Slow relaxation in quasi-periodically driven random spin chains

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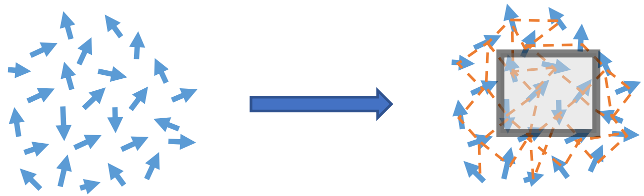
Phases in Isolated Quantum Systems

- Interested in non-equilibrium properties of highly excited many-body states
- New Quantum Dynamic Phases:
unitary evolution + coherent external fields
- Many recent experiments on systems which are well isolated from dissipation:

**Ultracold Atoms, Trapped Ions,
Superconducting Qubits, NV Centers**

Thermalization of Quantum Systems

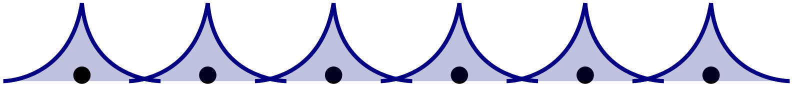
- ETH for Static Unitary Evolution:
  pure state stay pure, but subsystems thermal
  [Srednicki; Deutsch]



- Driven Quantum Systems:
  Thermalize to infinite temperate, governed by (entanglement) Hydrodynamics
  [Nahum, Ruhman, Vijay, Haah; von Keyserlingk, Rakovszky, Pollmann, Sondhi]

Many-Body Localization in 1D (Static)

- Complete failure to thermalize
- Eigenstates are product states of local integrals of motion (LIOM / L-bits)



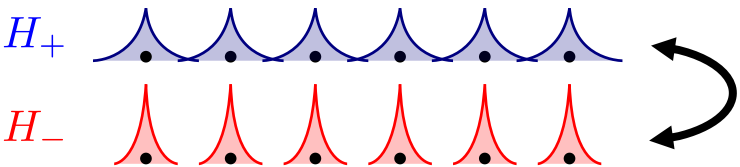
$$|\Psi\rangle = |\tau_1^z \tau_2^z \cdots \tau_L^z\rangle$$

- Excited states ~ equilibrium ground states
  - Area Law Entanglement: $S_{bp} \sim \text{const.}$
  - Have Phase Transitions [ETH-MBL / MBL-MBL]
  - Support symmetry breaking / topological phases

[Anderson; Fleishman-Anderson; Basko, Aleiner, Altshuler;
Gornyi, Mirlin, Polyakov; Pal, Huse; Serbyn, Papić, Abanin; Huse, Nandkishore, Oganesyan]

MBL with Periodic Driving

- Rapid, periodic, local driving keeps system MBL
  [Ponte, Papić, Huveneers, Abanin; Lazarides, Das, Moessner]

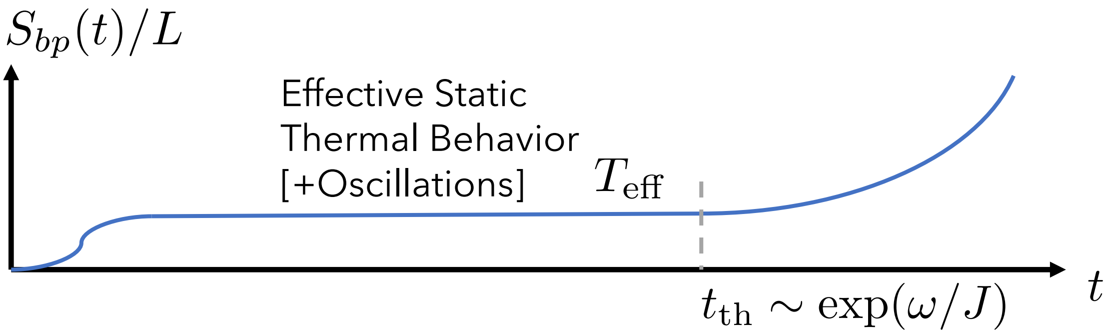


$$U_F = e^{-i\lambda H_+} e^{-i\lambda H_-}$$
$$= e^{-i(2\lambda) H_{\mathrm{F}}}$$

- Stability: can there be many-body resonances?
  - Single frequency (perturbative): exponential matrix element suppression, finite energy difference
  - No energy absorption; Modified L-bit structure
- Actual resonances with random driving or at low frequencies (Many-Body Landau-Zener)

Clean Pre-Thermal Floquet Phases

- No disorder, but heating rate can be exponentially suppressed with fast driving
  [Abanin, De Roeck, Ho, Huveneers; Else, Bauer, Nayak]



- Can realize Floquet phases
- Cool pre-thermal regime: quantum phases

Quasiperiodic Sequences

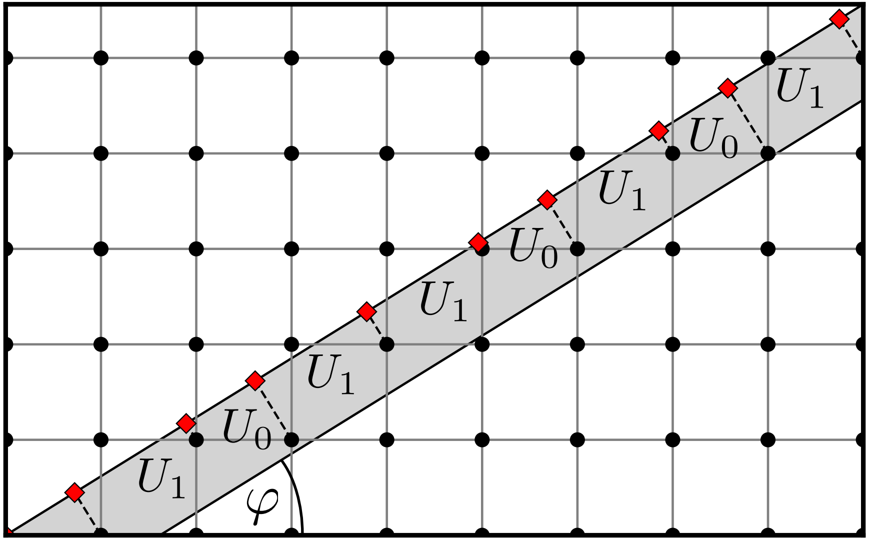
- Fibonacci Drive: $U_n = U_{n-2}U_{n-1}$

$$
\begin{aligned}
U_0 \\
U_1 \\
U_0U_1 &= U_2 \\
U_1U_0U_1 &= U_3 \\
U_0U_1U_1U_0U_1 &= U_4 \\
U_1U_0U_1U_0U_1U_1U_0U_1 &= U_5
\end{aligned}
$$

- Efficient to simulate: need $\sim \log n$ multiplications

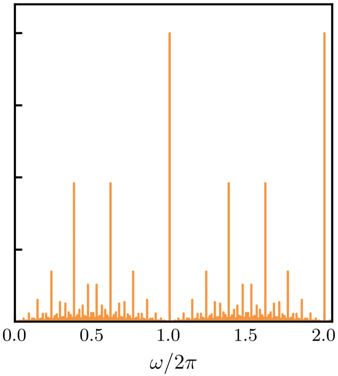
Quasiperiodicity From Projection

- Project 2D lattice at irrational angle φ:



$$U_5 = U_1 U_0 U_1 U_0 U_1 U_1 U_0 U_1$$

Quasiperiodic Spectrum



[Levine-Steinhardt]

- Incommensurate Frequencies: $1, \varphi$
- Spectrum is dense, but discreet

- What happens to MBL?
  All transitions possible but suppressed

$$\Delta \sim |E - n - m\varphi| \quad \text{vs} \quad \Gamma \sim (\gamma_1/J)^n(\gamma_2/J)^m$$

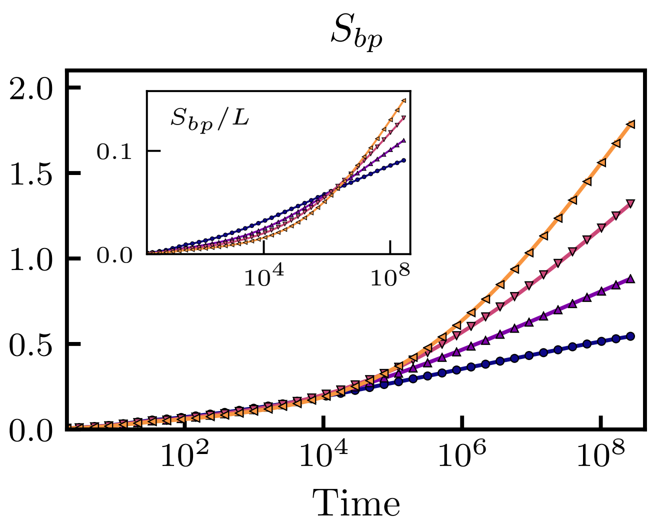
- Protocol:

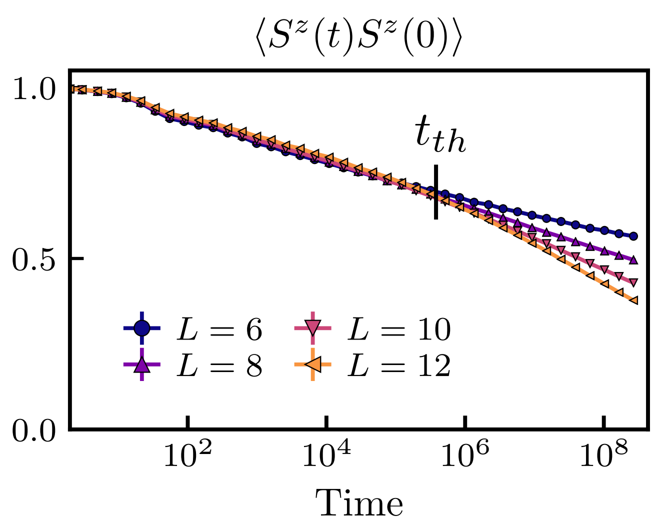
$$U_0 = e^{-i\lambda H_+}, U_1 = e^{-i\lambda H_-}$$

$$H_\pm = \pm\delta J \sum_i \mathbf{S}_i \cdot \mathbf{S}_{i+1} + \sum_i h_i S_i^z, \quad h_i \in [-2\pi, 2\pi)$$

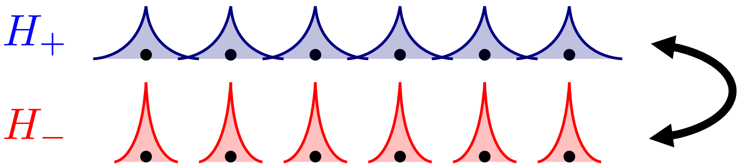
Slow Thermalization

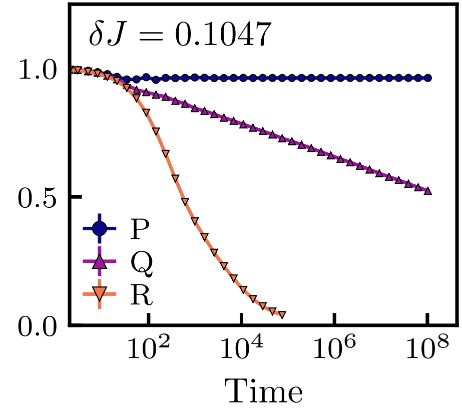
- Three Regimes:
  Initially Coherent, Glassy Decay, Fast Thermalization





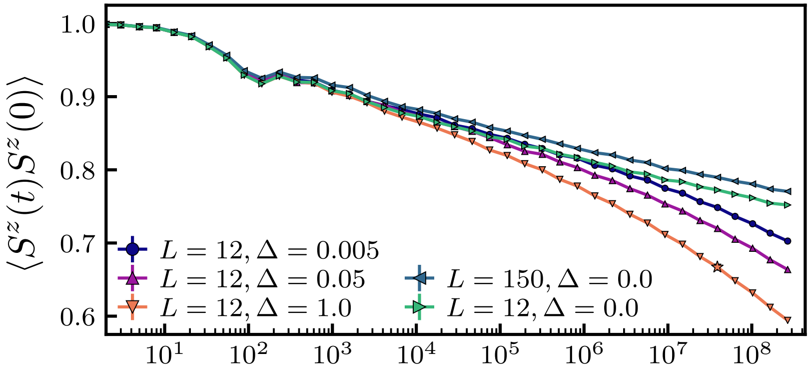
Are there L-Bits?





- $\langle S^z(t) S^z(0) \rangle$ does not saturate: no conserved spin component

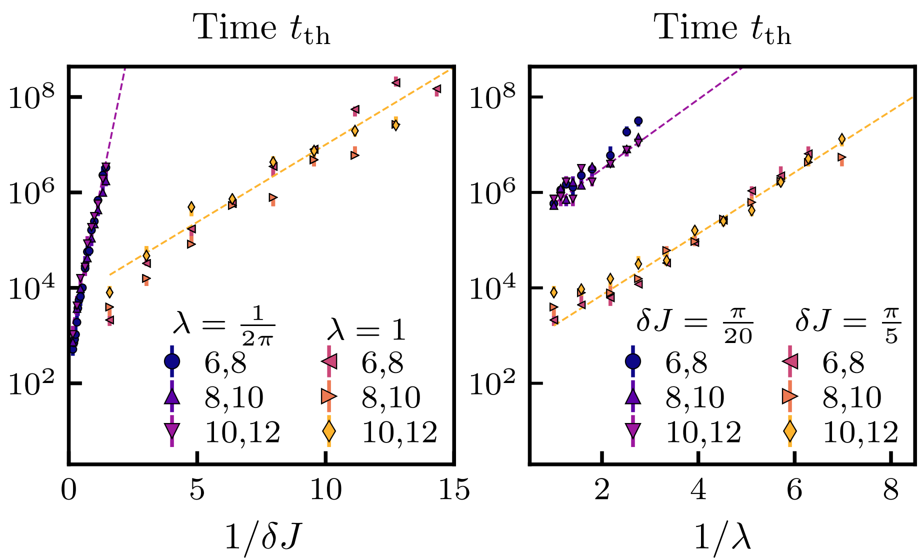
- But log decay in non-interacting limit:



Glassy Regime is Long-lived

- Thermalization time t_{th} depends exponentially on parameters

$$t_{\mathrm{th}} \sim e^{1/\lambda}, t_{\mathrm{th}} \sim e^{1/\delta J}$$



Effective Hamiltonian

- Effective Hamiltonian for the glassy regime?
- Periodic Driving: Floquet Operator

$$U_F = e^{\Omega_0} e^{\Omega_1} = e^{-iH_{\text{eff}}T},$$

$$U_F \sim e^{\Omega_0+\Omega_1+\frac{1}{2}[\Omega_0,\Omega_1]+\frac{1}{12}([\Omega_0,[\Omega_0,\Omega_1]]+[[\Omega_0,\Omega_1],\Omega_1])+\cdots}$$

- For Fibonacci Drive:

$$\left.\begin{aligned} U_0 &= U_0 \\ U_1 &= U_1 \\ U_0U_1 &= U_2 \\ U_1U_0U_1 &= U_3 \\ U_0U_1U_1U_0U_1 &= U_4 \\ U_1U_0U_1U_0U_1U_1U_0U_1 &= U_5 \end{aligned}\right\} \begin{aligned} & t_n = F_n \sim \varphi^n \\ & U_n = e^{\Omega_n} \to e^{-iH_{\text{eff}}\varphi^n} \end{aligned}$$

Recursive Magnus Expansion

- Fibonacci Drive has 'deflation' property.
  Rule $U_0 \to U_1, U_1 \to U_0 U_1$ generates next term:

$$\Omega_{n+1}(\Omega_0, \Omega_1) = \Omega_n(\Omega_1, \Omega_0 * \Omega_1)$$
$$\Omega_0 * \Omega_1 = \log(\exp \Omega_0 \exp \Omega_1)$$

- Expand onto all possible commutators

$$\begin{aligned}\Omega_n =& a_n \Omega_0 + b_n \Omega_1 + h_n [\Omega_0, \Omega_1] \\ &+ f_n [\Omega_0, [\Omega_0, \Omega_1]] + g_n [[\Omega_0, \Omega_1], \Omega_1] + \ldots\end{aligned}$$

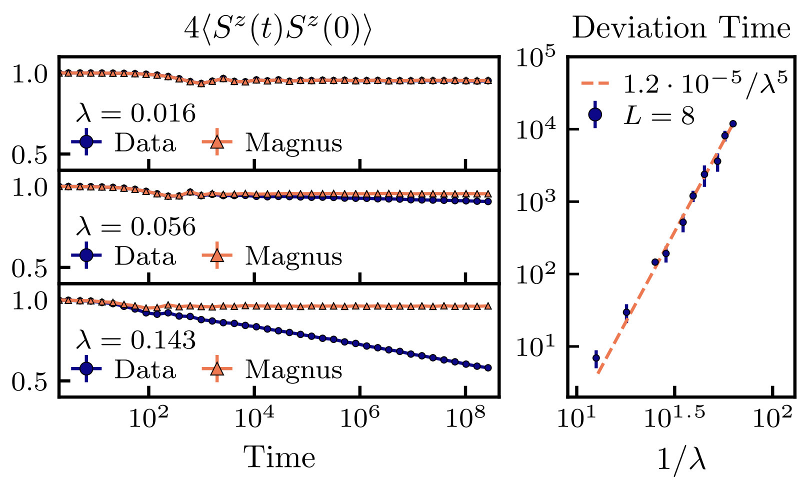
  Order-by-order difference equations are solved recursively

Breakdown of Magnus Expansion

- Asymptotic properties, order k of expansion

$$a_n \sim \varphi^{n|k-2|}$$

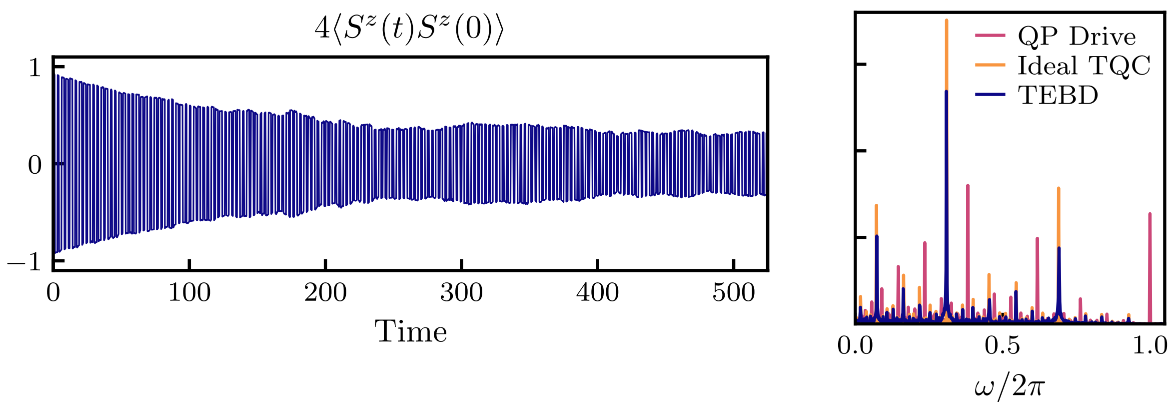
- Hamiltonian interpretation breaks down at *k=4*
- Up to *k=3:*
  - Expansion works
  - No Decay
  - Resummation possible?



Time Quasicrystal

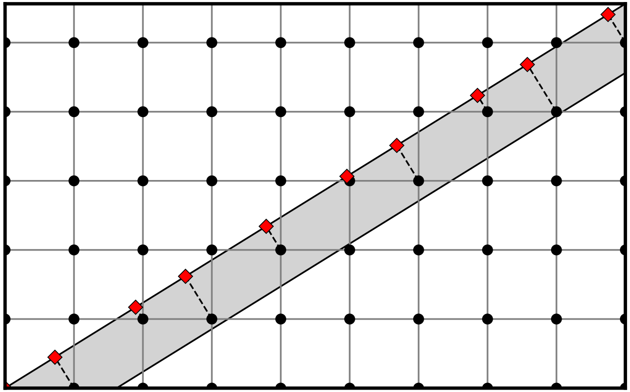
- Use glassy regime for new quantum phases [holds as good as log decay]
- Simplest case: break down a symmetry of the quasiperiodic time drive ['time quasicrystal']

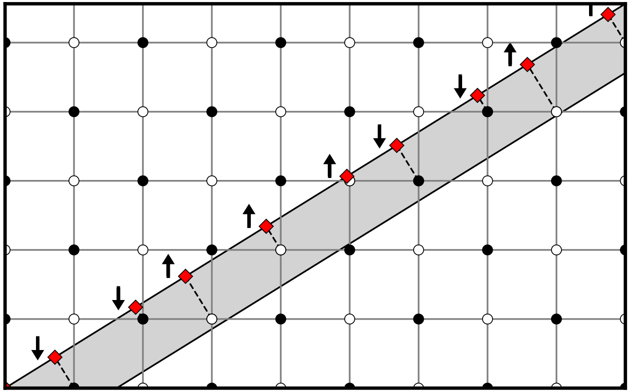
$$U_0 = e^{-i\theta \sum_i S_i^x}, U_1 = e^{-i\lambda \sum_i (J_i S_i^z S_{i+1}^z + h_i^z S_i^z + h_i^x S_i^x)}.$$



Time Quasicrystal Symmetry

- Time Quasicrystal is symmetry breaking in the 2d space defining sequence
- Pattern survives the projection to 1d
- Dual to QP Majoranas [Peng, Refael]
- Generalize to other topological phases, but projection





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