Entanglement Features of Random Hamiltonian Dynamics

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Novel Approaches to Quantum Dynamics

KITP, Aug. 2018

Outline

- Motivation and definition of Entanglement Features.
- Entanglement features of States
  - Relation to Random Tensor Networks (RTN)

YZ You, Z Yang, XL Qi, 1709.01223
R Vasseur, AC Potter, YZ You, AWW Ludwig, 1807.07082

- Entanglement features of Unitaries
  - Random Hamiltonian generated unitaries ☆

YZ You, Y Gu, 1803.10425

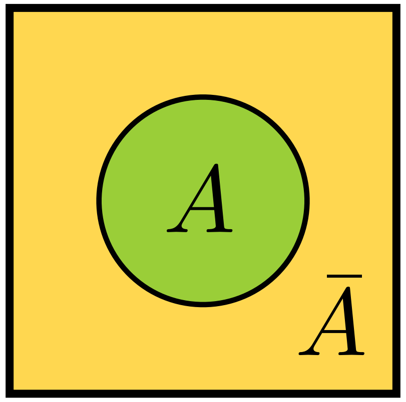
  - Floquet dynamics

WT Kuo, D. Arovas, YZ You (in progress)

Motivation

- Goal: to describe the structure and dynamics of quantum many-body entanglement.
- Entanglement Entropy - a tool to quantify entanglement
  - Quantum many-body system in a pure state $|\Psi\rangle$
  - Reduced density matrix of subsystem A

$$\rho_A = \mathrm{Tr}_{\bar{A}} |\Psi\rangle\langle\Psi|$$



  - Entanglement (Renyi) Entropy

$$S_\Psi^{(n)}(A) = \frac{1}{1-n} \ln \operatorname*{Tr}_A \rho_A^n$$

    measures the degree of entanglement between A and $\bar{A}$
    - Renyi index n. Limit of $n \to 1$, von Neumann entropy

$$S_\Psi^{(1)}(A) = -\mathrm{Tr}_A \rho_A \ln \rho_A$$

    - Entanglement region A (can be disconnected in general)

Motivation

- Entangleme entropy is useful to construct many quantum information measures
  - Different Renyi indices → entanglement spectrum
  - Different entanglement regions → mutual information …

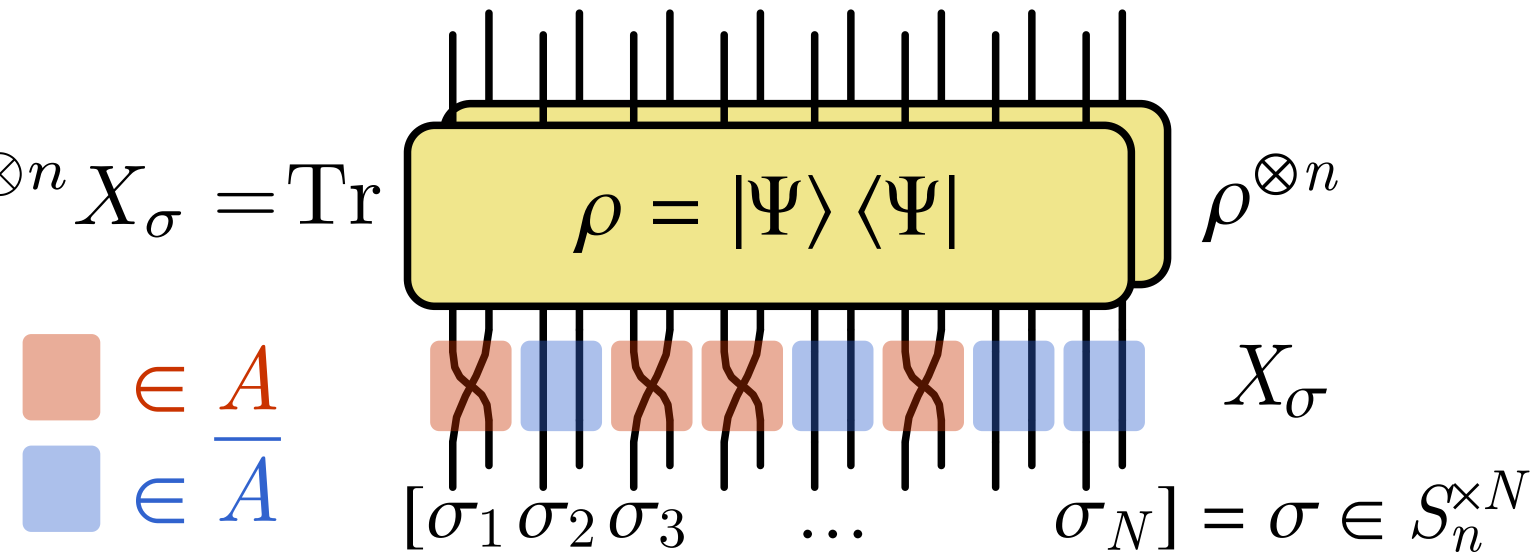
$$I_\Psi^{(n)}(A,B) = S_\Psi^{(n)}(A) + S_\Psi^{(n)}(B) - S_\Psi^{(n)}(A \cup B)$$

- They are useful in describing structures and dynamics of quantum many-body entanglement
  - Ryu-Takayanagi formula and holographic duality
  - Design tensor / neural network structures
  - Analyze quantum dynamics (many-body localization, thermalization, driven systems)
  - …

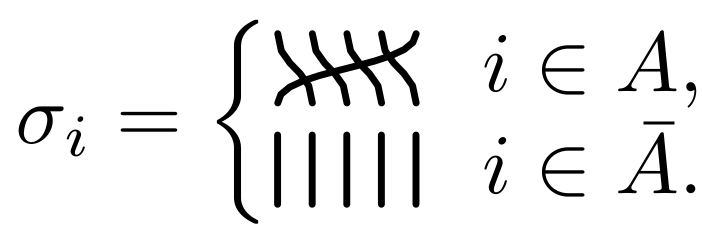
Permutation Group Formulation

- Partial trace can be evaluated using permutations

$$\operatorname{Tr}_A \rho_A^n = \operatorname{Tr} \rho^{\otimes n} X_\sigma = \operatorname{Tr}$$



  - A system of N qudits (d-dim local Hilbert space)
  - Renyi index n sets the degree of the permutation
  - Entanglement region specified by group element acting on each qudit channel



$\sigma = \sigma_1 \times \sigma_2 \times \cdots \times \sigma_N \in S_n^{\times N}$ represented as X_σ

Entanglement Features

- Entanglement Features (EF) of a pure state $|\Psi\rangle$

$$W_{\Psi}^{(n)}[\sigma] = \mathrm{Tr}(|\Psi\rangle\langle\Psi|)^{\otimes n} X_{\sigma}$$

- or equivalently as exponentiate entanglement entropy

$$W_{\Psi}^{(n)}[\sigma] = \exp(-(n-1)S_{\Psi}^{(n)}[\sigma])$$

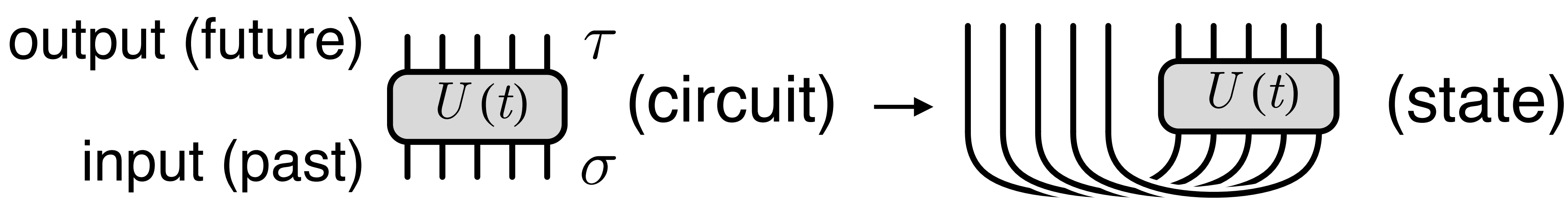
- mapping to statistical mechanics problems

entanglement region	$\longleftrightarrow$	permutation configuration
entanglement entropy	$\longleftrightarrow$	energy
entanglement feature	$\longleftrightarrow$	Boltzmann weight

- All entanglement features: exponentiated entanglement entropies of over all entanglement regions and to all Renyi indices.

Entanglement Features

- Is entanglement feature just a rewriting of entanglement entropy?
  - Basically yes.
  - But organized as partition functions (leads to new insights)
  - And becomes useful when we discuss quantum information dynamics
- Quantum dynamics: described by unitary evolution U



- Entanglement Features (EF) of a unitary evolution U

$$W_U^{(n)}[\sigma, \tau] = \operatorname{Tr} U^{\otimes n} X_\sigma (U^{\otimes n})^\dagger X_\tau$$

where $\sigma, \tau \in S_n^{\times N}$ act on the past and future respectively

Entanglement Features

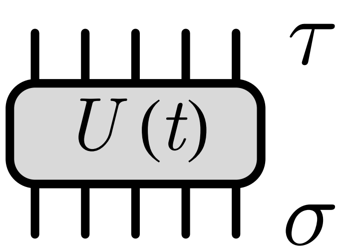
- Any relation between state EF and unitary EF?

state: $$W_\Psi^{(n)}[\sigma] = \operatorname{Tr}(|\Psi\rangle\langle\Psi|)^{\otimes n} X_\sigma$$

unitary: $$W_U^{(n)}[\sigma,\tau] = \operatorname{Tr} U^{\otimes n} X_\sigma (U^{\otimes n})^\dagger X_\tau$$

- Growth of entanglement entropy from product state
  - Evolution of state $|\Psi(t)\rangle = U(t)|\Psi(0)\rangle$
  - Evolution of entanglement features

$$W_{\Psi(t)}^{(n)}[\tau] = \frac{\Gamma(d)^N}{\Gamma(d+n)^N} \sum_{[\sigma]} W_{U(t)}^{(n)}[\sigma,\tau]$$



YZ You, Y Gu, 1803.10425; YD Lensky, XL Qi, 1805.03675

- More generally, how does the unitary evolution of state induces (non-unitary) evolution of entanglement features?

$$\mathrm{i}\partial_t|\Psi\rangle = H|\Psi\rangle \quad \Rightarrow \quad -\partial_t W_\Psi^{(n)} \stackrel{?}{=} \hat{D} W_\Psi^{(n)}$$

c.f. C Jonay, D Huse, A Nahum 1803.00089; T Zhou, A Nahum 1804.09737

Random Tensor Network

- Focus on the 2nd Renyi entanglement features ($n = 2$)
  - $\tau \in S_2^{\times N}$: Ising variables (identity = 1, swap = -1)

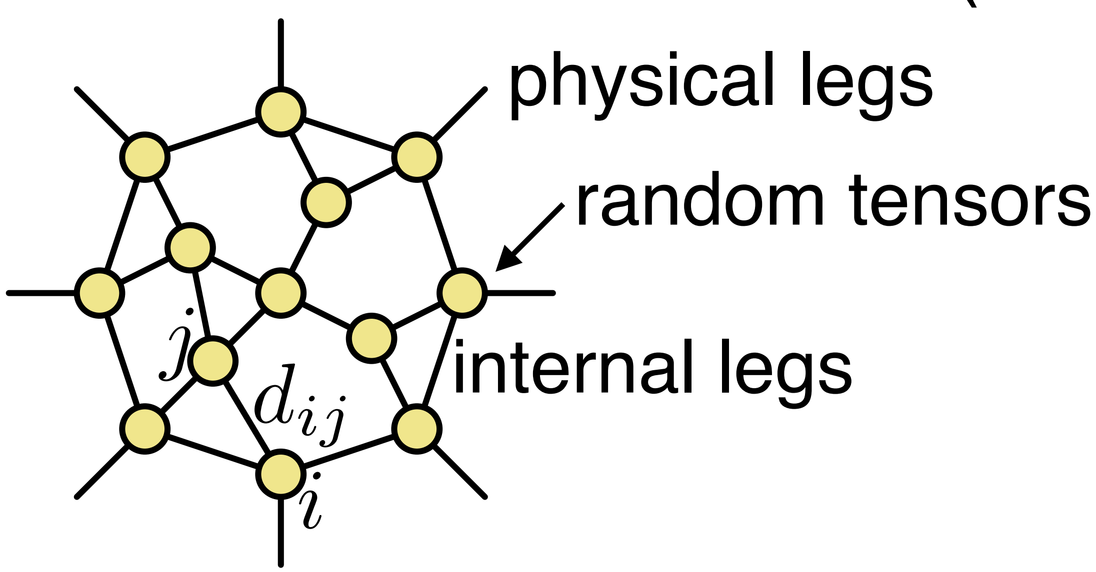
$$W_\Psi^{(2)}[\tau] = e^{-S_\Psi^{(2)}[\tau]}$$

$$S_\Psi^{(2)}[\tau] = S_0 - \sum J_{ij}\tau_i\tau_j - \sum J_{ijkl}\tau_i\tau_j\tau_k\tau_l + \cdots$$

  - What is the structure of this Ising model?
    - Random Tensor Network (RTN) provides a microscopic construction

Hayden, Nezami, Qi, Thomas, Walter, Yang 2016

- Random Tensor Network (RTN)

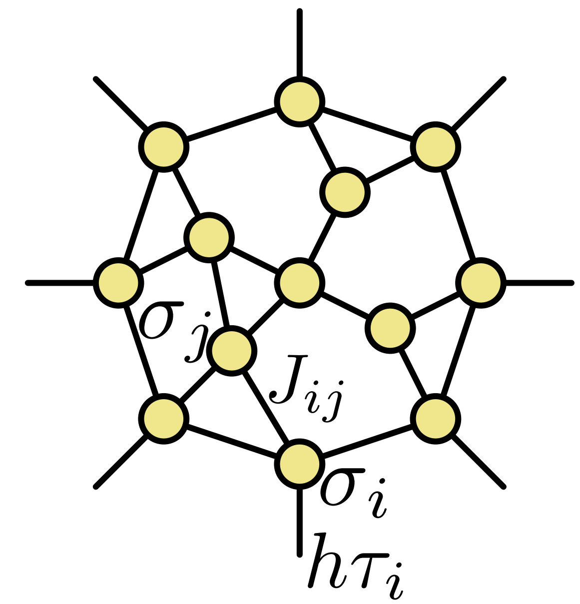


- On each vertex: rand. tensor
- On each link: bond dim. d_{ij}
- RTN: an ensemble of quantum many-body states

Random Tensor Network

- Entanglement Features of RTN

$$W^{(2)}_{\mathrm{RTN}}[\tau] = e^{-S^{(2)}_{\mathrm{RTN}}[\tau]} = \mathrm{Tr}(|\mathrm{RTN}\rangle\langle\mathrm{RTN}|)^{\otimes 2} X_\tau$$



  - Ensemble average over random tensors

$$\mathbb{E}W^{(2)}_{\mathrm{RTN}}[\tau] = \sum_{[\sigma]} e^{-E[\sigma,\tau]}$$

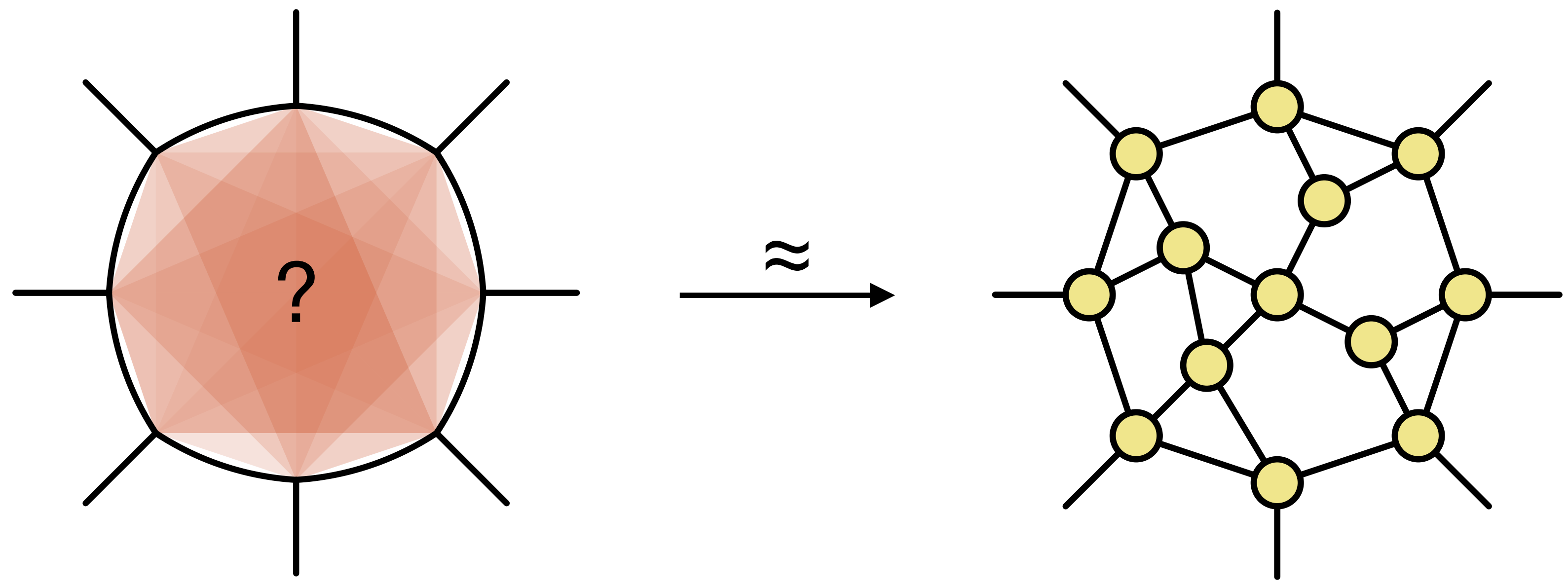
$$E[\sigma,\tau] = -\sum_{\langle ij\rangle} J_{ij}\sigma_i\sigma_j - \sum_{i\in\partial} h\tau_i\sigma_i \quad (J_{ij} = \tfrac{1}{2}\ln d_{ij})$$

  - Boundary spins → entanglement region $\tau_i = \begin{cases} -1 & i \in A, \\ +1 & i \in \bar{A}. \end{cases}$
  - Entanglement ~ spin correlation
  - Entanglement entropy ~ free energy (tracing out bulk σ)

$$S^{(2)}_{\mathrm{RTN}}[\tau] = S_0 - \sum J_{ij}\tau_i\tau_j - \sum J_{ijkl}\tau_i\tau_j\tau_k\tau_l + \cdots$$

Holographic Duality

- Multi-spin interaction in the Ising model reflects the non-local structure of many-body entanglement



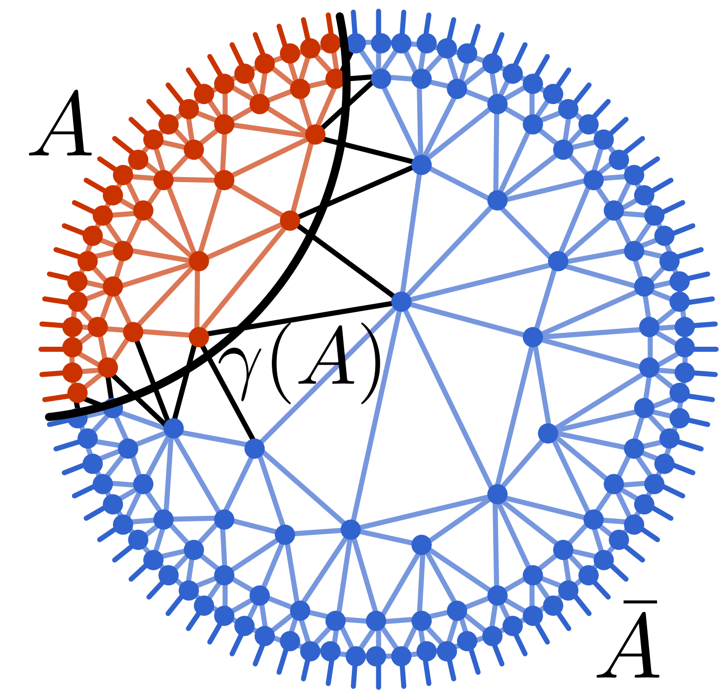
- Entanglement structure approximately resolved by tensor network (PEPS)
  - as entanglement pairs (two-spin interaction)
  - at the price of introducing bulk tensors (projections)
  - bulk network geometry ~ emergent holographic geometry

Machine Learning Spacial Geometry

- Ryu-Takayanagi: entanglement = area

$$S_\Psi(A) = \frac{1}{4G_N}|\gamma(A)|$$

  - area of minimal surface in the bulk ~ domain wall energy in the Ising model
  - ER = EPR = Ising coupling

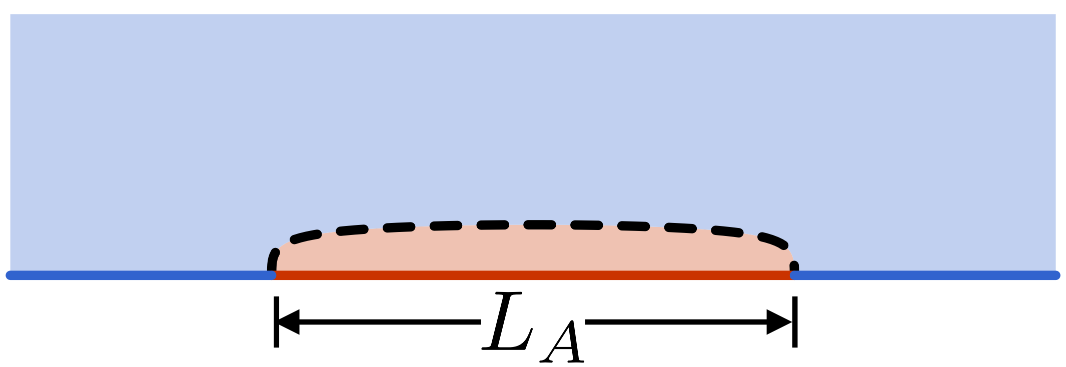


- Deep Learning: introducing hidden (bulk) variables to resolve complicated correlation in visible (boundary) variables
  - Ising model → Deep Boltzmann Machine
  - Input: entanglement features
  - Train: network connectivity
  - Result: holographic geometry

YZ You, Z Yang, XL Qi, 1709.01223

Entanglement Transition from Holographic RTN

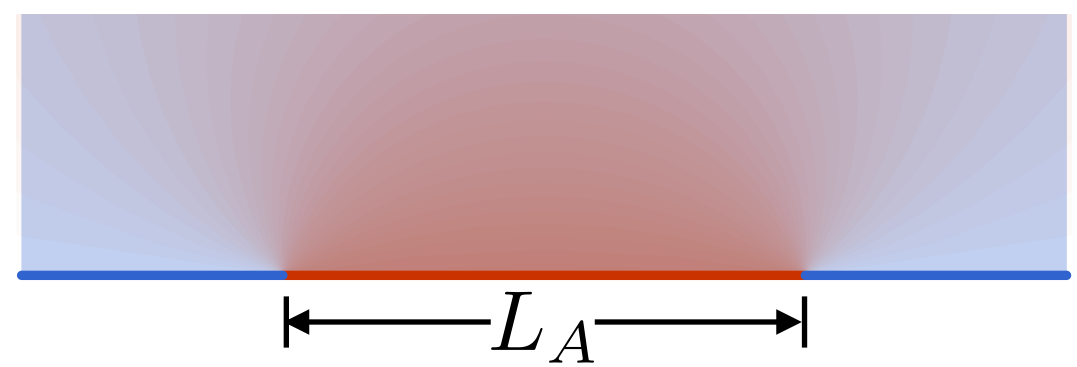
- Entanglement Transitions: area-law to volume-law transition, e.g. many-body localization to thermalization transition
  - Phase transition in holographic Ising model



Ferromagnet (ordered)

domain wall energy ~ L_A

volume-law entanglement



Paramagnet (disordered)

domain wall energy ~ const.

area-law entanglement

  - Transition driven by bond dimension of RTN

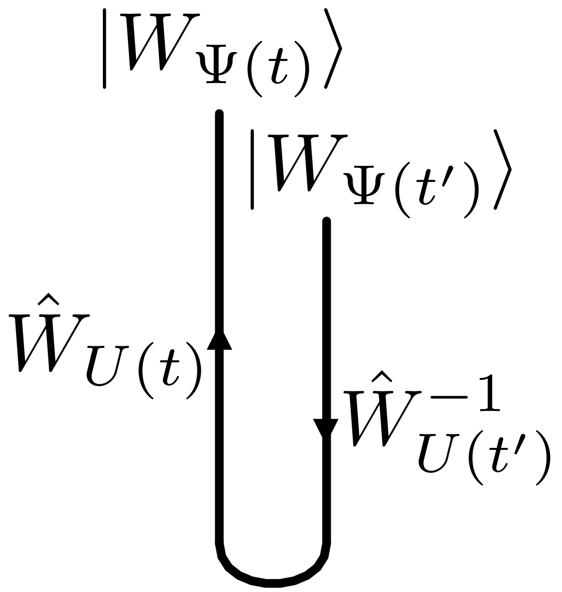


R Vasseur, AC Potter, YZ You, AWW Ludwig, 1807.07082

Dynamics of Entanglement Feature

- We can "Quantize" the entanglement features
  - State EF → vector: $|W_\Psi\rangle = \sum_{[\sigma]} W_\Psi^{(2)}[\sigma] |[\sigma]\rangle$
  - Unitary EF → matrix: $\hat{W}_U = \sum_{[\sigma,\tau]} W_U^{(2)}[\sigma,\tau] |[\tau]\rangle\langle[\sigma]|$
- Unitary evolution of a state

YZ You, Y Gu, 1803.10425



$$|\Psi(t)\rangle = U(t)|\Psi(0)\rangle$$

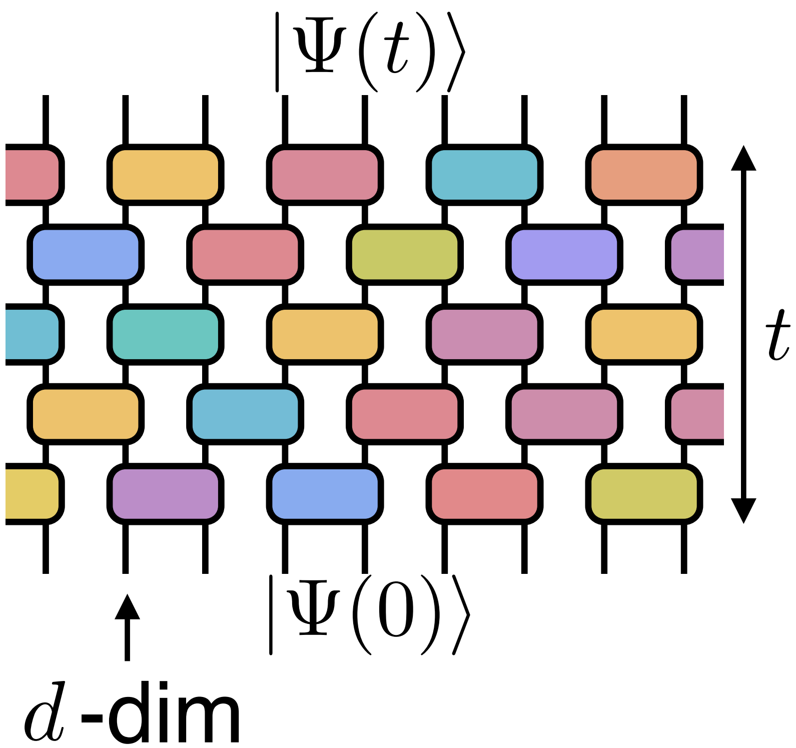
will induce a (generally) non-unitary evolution of its entanglement features

$$|W_{\Psi(t)}\rangle = \hat{W}_{U(t)} \hat{W}^{-1}_{U(t')} |W_{\Psi(t')}\rangle$$

  - As long as we know how to compute $\hat{W}_U$, we know everything about the full entanglement dynamics.
  - In general a hard problem, but for random unitary circuit, the answer is known.

Random Unitary Circuit

- Independent Haar random unitary gates (with locality)



$$|W_{\Psi(t+1)}\rangle = \hat{F}|W_{\Psi(t)}\rangle \quad \text{(Floquet-like)}$$

- In each Floquet cycle: two steps

$$\hat{F} = \prod_{\langle ij\rangle} \hat{Q}_{ij}\hat{P}_{ij}$$

- Projection (proj. out domain walls)

$$\hat{P}_{ij} = \frac{1}{2}(1 + Z_i Z_j)$$

- followed by Quantum fluctuations

$$\hat{Q}_{ij} = 1 + \frac{d}{d^2+1}(X_i + X_j)$$

A Nahum, J Ruhman, S Vijay, J Haah 1608.06950; C Keyserlingk, T Rakovszky, F Pollmann, S Sondhi 1705.08910 …

- $-\nabla_t|W_\Psi\rangle = \hat{H}_F|W_\Psi\rangle$ enhance ferromagnetic correlations
  - Thermalization: para = area-law → ferro = volume-law
  - Mode decay $\langle[\sigma]|W_\Psi\rangle = e^{-S_\Psi[\sigma]} \sim e^{-t/\tau}$ (linear S growth)

c.f. T Zhou, A Nahum 1804.09737

Random Hamiltonian Dynamics

- Random Hamiltonian dynamics: unitary evolution generated by random Hamiltonian

YZ You, Y Gu, 1803.10425

$$U(t) = e^{-\mathrm{i}Ht}$$

- A quantum many-body system of N qudits
  - each qudit: d dimensional Hilbert space
  - Total Hilbert space dimension $D = d^N$
- Radom Hamiltonian H
  - a $D \times D$ Hermitian matrix acting on all qudits
  - randomly drawn from Gaussian unitary ensemble

$$P(H) \propto e^{-\frac{D}{2}\operatorname{Tr} H^2} \qquad \text{(fixed spectral radius)}$$

  - Hamiltonian is non-local (i.e. not a sum of local / few-body operators), modeling a strongly thermalizing / chaotic system.

Random Hamiltonian Dynamics

- Random Hamiltonian v.s. Random Unitary
  - Random Unitary: locality, energy not conserved
  - Random Hamiltonian: energy conserved, non-local
- Tensor product structure of the Hilbert space still allows us to specify entanglement regions and define entanglement features
- Goal: Entanglement Features of $U(t) = e^{-\mathrm{i}Ht}$

$$W_U^{(2)}[\sigma, \tau] = \mathrm{Tr}\, U^{\otimes 2} X_\sigma (U^{\otimes 2})^\dagger X_\tau$$

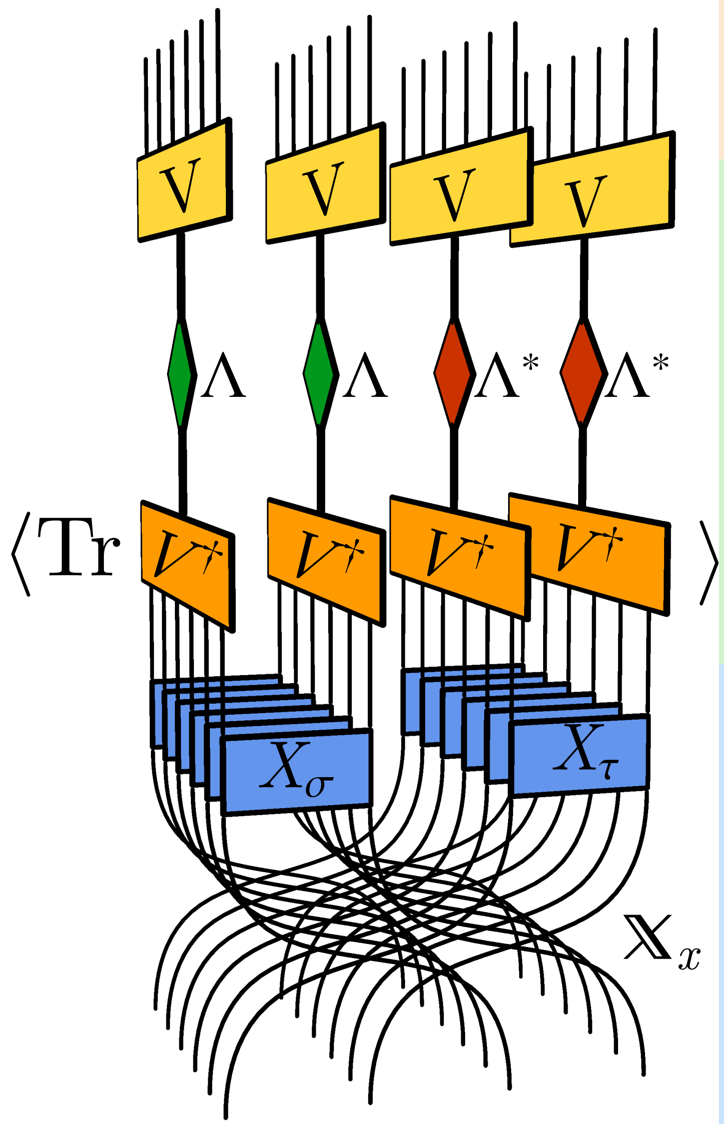
  - averaged over ensemble $\mathcal{E}(t) = \{U(t) = e^{-\mathrm{i}Ht} | H \in \mathrm{GUE}\}$

$$W^{(2)}[\sigma, \tau] = \langle W_U^{(2)}[\sigma, \tau] \rangle_{U \in \mathcal{E}(t)}$$

  - focused on Renyi index = 2 case.

Result of Entanglement Features

$$W^{(2)}[\sigma,\tau] =$$



$$W^{(2)}[\sigma,\tau] = W_{\text{early}}[\sigma,\tau] + W_{\text{late}}[\sigma,\tau],$$

$$W_{\text{early}}[\sigma,\tau] = \sum_{\upsilon=\pm1} D^{\frac{1}{2}(\upsilon\bar{\sigma}\bar{\tau}+\upsilon)} F_{\text{early}}(\upsilon), \quad F_{\text{early}}(\upsilon) = \frac{f_{\text{early}}(\upsilon)}{Z_4(D)}$$

$$W_{\text{late}}[\sigma,\tau] = \sum_{\upsilon_{1,2}=\pm1} D^{\frac{1}{2}(\upsilon_1\bar{\sigma}+\upsilon_2\bar{\tau}+\upsilon_1\upsilon_2)} F_{\text{late}}(\upsilon_1\upsilon_2). \quad F_{\text{late}}(\upsilon) = \frac{f_{\text{late}}(\upsilon)}{Z_4(D)}$$

$$\begin{aligned}
f_{\text{early}}(+1) =& D^3\big(4(D^2+6)(\mathcal{R}_{[00]}-\mathcal{R}_{[0]}) + 16(2D^2-3)\mathcal{R}_{[1\bar{1}]} + (D^2-3)(D^2-4)\mathcal{R}_{[2\bar{2}]} - 4D^2(D^2+1)\mathcal{R}_{[1\bar{1}0]} \\
&- 4D^2(D^2-4)\mathcal{R}_{[2\bar{1}\bar{1}]} + D^2(D^2-3)(D^2-4)\mathcal{R}_{[11\bar{1}\bar{1}]}\big),\\
f_{\text{early}}(-1) =& 2D^5\big(10(\mathcal{R}_{[0]}-\mathcal{R}_{[00]}) - 4(D^2+1)\mathcal{R}_{[1\bar{1}]} - (D^2-4)\mathcal{R}_{[2\bar{2}]} \\
&+ 4(2D^2-3)\mathcal{R}_{[1\bar{1}0]} + (D^2-3)(D^2-4)\mathcal{R}_{[2\bar{1}\bar{1}]} - D^2(D^2-4)\mathcal{R}_{[11\bar{1}\bar{1}]}\big),\\
f_{\text{late}}(+1) =& D^{9/2}\big(-2(D^2-14)\mathcal{R}_{[0]} + (D^4-11D^2+8)\mathcal{R}_{[00]} - 40\mathcal{R}_{[1\bar{1}]} - (D^2-4)\mathcal{R}_{[2\bar{2}]} + 4(D^2+6)\mathcal{R}_{[1\bar{1}0]} \\
&+ 6(D^2-4)\mathcal{R}_{[2\bar{1}\bar{1}]} - D^2(D^2-4)\mathcal{R}_{[11\bar{1}\bar{1}]}\big),\\
f_{\text{late}}(-1) =& D^{9/2}\big((D^2+1)(D^2-12)\mathcal{R}_{[0]} - 2(D^4-12D^2+12)\mathcal{R}_{[00]} + 8(D^2+6)\mathcal{R}_{[1\bar{1}]} + 3(D^2-4)\mathcal{R}_{[2\bar{2}]} \\
&- 20D^2\mathcal{R}_{[1\bar{1}0]} - 2D^2(D^2-4)\mathcal{R}_{[2\bar{1}\bar{1}]} + 3D^2(D^2-4)\mathcal{R}_{[11\bar{1}\bar{1}]}\big).
\end{aligned}$$

$$\begin{aligned}
\mathcal{R}_{[0]}(t) =& \mathcal{R}_{[00]}(t) = 1,\\
\mathcal{R}_{[1\bar{1}]}(t) =& \mathcal{R}_{[1\bar{1}0]}(t) = r_1(t)^2 + (1-r_2(t))/D,\\
\mathcal{R}_{[2\bar{2}]}(t) =& \mathcal{R}_{[1\bar{1}]}(2t),\\
\mathcal{R}_{[21\bar{1}]}(t) =& r_1(2t)r_1(t)^2 + (-r_1(2t)r_2(t)r_3(2t) - 2r_1(t)r_2(2t)r_3(t) + +r_1(2t)^2 + 2r_1(t)^2)/D\\
&+ (2r_2(3t) - r_2(2t) - 2r_2(t) + 1)/D^2,\\
\mathcal{R}_{[11\bar{1}\bar{1}]}(t) =& r_1(t)^4 + (-2r_1(t)^2r_2(t)r_3(2t) - 4r_1(t)^2r_2(t) + 2r_1(2t)r_1(t)^2 + 4r_1(t)^2)/D\\
&+ (2r_2(t)^2 + r_2(t)^2r_3(2t)^2 + 8r_1(t)r_2(t)r_3(t) - 2r_1(2t)r_2(t)r_3(2t) - 4r_1(t)r_2(2t)r_3(t)\\
&\quad + r_1(2t)^2 - 4r_1(t)^2 - 4r_2(t) + 2)/D^2\\
&+ (-7r_2(2t) + 4r_2(3t) + 4r_2(t) - 1)/D^3,
\end{aligned}$$

$$r_1(t) = \frac{J_1(2t)}{t}, \quad r_2(t) = \Big(1-\frac{|t|}{2D}\Big)\Theta\Big(1-\frac{|t|}{2D}\Big), \quad r_3(t) = \frac{\sin(\pi t/2)}{\pi t/2}$$

Result of Entanglement Features

- To the leading order in $D = d^N$

$$
\begin{aligned}
W^{(2)}[\sigma,\tau] = \mathcal{R}_{[11\bar{1}\bar{1}]} D^{\frac{3+\overline{\sigma\tau}}{2}} \\
- 2(\mathcal{R}_{[11\bar{1}\bar{1}]} - \mathcal{R}_{[2\bar{1}\bar{1}]}) D^{\frac{1-\overline{\sigma\tau}}{2}} \\
+ (\mathcal{R}_{[00]} - \mathcal{R}_{[11\bar{1}\bar{1}]})(D^{\frac{2+\overline{\sigma}+\overline{\tau}}{2}} + D^{\frac{2-\overline{\sigma}-\overline{\tau}}{2}}) \\
- (2\mathcal{R}_{[00]} - \mathcal{R}_{[0]} + 2\mathcal{R}_{[2\bar{1}\bar{1}]} - 3\mathcal{R}_{[11\bar{1}\bar{1}]}) \\
\times (D^{\frac{\overline{\sigma}-\overline{\tau}}{2}} + D^{\frac{-\overline{\sigma}+\overline{\tau}}{2}}) + \cdots ,
\end{aligned}
$$

- Time dependence

$$\mathcal{R}_{[k]}(t) = \frac{1}{D^l} \langle \prod_a \operatorname{Tr} U(t)^{k_a} \rangle$$

Spectral form factor of GUE

J Cotler, N Hunter-Jones, J Liu, B Yoshida, 1706.05400

- Region dependence

$\overline{\sigma} = N^{-1} \sum_i \sigma_i$ (input)

$\overline{\tau} = N^{-1} \sum_i \tau_i$ (output)

$\overline{\sigma\tau} = N^{-1} \sum_i \sigma_i \tau_i$ (coupling)

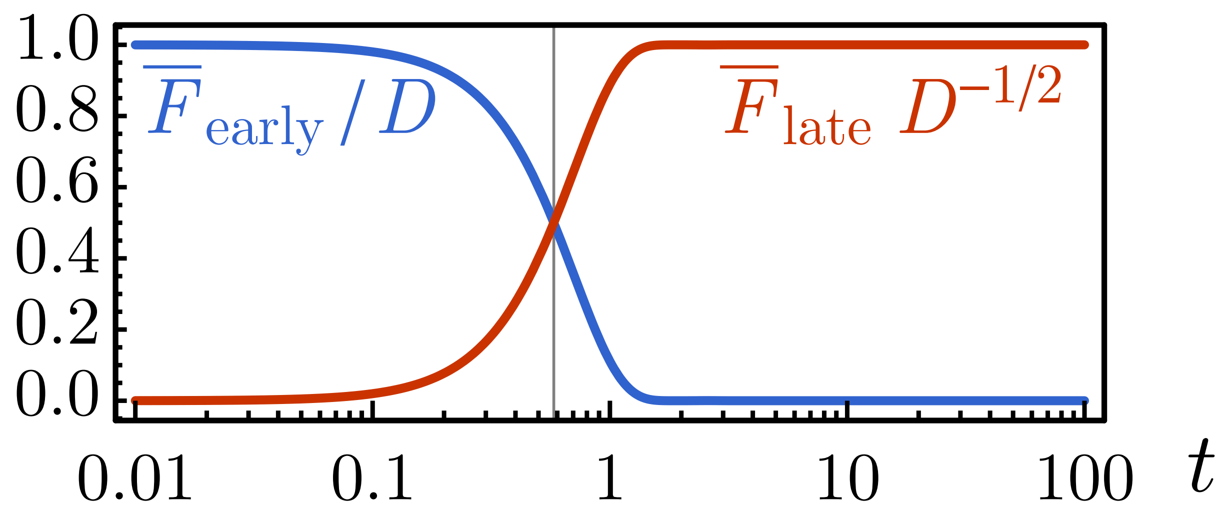
Holographic Ising Model

- Given $W^{(2)}[\sigma,\tau]$ as a Boltzmann weight, what kind of Ising model does it describe?
- Introduce hidden (bulk) variables to simplify the result

$$W^{(2)}[\sigma,\tau] = W_{\text{early}}[\sigma,\tau] + W_{\text{late}}[\sigma,\tau]$$

$$W_{\text{early}}[\sigma,\tau] = \sum_{\upsilon=\pm 1} D^{\frac{1}{2}(\upsilon\overline{\sigma\tau}+\upsilon)} F_{\text{early}}(\upsilon),$$

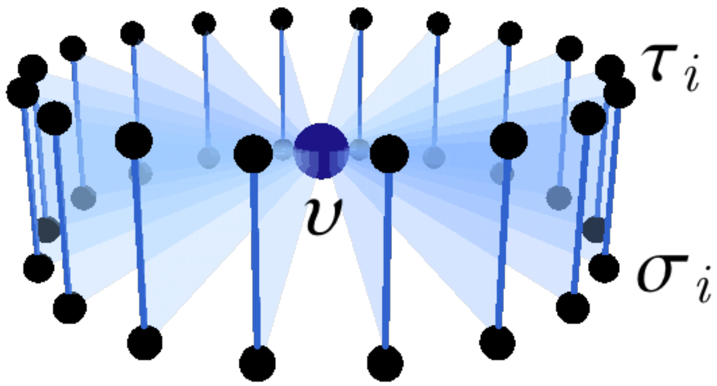
$$W_{\text{late}}[\sigma,\tau] = \sum_{\upsilon_{1,2}=\pm 1} D^{\frac{1}{2}(\upsilon_1\overline{\sigma}+\upsilon_2\overline{\tau}+\upsilon_1\upsilon_2)} F_{\text{late}}(\upsilon_1\upsilon_2).$$



Holographic Ising Model

- Early-time Ising model

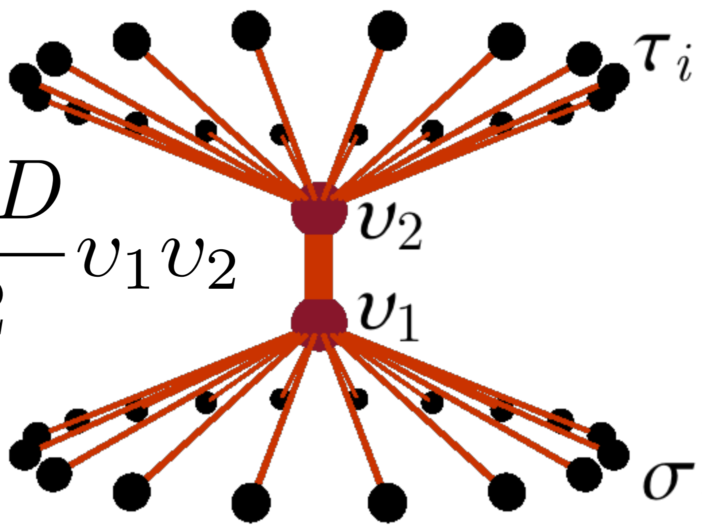
$$E_{\text{early}}[\sigma,\tau;\upsilon] = -\frac{\ln d}{2}\sum_i \upsilon\sigma_i\tau_i - \frac{\ln D}{2}\upsilon$$



  - Strong pinning field → $\upsilon = +1$
  - Direct coupling (max entanglement) between past & future
  - Spacial geometry is fragmented (independent channels)

- Late-time Ising model

$$E_{\text{late}}[\sigma,\tau;\upsilon] = -\frac{\ln d}{2}\sum_i (\upsilon_1\sigma_i + \upsilon_2\tau_i) - \frac{\ln D}{2}\upsilon_1\upsilon_2$$



  - RTN: information falls into tensor υ_1, gets scrambled, emits from tensor υ_2
  - A pair of temporally entangled black / white holes

- Random Hamiltonian dynamics → black hole formation

Thermalization and Quantum Chaos

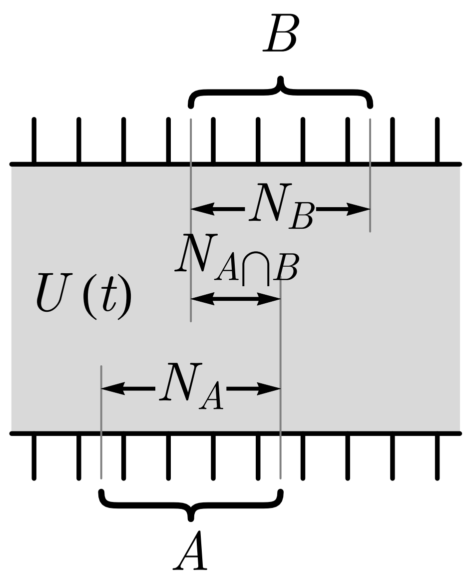
- Two approaches to describe Thermalization
  - Equilibrium (static) approach: eigenstate thermalization hypothesis, level statistics, volume-law entanglement …
  - Dynamical approach: quantum chaos, information scrambling, OTOC (butterfly effect), entropy growth …
- Random Hamiltonian: (over)simplified model of ETH
- Many measures of quantum chaos (OTOC, entropy growth) can be formulated as entanglement features of unitary.
- Goal: learn about typical quantum chaotic behavior of many-body systems that exhibit eigenstate thermalization.
- Tool: Entanglement features of random Hamiltonian dynamics

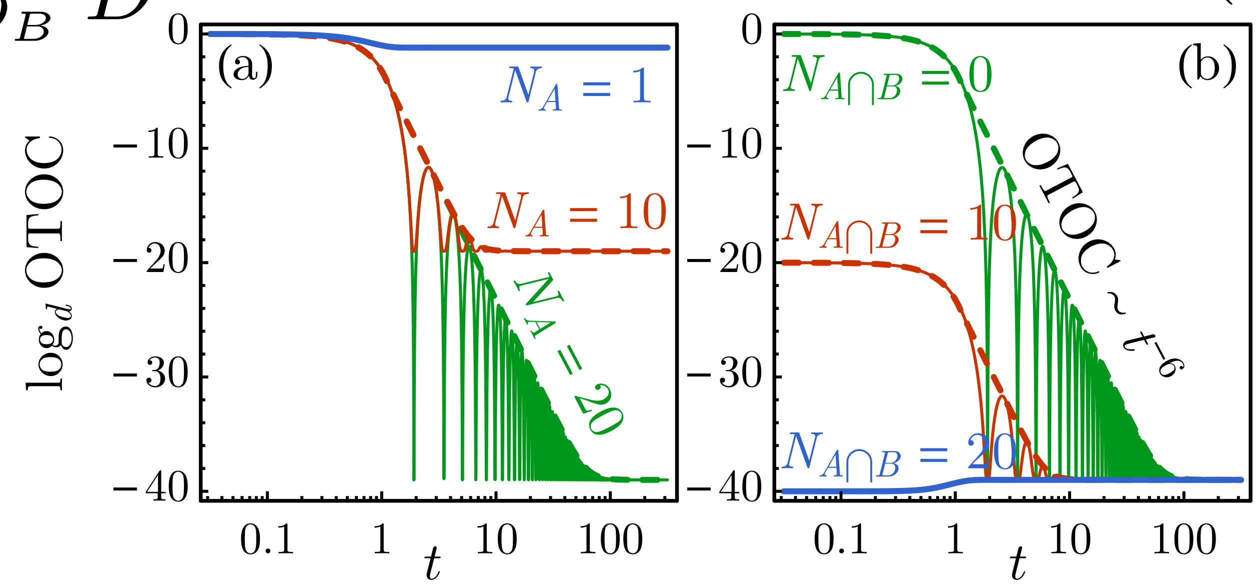
$$W^{(2)}[\sigma,\tau] = \langle W_U^{(2)}[\sigma,\tau]\rangle_{U\in\mathcal{E}(t)}$$

Thermalization and Quantum Chaos

- Operator-averaged OTOC P Hosur, XL Qi, DA Roberts, B Yoshida, 1511.04021

$$\text{OTOC}(A,B) = \underset{O_A,O_B}{\text{avg}} \frac{1}{D} \operatorname{Tr} O_A(t) O_B O_A(t) O_B = W^{(2)}_{U(t)}(A,\bar{B})$$





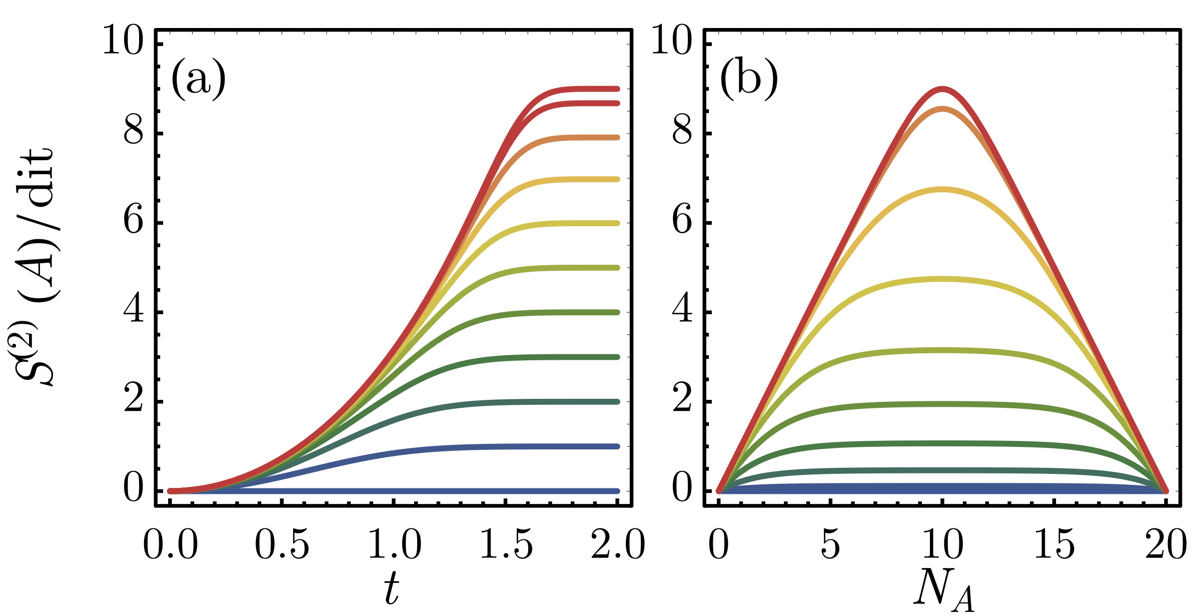
c.f. S Vijay, A Vishwanath, 1803.08483

- Entropy growth from product state

$$W^{(2)}_{\Psi(t)}[\tau] \propto \sum_{[\sigma]} W^{(2)}_{U(t)}[\sigma,\tau]$$

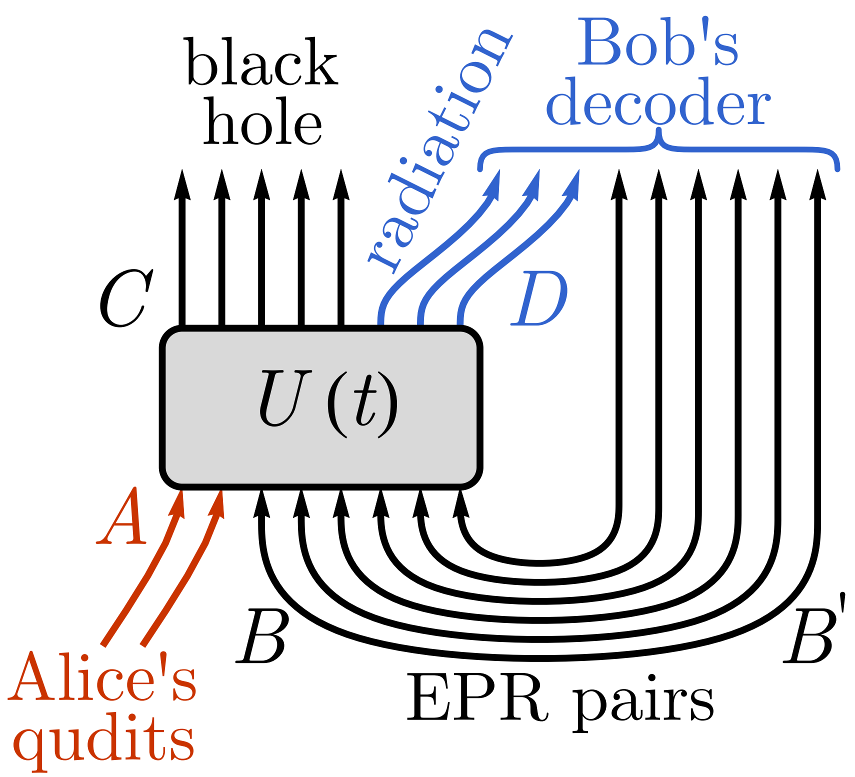
c.f. YD Lensky, XL Qi, 1805.03675

(not a linear growth in time, due to non-local Hamiltonian)



Hayden-Preskill Problem

- Can Bob decode Alice's qudits?



- Alice throws qudits to black hole B
- B was maximally entangled with B'
- Bob collects radiation D at time t

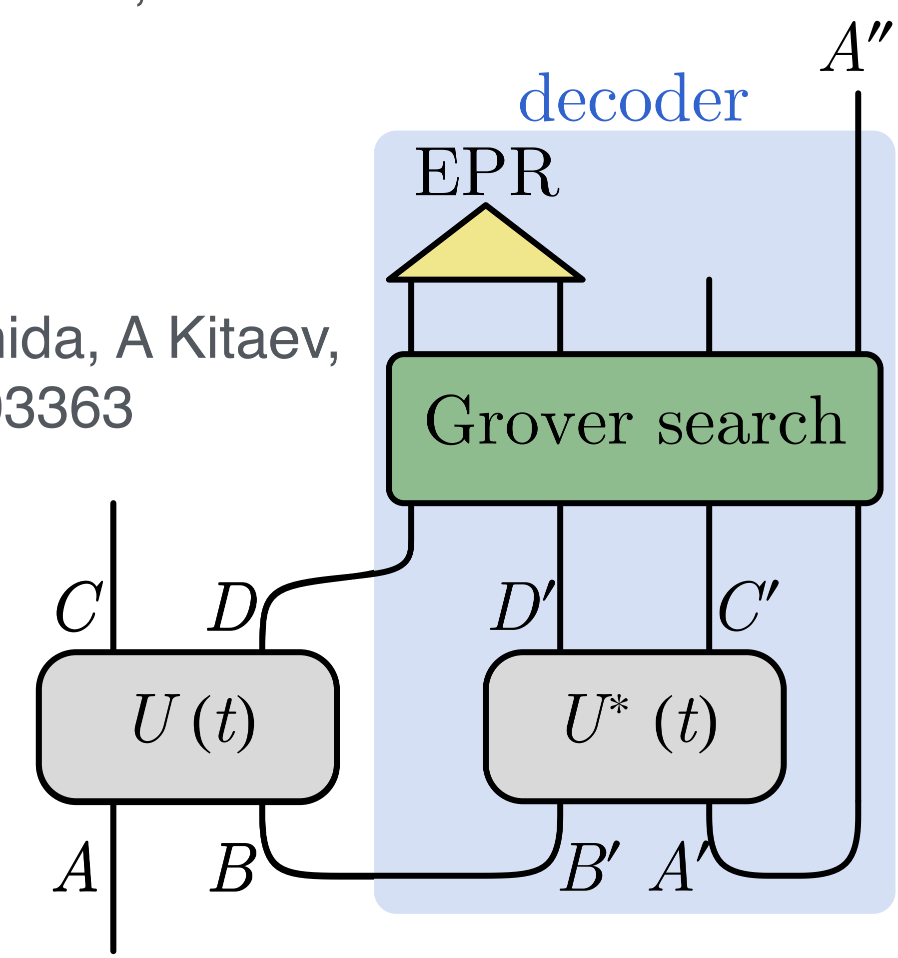
P Hayden, J Preskill, 0708.4025

- Yoshida-Kitaev protocol
  - Teleportation fidelity

$$F = \langle A | A'' \rangle^2 = e^{-I^{(2)}(A,C)}$$

$$\geq \frac{1}{1 + d^{2(N_A - N_D)}} \xrightarrow{d^{N_D} \gg d^{N_A}} 1$$

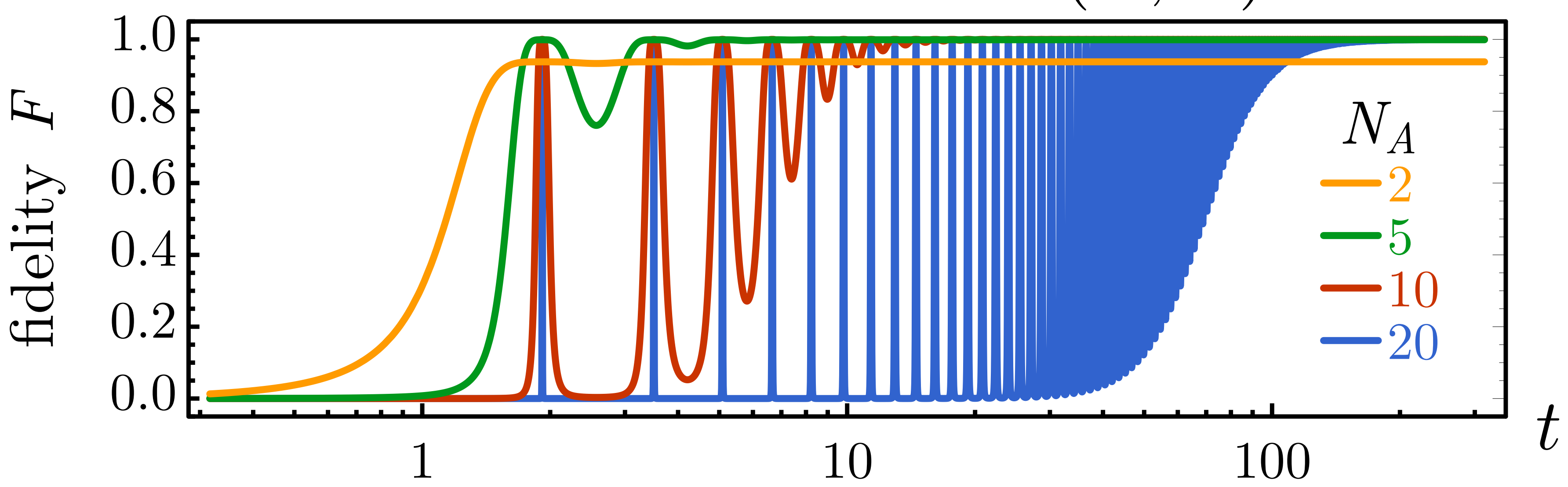
B Yoshida, A Kitaev, 1710.03363



Hayden-Preskill Problem

- Modeling back hole dynamics by
  - Haar random unitary   P Hayden, J Preskill; B Yoshida, A Kitaev
  - unitary generated by random Hamiltonian ☆
- Teleportation fidelity in terms of entanglement features

$$F = e^{-I^{(2)}(A,C)} = \frac{d^{N_B} d^{N_D}}{W^{(2)}(A,C)}$$

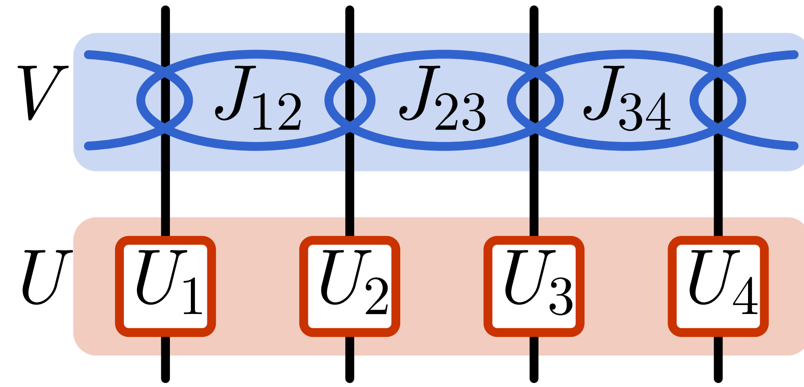


  - Full scrambling takes a long time $t_s = (d^{N_A}/\pi)^{1/3}$
  - A sequence of time windows (Bob must seize the moment)

Random Floquet Dynamics

- Chan-Luca-Chalker Model   A Chan, AD Luca, JT Chalker, 1803.03841

$U_F = VU =$



$$V = \exp\left(\mathrm{i}\sum J_{ij}^{ab} E_i^a E_j^b\right)$$

$$U = \bigotimes_i U_i$$

  - On-site scrambling, followed by inter-site coupling
  - U_i: Haar random, J_{ij}: Gaussian random $\langle J_{ij}^2 \rangle \equiv J^2$
  - Locality + quasi-energy conservation
- Entanglement Features of random Floquet dynamics

$$W^{(2)}_{U_F^t}[\sigma,\tau] = \mathrm{Tr}(U_F^t)^{\otimes 2} X_\sigma (U_F^{-t})^{\otimes 2} X_\tau = e^{-E[\sigma,\tau]}$$

$$\begin{aligned} E[\sigma,\tau] = & -\frac{\ln d}{2}\sum_i(\sigma_i\tau_i + 3) \\ & -\frac{J^2 t}{4}\sum_{\langle ij\rangle}((\sigma_i-\sigma_j)(\tau_i-\tau_j) \\ & +(1-\sigma_i\sigma_j)(1-\tau_i\tau_j)) \end{aligned}$$

WT Kuo, D. Arovas, YZ You (in progress)

Summary

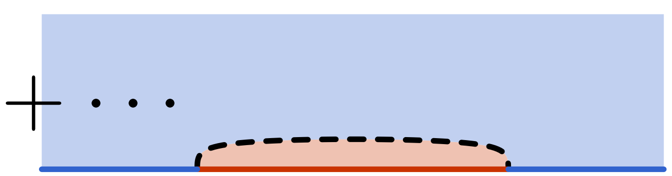
- Entanglement Features

$$W^{(n)}[\sigma] = \exp\big(-(n-1)S^{(n)}[\sigma]\big)$$

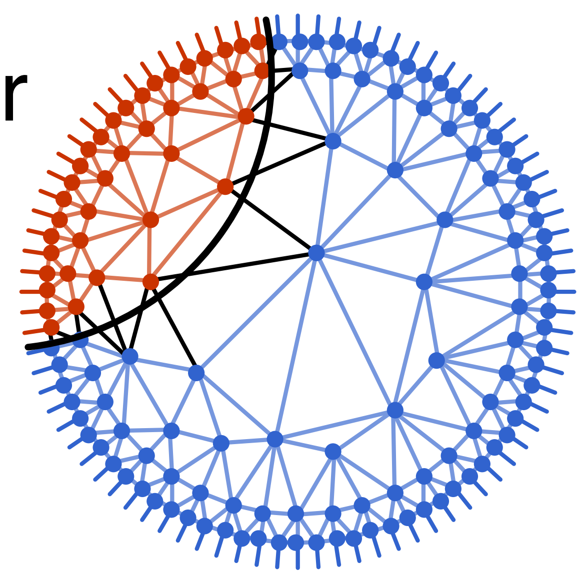
  - Defined for states and unitary operators
    They are related by $|W_{\Psi(t)}\rangle = \hat{W}_{U(t)}\hat{W}^{-1}_{U(t')}|W_{\Psi(t')}\rangle$

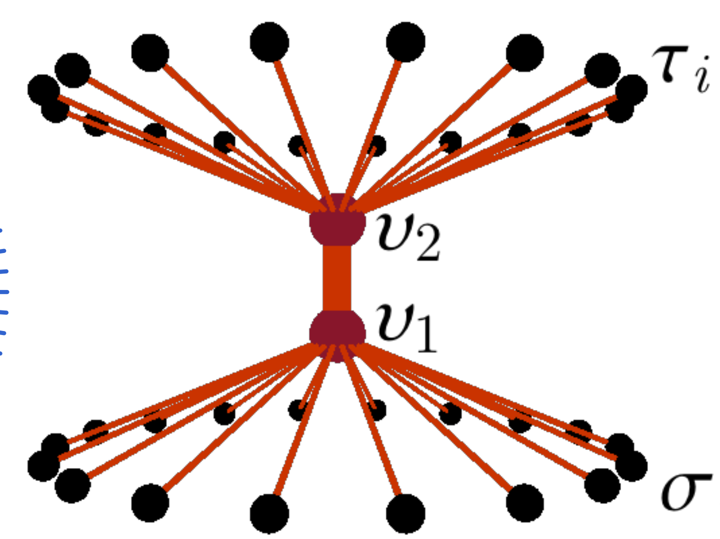
- Map to Ising model (or more general models)

$$S^{(2)}_{\Psi}[\tau] = S_0 - \sum J_{ij}\tau_i\tau_j - \sum J_{ijkl}\tau_i\tau_j\tau_k\tau_l + \cdots$$



- Make connections to tensor networks and holography
- Apply to random unitary / Hamiltonian dynamics …





Thanks for your attention!