Center for Theory of Quantum Matter
CTQM

Dirty interacting two-dimensional topological insulator edges

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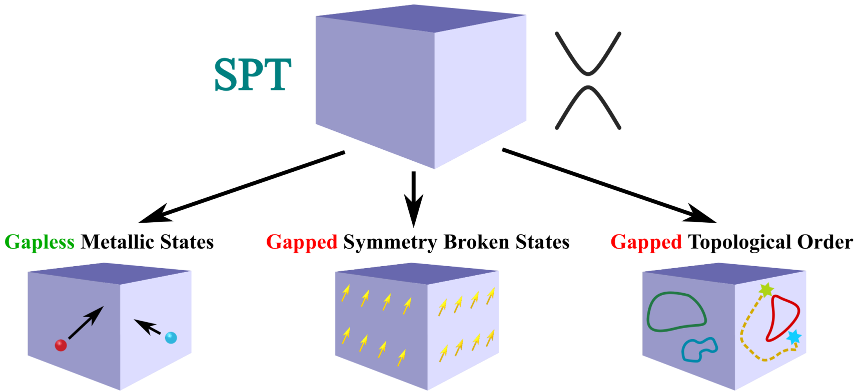
Topological Insulators with Interactions



See reviews: Hasan and Kane (2010); Qi and Zhang (2011); Senthil (2015); Wen (2017), ...

Boundaries of topological insulators with disorder:

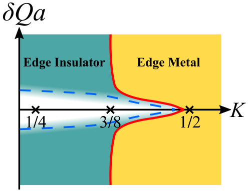
- Gapless metallic, avoidance of Anderson localization

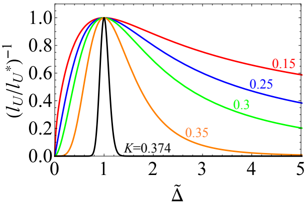
Boundaries of topological insulators with interactions:

- Gapless metallic
- Gapped, symmetry broken
- Gapped, forming topological order

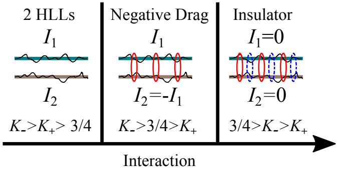
Q: Are these exhaustive with disorder and interaction?

- Gapless insulating edges of dirty interacting topological insulators
  Ref: PRB 98, 054205 (2018)





- Localization-driven correlated states of two isolated interacting helical edges
  Ref: https://arxiv.org/abs/1806.02353



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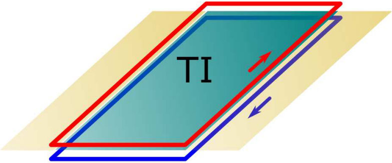
Gapless insulating edges of dirty interacting TIs

Ref: Chou, Nandkishore, Radzihovsky, PRB 98, 054205 (2018)

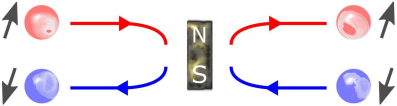
2D $\mathbb{Z}_2$ Topological Insulator



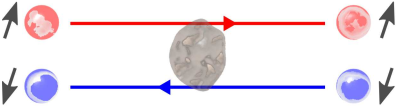
2D $\mathbb{Z}_2$ Topological Insulators:

- Kane-Mele, BHZ models
- HgTe, InAs/GaSb, WTe_2, WSe_2, ...
- Time reversal symmetry
- Robust edge states against TRS disorder
- Potential platform for Majorana zero mode

$\mathcal{T}$-breaking perturbation



Non magnetic impuritiy



Dirty Interacting TI Edges

A minimal generic model for 2D TI edge states:

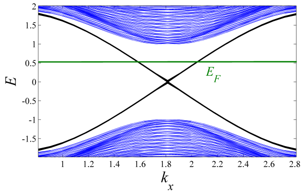
Helical Luttinger liquid

Time reversal symmetric (TRS) perturbations:

TRS disorder + **TRS umklapp interaction**

Not including *material specific issues*, *external perturbations*, ...

Model



$$\hat{H}_0 = v_F \int_x \left[R^\dagger \left(-i\partial_x R\right) - L^\dagger \left(-i\partial_x L\right) \right].$$

Time-reversal operation:

$$R \to L,\ L \to -R,\ i \to -i.$$

Normal impurity backscattering:

$L^\dagger R + R^\dagger L \to -R^\dagger L - L^\dagger R$

TRS breaking!

TRS Disorder:

$$\hat{H}_V = \int_x V(x) \left[R^\dagger(x) R(x) + L^\dagger(x) L(x) \right].$$

$$\overline{V(x)V(y)} = \Delta\delta(x-y),\ \overline{V(x)} = 0.$$

TRS umklapp term:

$$H_U = U \int_x \left[e^{-i\delta Q x} : \left(L^\dagger R\right)^2 : + \text{H.c.} \right].$$

$$\delta Q = Q - 4k_F,\ Q = 2\pi/a$$

Luttinger Interactions:

TRS marginal perturbation

Helical Luttinger Liquid + TRS Perturbations

TRS disorder only:

Spatially varying chemical potential

Forward scattering → **Ballistic**

TRS umklapp only:

Commensurate-incommensuarate CDW
Pokrovsky and Talapov (1979)

$\delta Q \neq 0$ generically → **Ballistic**

TRS disorder + **TRS umklapp**:

Disorder-assisted backscattering for $T \neq 0$.
Fiete, *et al* (2006); Kainaris, *et al* (2014); **Chou**, *et al* (2015).

Scatterings rate Γ:
Chou, Levchenko, Foster (2015)

$$\Gamma \propto \begin{cases} e^{-\frac{v\delta Q}{2T}}, & \text{clean} \\ T^{8K-2}, & \text{disorder} \end{cases}$$

δQ is compensated by $V(x)$ *randomly* → Local commensuration

→ **Localization at zero T?**

Bosonized Action

Bosonization: $n = \frac{1}{\pi}\partial_x\theta, \quad j = -\frac{1}{\pi}\partial_t\theta.$

$$\mathcal{S} = \int_{\tau,x} \frac{1}{2\pi v K}\left[(\partial_\tau\theta)^2 + v^2(\partial_x\theta)^2\right] + \int_{\tau,x} V(x)\frac{1}{\pi}\partial_x\theta + \tilde{U}\int_{\tau,x} \cos\left[4\theta - \delta Q x\right].$$

$\mathcal{S}_0,\ K < 1$ **TRS disorder** **TRS umklapp**

Under a linear transformation of θ, TRS disorder is eliminated

$$\mathcal{S} \to \mathcal{S}_0 + \frac{\tilde{U}}{2}\int_{\tau,x}\left\{\eta(x)\,e^{i4\theta(\tau,x)} + \eta^*(x)\,e^{-i4\theta(\tau,x)}\right\} = \mathcal{S}_0 + \tilde{U}\int_{\tau,x}\cos\left[4\theta + \chi(x)\right],$$

- $\eta(x) = e^{i\chi(x)}$
- $\chi(x) = -\frac{4K}{v}\int_{-\infty}^{x} ds V(s) - \delta Q x$
- $\overline{\eta^*(x')\eta(x)} = e^{-\frac{8K^2}{v^2}\Delta|x-x'|}e^{-i\delta Q(x-x')}$, $\overline{\eta(x')\eta(x)} = \overline{\eta(x)} \to 0$

Mapping to Giamarchi-Schulz Model

Effective disorder averaged action:

$$\mathcal{S}_{U,dis} = -\Delta_U \sum_{a,b} \int_{\tau,\tau',x} \cos\left[4\left(\theta_a(\tau,x) - \theta_b(\tau',x)\right)\right], \qquad \Delta_U = \tilde{U}^2 \frac{K^2\Delta/v^2}{16(K^2\Delta/v^2)^2 + \delta Q^2/4}.$$

Also see derivations in Kainaris, *et al* (2014).

- Giamarchi-Schulz Model with $K \to 4K$

  Giamarchi-Schulz Model: Bosonization form of 1D disordered Luttinger liquid

  $$\mathcal{S}_{\mathrm{GS},dis} = -\Delta_{\mathrm{GS}} \sum_{a,b} \int_{\tau,\tau',x} \cos\left[2\left(\theta_a(\tau,x) - \theta_b(\tau',x)\right)\right].$$

  Localization for $K < 3/2$.

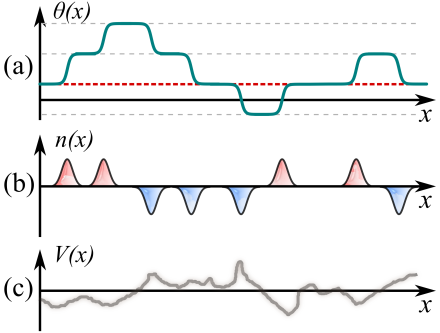
- 1D Bose glass RG with $K_c = 3/8$.

  Wu, Bernevig, and Zhang (2006); Xu and Moore (2006)

Physical intepretation? What is localized?

$K \to 0$, $\delta Q = 0$, Gaussian Disorder

$$H \approx \int_x V(x) \frac{1}{\pi} \partial_x \theta + \tilde{U} \int_x \cos(4\theta)$$



"Imry-Ma" argument:

$\to$ No long-ranged order

Diluted kinks and anti-kinks

Ground state:

Spontaneous **TRS** breaking

Spin-glass fashion

$\delta Q \neq 0 \to V(x)$ is shifted by $v\delta Q \to$ Similar ground state

Luther-Emery Point ($K = 1/4$): Refermionization

A 1D massive $e/2$ Dirac fermion with scalar disorder

$$\hat{H}_{LE} = -iv \int dx \left[\Psi_R^\dagger \partial_x \Psi_R - \Psi_L^\dagger \partial_x \Psi_L \right] + M \int dx \left[\Psi_R^\dagger \Psi_L + \Psi_L^\dagger \Psi_R \right] + \frac{1}{2} \int dx [V(x) + v\delta Q] \left[\Psi_R^\dagger \Psi_R + \Psi_L^\dagger \Psi_L \right],$$

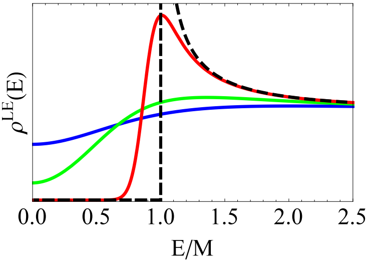
Anderson Localization Bocquet (1999)

$\rho^{\mathrm{LE}}(x) = \frac{1}{2} n(x)$ and $j^{\mathrm{LE}}(x) = \frac{1}{2} j(x)$.

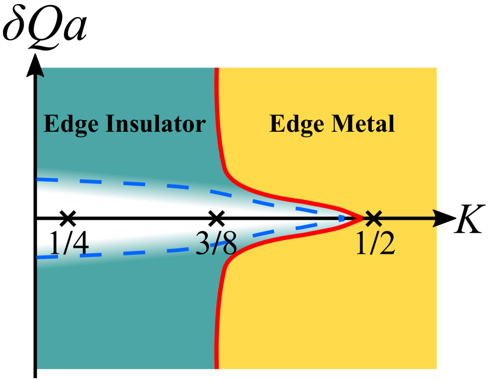
→ dc conductivity $= 0$

→ compressibility $\neq 0$

→ Gapless insulating state



Phase Diagram



Gaussain disorder:

$\delta Q \neq 0$: $K_c = 3/8$

Bounded disorder:

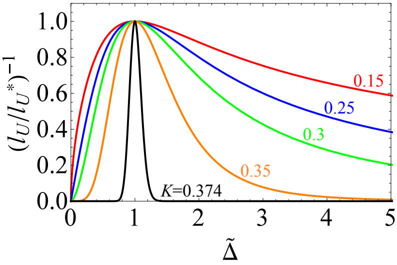
Fine structures near $\delta Q = 0$

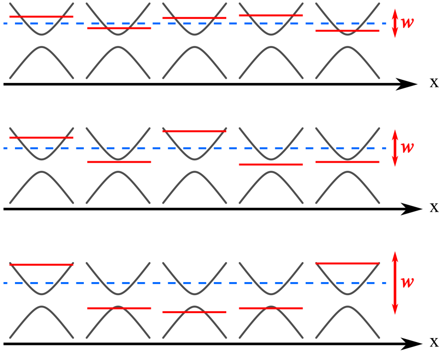
Gapped insulator - Glass - Metal

Non-monotonicity in Localization Length

$$\mathcal{S}_{U,dis} = -\Delta_U \sum_{a,b} \int_{\tau,\tau',x} \cos\left[4\left(\theta_a(\tau,x) - \theta_b(\tau',x)\right)\right], \qquad \Delta_U = \tilde{U}^2 \frac{K^2\Delta/v^2}{16(K^2\Delta/v^2)^2 + \delta Q^2/4}.$$

$$l_U \sim \alpha \Delta_U^{-1/(3-8K)}$$



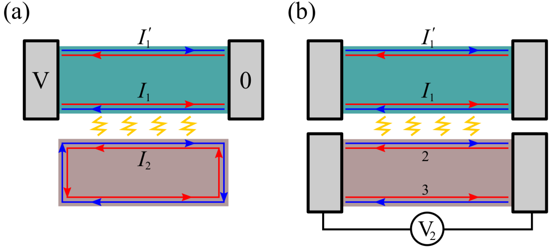


Localization-driven correlated states of two isolated interacting helical edges

Ref: Chou, arxiv.org/abs/1806.02353

Two Isolated Helical Luttinger Liquids

Two isolated TI edges with inter-edge Coulomb interaction.



- No single particle tunneling
- Interplay of inter-edge interaction and disorder
- Platform: Coulomb drag experiments for TI edges

Disorder and Interaction Terms

TRS Inter-edge Interactions

$$\hat{H}_- = U_- \int dx \left[e^{-i\delta Q_- x} L_1^\dagger R_1 R_2^\dagger L_2 + \text{H.c.} \right], \; \delta Q_- = Q - 2k_{F1} + 2k_{F2}$$

$$\hat{H}_+ = U_+ \int dx \left[e^{-i\delta Q_+ x} L_1^\dagger R_1 L_2^\dagger R_2 + \text{H.c.} \right], \; \delta Q_+ = Q - 2k_{F1} - 2k_{F2}$$

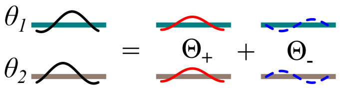
$$\hat{H}_{\text{LL}} = U \int dx \left(R_1^\dagger R_1 + L_1^\dagger L_1 \right) \left(R_2^\dagger R_2 + L_2^\dagger L_2 \right),$$

TRS disorder

$$\hat{H}_V = \sum_{a=1,2} \int dx \, V_a(x) \left[R_a^\dagger(x) R_a(x) + L_a^\dagger(x) L_a(x) \right],$$

Symmetric and Anti-symmetric Modes

$$\Theta_{\pm} = \frac{1}{\sqrt{2}}\left(\theta_1 \pm \theta_2\right)$$



TRS Inter-edge Backscatterings

$$\hat{H}_{b,-} = \frac{U_-}{2\pi^2\alpha^2}\int dx \cos\left[2\sqrt{2}\Theta_- - \delta Q_- x\right],$$

$$\hat{H}_{b,+} = \frac{U_+}{2\pi^2\alpha^2}\int dx \cos\left[2\sqrt{2}\Theta_+ - \delta Q_+ x\right],$$

Luttinger Interactions

$$\hat{H}_{b,\mathrm{LL}} = \frac{U}{2\pi^2}\int dx \left[\left(\partial_x\Theta_+\right)^2 - \left(\partial_x\Theta_-\right)^2\right].$$

Generically, $1 > K_- > K_+$. Klesse and Stern (2000)

TRS disorder

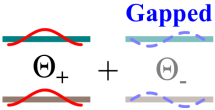
$$\hat{H}_{b,V} = \int dx\, V_+(x)\frac{1}{\pi}\partial_x\Theta_+ + \int dx\, V_-(x)\frac{1}{\pi}\partial_x\Theta_-, \quad V_{\pm} = \frac{1}{\sqrt{2}}\left(V_1 \pm V_2\right)$$

Clean Identical Edges ($k_{F,1} = k_{F,2}$)

TRS Inter-edge Backscattering

$$\hat{H}_{b,-} = \frac{U_-}{2\pi^2\alpha^2} \int dx \cos\left[2\sqrt{2}\Theta_-\right],$$
$$\hat{H}_{b,+} = \frac{U_+}{2\pi^2\alpha^2} \int dx \cos\left[2\sqrt{2}\Theta_+ - \delta Q_+ x\right].$$

δQ_+ is nonzero generically. $\hat{H}_{b,+}$ can be ignored.



- Θ_- is gapped when $K_- < 1$
- Θ_+ remains metallic
- Symmetric interlocked fluid → Infinite $T = 0$ drag resistivity
  Nazarov and Averin (1998); Klesse and Stern (2000)

Localization of Inter-edge Modes

Bosonized action:

$$\mathcal{S}_{0,\pm} = \frac{1}{2\pi v_\pm K_\pm} \int d\tau dx \, \left[(\partial_\tau \Theta_\pm)^2 + v_\pm^2 (\partial_x \Theta_\pm)^2 \right],$$

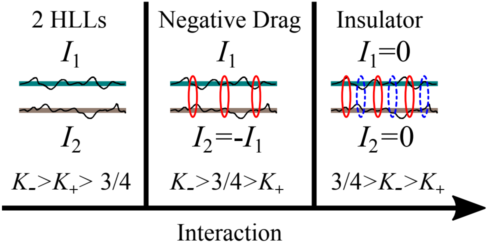
$$\mathcal{S}_{V,\pm} = \int d\tau dx \, V_\pm(x) \frac{1}{\pi} \partial_x \Theta_\pm,$$

$$\mathcal{S}_{U,\pm} = \frac{U_\pm}{2\pi^2 \alpha^2} \int d\tau dx \, \cos\left[2\sqrt{2}\Theta_\pm - \delta Q_\pm x \right].$$

For $K_\pm < 3/4$ and $\delta Q_\pm \neq 0$, $\longrightarrow$ inter-edge $\pm$ mode is **localized**.

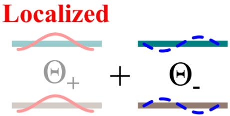
- $K_c = 3/4 > 3/8 \to$ Dominating over intra-edge instability
- Non-monotonic localization length in disorder strength
- Luther-Emery point at $K_\pm = 1/2$

Dirty TI Edges with Non-equal Denisties

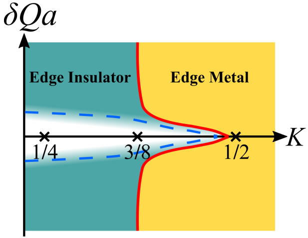


- Weak interactions $\rightarrow$ Two helical Luttinger liquids
- Both Θ_+ and Θ_- modes are localized,
  $\rightarrow$ A spontaneous TRS broken localized insulator
- Only Θ_+ mode is localized, $\rightarrow$ An interlocked fluid state.

New drag mechanism!



Summary (1)



Main Results

- Gapless insulating edges made of $e/2$ particles
- Non-monotonic localization length
- A possible scenario for InAs/GaSb systems

Experimental Signatures

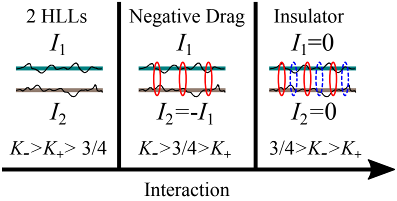
- Insulator-like finite T conductivity
- Mott-Berezinskii finite ac conductivity
- Collective depinning behavior in nonlinear I-V

Our theory also applies to helical hindge states of HOTIs.

Summary (2)

Main Results

- A localizing mechanism for correlation
- Inter-edge localized insulator
- Negative drag for non-identical edges
- Proposed experimental signatures



To be done ...

- Finite temperature and finite size
- ac conductivity for edge-loop setup
- WTe_2 and WSe_2