Deeply Focused Earthquakes as Phase Transformations

Michael Grinfeld
The U.S. Army Research Laboratory,
Weapon and Materials Research Directorate
Aberdeen Proving Ground, MD 21005

From the Atomic to the Tectonic: Friction, Fracture and Earthquake Physics

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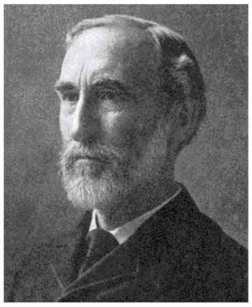
OUTLINE

- Seismological introduction
- Thermodynamic introduction
- Problem of nucleation at solid/solid phase transformation
- Lifshitz vs Eshelby
- Lifshitz's contribution
- The Eshelby solution for an ellipsoidal inclusion
- An ellipsoidal seed of a new solid phase
- Stability of an ellipsoidal inclusion
- Unsolved problems
- Intensively fractured zones
- Conclusion

THE MEN INSPIRED THIS WORK



Gibbs Josiah Willard

1839 - 1903

Лифшиц Илья Михайлович

1915 - 1982

Eshelby John Douglas

1914 - 1981

Садовский Михаил Александрович

1904 - 1994

1. Brief Seismology Introduction

The shallow and deep crustal and tectonic earthquakes

Crustal (shallow) earthquakes

These events result from the relative motion within tectonic plates at depths of 10 to 25 kilometers.

Deep earthquakes

These events, also referred to as intra-plate events, occur at depths of 40 to 60 kilometers.

Subduction zone earthquakes

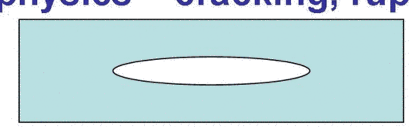
These events refer to the zone of contact between the two plates.

The underlying physics – cracking, rupture.

Inglis (1907, 1909),

↓

Griffiths (1921) ⟶ Kostrov, Rice ⟶



2. Brief Seismology Introduction

Phase transformations in the Earth's mantle:
The mantle earthquakes

- The 410 km discontinuity is interpreted as the transition from the α-phase (olivine) to the β-phase (wadsleyite) of $(Mg,Fe)_2SiO_4$.
- The 660 km discontinuity is interpreted as a phase transformation of ringwoodite (the γ-phase of $(Mg,Fe)_2SiO_4$) to perovskite ($(Mg,Fe)SiO_3$) and magnesiowustite ($(Mg,Fe)O$).
- The mechanism of the deepest earthquakes remains questionable. Brittle shear failure and frictional sliding - the mechanisms of shallow earthquakes - are unlikely to occur in unmodified form at depths as great as 660 km.

The Ringwood congecture: Phase nucleation ?!

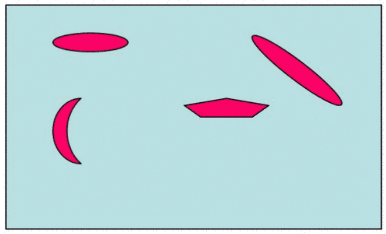
Brief Thermodynamics Introduction

- The Gibbs paradigm of heterogeneous systems and *nonhydrostatical thermodynamics* in the Earth sciences
- The central question of nonhydrostatical thermodynamics in the Earth sciences:

WHAT IS THE CHEMICAL POTENTIAL OF A NONHYDROSTATICALLY STRESSED SOLID?

- Theories of local and global chemical potential (Bridgman, Verhoogen, Kamb, Kumazava, Ida)

The Gibbs Physics of Heterogeneous Systems



Variables:

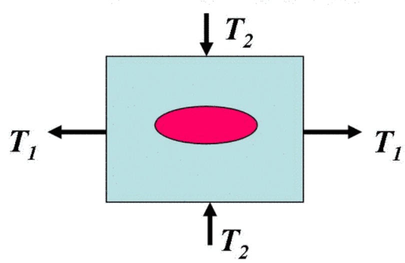
Displacements, local entropy, shape of inclusions

$$\begin{cases} \min E = \int_{\omega} d\omega e(u_{i,j}, \eta) \\ \mathrm{N} = \int_{\omega} d\omega\eta = const \end{cases}$$

Minimization:

Local or global?

The Problem of Equilibrium Shape of a Nucleus of New Solid Phase



Main publications:

Gibbs, On the equilibrium of heterogeneous substances, Trans. Connect. Acad. Sci. 1878;

Lifshitz, I.M., Gulida, L.S., On the theory of a local melting, Dokl. AN SSSR, **87**, 377-380, 1952 (in Russian).

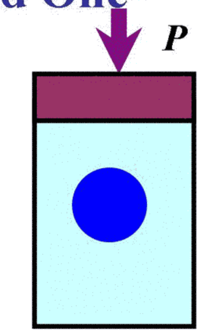
Eshelby, J.D., The determination of the elastic field of an ellipsoidal inclusion, Proc. Roy. Soc., **A241**, 376-396. 1957.

Grinfeld, M.A., Conditions for thermodynamic phase equilibrium in a nonlinear elastic material, Dokl. AN SSSR, **251**, 824-827, 1980, (in Russian).

Berdichevsky, V.L. The nuclei of a melt in solids, Dokl. AN SSSR, **273**, 80-84, 1983 (in Russian).

Grinfeld, M.A., Thermodynamic Methods in the Theory of Heterogeneous Systems, Longman, 1991.

The Gibbs Chemical Potential of Liquid One-Component Substance



Conditions of phase equilibrium $\left(e = e(\eta, v)\right)$

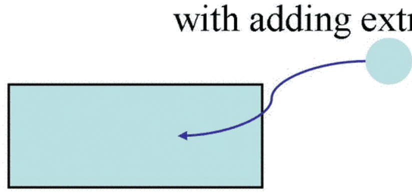
$$p_1 = p_2,\ T_1 = T_2\ ;$$

$$\mu_1 = \mu_2, \quad \mu \equiv e - \eta \frac{\partial e}{\partial \eta} - v \frac{\partial e}{\partial v}\ ;$$

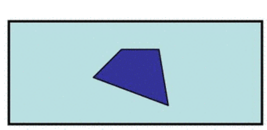
μ - the Gibbs chemical potential

Two Understandings of Chemical Potential

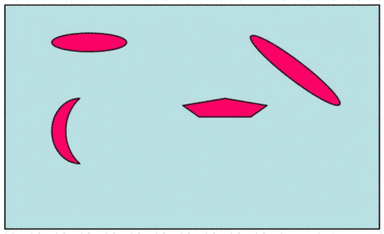
I. Chemical potential as the change of thermodynamic potential with adding extra mass.



II. Chemical potential as the condition of equilibrium at phase interfaces.



Theory of the Global Chemical Potential



$$\mu \equiv e - u_{i,j} \frac{\partial e}{\partial u_{i,j}} - \eta \frac{\partial e}{\partial \eta}$$

The Main Difficulty

$$e - u_{i,j} \frac{\partial e}{\partial u_{i,j}} - \eta \frac{\partial e}{\partial \eta} \neq e - v \frac{\partial e}{\partial v} - \eta \frac{\partial e}{\partial \eta}$$

Iliya Mikhailovich Lifshitz (1917 - 1982)

On macroscopic description of the phenomenon of twinning in crystals, Zh.ETP, 1948, 18, 1134-1143 (1948).

Some thoughts on twinning in calcite, Izv. AN SSSR, 12, 65-80 (1948).

Calculation of the shape of the twinning layer from the values of stresses on its boundaries, Uchenye Zapiski, Khar'kov Univ., 3, 7-10 (1952).

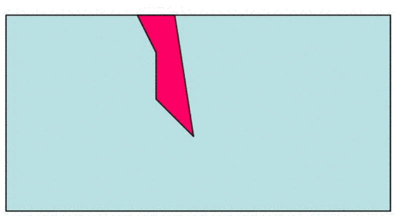
On the theory of local melting, Dokl. AN SSSR, 87, 377-380 (1952).

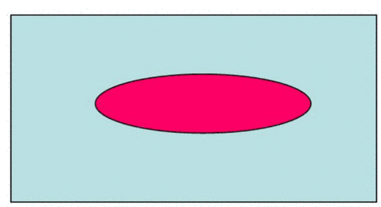
On development of nuclei of local melting, Dokl. AN SSSR, 87, 523 (1952).

John Douglas Eshelby (1916-1981)

Force on an elastic singularity, Phil. Tran. Roy. Soc. Lond., 244 A, 87-112 (1951).
The Determination of the Elastic Field of an Ellipsoidal Inclusion and Related Problems, Proc. R. Soc. London A, 241, 376 - 96 (1957).
The Elastic Energy-Momentum Tensor, J. Elasticity, 5, 321 – 335 (1975).
The Energy-Momentum Tensor of Complex Continua, In “Continuum Models of Discrete Systems”(1980).

Lifshitz versus Eshelby (1)





The key difference:

Eshelby: an ellipsoidal *shape* of a nucleus is given a priori !

Lifshitz: *shape* of a nucleus – the most important unknown !

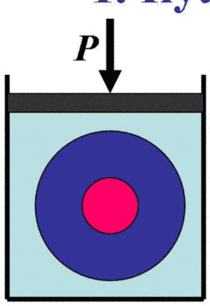
2. Lifshitz versus Eshelby

What is a consistent definition of chemical potential of nonhydrostatically stressed solid ?

Lifshitz and the local chemical potential…

Eshelby and the energy-momentum tensor…

1. Ilya Lifshitz on the "internal" melting of solids (1952)



$$\min \Phi = \int_v d\omega\, \psi_l(\rho_l, T) + \int_V d\omega\, \psi_s(u_{i,j}, T) + PV + \sigma S$$

An Infinite Isotropic Matrix

$$\Delta\Phi = \left(\varphi - \frac{q}{T^\circ}\Delta T\right)\frac{4\pi R^3}{3} + 4\pi R^2 S$$

$$\varphi = \frac{(k_l - k_s)(4\mu + 3k_s)}{2k_s^2(4\mu + 3k_l)}P^{\circ 2} + \frac{k_l(4\mu + 3k_s)}{k_s(4\mu + 3k_l)}\frac{\delta\rho}{\rho}P^\circ + \frac{2k_l\mu}{4\mu + 3k_l}\left(\frac{\delta\rho}{\rho}\right)^2$$

2. Ilya Lifshitz on the "internal" melting of solids (1952)

The "local" chemical potential of Lifshitz

$$\mu^{(N)} \equiv e - \eta \frac{\partial e}{\partial \eta} - \frac{P^{(N)}}{\rho} \left(u_{i,j} \frac{\partial e}{\partial u_{i,j}} \right)$$

The chemical potential of stressed solid (Gibbs (1878), Bridgman (1916), Kamb (1960), Nozieres (1990),…)

$$\mu \equiv e - \eta \frac{\partial e}{\partial \eta} + \frac{p}{\rho}$$

The Bowen's tensorial chemical potential (1963, 1967)

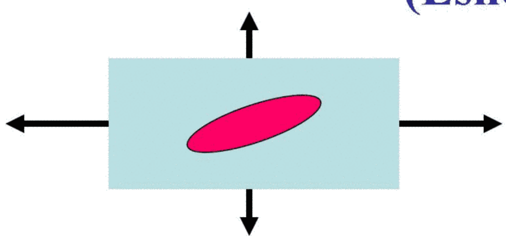
Nonhydrostatic configurations

$$\chi^{ij} \equiv \left(e - \eta \frac{\partial e}{\partial \eta} \right) \delta^{ij} - \frac{1}{\rho} P^{ij}$$

Hydrostatic configurations: $P^{ji} = -p\delta^{ij}$

$$\chi^{ij} \equiv \left(e - \eta \frac{\partial e}{\partial \eta} + pv \right) \delta^{ij}$$

1. An Ellipsoidal Inclusion in Isotropic Matrix (Eshelby, 1956)



Bulk equations:

$$p^{ij}_{,j} = 0; \quad p^{ij} = c^{ijkl} u_{k,l}$$

The matrix isotropy:

$$c^{ijkl} = \lambda \delta^{ij} \delta^{kl} + \mu\left(\delta^{ik} \delta^{jl} + \delta^{il} \delta^{jk}\right)$$

The interface clamping condition

$$\left[u^i\right]_-^+ = 0, \quad \left[p^{ji}\right]_-^+ n_j = 0$$

The conditions at infinity

$$u_i \to \kappa_{ij} x^j \quad at \quad |x^j| \to \infty$$

2. An Ellipsoidal Inclusion in Isotropic Matrix

Inside inclusion: $u_i = \boxed{\omega_{ij}} x^j$

Inside matrix:

$$u_i = \kappa_{ij} x^j + \frac{1}{4\pi\mu_+} \boxed{\tau_{ij}} \partial^j \Pi - \frac{1}{16\pi\mu_+ (1-\nu_+)} \boxed{\tau_{jk}} \partial_i \partial^j \partial^k \Pi^*$$

The harmonic and bi-harmonic potentials

$$\Pi(x) = \int_v d\,x^* \frac{1}{|x - x^*|}, \quad \Pi^*(x) = \int_v d\,x^* \,|x - x^*|$$

3. An Ellipsoidal Inclusion in Isotropic Matrix

Inside ellipsoidal inclusion: $\partial_j \Pi = -\boxed{N_{jk}}\, x^k \;,\; \partial_k \partial_j \partial_k \Pi = -\boxed{M_{ijkl}}\, x^l$

$$N_{II} = 2\pi a_1 a_2 a_3 \int_0^\infty \frac{dq}{D(q)\left(a_I^2 + q\right)},\; N_{IJ} = 0 \;\; at\ I \neq J$$

$$M_{IIII} = 6\pi a_1 a_2 a_3 \int_0^\infty \frac{q\,dq}{D(q)\left(a_I^2 + q\right)^2};$$

$$M_{IIJJ} = 2\pi a_1 a_2 a_3 \int_0^\infty \frac{q\,dq}{D(q)\left(a_I^2 + q\right)\left(a_J^2 + q\right)} \;\; at\ I \neq J$$

$$D(q) = \sqrt{q\left(a_1^2 + q\right)\left(a_2^2 + q\right)\left(a_3^2 + q\right)}$$

4. An Ellipsoidal Inclusion in Isotropic Matrix

The linear master system for ω^{ij} **and** τ^{ij}

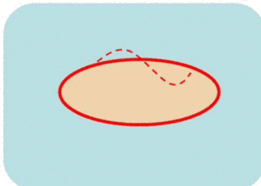
$$\boxed{\omega^{ij}}\, P_{klij} = \kappa_{ip} \;,\; \boxed{\tau^{kl}}\, P_{klij} = -[\lambda]_-^+ \kappa^{l}_{.l} \delta_{ij} - 2[\mu]_-^+ \kappa_{(ij)}$$

The material tensor P_{klij}

$$P_{klij} \equiv -\delta_{ki}\delta_{ij} + \frac{[\lambda]_-^+}{4\pi\left(\lambda_+ + 2\mu_+\right)} N_{kl}\delta_{ij} +$$

$$\frac{[\mu]_-^+\left(\lambda_+ + \mu_+\right)}{4\pi\mu_+\left(\lambda_+ + 2\mu_+\right)} \delta_{k(i} N_{j)l} - \frac{[\mu]_-^+\left(\lambda_+ + \mu_+\right)}{4\pi\mu_+\left(\lambda_+ + 2\mu_+\right)} M_{ijkl}$$

Phase Equilibrium at Coherent Phase Transformations



The "chemical" equilibrium

$$\left[\mu^{ij}\right]_{-}^{+} n_i n_j = 0$$

The asymmetric tensorial "chemical" potential

$$\mu^{ij} = \left(e - \eta \frac{\partial e}{\partial \eta}\right)\delta^{ij} + \frac{\partial e}{\partial u_{k,j}}\left(\delta_k^i + u_{k,\cdot}^{\cdot\, i}\right)$$

The tensorial chemical potential μ^{ij} and the Eshelby energy-momentum tensor e^{ij}

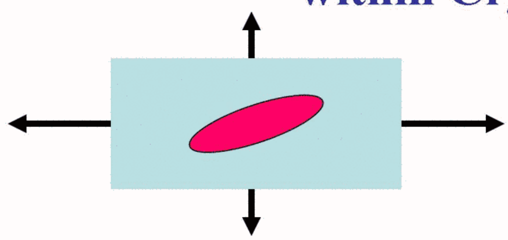
$$\mu^{ij} = \left(e - \eta \frac{\partial e}{\partial \eta}\right)\delta^{ij} - \frac{\partial e}{\partial u_{k,j}}\left(\delta_k^i + u_{k,\cdot}^{\cdot\, i}\right)$$

$$e^{ij} = e\delta^{ij} - u_{k,\cdot}^{\cdot\, i} \frac{\partial e}{\partial u_{k,j}}$$

Hydrostatic configurations

$$\mu^{ij} = \mu\delta^{ij}\ ,\ e^{ij} \neq \mu\delta^{ij}\ ,\ \mu_{class} \equiv e - \eta T - \nu p$$

1. An Equilibrium Shape of New Solid Phase within Crystalline Solid



Bulk equations:

$$\left(\frac{\partial e}{\partial u_{i,j}}\right)_{,j} = 0\,;\quad \frac{\partial e}{\partial \eta} = const$$

Interface clamping condition

$$\left[u^i\right]_-^+ = \Delta_{ij} x^j,\quad \left[\frac{\partial e}{\partial u_{i,j}}\right]_-^+ n_j = 0$$

Interface "chemical" condition

$$\left[\mu^{ij}\right]_-^+ n_i n_j = 0$$

The conditions at infinity

$$u_i \to \kappa_{ij} x^j \quad at \quad |x^j| \to \infty$$

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2. An Equilibrium Shape… at Small Elastic Displacements ($|\Delta_{ij}| \sim |\kappa_{ij}| << 1$)

Bulk equations: $p^{ij}_{,j} = 0;\quad p^{ij}_{\pm} = c^{ijkl}_{\pm} u_{k,l} + \alpha^{ij}_{\pm}\theta\,.$

The interface clamping condition $\left[u^i\right]_-^+ = \Delta^{ij} x_j,\quad \left[p^{ji}\right]_-^+ n_j = 0$

The interface "chemical" condition:

$$[\eta]_-^+ T = \frac{1}{2}\left[c^{ijkl} u_{i,j} u_{k,l}\right]_-^+ - \left[c^{ijkl} u_{k,l}\left(u_{(i,)}^{\;l} + \Delta_{(i,)}^{\;l}\right)\right]_-^+ n_j n_q$$

The conditions at infinity $u_i \to \kappa_{ij} x^j \quad at \quad |x^j| \to \infty$

The matrix isotropy:

$$c^{ijkl} = \lambda \delta^{ij}\delta^{kl} + \mu\left(\delta^{ik}\delta^{jl} + \delta^{il}\delta^{jk}\right),\quad \alpha^{ij} = \alpha K \delta^{ij}$$

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3. An Equilibrium Ellipsoidal Nuclei…

$$u_i = \kappa_{ij} x^j + \frac{1}{4\pi\mu_+}\tau_{ij}\partial^j \Pi - \frac{1}{16\pi\mu_+(1-\nu_+)}\tau_{jk}\partial_i\partial^j\partial^k\Pi^*$$

The Berdichevsky (1983) anzatz:

$$\tau_{ij} \sim \delta_{ij} \quad \rightarrow u_i = \kappa_{ij} x^j + \frac{\gamma}{4\pi}\Pi_{,i}$$

The system for ω_{ij} , κ_{ij}

$$\begin{cases} \sigma_\infty^{ij} + 2\mu_+ \gamma \left(\delta^{ij} - \frac{1}{4\pi}N^{ij}\right) - c_-^{ijkl}\,\omega_{kl} = 0 \\ \frac{\gamma}{4\pi}N_{ij} - \kappa_{ij} + \omega_{ij} + \Delta_{ij} = 0 \end{cases}$$

4. An Equilibrium Ellipsoidal Nuclei…

(isotropic nucleus, arbitrary "misfit" Δ_{ij})

$$\omega_{(ij)} = \delta_{ij}\left\{\frac{\lambda_+ + 2\mu_+}{3K_- + 4\mu_+}\kappa_k^k + \frac{\mu_+(\lambda_- + 2\mu_-)}{(3K_- + 4\mu_+)(\mu_+ - \mu_-)}\Delta_k^k\right\} - \frac{\mu_+}{\mu_+ - \mu_-}\Delta_{(ij)}$$

$$\omega_{[ij]} = \kappa_{[ij]} - \Delta_{[ij]}$$

$$\frac{N_{ij}}{4\pi} = -\frac{K_- + 4\mu_+/3}{(K_+ - K_-)\kappa_k^k + K_-\Delta_k^k}\left(\kappa_{(ij)} + \frac{\mu_-}{\mu_+ - \mu_-}\Delta_{ij}\right) + \\ + \delta_{ij}\frac{1}{3}\frac{\lambda_+ + 2\mu_+}{(K_+ - K_-)\kappa_k^k + K_-\Delta_k^k}\left(\kappa_k^k + \frac{\mu_-}{\mu_+ - \mu_-}\frac{\lambda_- + 2\mu_-}{\lambda_+ + 2\mu_+}\Delta_{ij}\right)$$

5. An Equilibrium Ellipsoidal Nuclei…

(isotropic nucleus, arbitrary “misfit” Δ_{ij})

Shift in the phase transformation temperature

$$[\eta]_-^+ T = -\frac{\mu_+\mu_-}{\mu_+ - \mu_-}\Delta^{(ij)}\Delta_{(ij)} + \frac{\mu_+(3K_-\mu_- - 2\lambda_-\mu_+)}{(\mu_+ - \mu_-)(3K_- + 4\mu_+)}\Delta_k^k\Delta_l^l + \frac{3K_-(\lambda_+ + 2\mu_+)}{3K_- + 4\mu_+}\kappa_k^k\Delta_l^l + \frac{3(\lambda_+ + 2\mu_+)(K_+ - K_-)}{2(3K_- + 4\mu_+)}\kappa_k^k\kappa_l^l$$

6. An Equilibrium Ellipsoidal Nuclei…

(arbitrary “misfit” Δ_{ij}, no loading at infinity $\kappa_{ij} = 0$)

$$\omega_{(ij)} = \frac{\mu_+}{\mu_+ - \mu_-}\left(\delta_{ij}\frac{\lambda_- + 2\mu_-}{3K_- + 4\mu_+}\Delta_k^k - \Delta_{(ij)}\right)$$

$$\frac{N_{ij}}{4\pi} = \frac{\mu_+}{\mu_+ - \mu_-}\left(\delta_{ij}\frac{\lambda_- + 2\mu_-}{3K_-} - \frac{3K_- + 4\mu_+}{3K_-}\frac{\mu_-}{\mu_+}\frac{1}{\Delta_k^k}\Delta_{ij}\right)$$

$$[\eta]_-^+ T = -\frac{\mu_+\mu_-}{\mu_+ - \mu_-}\Delta^{(ij)}\Delta_{(ij)} + \frac{\mu_+(3K_-\mu_- - 2\lambda_-\mu_+)}{(\mu_+ - \mu_-)(3K_- + 4\mu_+)}\Delta_k^k\Delta_l^l$$

7. An Equilibrium Ellipsoidal Nuclei…

(volumetric "misfit"$\Delta_{ij} = \delta\delta_{ij}$)

$$\omega_{(ij)} = \frac{\mu_+}{\mu_+ - \mu_-}\left(\delta_{ij}\frac{\lambda_- + 2\mu_-}{3K_- + 4\mu_+}\Delta_k^k - \Delta_{(ij)}\right)$$

$$N_{ij} = \delta_{ij}\frac{\lambda_- + 2\mu_-}{3K_-}\frac{\mu_-}{\mu_+ - \mu_-} - \frac{4\pi(K_- + 4\mu_+/3)}{3K_-}\frac{\mu_-}{\mu_+ - \mu_-}$$

Volumetric misfit: the Lifshitz-Gulida formule

$$[\eta]_-^+ T = -\frac{18(\lambda_+ + 2\mu_+)(K_+ - K_-)}{3K_- + 4\mu_+}\delta^2 - \frac{9K_-(\lambda_+ + 2\mu_+)}{3K_- + 4\mu_+}\delta\kappa_k^k + \frac{3(K_+ - K_-)(\lambda_+ + 2\mu_+)}{2(3K_- + 4\mu_+)}\kappa_k^k\kappa_l^l$$

8. An Equilibrium Ellipsoidal Nuclei…

(existence of the solution)

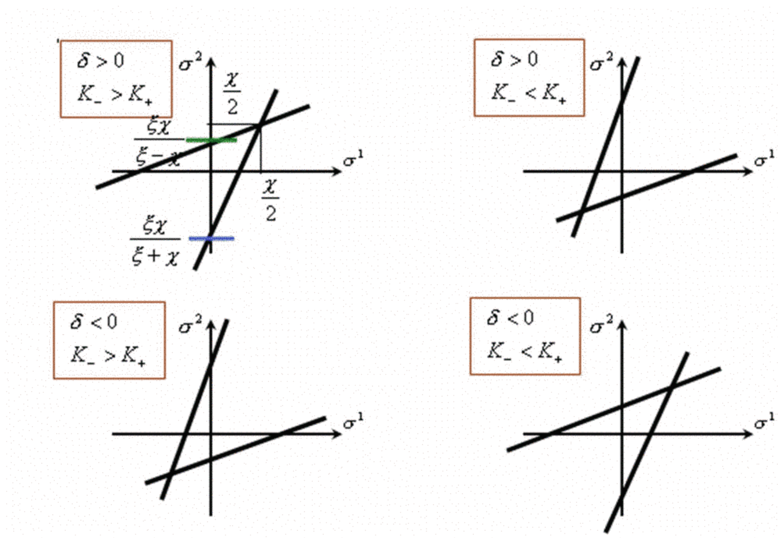
Necessary conditions: $N_{II} > 0 \quad for \quad I = 1, 2, 3; \; N^I{}_I = 4\pi$

$$\frac{6\mu_+K_+K_-\delta + (K_-\lambda_+ + 2\mu_+K_+)\sigma^k_{.k\infty}}{(K_+ - K_-)\sigma^k_{.k\infty} + 9K_+K_-\delta} > \frac{(3K_- + 4\mu_+)K_+}{(K_+ - K_-)\sigma^k_{.k\infty} + 9K_+K_-\delta}\sigma^{II}_\infty$$

$$\begin{cases} \frac{K_-\lambda_+ + 2\mu_+(K_+ + K_-)}{K_-\lambda_+ + 2\mu_+K_+}\sigma^{11} - \sigma^{22} - \sigma^{33} \begin{array}{c}\geq\\ \leq\end{array} \frac{6\mu_+K_+K_-\delta}{K_-\lambda_+ + 2\mu_+K}, \\ -\sigma^{11} - \frac{K_-\lambda_+ + 2\mu_+(K_+ + K_-)}{K_-\lambda_+ + 2\mu_+K_+}\sigma^{22} - \sigma^{33} \begin{array}{c}\geq\\ \leq\end{array} \frac{6\mu_+K_+K_-\delta}{K_-\lambda_+ + 2\mu_+K}, \\ -\sigma^{11} - \sigma^{22} - \frac{K_-\lambda_+ + 2\mu_+(K_+ + K_-)}{K_-\lambda_+ + 2\mu_+K_+}\sigma^{33} \begin{array}{c}\geq\\ \leq\end{array} \frac{6\mu_+K_+K_-\delta}{K_-\lambda_+ + 2\mu_+K} \end{cases}$$

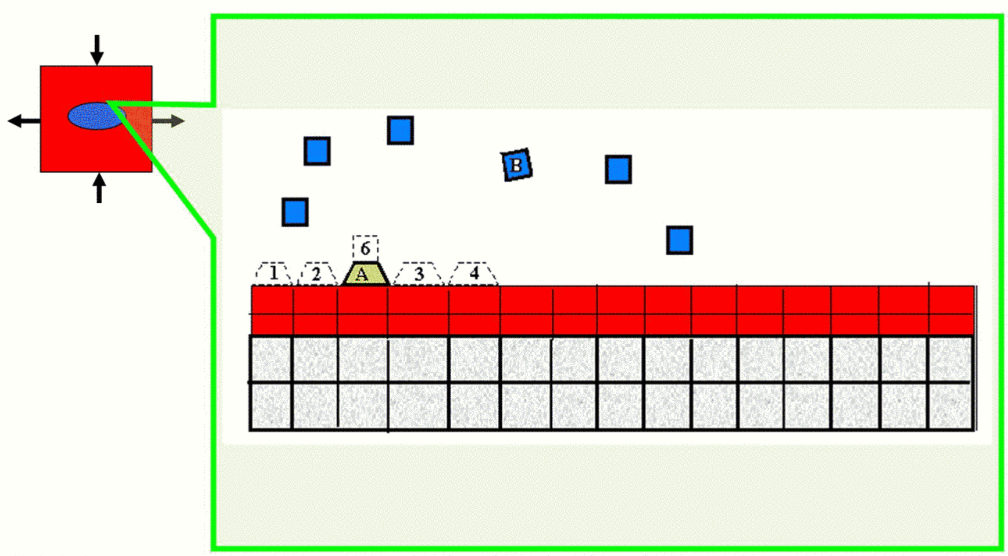
9. An Equilibrium Ellipsoidal Nuclei…

(existence of the solution)



Is the ellipsoidal nucleus stable ?

The morphological instability "stressed crystal – melt"



Nucleation of Solid Phases: Open Questions

- Is the ellipsoidal solution unique?
- Is the ellipsoidal solution unique in the class of finite inclusions?
- Is the ellipsoidal solution unique in the class of ellipsoids?
- Are there any bifurcations in the system?
- Can 2 (N) compact phase inclusions co-exist in stable equilibrium?
- Are there any elementary 3D solutions for incoherent phase nuclei?
- What are the influences of surface energy, gravity, finite size of the system, unharmonicity?
- How the describe quasi-statics, dynamics?
- Et cetera ad infinitum…