ECOLE NORMALE SUPERIEURE DE LYON

Failure time, Critical Behaviour and Activation Processes in Crack Formation

Models and experiments

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Failure time, Critical Behaviour and Activation Processes in Crack Formation

Motivations:

- May simple models * describe the formation of a crack ?
  *(percolation models, SOC, etc.)
- Is the crack formation similar to a critical phenomenon?
- If yes,what are the experimental conditions?
- Is the life time of a sample submitted to a tensile stress predictable?

OUTLINE

1) Microcrack localization and statistical analysis
   - Experimental apparatus
   - Comparison with percolation models

2) Sample life time
   - Activation models for crack formation
   - Comparison with experiments
   - Problems

3) The thermally activated fiber bundle model
   - Properties and results:
     * The disorder strongly reduces the sample life time but it increases the life time predictability.
     * The Omori law and the divergence of the damage rate are natural consequences of this simple model.

L. Vanel talk 2 weeks ago

4) Extension of the fiber bundle in 2D
   - The slow propagation of a single crack.
   - The crack tip equation of motion

5) Comparison with a 2D experiment.

6) Conclusions

Experimental Set up

What do we measure ?

-The applied pressure and the sample strain
A feedback loop allows us to impose either the stress or the strain.

- Acoustic emissions of microcracks.

- Microcrack localization

- Acoustic energy.

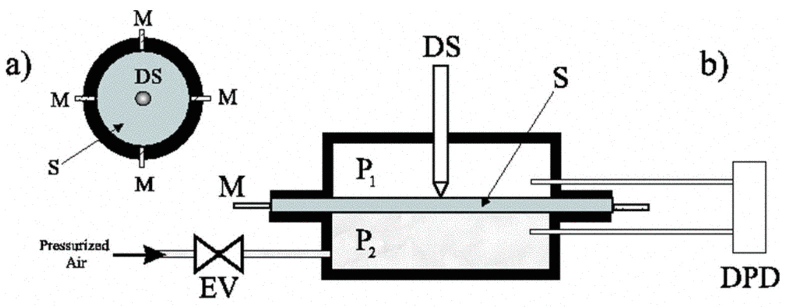
- X-Ray Tomography

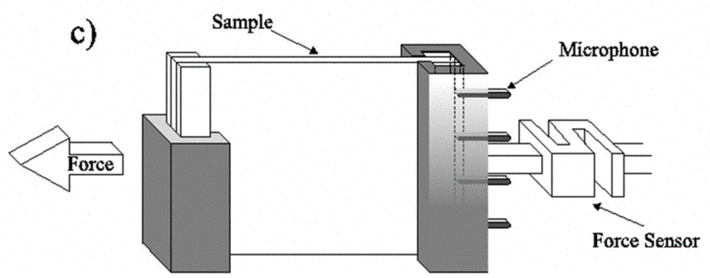
Materials :
chipboard wood panel, fiber glass, PU foams

Young modulus

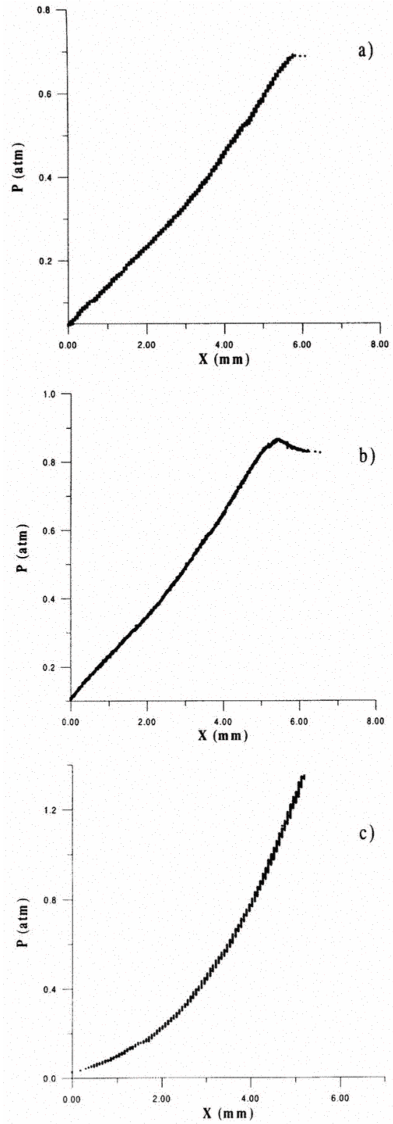
wood panels	$Y = 2\ 10^8\ N/m^2$
fiber glass	$Y = 10^{10}\ N/m^2$
PU and epoxy solids foams	$Y \sim 10^9\ N/m^2$

Experimental Apparatus





Pressure versus strain

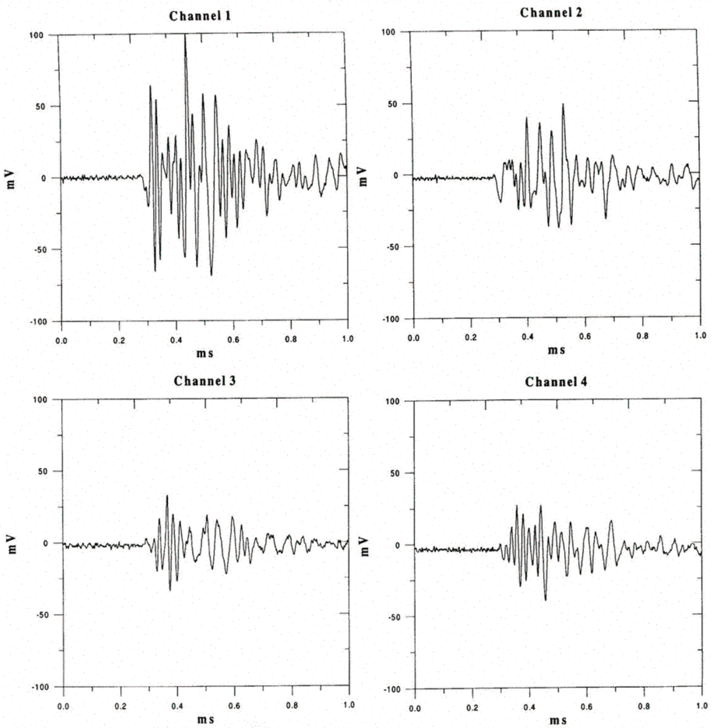


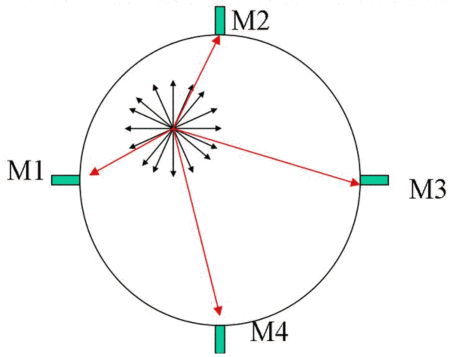
Chipboard wood panel
Wood imposed pressure

Wood imposed strain

Fiber glass
imposed pressure

Sound detected by the piezoelectric detectors

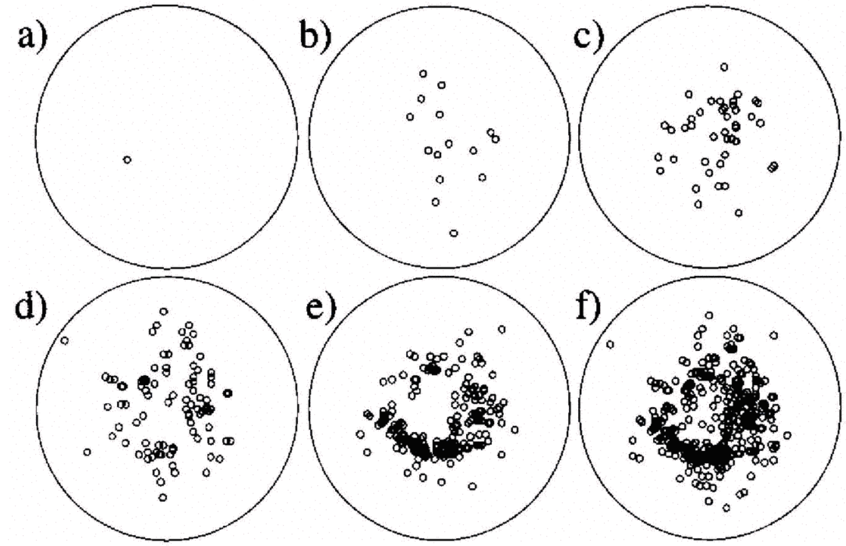




M1,M2,M3,M4 microphones

EVENTS LOCALISATION FOR INCREASING PRESSURE

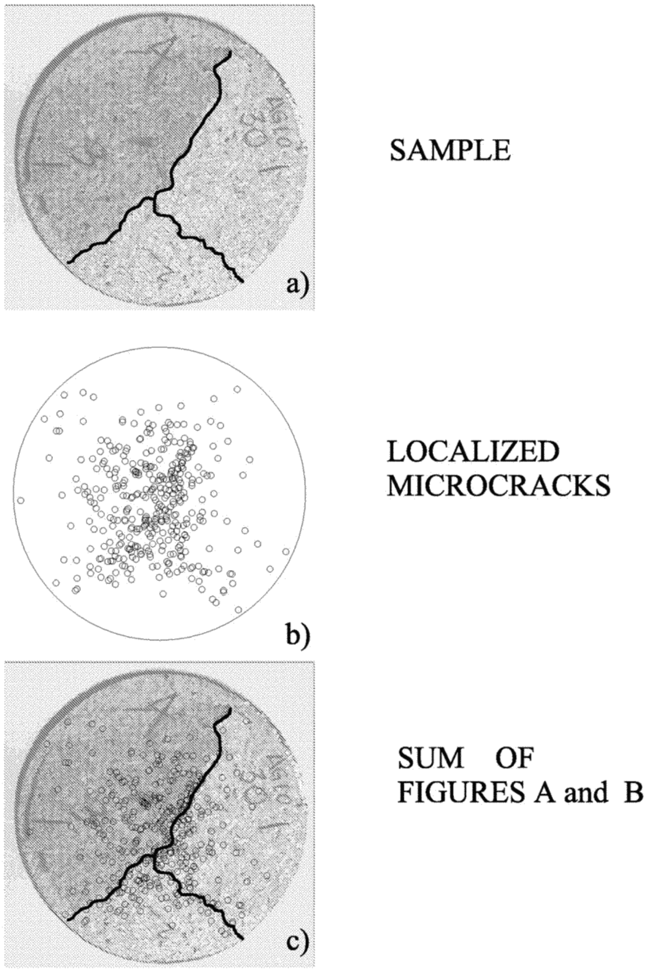
applied pressure: $P = A\ t$



From a) to e) the applied pressure increases

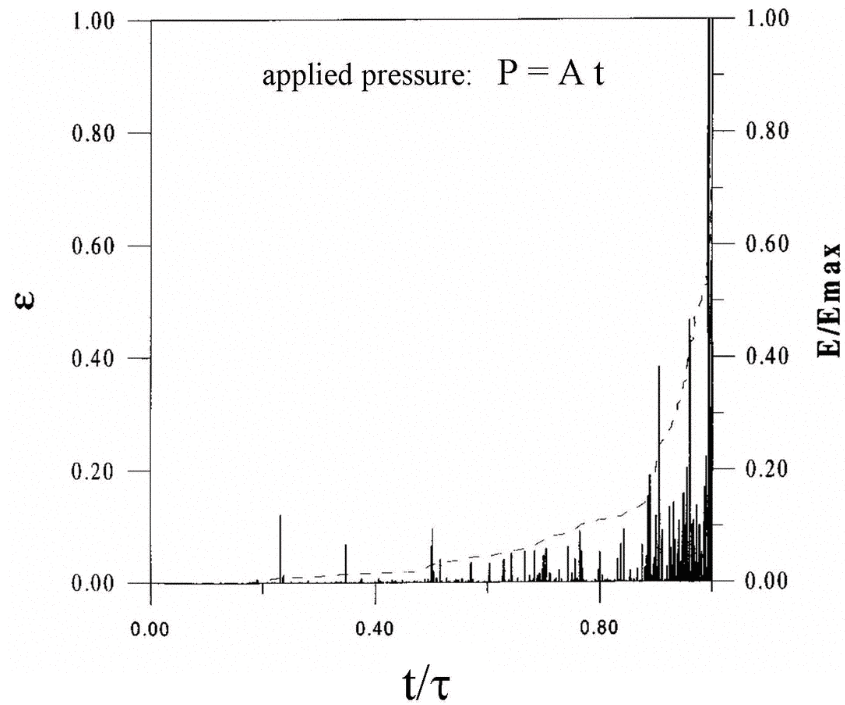
In f) the total number of events is shown.

Micro Crack Localisation



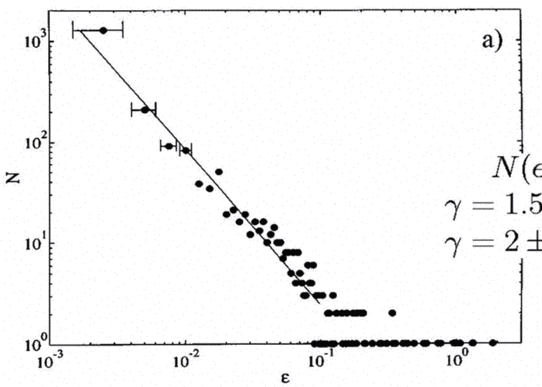
Acoustic energy versus time

A linear ramp of pressure (strain) is applied to the sample



τ is the sample life time and $E = \int_0^t \epsilon(t')\, dt'$

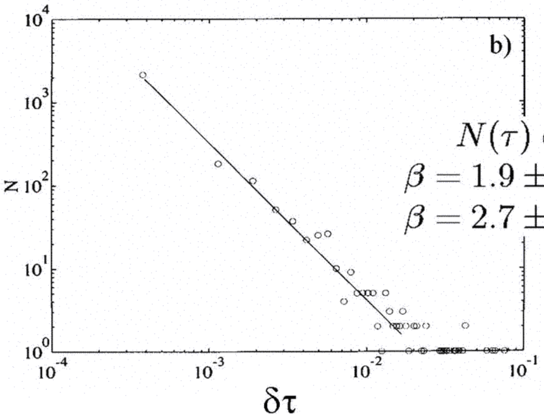
Event distribution during the pressure ramp



$N(\epsilon) \propto \epsilon^{-\gamma}$

$\gamma = 1.5 \pm 0.1$ for wood

$\gamma = 2 \pm 0.1$ for fiberglass



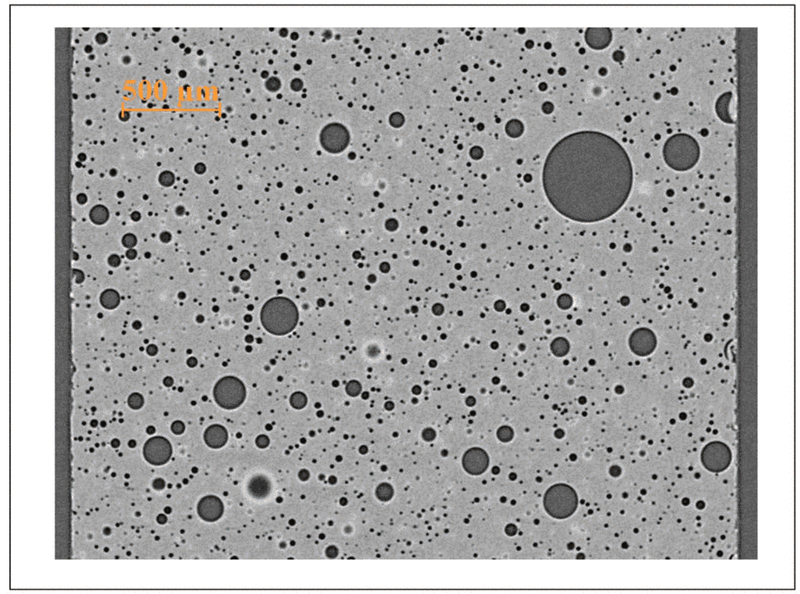
$N(\tau) \propto \delta\tau^{-\beta}$

$\beta = 1.9 \pm 0.1$ for wood

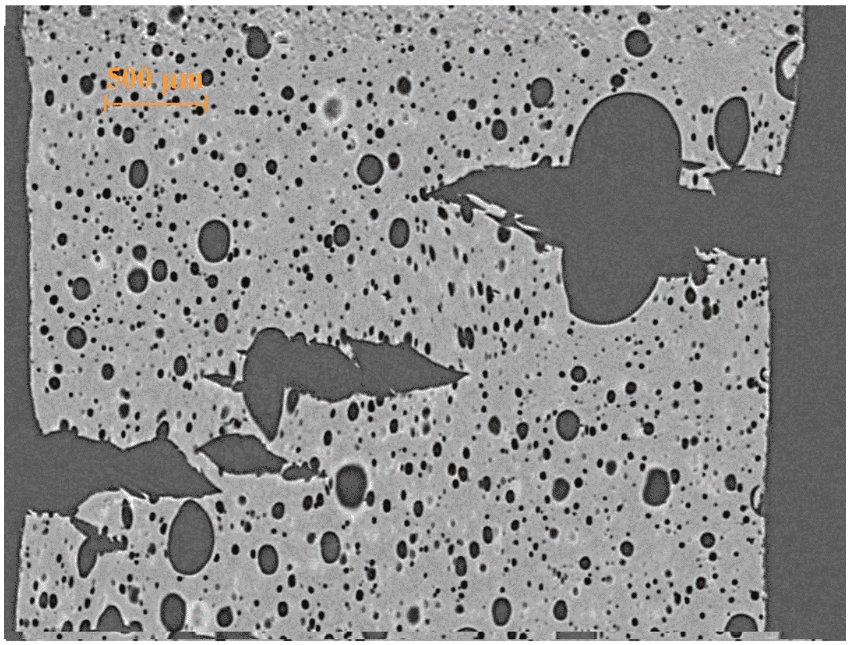
$\beta = 2.7 \pm 0.1$ for fiberglass

Uniaxial tensile test under X-ray tomography of PU solid foams

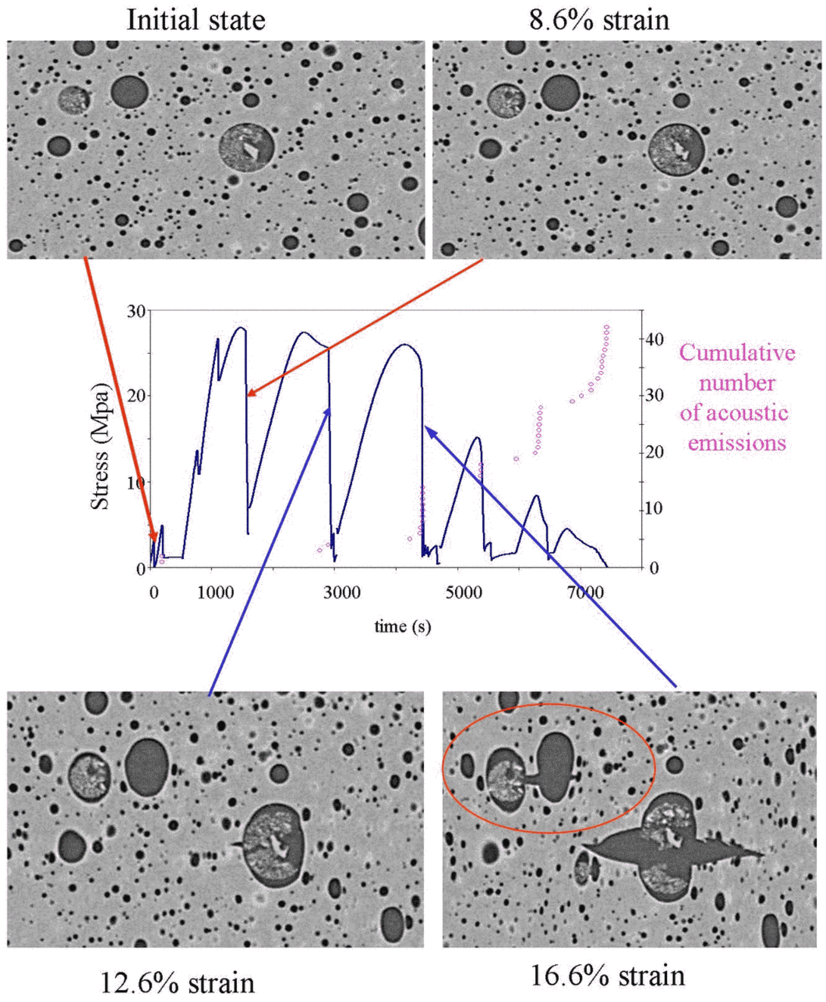
S. Deschanel, G. Vigier (INSA de Lyon)



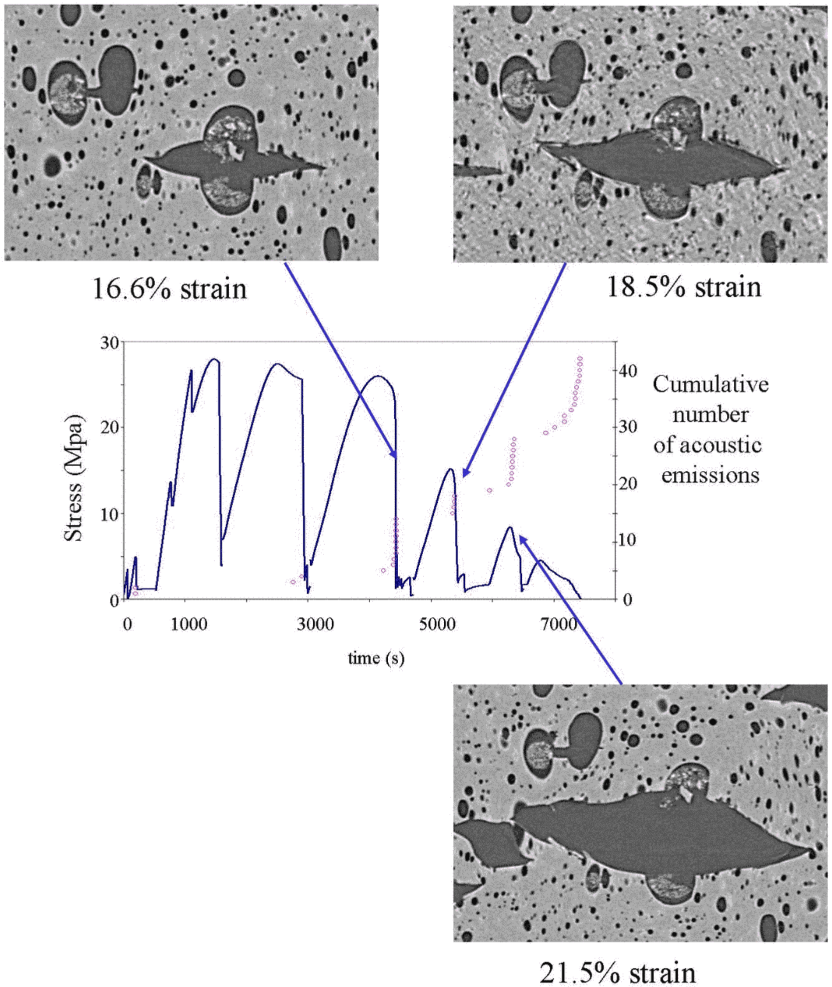
Time evolution



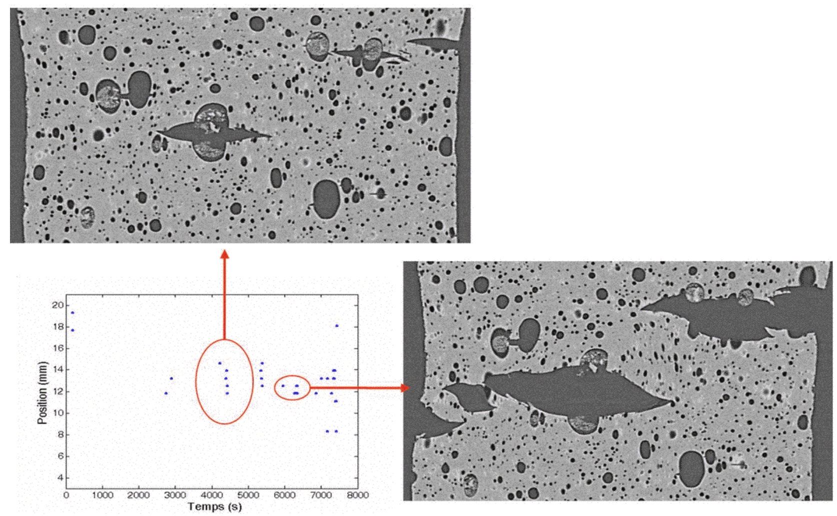
Uniaxial tensile test (1)



Uniaxial tensile test (2)

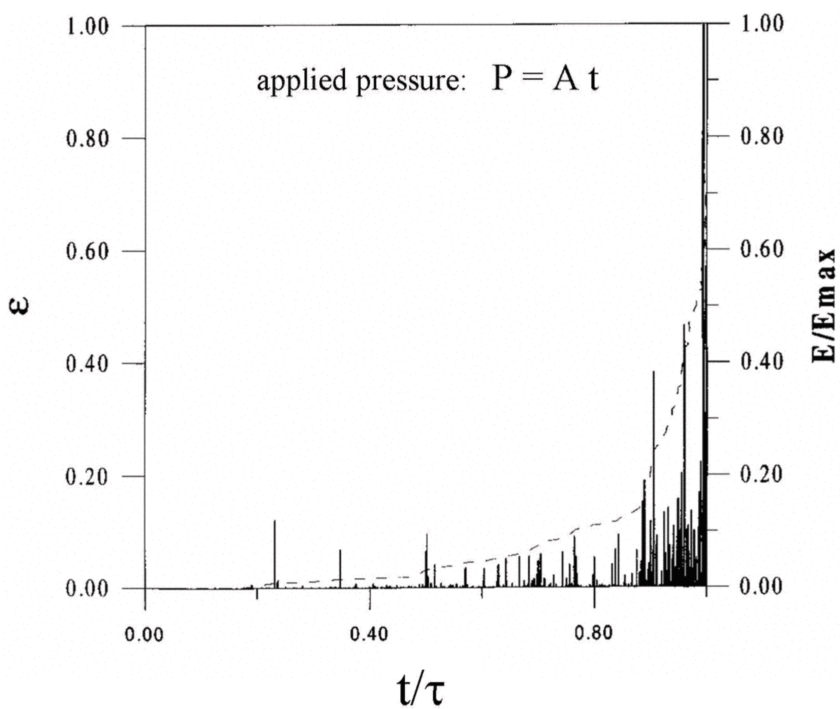


Acoustic Localisation and tomography



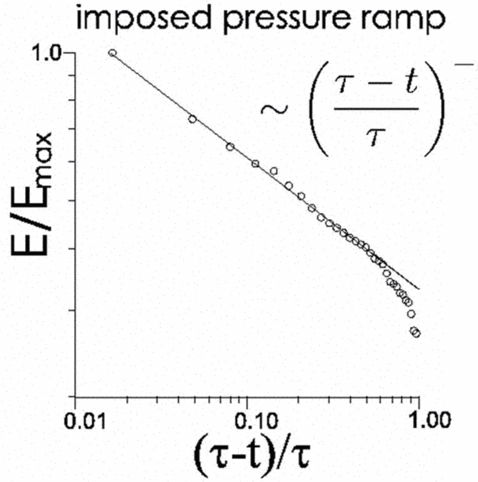
Acoustic energy versus time

A linear ramp of pressure (strain) is applied to the sample



τ is the sample life time and $E = \int_0^t \epsilon(t')\, dt'$

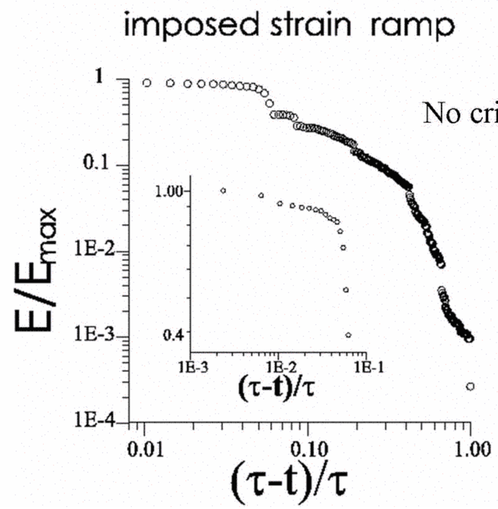
Acoustic energy versus reduced time



$\sim \left(\frac{\tau - t}{\tau}\right)^{-\alpha}$

α=0.22 for wood

α =0.27 for fibeglass



No critical behaviour

Summary of the experimental results

1) Experimental data show a strong analogy between crack formation in composite materials and percolation models.

2) When the system is driven at imposed pressure a critical behavior is found near the critical pressure.

3) These models do not seem to have a good predicatability of the life time of the sample:
   - the critical divergence appear very close to τ
   - The localisation is hard to detect in a quantitative way (minimum spanning tree, localisation entropy)

May life time be predicted ?

Activation processes and crack formation

Zurkov (1960), Golubovic (1993), Pomeau (1993)

The main physical hypothesis behind is that the macroscopic failure of a material is produced by a thermal activation of micro cracks. Specifically

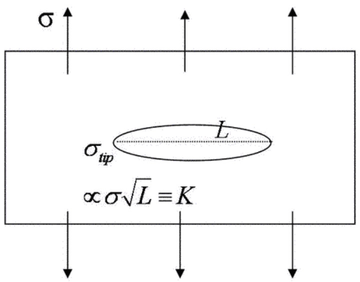
$$\tau = sample\ life\ time = \exp(\frac{W}{kT})$$

W=activation energy, k=Boltzmann constant, T=temperature

How W can be computed for a crack ?

Using the Griffith criterion for the crack stability.

Activation model in 2D

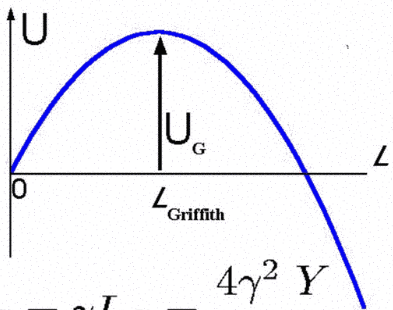


K is the stress intensity factor

Griffith Potential energy

$$U = -\frac{\pi L^2 \sigma^2}{4Y} + 2\gamma L + U_0$$

Y : Young modulus, γ :surface energy
Uo: elastic energy without fracture



$$U_G = \gamma L_G = \frac{4\gamma^2\, Y}{\pi\sigma^2}$$

The time of life of the sample
time needed to nucleate a crack of critical length L_G

$$\tau \propto \exp(\frac{W}{kT}) \quad \text{with} \quad W = U_G$$

Results of Pomeau's conjectures

2D

$$\tau = \tau_o \exp\left(\frac{P_o}{P}\right)^2 \quad \textit{(verified in 2D crystals)}$$

3D

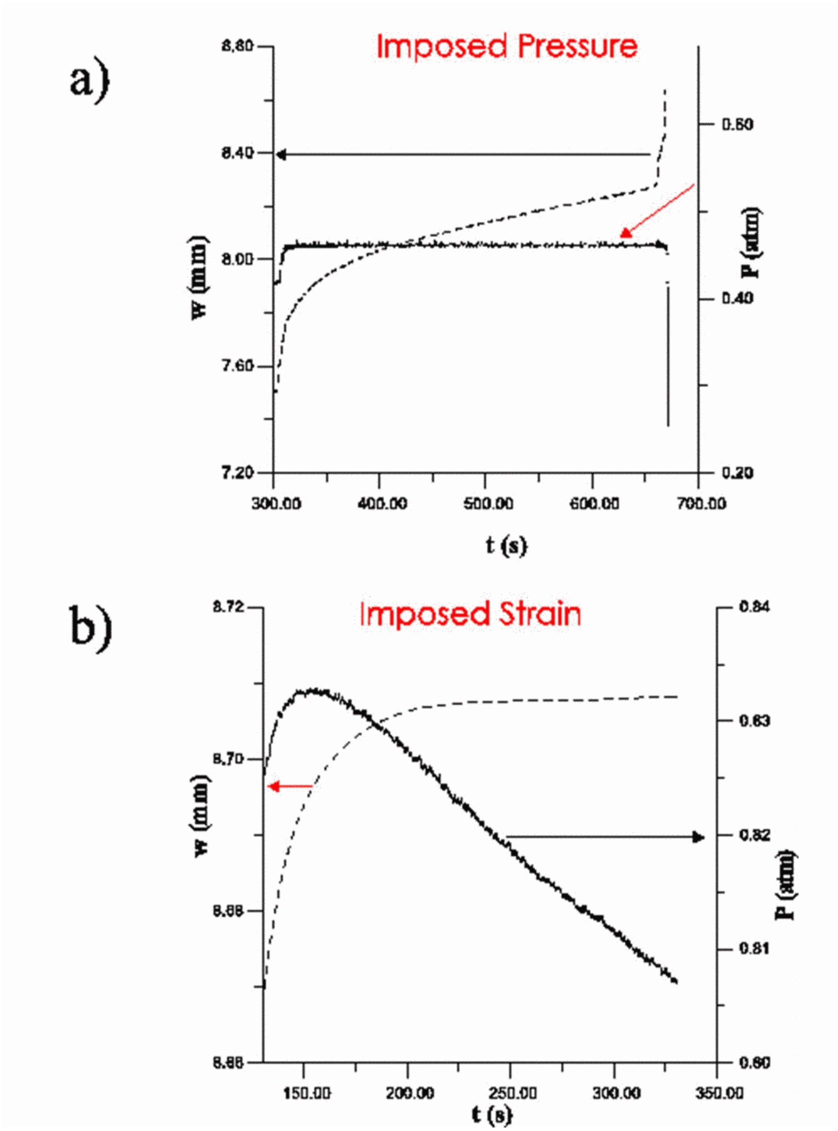
$$\tau = \tau_o \exp\left(\frac{P_o}{P}\right)^4 \quad \text{with} \quad P_o^4 = \frac{\alpha\Gamma^3 Y^2}{K_B T}$$

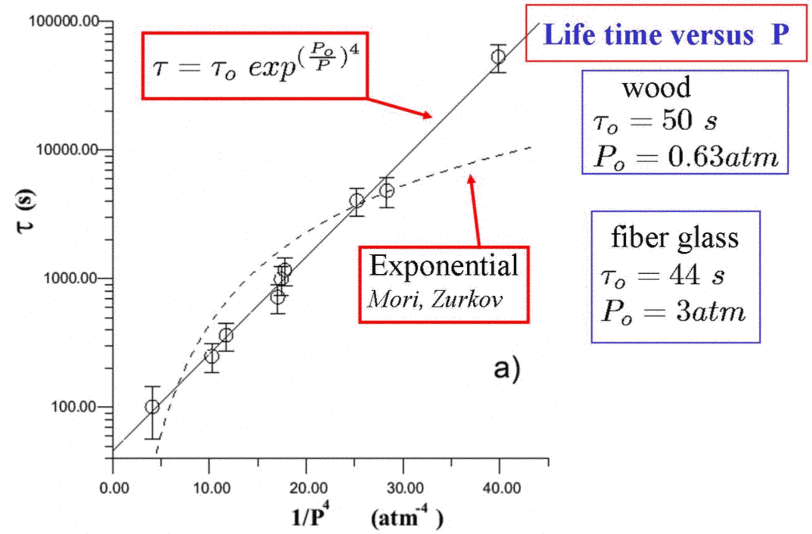
We checked this idea in our experiment

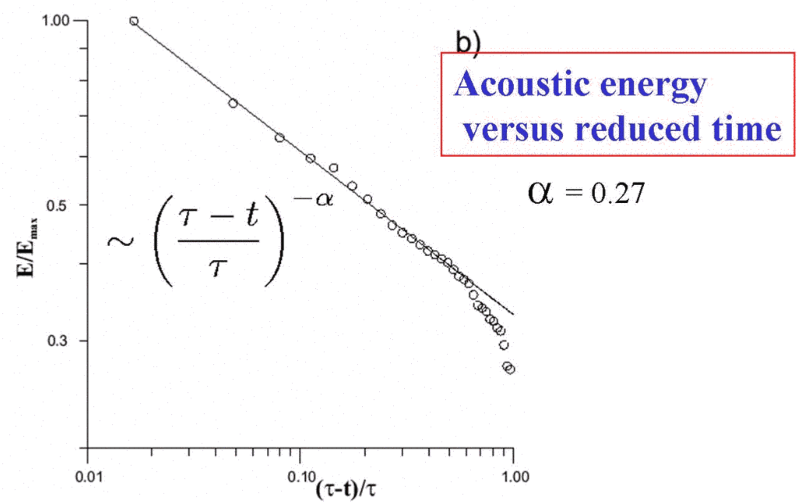
Does this hold in a cumulative way and for a time dependent pressure ?

If pressure is time dependent then:

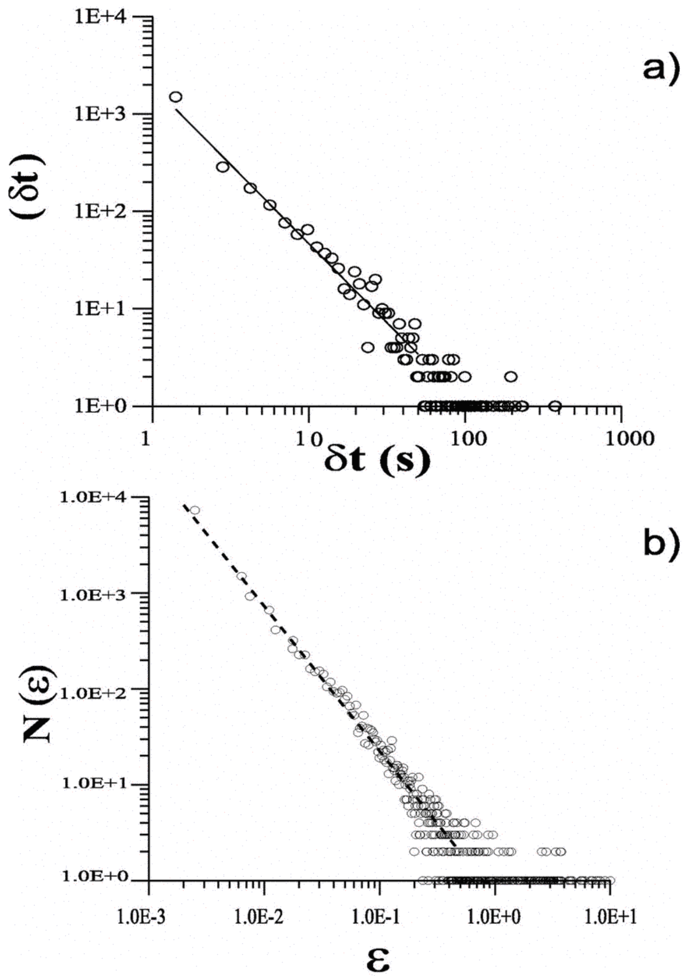
$$1 = \frac{1}{\tau_o}\int_0^{\tau} \exp\left[-\left(\frac{P_o}{P}\right)^4\right] dt$$


a)
Imposed Pressure
8.80
8.40
8.00
7.60
7.20
w (mm)
0.60
0.40
0.20
P (atm)
300.00
400.00
500.00
600.00
700.00
t (s)
b)
Imposed Strain
8.72
8.70
8.68
8.66
w (mm)
0.84
0.83
0.82
0.81
0.80
P (atm)
150.00
200.00
250.00
300.00
350.00
t (s)

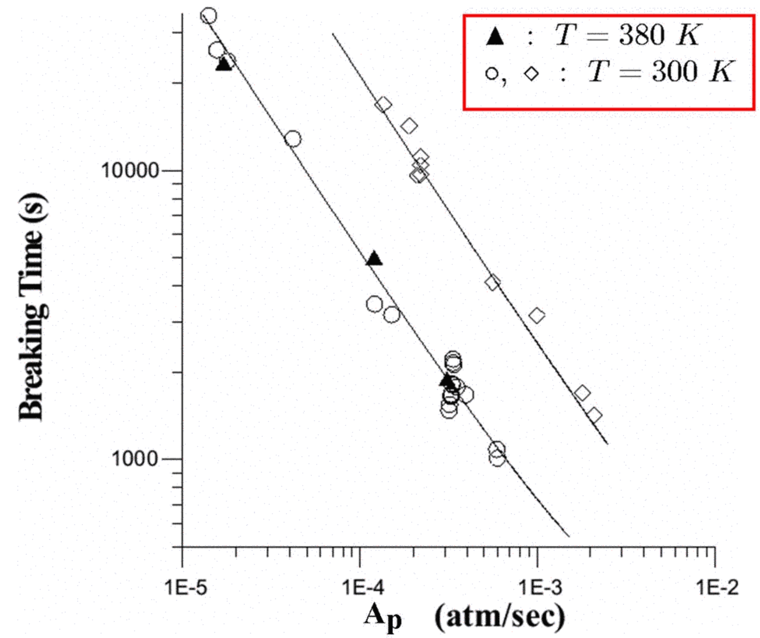

$\tau = \tau_o \; exp^{(\frac{P_o}{P})^4}$
Life time versus P
wood
$\tau_o = 50 \; s$
$P_o = 0.63atm$
fiber glass
$\tau_o = 44 \; s$
$P_o = 3atm$
Exponential
Mori, Zurkov
a)
100000.00
10000.00
1000.00
100.00
τ (s)
0.00
10.00
20.00
30.00
40.00
$1/P^4$ (atm^{-4})


b)
Acoustic energy
versus reduced time
$\alpha = 0.27$
$\sim \left(\frac{\tau - t}{\tau}\right)^{-\alpha}$
1.00
0.5
0.3
E/E_{max}
0.01
0.10
1.00
(τ-t)/τ

Events distribution at imposed constant pressure



Breaking time as a function of the pressure ramp Rate



The applied pressure is $P = A_p\ t$

Continuous lines are computed using:

$$1 = \frac{1}{\tau_o} \int_0^{\tau} \exp\left[-\left(\frac{P_o}{P} \right)^4 \right] dt$$

OPEN PROBLEMS

1) Temperature has not a big influence on τ

For a temperature variation $\Delta T = 80K$ the expected variation of τ is about:

50% at the smallest pressure
100% at the largest pressure

To have $\Delta\tau/\tau < 10\%$ for a $\Delta T \sim 80$ K one has to assume an effective temperature:

$$T_{eff} \sim 3000 \text{ K}$$

What is the origin of such a high temperature ?

2) Very large accuracy on the life time estimation

2a)
In activation process the time statistics is Poissonian whereas in our experiment it is Gaussian

2b)
In many creep tests done in many materials (e.g. stainless steel) the life time of samples submitted to a tensile stress has fluctuations of about 100%.
In our experiment the error is less than 10%.

Where these big differences come from ?

Discussion

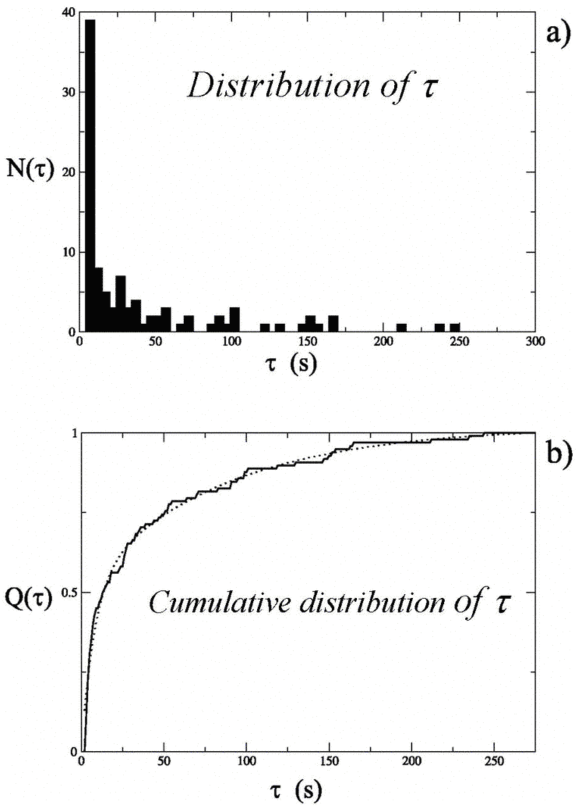
The sample disorder may play an important role

- The original Pomeau description was based on a single crack.
- In a real sample the final crack is the sum over many microcracks. This process may produce a Gaussian statistic starting from non-Gaussian ones.
- This could also explain why in small samples the time is not reproducible.

A series of experiments done on samples of different sizes confirm this conjecture

Life time in small samples

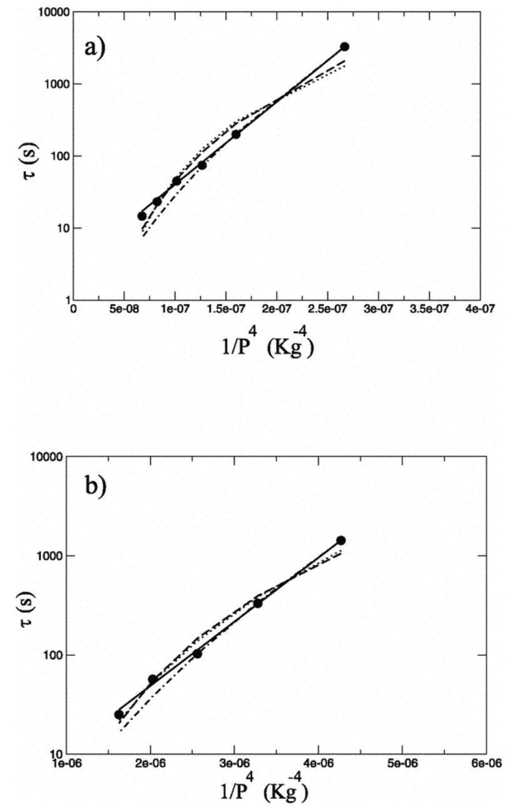
M. Zei



Distribution of the lifetimes τ of 100 samples in fiberglass broken with the BM (clamped edges) with a load $P = 54\,Kg$. The sample's size are $22 \times 2 \times 0.2\,cm$. a) The histogram of τ shows that the distribution of lifetimes is not gaussian. b) The cumulative distribution $Q(\tau) = \frac{\int_0^\tau N(t)dt}{\int_0^\infty N(t)dt}$ (solid line) is best fitted by the sum of two exponential terms (dotted line).

Life time in small samples

M. Zei



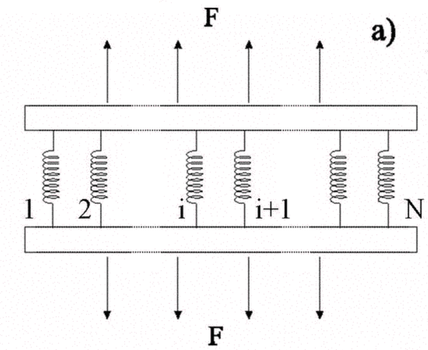
Failure time τ of the samples in fiberglass broken with the FM (clamped edges). The sample's size are $22 \times 2 \times 0.2\,cm$ (a) and $22 \times 1 \times 0.2\,cm$ (b). Each point represents the mean value of 20 measures. Lines represent the best fit with $\tau = \tau_o \exp\left(\frac{P_o}{P}\right)^4$ (solid line), $\tau = \tau_o \exp\left(\frac{P_o}{P}\right)^2$ (dashed-dotted line), $\tau = A \exp(-bP)$ (dotted line), and $\tau = A\ P^{-b}$ (dashed line).

Summary of experimental results

1) Experimental data show that the sample life time follows a well defined function of pressure. (Pomeau's model)
2) When the system is driven at constant pressure a critical divergence is found as a function of the reduced time
3) Event sizes and time delay between events are power law distributed.
4) The life time statistics depends on the system size
5) The life time does not depend on temperature

May a simple activation model take into account these observations ?

The thermally activated fiber bundle model



N springs in parallel submitted to a force F

The initial force per fiber is fo=F/N

1) Each fiber is submitted to a local force fi=fo/(1-n/N), where n is the number of broken fibers
2) Each fiber **i** is characterized by a critical strength $f_c(i)$. If, at time t, $f_i > f_c(i)$ the fiber i cracks.
3) The distribution $P(f_c)$ of f_c is characterized by a variance T_d and can be normal or uniform.
4) Each fiber is submitted to an additive random stress which follows a zero mean normal distribution of variance T. Thus the total force acting on each fiber is:

$$f_i = \frac{f_o}{1 - n/N} + \eta_a$$

The solution of the model (1)

We choose $P(f_c) = \frac{1}{\sqrt{2\pi T_d}} \exp\left[-\frac{(f_c-1)^2}{2T_d}\right]$

The relevant variables of the problem are:
The distribution of unbroken bonds $Q(f_c,t)$ with intitial condition $Q(f_c,0)=P(f_c)$.
The fraction of broken bonds is:

$$\Phi(t) = \lim_{N\to\infty} \frac{n}{N} \equiv 1 - \int_{-\infty}^{+\infty} Q(f_c,t)\ df_c$$

The average force fa exerted on each fiber is:

$$f_a = \frac{f_0}{1-\Phi}$$

The probability of breaking the fiber of critical strength f_c is:

$$G(f_c - f_a) = \frac{\gamma}{2}\left\{1 - \text{erf}\left[-\frac{(f_c-f_a)^2}{2T}\right]\right\}$$

for small T

$$G(f_c - f_a) = \frac{\gamma}{f_c-f_a}\sqrt{\frac{T}{2\pi}} \exp\left[-\frac{(f_c-f_a)^2}{2T}\right]$$

The solution of the model (2)

The dynamical equation for Q(f,t) is

$$\dot{Q}(f_c,t) = -Q(f_c,t)G(f_c - f_a)$$

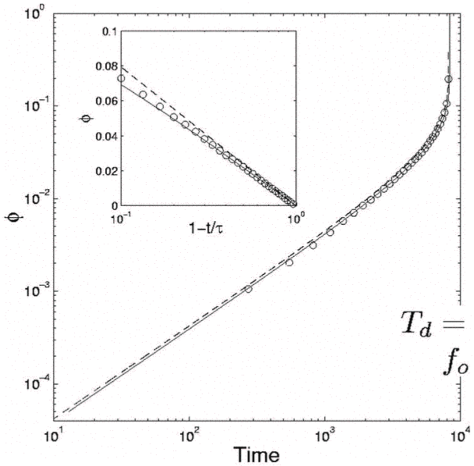
The difficulty of solving this model arises from the time dependence of $\mathbf{f_a}$ which is determined by the integral of **Q** over all f_c values.

$$f_a = \frac{f_0}{1-\Phi}$$

with

$$\Phi(t) \equiv 1 - \int_{-\infty}^{+\infty} Q(f_c,t)\ df_c$$

Numerical simulation

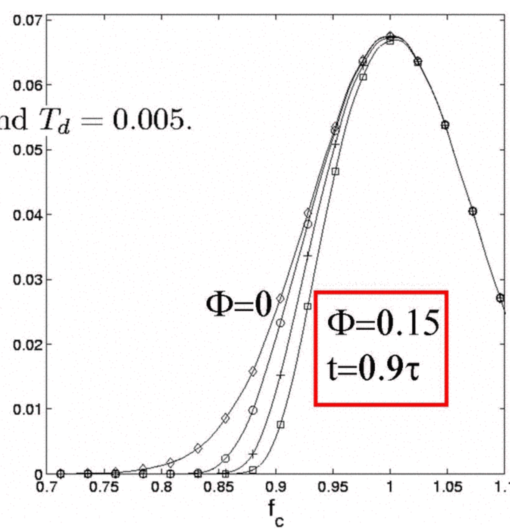


Evolution of the fraction of broken bonds

$T_d = 10^{-3}$ and $T = 8\ 10^{-3}$
$f_o = 0.6$

$f_o = 0.6, T = 0.006$ and $T_d = 0.005.$

Evolution of Q(fc,t)

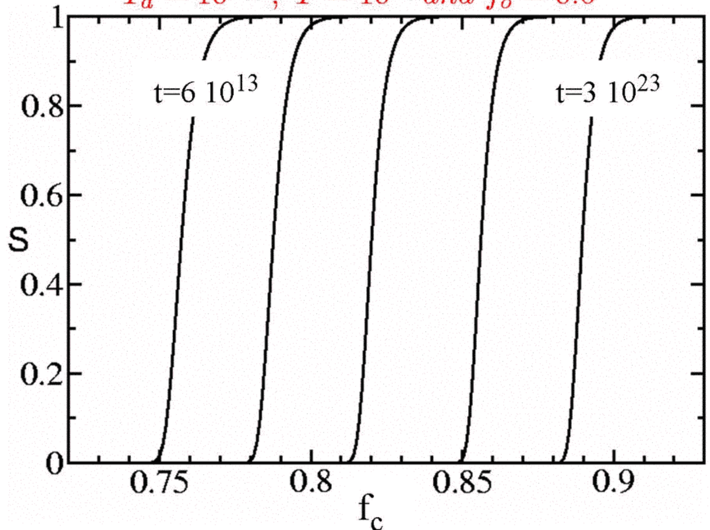


Time evolution of Q

We define:

$$S(f_c, t) = \frac{Q(f_c, t)}{P(f_c)}$$

$T_d = 10^{-2}$, $T = 10^{-3}$ and $f_o = 0.6$



For T ⟶ 0

the Heaviside shape is a good approximation for S

Approximated analytical solution for large t

$$\ln\left(\frac{\tau - t}{\tau}\right) = -C\ \phi$$

$$t = \tau\ [1 - \exp(-\phi\ C)]$$

where

$$\tau \simeq \tau_o \exp\left(\frac{(1-f_o)^2}{2\ T_{eff}}\right)$$

and

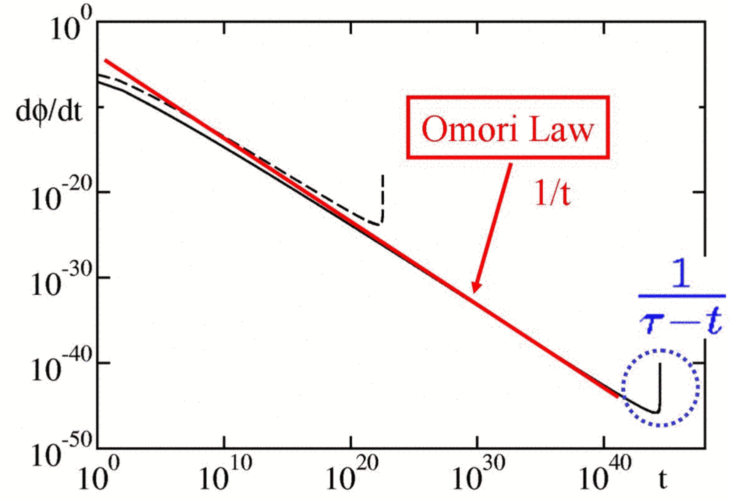
$$T_{eff} \simeq \frac{T}{\left(1 - \frac{\sqrt{\pi}\ \sigma_o}{2(1-f_o)}\right)^2}$$

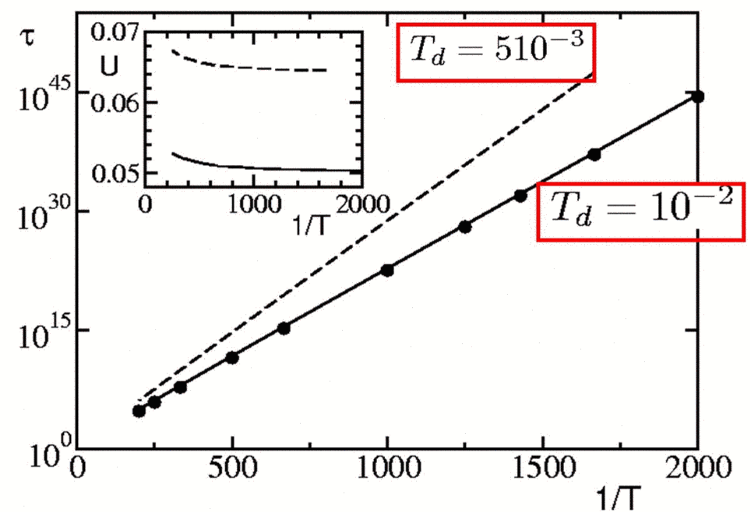
with $\sigma_o = \sqrt{T_d}$

$$\frac{d\phi}{dt} \propto \frac{1}{\tau - t}$$

Critical divergence of the damage rate

Comparison with the numerical simulation





Conclusions on the fiber bundle

The thermally activated fiber bundle model presents interesting features:

* The disorder strongly reduces the sample life time but it increases the life time predictability.
This property could explain our experimental results on life time

* The Omori law and the divergence of the damage rate are natural consequences of this simple model.

Problem

The dependence on P is not the one predicted by Pomeau

Is this due to the 1D structure of the model ?

Extension of the thermally activated fiber bundle in 2D

1) Noise induced dynamics of a single crack
(L. Vanel talk 2 weeks ago)

2) Damage growth in a desordered 2D network

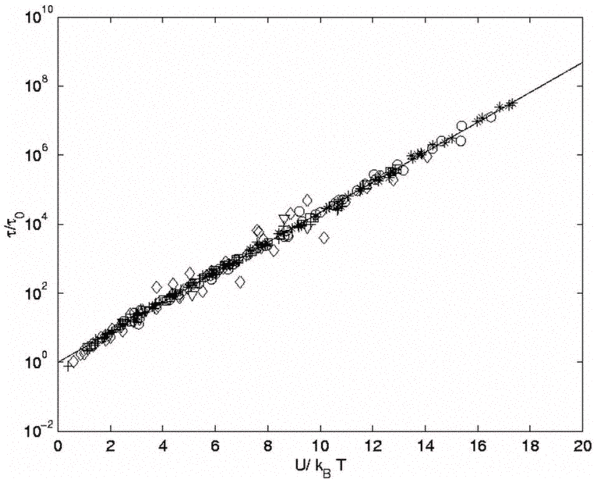
Damage growth in a desordered 2D network

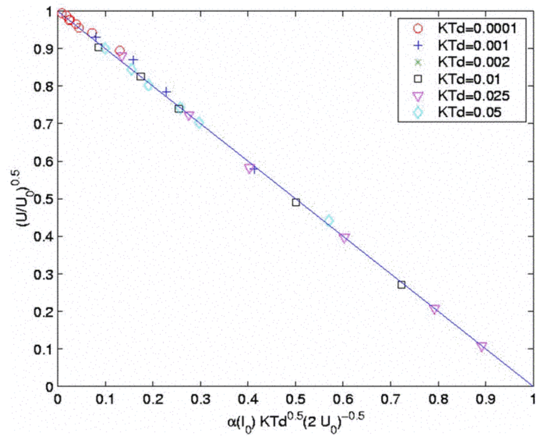
In the ordered network :

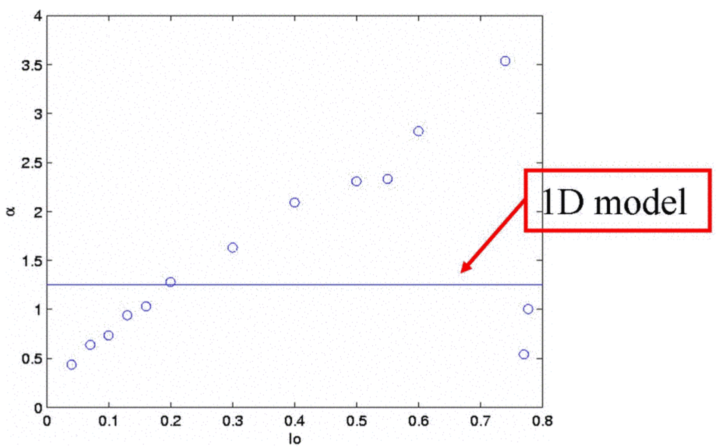
$$\tau = \tau_o \exp(\frac{U_o}{T}) \quad \text{with} \quad U_o = \frac{1}{2}(1 - I_o)^2$$

In the disordered network :

$$\tau = \tau \exp(\frac{U}{T}) \quad \text{with} \quad U = U_o(1 - \frac{\alpha(I_o)\ \sigma_o}{\sqrt{2U_o}})$$







Conclusions

Activation models of crack explain many experimental results

In the case of damage growth they justify :

- The existence of an effective temperature *(reduction of the energy barrier)* induced by the sample disorder
- The transition from a Poissonian to a Gaussian statistics of the life time, when the sample size is increased.
- The critical divergence of the damage rate close to τ

In the case of a single crack growth in a heterogenous material

- The dynamics of the crack tip.
- The statistics of jumps

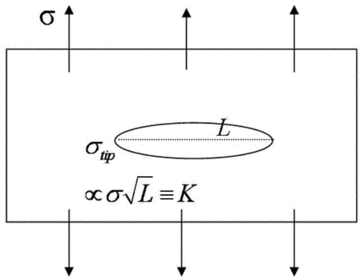
Activation models can probably be used to predict the life time in large hetereogenous structures

Problems and perspectives

- The dependence in P^{-4} of the life time of 3D heterogenous sample is not explained by our model.
- The role of disorder in samples has to be studied experimentally.

  ** work is in progress using solid foams and other materials*

2D models



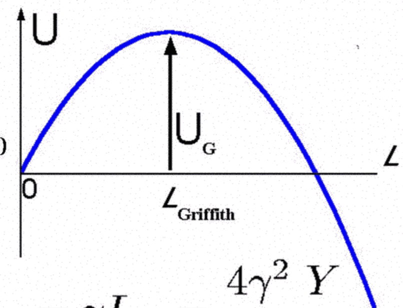
$$\sigma_L = \sigma\sqrt{\frac{L}{\lambda}} = \frac{K}{\sqrt{\lambda}}$$

K is the stress intensity factor

Griffith Potential energy

$$U = -\frac{\pi L^2 \sigma^2}{4Y} + 2\gamma L + U_0$$

Y : Young modulus, γ :surface energy
Uo: elastic energy without fracture



$$U_G = \gamma L_G = \frac{4\gamma^2\, Y}{\pi\sigma^2}$$

The time of life of the sample

time needed to nucleate a crack of critical length L_G

$$\tau \propto \exp\left[\frac{U_G}{kT}\right]$$

In the Pomeau model crack is reversible

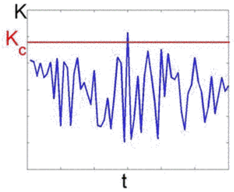
Irreversible crack

Hypothesis of the model

a) The local breaking threshold is overcome by the fluctuations of the applied stress.

$$\langle \Delta\sigma \rangle^2 \propto kT$$

k constante de Boltzmann,
T température



b) The damage processes is irreversible

c) The velocity V of the crack tip is proportional to the probability P of having a fluctuation K>Kc

Analytical results

Time evolution of a crack of initial length L_i :

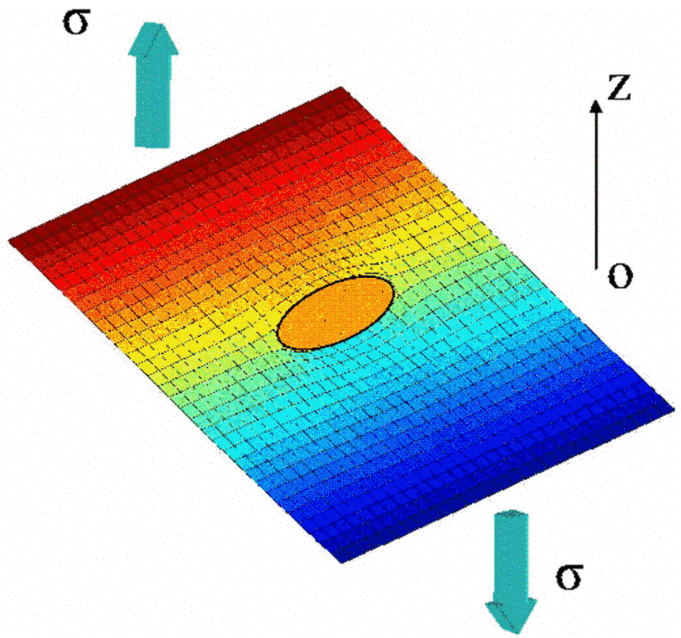
$$t = \tau \left[1 - \exp\left(-\frac{L-L_i}{\zeta}\right)\right]$$

Time of life	*Characteristic length*
$\tau = \tau_0 \exp\left[\frac{(\sigma_c-\sigma_i)^2 V}{2Y k_B T}\right]$	$\zeta = \frac{2Y k_B T}{V} \frac{L_i}{\sigma_i(\sigma_c-\sigma_i)}$

Numerical simulation on a spring network

(S.Santucci et al, Europhysics Letters (2003))

Square network in antiplane deformation (same as fuse network)
Shearing load σ (mode III)
No disorder : constant spring breaking threshold σ_c

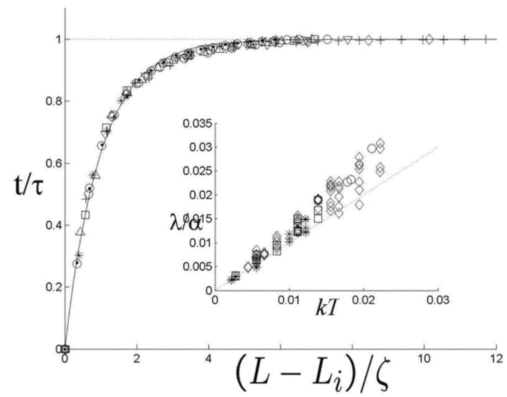


This is equivalent to a 2D fuse network driven at constant current

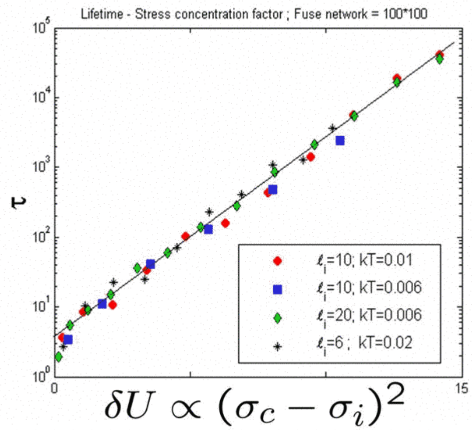
Results of the 2D numerical simulation

(S.Santucci et al, Europhysics Letters (2003))

Average over 50 realisations

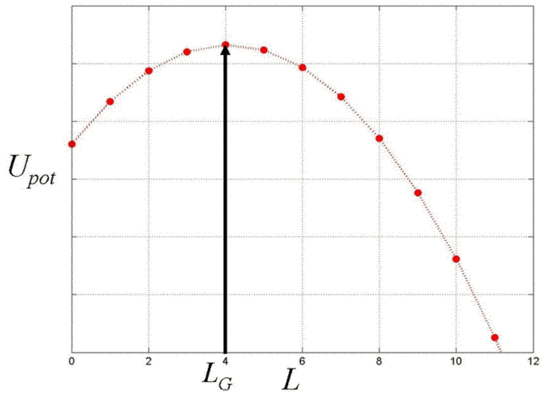


ime evolution
f the crack tip



Mean life time of the sample

Potential energy U_{pot} in the lattice model

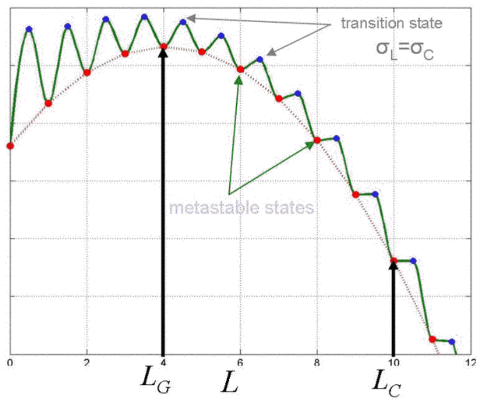


in agreement with expected Griffith energy !

Minimum energy cost : L → L+1 ?

Potential energy U_{pot} in the lattice model

"LATTICE TRAPPING" : *(R. Thomson , Solid State Physics (1986))*



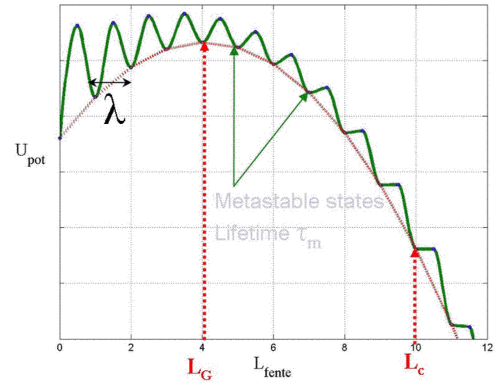
Dynamics controlled by thermal noise :

***L < L_G* :** crack closes
L_G < L < L_C* :** ***irreversible crack growth
***L_C < L* :** fast rupture

Probability distribution P_S of crack jump sizes S

(S.Santucci et al, P.R.L (2004))

Jump distribution P_S ⟷ Distribution of stress fluctuations $G(\sigma_f)$



- **Mean growth velocity** *V(L)*

$$\begin{cases} V(L) \propto P(\sigma_f > \sigma_c) \\ V(L) = \frac{\langle s \rangle}{\tau_m} = \frac{\int s P_s ds}{\tau_m} \end{cases}$$

- **Crack arrest mechanism**

dissipation : barrier + elastic release

$$s = \frac{\delta U_f}{\delta U_c}\lambda = \frac{(\sigma_f - \sigma_m)^2}{(\sigma_c - \sigma_m)^2}\lambda$$

distribution of crack jump sizes

$$P_s = N(V, \lambda)\ s^{-3/2} e^{-s/\xi}$$

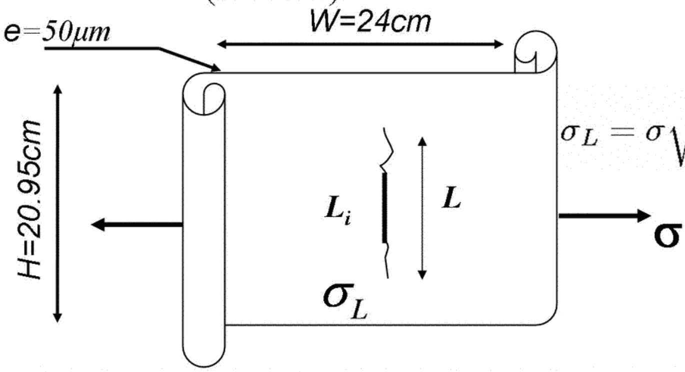
Cut-off length $$\xi = \lambda \frac{2 Y k_B T}{(\sigma_c - \sigma_m)^2 V}$$

V volumic scale of damge
λ discretization scale

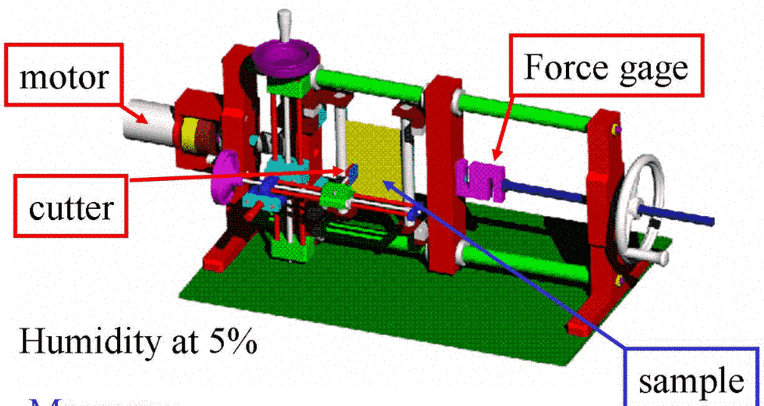
Similar to « critical point theory in percolation»

Experiment in 2D

Material: Sheets of fax paper under tensile stress (mode 1).



$$\sigma_L = \sigma\sqrt{\frac{L}{\lambda}} = \frac{K}{\sqrt{\lambda}}$$

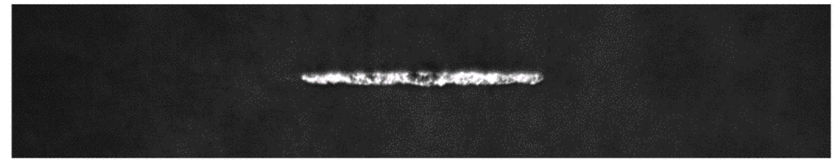


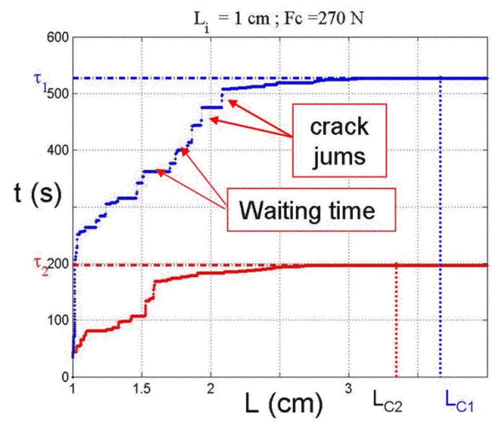
Humidity at 5%

Measures:

- Applied strain and stress,
- length of the crack as a function of time
- time of life

Crack Length versus time





Evolution of two cracks having the same initial conditions

For L > Lc the crack propagates very fast

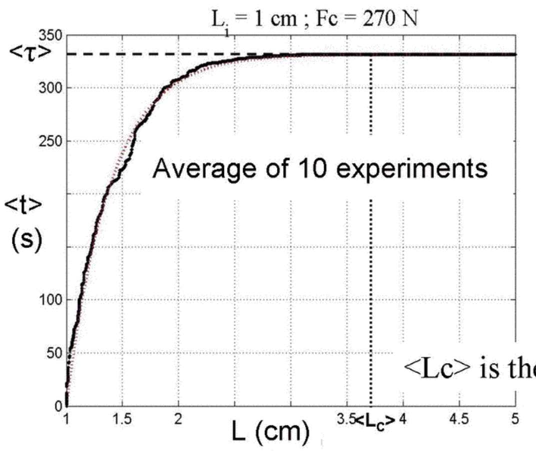
Extended study for:

1 cm < L_i < 4 cm

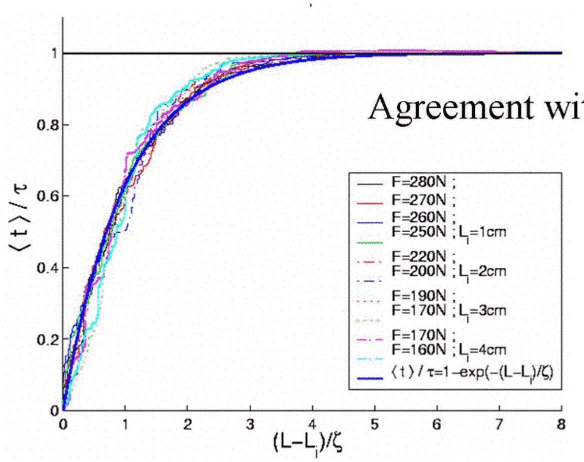
140 N < F < 280 N

Mean dynamics

$$\frac{\langle t\rangle}{\tau} = \left[1 - \exp\left(-\frac{L-L_i}{\zeta}\right)\right]$$ Prediction of the model

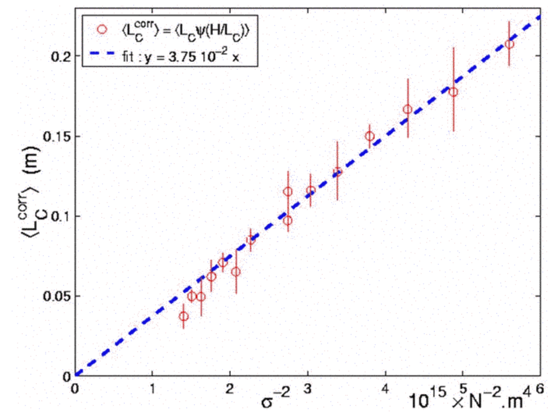


<Lc> is the critical length

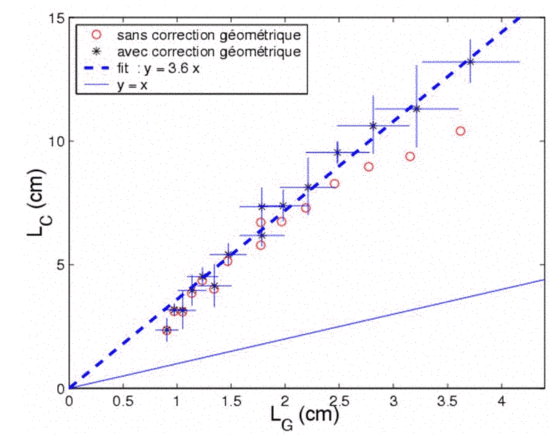


Agreement with the model

L_c dependence on σ



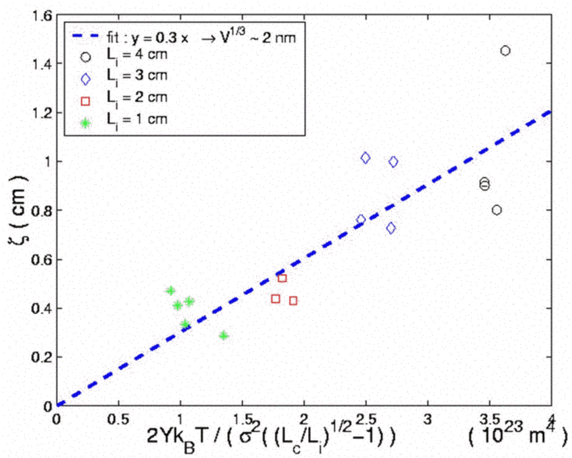
Comparison with $$L_G = \frac{4\langle\gamma\rangle Y}{\pi\sigma^2}$$



Characteristic length

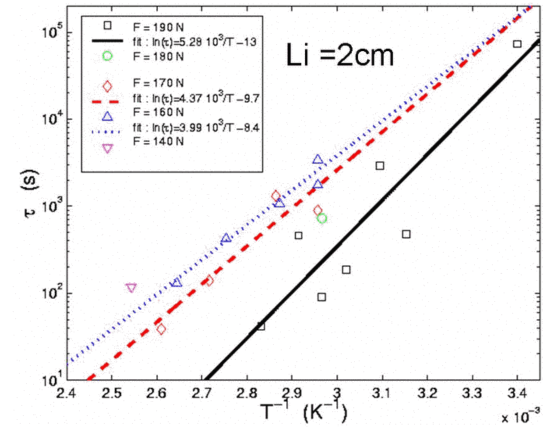
Prediction of the model:

$$\zeta = \frac{2Yk_BT}{V}\frac{L_i}{\sigma_i(\sigma_c-\sigma_i)}$$

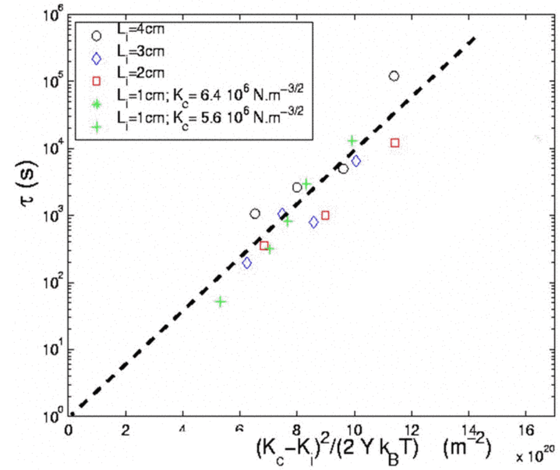


Life Time

Prediction of the model: $\tau = \tau_0 \exp\left[\frac{(\sigma_c-\sigma_i)^2V}{2Yk_BT}\right]$



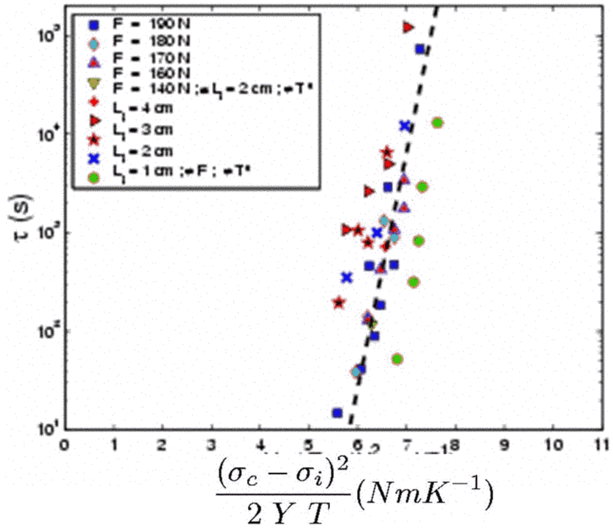
Temperature dependence



K dependence

Life time

Prediction of the model: $\tau = \tau_0 \exp\left[\frac{(\sigma_c - \sigma_i)^2 V}{2 Y k_B T}\right]$



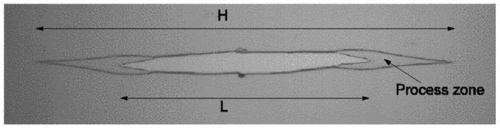
The activation model predicts the average dynamics of the crack.
The jumps statistics can also be derived

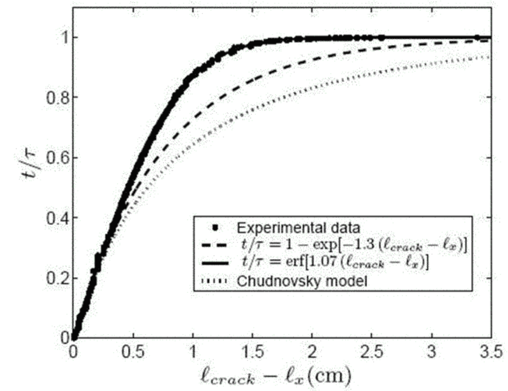
Problem

If $\lambda \sim 50 \mu m$ (size of the fibers) the only free parameter of the fits is V, which can be determined by the fit of ζ and τ.

These two independent fits gives $V^{1/3} \sim 1$ nm

Average growth dynamics in polycarbonate





$$\frac{\langle t \rangle}{\tau} \neq \left[1 - \exp\left(-\frac{L - L_i}{\zeta}\right)\right]$$

(Cortet et al, EPL 2005)