Hyperbolic and Flat-Band Lattices in Circuit QED

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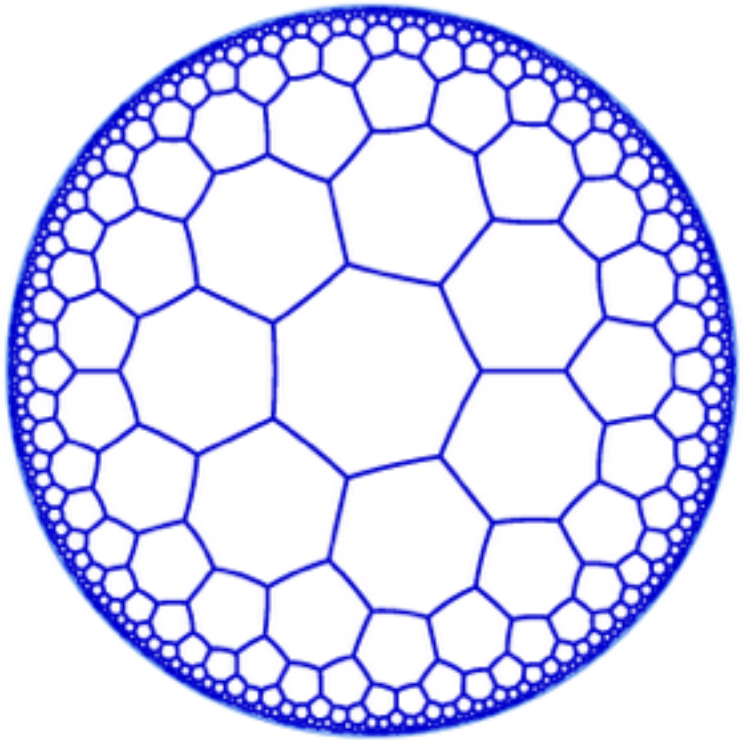
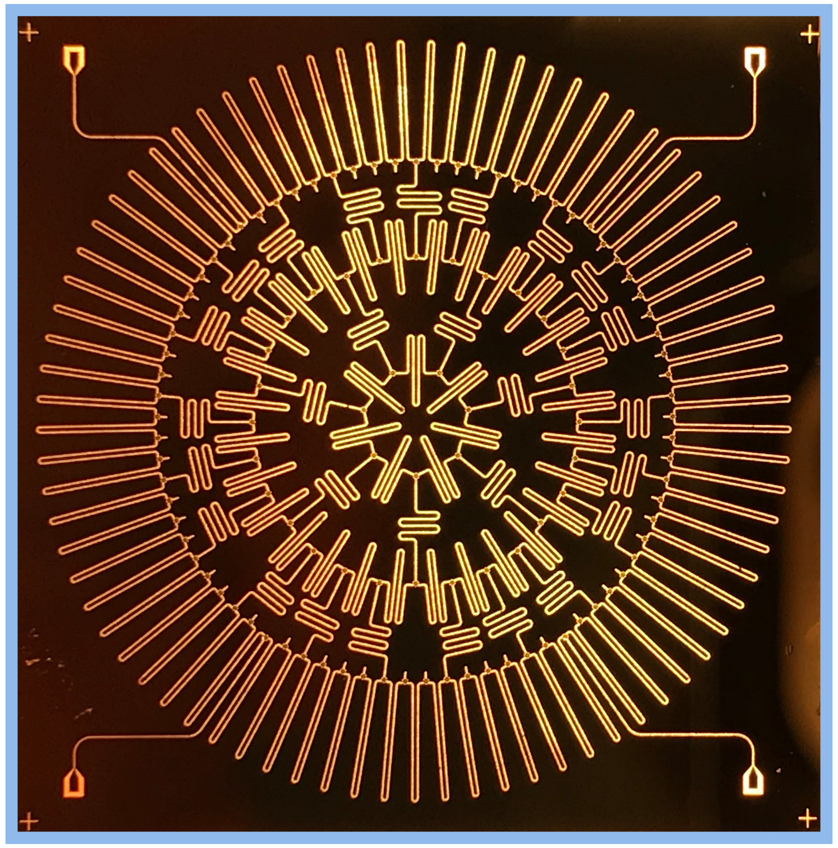
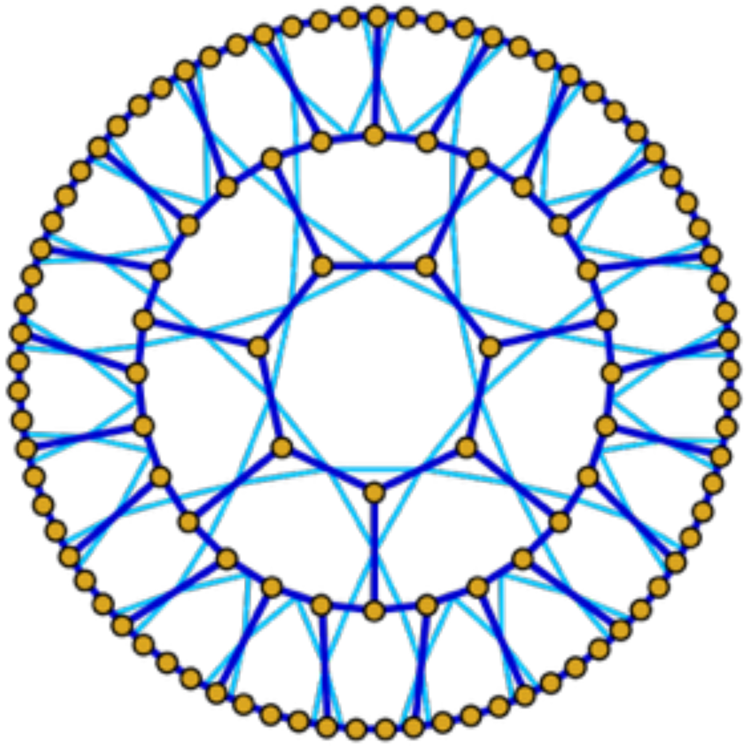






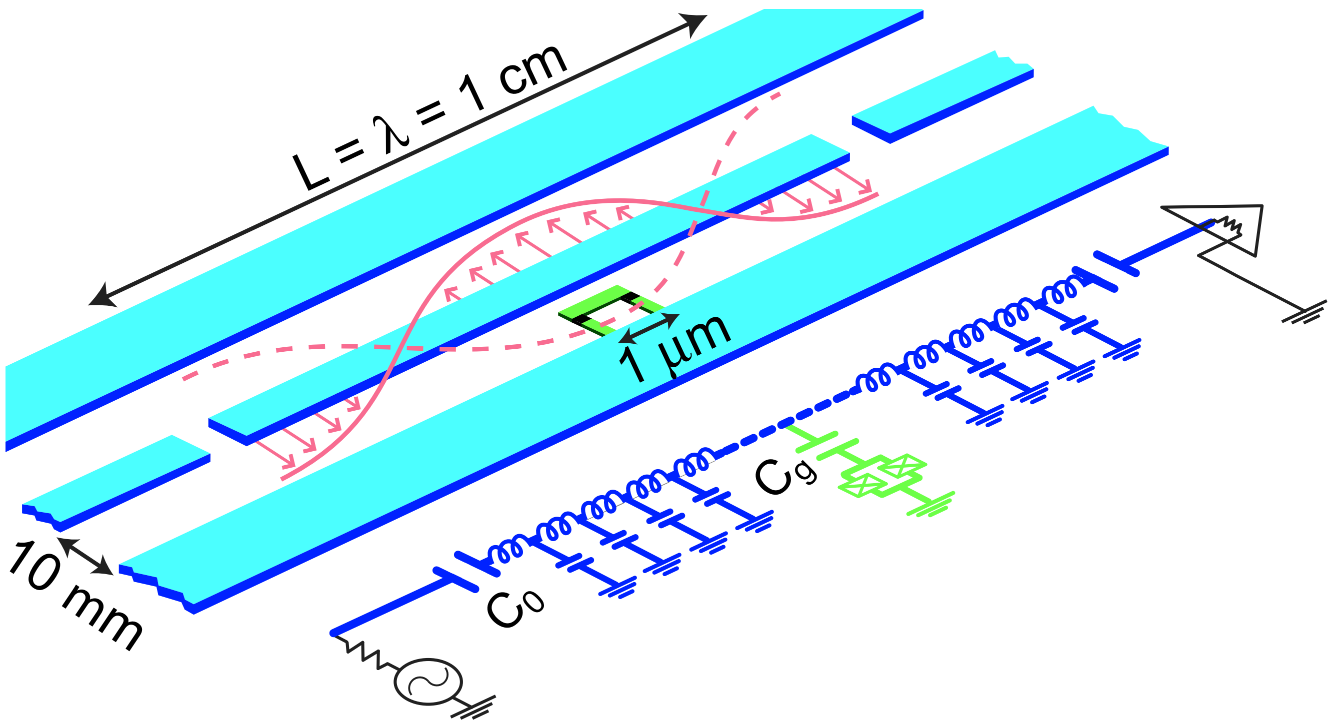
KITP, Sept 12th 2018

Outline

- Quantum simulation with circuit QED lattices
  - Microwave resonators
  - Superconducting qubits
  - Interacting photons
- Hyperbolic lattices
  - Connections to GR, AdS, Comp Sci, Math
  - Projection to flat space
  - Deformable resonators
- Flat-band lattices
  - Line graphs
  - Maximal Gaps

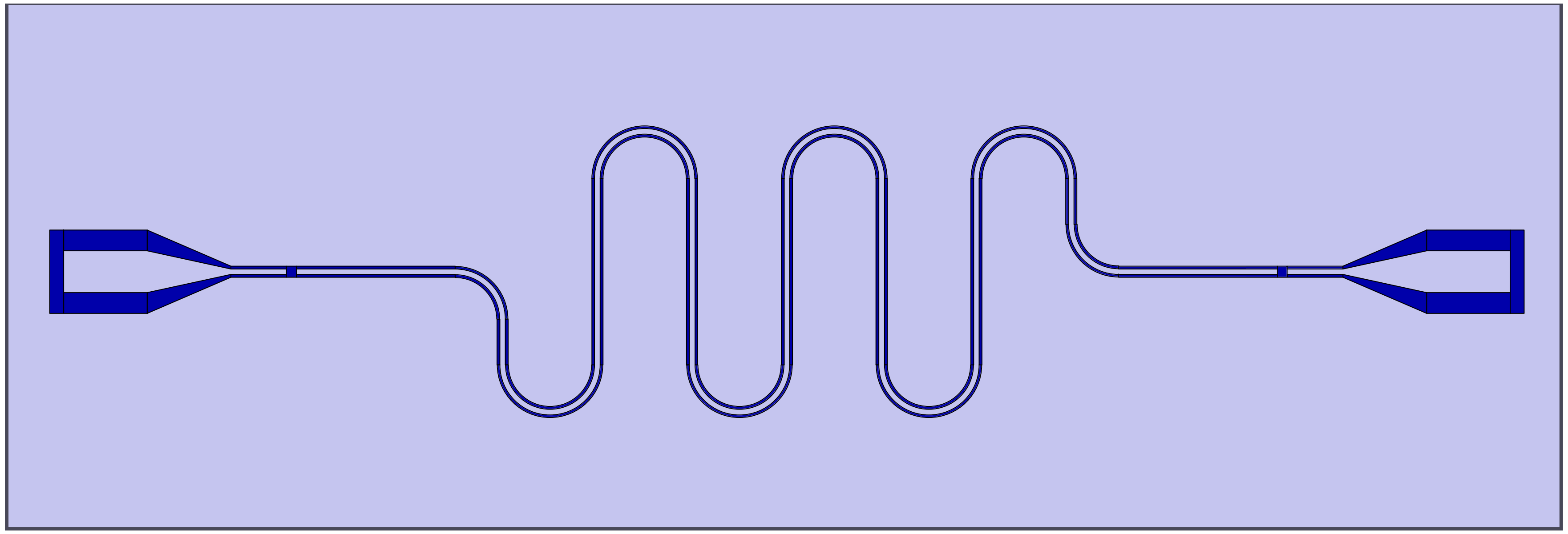
Microwave Coplanar Waveguide Resonators

- 2D analog of coaxial cable
- Cavity defined by cutting center pin
- Voltage antinode at "mirror"



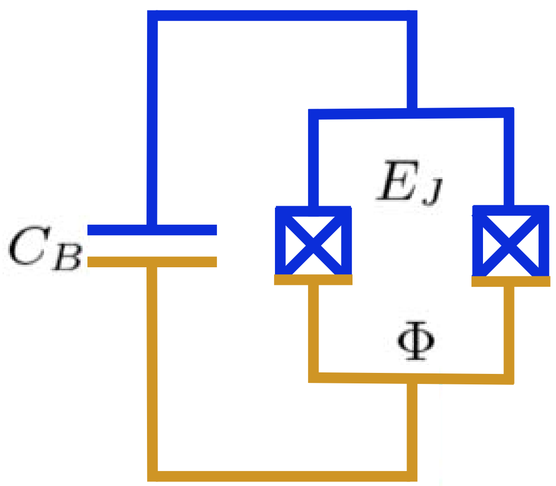
Harmonic oscillator

$$\hat{H} = \frac{1}{2C}\hat{n}^2 + \frac{1}{2L}\hat{\varphi}^2$$



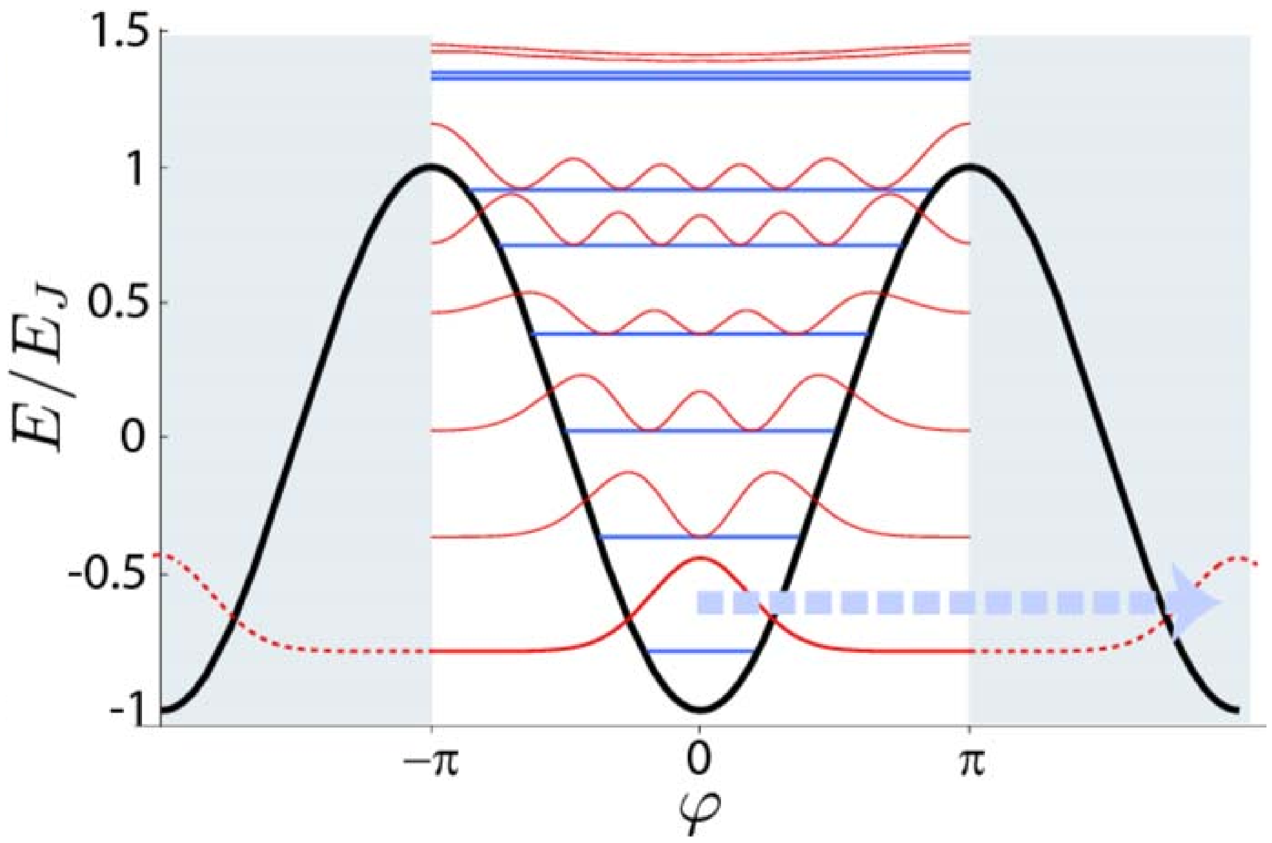
Blais *et al.*, PRA **69**, 062320 (2014)

Transmon Qubit



Anharmonic oscillator

$$\hat{H} = 4E_C\,\hat{n}^2 - E_J \cos\hat{\varphi}$$



Koch *et al*. PRA **76**, 042319 (2007)

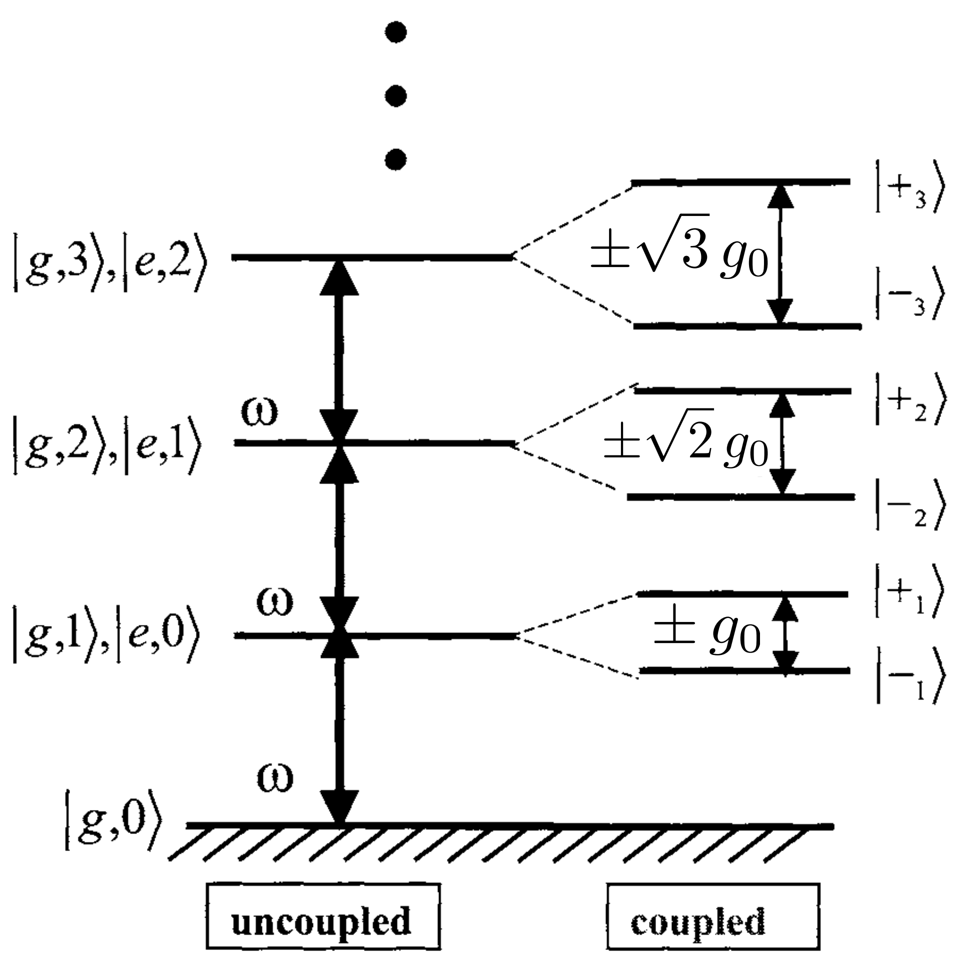
Non-Linearities and Photon-Photon Interactions

Qubit-Cavity

(Jaynes-Cummings Model)

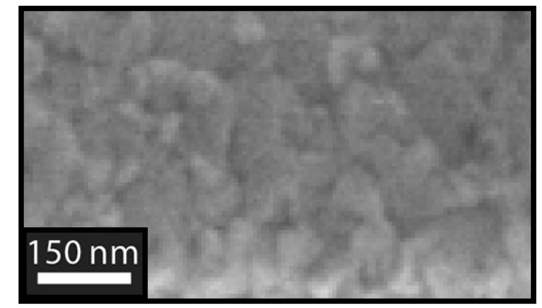
$$H_{JC} = \omega_c a^\dagger a + \frac{1}{2}\omega_q \sigma_z + g_0(a^\dagger \sigma^- + a\,\sigma^+)$$

$$|\pm_n\rangle = \frac{1}{\sqrt{2}}\left(|g,n\rangle \pm |e,n-1\rangle\right),$$



Ye *et al*. Advances in AMO Physics **49** (2003)

Al Film



- Oxide not perfectly uniform

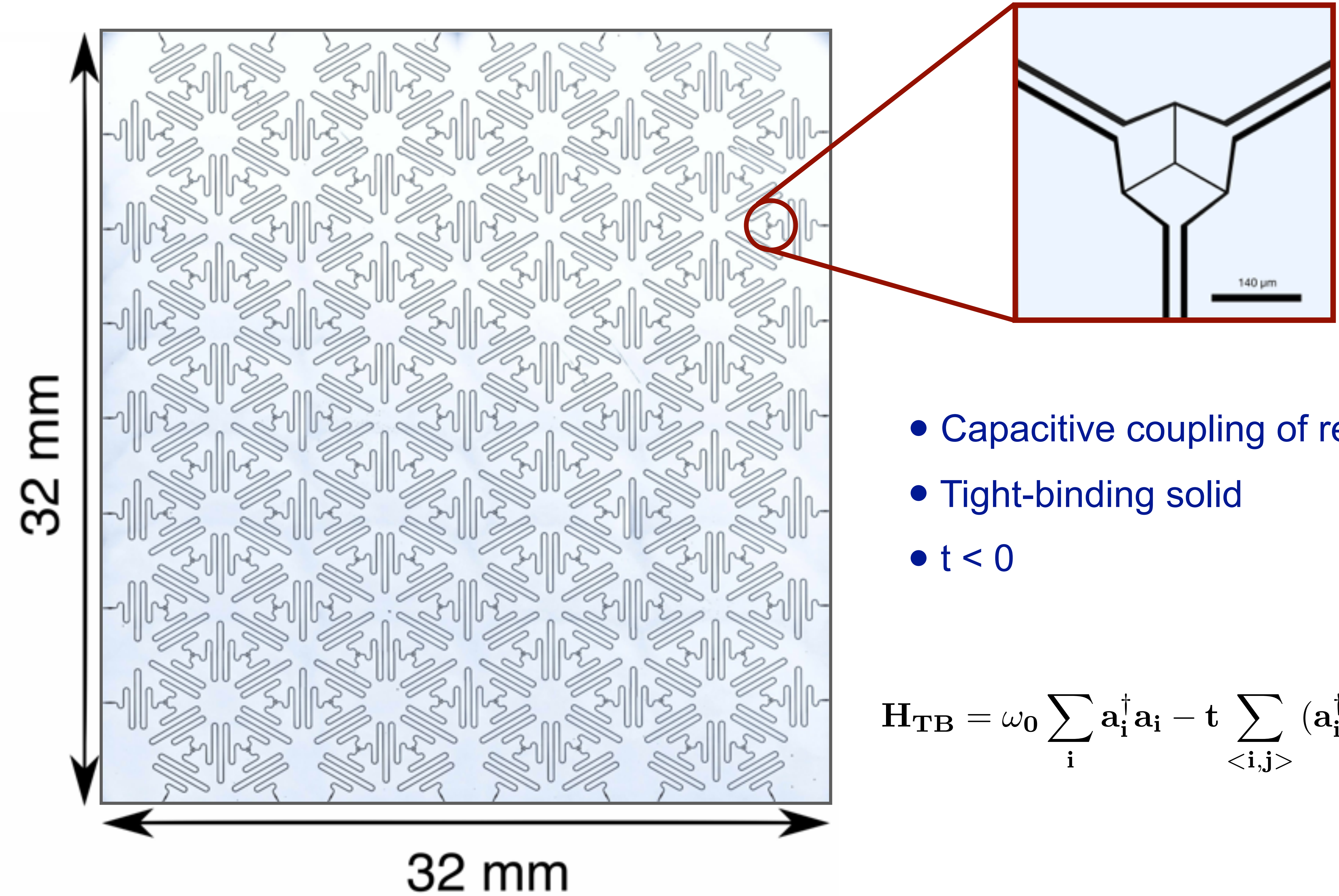
Kinetic Inductor

- Inductance from electron momentum
- Dependent on carrier density

$$H_{KI} = \left(\omega_c + \chi_{eff}\, a^\dagger a\right) a^\dagger a$$

Vissers *et al*. APL **107**, 062601 (2015)

CPW Lattices

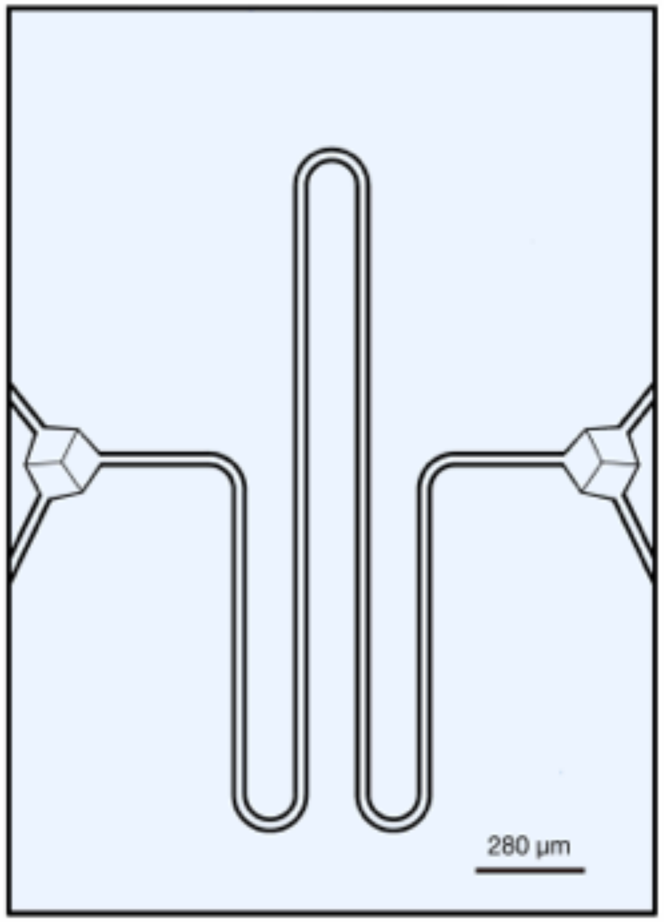


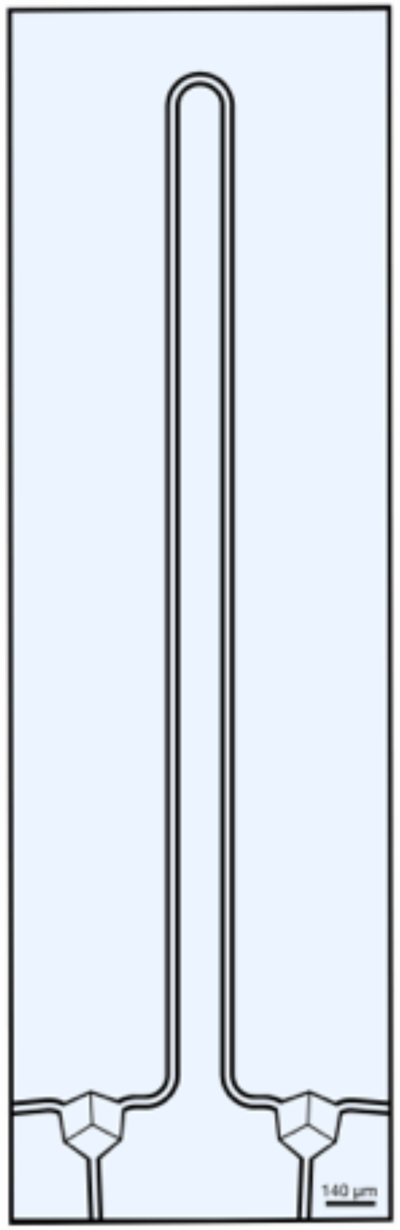
- Capacitive coupling of resonators
- Tight-binding solid
- $t < 0$

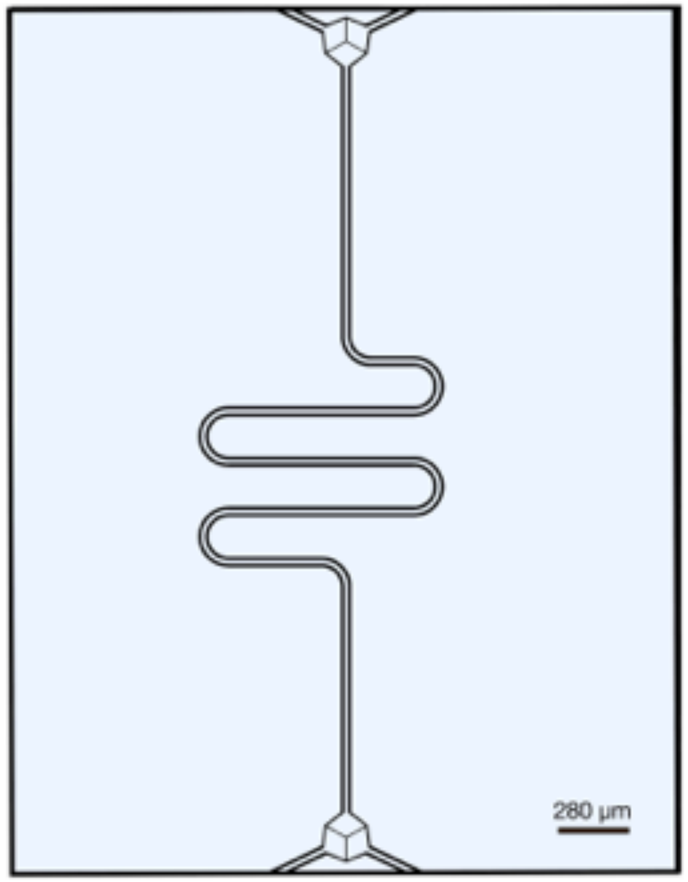
$$\mathbf{H_{TB}} = \omega_0 \sum_{\mathbf{i}} \mathbf{a_i^\dagger a_i} - \mathbf{t} \sum_{<\mathbf{i,j}>} (\mathbf{a_i^\dagger a_j} + \mathbf{a_j^\dagger a_i})$$

Houck *et al*. Nat Phys **8**, (2012)
Underwood *et al*. PRA **86**, 023837 (2012)

Deformable Resonators



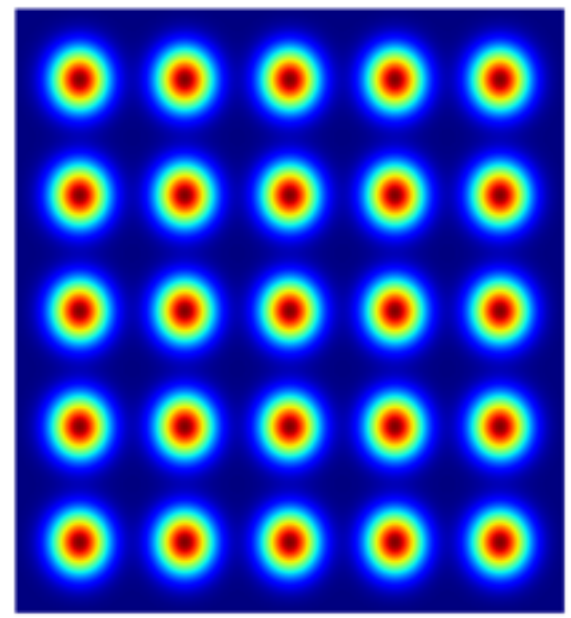




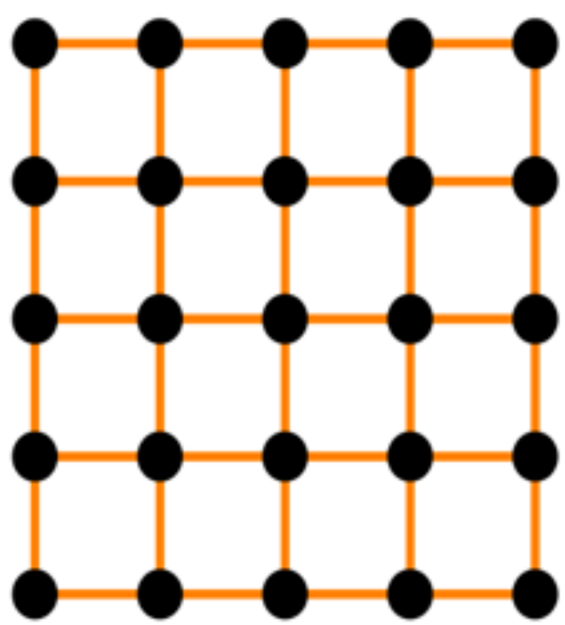
- Frequency depends only on length
- Coupling depends on ends
- “Bendable”

The Graph is Everything

Regular Lattice



Regular Tight-Binding Graph



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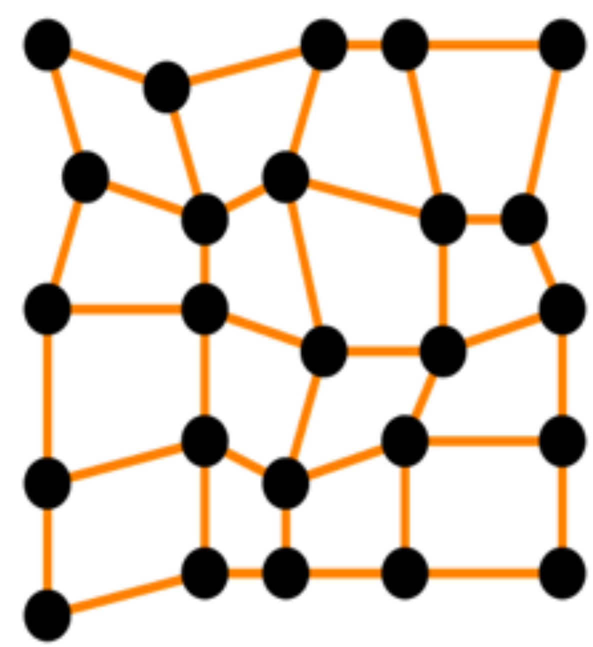
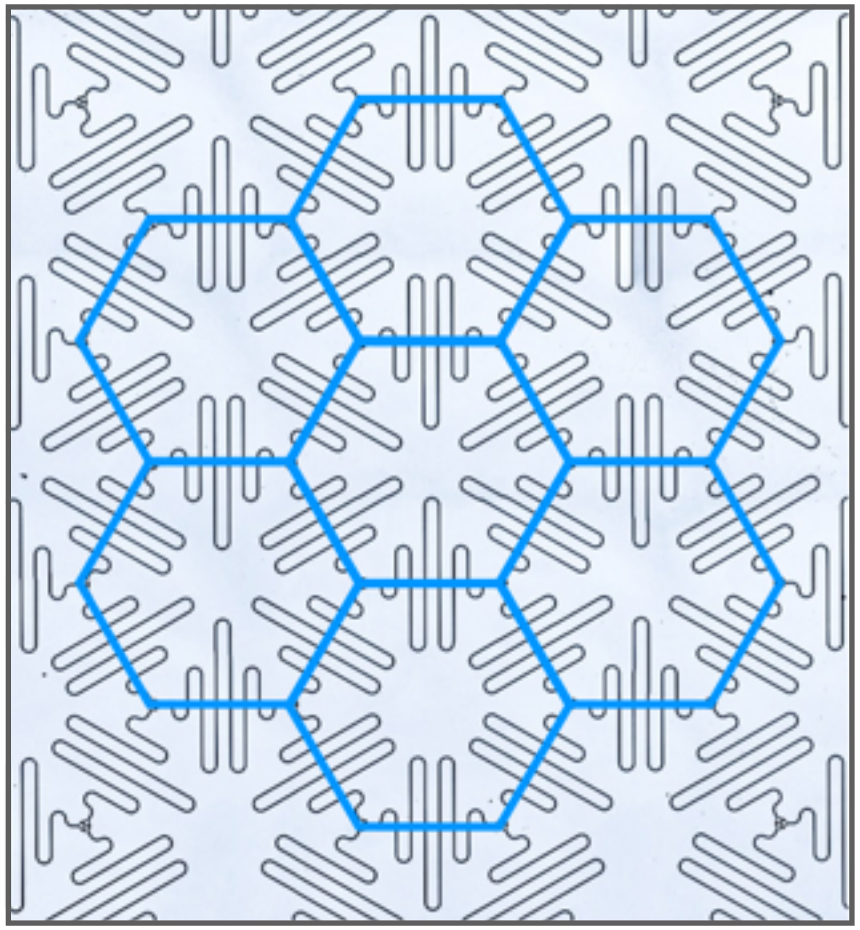
Alternate Tight-Binding Graph



Layout and Effective Lattices

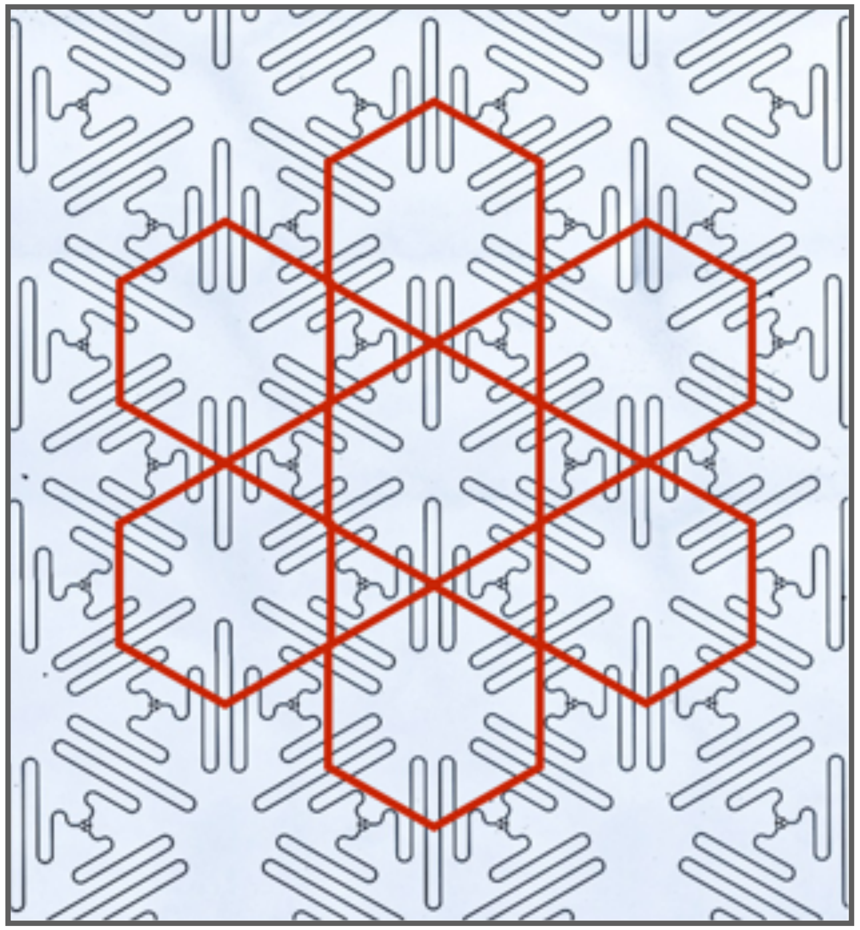
Resonator Lattice



- An *edge* on each resonator

Layout X

Effective Photonic Lattice



- A *vertex* on each resonator

Line Graph $L(X)$

Outline

- Quantum simulation with circuit QED lattices
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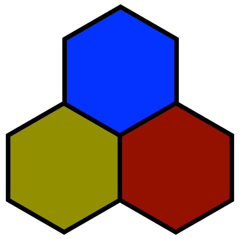
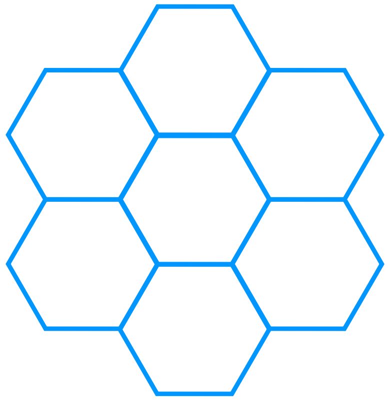
  - Deformable resonators
- Flat-band lattices
  - Line graphs
  - Maximal Gaps

Projecting to Flat 2D

n = 6

flat





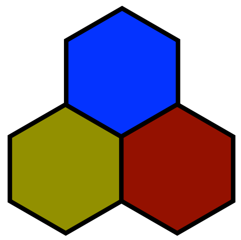
Distance is not preserved.

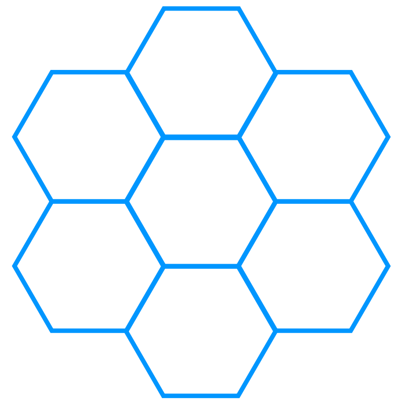
Distance is not preserved.

Planar and Non-Planar Lattices

n = 6

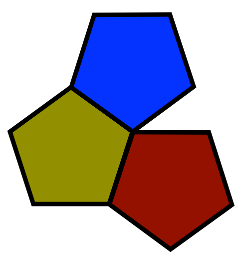
flat

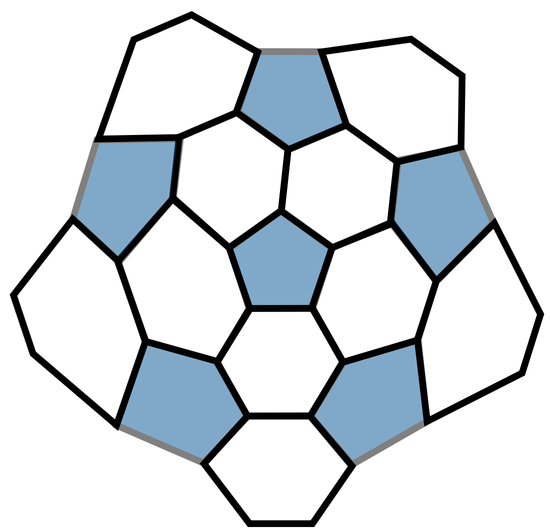




n = 5

spherical



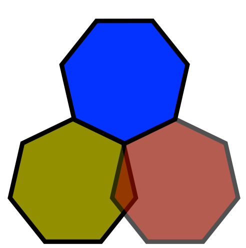


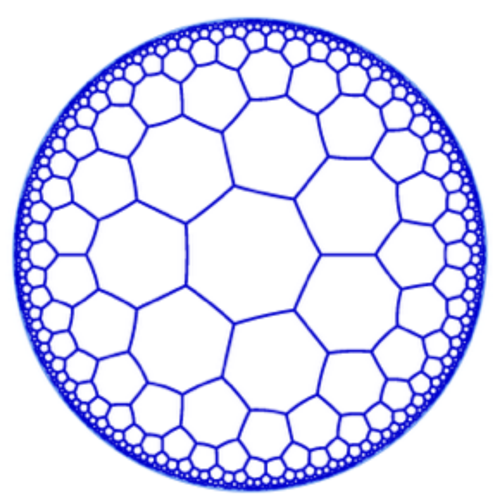
Distance is not preserved.

t *is* preserved.

n = 7

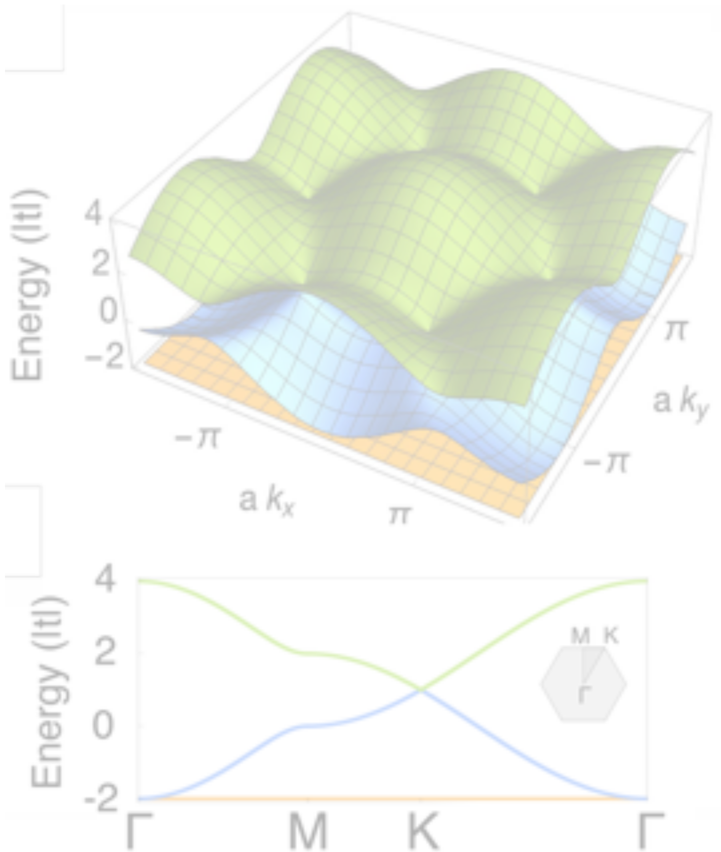
hyperbolic





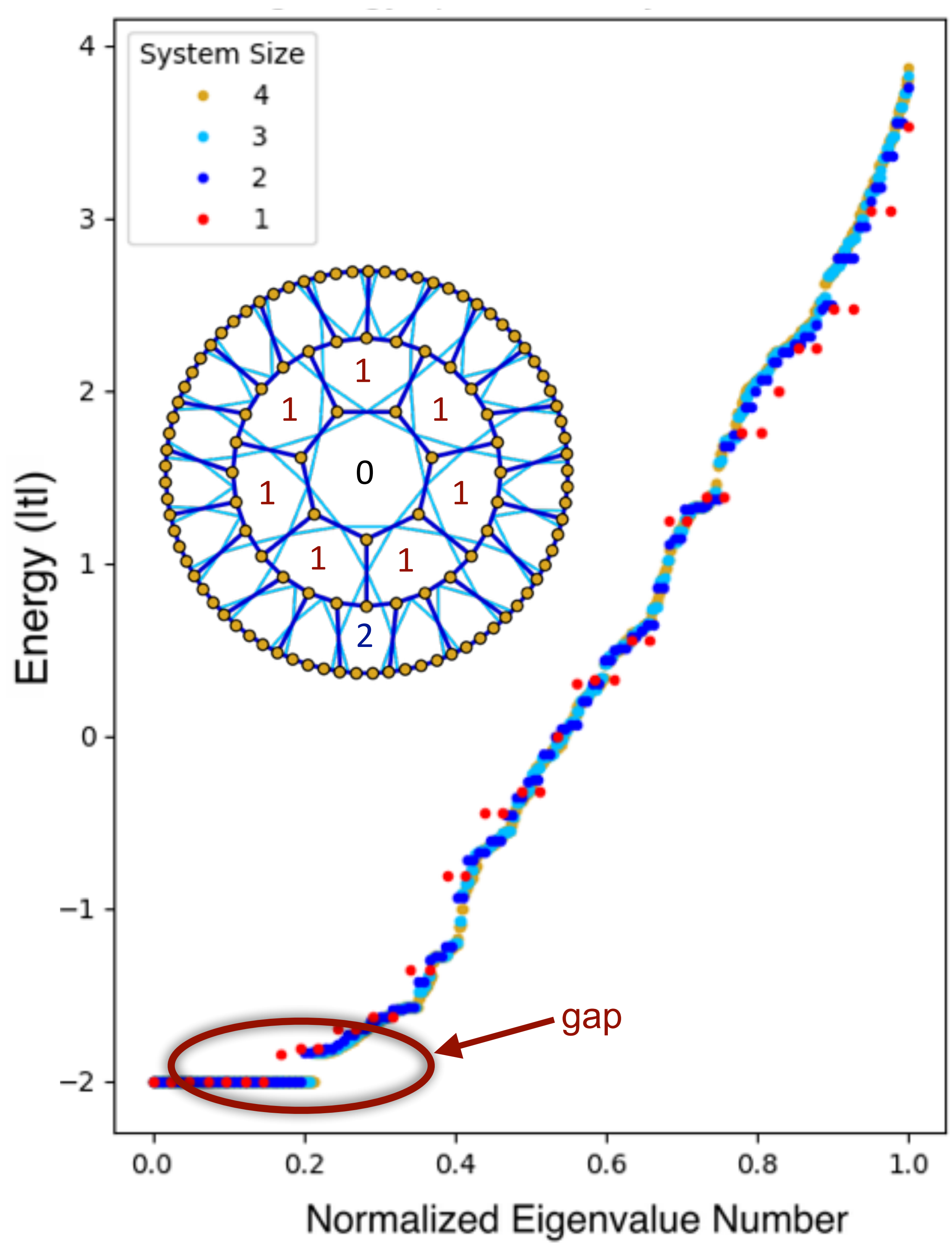
Graph *is* preserved.

Band Structure Calculations

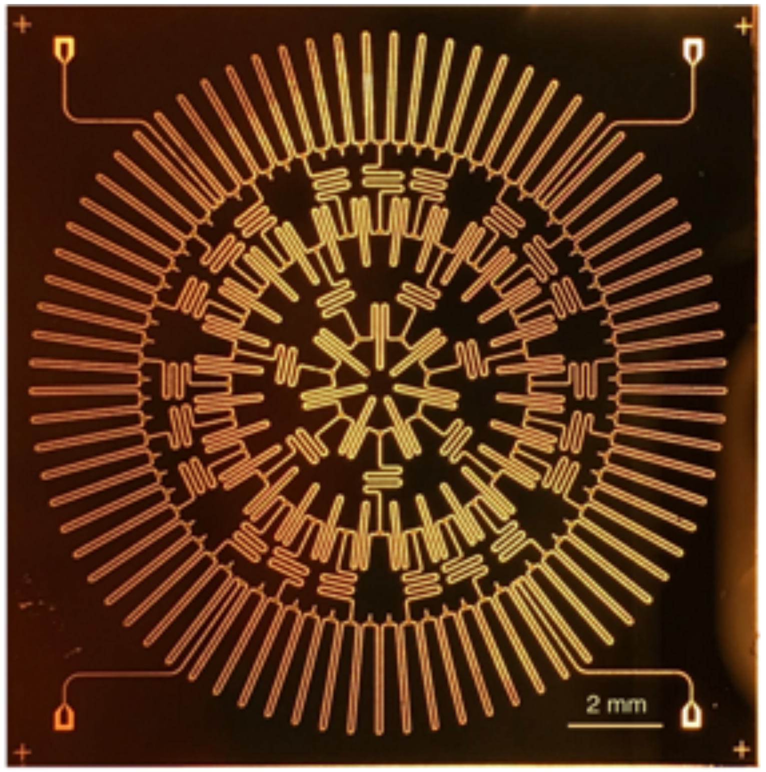


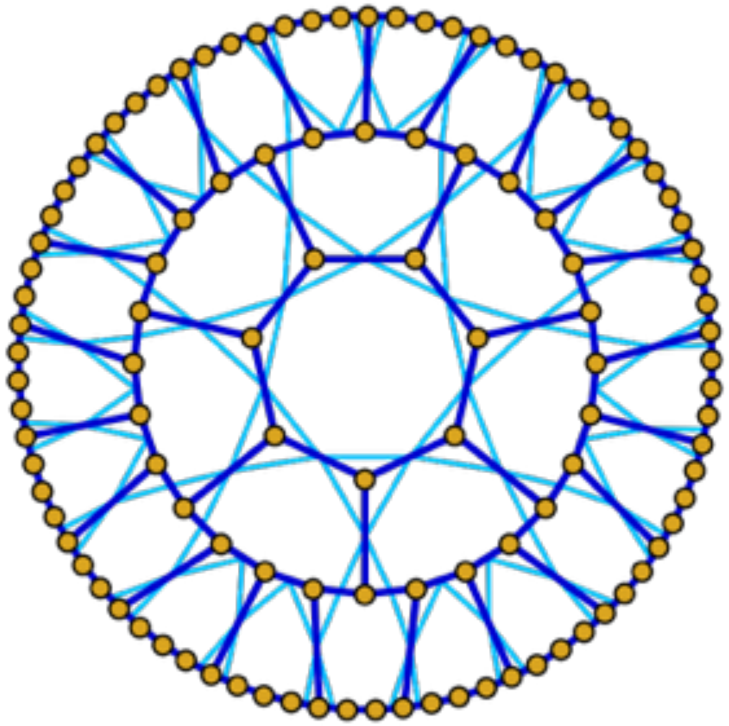
Hyperbolic geometry is non-commutative

- No Bravais lattice
- No Bloch theory
- Graph theory
- Brute force TB numerics

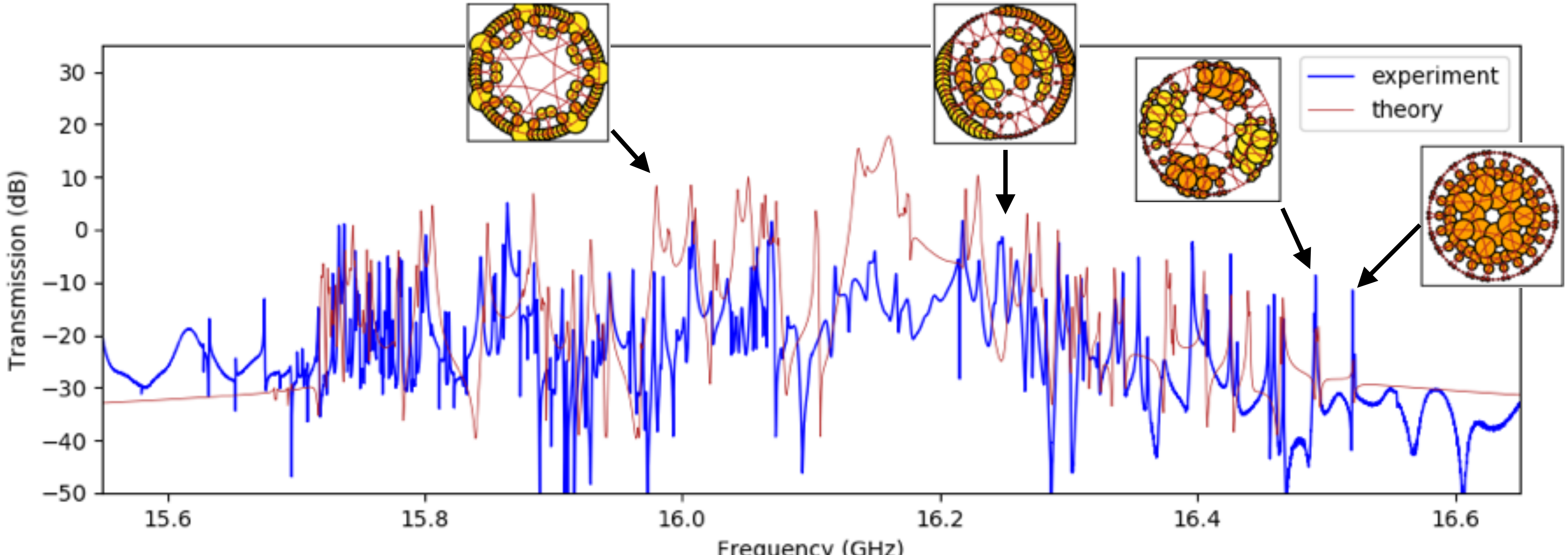


Heptagon-Kagome Device





- 2 shells
- Operating frequency: 16 GHz
- 4 input-output ports

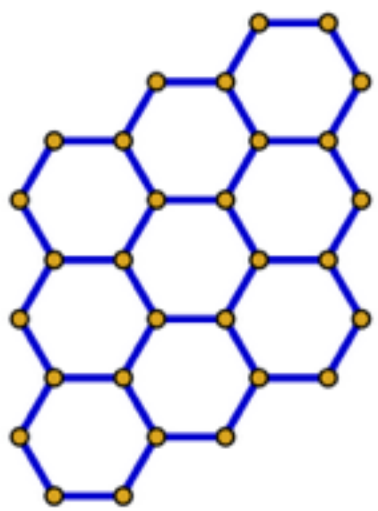


Kollár *et al*. arXiv:1802.09549 (2018)

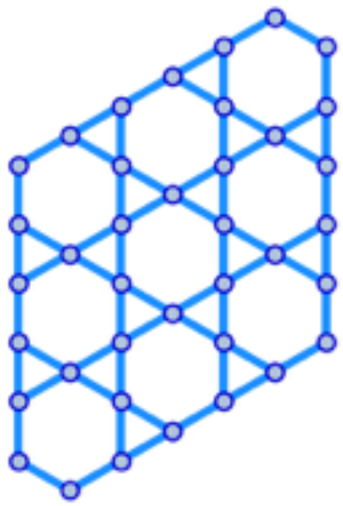
Line-Graph Lattices

Graphene

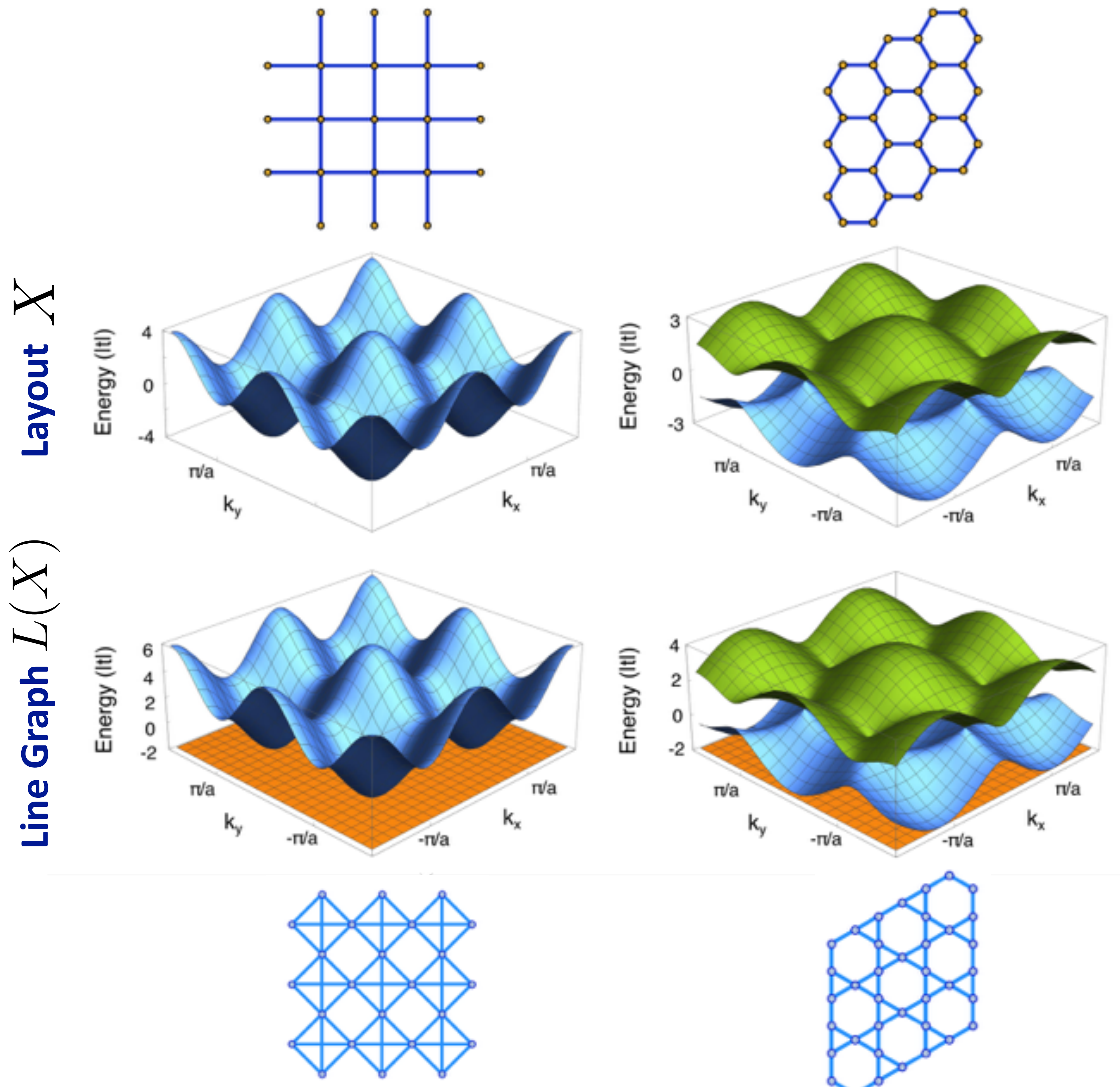
Layout X



Line Graph $L(X)$



Band Structure Correspondence



Band Structure Correspondence

Layout Tight-Binding Hamiltonian

- Bounded self-adjoint operator on X

$$H_X$$

Effective Hamiltonian

- Bounded self-adjoint operator on L(X)

$$\bar{H}_s(X) = H_{L(X)}$$

Incidence Operator

- From X to L(X)

$$M : \ell^2(X) \to \ell^2(L(X))$$

$$M(v,e) = \begin{cases} 1, & \text{if } e \text{ and } v \text{ are incident,} \\ 0 & \text{otherwise.} \end{cases}$$

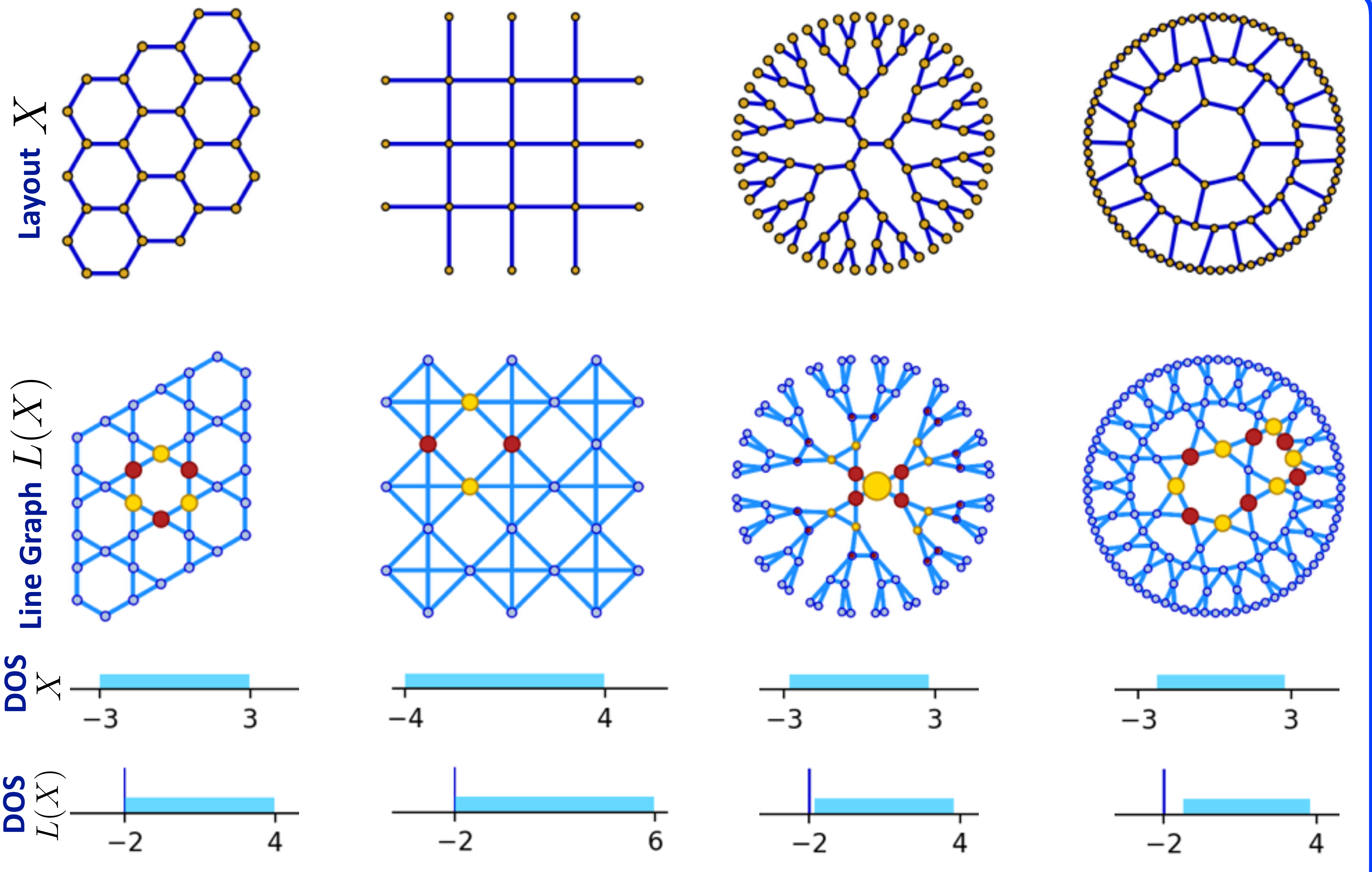
$$M^t M = D_X + H_X$$

$$M M^t = 2I + \bar{H}_s(X)$$

$$D_X + H_X \simeq 2I + \bar{H}_s(X)$$

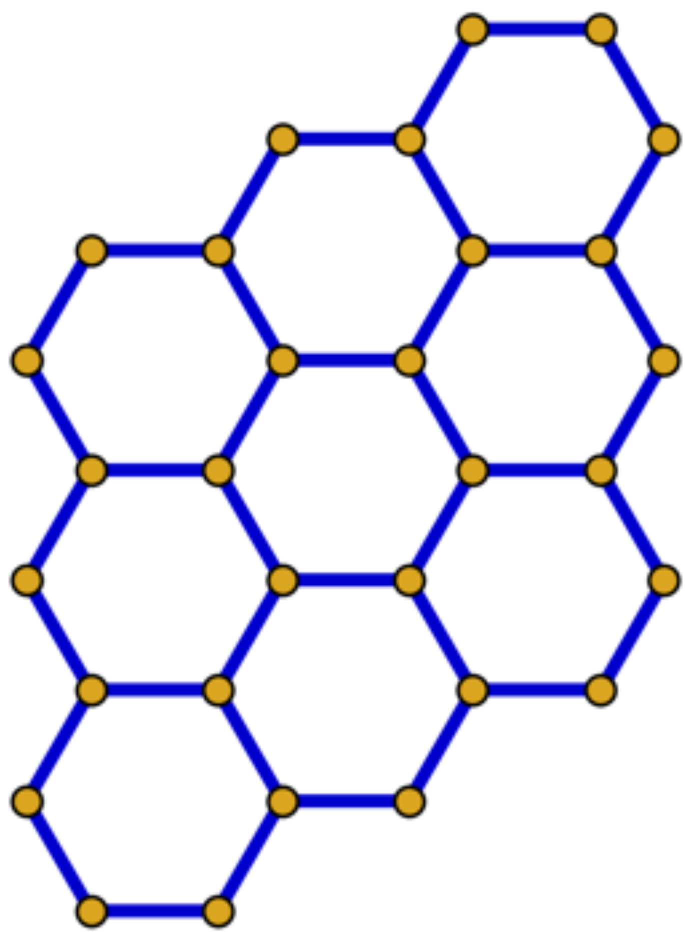
$$E_{\bar{H}_s} = \begin{cases} d - 2 + E_{H_X} \\ -2 \end{cases}$$

Density of States and Flat-Band States

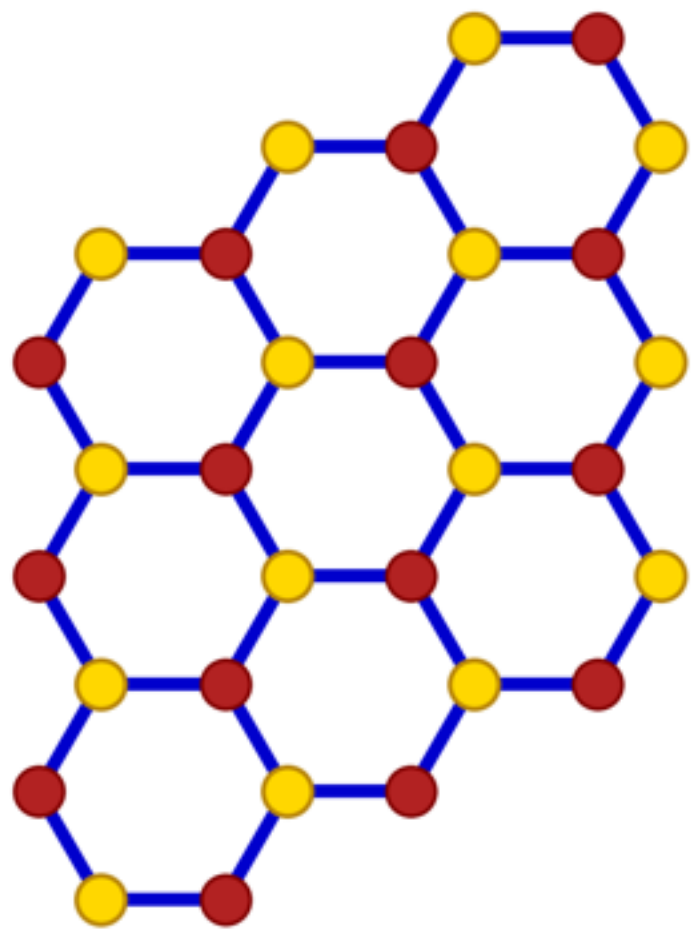


Bipartite and Non-Bipartite Graphs

Bipartite

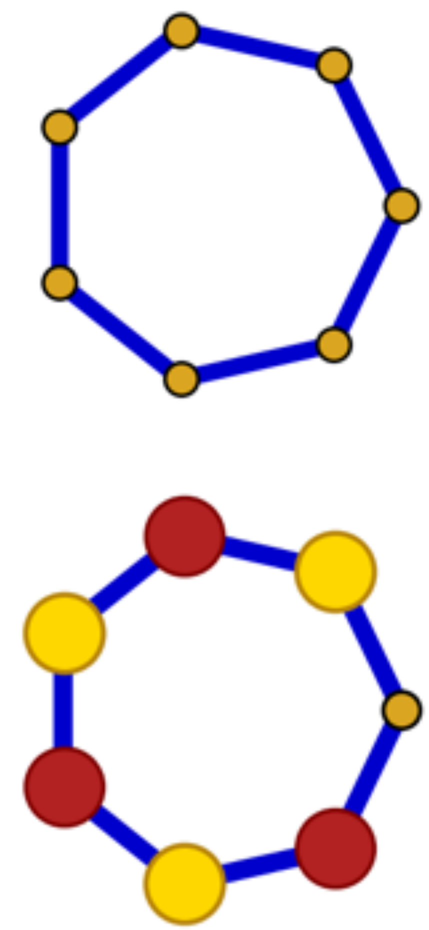


E = -3



- All neighbors opposite sign

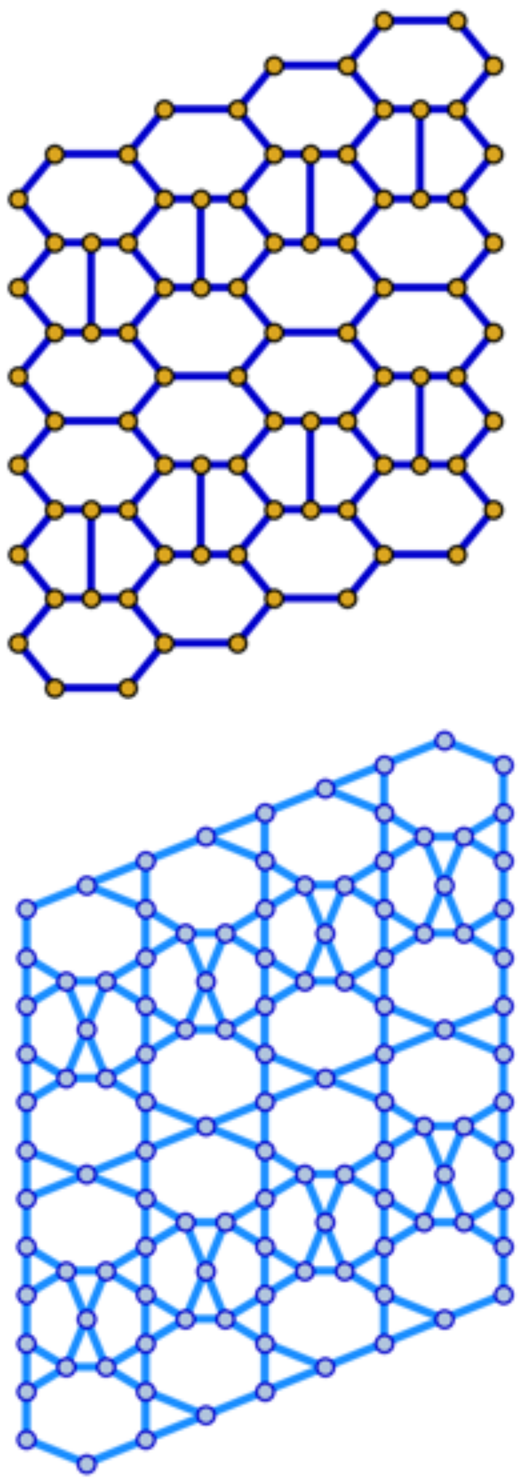
Non-Bipartite

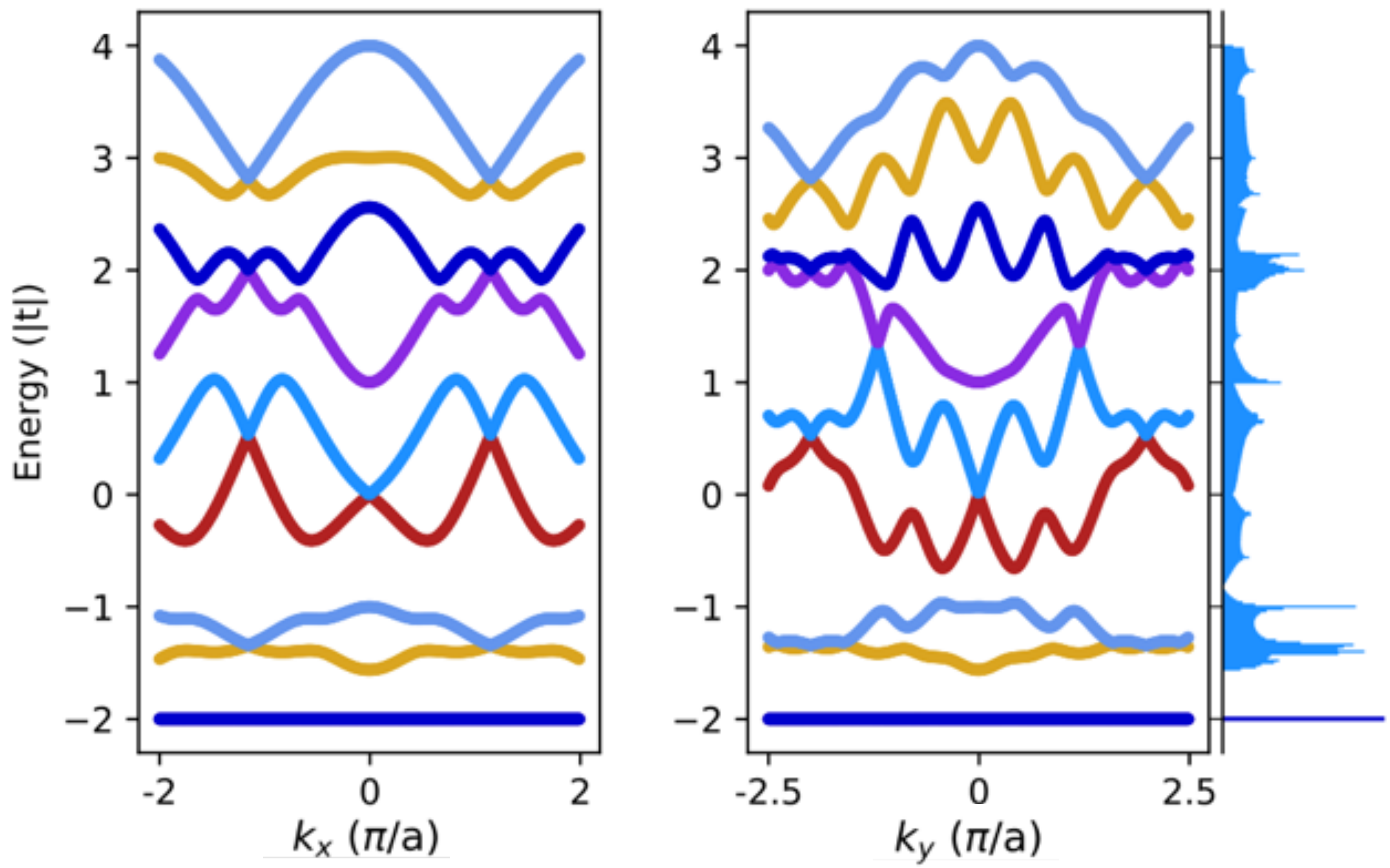


- Not all neighbors can be opposite sign

Heptagon-Pentagon-Kagome Lattice

- Modified graphene with interstitials
- Heptagonal and pentagonal plaquettes
- Non-bipartite
- Tripled 12-site unit cell

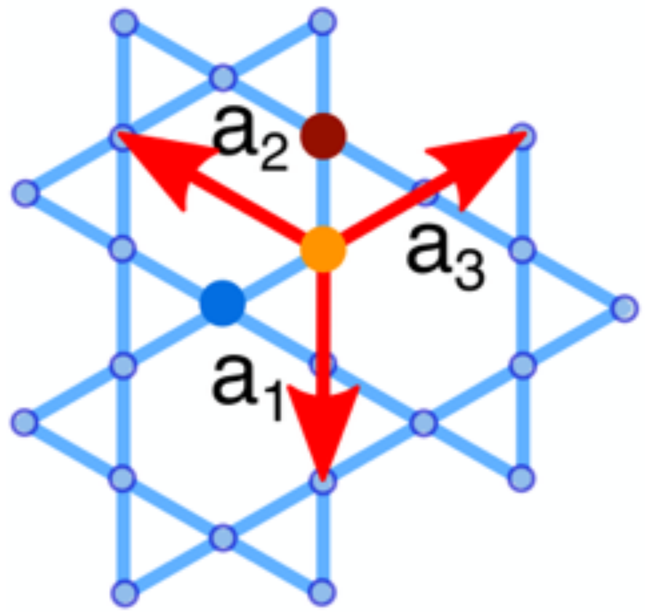


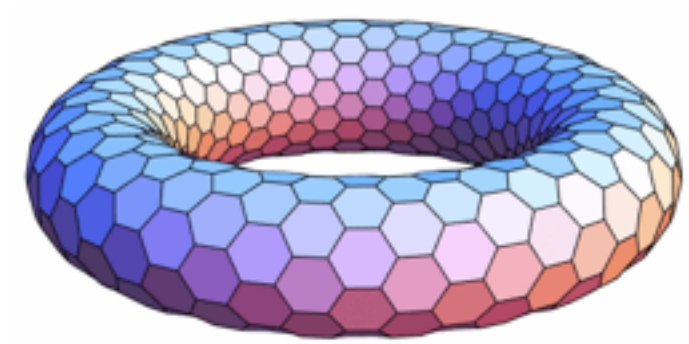


Real-Space Topology and Band Touches

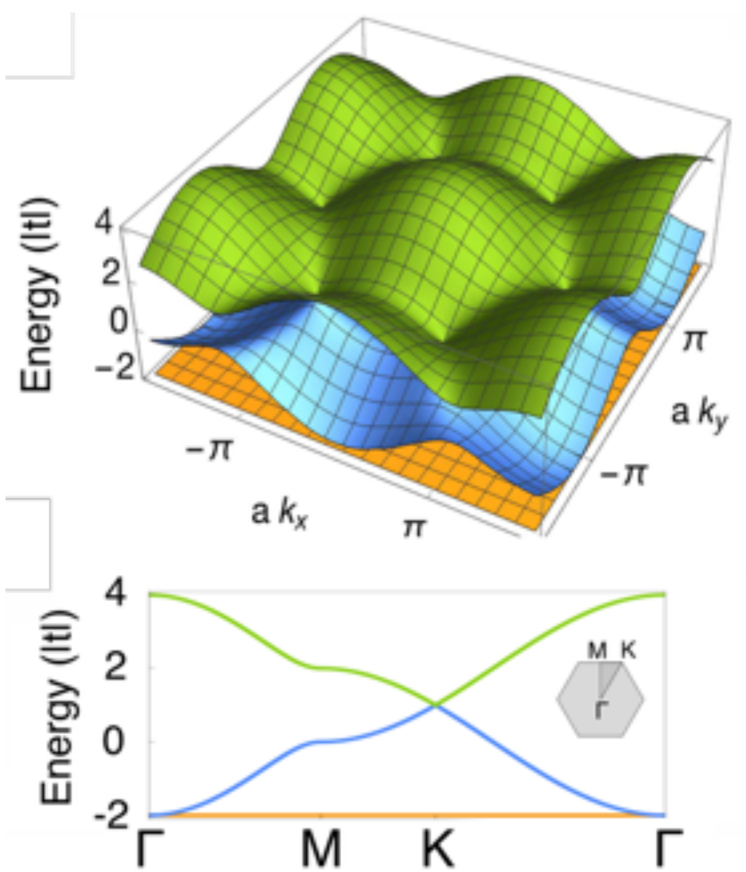
Kagome lattice

- Triangular Bravais lattice
- 3 site unit cell

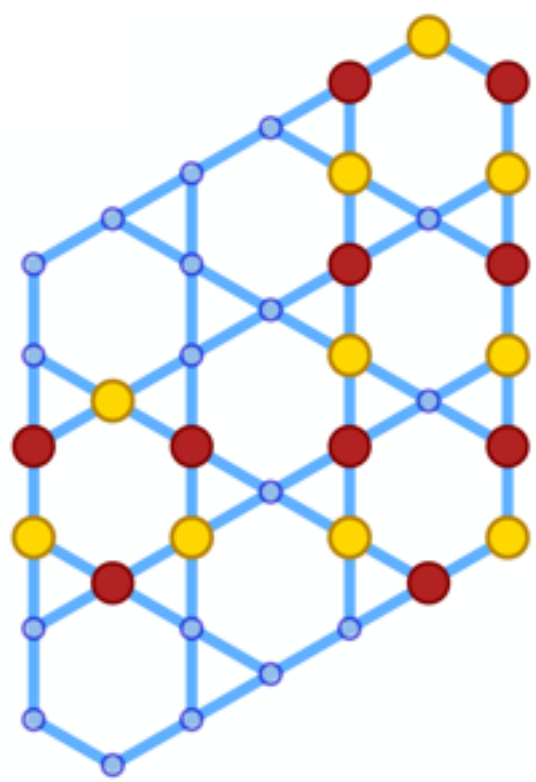




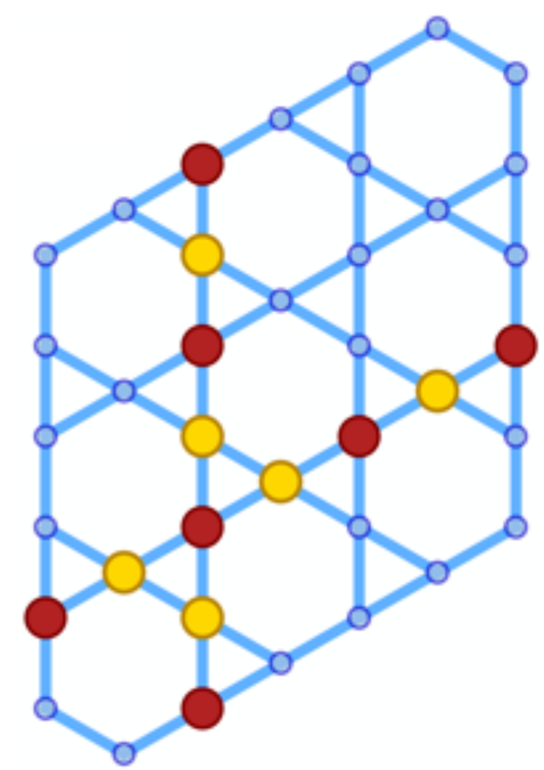
Band Structure



Flat-band States

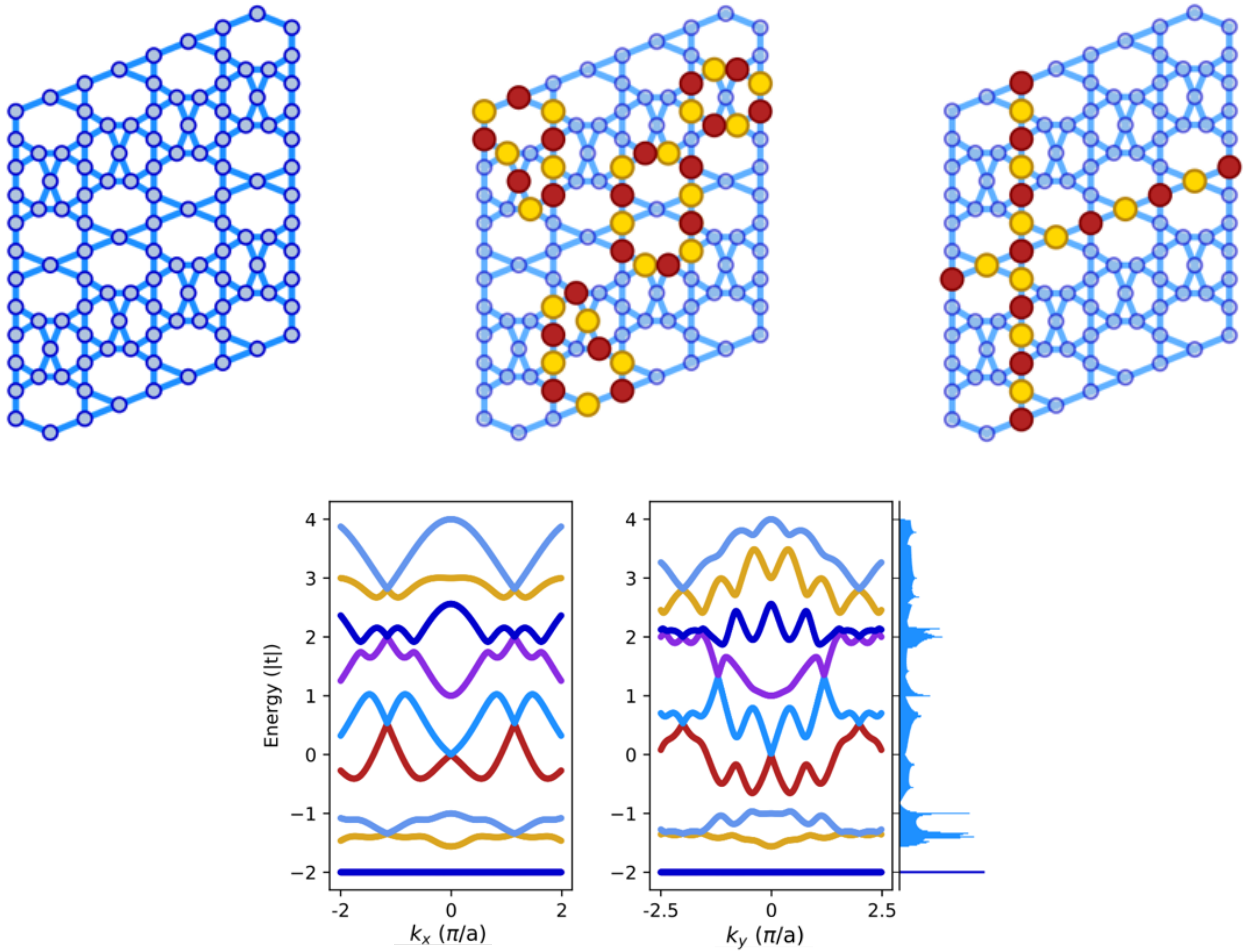


Incontractible Loop States



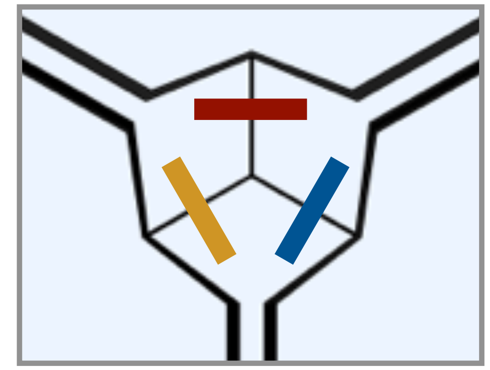
Bergman *et al*. PRB **78**, 125104 (2008)

Real-Space Topology and Band Gaps

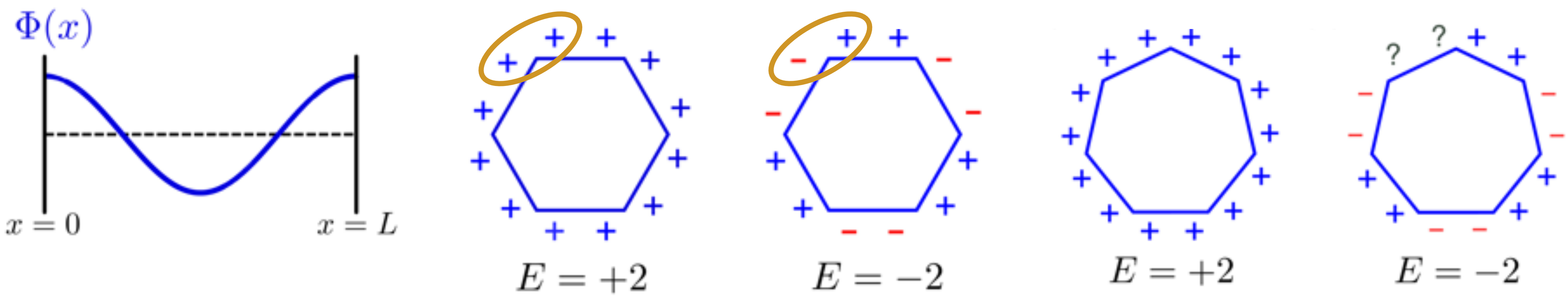


S-Wave and P-Wave On-Site Wave Functions

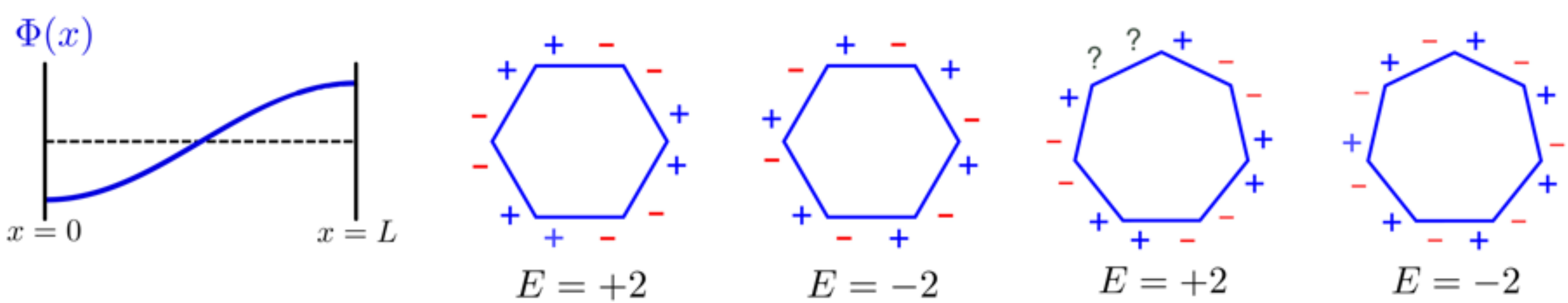
$$\mathcal{H} = \sum_{\substack{coupling \\ capacitors}} \omega C_c \Phi^+ \Phi^-$$



Full-wave



Half-wave



Half-Wave Band Structure Correspondence

Layout Tight-Binding Hamiltonian

- Bounded self-adjoint operator on X

$$H_X$$

Incidence Operator

- From X to L(X)

$$N : \ell^2(X) \to \ell^2(L(X))$$

$$N(v,e) = \begin{cases} 1, & \text{if } e^+ = v, \\ -1 & \text{if } e^- = v, \\ 0 & \text{otherwise.} \end{cases}$$

Effective Hamiltonian

- Bounded self-adjoint operator on L(X)
- Mixed positive and negative hopping

$$\bar{H}_a(X) \neq H_{L(X)}$$

$$N^t N = D_X - H_X$$

$$N N^t = 2I + \bar{H}_a(X)$$

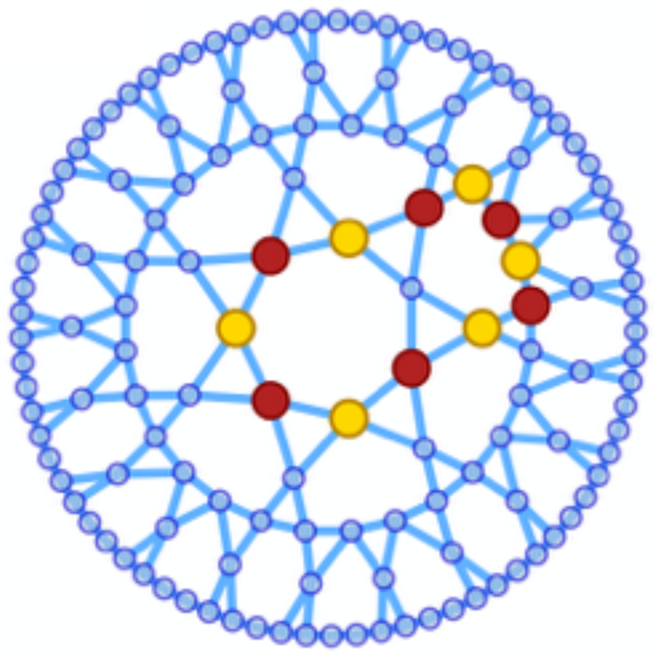
$$D_X - H_X \simeq 2I + \bar{H}_a(X)$$

$$E_{\bar{H}_a} = \begin{cases} d - 2 - E_{H_X} \\ -2 \end{cases}$$

- Identical on bipartite graphs
- Inverted otherwise

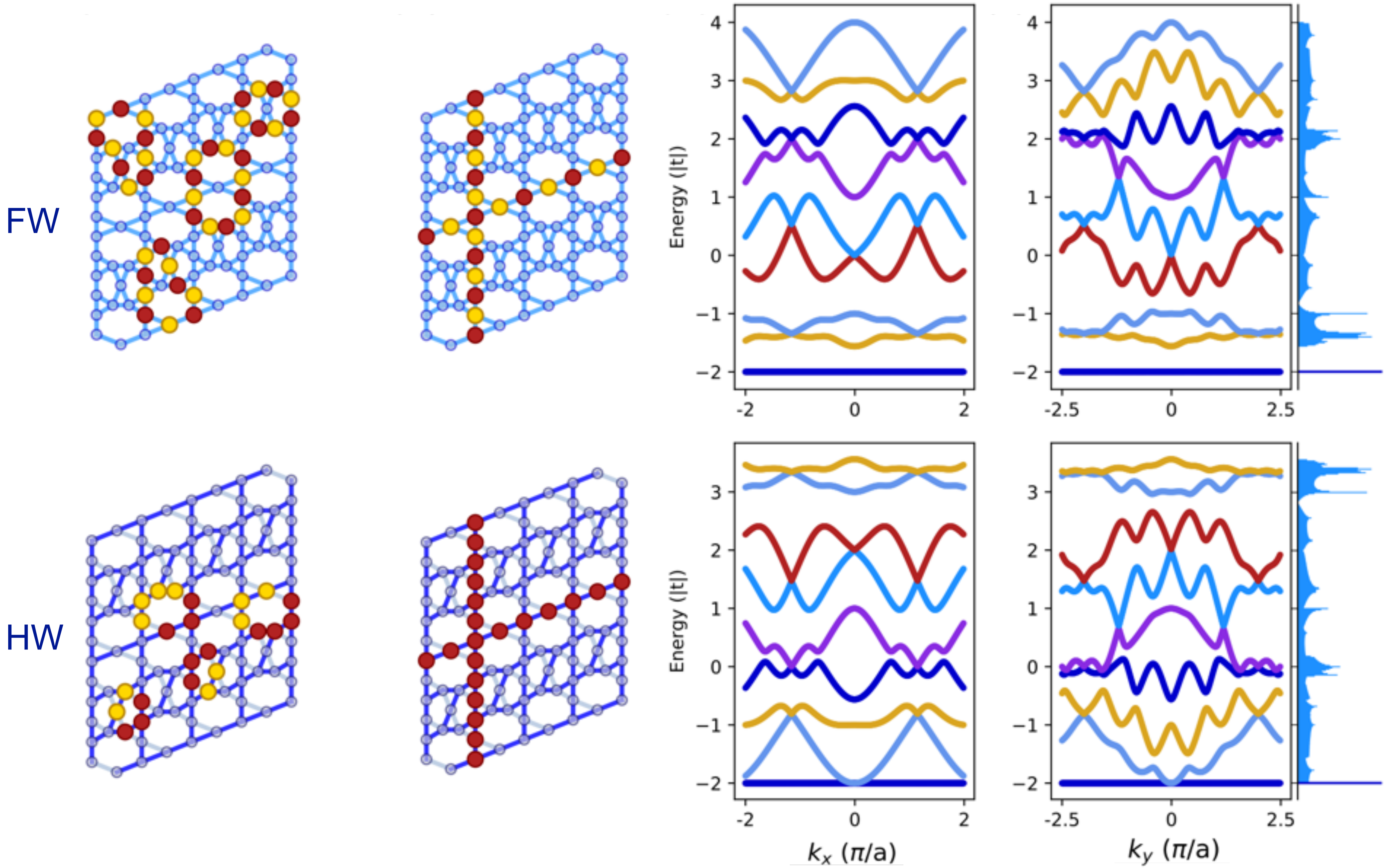
Full-Wave v Half-Wave Flat Band States

FW



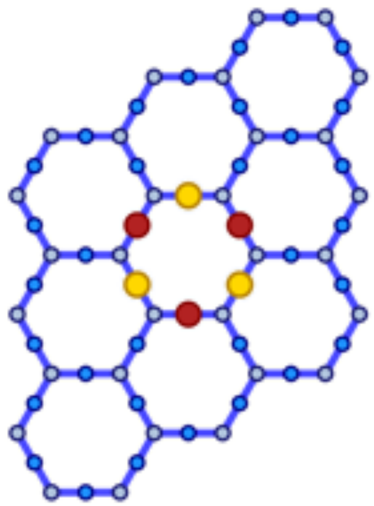
- Full-wave has localized states on only even cycles of the layout.
- Half-wave has localized states on any cycle of the layout.

Full-Wave Half-Wave Correspondence

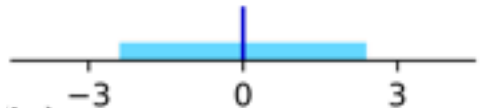


Subdivision Graphs: Flat Bands at 0

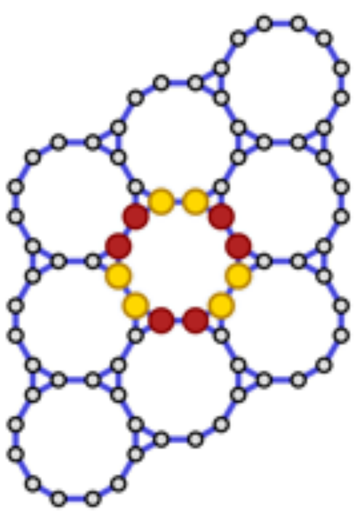
Subdivision $\mathbb{S}(X)$



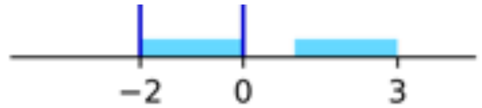
DOS $\mathbb{S}(X)$



Line Graph $L(\mathbb{S}(X))$

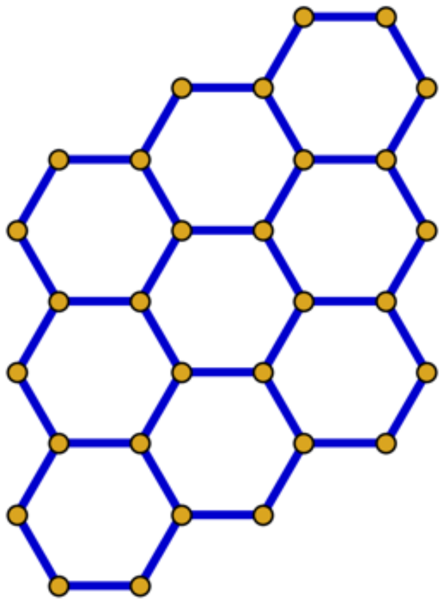


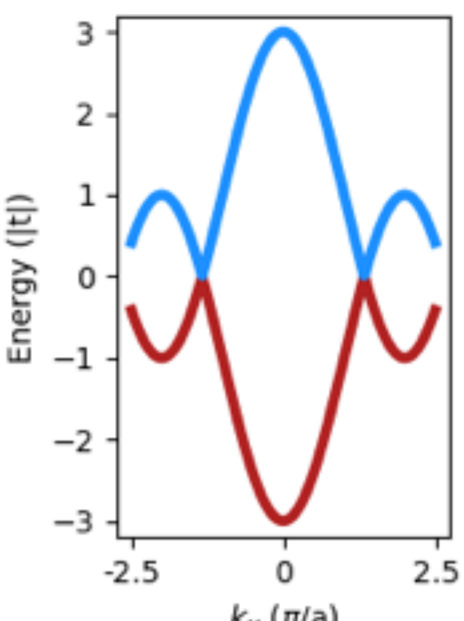
DOS $L(\mathbb{S}(X))$



Subdivision Graphs and Optimally Gapped Flat Bands

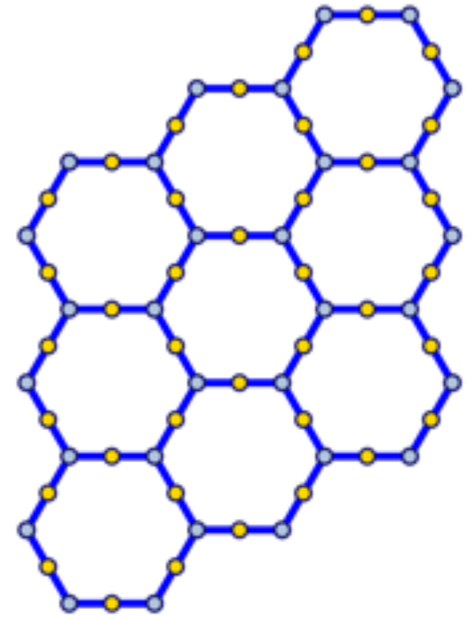
X

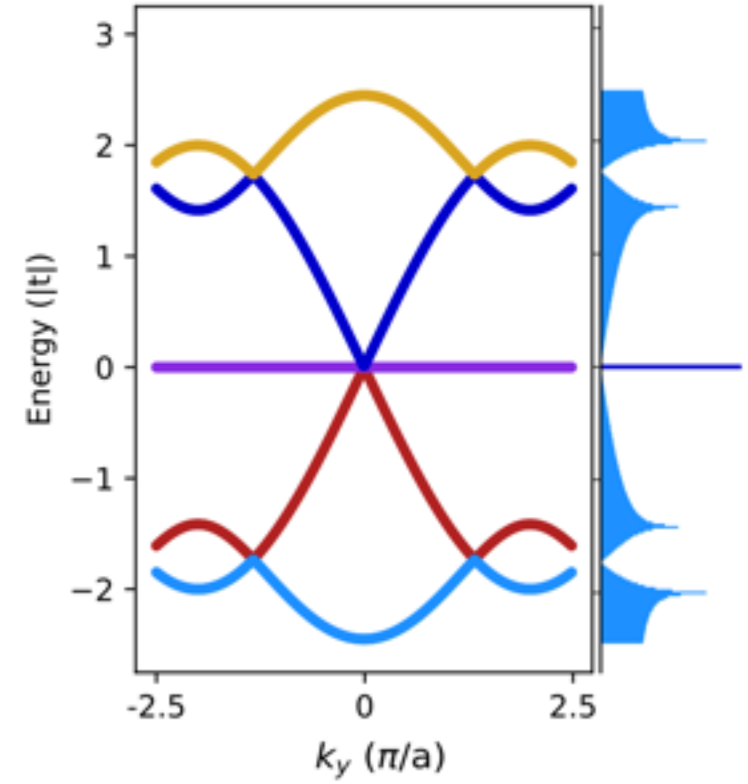




$$E_{\mathbb{S}(X)} = \begin{cases} \pm\sqrt{E_X + 3} \\ 0 \end{cases}$$

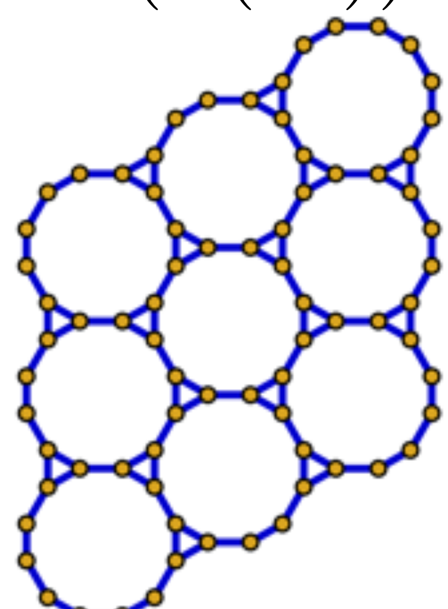
$\mathbb{S}(X)$

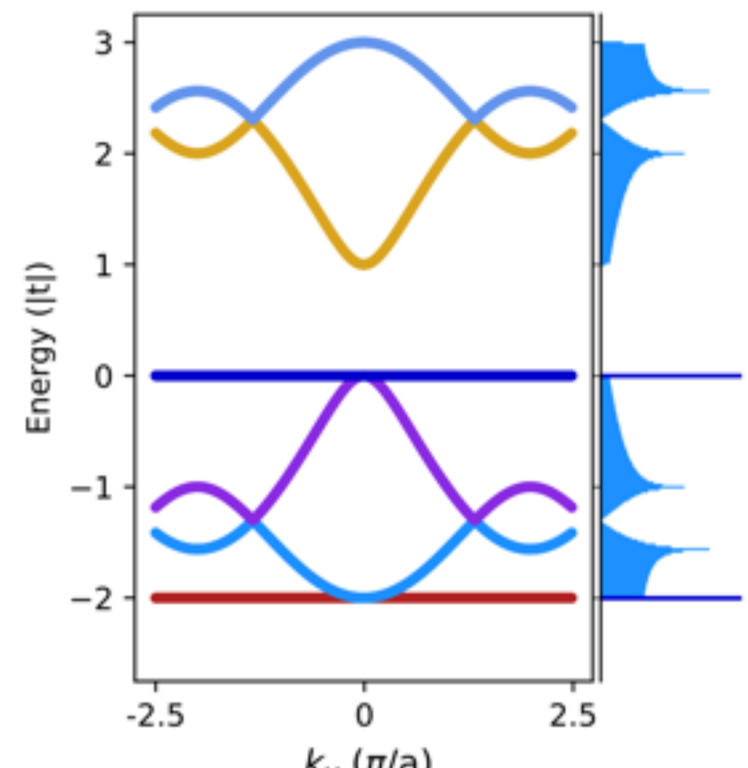




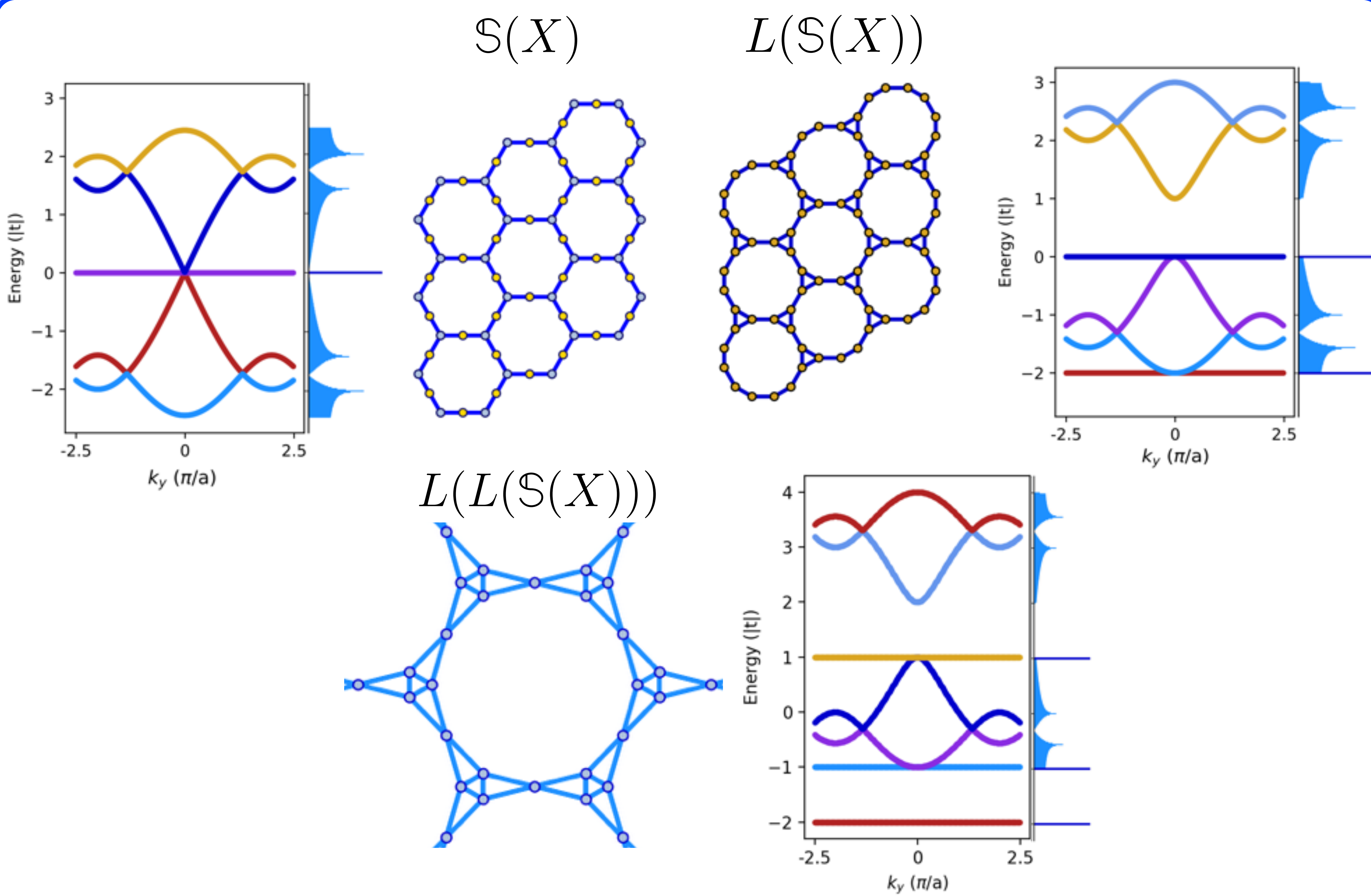
$$E_{L(\mathbb{S}(X))} = \begin{cases} \frac{1 \pm \sqrt{1+4(E_X+3)}}{2} \\ 0 \\ -2 \end{cases}$$

$L(\mathbb{S}(X))$



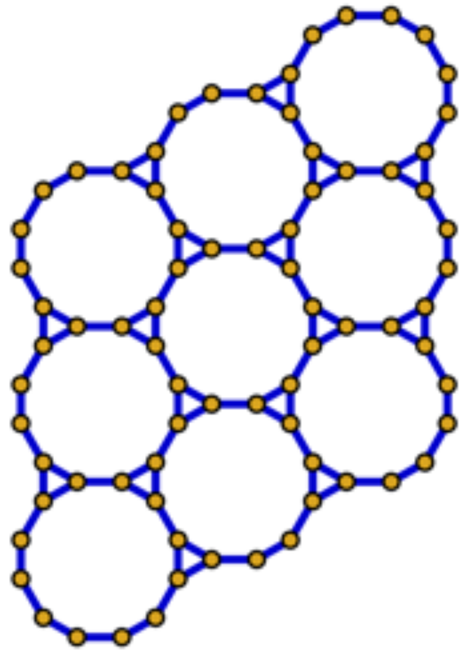


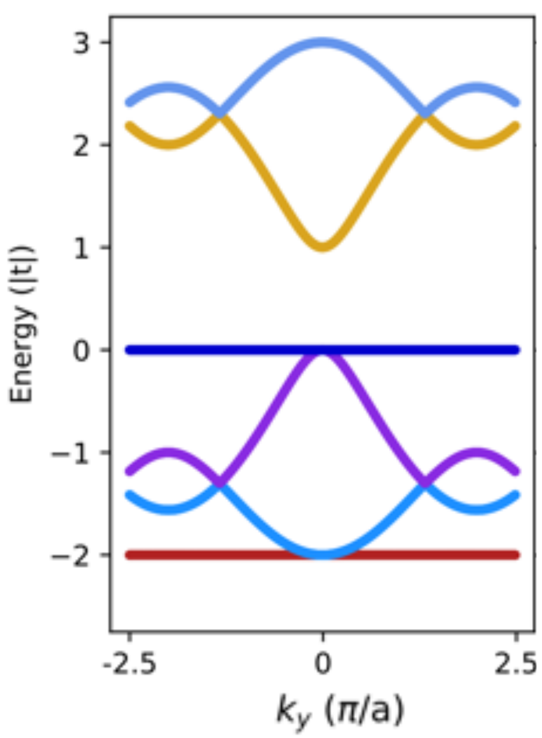
Subdivision Graphs and Optimally Gapped Flat Bands



Subdivision Graphs and Optimally Gapped Flat Bands

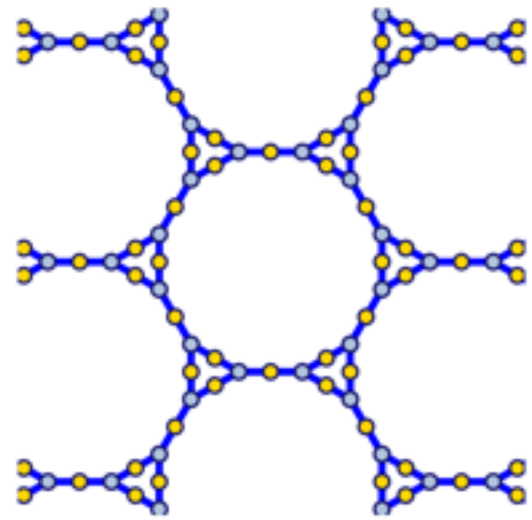
X

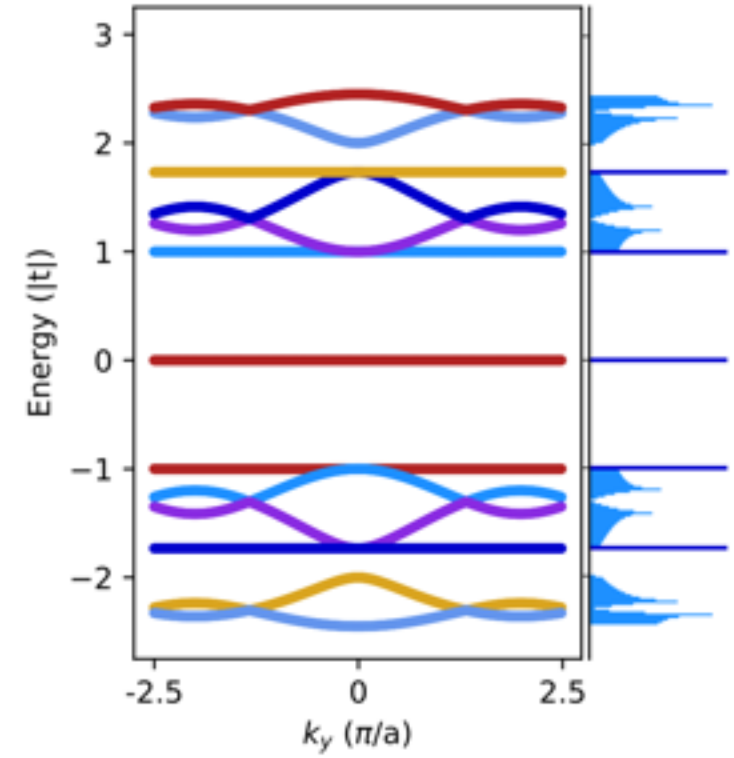




$\mathbb{S}(X)$

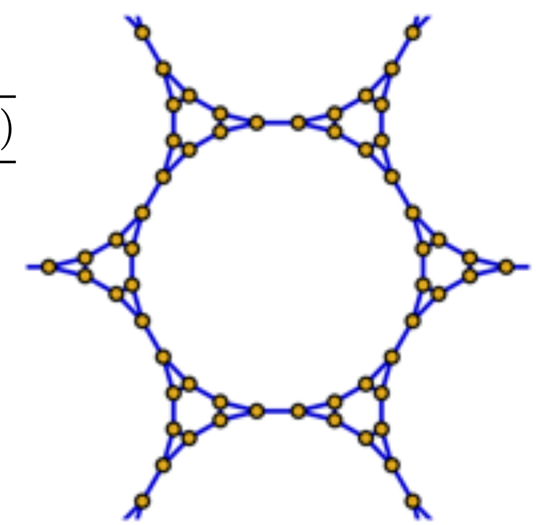
$$E_{\mathbb{S}(X)} = \begin{cases} \pm\sqrt{E_X + 3} \\ 0 \end{cases}$$

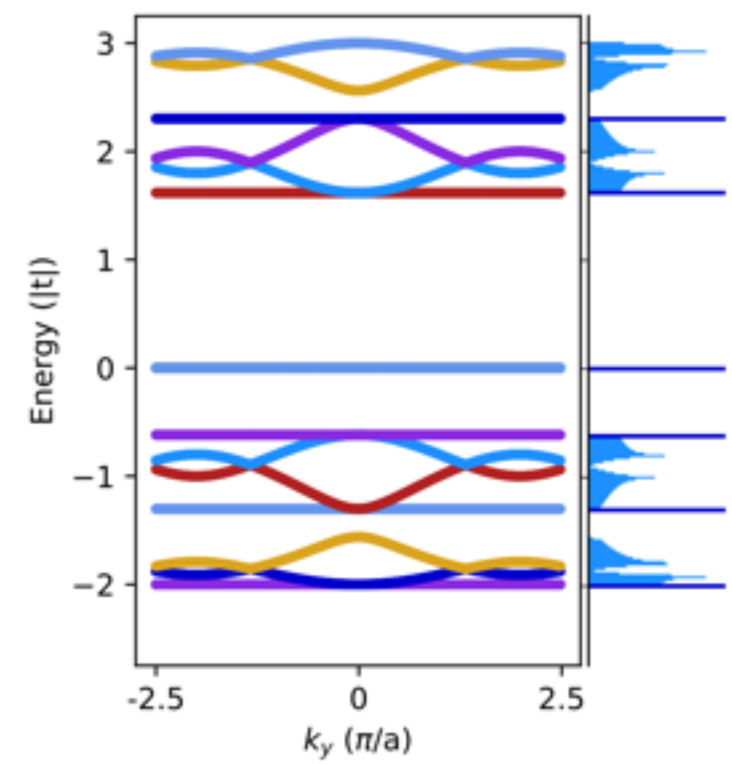




$L(\mathbb{S}(X))$

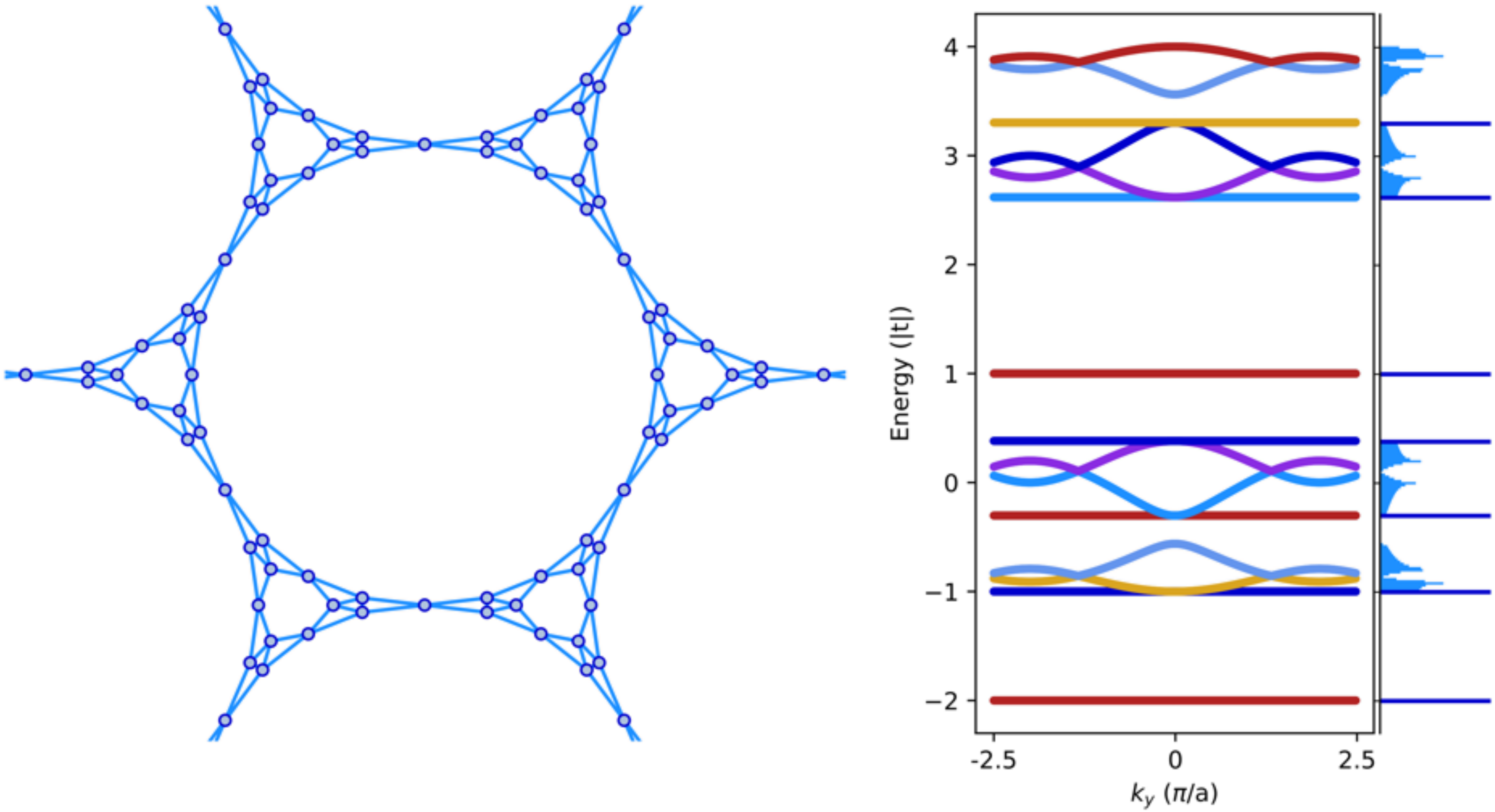
$$E_{L(\mathbb{S}(X))} = \begin{cases} \frac{1\pm\sqrt{1+4(E_X+3)}}{2} \\ 0 \\ -2 \end{cases}$$





Subdivision Graphs and Optimally Gapped Flat Bands

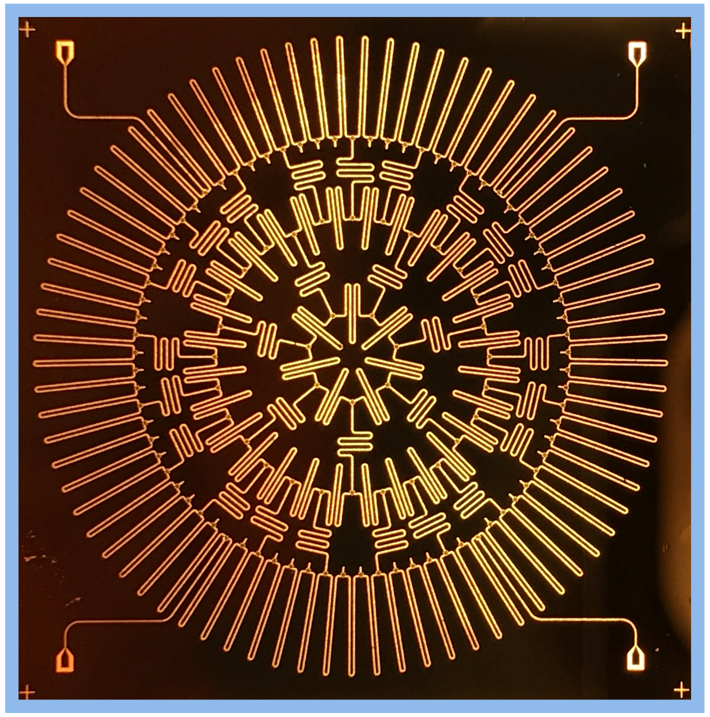
$L(L(\mathbb{S}(X)))$

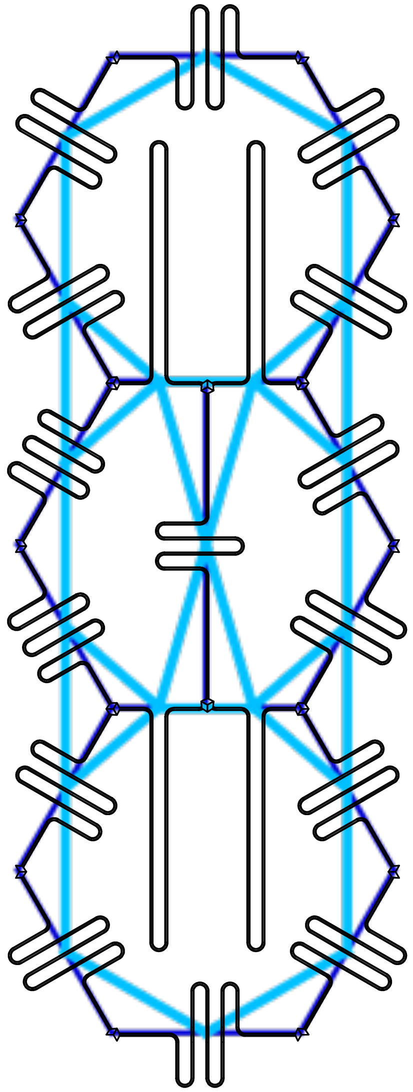


Conclusion and Outlook

- Circuit QED lattices
  - Artificial photonic materials
  - Interacting photons
- Hyperbolic lattices
  - Unusual band structures
  - On-chip fabrication
- Flat-band lattices
  - 0, -2
  - Optimal gaps

- Outlook
  - Interacting photons in curved space
  - Many-body physics in flat bands





Kollár *et al*. arXiv:1802.09549 (2018)

Hyperbolic and Flat-Band Lattices in Circuit QED

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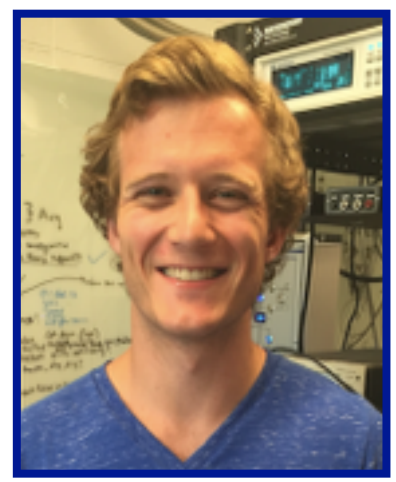

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