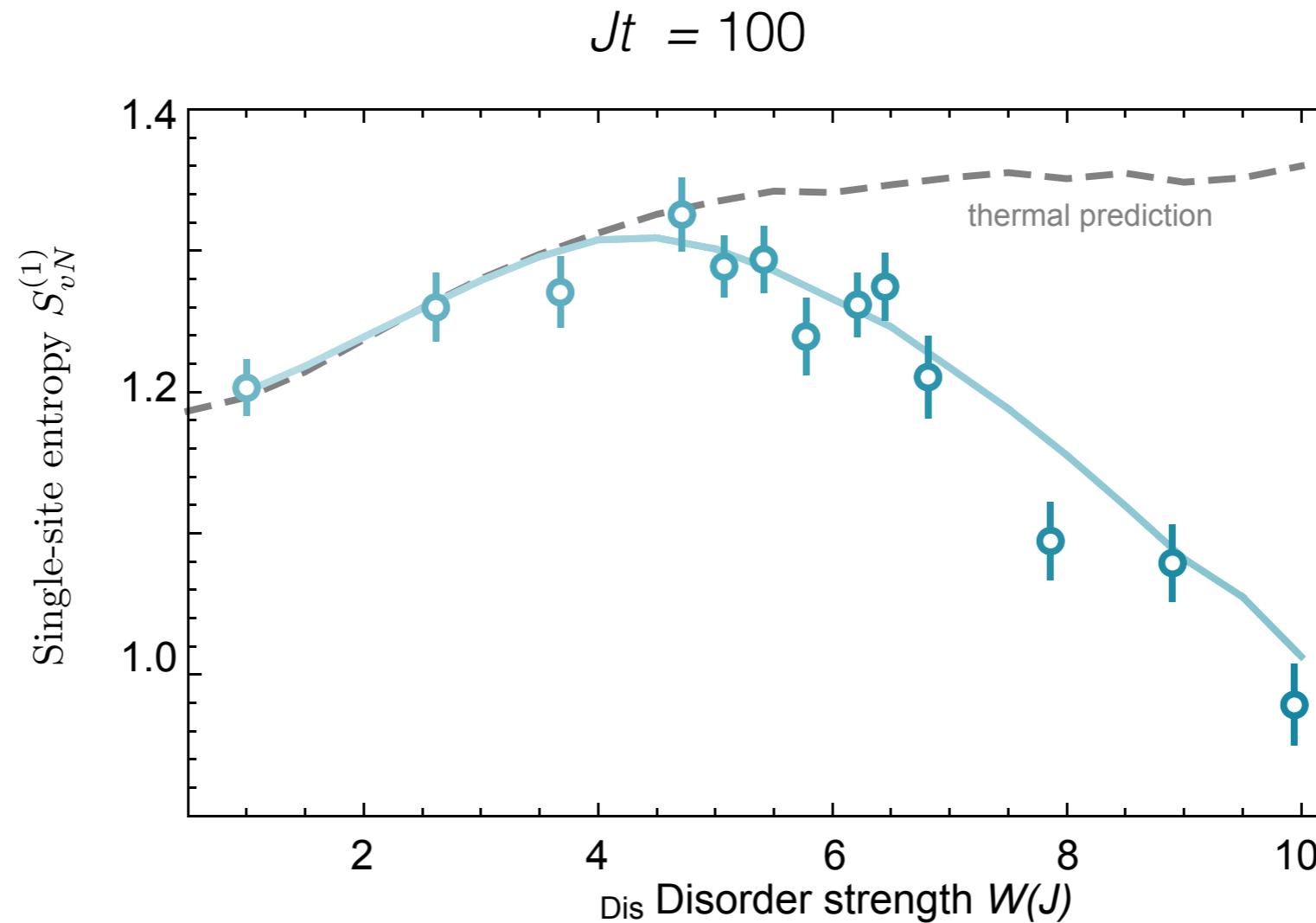
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$$S_{vN}^{(1)} = \sum_n P(n) \text{Log}(P(n))$$

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Breakdown of thermalization

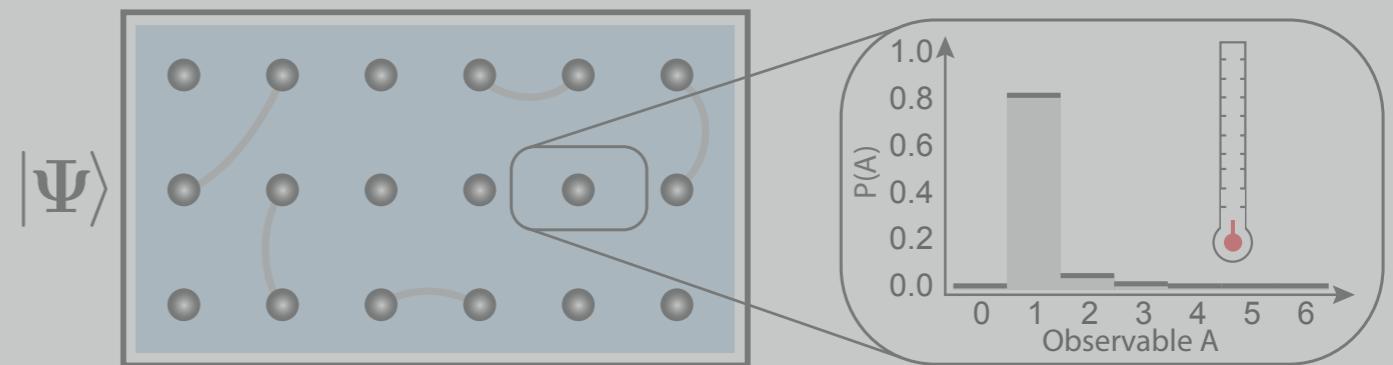


Key features of MBL

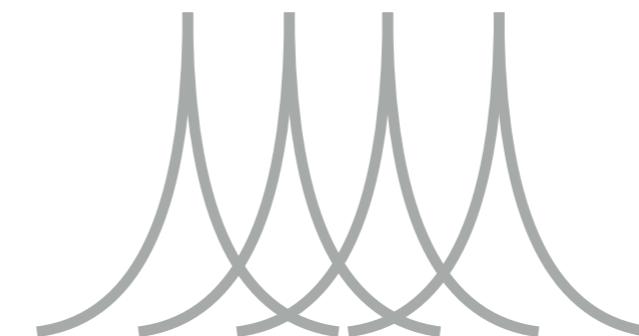
- Experimental setup



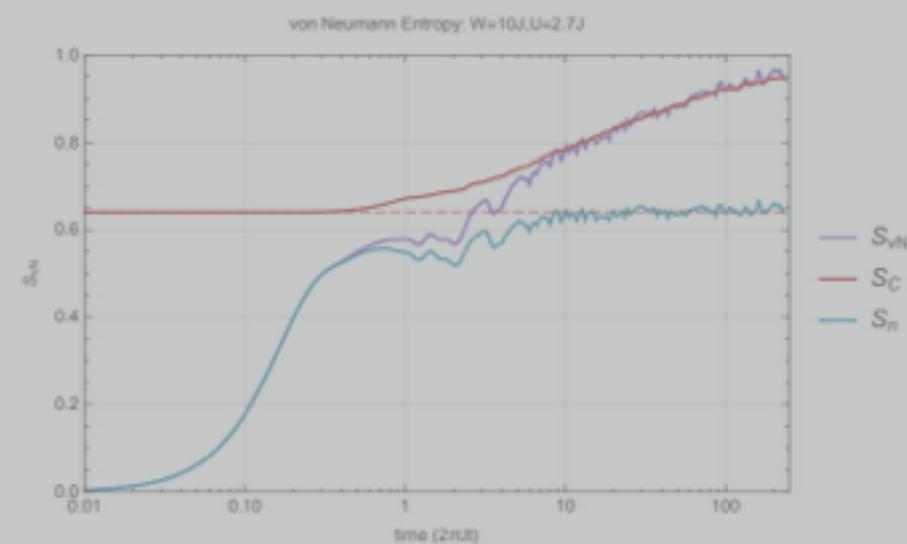
- Breakdown of thermalization



- Spatial localization



- Entanglement growth



Spatial localization

$$G^{(2)}(d) = \langle n_i n_{i+d} \rangle - \langle n_i \rangle \langle n_{i+d} \rangle$$

Spatial localization

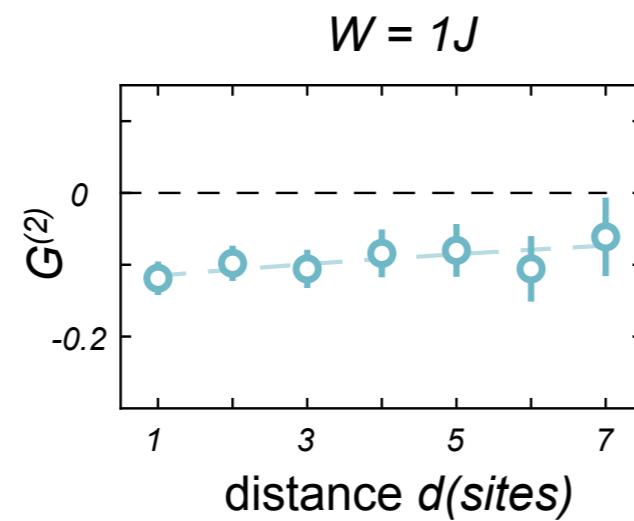
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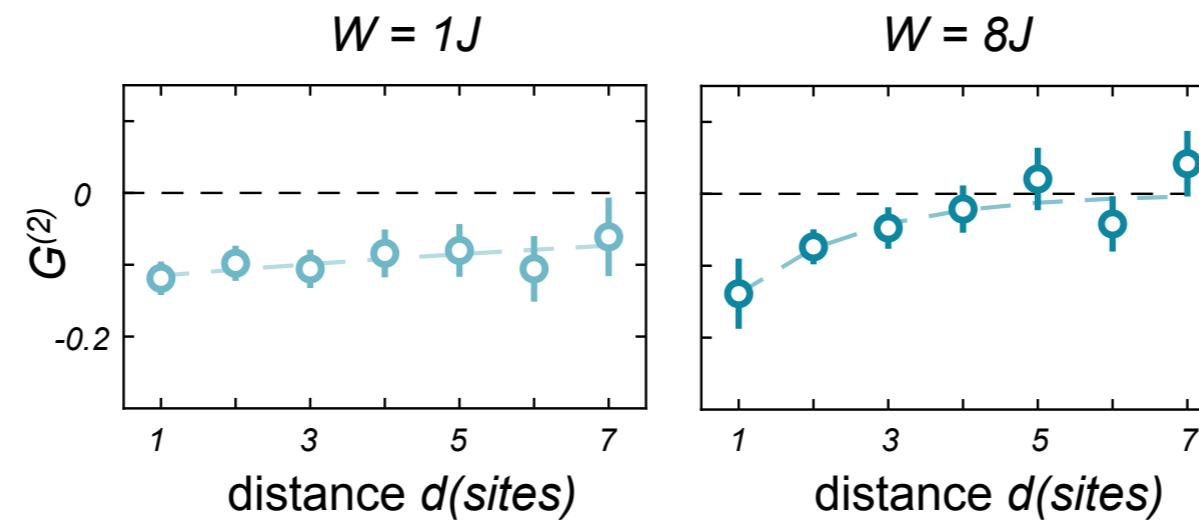


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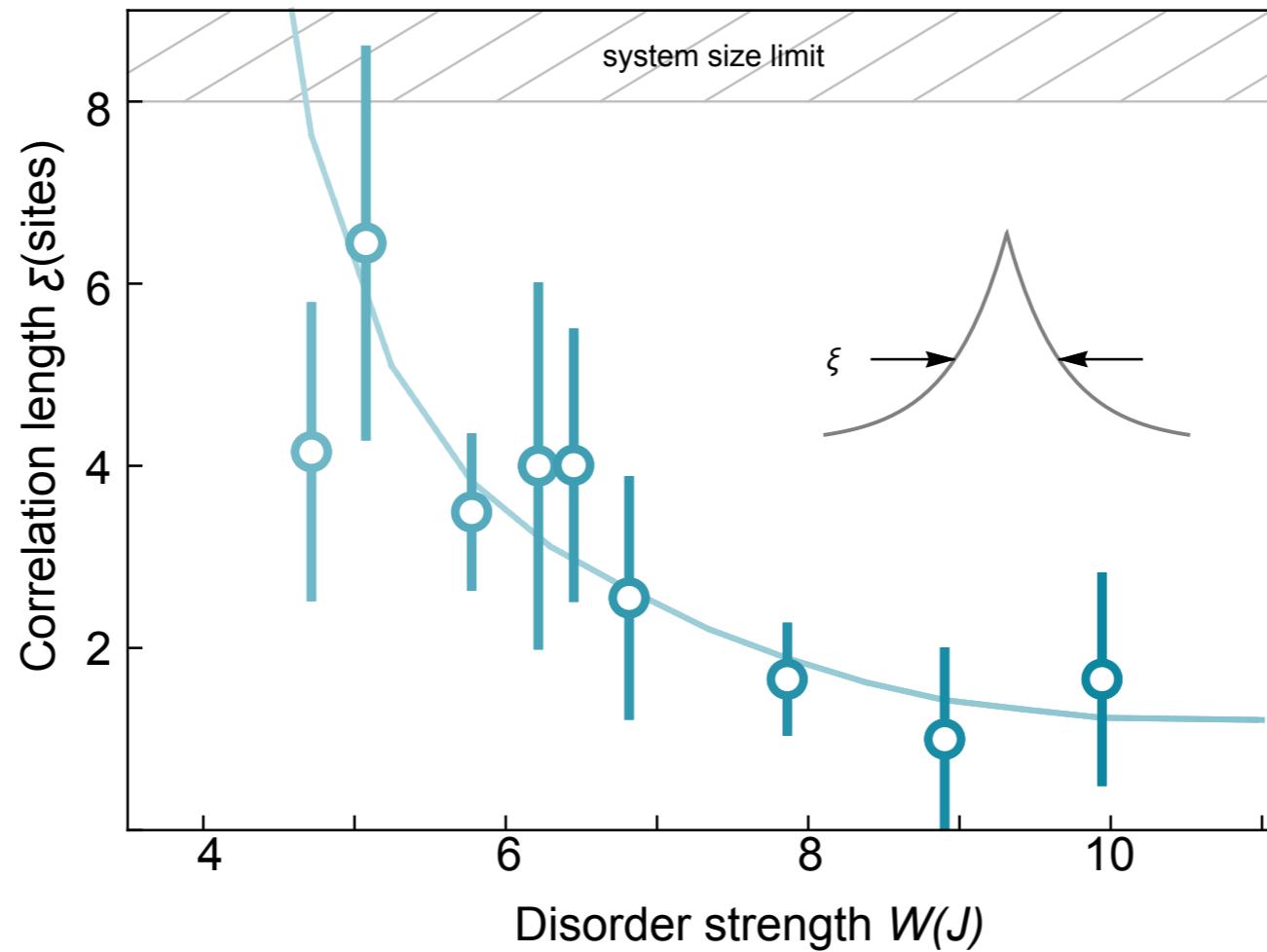
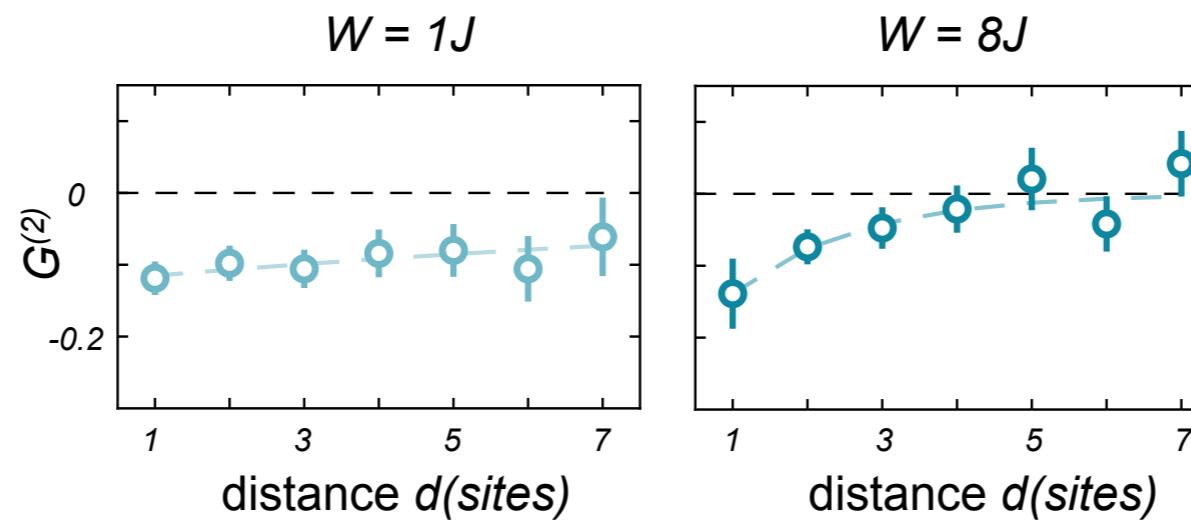


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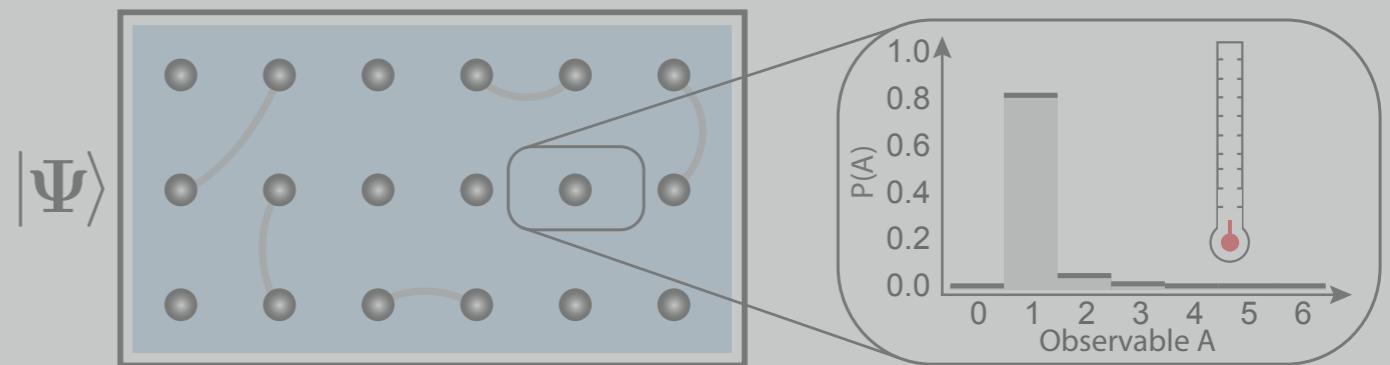


Key features of MBL

- Experimental setup



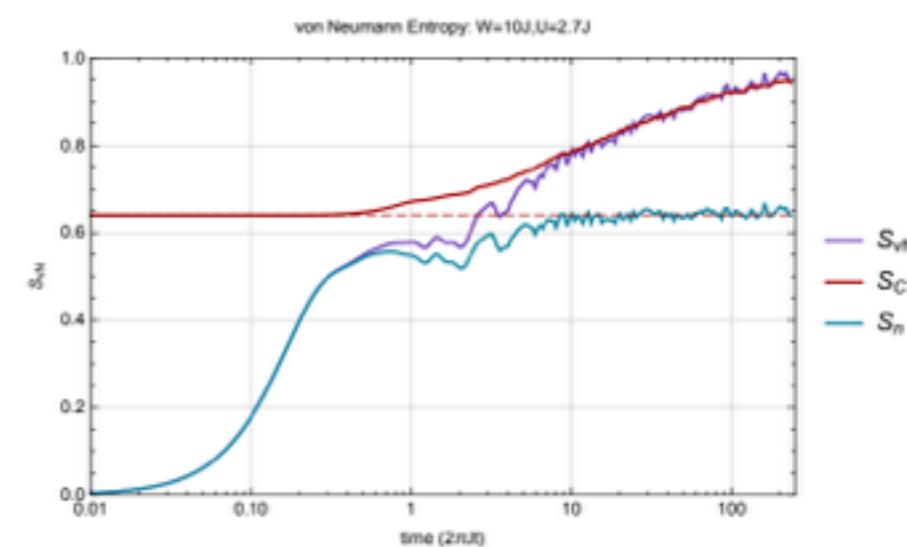
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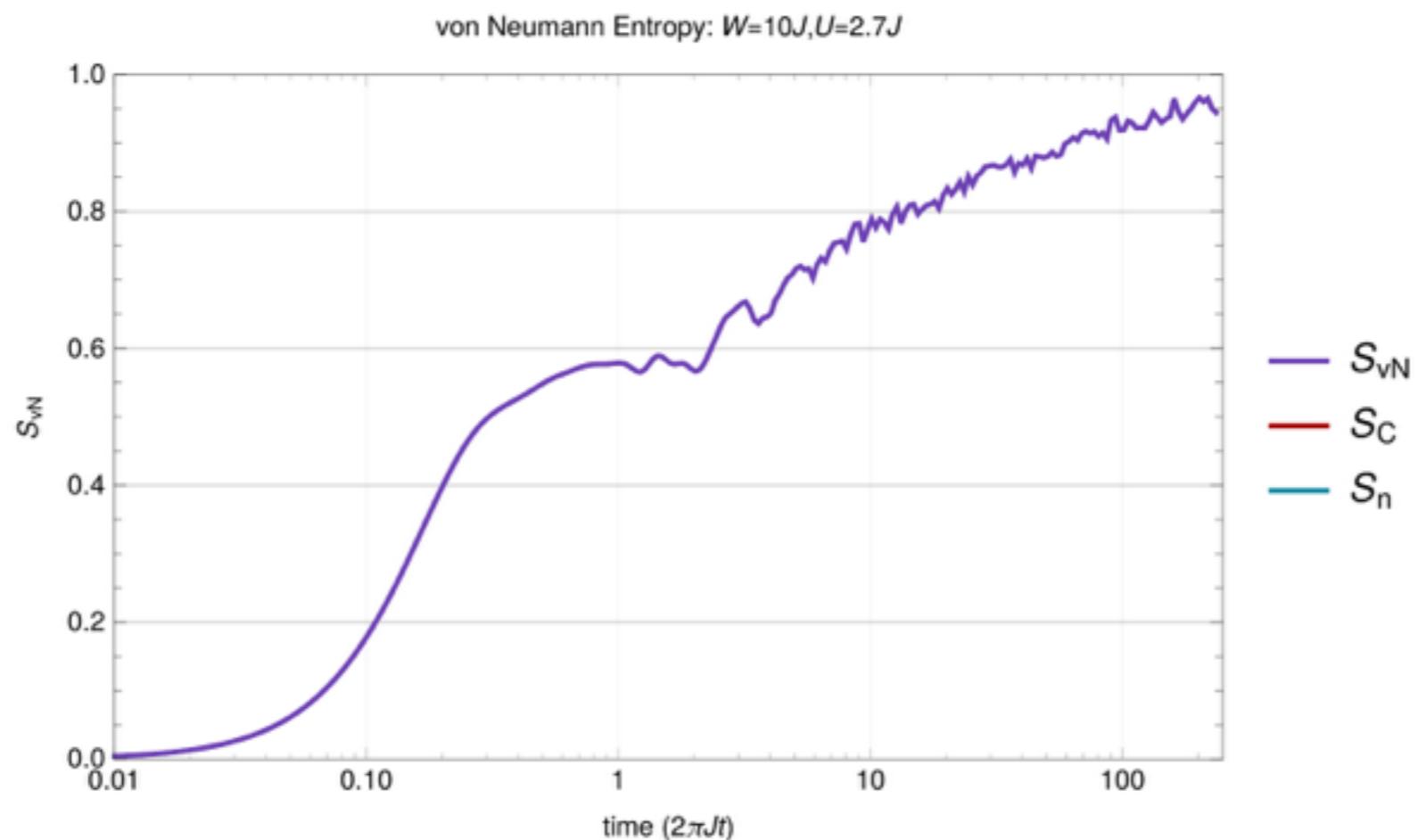
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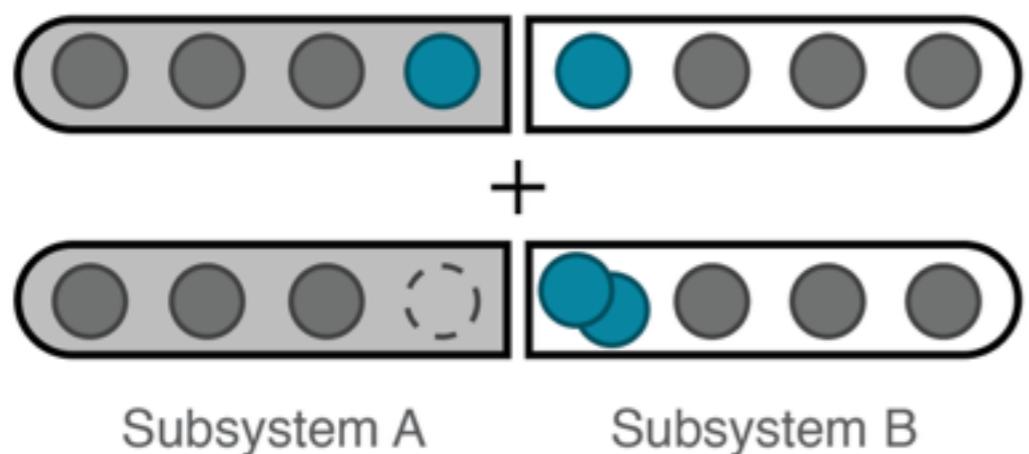


Logarithmic grows of entanglement



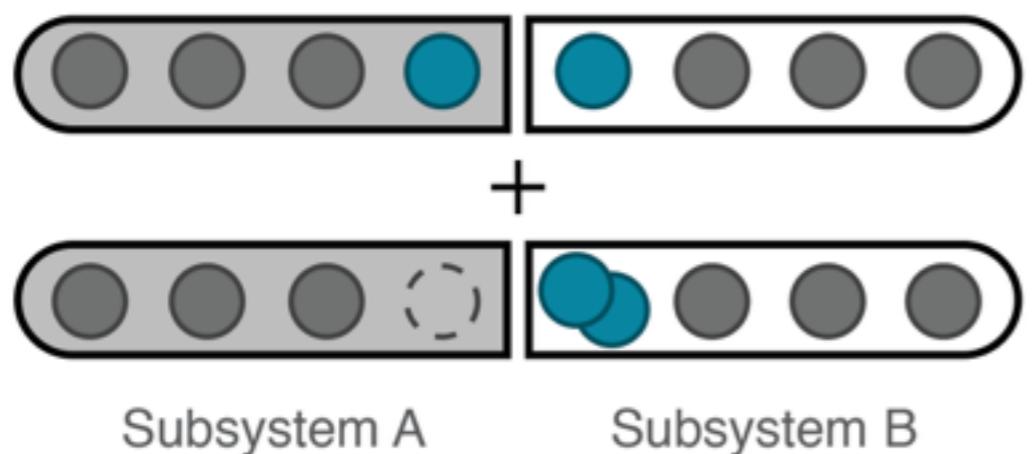
Two types of entanglement

Number entanglement



Two types of entanglement

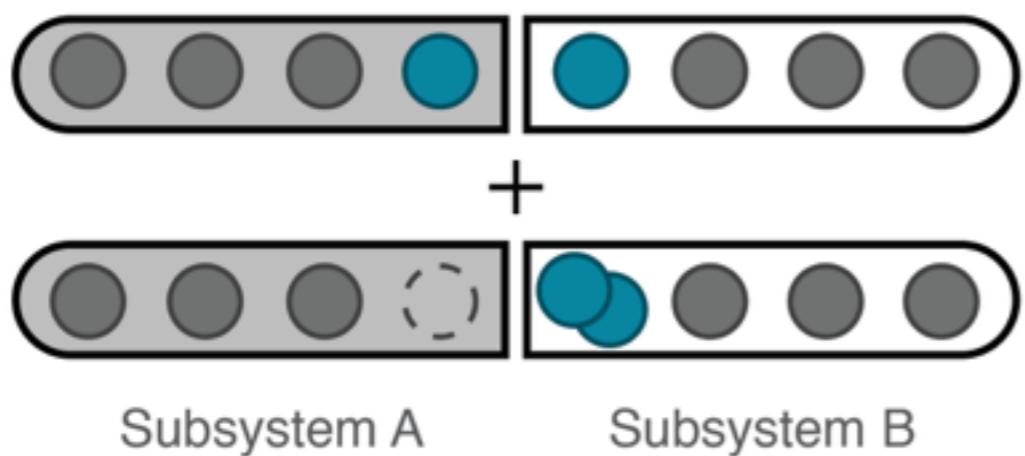
Number entanglement



particle tunneling

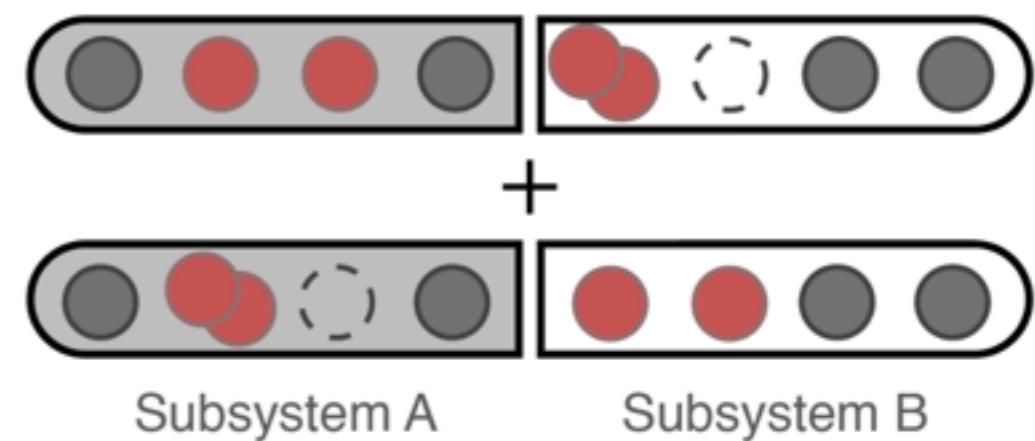
Two types of entanglement

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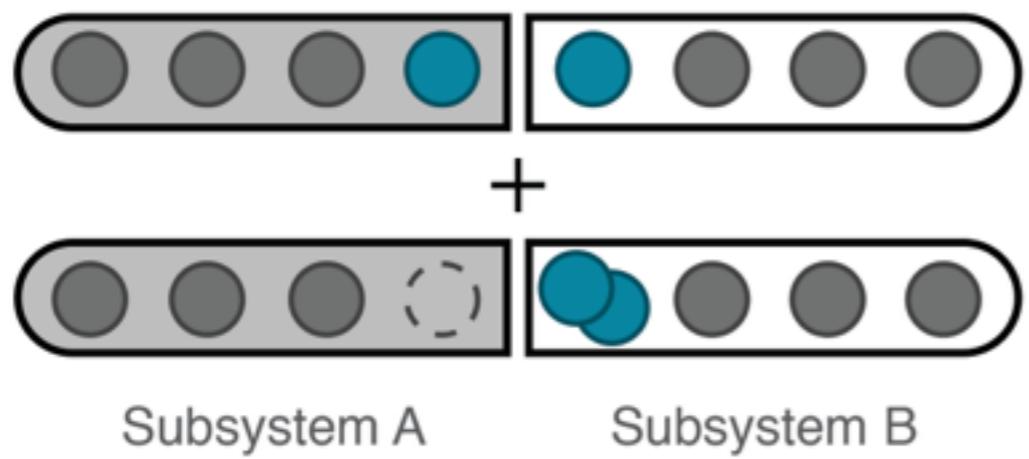
particle tunneling

Configurational entanglement



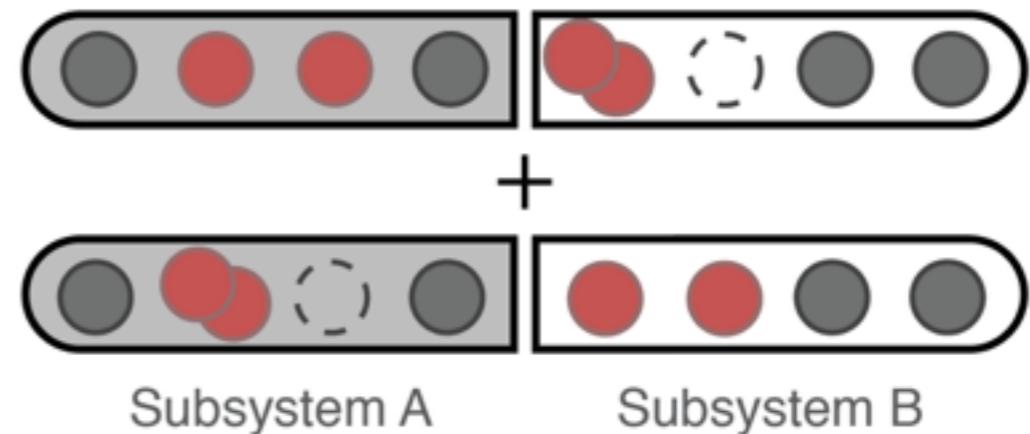
Two types of entanglement

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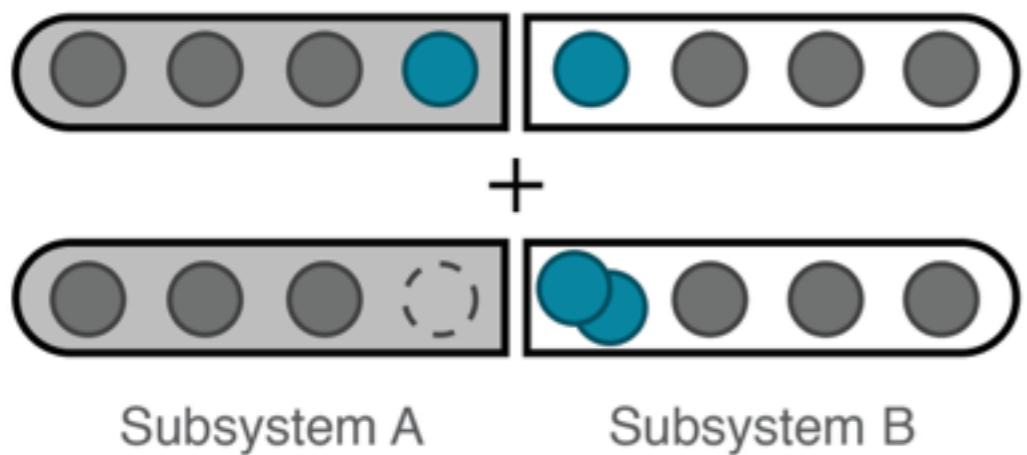
Configurational entanglement



non-local quantum correlations

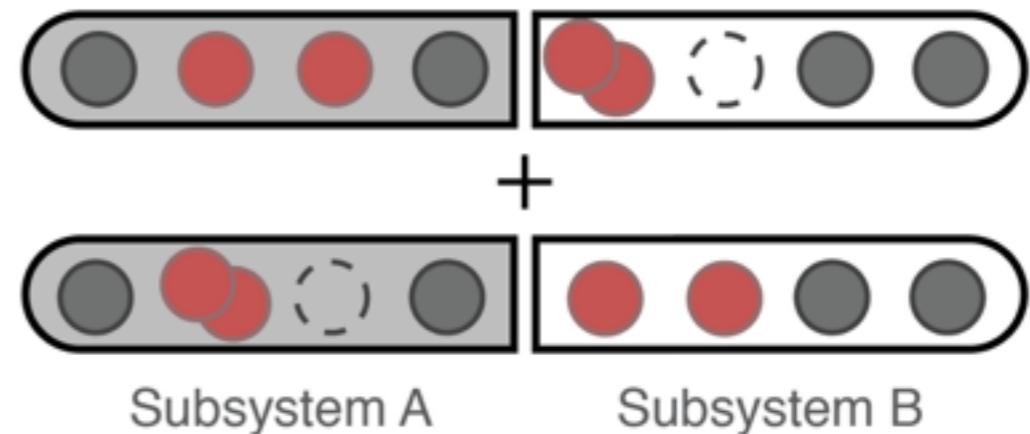
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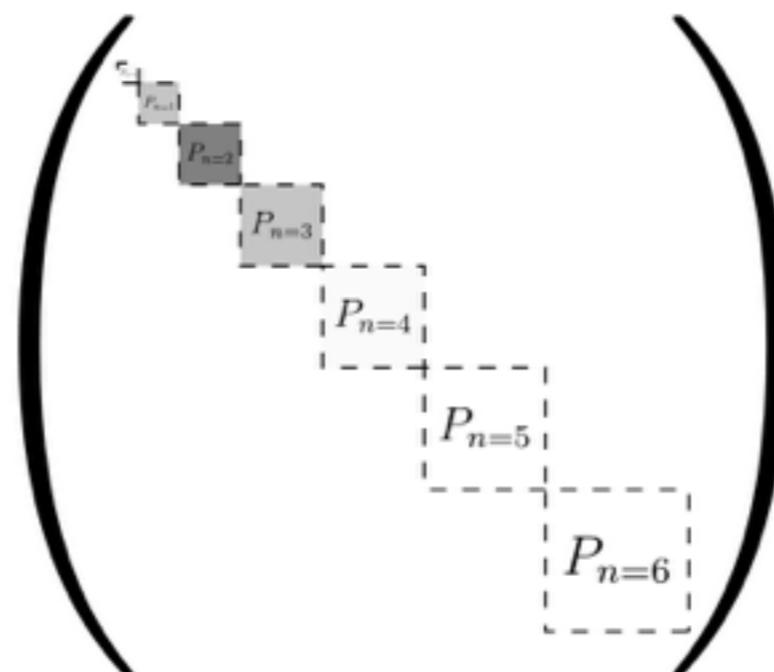


particle tunneling

Configurational entanglement

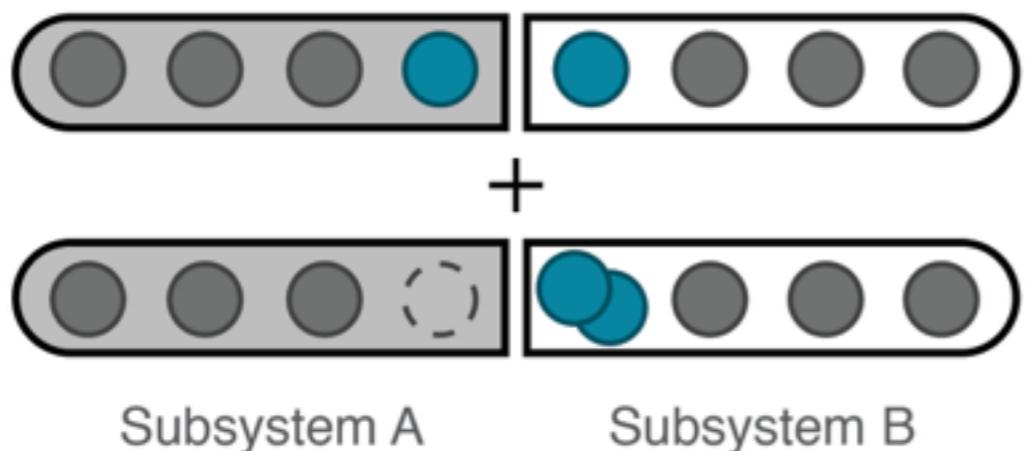


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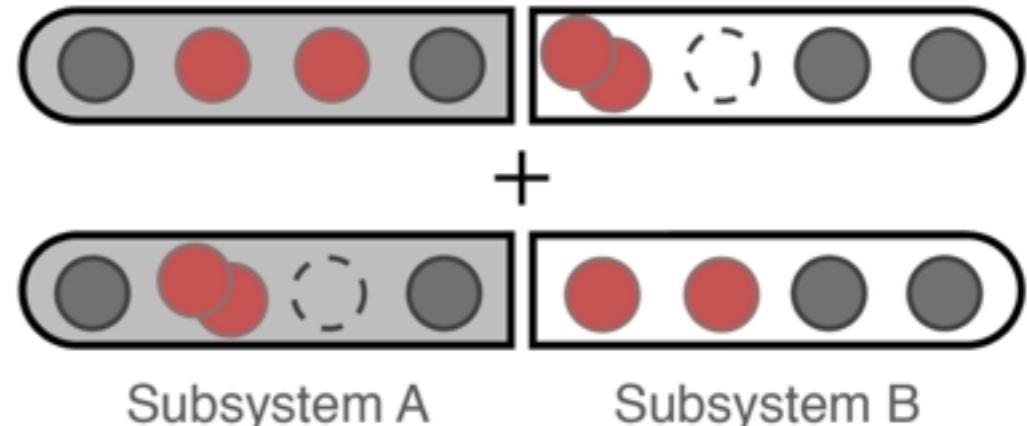
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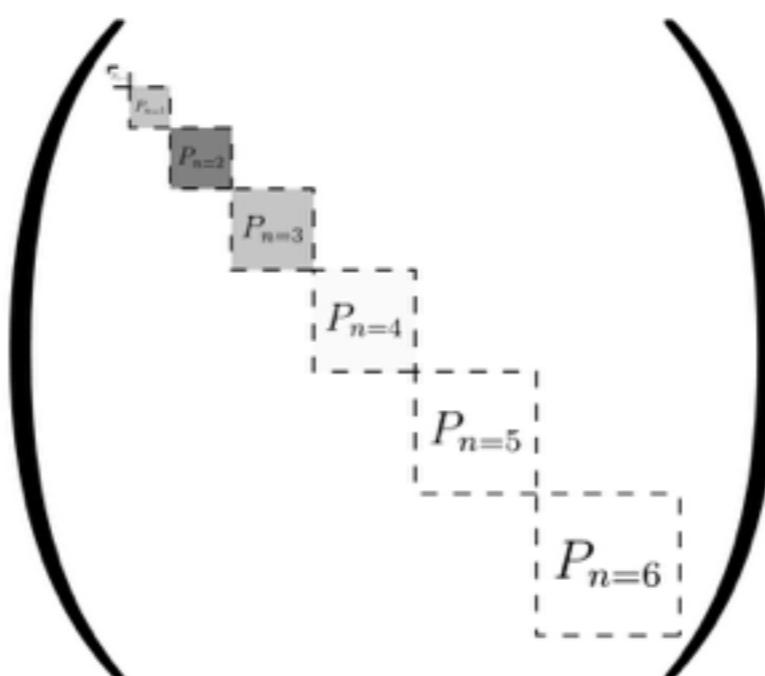
particle tunneling

Configurational entanglement



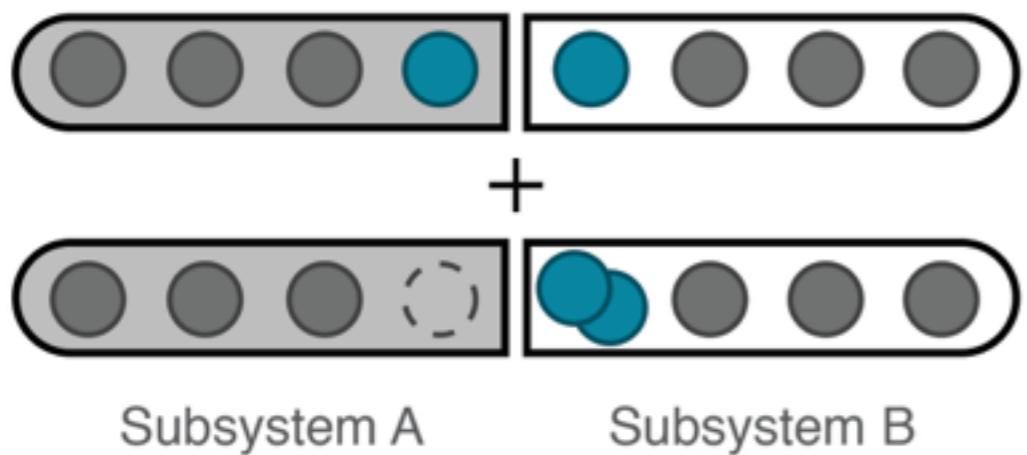
non-local quantum correlations

distribution
between the blocks



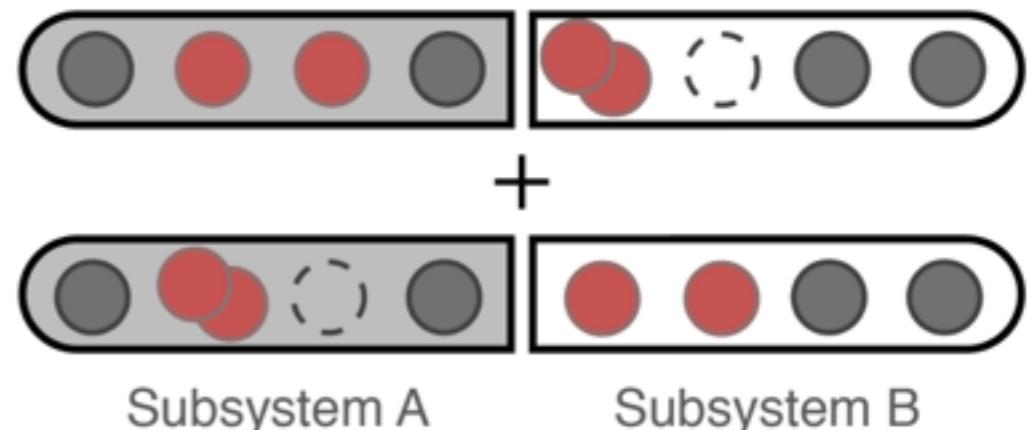
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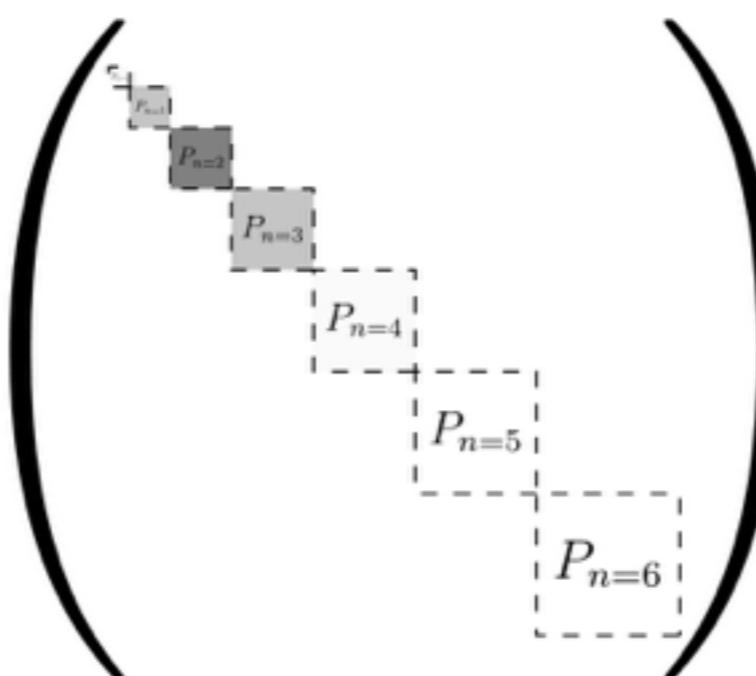
particle tunneling

Configurational entanglement



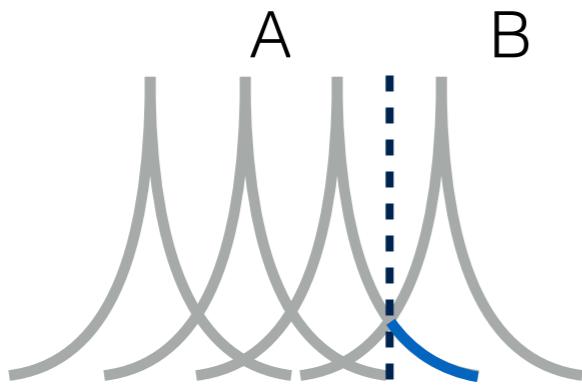
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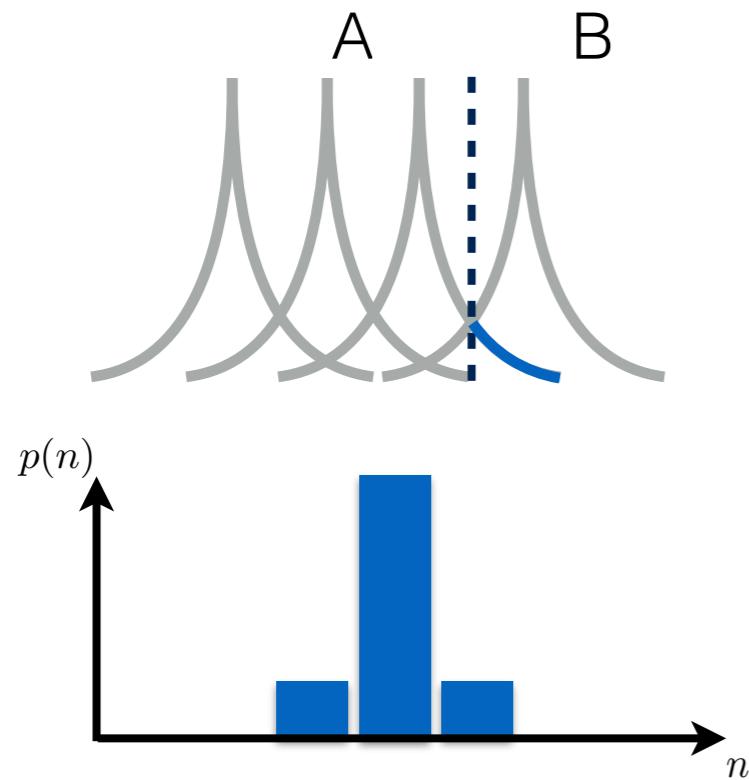


number of populated states
within the blocks

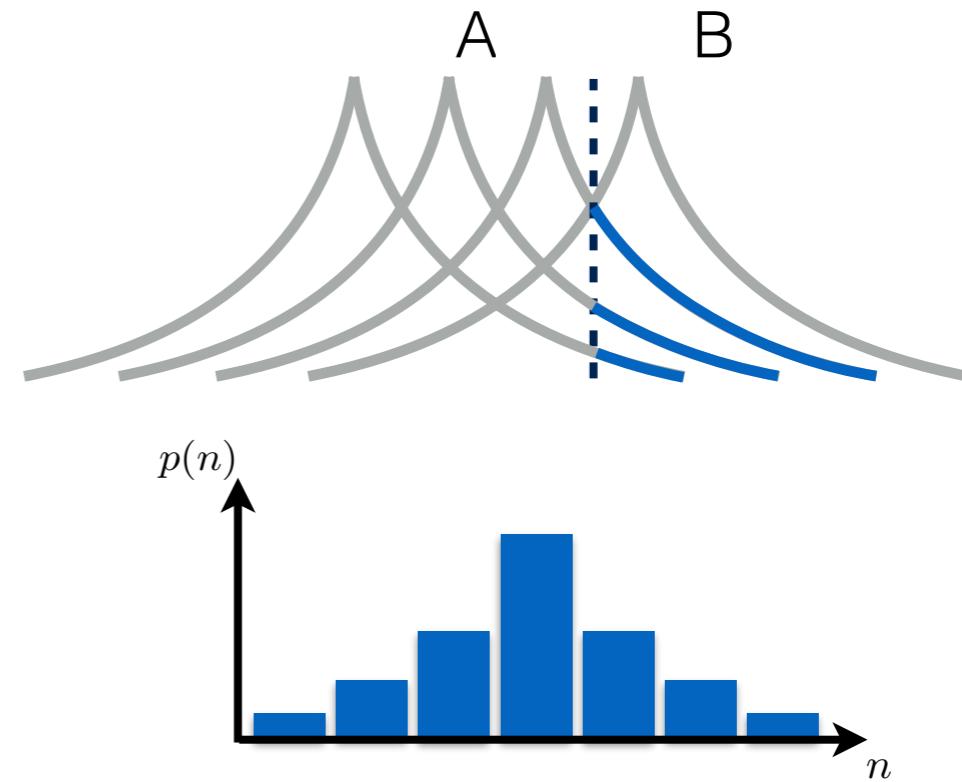
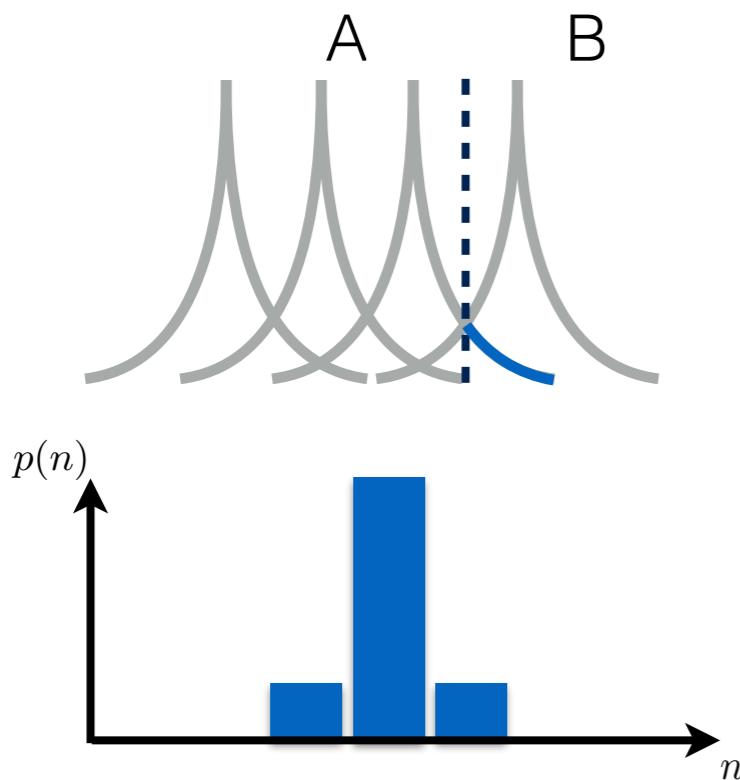
Number entanglement in MBL



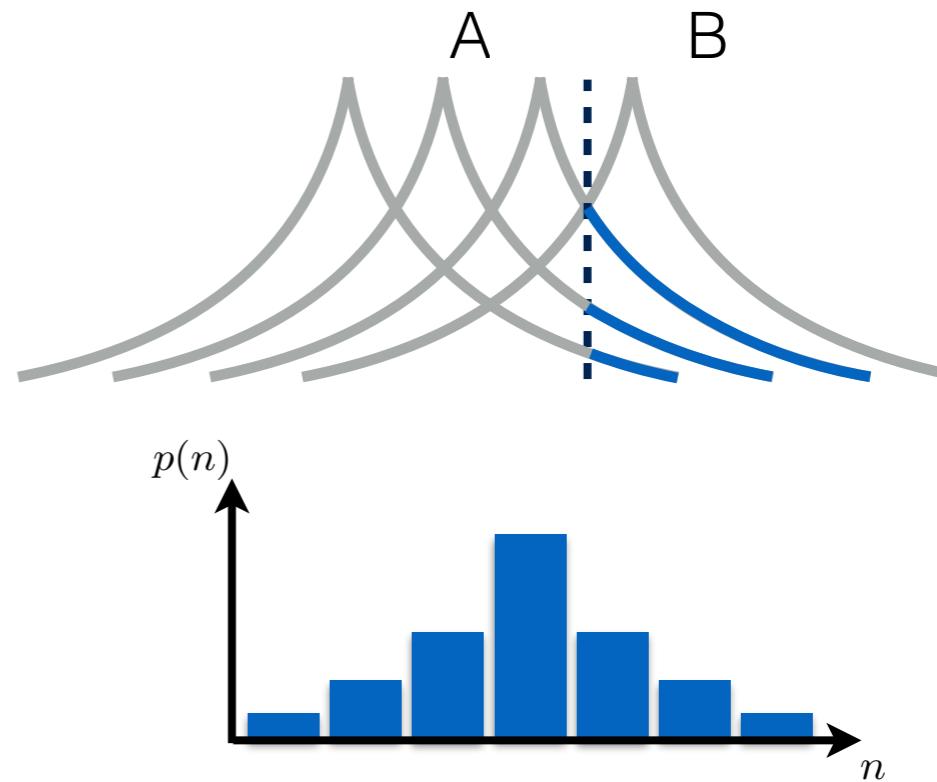
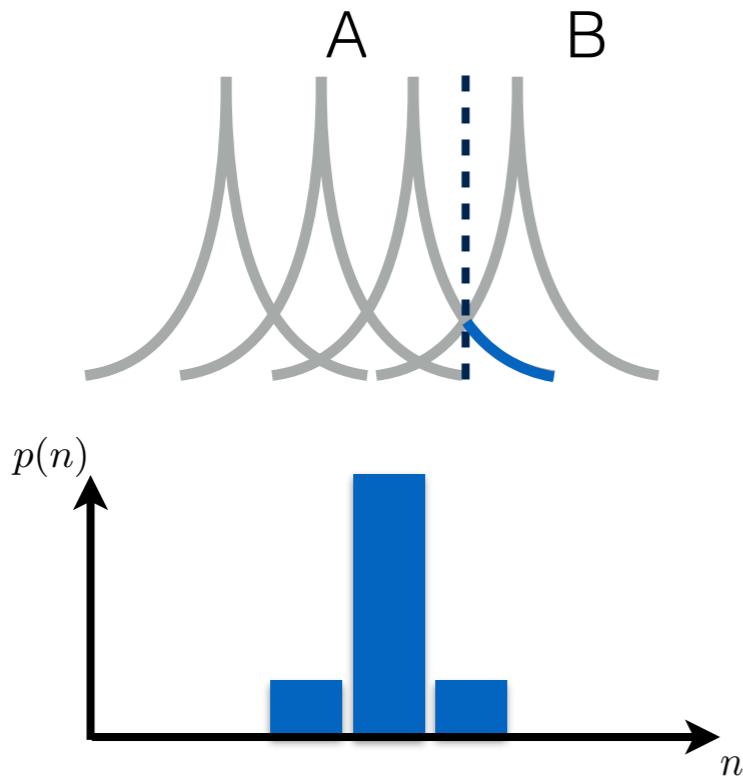
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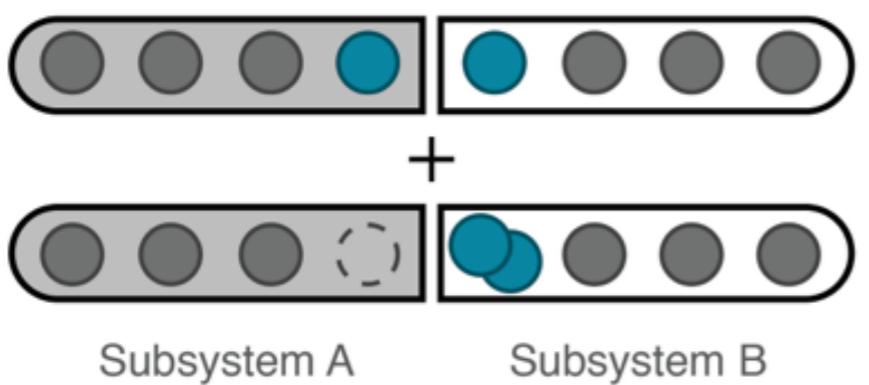


Number entanglement in MBL

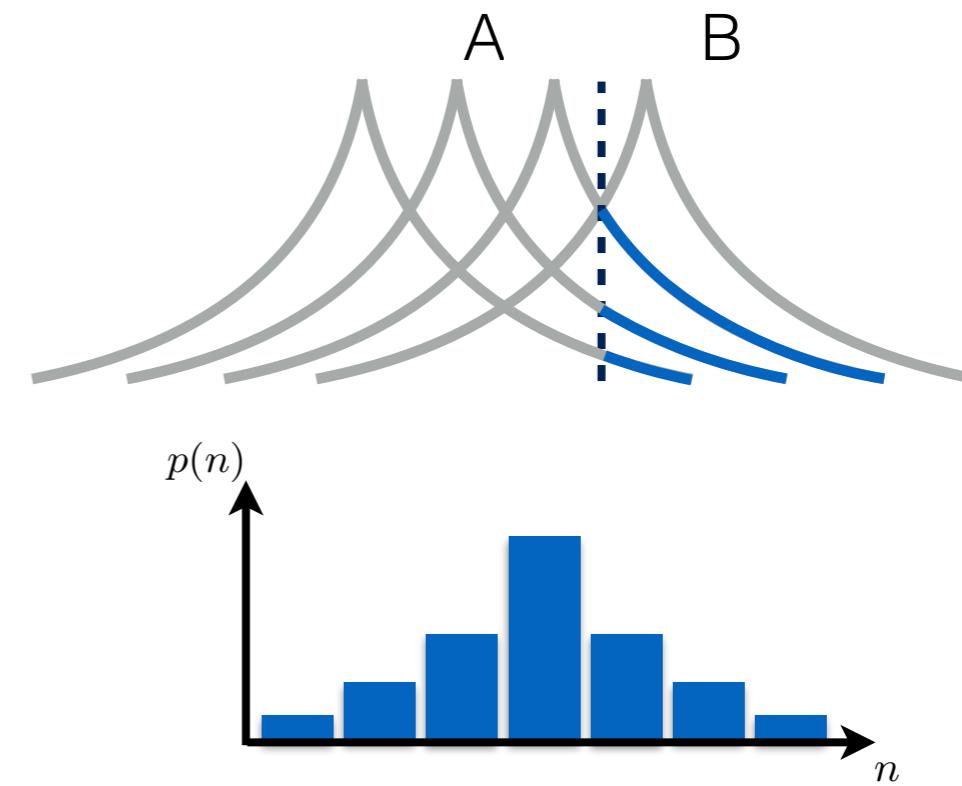
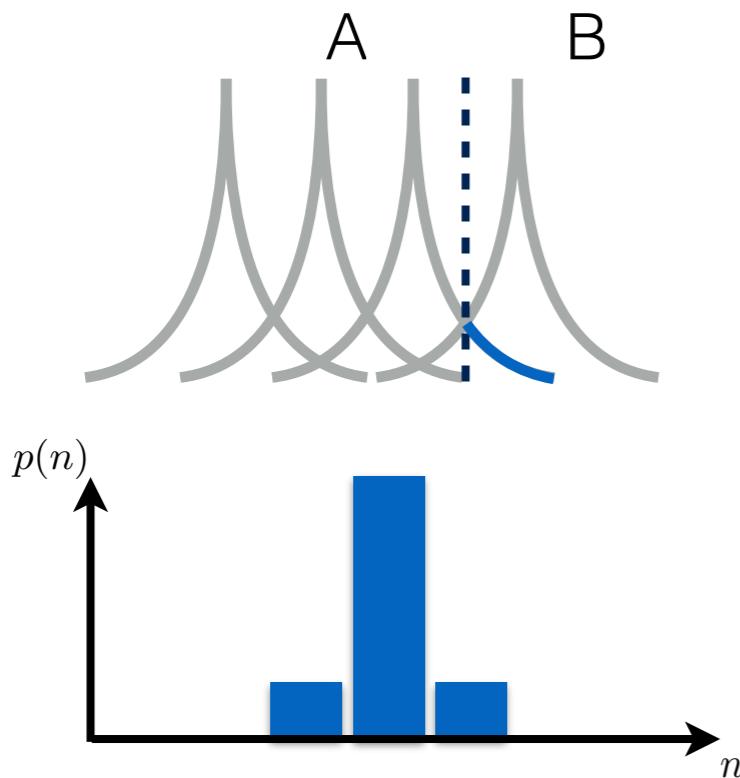


$$S_n = - \sum_n p_n \log (p_n)$$

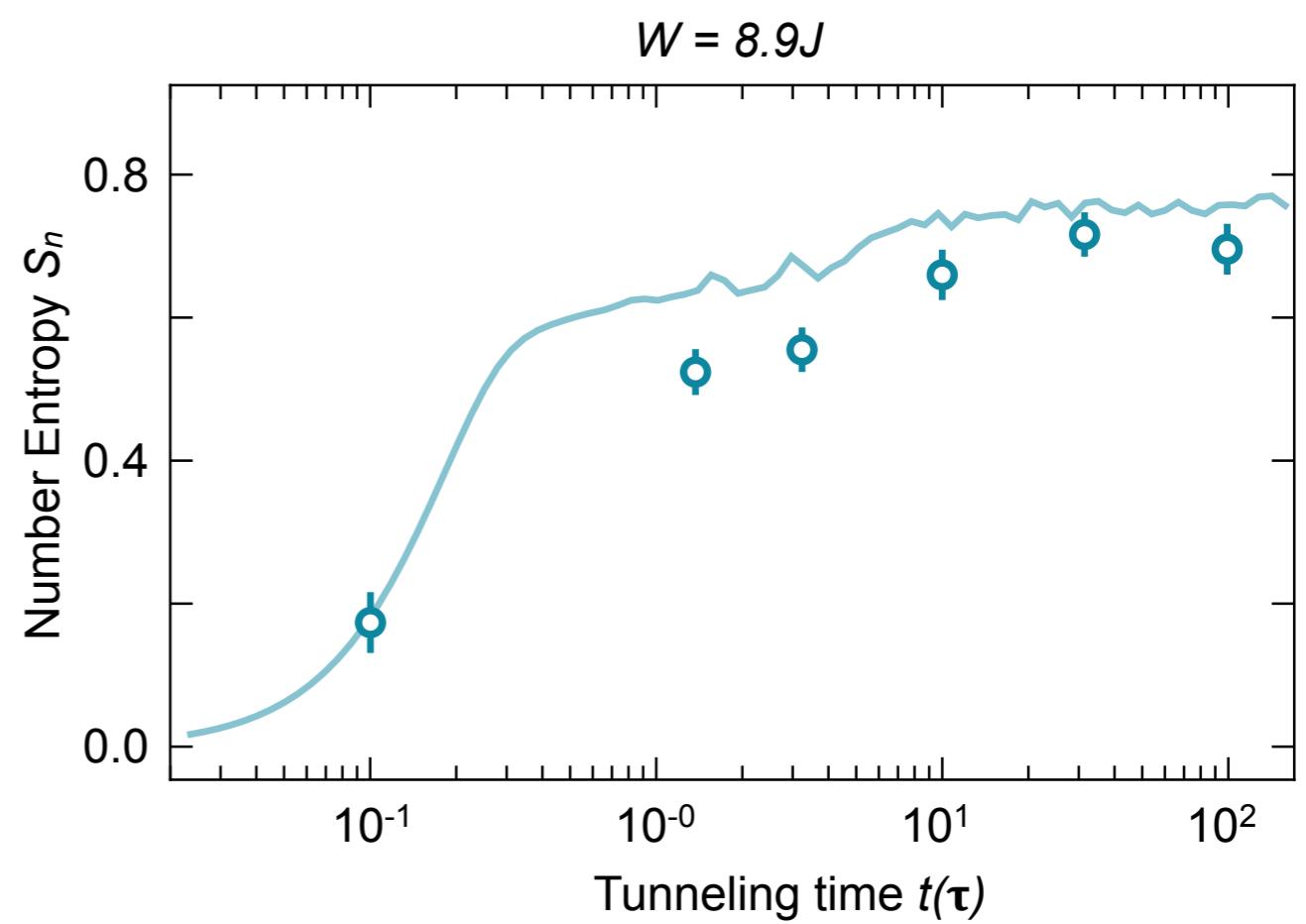
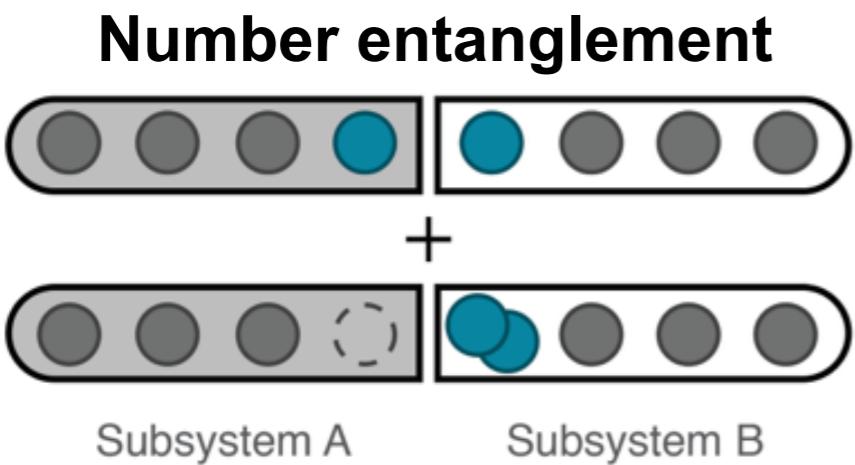
Number entanglement



Number entanglement in MBL

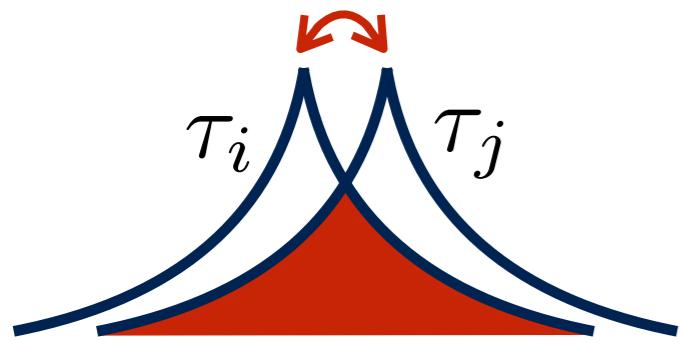


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Configurational entanglement in MBL

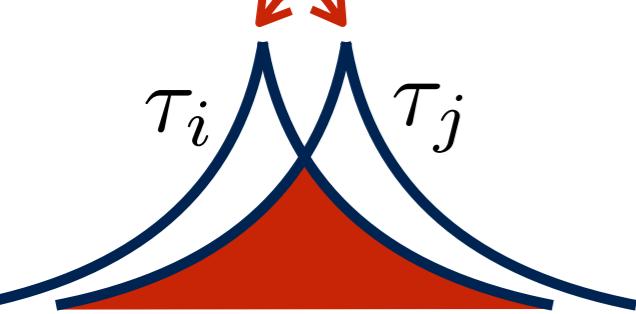
U_{eff}



$$\hat{\mathcal{H}} = \sum_{i,j} U_{eff}^{ij} \tau_i \tau_j$$

Configurational entanglement in MBL

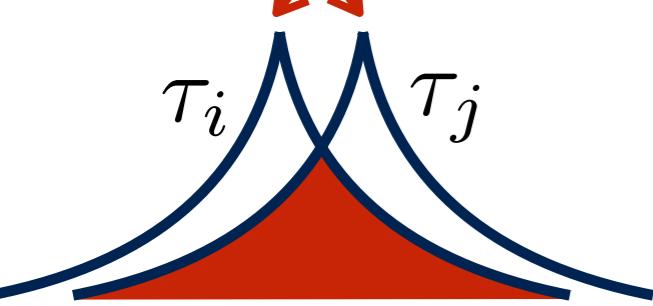
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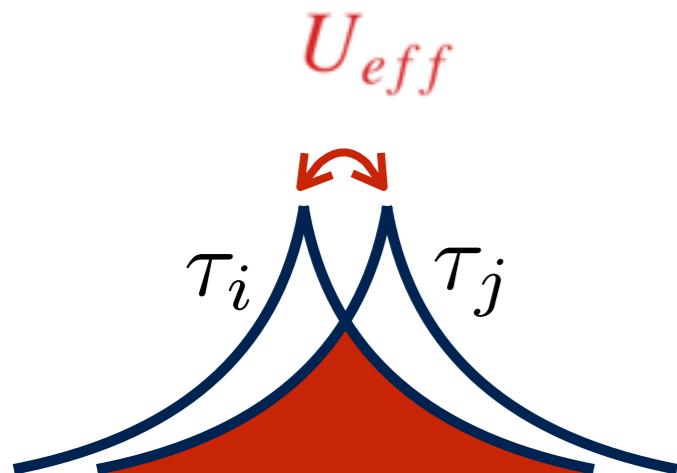


unentangled product state

$$\tau_i \otimes \tau_j = (|0\rangle + |1\rangle)_i \otimes (|0\rangle + |1\rangle)_j = |00\rangle + |01\rangle + |10\rangle + |11\rangle$$

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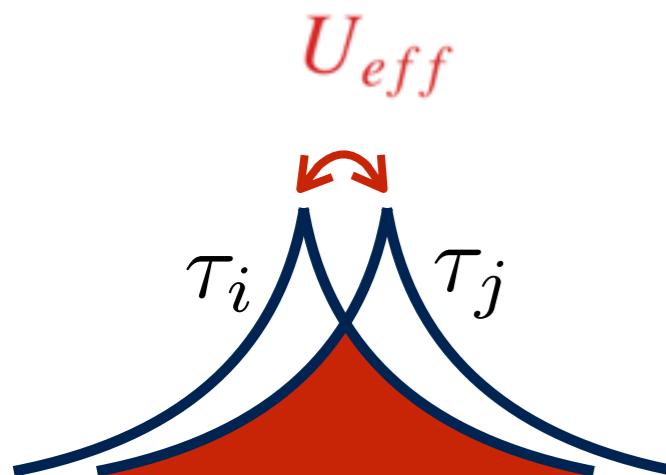
Configurational entanglement in MBL



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Configurational entanglement in MBL



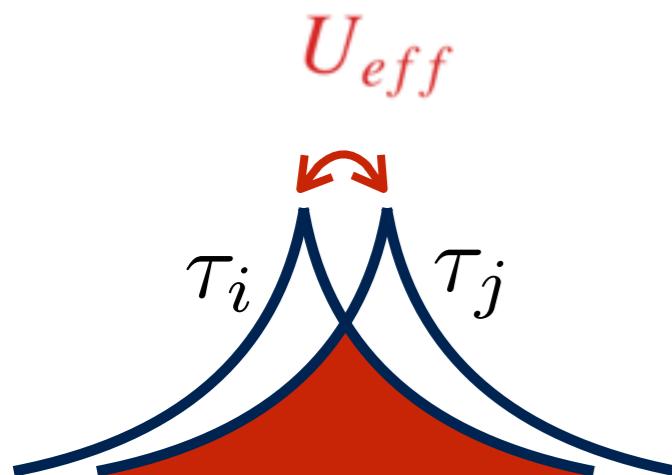
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Configurational entanglement in MBL



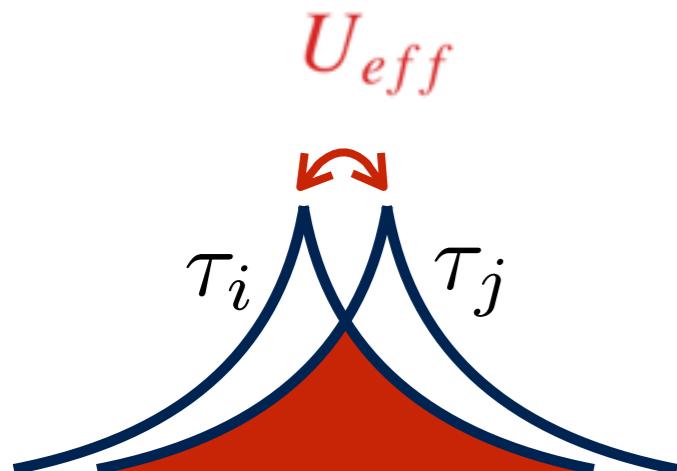
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Configurational entanglement in MBL



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unentangled product state

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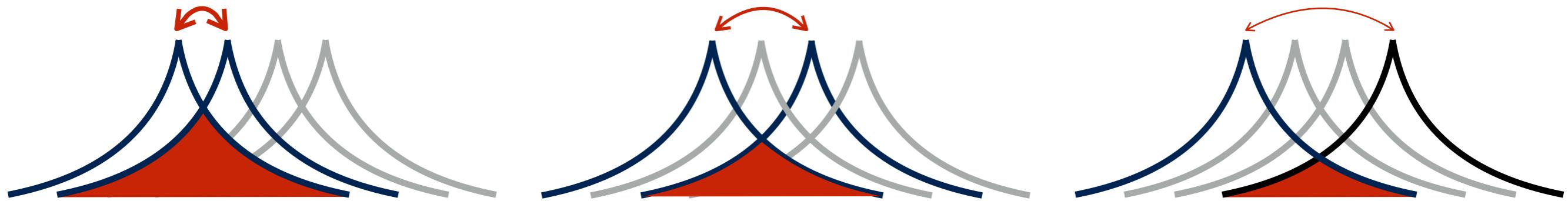
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$$= |0\rangle (|0\rangle + |1\rangle) + |1\rangle (|0\rangle - |1\rangle) \rightarrow |0\rangle |0\rangle + |1\rangle |1\rangle$$

entangled state

Configurational entanglement in MBL

$$U_{eff} \sim U_o e^{-|x_i - x_j|/\xi}$$

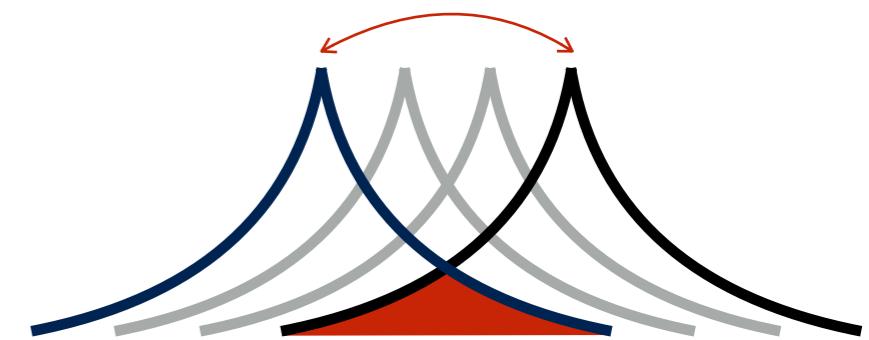
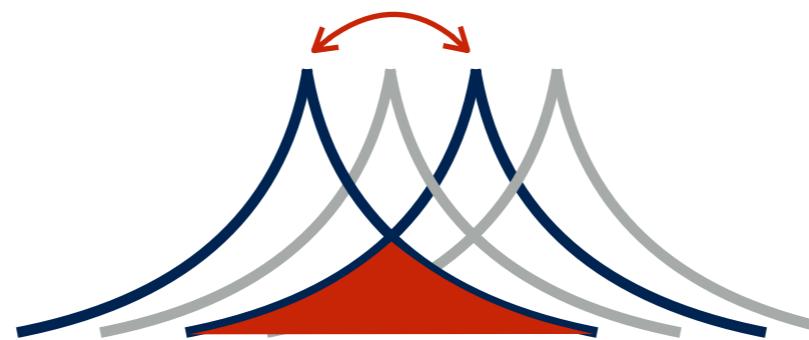
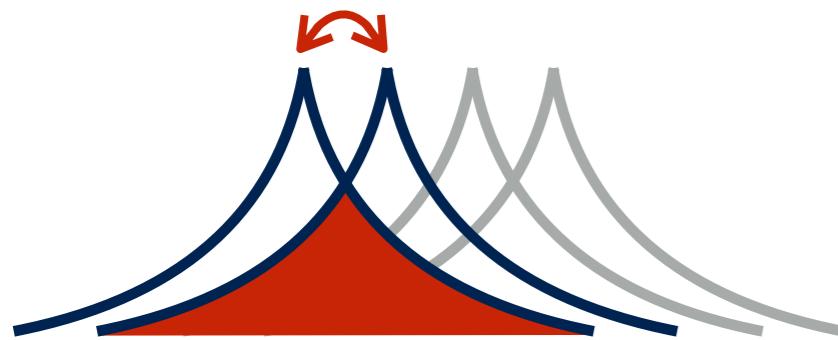


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Configurational entanglement in MBL

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phase effects are cumulative



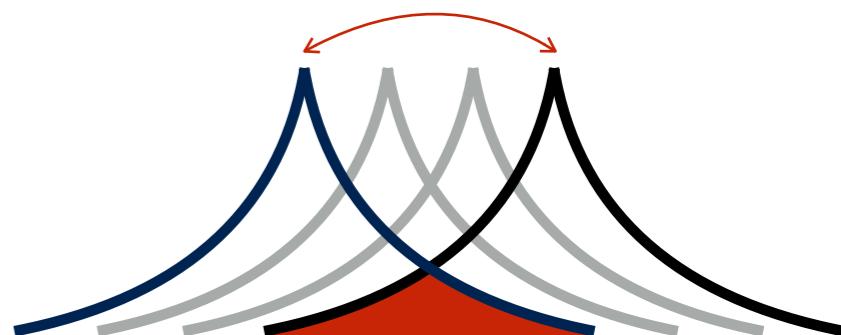
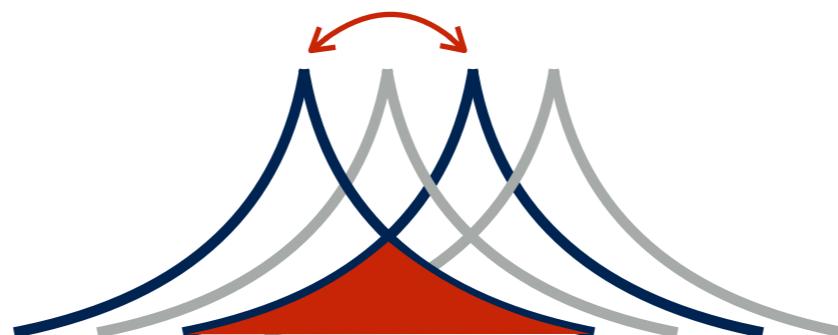
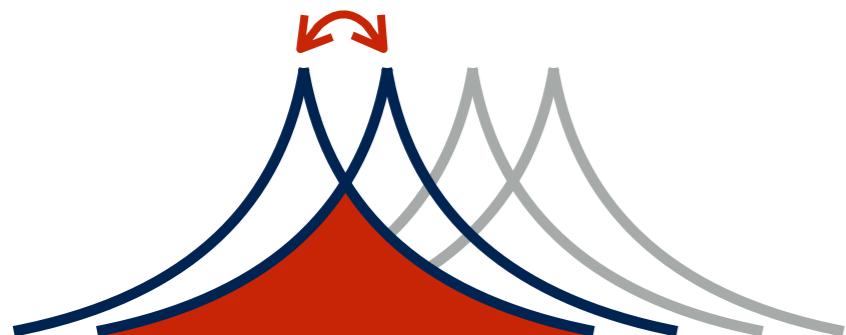
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$$|00\rangle + |01\rangle + |10\rangle + |11\rangle \rightarrow |0\rangle |0\rangle + |1\rangle |1\rangle$$

Configurational entanglement in MBL

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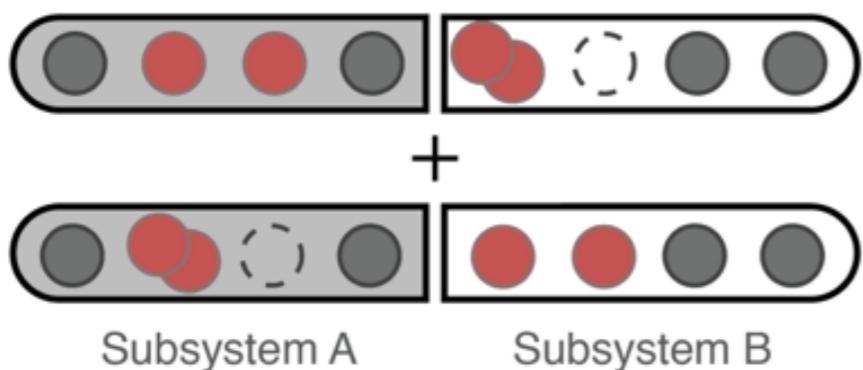


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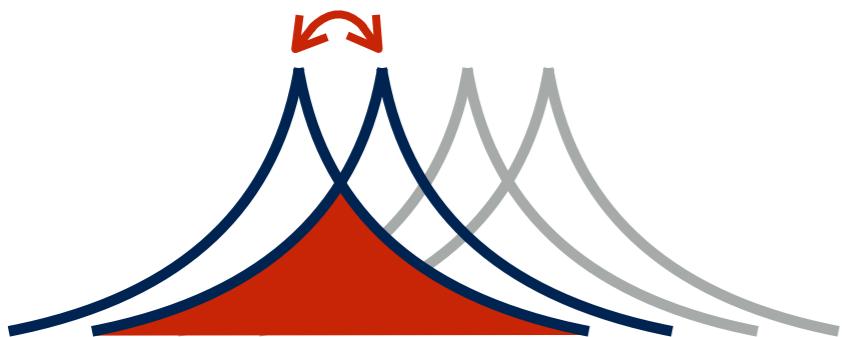
$$C = \sum_{\{A_n\}, \{B_n\}} |p(A_n \otimes B_n) - p(A_n)p(B_n)|$$

Configurational entanglement

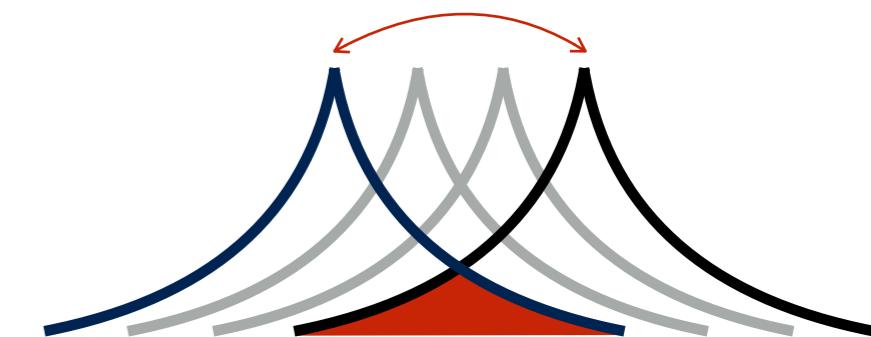
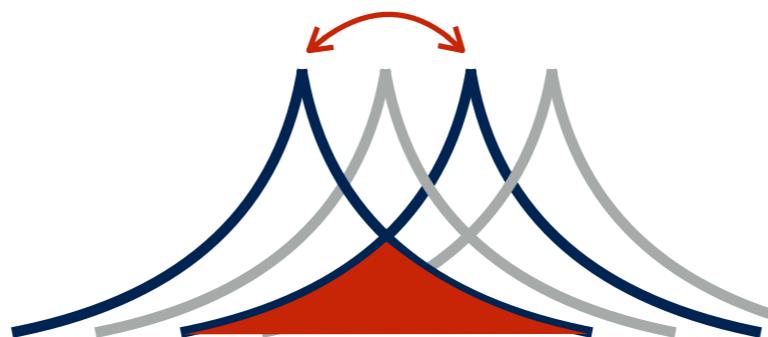


Configurational entanglement in MBL

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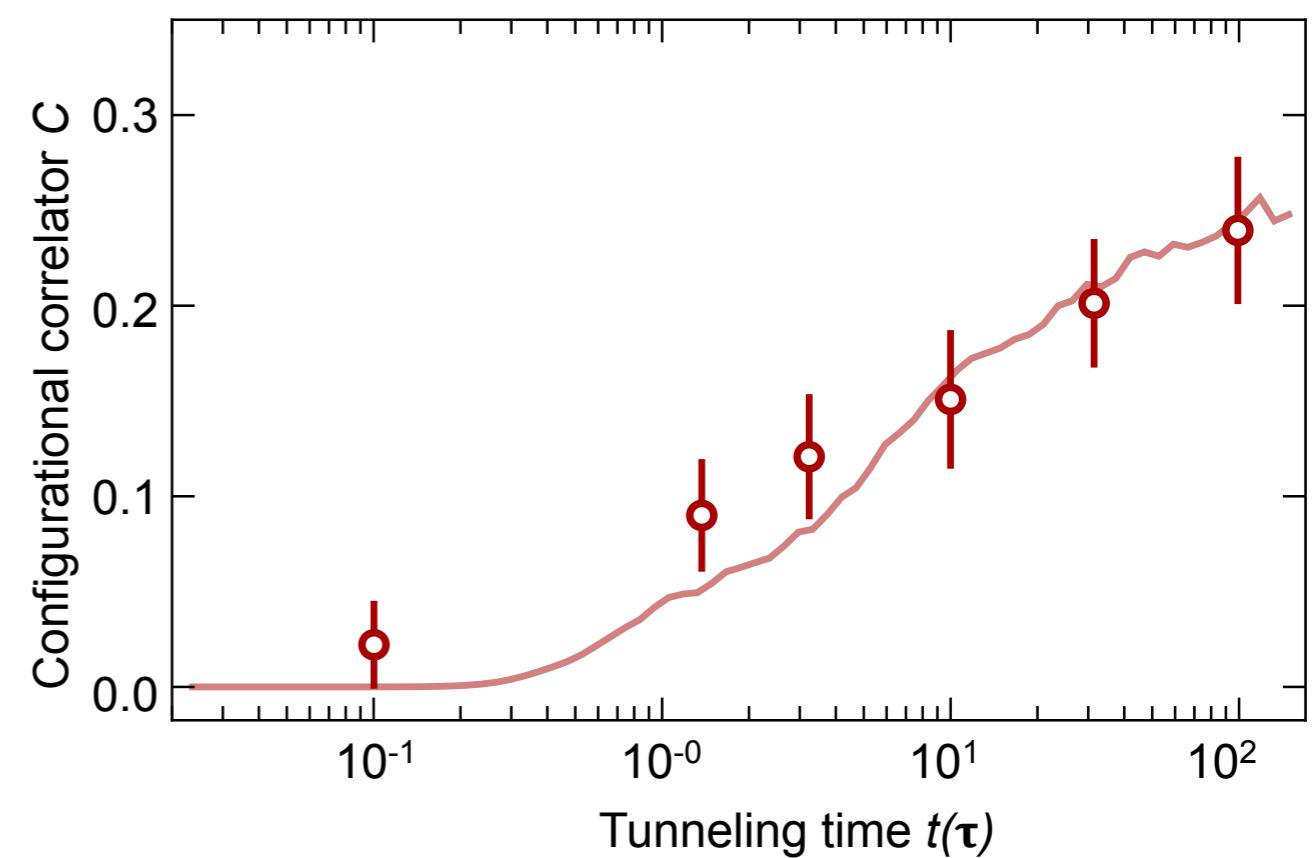
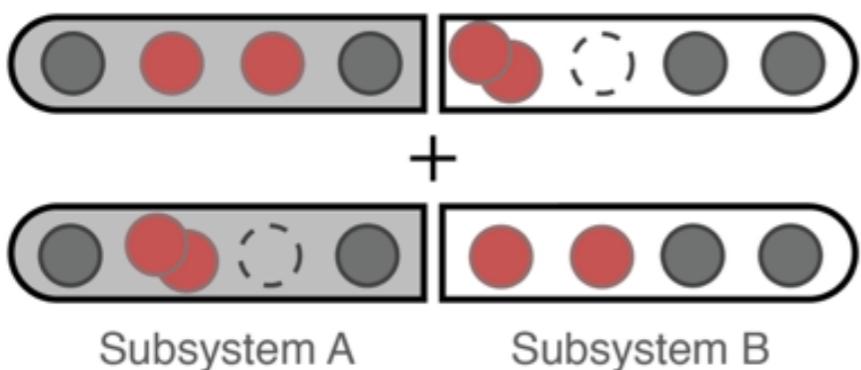


$$\hat{\mathcal{H}} = \sum_{i,j} U_{eff}^{ij} \tau_i \tau_j$$

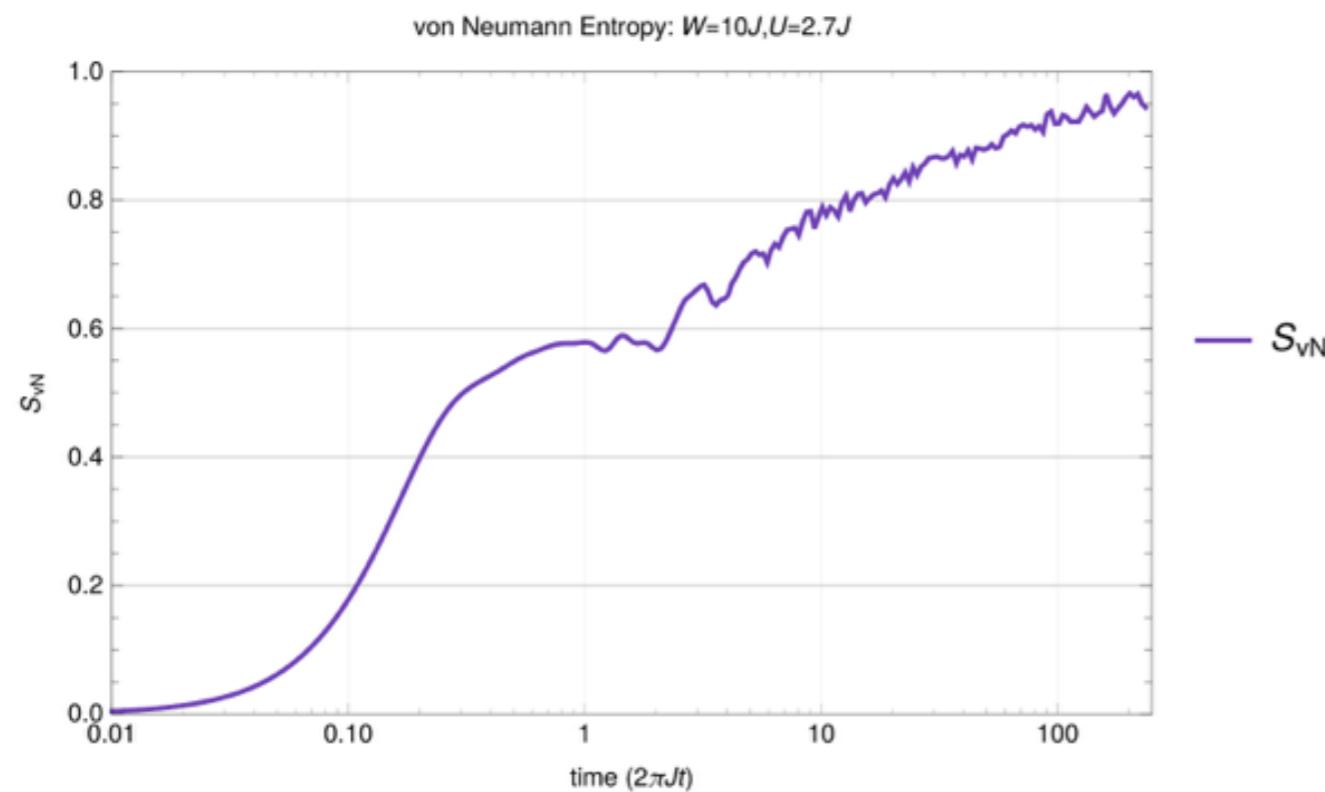
$$|00\rangle + |01\rangle + |10\rangle + |11\rangle \rightarrow |0\rangle |0\rangle + |1\rangle |1\rangle$$

$$C = \sum_{\{A_n\}, \{B_n\}} |p(A_n \otimes B_n) - p(A_n)p(B_n)|$$

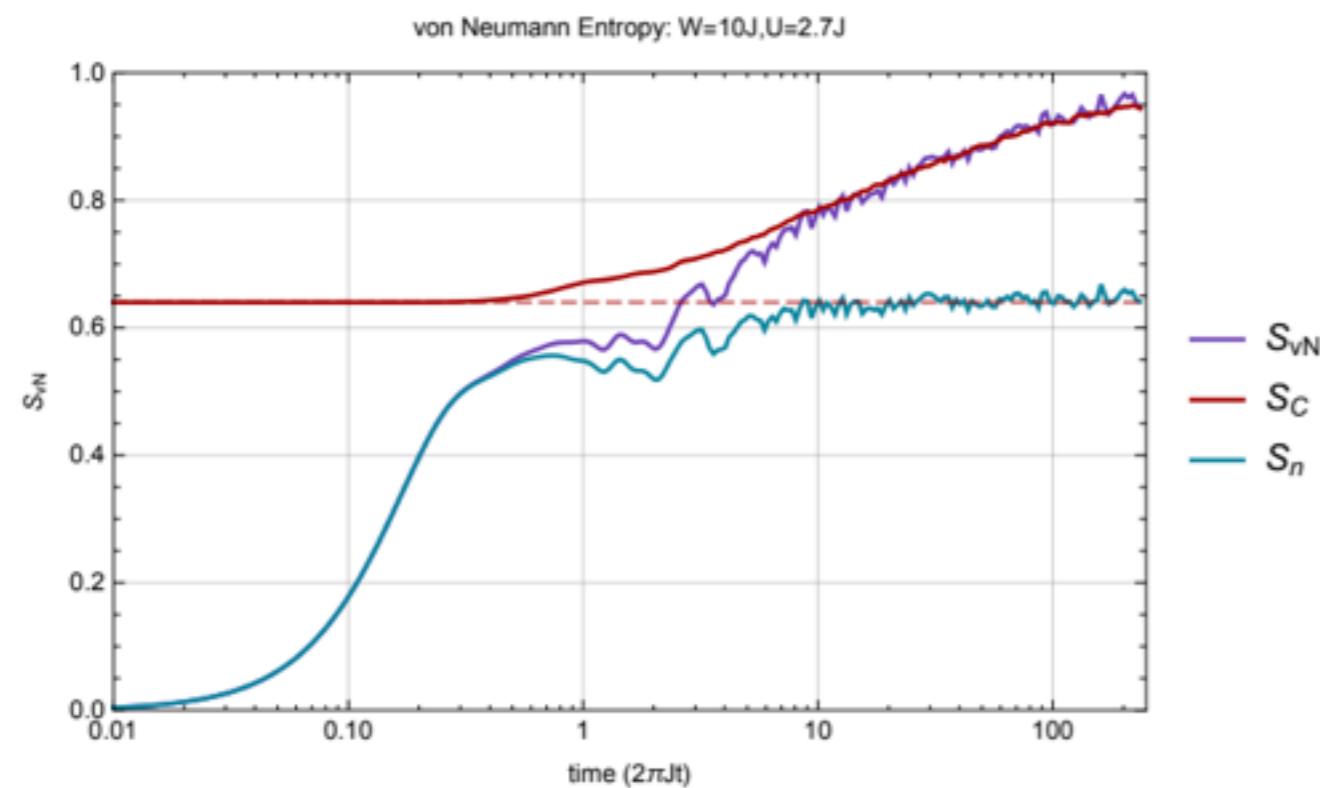
Configurational entanglement



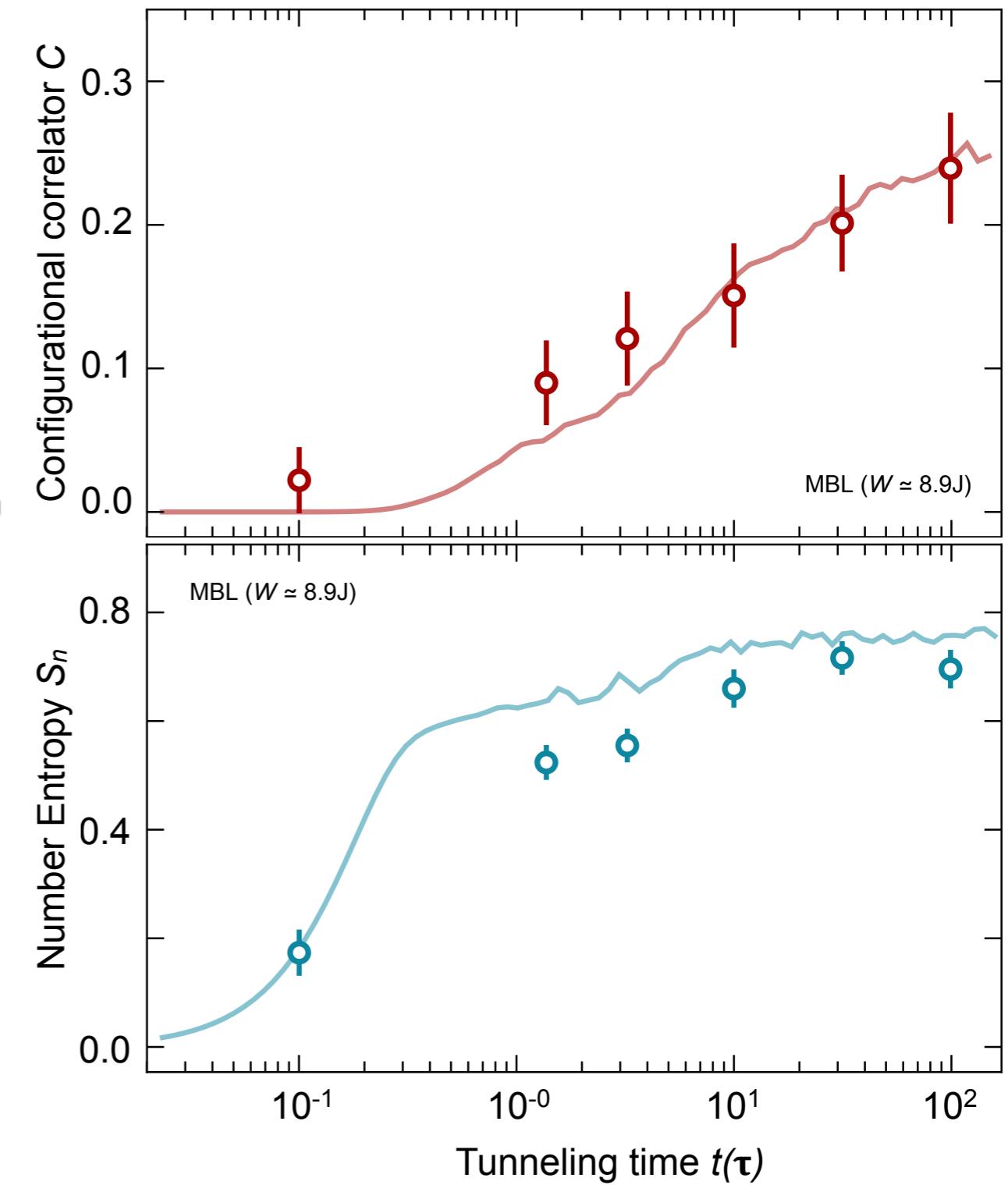
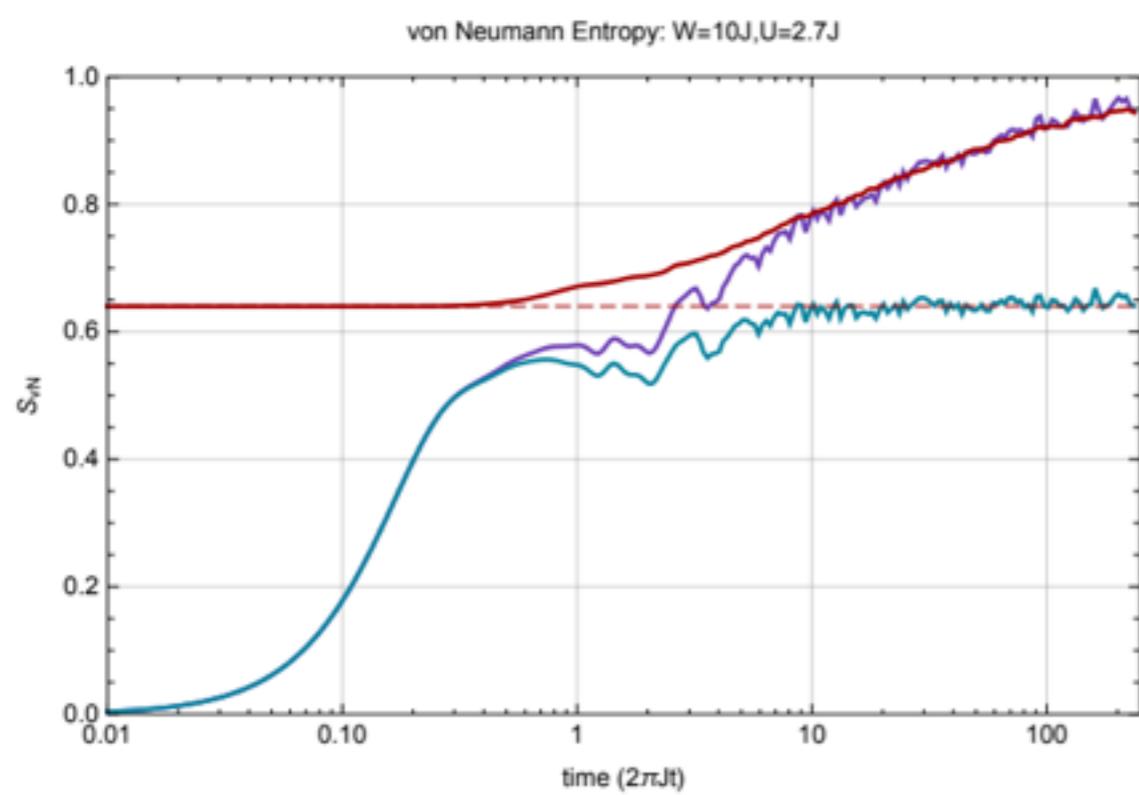
Logarithmic grows of entanglement



Logarithmic grows of entanglement



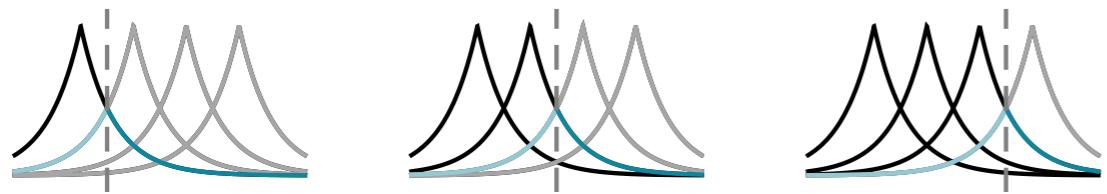
Logarithmic grows of entanglement



system is limited by finite evolution time, rather then size

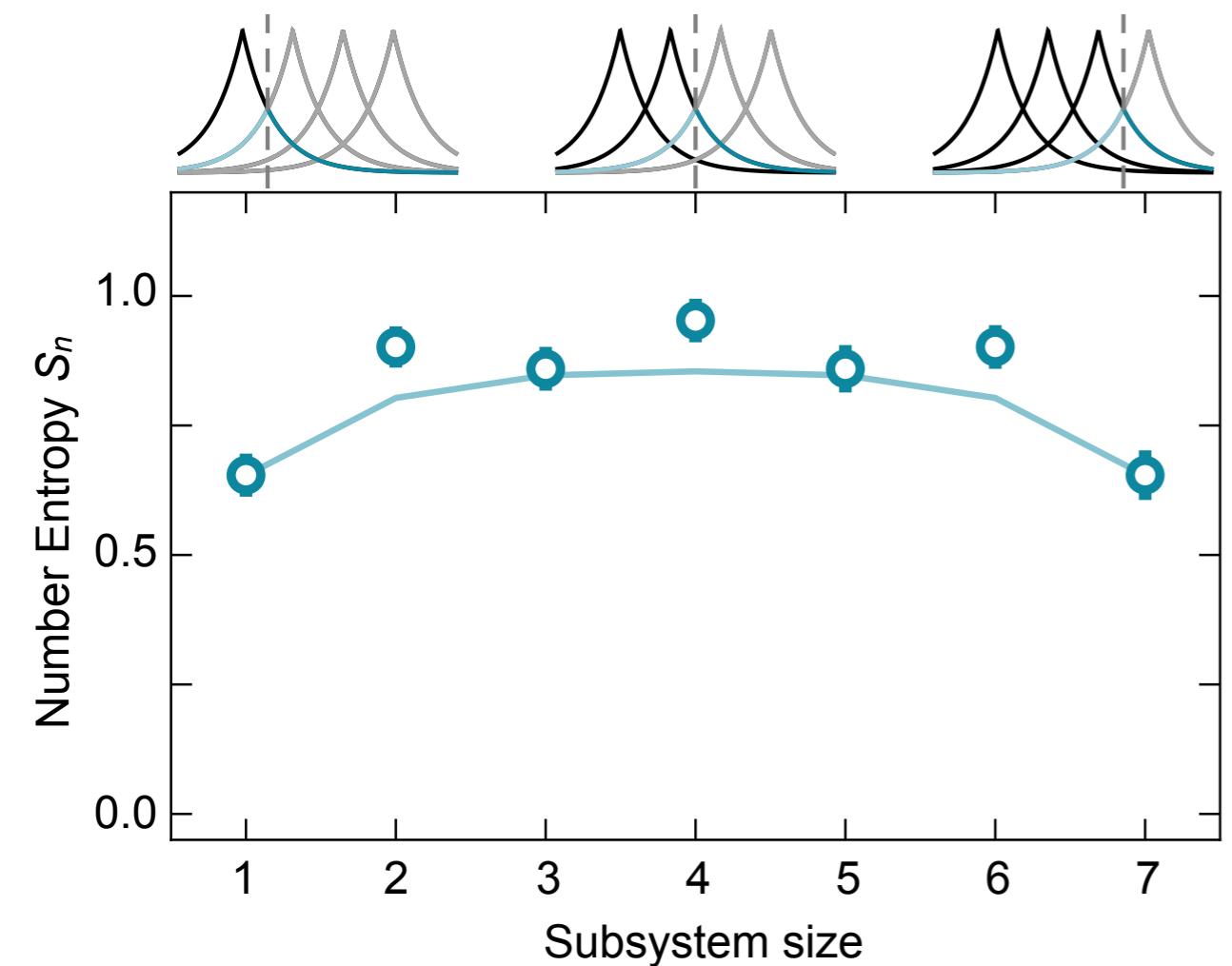
Entanglement scaling

Area law



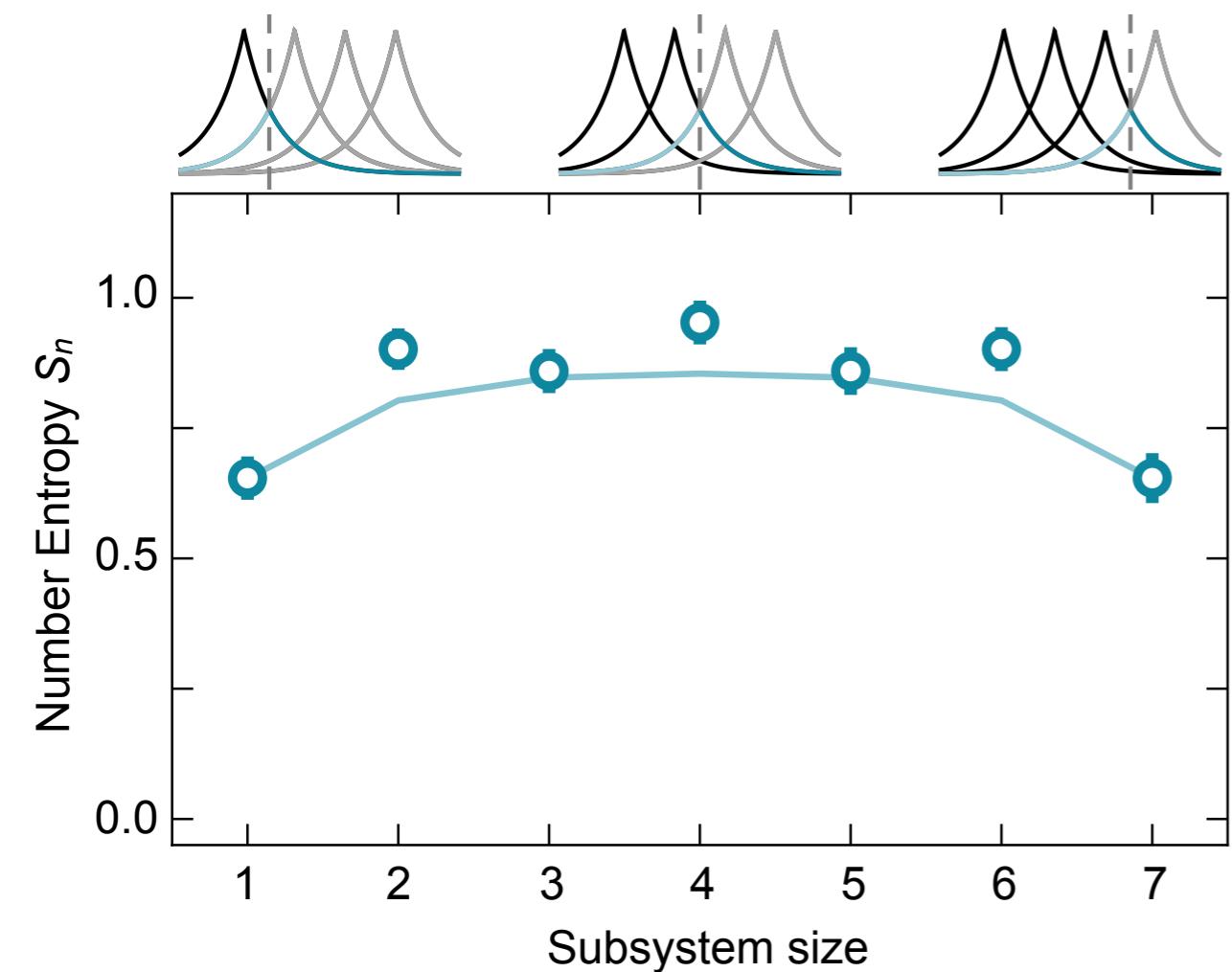
Entanglement scaling

Area law

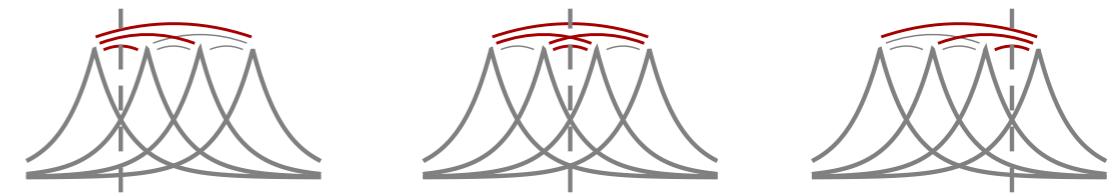


Entanglement scaling

Area law

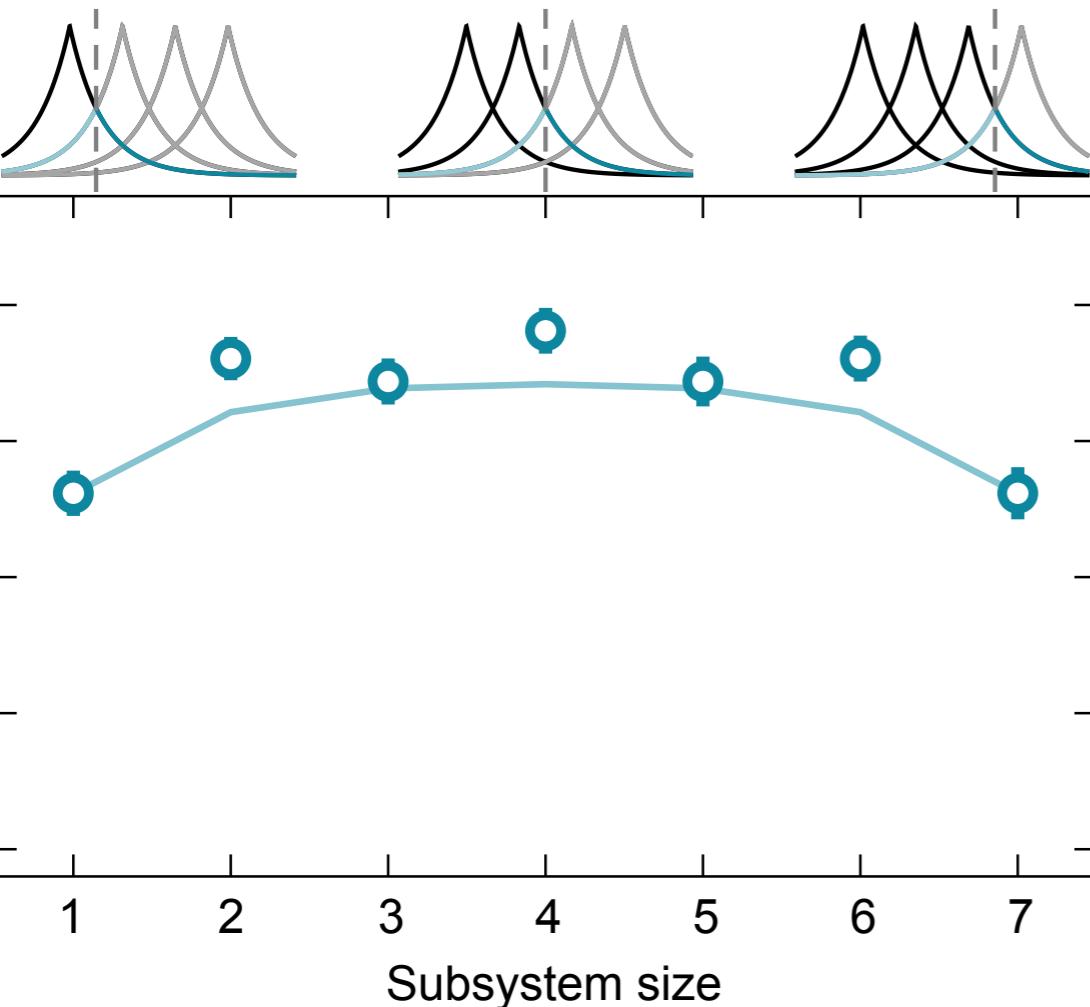


Volume law

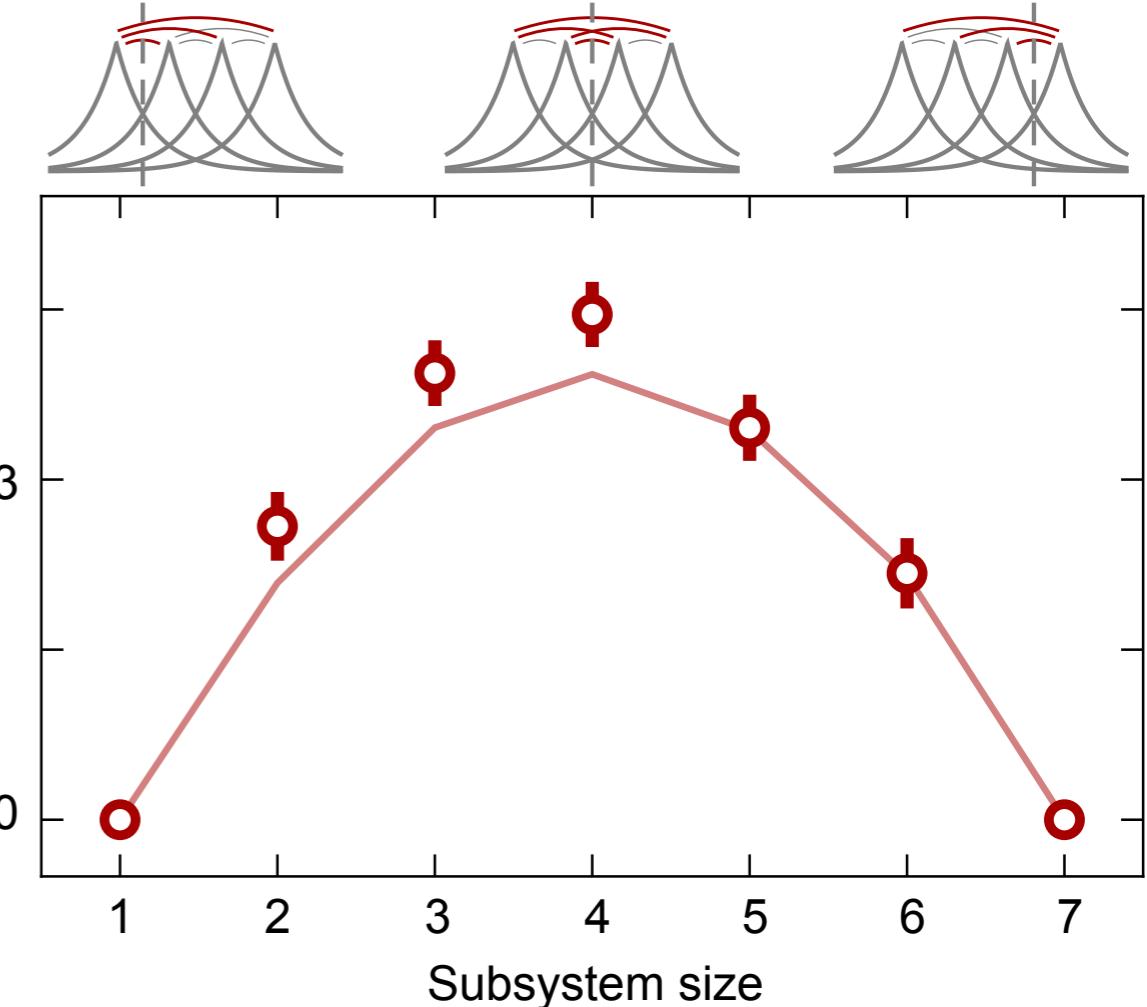


Entanglement scaling

Area law

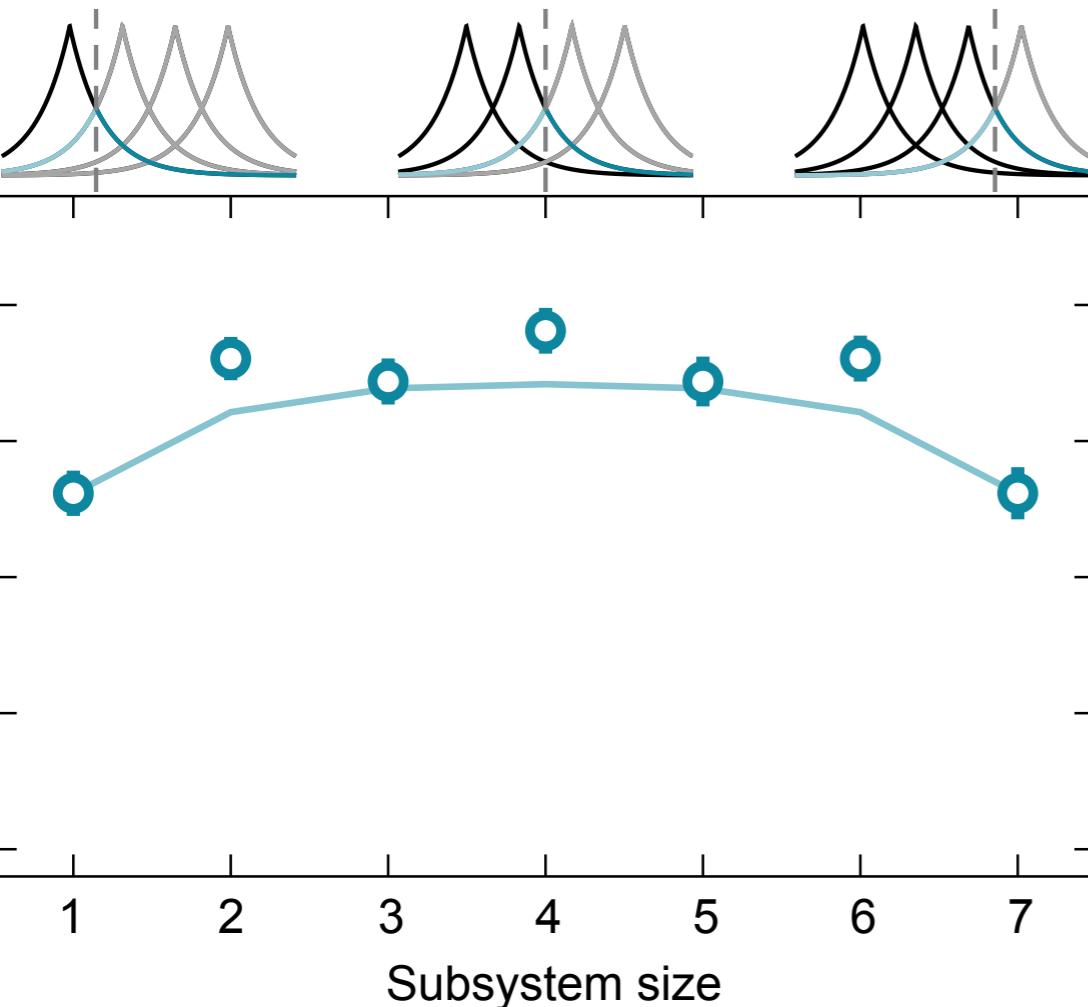


Volume law

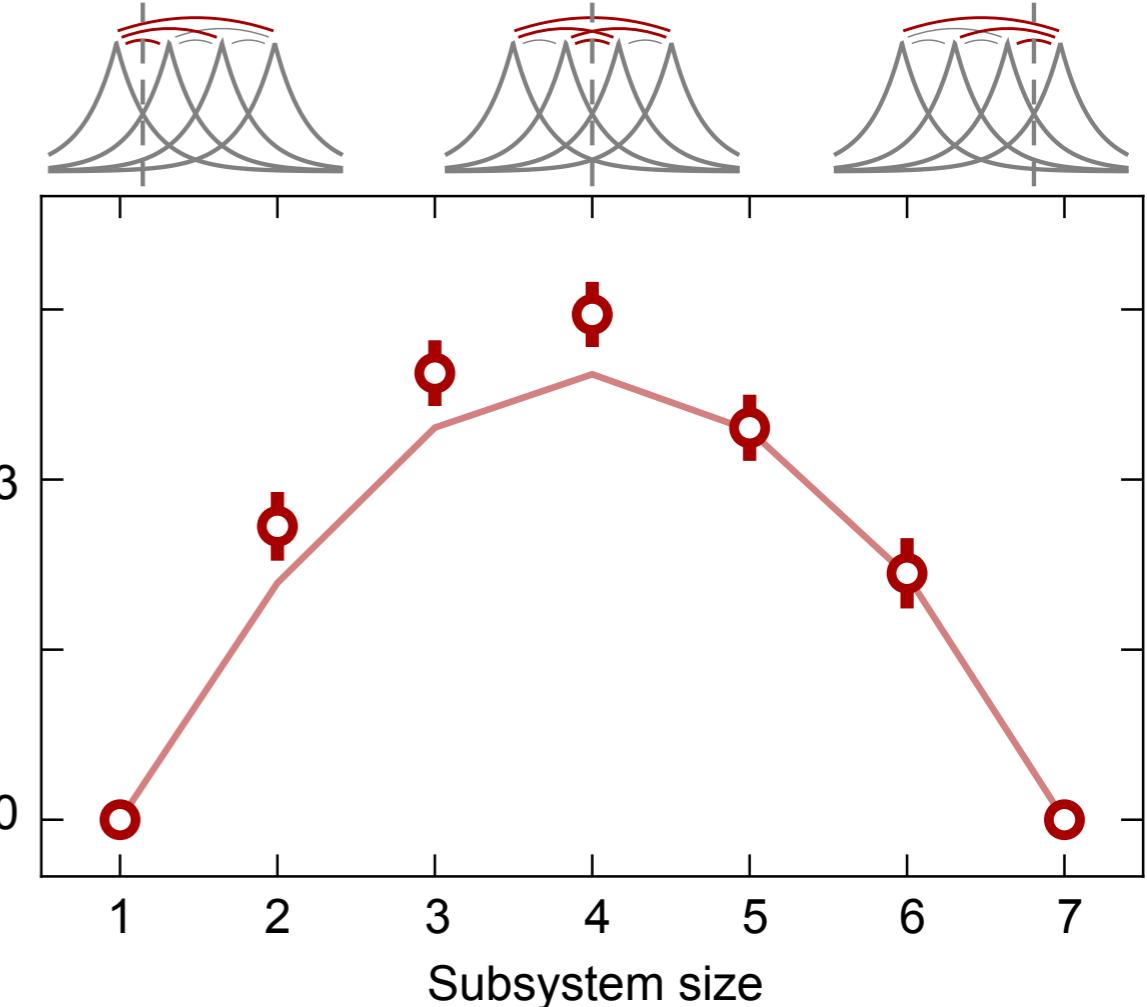


Entanglement scaling

Area law



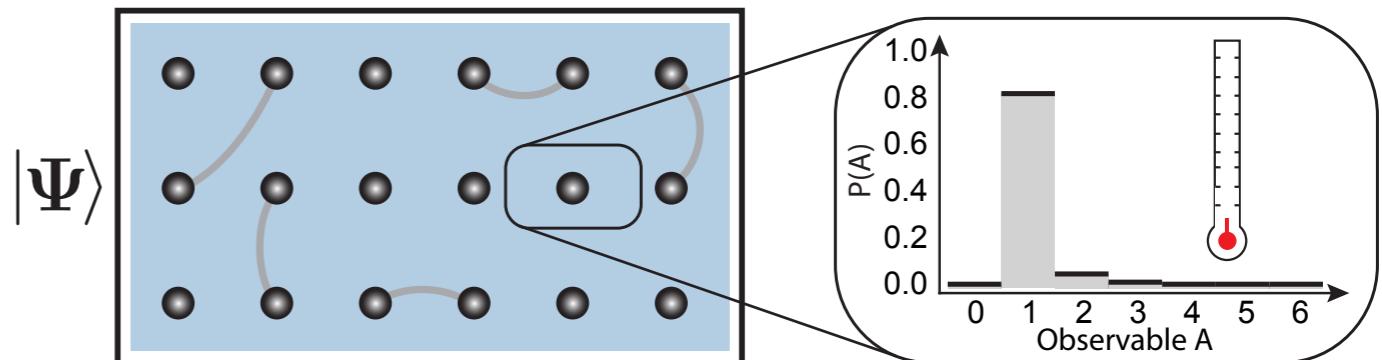
Volume law



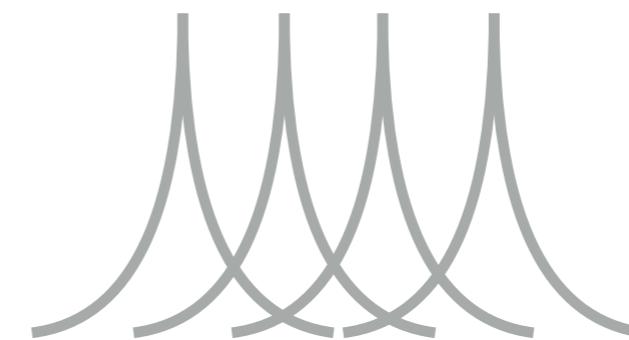
long range correlated state

Conclusions

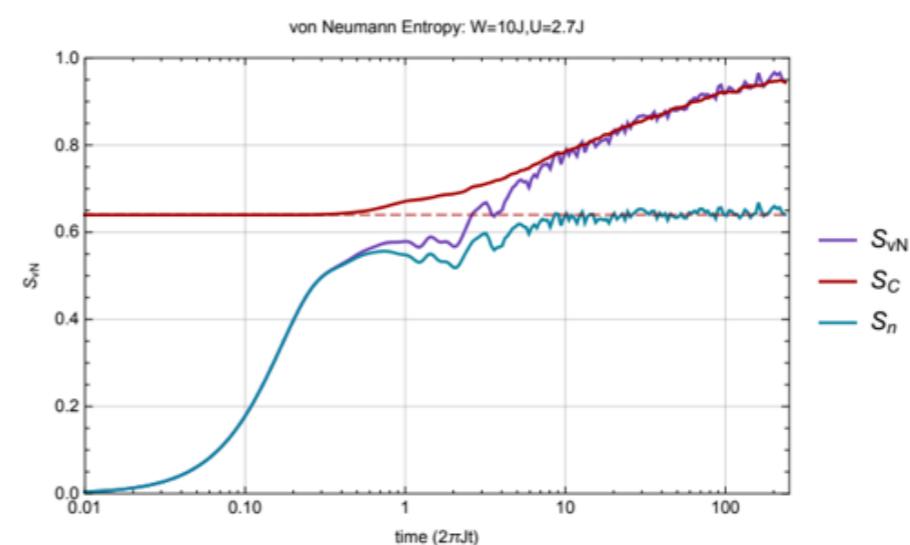
✓ • Breakdown of thermalization



✓ • Spatial localization

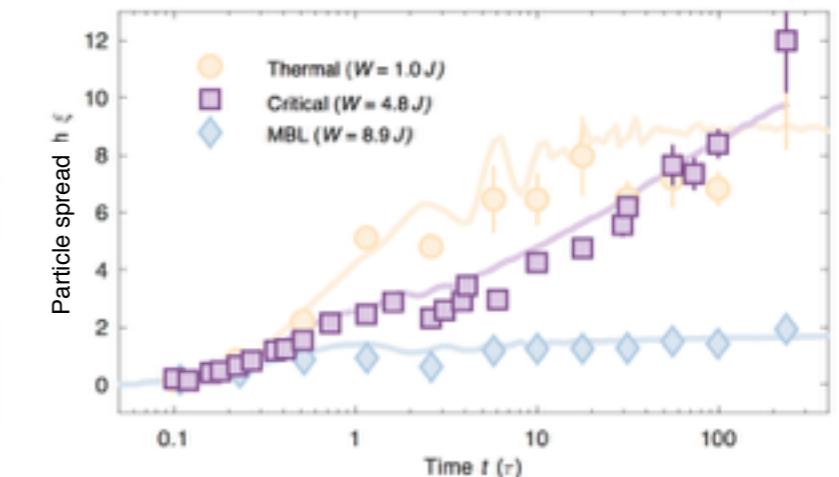
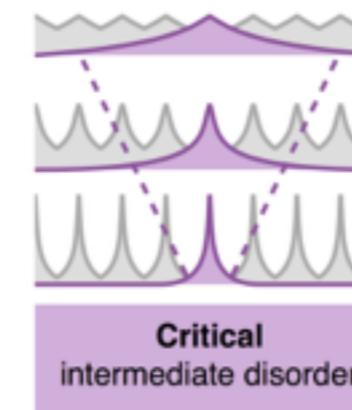
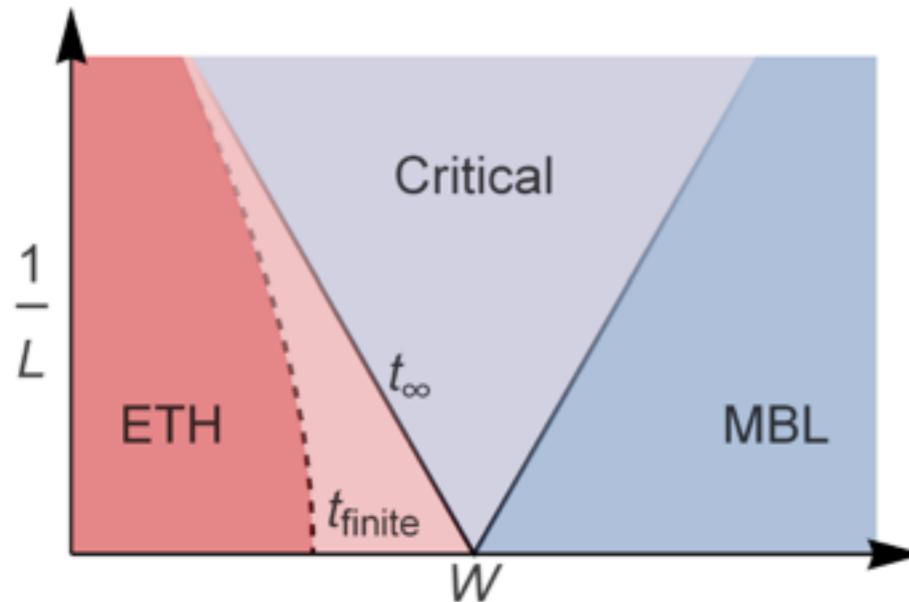


✓ • Entanglement growth



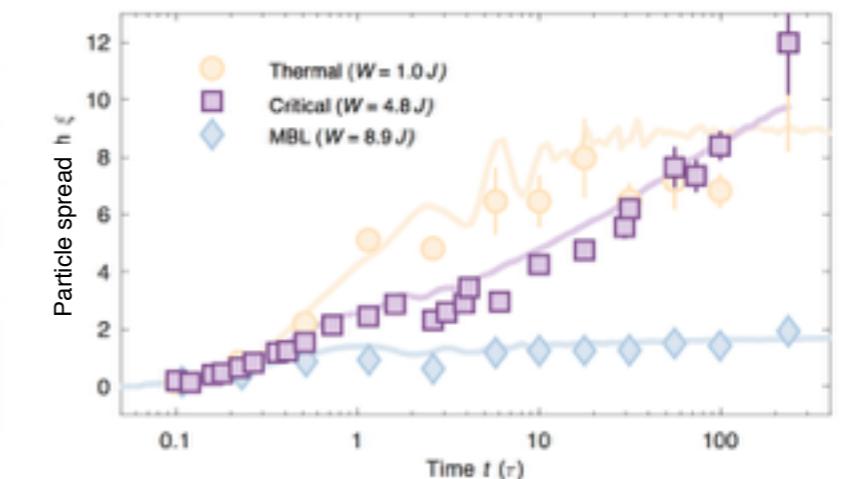
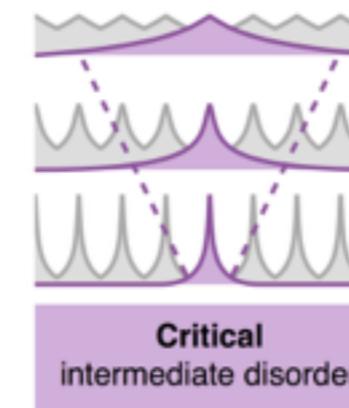
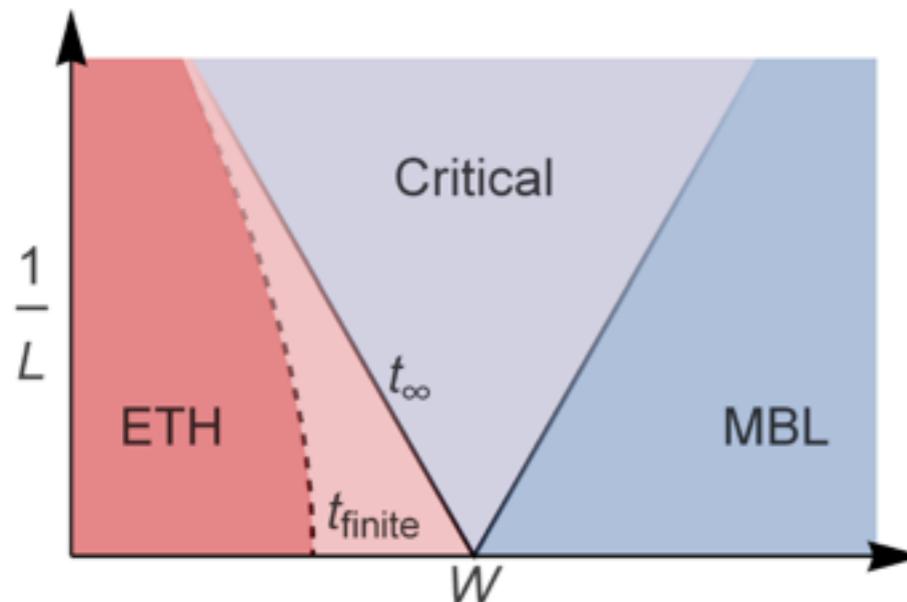
Outlook

- Study critical dynamics at the phase transition

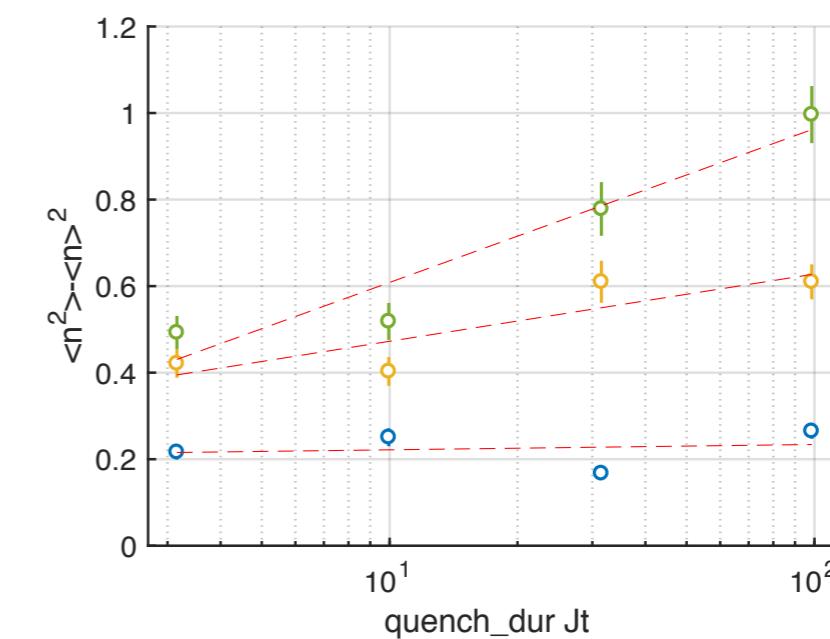
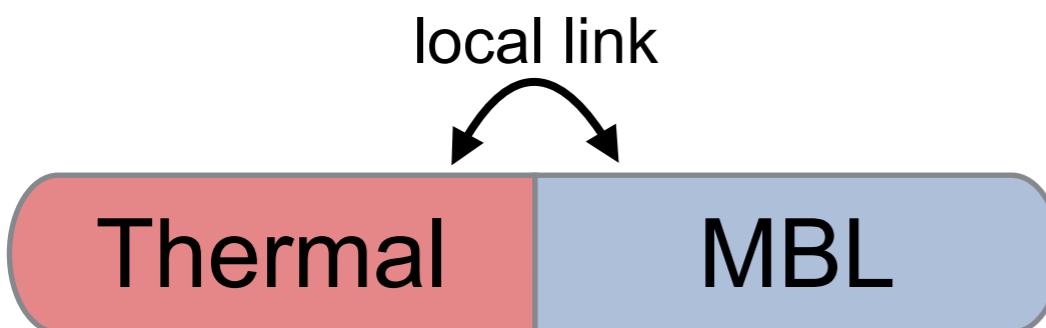


Outlook

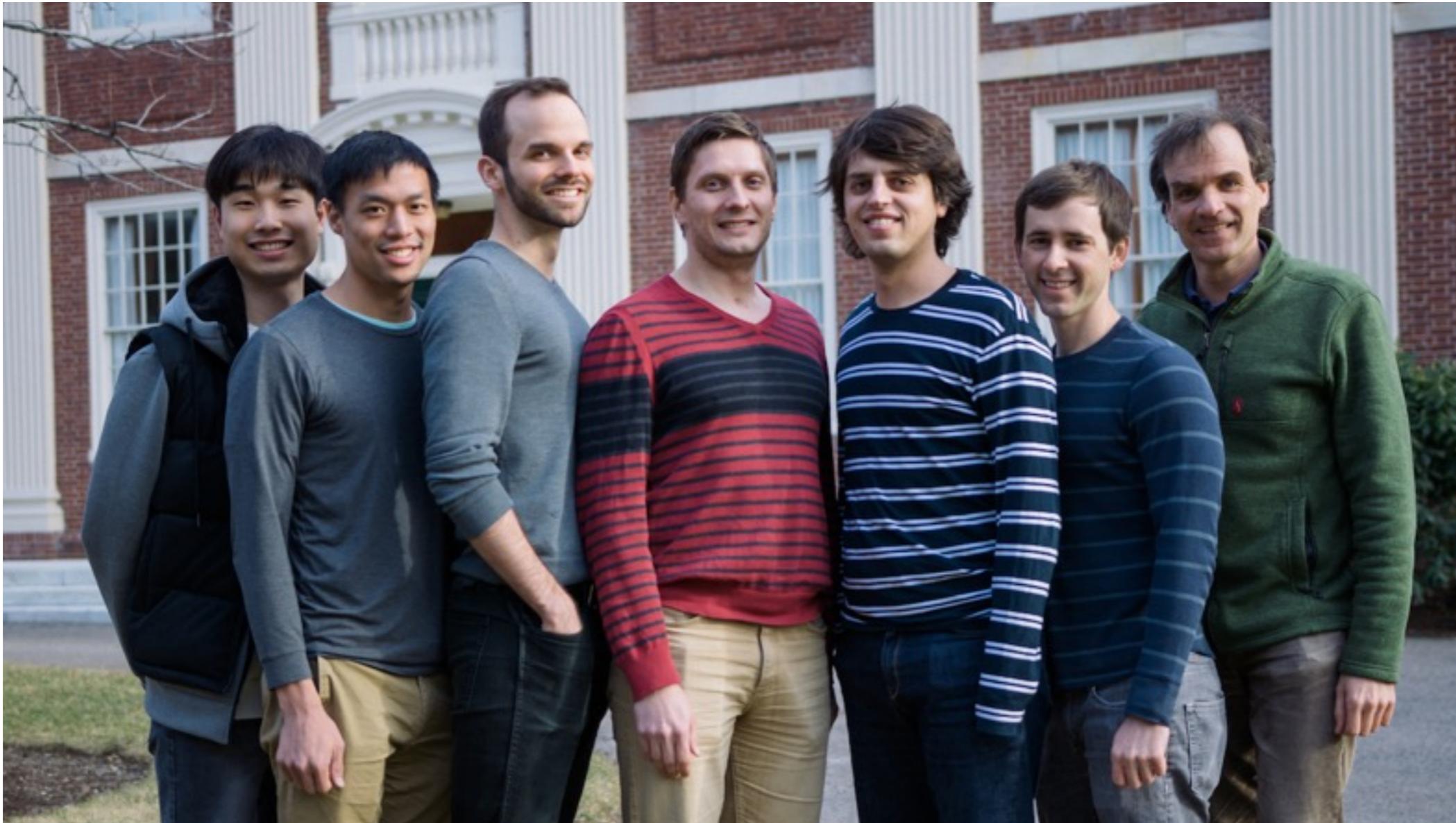
- Study critical dynamics at the phase transition



- Explore quantum thermodynamics by locally coupling MBL to a thermal region.



Thank you!



Soochin
Kim

Eric
Tai

Robert
Schittko

A.L.

Julian
Leonard

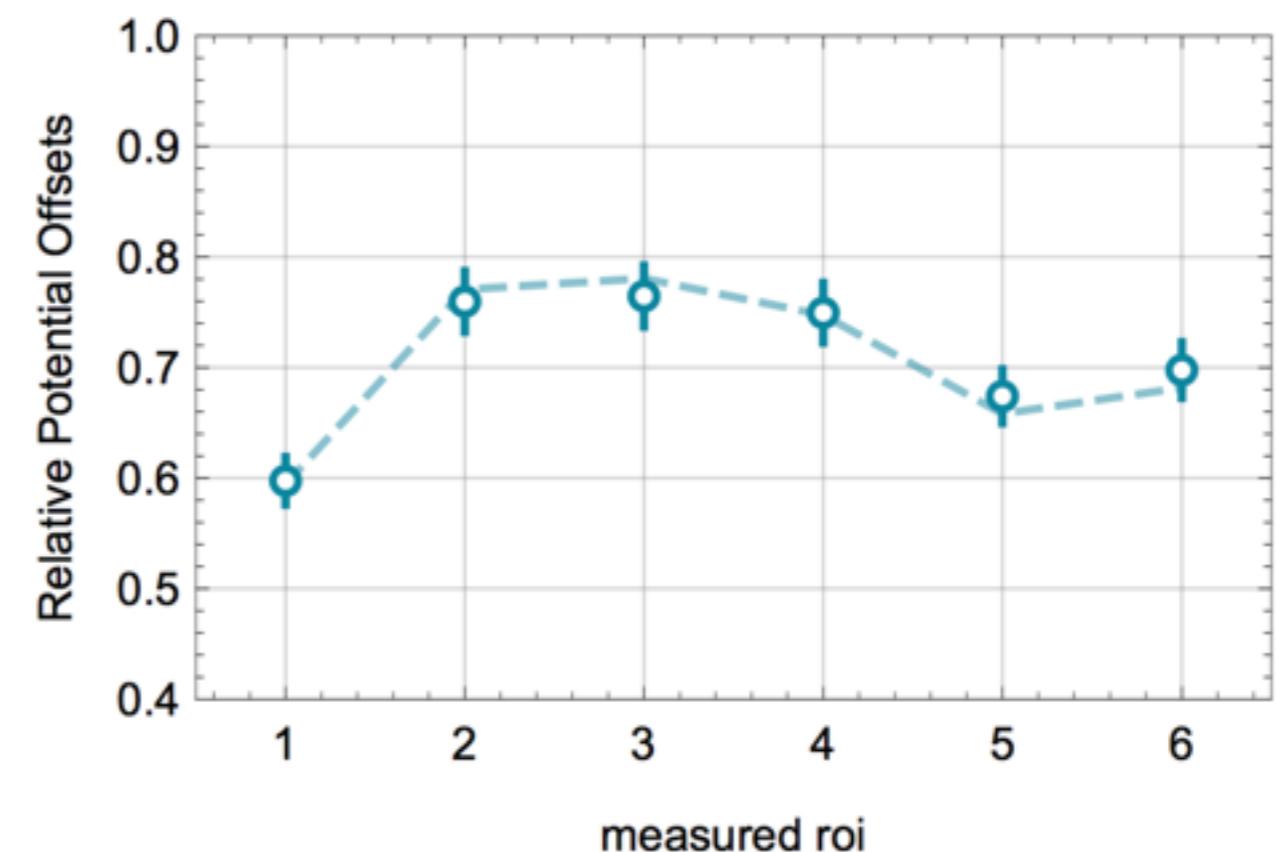
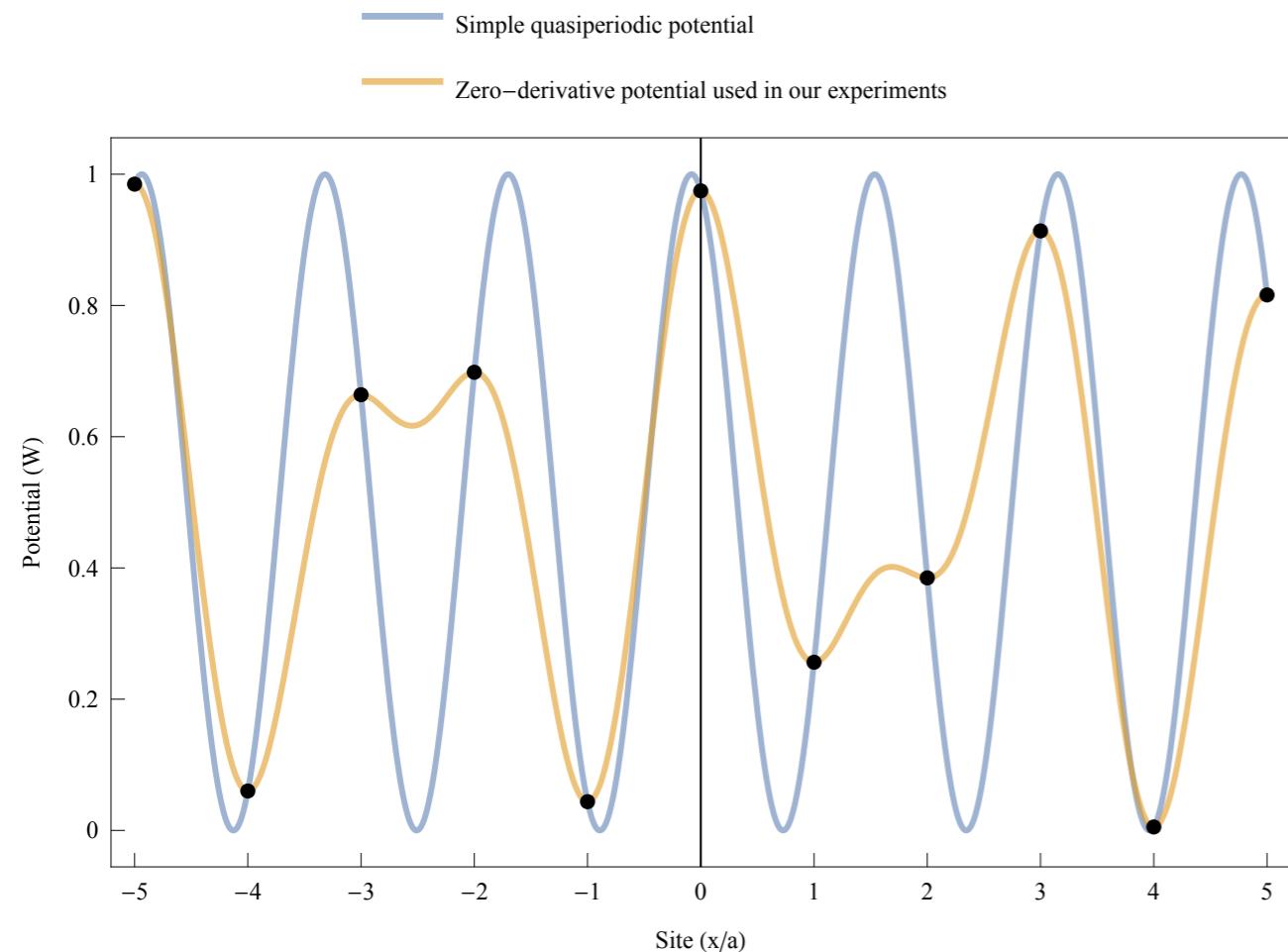
Matthew
Rispoli

Markus
Greiner

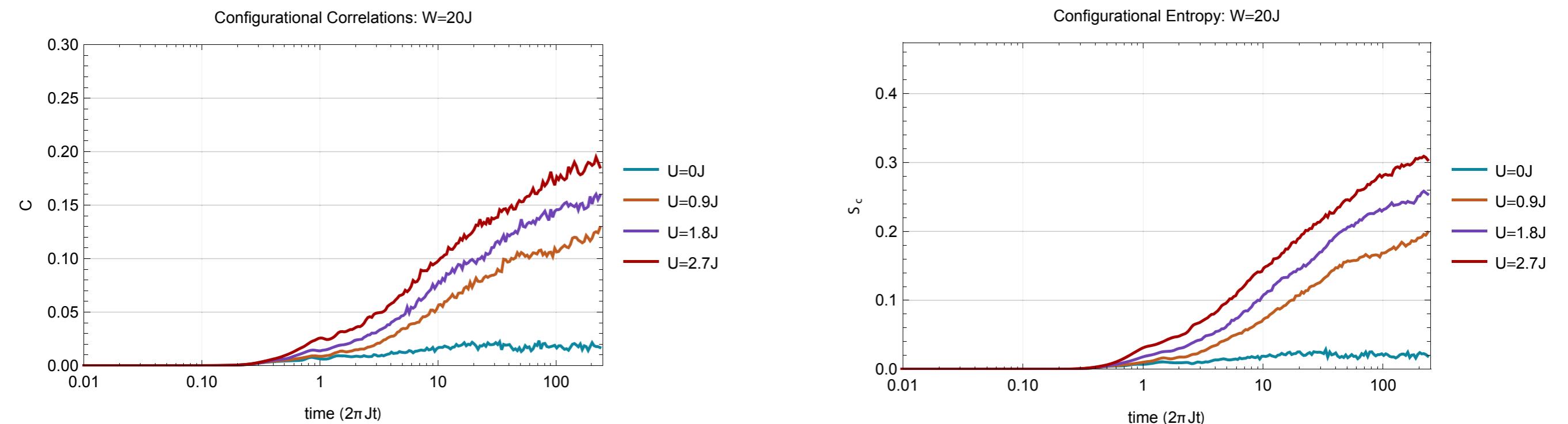
in collaboration with Soonwon Choi, Vedika Khemani and Adam Kaufman



Disorder potential



Configurational entropy and correlator



Disorder potential calibration

