UNIVERSITY OF CAMBRIDGE

Yale University

Turbulence in quantum gases

scale invariance, synthetic dissipation and cascade fluxes

Nir Navon

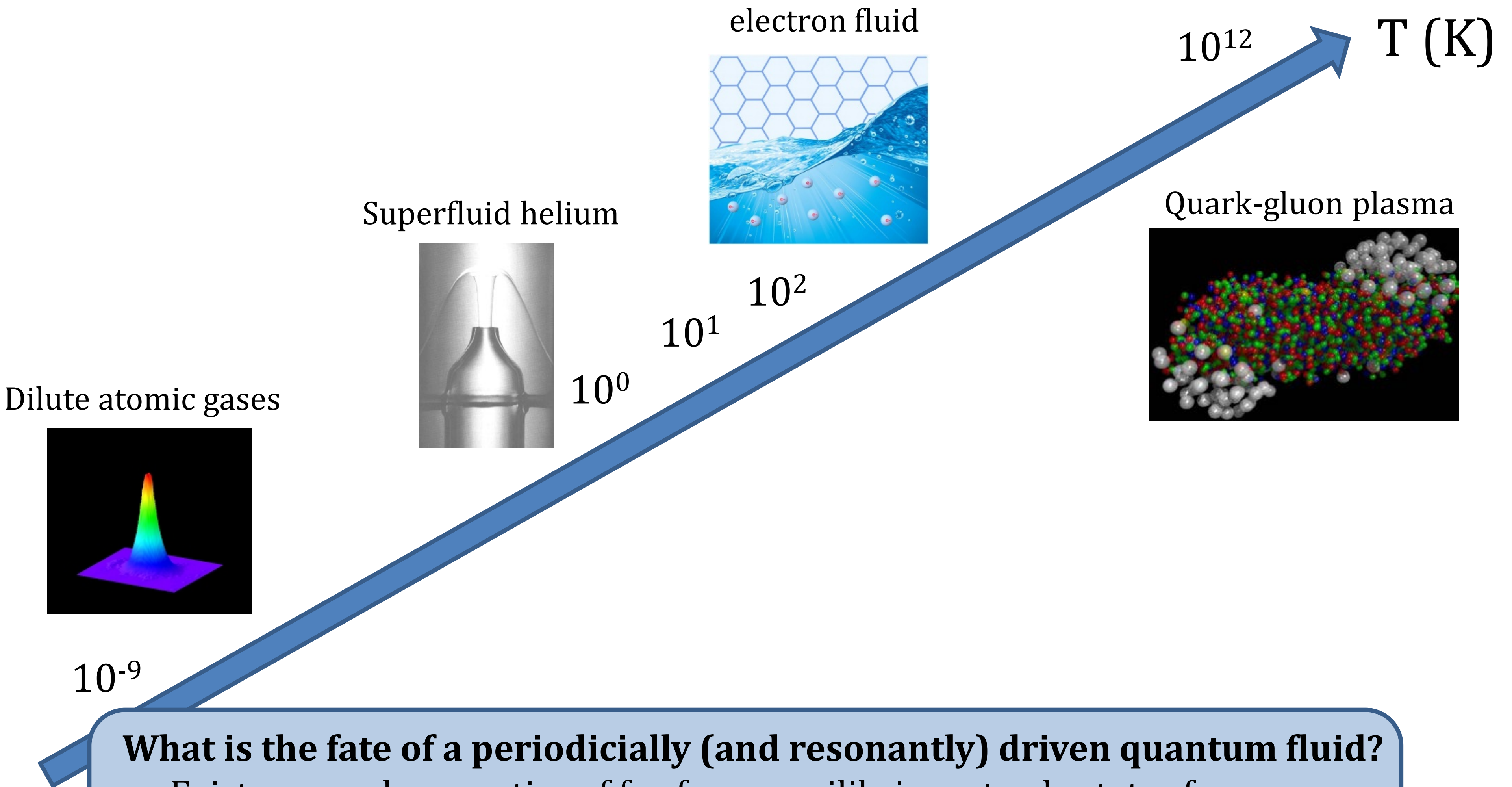
- Navon et al, *Nature* **539**, 72 (2016)
- Navon et al, *arXiv:1807.07564* (2018)
- Garratt et al, arXiv:1810.08195 (2018)

KITP, 24th October 2018

Outline

- Turbulence in quantum gases
- The optical box trap: a new tool for ultracold atoms
- Low-energy excitations of a box-trapped BEC
- A turbulent steady state
- A synthetic dissipation scale and turbulent-cascade fluxes

Hydrodynamics of Quantum fluids

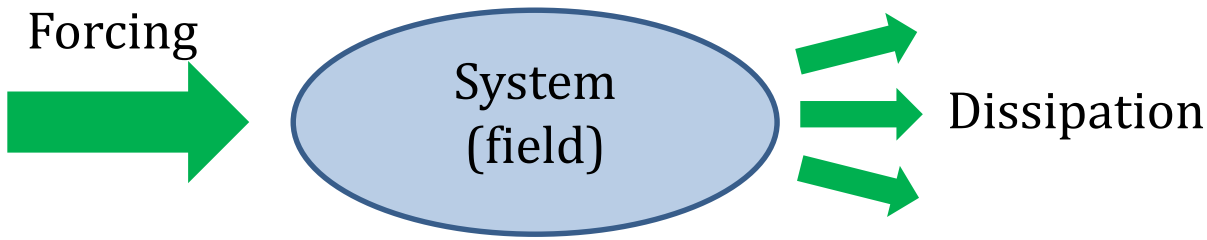


What is the fate of a periodicially (and resonantly) driven quantum fluid?

- Existence and properties of far-from-equilibrium steady states for continuously driven-dissipative quantum fluid?
- Emergence of *universal* behavior?

What is Turbulence ?

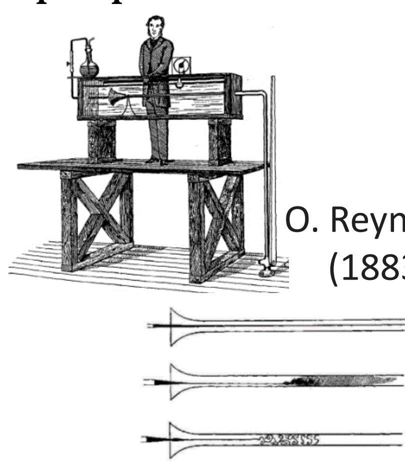
Quintessential phenomenon of out-of-equilibrium physics



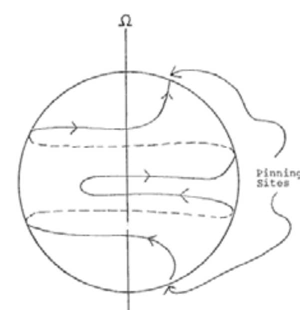
Far-from-equilibrium states that (usually) involves

- drive + dissipation
- steady in a statistical sense
- local restoration of symmetries (isotropy/homogeneity)
- many interacting degrees of freedom
- chaotic properties

Leonardo da Vinci (1509)



O. Reynolds (1883)

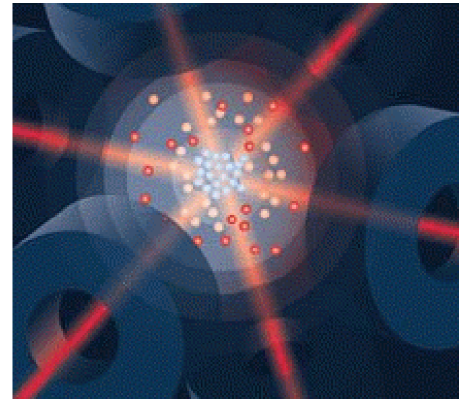


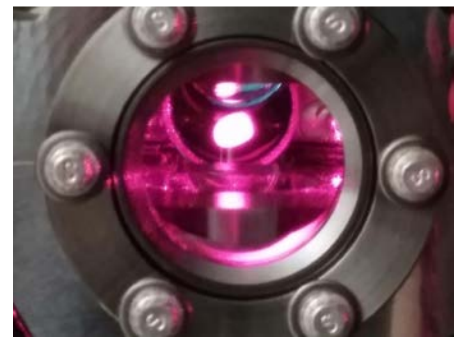
SF turbulence neutron stars / glitches? (1969)

Quantum gases as toy fluids

- Perfectly pure
- Resolvable from system size (typ. 100 μm) to interparticle spacing (typ. 1 μm)
- Easy to set out of equilibrium
- Real-time dynamics
- Highly controllable

  (i) Interparticle interactions
  (ii) Trapping geometry
  (iii) Custom forcing
  (iv) Tunable dissipation





→ Good candidates for simple turbulence experiments, « new knobs »

Described from first principles possible, e.g. GPE for Bose gas $$i\hbar\frac{\partial\psi}{\partial t} = \left(-\frac{\hbar^2}{2m}\Delta + g|\psi|^2\right)\psi$$

Experimental activity: V. Bagnato, B. P. Anderson, Y. Shin, P. Engels, K. Helmerson and T. Neely

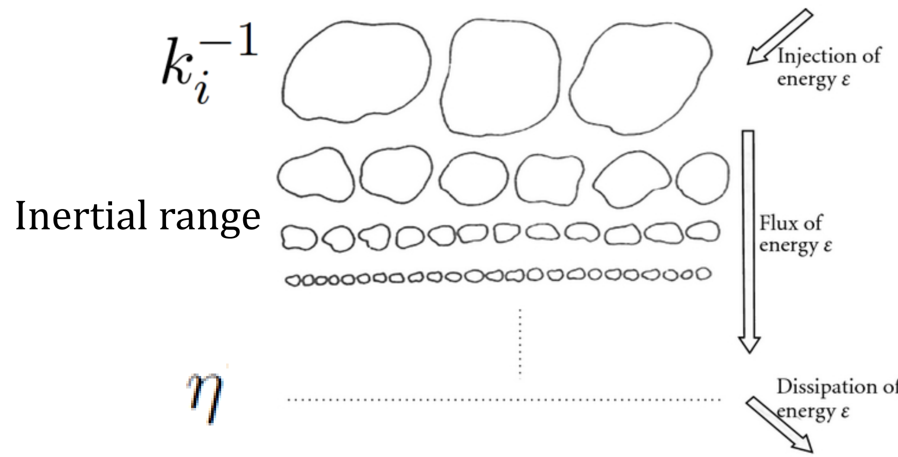
Turbulent cascades

A central phenomenology of turbulence: cascades of *<u>conserved quantities</u>*, *<u>gradually</u>* transported across different length scales

→ Generic prediction: scale invariant fluxes and power-law spectra

Famous example: "Kolmogorov 5/3" (hydro)

$$E_k = C_K \epsilon^{2/3} k^{-5/3}$$



kinetic energy cascade of incompressible flow

ϵ scale-invariant energy flux

Power-law turbulent spectra measured in many types of flows
Fluxes more challenging to measure (often indirect)

Can we observe in real time a turbulent cascade?
Can we directly measure a turbulent flux in momentum space?

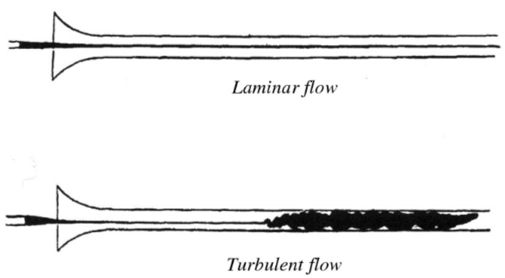
How does turbulence look like in the quantum realm?
How does a turbulent cascade end in a dissipationless quantum fluid?
Is there a generic scenario for a turbulent cascade of excitations?

Hydrodynamic vs. Wave Turbulence

Hydrodynamic turbulence

Hydrodynamic (HD) equations for **v**(**r**,t)

- Static solutions (e.g. Poiseuille)
- Static solutions can be unstable
  - → time invariance is broken
  - → chaotic, turbulent state can emerge



Wave turbulence

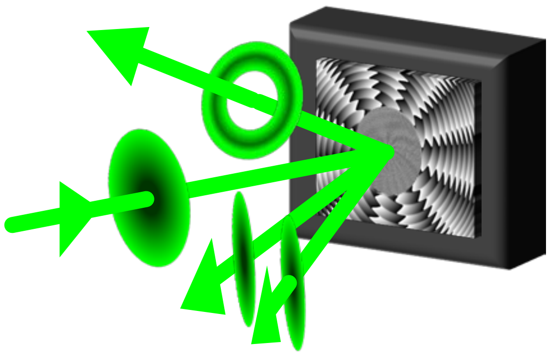
Eq of motion for ***wave*** occupations e.g. **n**(**k**,t)

Nonlinearity: waves ~~eigenmodes~~

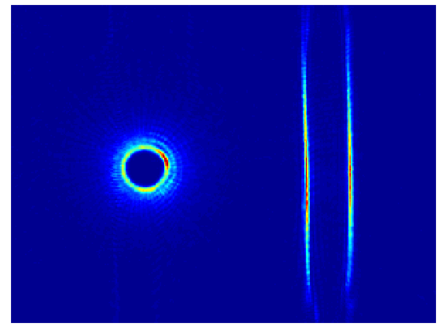
Analytic insights for weak interactions (weak wave turbulence)

Experimental protocol I

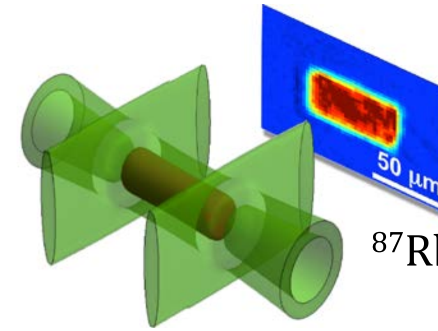
Spatial Light Modulator



repulsive light



Uniform density gas in hard-wall potential



^{87}Rb

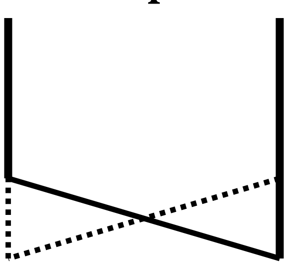
$N \sim 10^5$
$T \sim 5$ nK
$L \sim 30$ μm

$\mu \sim 2$ nK
$\xi \sim 1$ μm

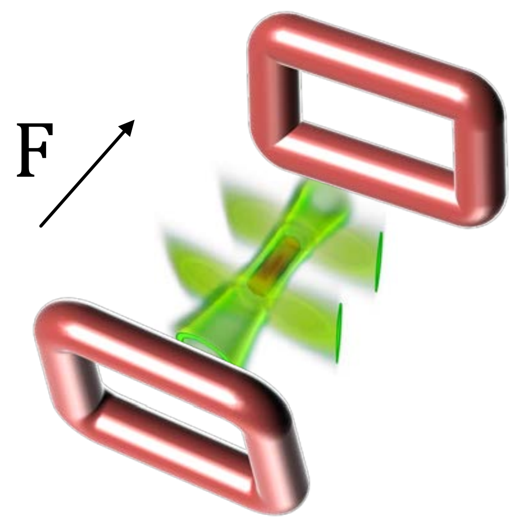
Clean setting for many-body physics, e.g.:

- Thermodynamics of uniform Bose gas, (Gaunt et al PRL 2013, Schmidutz et al PRL 2014)
- Kibble-Zurek mechanism in quenched gas, (NN et al, Science 2015)
- Bogoliubov quantum depletion of a Bose gas (Lopes et al, PRL 2017)
- Unitary Bose gas (Eigen et al PRX 2017, Eigen et al arXiv 2018)
- ...

Excite gas on system-size scale with spatially uniform force



$$V_{\text{shake}}(\mathbf{r}, t) = \frac{\Delta U}{L} \sin(\omega t) x$$



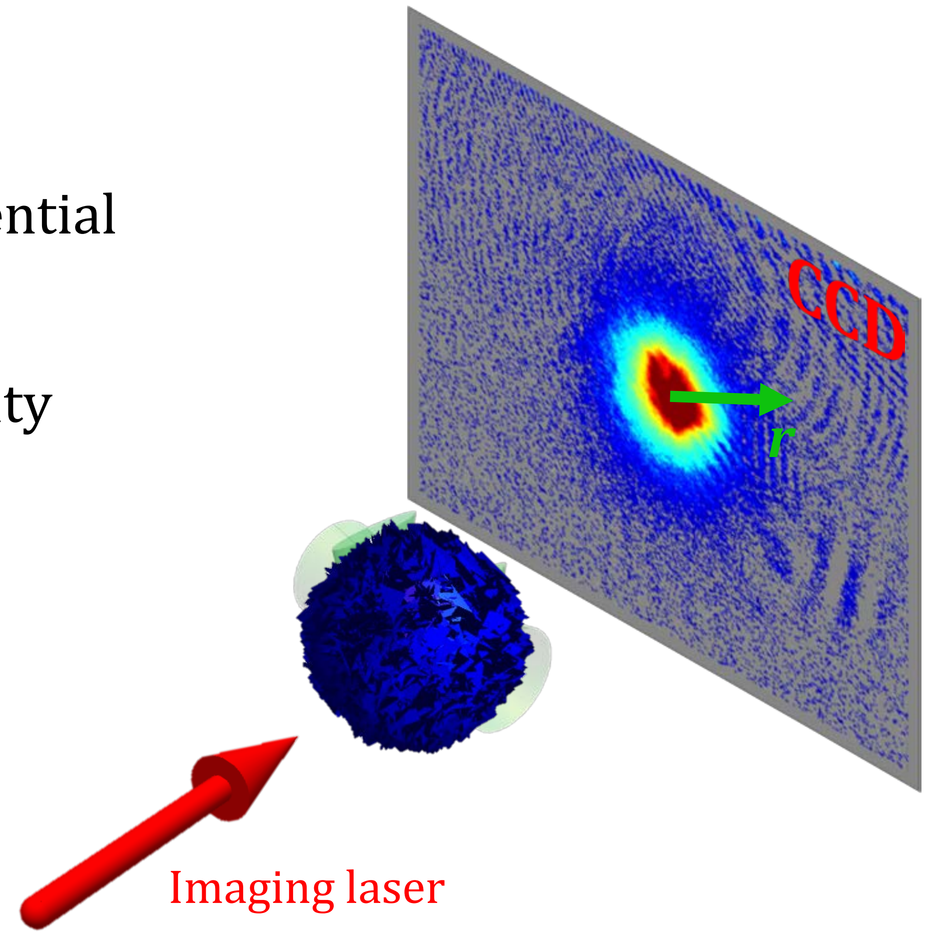
Experimental protocol II

For MWT, crucial quantity: $n(\boldsymbol{k})$

1) Switch off *abruptly* trapping potential

2) Let gas expand for time t_{TOF}

3) Take image of the resulting density by absorption imaging along **z** axis



Position $\boldsymbol{r}$ → velocity $\boldsymbol{v}=\boldsymbol{r}/t_{\mathrm{TOF}}$

Mapping to momentum space

→ 2D distribution $\tilde{n}(k_x,k_z)$

Important caveats:

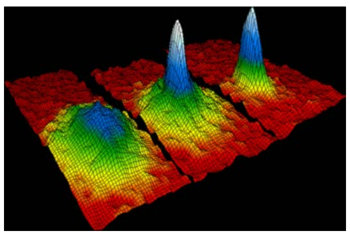
(i) *free* flight: no residual forces nor interactions in flight

(ii) long flights to avoid in-trap density convolution

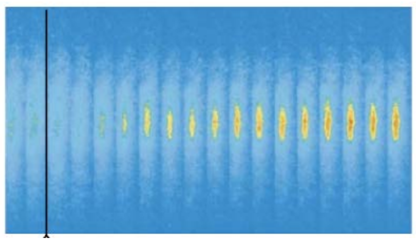
Harmonic vs. Uniform BEC

Equilibrium

BEC occurs both in real and momentum space



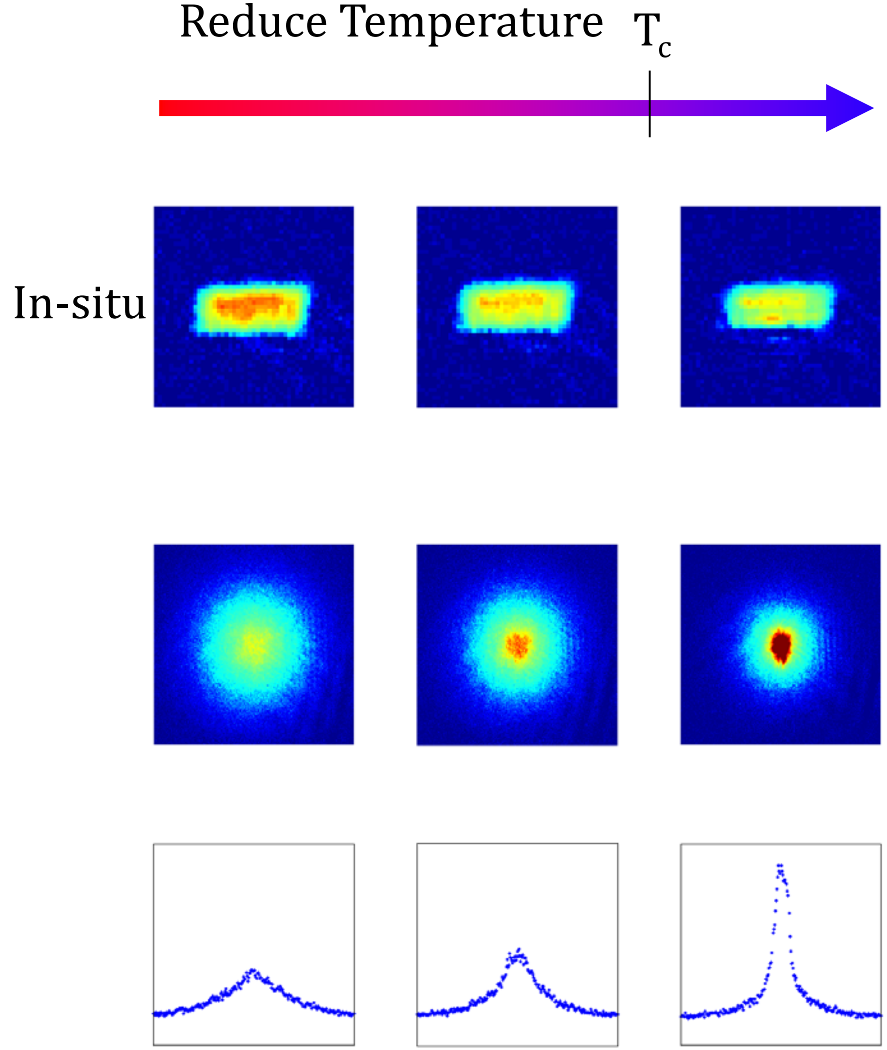
Time of Flight (JILA)



In-situ (MIT)

C. Wieman, E. Cornell

W. Ketterle

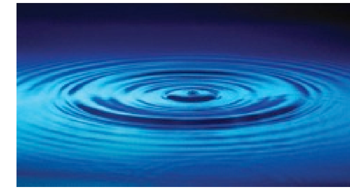


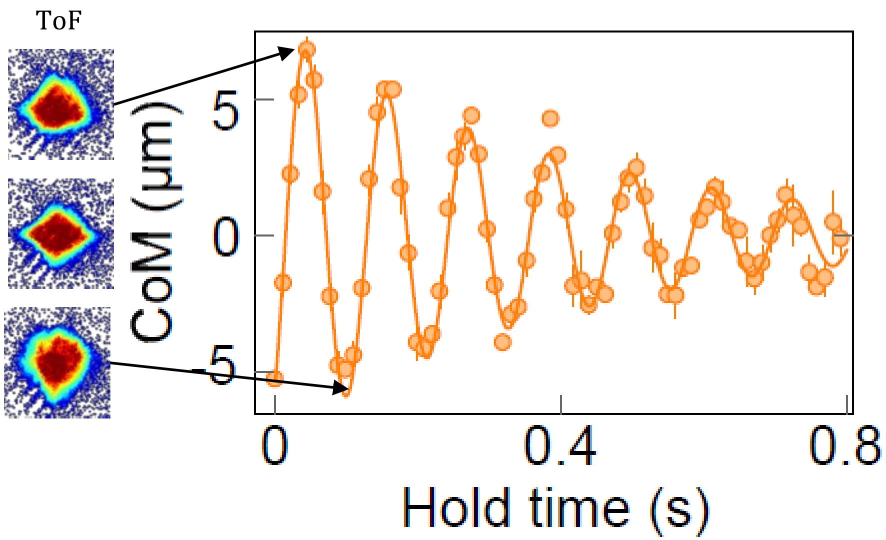
Only in momentum space !

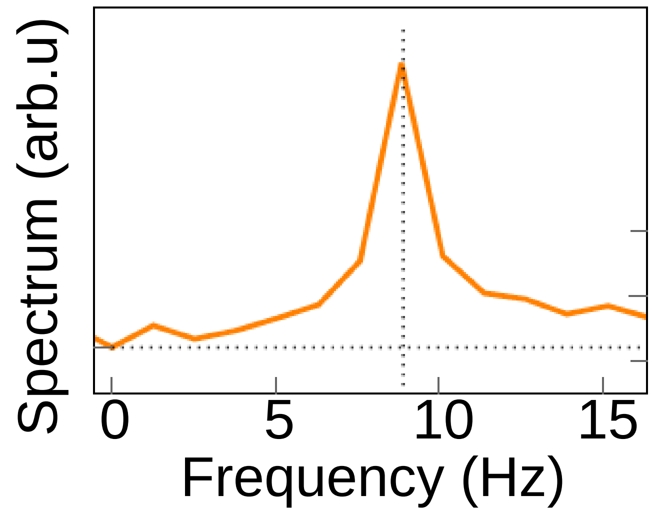
Linear regime: lowest-lying phonon

Near-equilibrium

Single kick

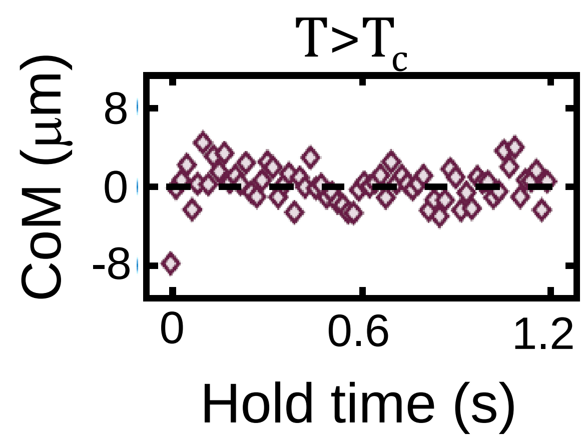






Note: this mode exists only below T_c (*mfp* ~ 10L for $T>T_c$)

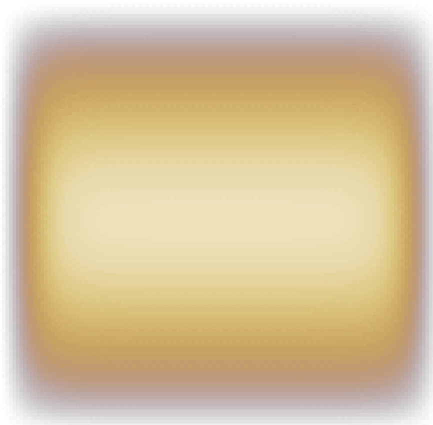
Ideal to start a direct matter-wave cascade (largest scale)



See Garratt et al, arXiv:1810.08195 (2018)

Emergence of Turbulence

(numerical simulations)

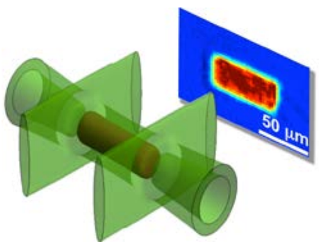


In situ density

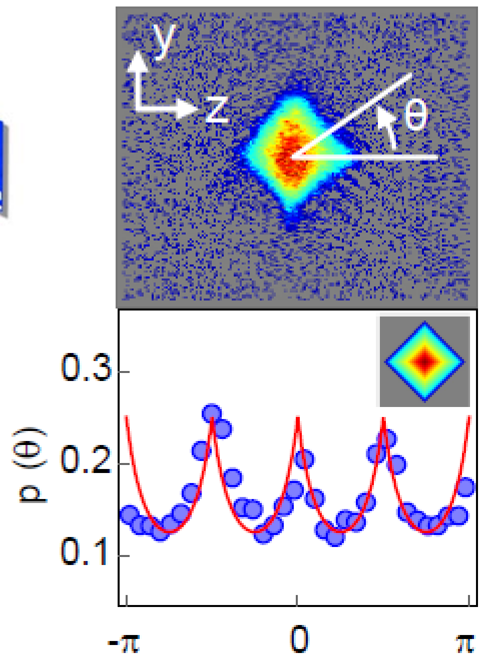
GPE in 256^3 grid with 10 μs steps for up to 4s (GPU)

30 min computation time per 1 sec experimental time

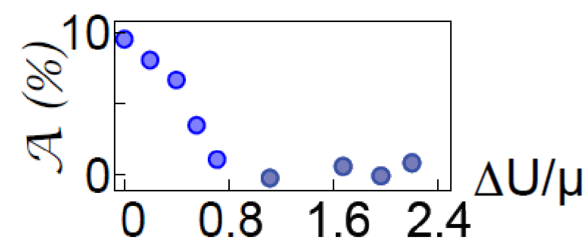
Emergence of turbulence



Initial BEC



Anistropy parameter $$\mathcal{A} = \frac{1}{2}\int \mathrm{d}\theta\, \left| p(\theta) - \frac{1}{2\pi} \right|$$

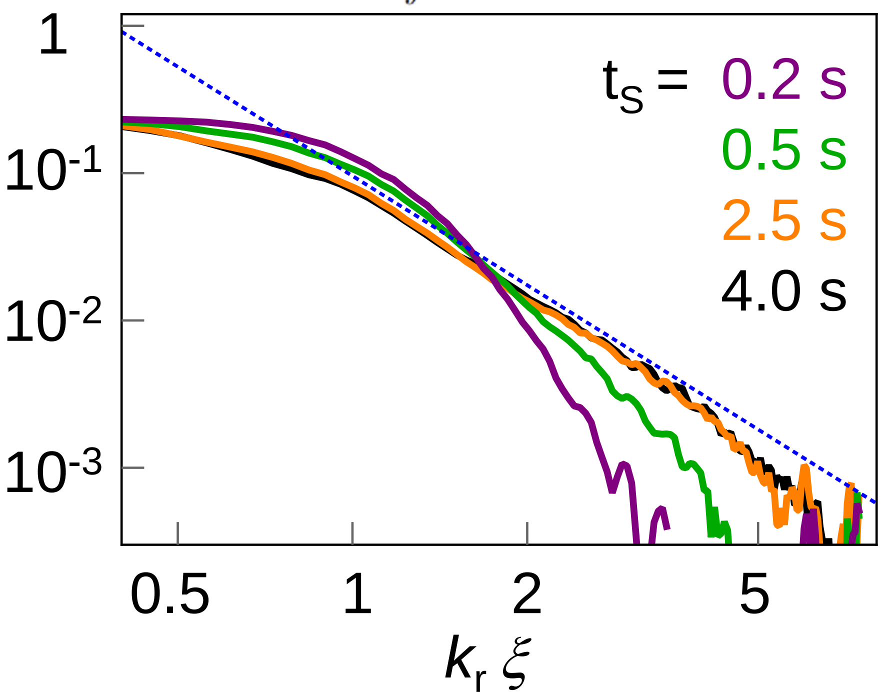


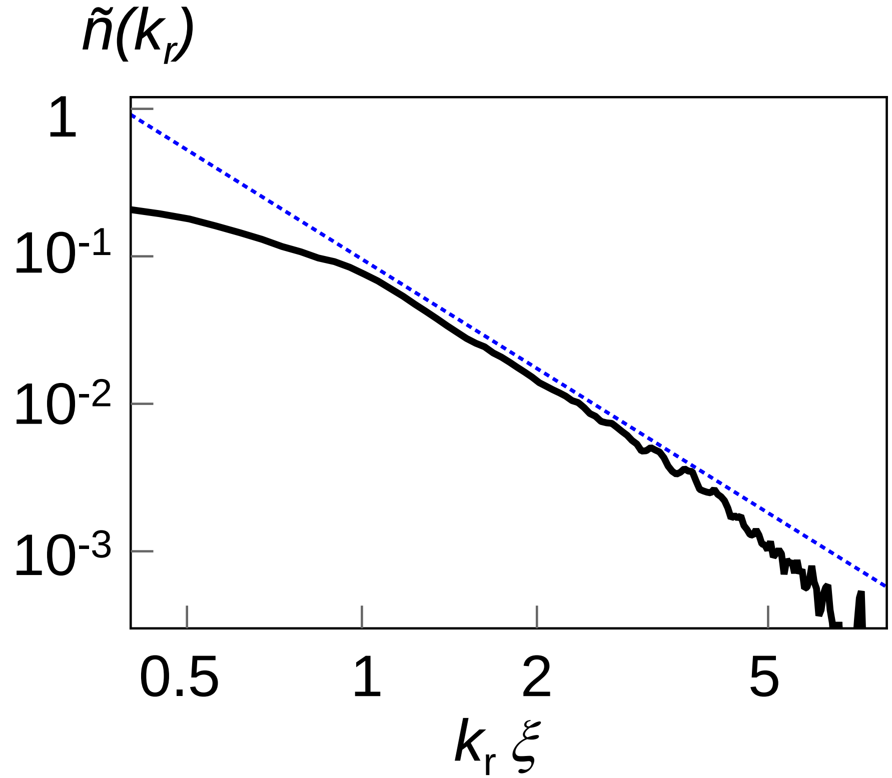
Isotropic state yet highly degenerate (not thermal!)

Matter-wave Turbulent Cascade

N. Navon *et al*, *Nature* **539** 72 (2016)

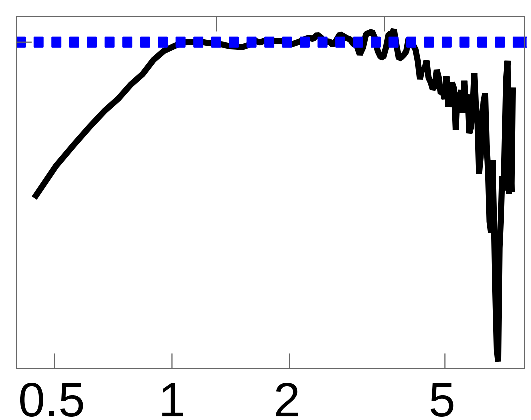
$$\tilde{n}(k_r) = \int \mathrm{d}k_z n(\sqrt{k_r^2 + k_z^2})$$





Compensated spectrum $k_r^{\gamma-1}\,\tilde{n}(k_r)$

$n(\mathbf{k}) \propto k^{-\gamma}$ with $\gamma = 3.5(1)$



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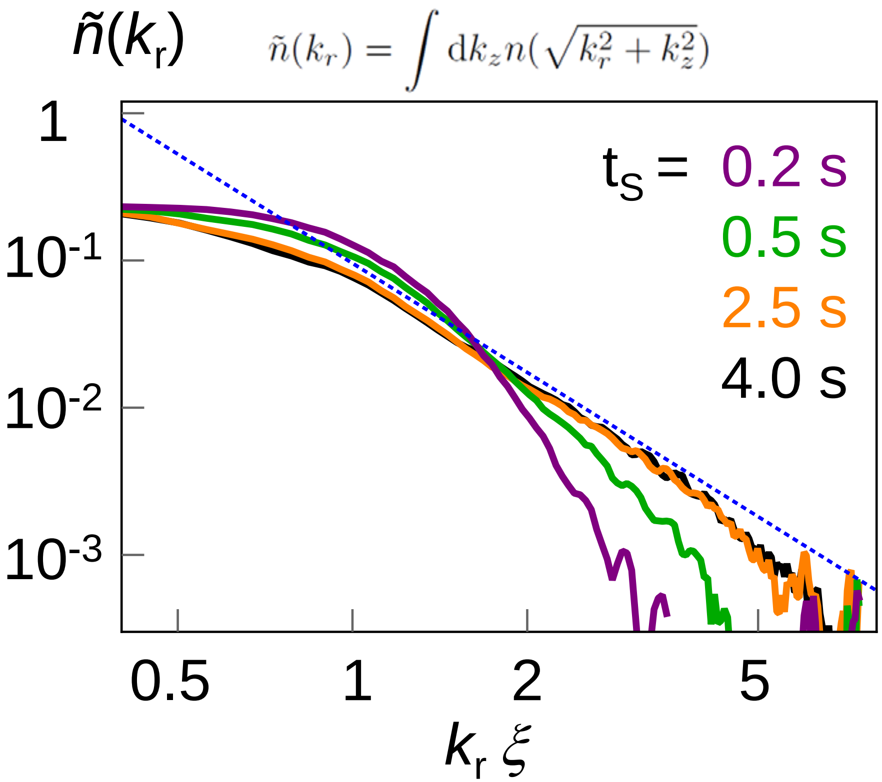
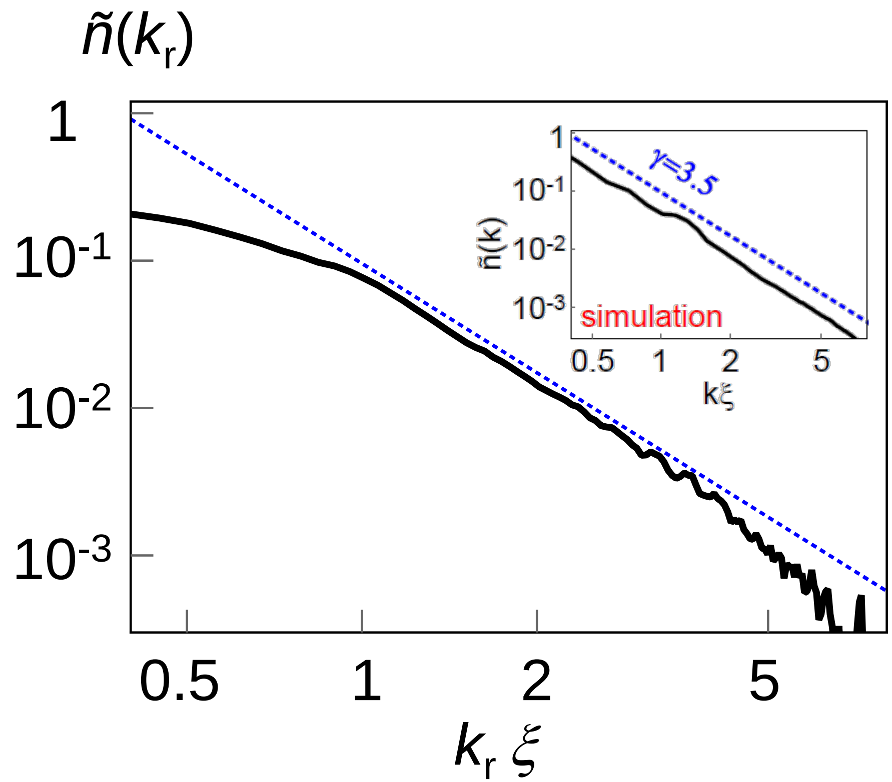
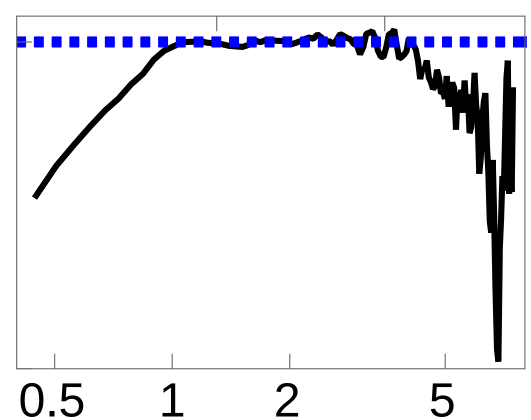




Compensated spectrum $k_r^{\gamma-1} \, \tilde{n}(k_r)$



$n(\mathbf{k}) \propto k^{-\gamma}$ with $\gamma = 3.5(1)$

Analytical prediction for γ?

Weak wave turbulence I

from Peierls to Zakharov

(a two-slide crash course)

- Starting from GP, analytical theory of WT in the weak coupling limit

$$i\hbar\frac{\partial\psi}{\partial t} = \left(-\frac{\hbar^2}{2m}\Delta + g|\psi|^2\right)\psi$$

(classical Bose field, not BEC wavefunction!)

- With assumption of weak coupling and 'random phase approximation':

$$\frac{\partial n(\mathbf{k},t)}{\partial t} = g^2\mathcal{I}_{\rm coll}[n(\mathbf{k},t)]$$

where $\psi(\mathbf{r},t) \propto \int \mathrm{d}\mathbf{k}\; c(\mathbf{k},t)e^{-i\mathbf{k}.\mathbf{r}}$ and $\langle c(\mathbf{k},t)c^*(\mathbf{k}',t)\rangle \approx n(\mathbf{k},t)\delta(\mathbf{k}-\mathbf{k}')$
higher order Gaussian correlators (Wick)

- 'Trivial' steady states: thermodynamic equilibrium $n_k^{\rm eq} = \dfrac{T}{\frac{k^2}{k_L^2} - \mu}$

- Exact turbulent cascade solutions $n_k \sim k^{-\gamma}$

Weak wave turbulence II

- If collision integral converges: KE is continuity equation (in **k** space)

$$\frac{\partial n(\mathbf{k},t)}{\partial t} = -\nabla_{\mathbf{k}}.\mathbf{\Pi}_n$$

- Continuity equation for energy $\mathcal{E}(\mathbf{k},t) = \hbar\omega_k n(\mathbf{k},t)$

$$\frac{\partial \mathcal{E}(\mathbf{k},t)}{\partial t} = -\nabla_{\mathbf{k}}.\mathbf{\Pi}_{\mathcal{E}}$$

- WWT theory of GP predicts two cascade solutions

?!?

(i) Direct energy cascade $n(k) \sim k^{-3}$

$$\begin{aligned} \Pi_{\mathcal{E}} &= \epsilon \\ \Pi_n &= 0 \end{aligned}$$

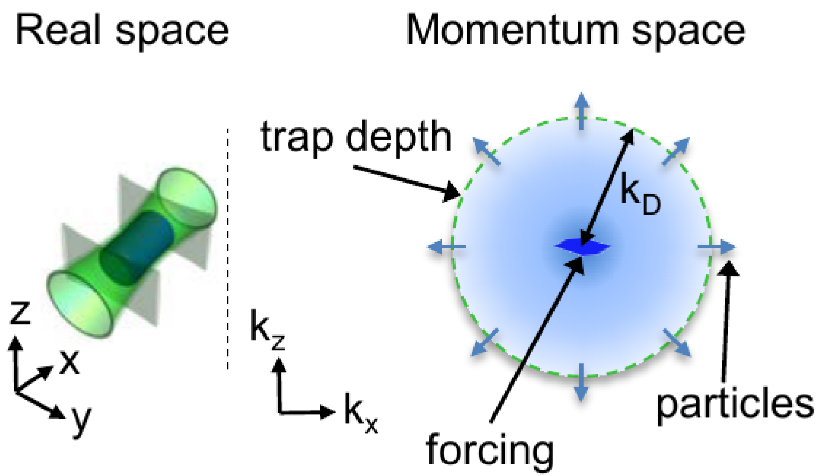
(ii) Inverse particle cascade $n(k) \sim k^{-7/3}$

$$\begin{aligned} \Pi_{\mathcal{E}} &= 0 \\ \Pi_n &= Q \end{aligned}$$

experimental relevance not obvious! source and sink terms…
Link to 0^{th} law of turbulence (phenomenological): keeping everything else the same, dissipation rate ε → constant when ν → 0 (anomalous limit)
"independence of forcing and dissipation"

Probing fluxes in a turbulent quantum gas

- Exotic feature: box depth introduces a momentum cutoff k_{D}
  a *synthetic* dissipation scale



- Direct measurement of particle flux through shell of *tunable* radius k_{D}

$$\frac{\partial N}{\partial t} = \frac{\partial}{\partial t} \int_0^{k_{\mathrm{D}}} n^{1\mathrm{D}}(k)\mathrm{d}k = -\Pi_n(k_{\mathrm{D}}, t)$$

We measure total E (not just HD) → dissipation = particle loss

- Note: in HD turbulence, dissipation (Kolmogorov) scale $\eta \propto \nu^{3/4}$

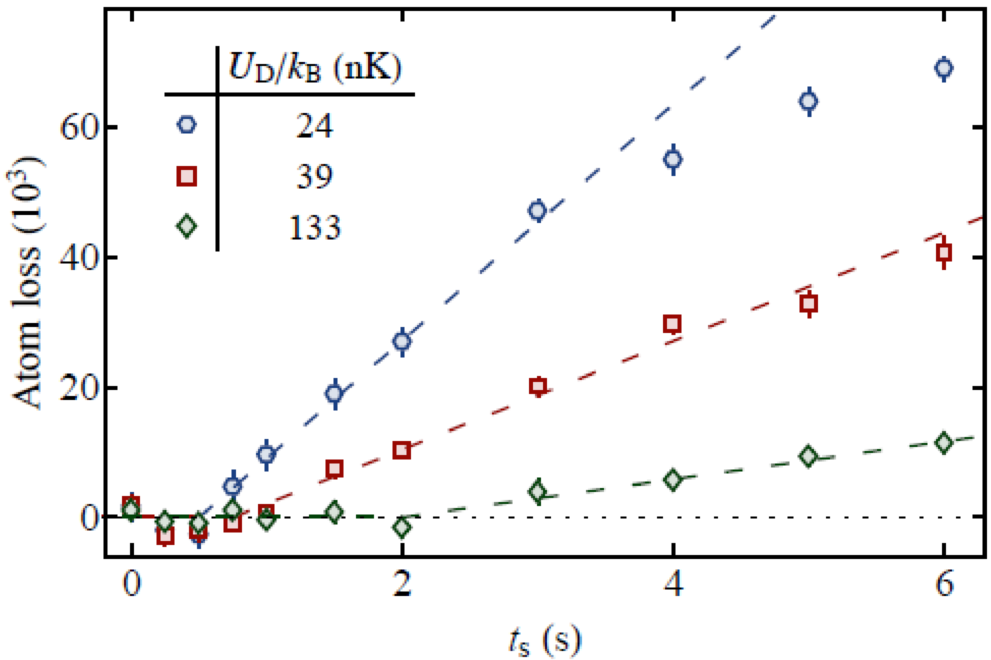
Measurement of remaining atoms

Navon et al, *arXiv:1807.07564* (2018)

- We measure the atom lost during the shaking

$$N_{\rm lost} = N(t_s)|_{\Delta U} - N(t_s)|_{\Delta U=0}$$

Differential measurement factors out important systematic effects:
residual background gas induced losses, slow drifts



$\mu \approx 2$ nK

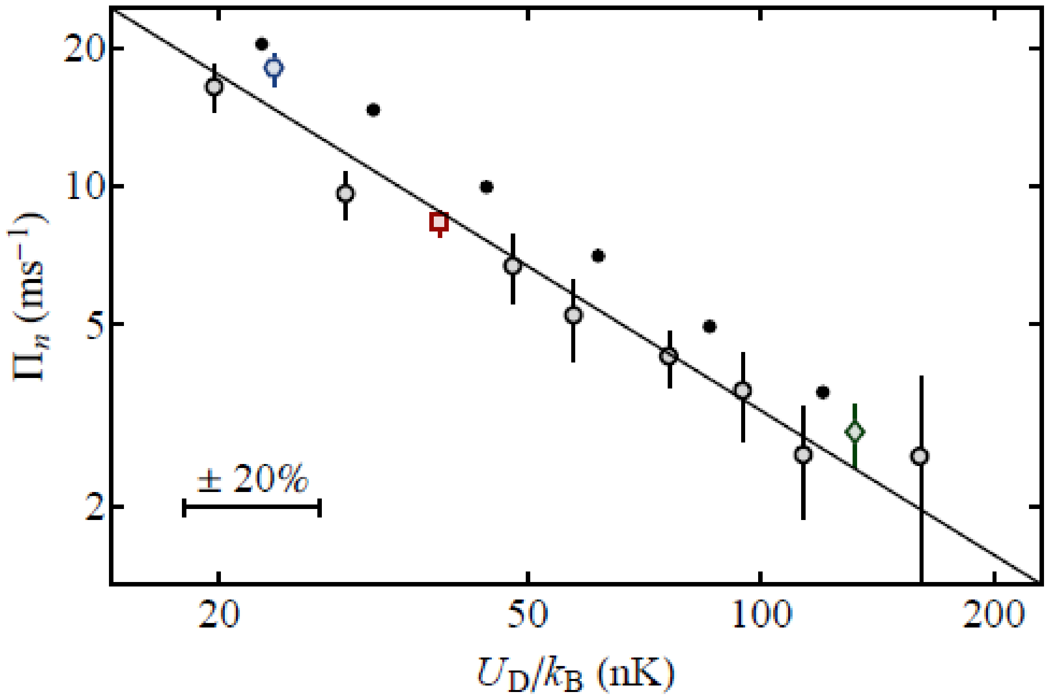
$\Delta U = 1.3\mu$

$$U_D = \frac{\hbar^2 k_D^2}{2m}$$

Two features: (1) delayed onset of losses
(2) lost rate constant afterwards
Both depend on the dissipation scale

The particle flux

Navon et al, *arXiv:1807.07564* (2018)



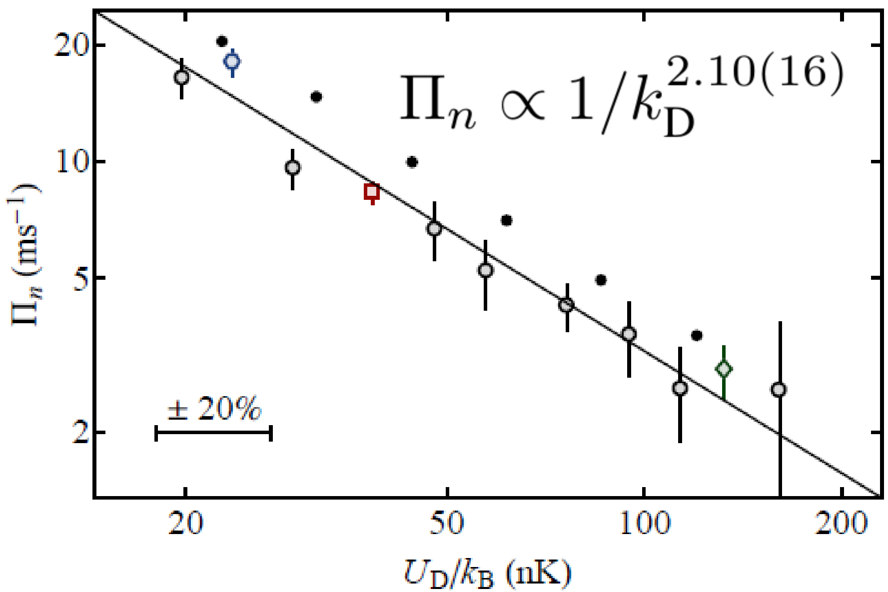
$$\Pi_n \propto U_D^\alpha$$
$$\alpha = -1.05(8)$$
$$\Rightarrow \ \Pi_n \propto 1/k_D^{2.10(16)}$$

Simulation of GPE + dissipation (no free parameters)

$$\alpha_{sim} = -1.06(1)$$

→ Particle flux vanishes in the limit of zero dissipation scale

Relation between particle and energy fluxes



For weak interactions $\mathcal{E}(k) = \hbar\omega(k) n(k)$

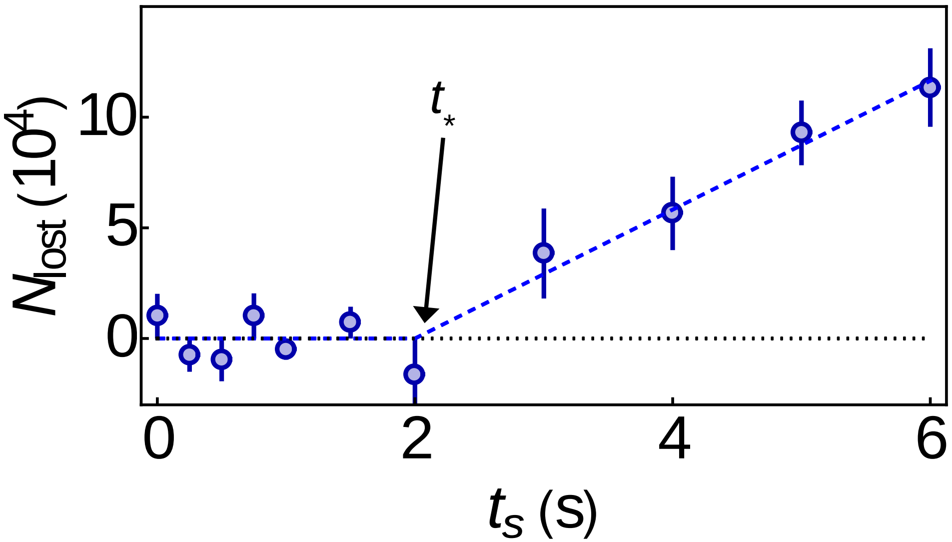
Then, naively $\Pi_{\mathcal{E}}(k) = \hbar\omega(k)\Pi_n(k)$ *can't be true in steady state!*

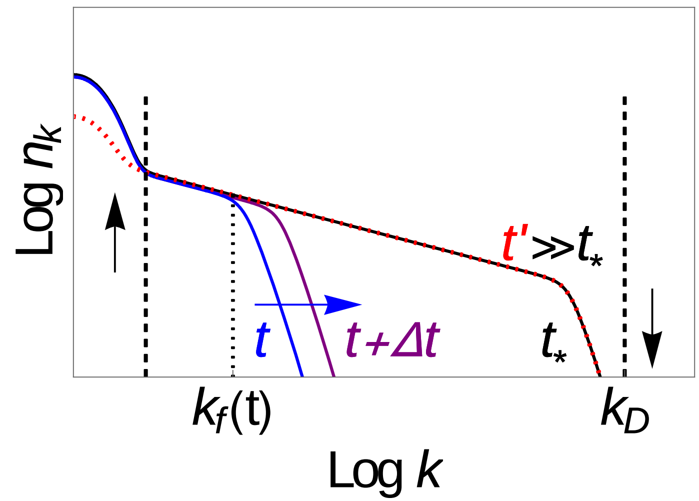
From continuity equations $\Pi_{\mathcal{E}}(k_{\rm D}) = \hbar\omega(k_{\rm D})\Pi_n(k_{\rm D})$

We thus find $\Pi_{\mathcal{E}}(k_{\rm D}) = \Pi_{\mathcal{E}} \quad (= \epsilon)$ energy cascade

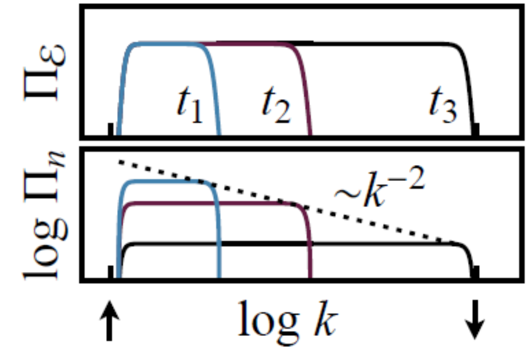
« 0^{th} law » of turbulence: for fixed forcing, energy input/dissipation rate tends to a constant when $\nu \rightarrow 0$

Consistent picture of the energy cascade (and its fluxes)





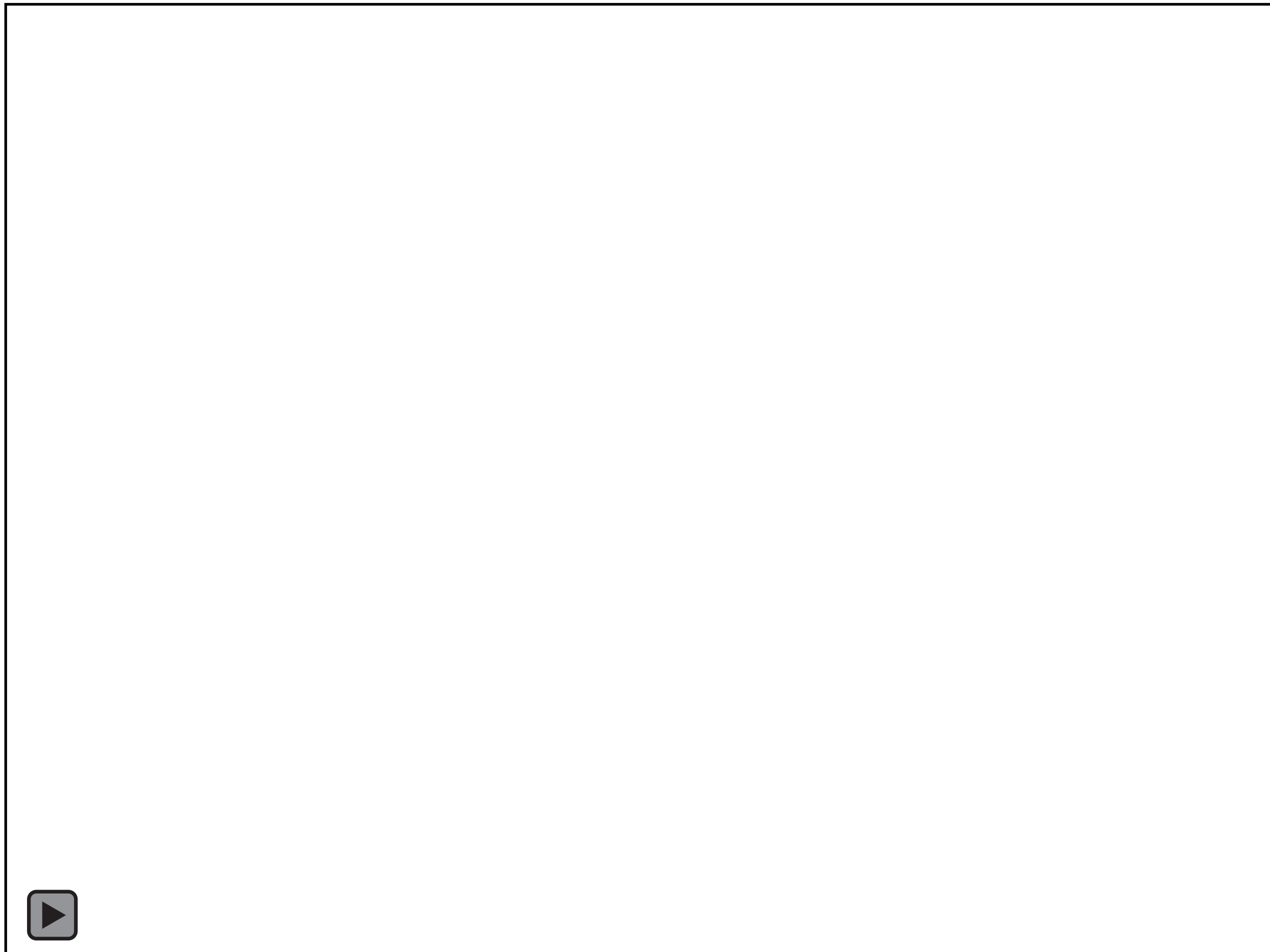
Real-time observation of turbulent cascade



Particle flux has to "adjust" to sustain the direct energy cascade

Testable prediction of cascade front dynamics (pre-steady state)

Momentum distribution dynamics (numerical simulations)



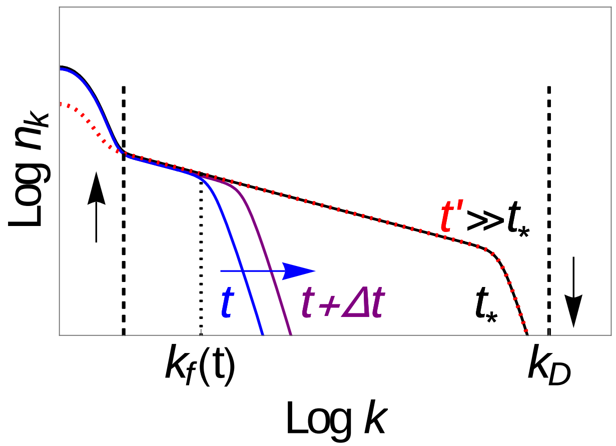
nd M. Tsubota

Amazing that such complex real-space dynamics is captured by simple Fourier space dynamics

Cascade front propagation

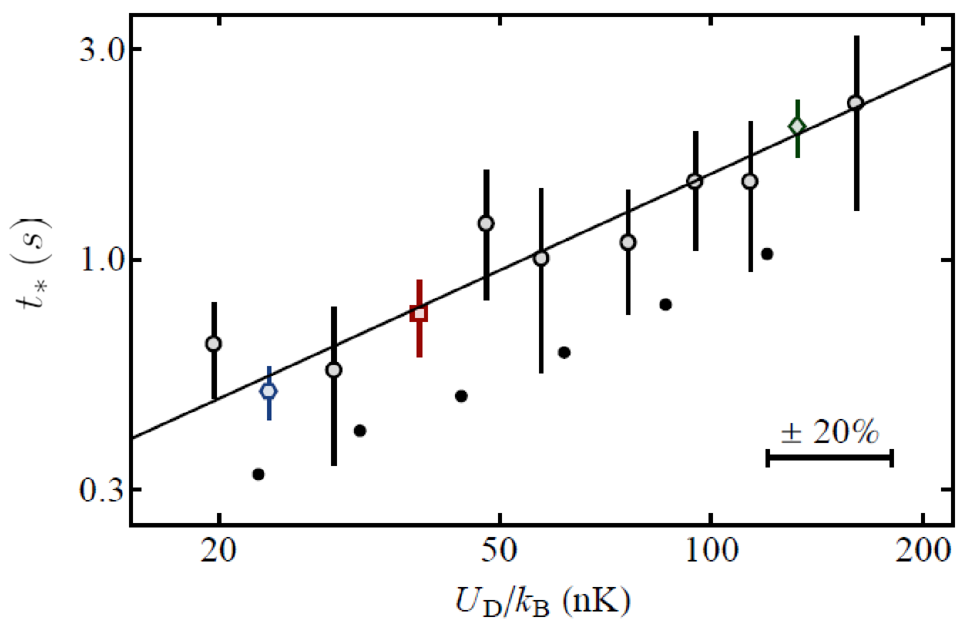
Energy in inertial range up to the cascade front:

$$E_{\mathrm{inert}}(t) = \int_{k_{\mathrm{low}}}^{k_{\mathrm{f}}(t)} \mathcal{E}(k)\mathrm{d}\mathbf{k}$$



Energy injected pushes cascade front forward

$$\epsilon \equiv \frac{\mathrm{d}E_{\mathrm{inert}}}{\mathrm{d}t} = \frac{\mathrm{d}k_{\mathrm{f}}}{\mathrm{d}t} k_{\mathrm{f}}^2.(k_{\mathrm{f}}^2 n_{k_{\mathrm{f}}})$$ with $n_k \sim k^{-\gamma}$ leads to $t_* \propto U_{\mathrm{D}}^{\beta}$



$$\beta = \frac{5-\gamma}{2} = 0.75(5)$$

$$\beta_{\mathrm{exp}} = 0.73(6)$$

$$\beta_{\mathrm{sim}} = 0.68(2)$$

Summary/Outlook

- Turbulent steady-state cascade in quantum gas

- Measurement of the fluxes
  - Particle flux vanishes in the zero-dissipation limit
  - Energy flux is constant
  - Quantitative understanding of cascade front propagation

- Simple technique specific to ultracold gases

- No-free parameters GPE simulations including realistic dissipation

- Open questions:
  - (i) Cascade exponent value from analytics?
  - (related: Quantify deviation from WWT theory?)
  - (ii) Ultimate dissipation mechanism for cascade? (effective dissipation beyond the classical regime?)
  - (iii) Establish the validity of classical field methods
  - (iv) Interplay of compressible and incompressible-fluid turbulence

The Team(s)

Color scale: Prof, Dr, Ph.D student

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Zoran Hadzibabic


Rob Smith

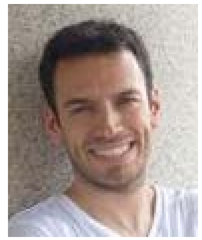

Alex Gaunt


Rapha Lopes

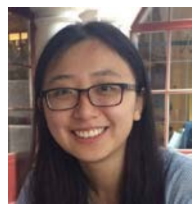

Chris Eigen

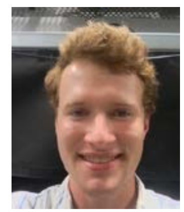

Jinyi Zhang

Yale

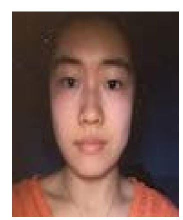

NN


Franklin Vivanco


Yunpeng Ji


Grant Schumacher


Gabriel Assumpcao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