Friction, Failure, and Dynamics of Granular Materials

KITP, Friction Fracture and Earthquakes
December 8, 2005

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OUTLINE

- Why granular materials?
- Where granular materials and molecular matter part company—open questions of relevant scales
- Review of models for force transmission

- Obtaining vector forces
- Force distributions and correlations
- Diffusion, dynamics, fluctuations in shear
- Dynamical transition: freezing by heating in a sheared and shaken system
- Force Transmission (via an older experimental approach)

- Conclusions

Examples of Granular Materials

- Earthquake gouge
- Avalanches and mudslides
- Food and other natural grains: wheat, rice,…
- Industrial materials: coal, ores,…
- Soils and sands
- Pharmaceutical powders
- Dust
- Chemical processing—e.g. fluidized beds

What are Granular Materials?

- Collections of macroscopic ‘hard’ (but not rigid) particles: interactions are dissipative
  - Classical $h \rightarrow 0$
  - A-thermal $T \rightarrow 0$
  - Draw energy for fluctuations from macroscopic flow
  - Exist in phases: granular gases, fluids and solids
  - Large collective systems, but outside normal statistical physics

Questions

- Fascinating and deep statistical questions
  - What is the nature of granular friction?
  - What is the nature of granular fluctuations—what is their range?
  - Is there a granular temperature?
  - Phase transitions
  - Jamming and connections to other systems: e.g. colloids, foams, glasses,…
  - The continuum limit and ‘hydrodynamics—at what scales?
  - What are the relevant macroscopic variables?
  - Novel instabilities and pattern formation phenomena

Practical Issues

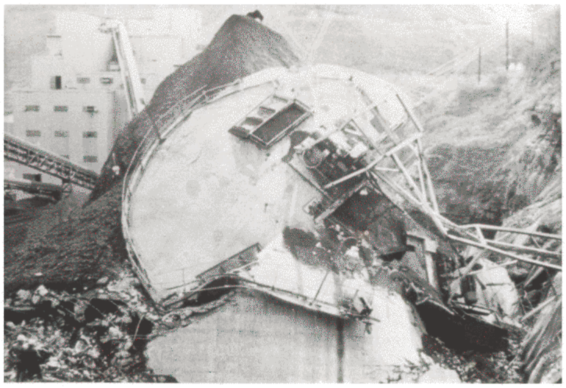
o Massive financial costs Claim:
~$1 Trillion/year in US for granular handling

o Failures are frequent, typical facilities operate at only ~65% of design

o Soil stability is difficult to predict/assess

o How is stress/information transmitted in granular materials?

Problems closer to (my) home



…And a bit further from home…

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Images from Mars rover reveal mysterious clumps

Submitted by GOENKARBOY on 18 January, 2004 - 12:55am.

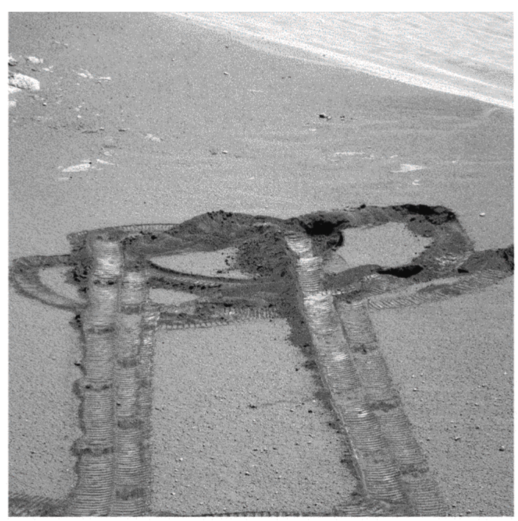
Title: Images from Mars rover reveal mysterious clumps
Scientists baffled by sandpaper-like patches on surface

Publication:
Author: By David Perlman, Chronicle Staff Editor

Short description: The images that Spirit sent down from its Martian parking spot, a few feet in front of its landing pad, was a flat patch of fine- and coarse-grained sand -- much of it stuck together in clumpy patches that scientists conceded they did not yet understand.

Article:

Pasadena -- Ecstatic scientists used the Mars rover's powerful camera Friday to take the first close-up images ever made of the Martian surface and immediately confronted a new mystery over what they saw. The images that Spirit sent down from its Martian parking spot, a few feet in front of its landing pad, was a flat patch of fine- and coarse-grained sand -- much of it stuck together in clumpy patches that scientists conceded they did not yet understand.



Assessment of theoretical understanding

- Basic models for dilute granular systems are reasonably successful
- For dense granular states, theory is far from settled, and under intensive debate and scrutiny
- Current ability to predict for dense granular states is poor relative to other systems—e.g. fluids

Granular Material Phases-Dense Phases

Granular Solids and fluids much less well understood than granular gases

Forces are carried preferentially on force chains→multiscale phenomena

Deformation leads to large spatio-temporal fluctuations

Granular Material Phases-Dense Phases Continued

Friction and extra contacts → preparation history matters

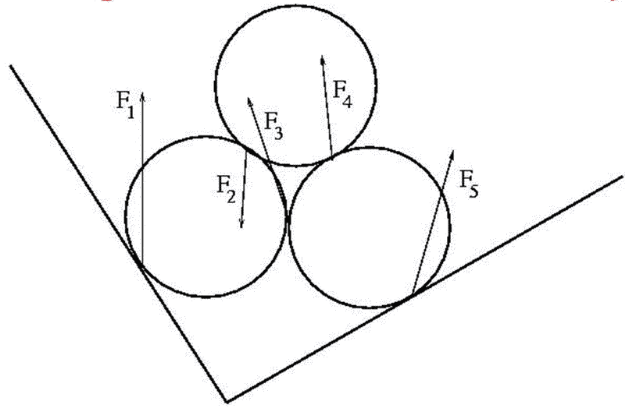
Jamming/glassy behavior near solid-fluid transition (Liu, Nagle, O'Hern, Bouchaud et al.)

--interesting connections to plasticity in disordered solids (e.g. Falk, Langer, Lemaitre, Maloney…)

In most cases, a statistical approach may be the only possible description

A look at some properties that affect dense granular states

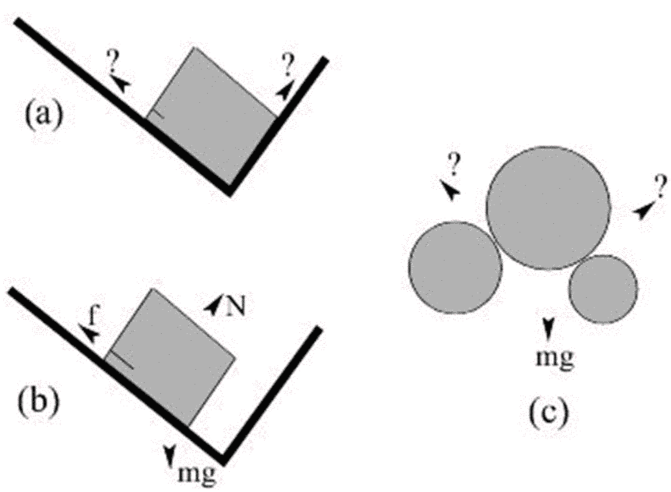
Multiple contacts => indeterminacy



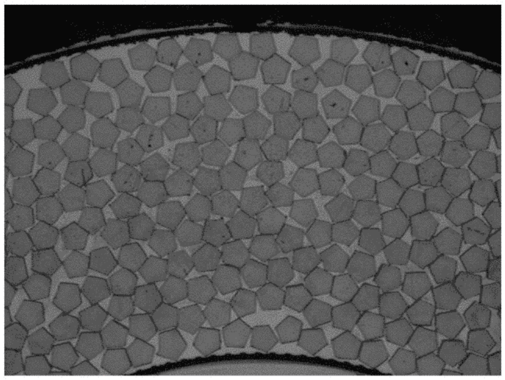
Note: 5 contacts => 10 unknown force components.

3 particles => 9 constraints

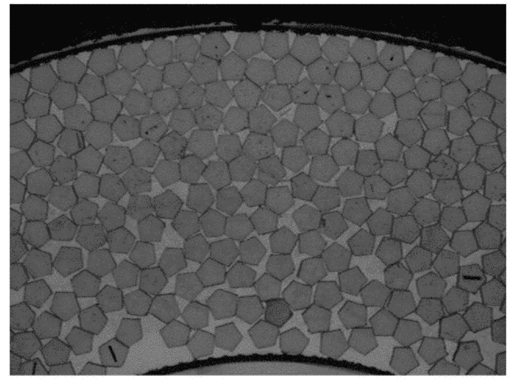
Frictional indeterminacy => history dependence



Dilation under shear

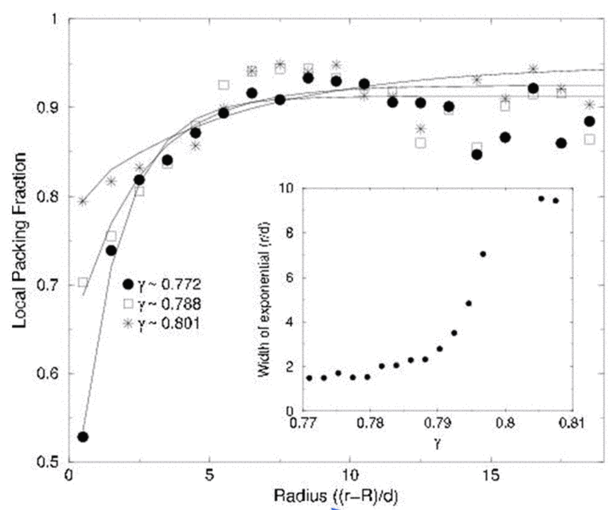


Before shearing



After sustained shearing

Density profiles following sustained shearing

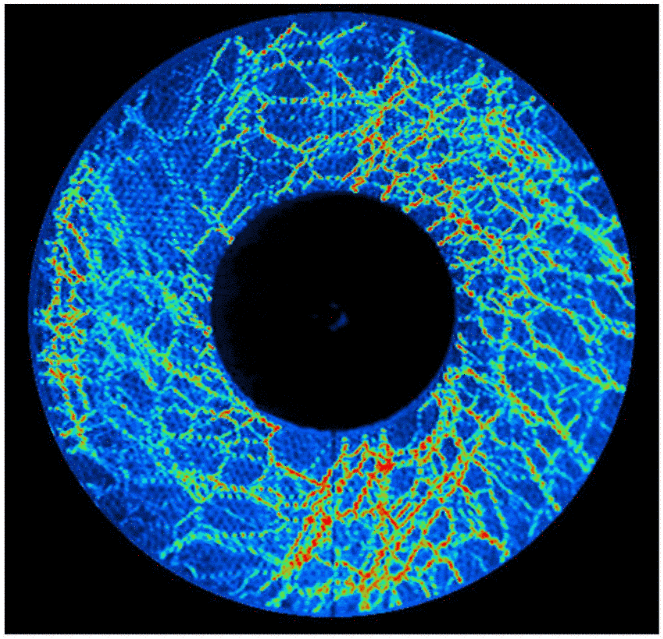


γ = mean packing fraction

r-R = distance from shearing surface

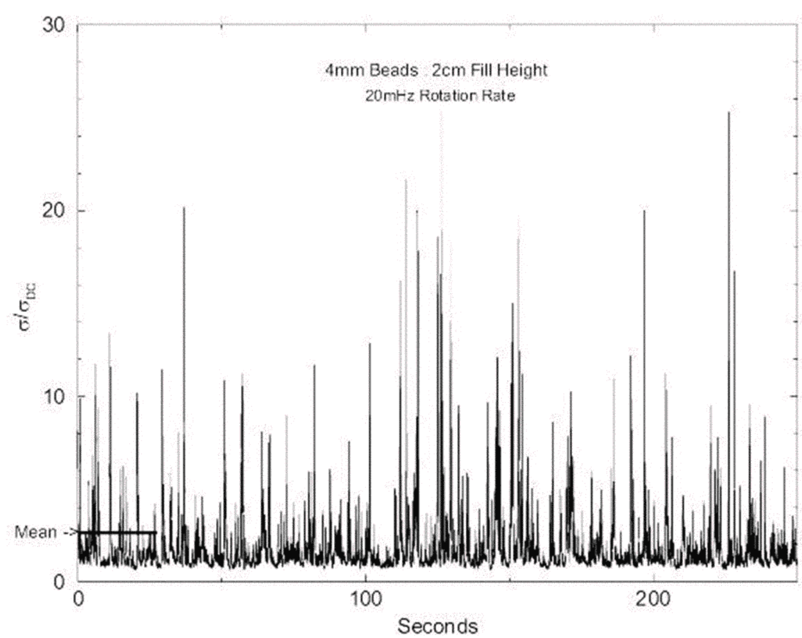
Example of Force Chains—Shear Experiment

Howell et al. PRL 82, 5241 (1999)



Stress Fluctuations in 3D Shear Flow

Miller et al. PRL 77, 3110 (1996)



Video of 2D shear flow

Understanding Static Stress Balance—Ideally from Micromechanics

- Four unknown stress components (2D)
- Three balance equations
  - Horizontal forces
  - Vertical forces
  - Torques
- Need a constitutive equation

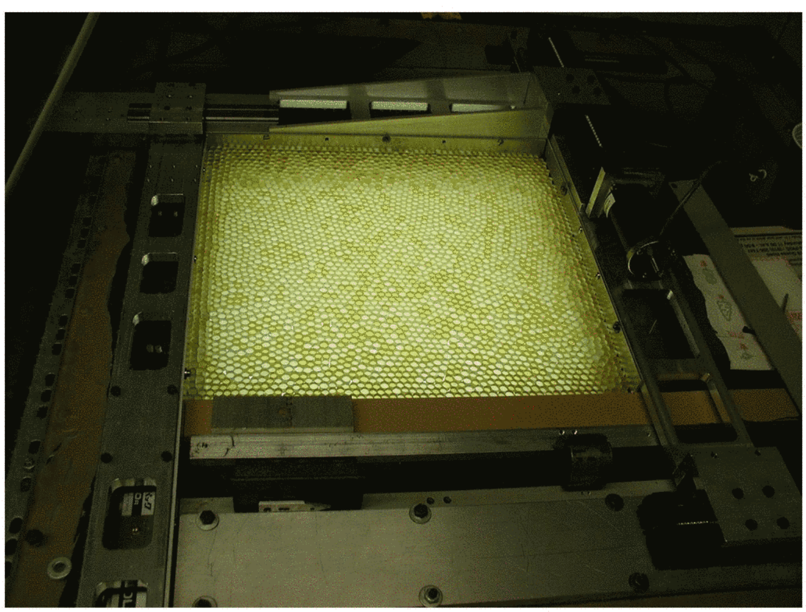
$$\frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \sigma_{xz}}{\partial z} = 0 \qquad \frac{\partial \sigma_{xz}}{\partial x} + \frac{\partial \sigma_{zz}}{\partial z} = 0 \qquad \sigma_{xz} = \sigma_{zx}$$

Some approaches to describing stresses

- Elasto-plastic models (Elliptic, then hyperbolic)
- Lattice models
  - Q-model (parabolic in continuum limit)
  - 3-leg model (hyperbolic (elliptic) in cont. limit)
  - Anisotropic elastic spring model
- OSL model (hyperbolic)
- Telegraph model (hyperbolic)
- Double-Y model (type not known in general)

Experiments to determine vector contact forces

(Trush Majmudar and RPB, Nature, June 23, 2005)



Experiments Use Photoelasticity:

Biax schematic

Compression

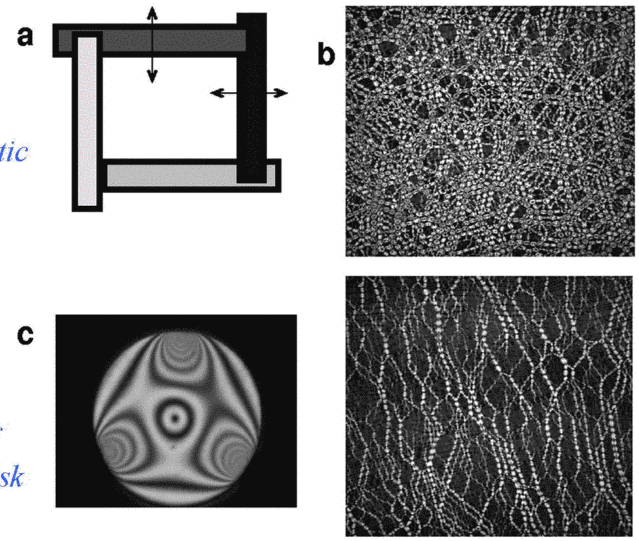
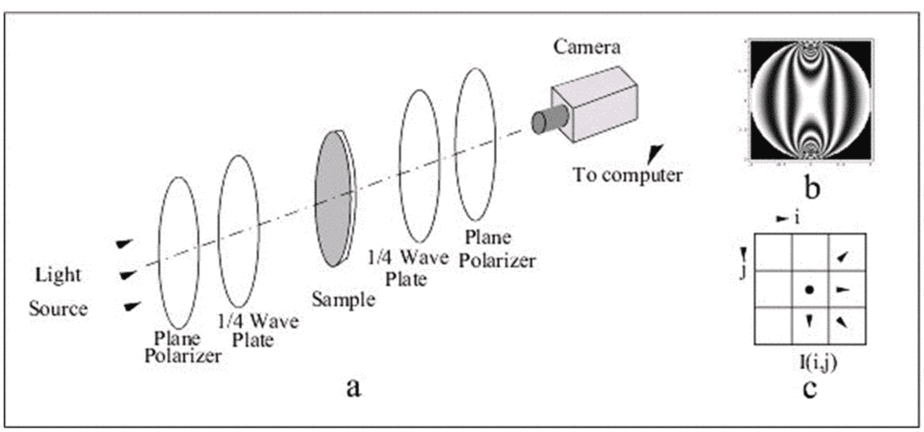


Image of Single disk

Shear

~2500 particles, bi-disperse, $d_L = 0.9cm$, $d_S = 0.8cm$, $N_S / N_L = 4$

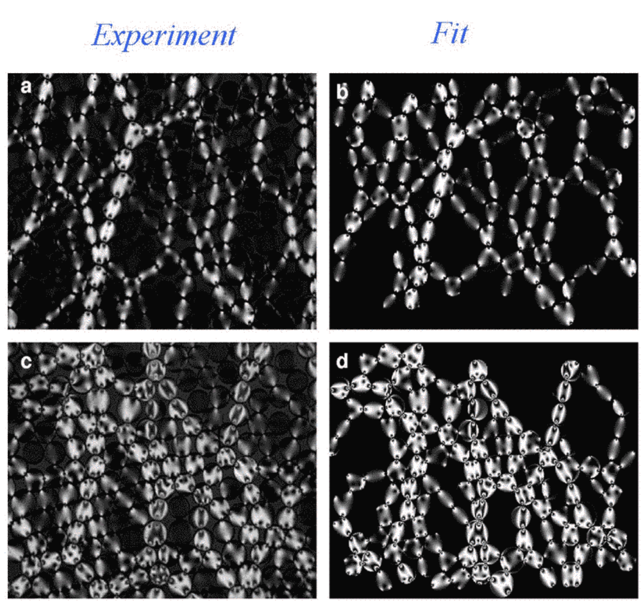
Measuring forces by photoelasticity



Basic principles of technique

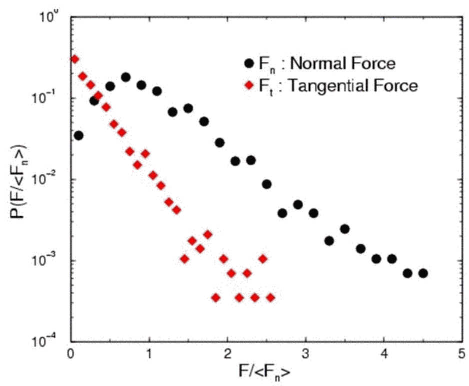
- Process images to obtain particle centers and contacts
- Invoke exact solution of stresses within a disk subject to localized forces at circumference
- Make a nonlinear fit to photoelastic pattern using contact forces as fit parameters
- $I = I_o \sin^2[(\sigma_2 - \sigma_1)CT/\lambda]$
- In the previous step, invoke force and torque balance
- Newton's 3d law provides error checking

Examples of Experimental and 'Fitted' Images



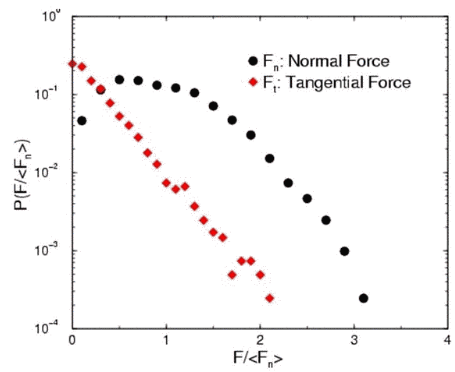
Force distributions for shear and compression

Shear



$\varepsilon_{xx} = -\varepsilon_{yy} = 0.04; \quad Z_{avg} = 3.1$

Compression



$\varepsilon_{xx} = -\varepsilon_{yy} = 0.016; \quad Z_{avg} = 3.7$

Edwards Entropy-Inspired Models for P(f)

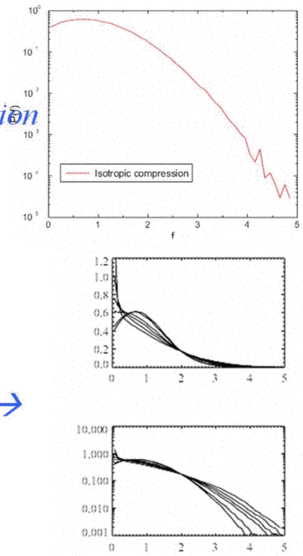
- Consider all possible states consistent with applied forces
- Compute Fraction where at least one contact force has value f→ P(f)

- E.g. Snoeier et al. PRL 92, 054302 (2004)
- Tighe et al. preprint (Duke University)

Some Typical Cases—isotropic compression and shear

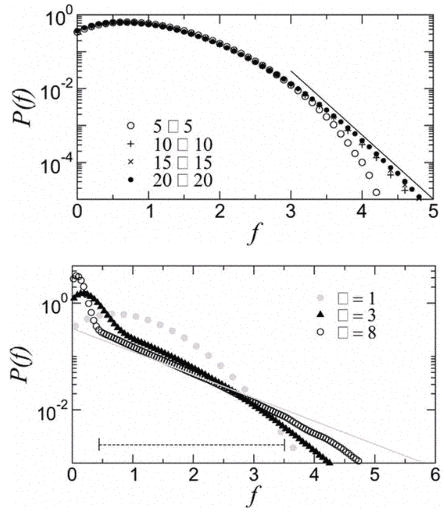
Snoeijer et al. ↓

Compression →



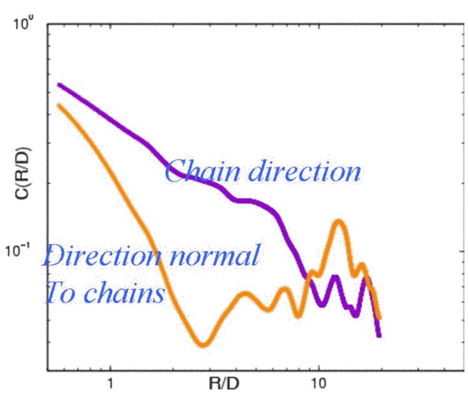
Shear →

Tigue et al ↓.

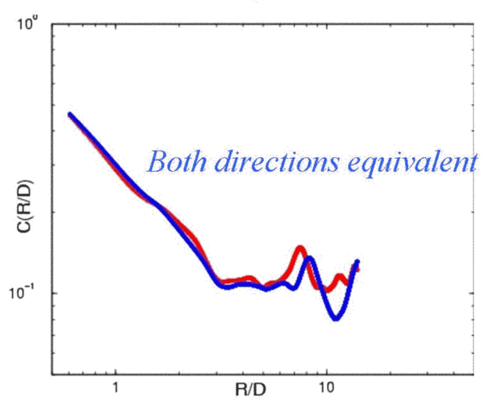


Spatial correlations of forces—angle dependent

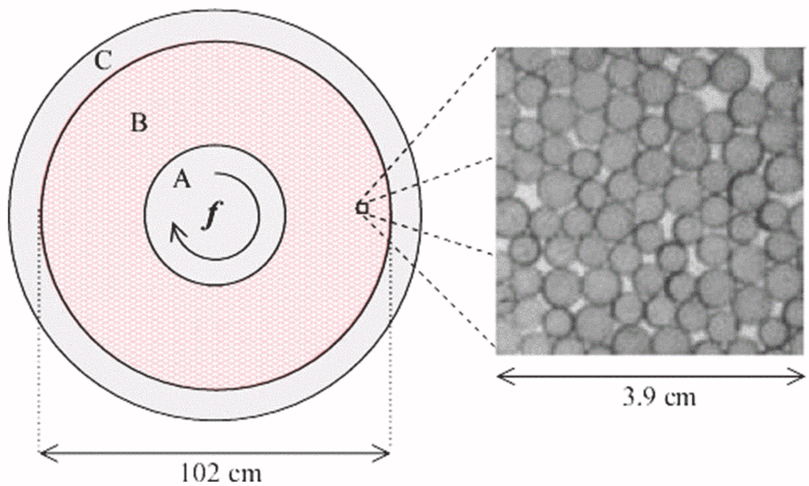
Shear



Compression

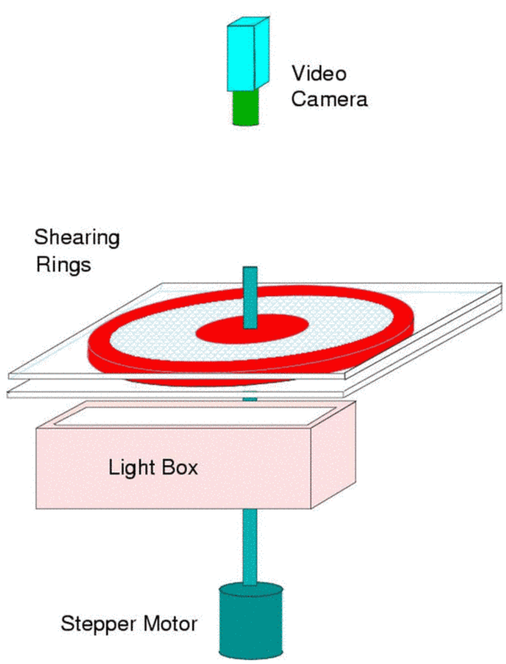


Granular friction and dynamics in a 2D sheared system

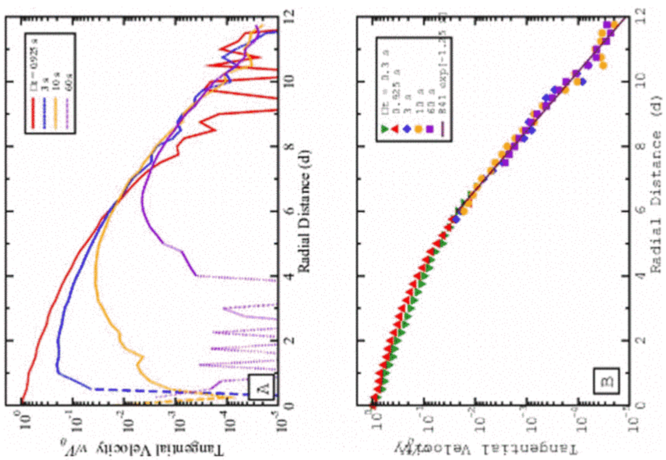


B.Utter and RPB PRE 69, 031308 (2004)
Eur. Phys. J. E 14, 373 (2004)

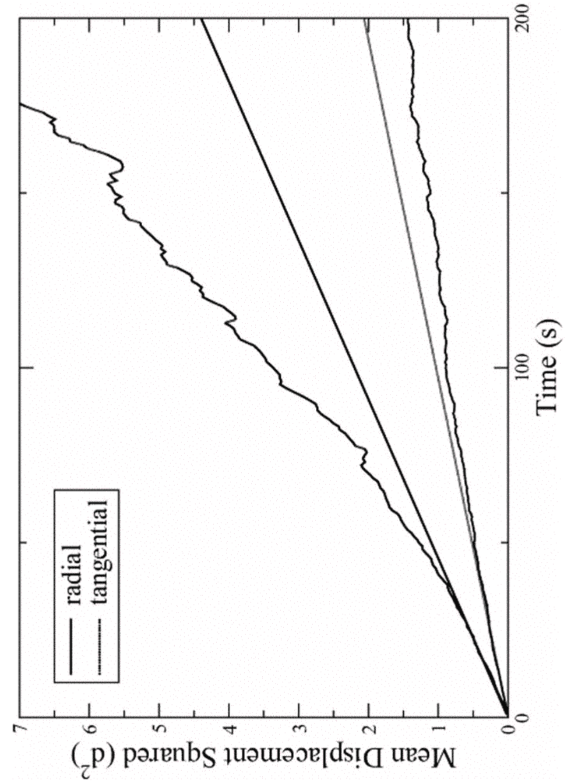
Schematic of apparatus



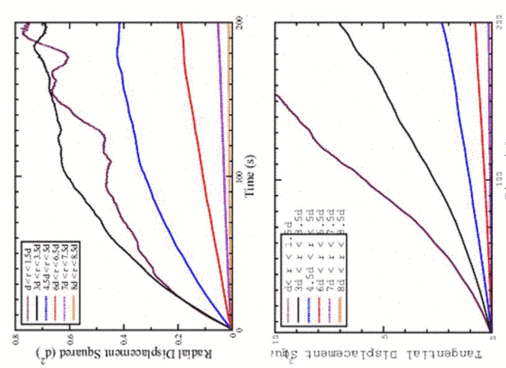
Typical near-exponential velocity profile



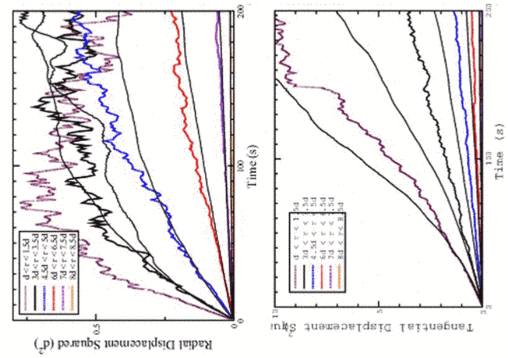
Variances in radial and tangential direction yield diffusivities



Diffusivities only appear sub- or super-diffusive due to Taylor-like dispersion and rigid boundary

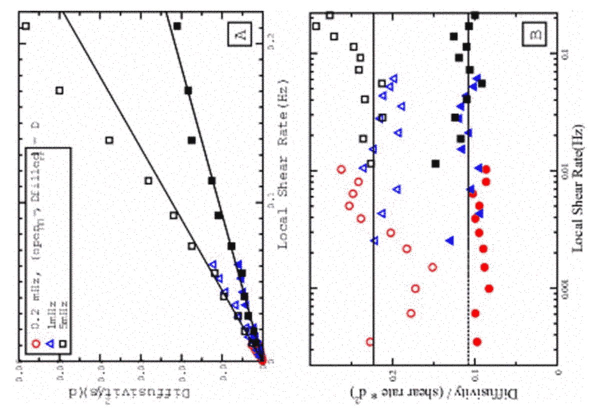


Experiment



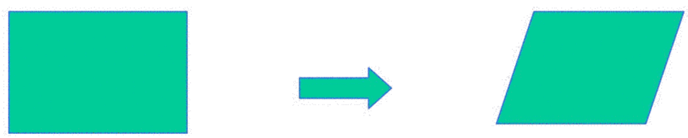
Simulations of random walk, with velocity profile, etc

Local shear rate determines diffusivities



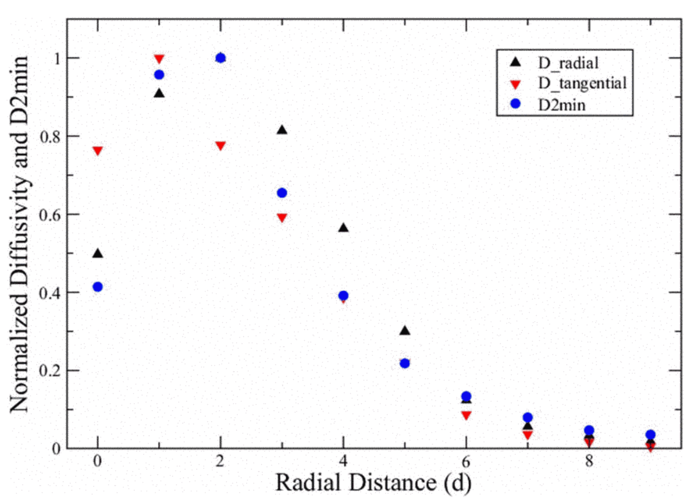
Parameterization of non-affine motion—after Falk and Langer

Local affine deformation



D^2_{min} characterizes local non-affine part of deformation

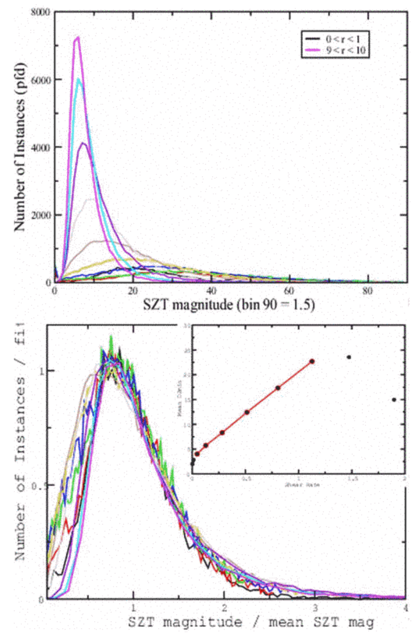
Relation to D^2_{min} of Falk and Langer



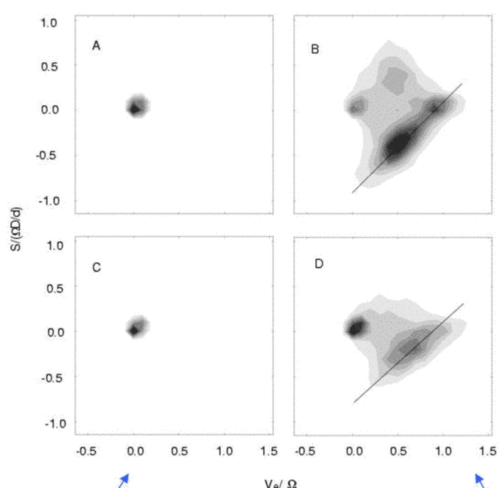
D^2_{min} measures non-affine particle motion

Distributions of D^2_{min} for different distances from shearing wheel

Useful candidate for measure of disorder?



What role does particle rotation play in granular friction?
See greyscale representations of velocity and angular velocity



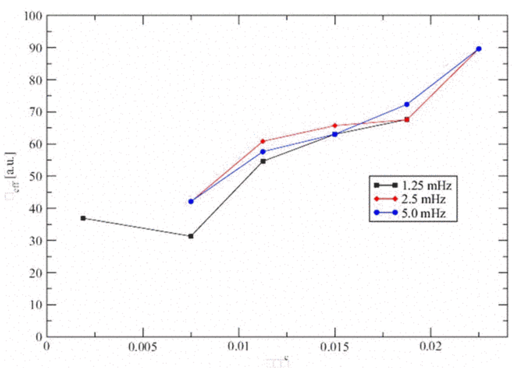
Disks

Pentagons

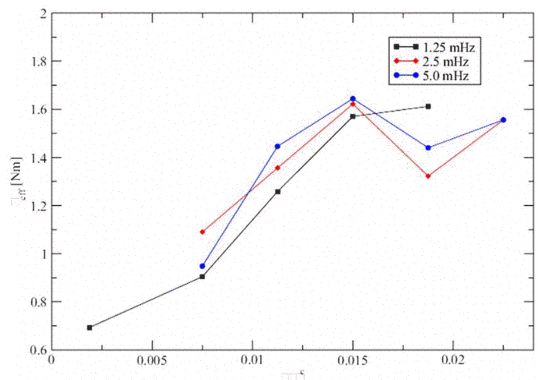
Low density

High density

Measure pressure and shear stress, compare ratio, for different densities

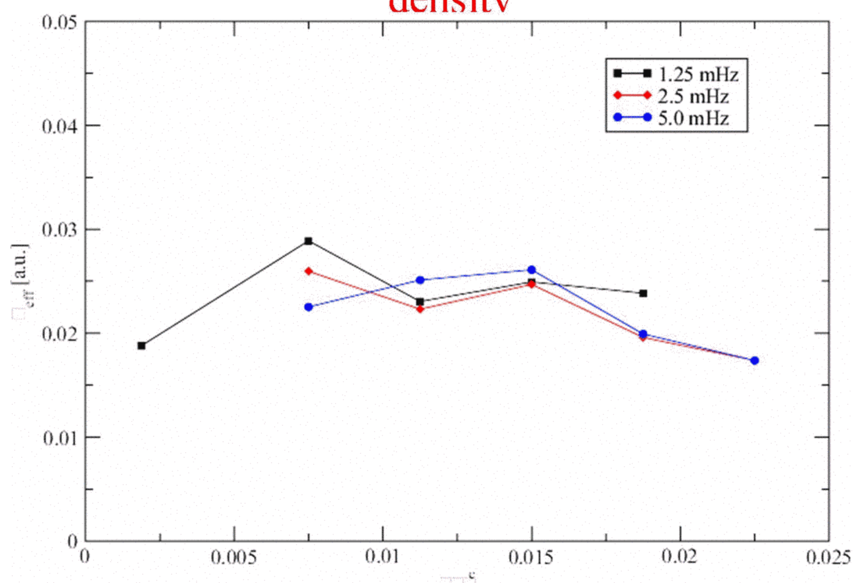


Mean P vs. density



Mean torque vs. density

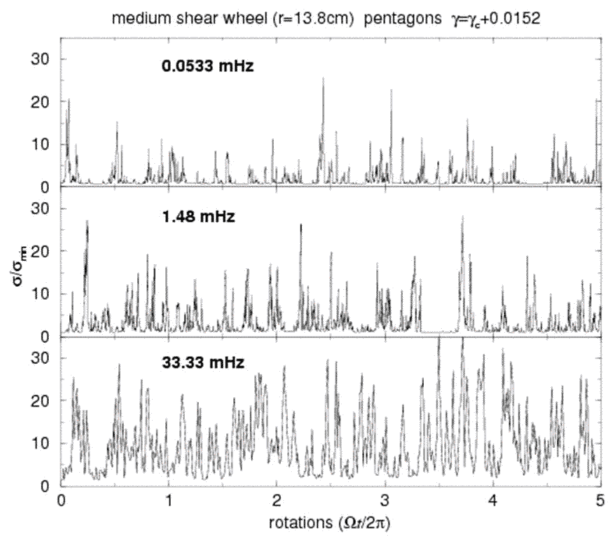
Effective friction coefficient does not change with density



Effective friction coefficient vs. density

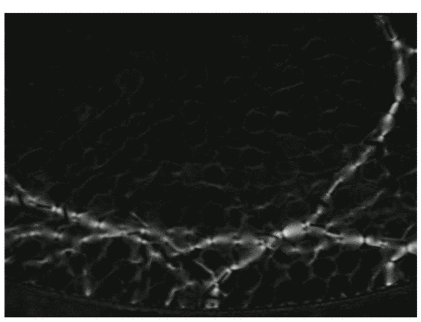
Rate Dependence in Granular Shear

γ gives density

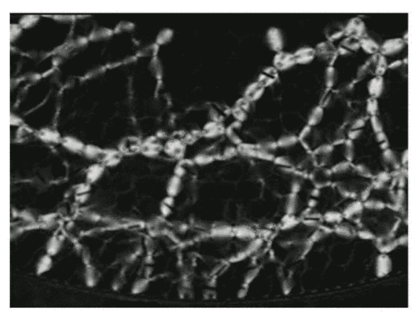


Stress fluctuations vs. time for different shear rates

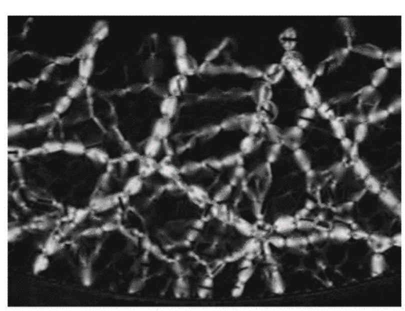
Videos at different shear rates



γ = 0.0027Hz

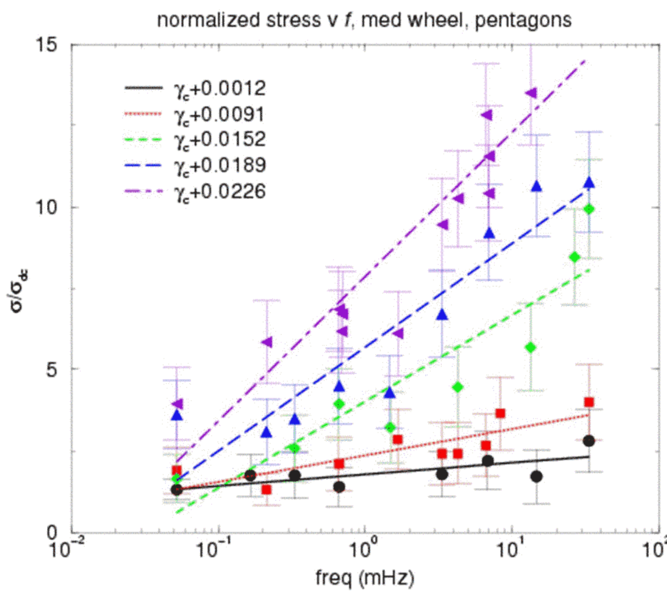


γ = 0.027Hz



γ = 0.27Hz

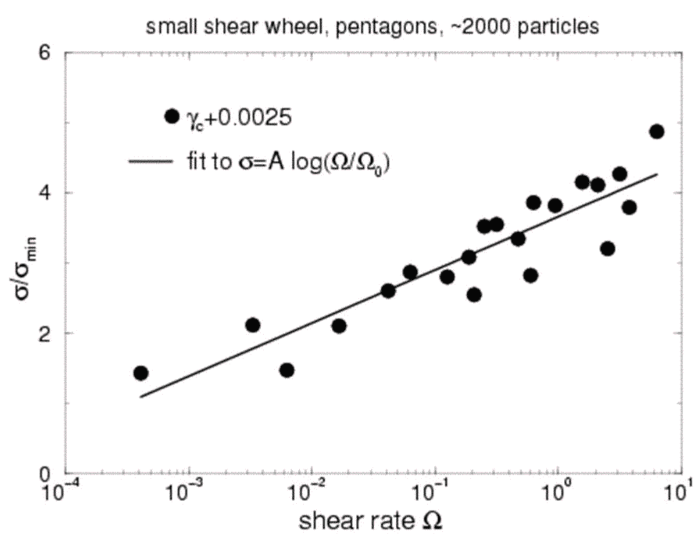
Time-averaged pressure vs. shear rate, and density



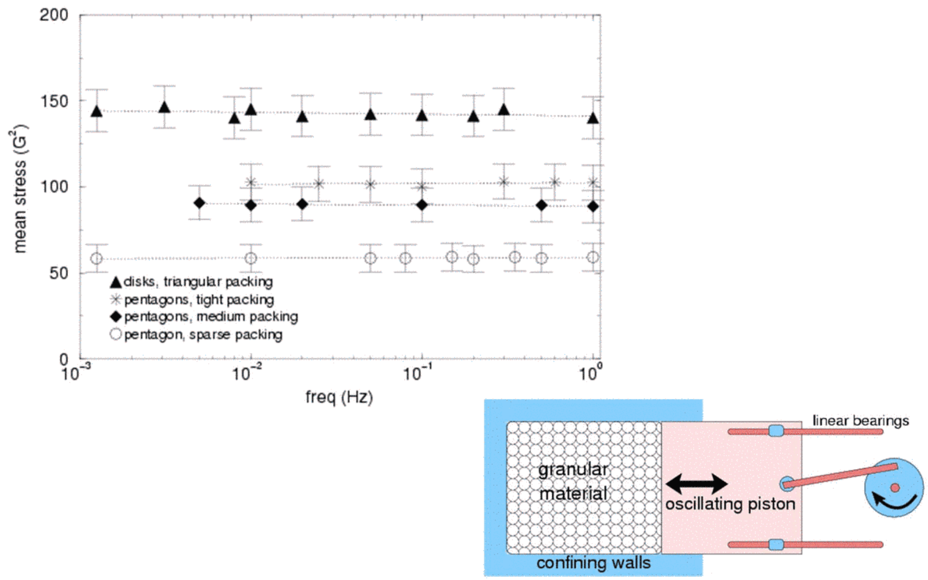
γ = packing fraction

Note, γ_c = Jamming point

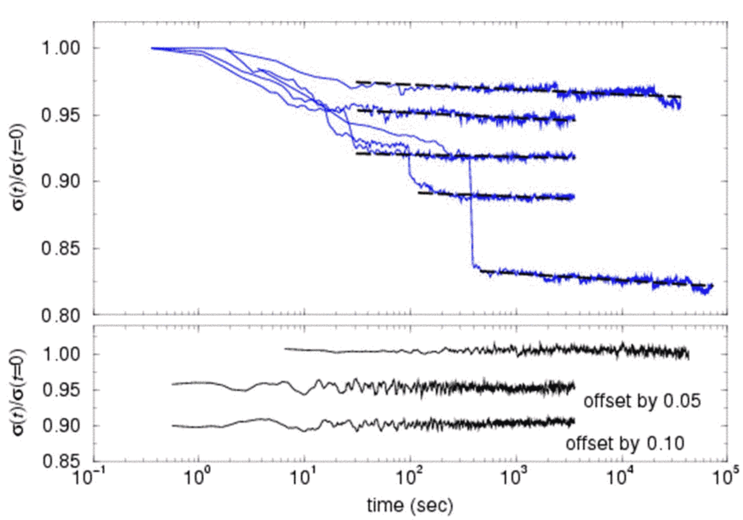
Time-averaged pressure vs. shear rate: 5 decades



No rate dependence for periodic compression



(Un?)Related Creep effects for sheared samples



Top: shear and hold

Bottom: compress and hold

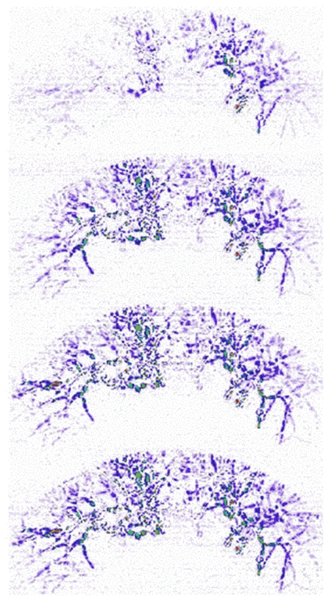
Visible relaxation in sheared sample

3s

36s

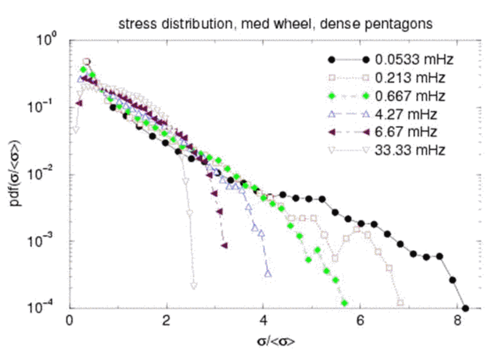
362s

36,000s



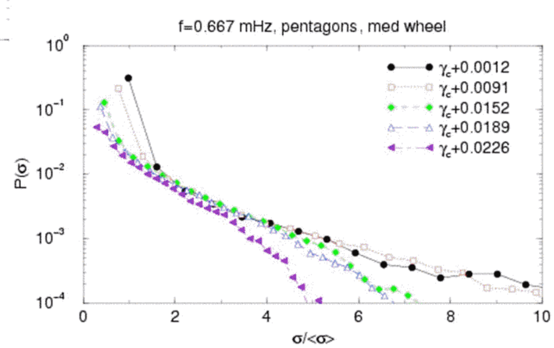
Changes in force network: Time increases downwards

Stress statistics, rate-dependence and 'avalanches'



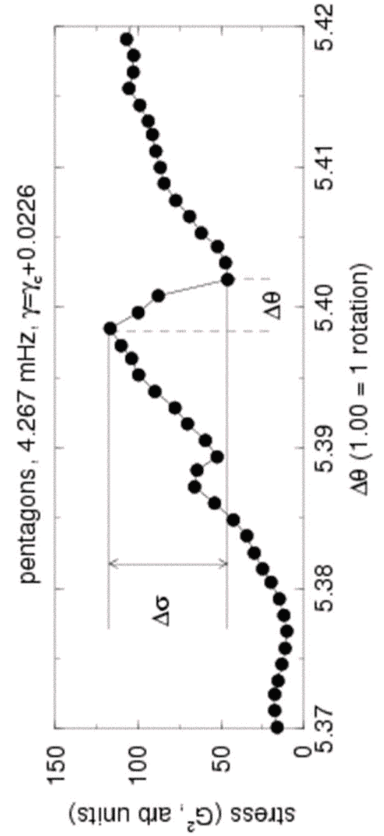
Effect of varying the shear rate

Exponential tail longer for: slower rates, and closer to critical density

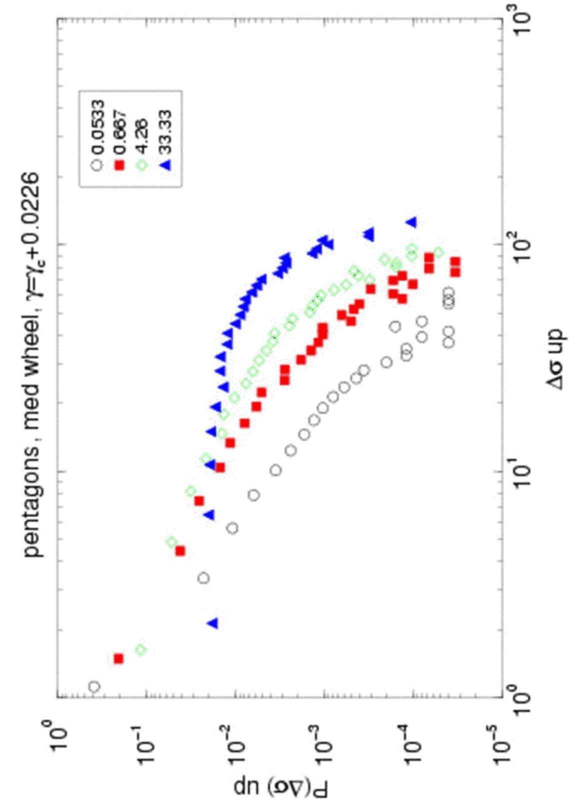


Effect of varying the density

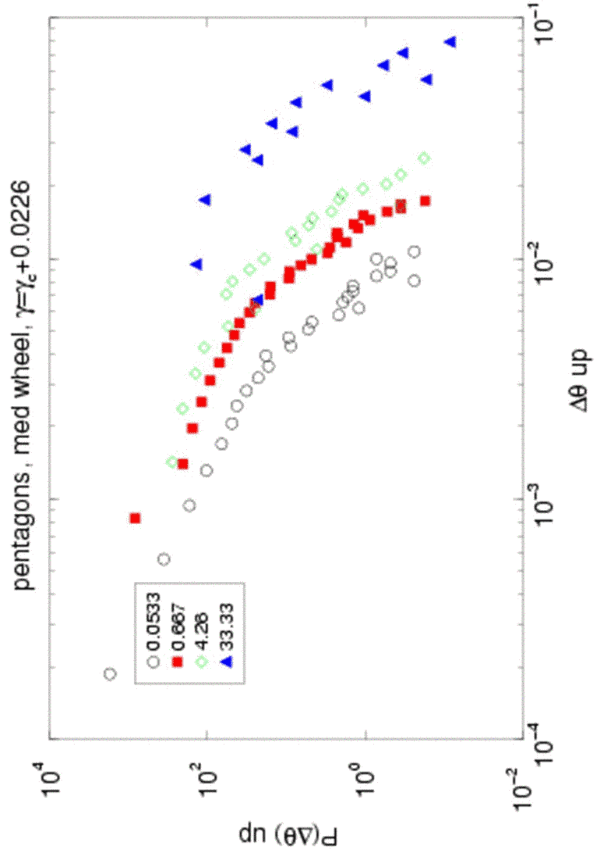
Stress Avalanches



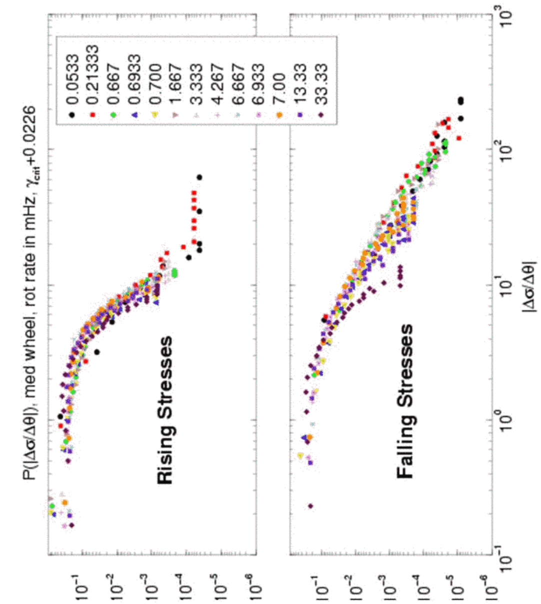
Pdf's for stress changes



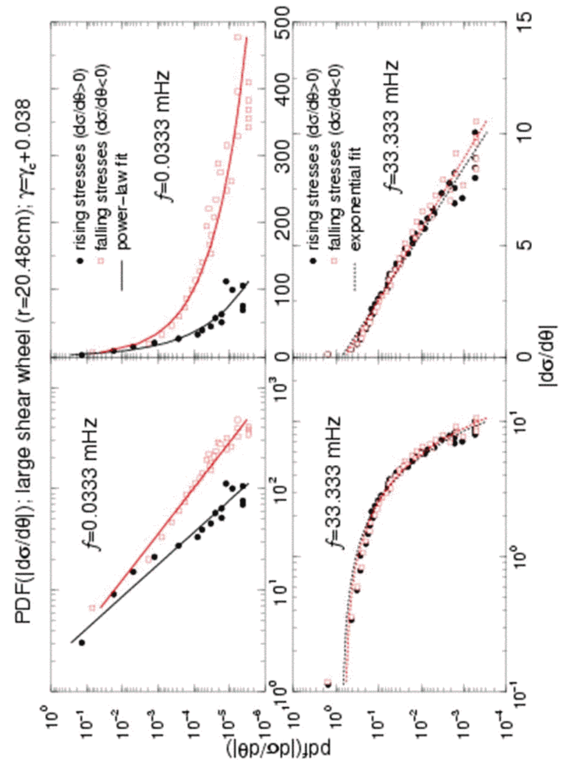
Pdf's for $\Delta\theta$



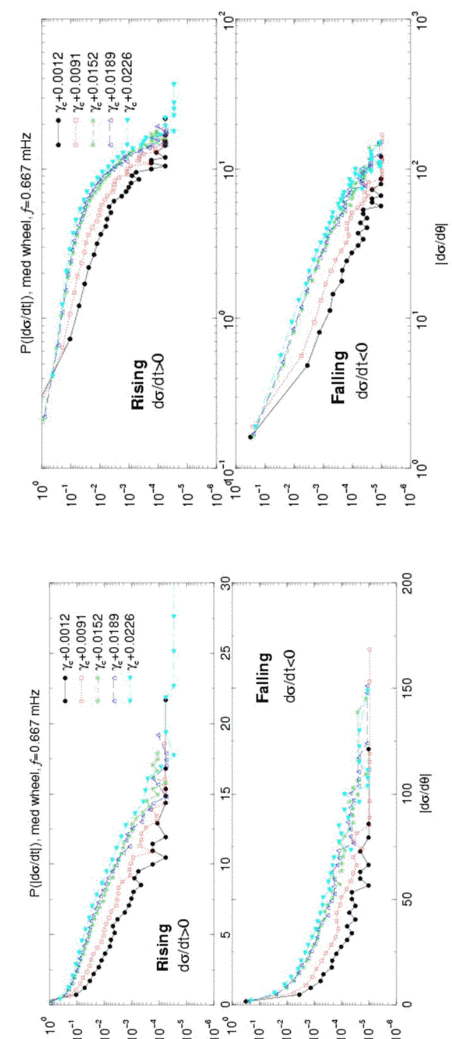
Pdf's for $d\sigma/d\theta$: for various rates



Pdf's for dσ/dθ: for various rates



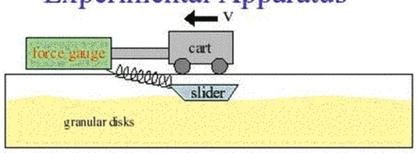
Pdf's for dσ/dθ: for various densities



Granular Rheology—a slider experiment

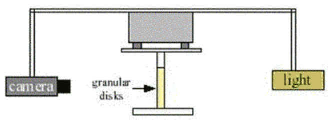
Experimental Apparatus

side view:



- Cart and force gauge move at constant speed v.
- Slider exhibits stick-slip motion on granular bed.

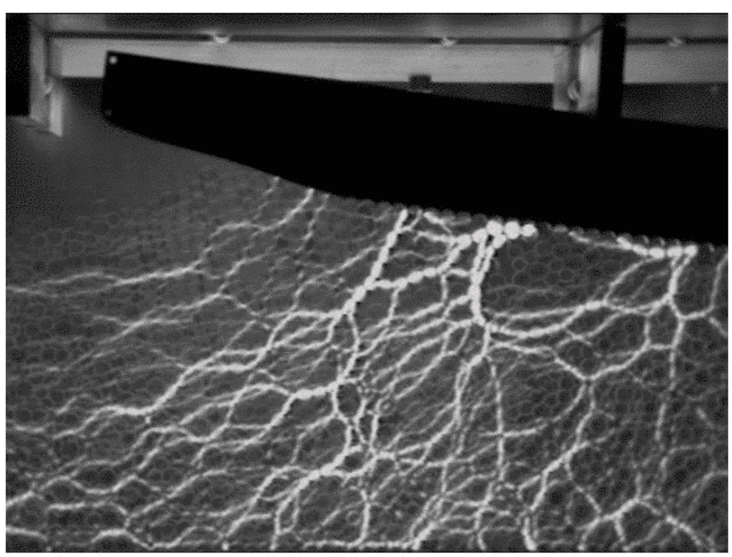
end view:



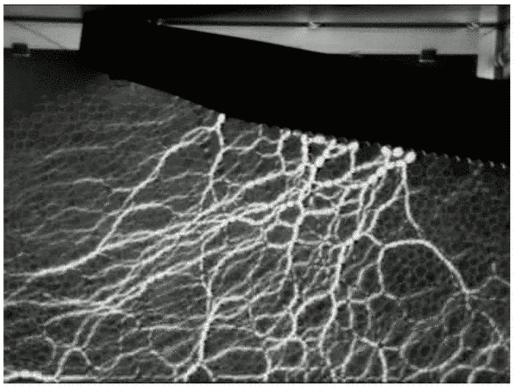
- Camera moves with the cart/slider.

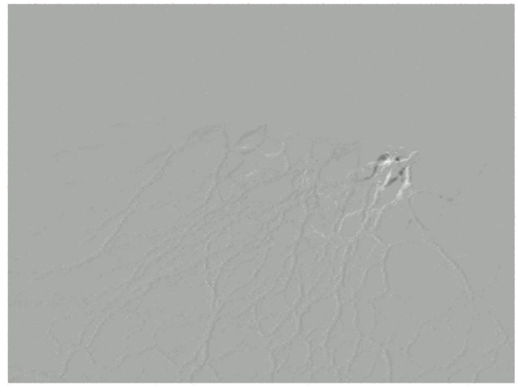
Granular bed = 500 d × 20 d deep, d = 0.41 cm and 0.51 cm bidisperse photoelastic disks.
Typical speeds = 0.1-2 d/s. Slider length = 30-40 d.
Dragging force = 0-100 grams (0-1 Newtons).

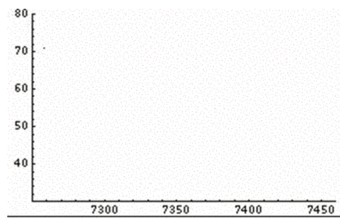
What is the relation between stick slip and granular force structure?



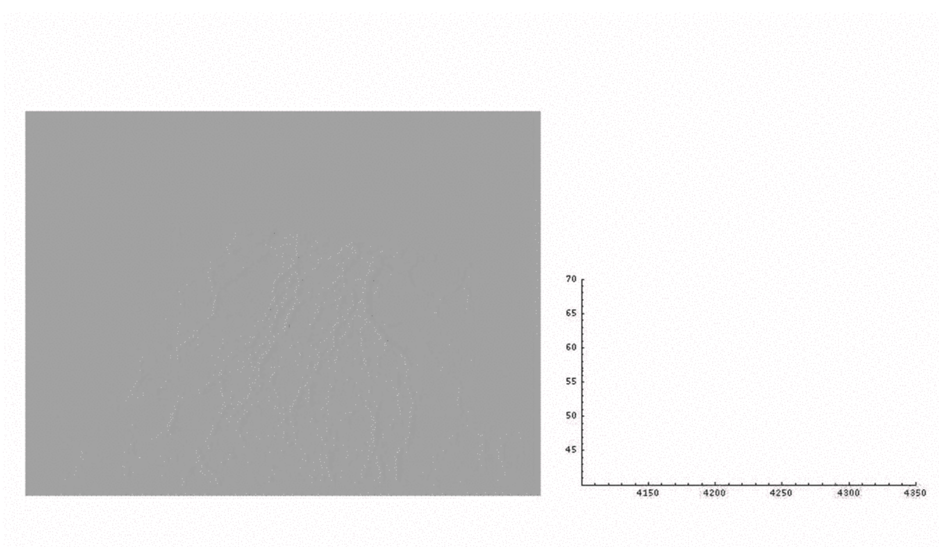
Videos of force evolution





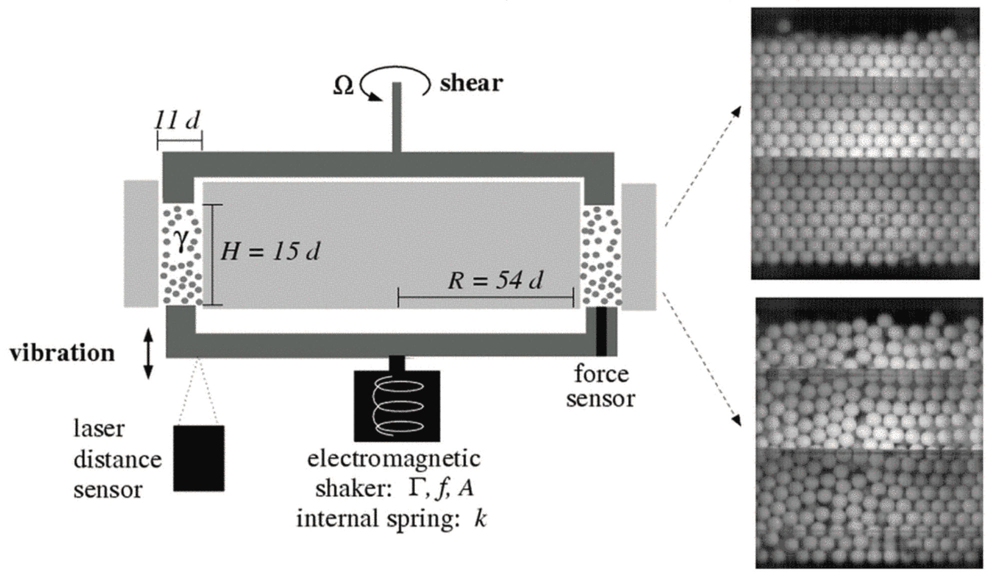


Composite force and photoelastic video: Relation between internal force network and force avalanches

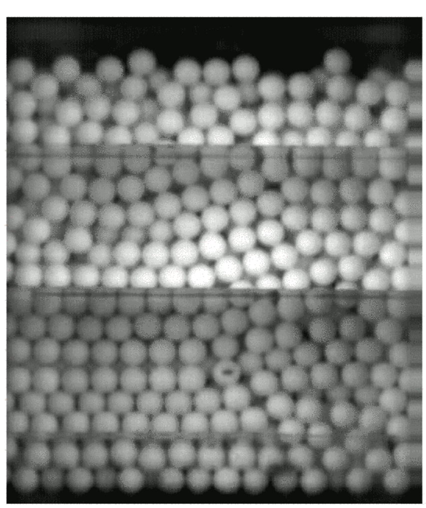


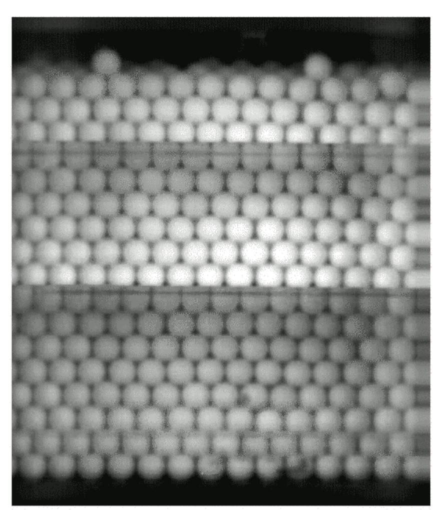
Order-disorder:
Transition from solid to dense fluid
Jamming/unjamming

K. Daniels and RPB, PRL 94 168001 (2005)

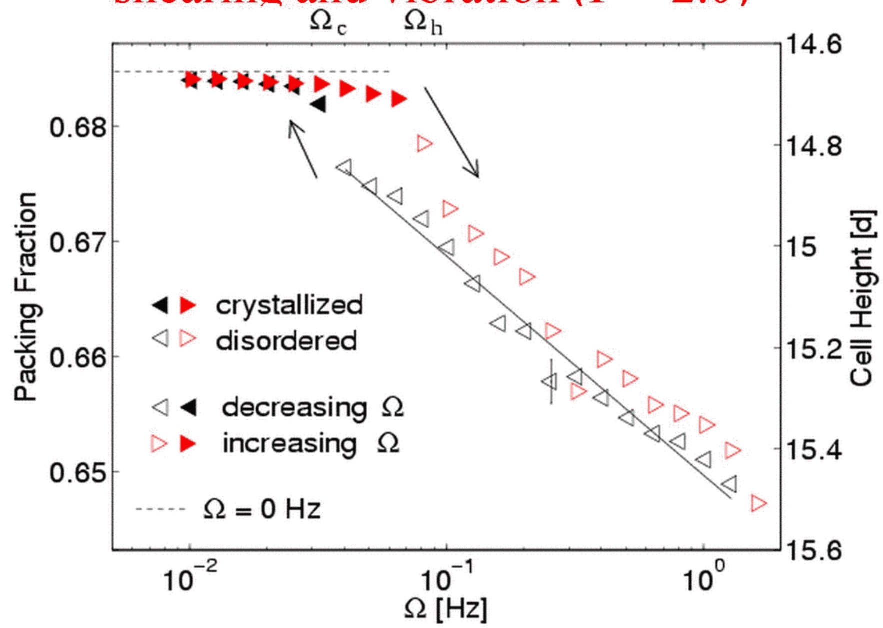


Videos of ordered/disordered states

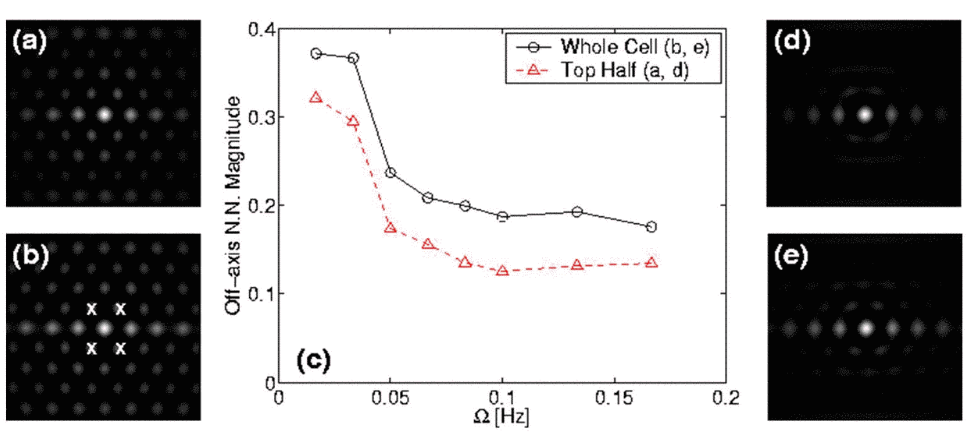




Freezing by Heating—Competition between shearing and vibration ($\Gamma = 2.0$)



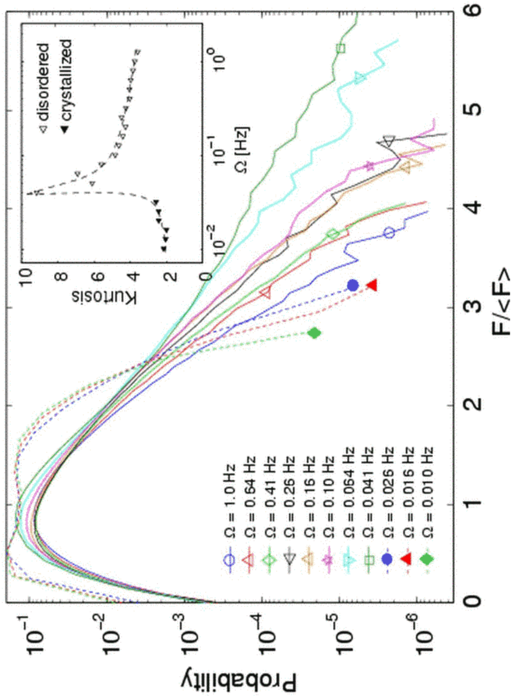
Spatial Autocorrelations show disorder with shear (a, b, d, e) and more quantitatively, c.



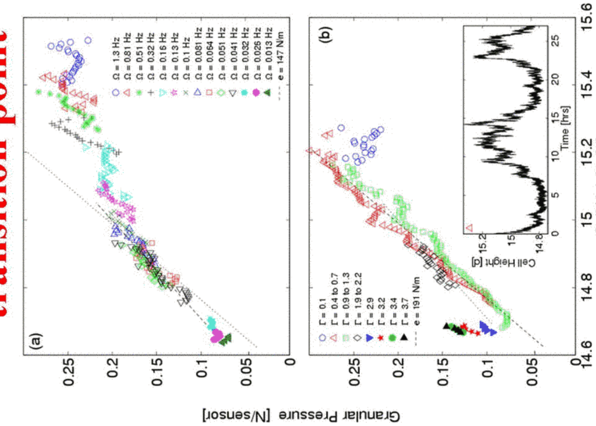
$\Omega = 0.0167$ Hz

$\Omega = 0.167$ Hz

Force Probability Distributions: Singular behavior in the Kurtosis

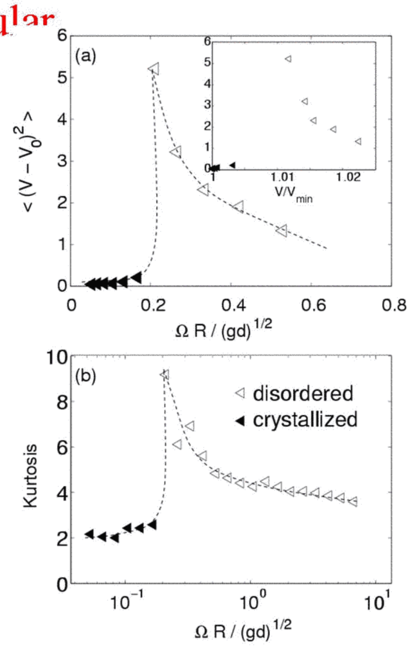


Large-scale Fluctuations, particularly near transition point

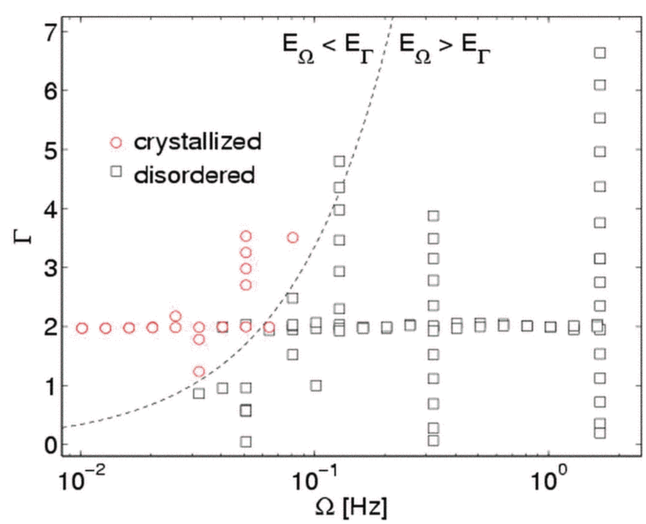


Kurtosis for force pdf and volume fluctuations are singular

Note: volume fluctuations in Edwards entropy picture are analogous to energy fluctuations in Canonical Ensemble → specific heat



Phase Diagram in Shear Rate (Ω) and Shaking Amplitude (Γ)



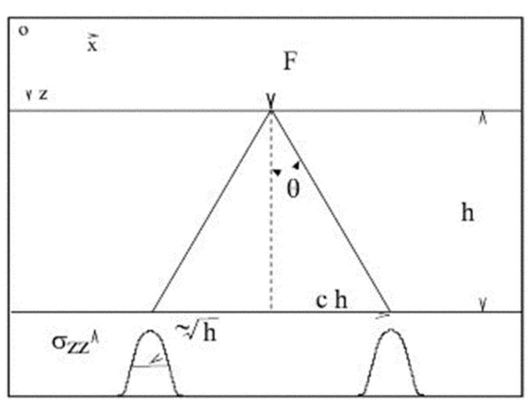
Some approaches to describing stresses

- Elasto-plastic models (Elliptic, then hyperbolic)
- Lattice models
  - Q-model (parabolic in continuum limit)
  - 3-leg model (hyperbolic (elliptic) in cont. limit)
  - Anisotropic elastic spring model
- OSL model (hyperbolic)
- Telegraph model (hyperbolic)
- Double-Y model (type not known in general)

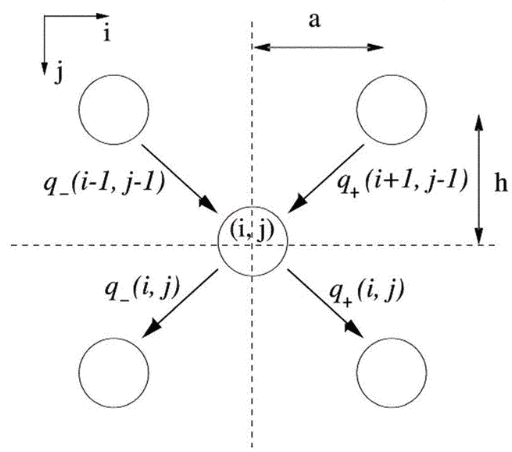
OSL model—hyperbolic—wave equation

$$\sigma_{xx} = \eta\sigma_{zz} + \mu\sigma_{xz}$$ *η, μ: phemonological parameters*

$$\sigma_{zz}(x,z) = \frac{F}{2}[\delta(x+cz) + \delta(x-cz)]$$



q-model (e.g. in 2D)



q's chosen from uniform distribution on [0,1]

Predicts force distributions ~ exp(-F/F$_o$)

Long wavelength description is a diffusion equation

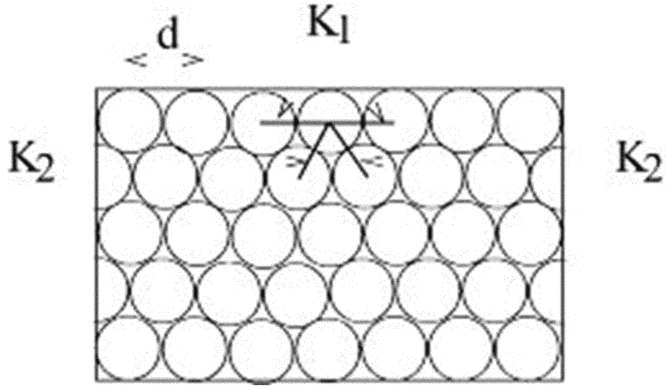
$$\frac{\partial w(z,j)}{\partial z} = \beta[w(z,j+1) + w(z,j-1) - 2w(z,j)]$$

$$\frac{\partial w}{\partial z} = D\frac{\partial^2 w}{\partial x^2}$$

Expected stress variation with depth

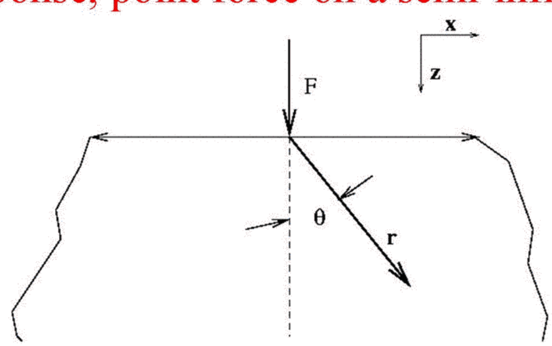
$$\sigma_{zz}(x,z) = \frac{F}{2\sqrt{\pi D z}}\exp(-x^2/4Dz)$$

Anisotropic elastic lattice model



Expect progagation along lattice directions
Linear widening with depth—e.g. Goldenberg and Goldhirsch, Nature 435, 188 (2005)

Elastic response, point force on a semi-infinite sheet



$$\sigma_{rr} = \frac{2F\cos\theta}{r\pi} \qquad \sigma_{r\theta} = \sigma_{\theta\theta} = 0$$

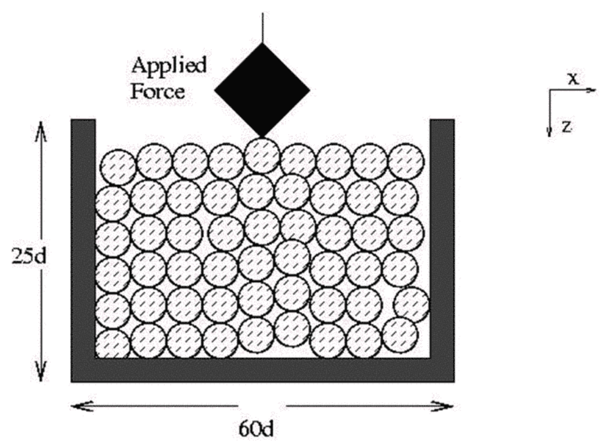
In Cartesian coordinates:

$$\sigma_{ii} = 1/[z(1+(x/z)^2]^p \qquad p = 1,2$$

Force response/transmission: what is the mechanical response to a small point force?

Physica D 182, 274 (2003)

Schematic of greens function apparatus



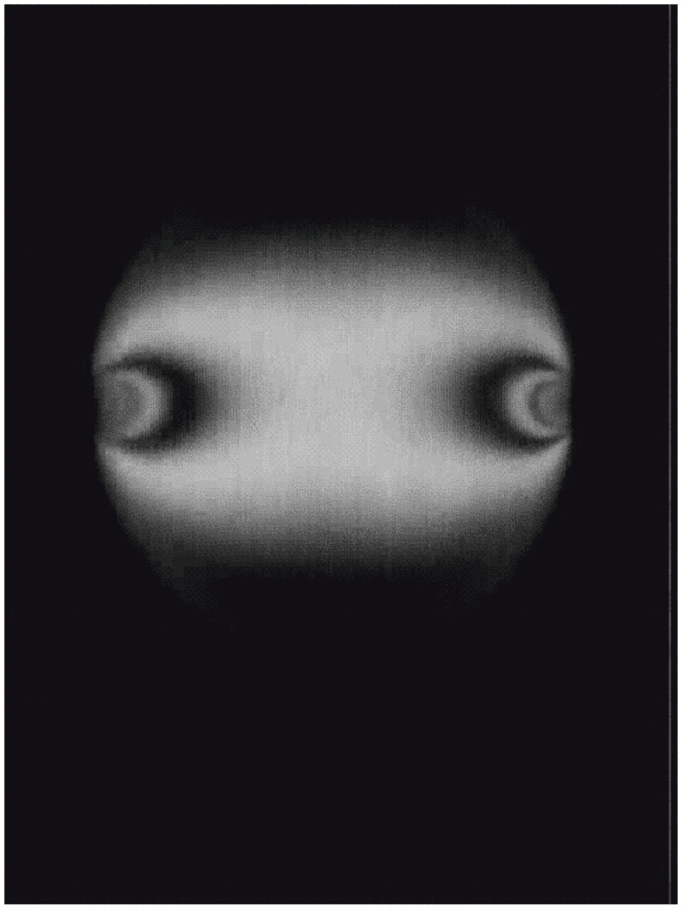
d = grain size

Use: 1) Monodisperse disks (spatially ordered)

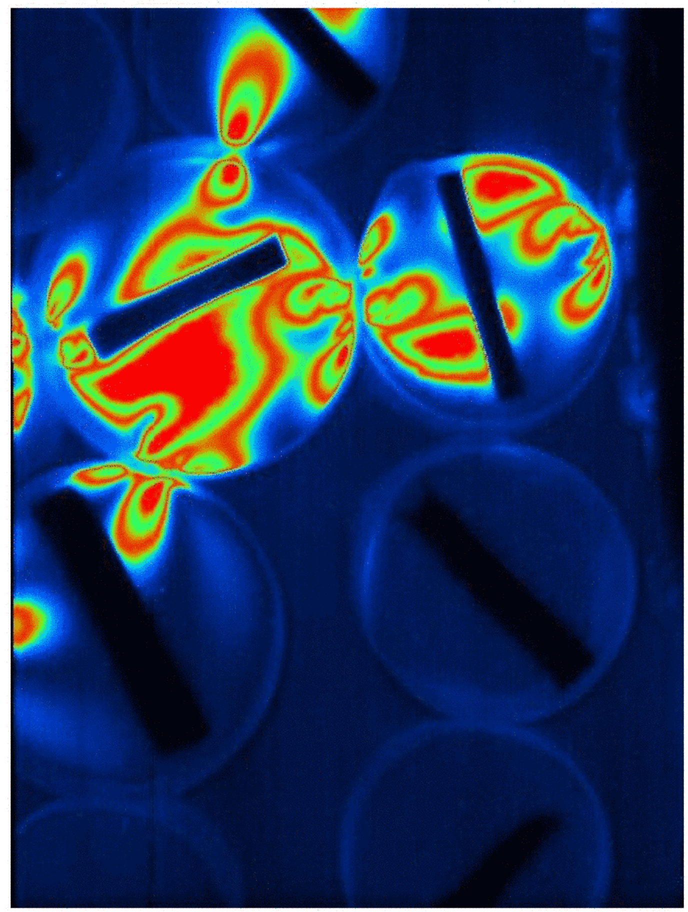
2) Bidisperse disks (weakly disordered)

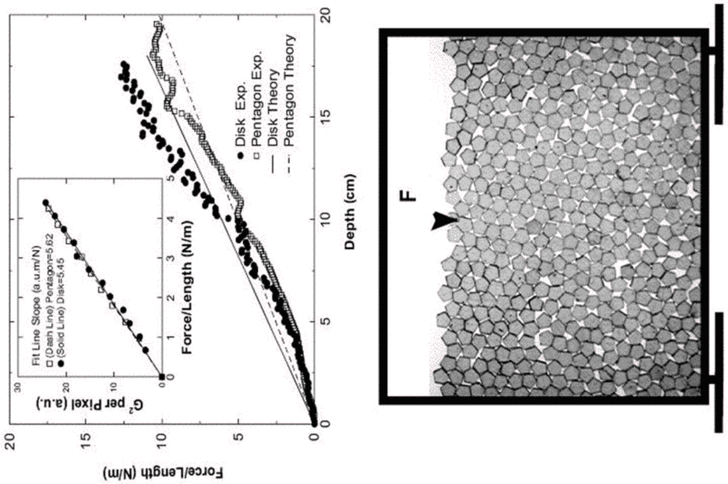
3) Pentagons (strongly disordered)

Diametrically opposed forces on a disk



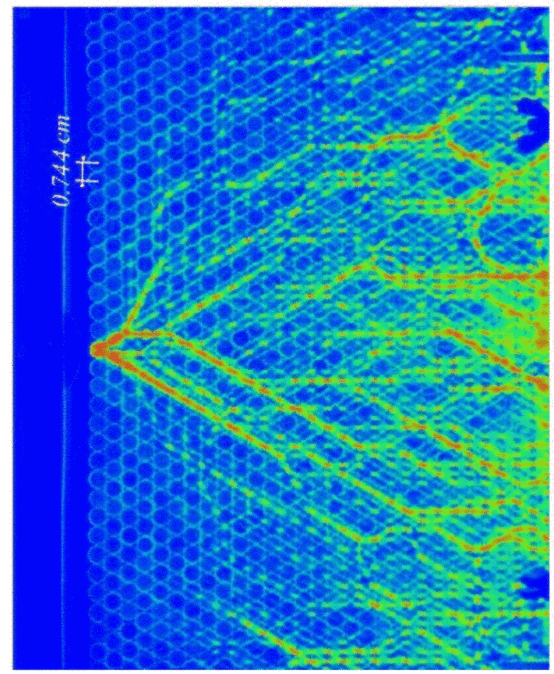
A gradient technique to obtain grain-scale forces



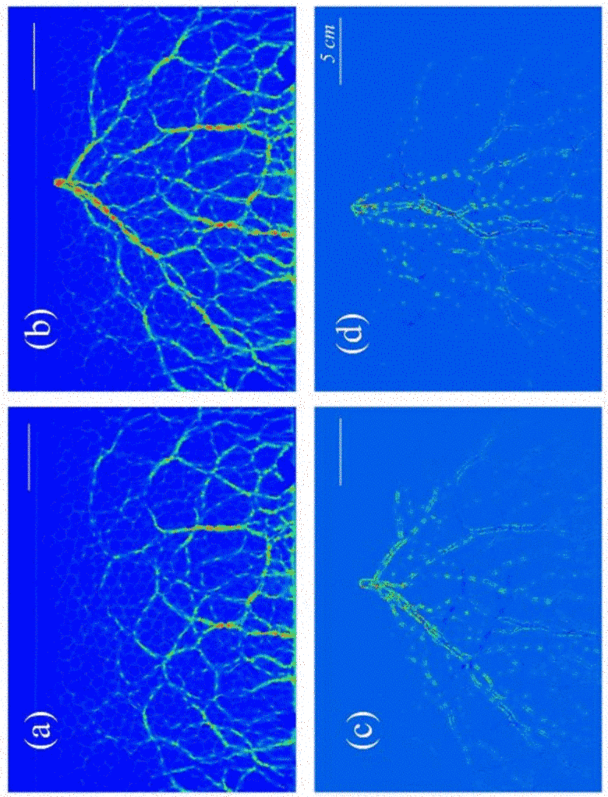


calibration

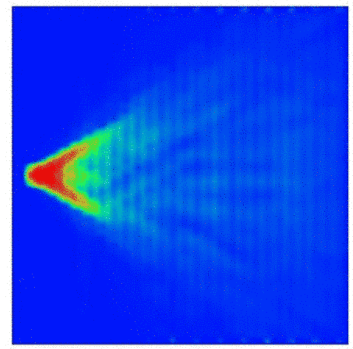
Disks-single response

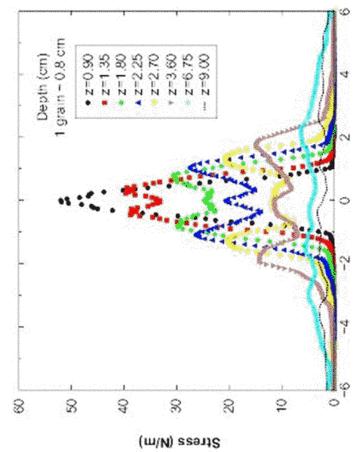


Before-after

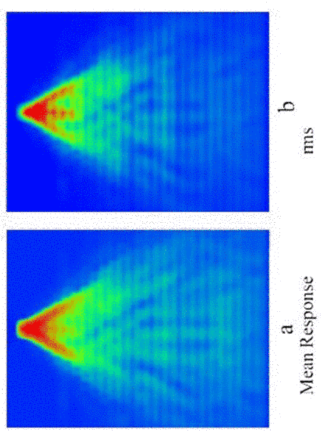


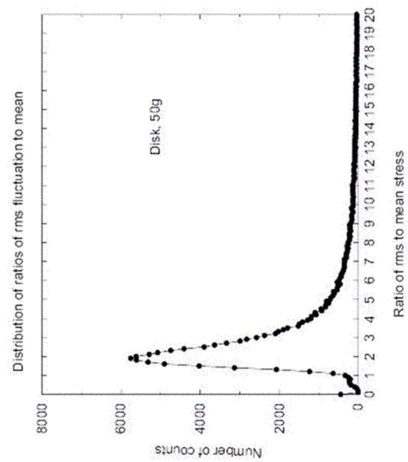
disk response mean



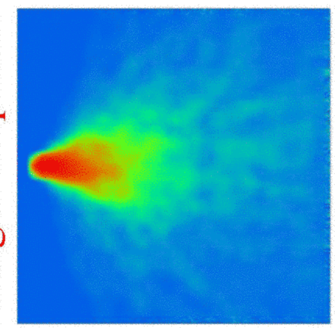


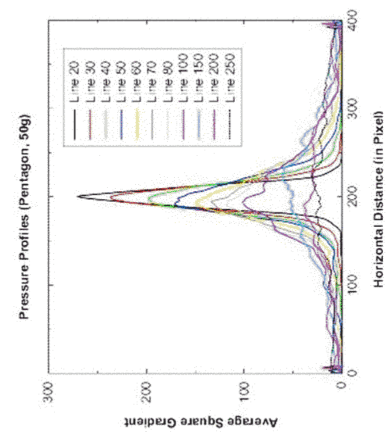
Large variance of distribution



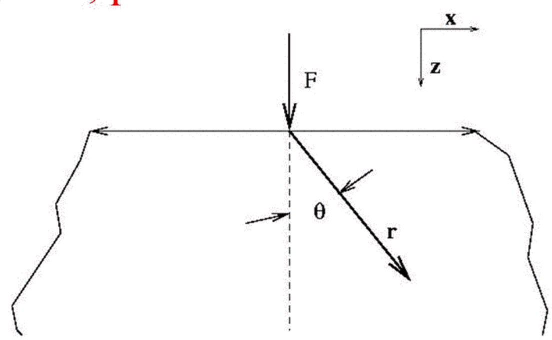


Pentagon response





Elastic response, point force on a semi-infinite sheet

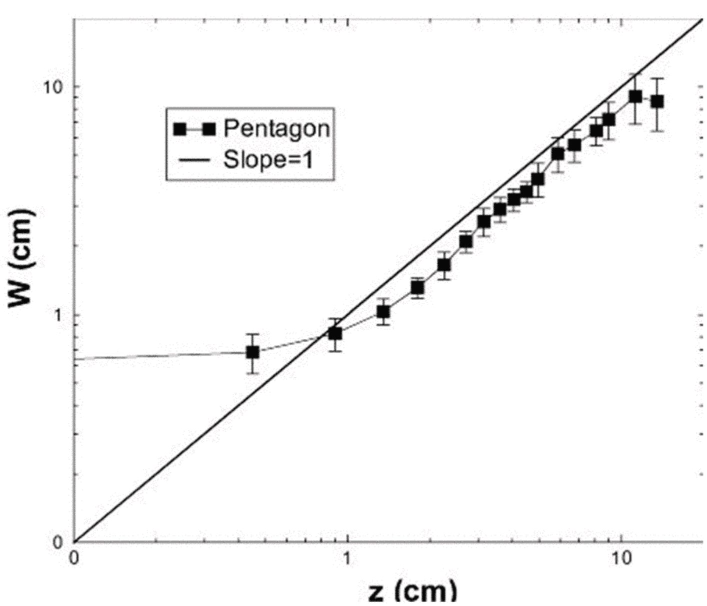


$$\sigma_{rr} = \frac{2F\cos\theta}{r\pi} \qquad \sigma_{r\theta} = \sigma_{\theta\theta} = 0$$

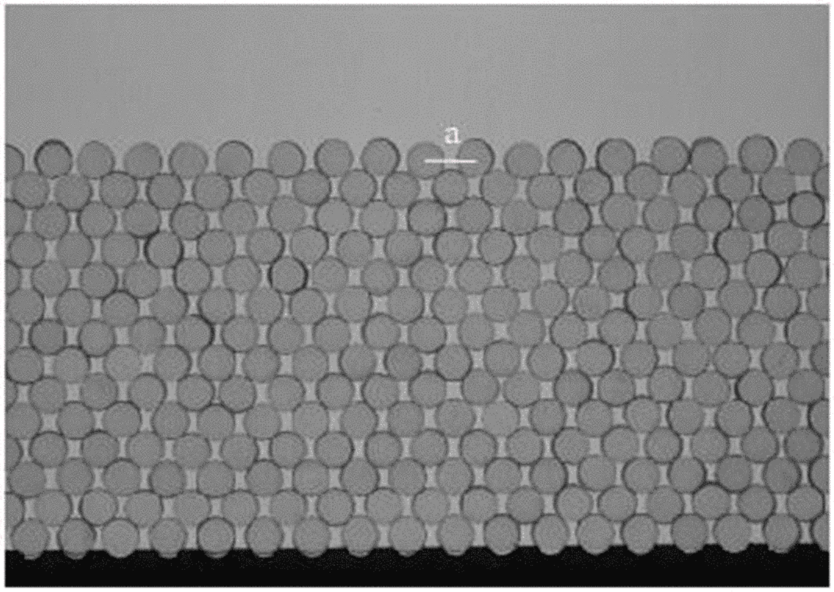
In Cartesian coordinates:

$$\sigma_{ii} = 1/[z(1+(x/z)^2]^p \qquad p = 1,2$$

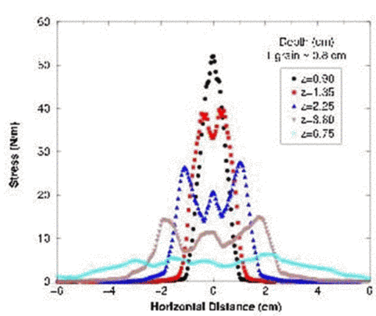
Pentagons, width vs. depth



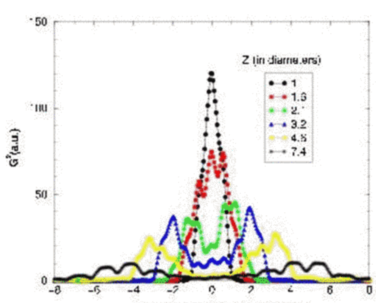
Rectangular packing reduces contact disorder



Hexagonal vs. square, data

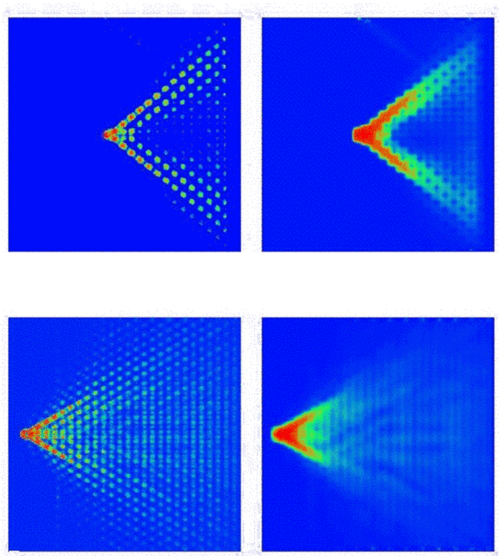


Hexagonal Packing

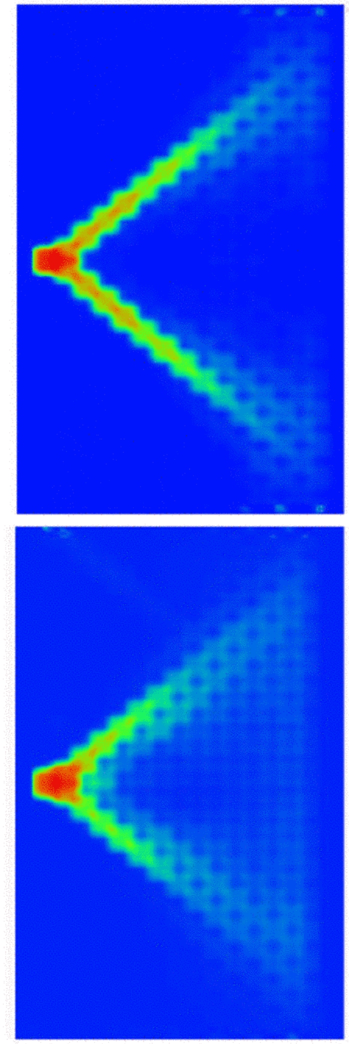


Square–lattice Packing

Hexagonal vs. square packing

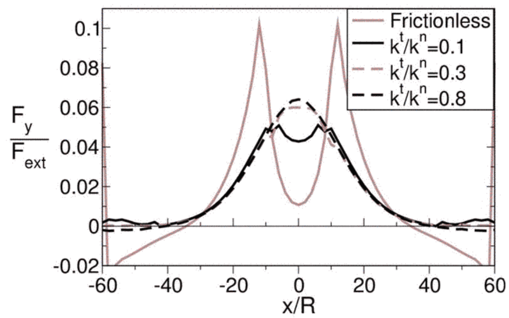


Square packs, varying friction

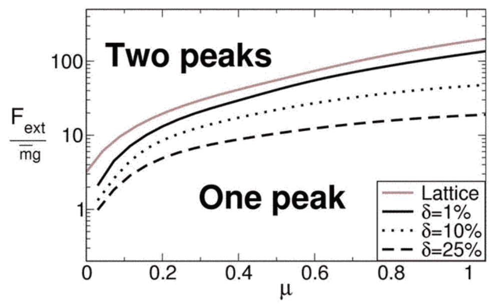


a b

Friction 'enhances' elasticity—e.g. tends to produce single-peaked response (Goldenberg/Goldhirsch)

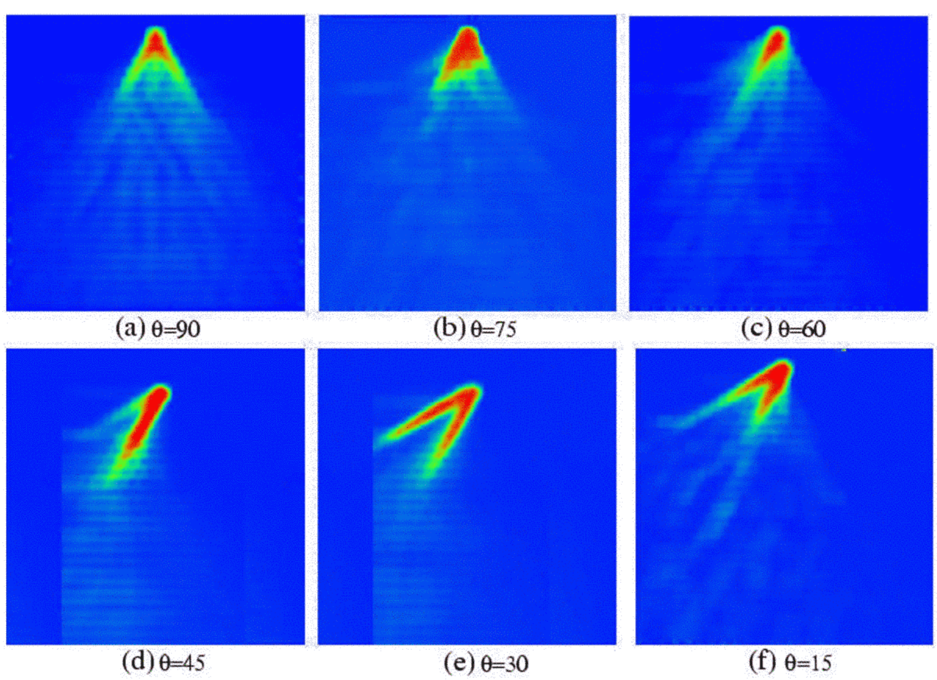


Change from 2 to 1 Peak as friction increase

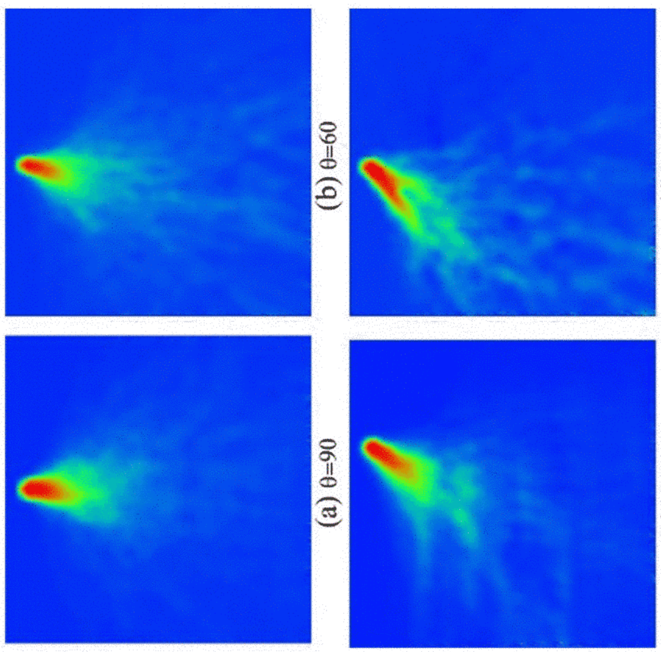


'Phase diagram' in Applied force, Friction space

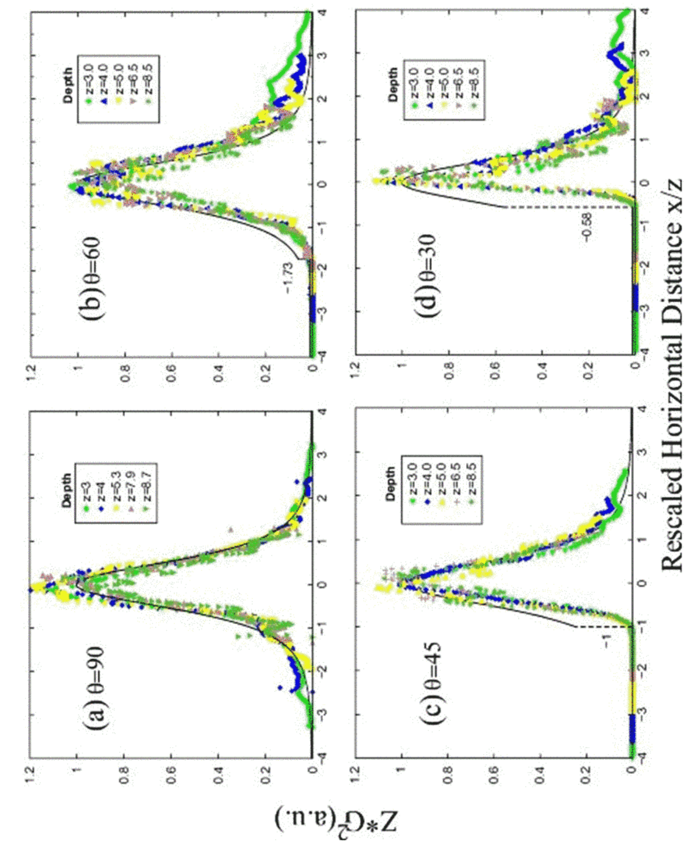
Non-normal response, disks, various angles



Non-normal responses, pentagons



Non-normal response, pentagons, rescaled



Conclusions

- Normal force distributions are sensitive to stress state
- Long-range correlations for forces in sheared systems—thus, force chains can be mesoscopic at least
- Diffusion in sheared systems: insights into microscopic statistics of driven granular materials
- Logarithmic rate dependence is seen in sheared granular systems
- Interesting connections to avalanches/earthquakes…
- Order-disorder transition—first order

  characterizes jamming-unjamming, contradictions notions of vibration→temperature in granular systems
- Strong effects on transmission from order/disorder (spatial and force-contact)—overall response is mostly elastic

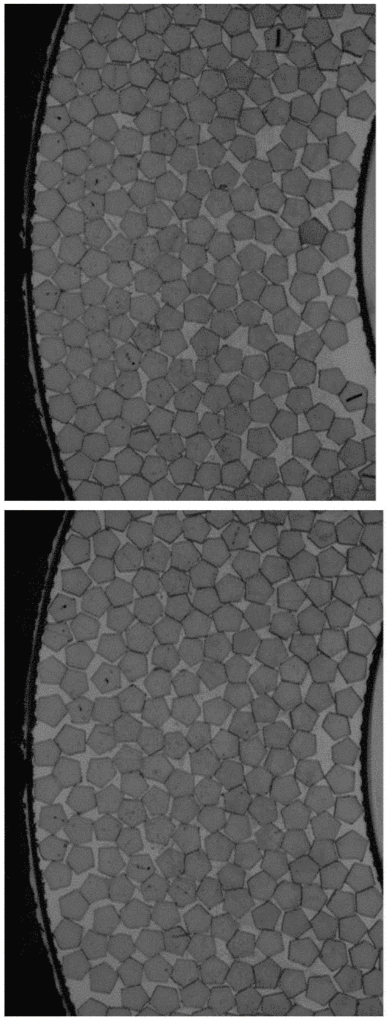
What are important questions? (Dense materials)

- What are statistical properties/variability of granular systems?
- What is the nature of spatio-temporal correlations/fluctuations?

  The answer to this requires addressing the relevant multi-scale phenomena involved—something that is just now being considered
- Is there a universal description for stress, deformation, etc?

Extra Slides Follow

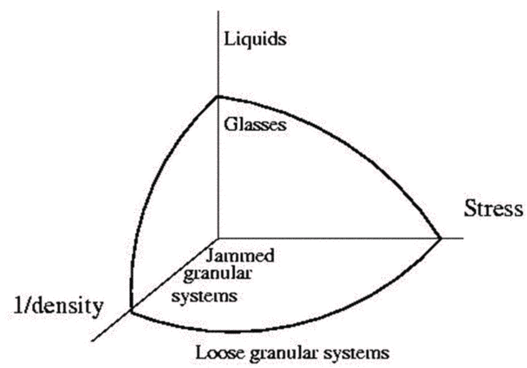
Reynolds Dilatancy



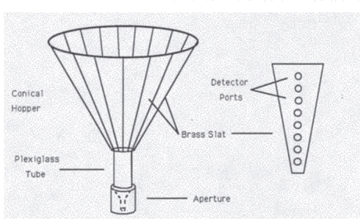
Jamming and fragility (Liu and Nagle, Bouchaud et al.)

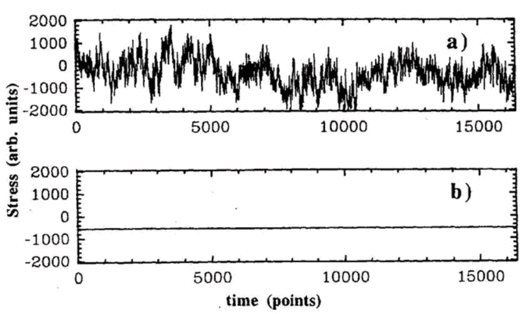
Class of systems that are constrained or jammed

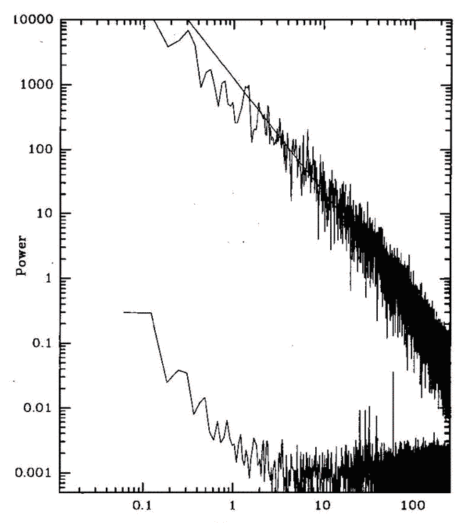
- Granular Materials
- Foams
- Colloids
- Glasses ....



Fluctuations in a Simple Granular Flow Baxter et al. Eur. J. Mech.B 1991

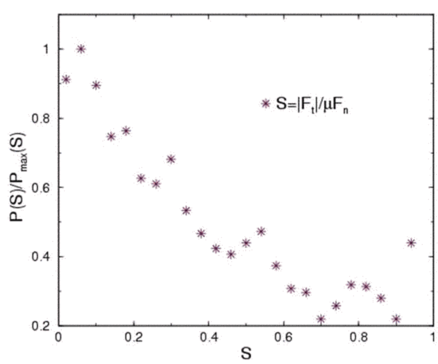




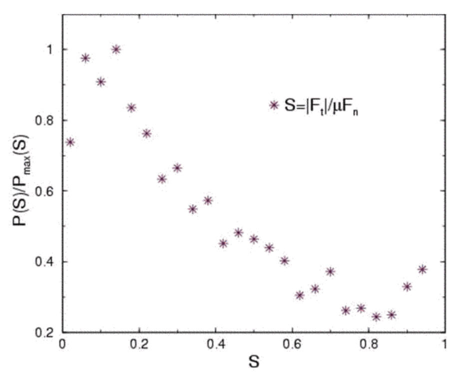


Distributions of frictional mobilization

Shear

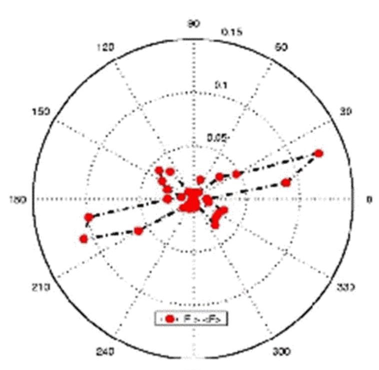


Compression



Distributions of contact angles for larger forces (Geometric anisotropy)

Shear



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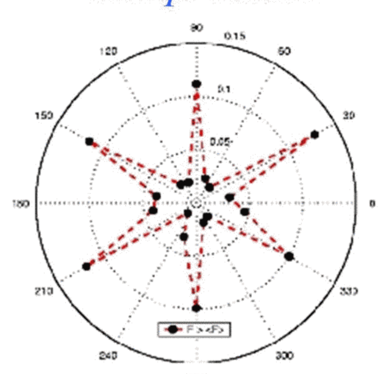
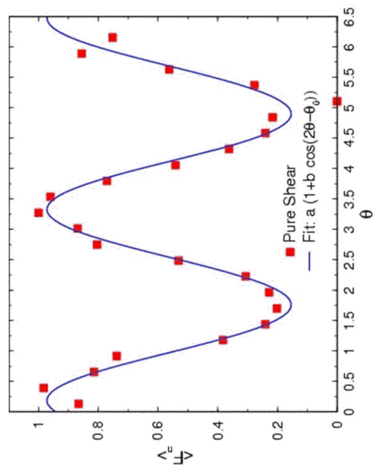
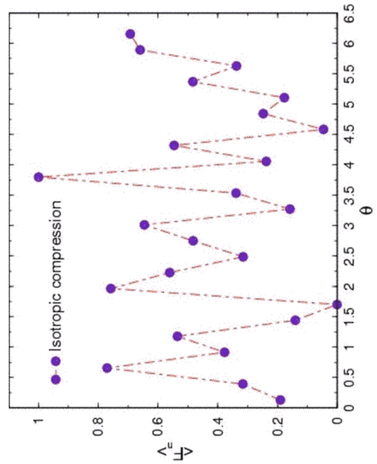


Variation with angle of mean normal force
(Force anisotropy)

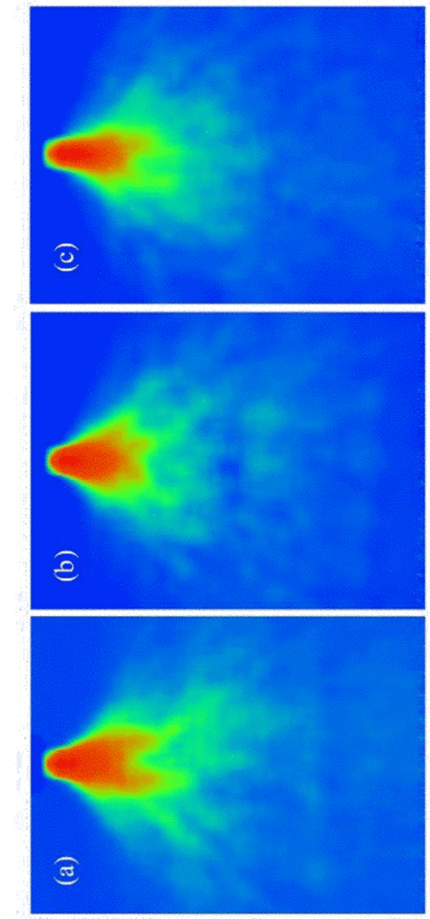
Shear



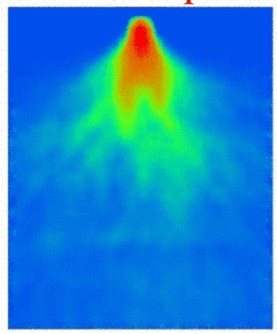
Compression

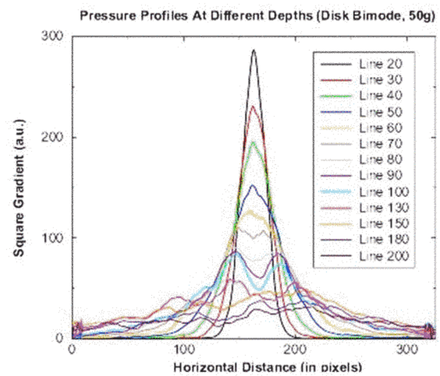


Bidisperse responses vs. A

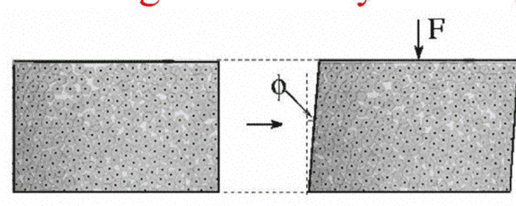


Weakly bi-disperse: two-peak structure remains

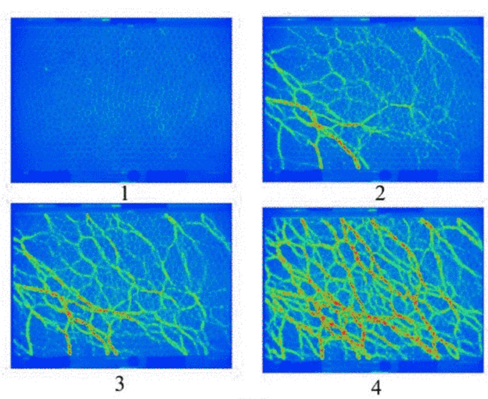




Creating a texture by shearing

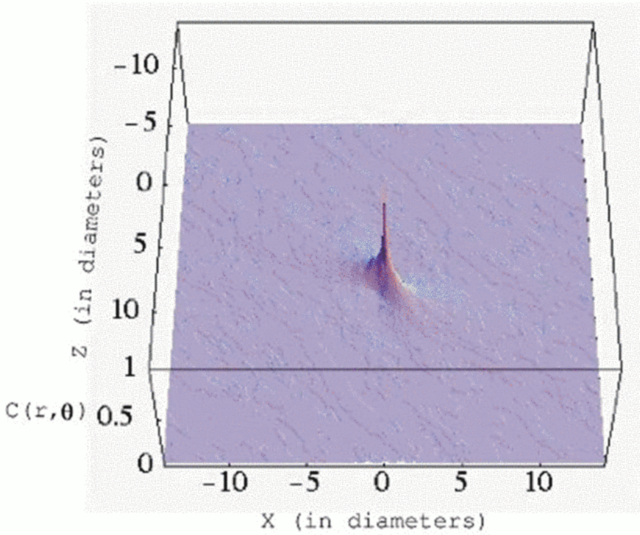


(a)

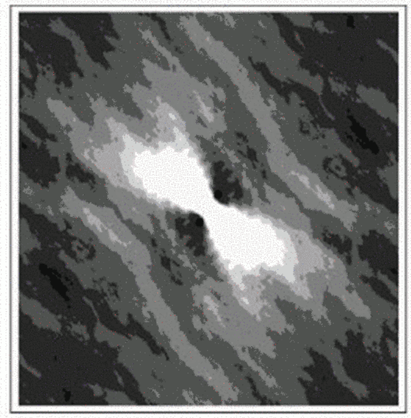


(b)

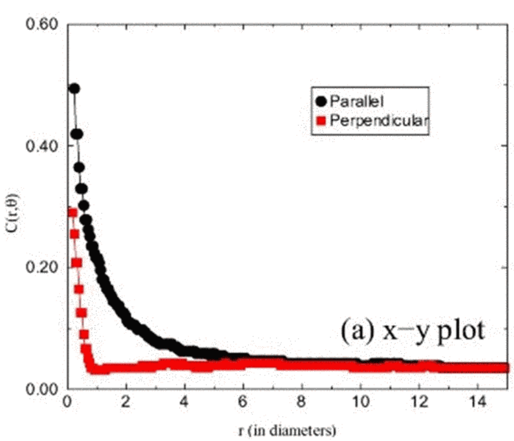
Force correlation function



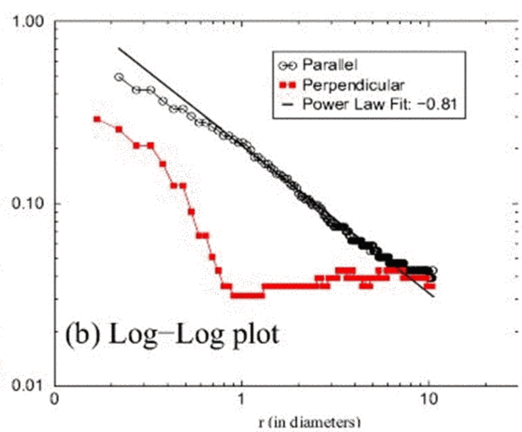
(a) 3D


(b) 2D

Correlation functions along specific directions



(a) x−y plot



(b) Log−Log plot

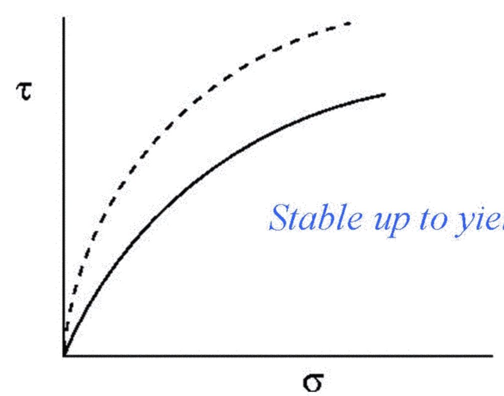
Features of elasto-plastic models

Conserve mass: $\partial\rho / \partial t + \partial_i(\rho v_i) = 0$

(Energy: lost by friction)

Conserve momentum: $\rho dv_i / dt = -\partial_j T_{ij}$

Concept of yield and rate-independence



Stable up to yield surface

τ => shear stress, σ => normal stress

Example of stress-strain relationship for deformation

$$T_{ij} = P\delta_{ij} kPV_{ij} / |V|$$

$$V_{ij} = -(\partial_j v_i + \partial_i v_j)/2$$

(Strain rate tensor with minus)

$$|V|^2 = \Sigma V_{ij}^2$$ *$|V|$ = norm of V*

Contrast to a Newtonian fluid:

$$T_{ij} = P\delta_{ij} + 2\eta[V_{ij} - Tr(V)/3] + (2\zeta/3)Tr(V)$$

Convection-diffusion/3-leg model

Applies for weak disorder

$$O^{+}O^{-}\sigma = 0$$

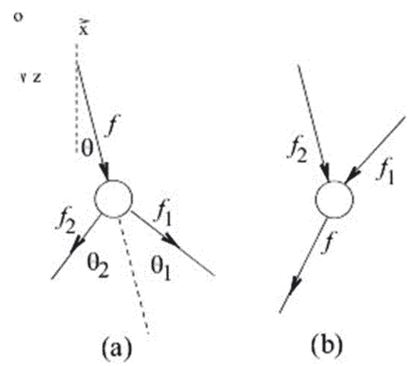
$$O^{\pm} = [\partial/\partial z \pm c\partial/\partial x - D\partial^2/\partial x^2]$$

Expected response to a point force:

$$\sigma_{zz} = \frac{F}{2}\frac{1}{\sqrt{4\pi Dz}}\{\exp[-(x+cz)^2/4Dz] + \exp[-(x-cz)^2/4Dz]\}$$

Double-Y model

Assumes Boltzmann equation for force chains



For shallow depths: One or two peaks
Intermediate depths: single peak-elastic-like
Largest depths: 2 peaks, propagative, with diffusive widening

A bit further from home

Granular Material Phases-Gases

Granular Gases:

Cool spontaneously, show clustering instability

$Tg = (1/2)m\langle v^2 \rangle$

Clustering in a Cooling Granular Gas (from work by S. Luding, H. Herrmann)

- Cooling