Earthquake Models that Account for Dynamic Weakening Mechanisms

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Emphasis of this work: ***Combining rate and state friction with strong dynamic weakening due to shear heating mechanisms***

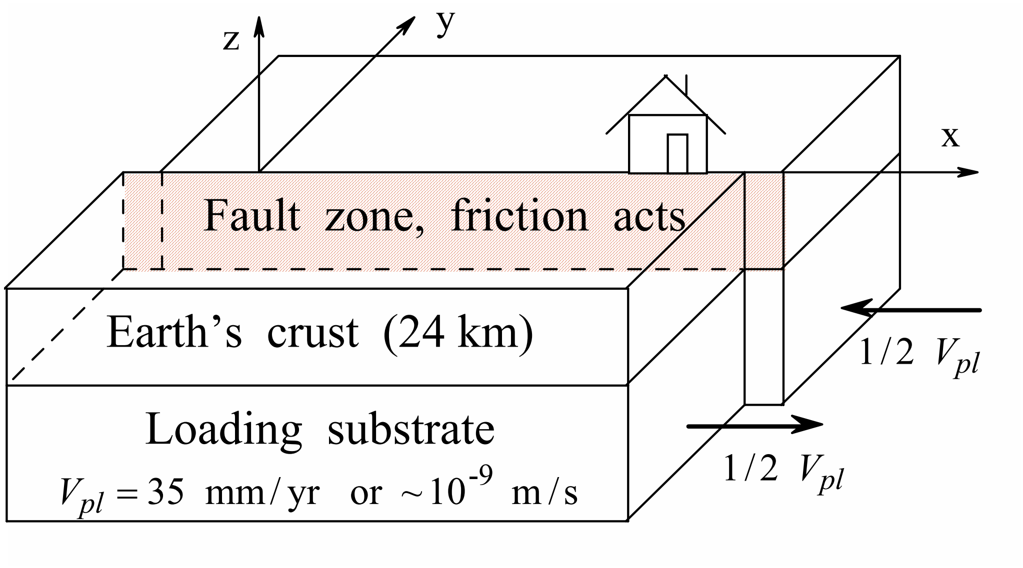
Observations we would like to match:

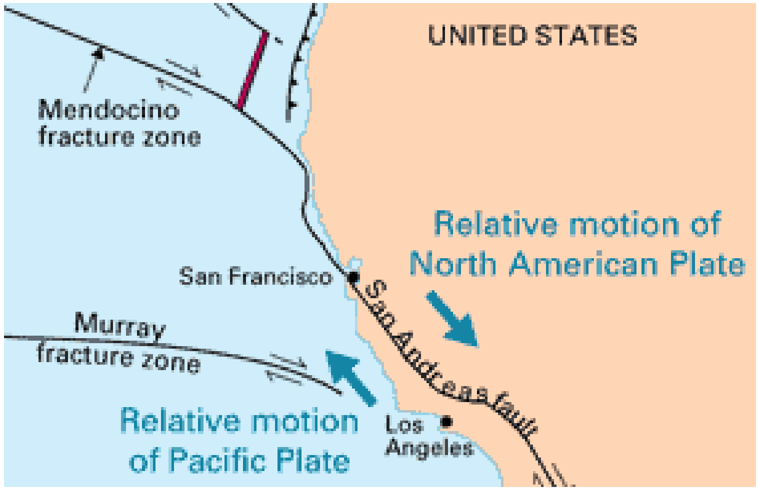
Laboratory reports of large friction coefficients in slow sliding
Theoretical and experimental evidence of dynamic weakening
Relatively small static stress drops (1-10 MPa)
Low-heat, low-stress operation of some major faults

We simulate sequences, and not just a single event

That allows us to produce and study earthquakes with features that result from the physics and geometry of the problem rather than initial conditions.

Model of a vertical strike-slip fault





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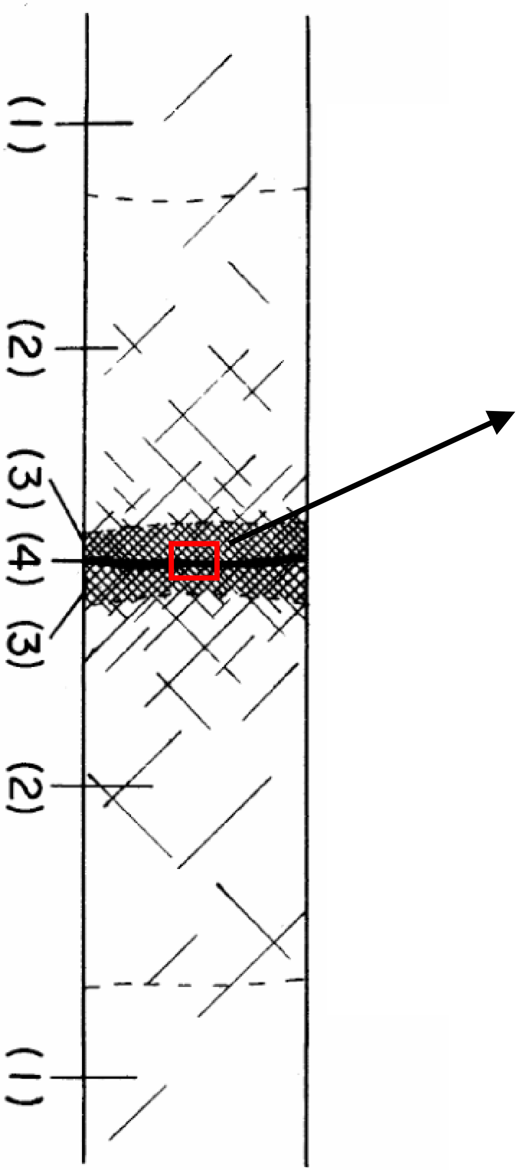
2D depth-variable model
Variations with z and y only,
no variation with x (Rice, 1993)

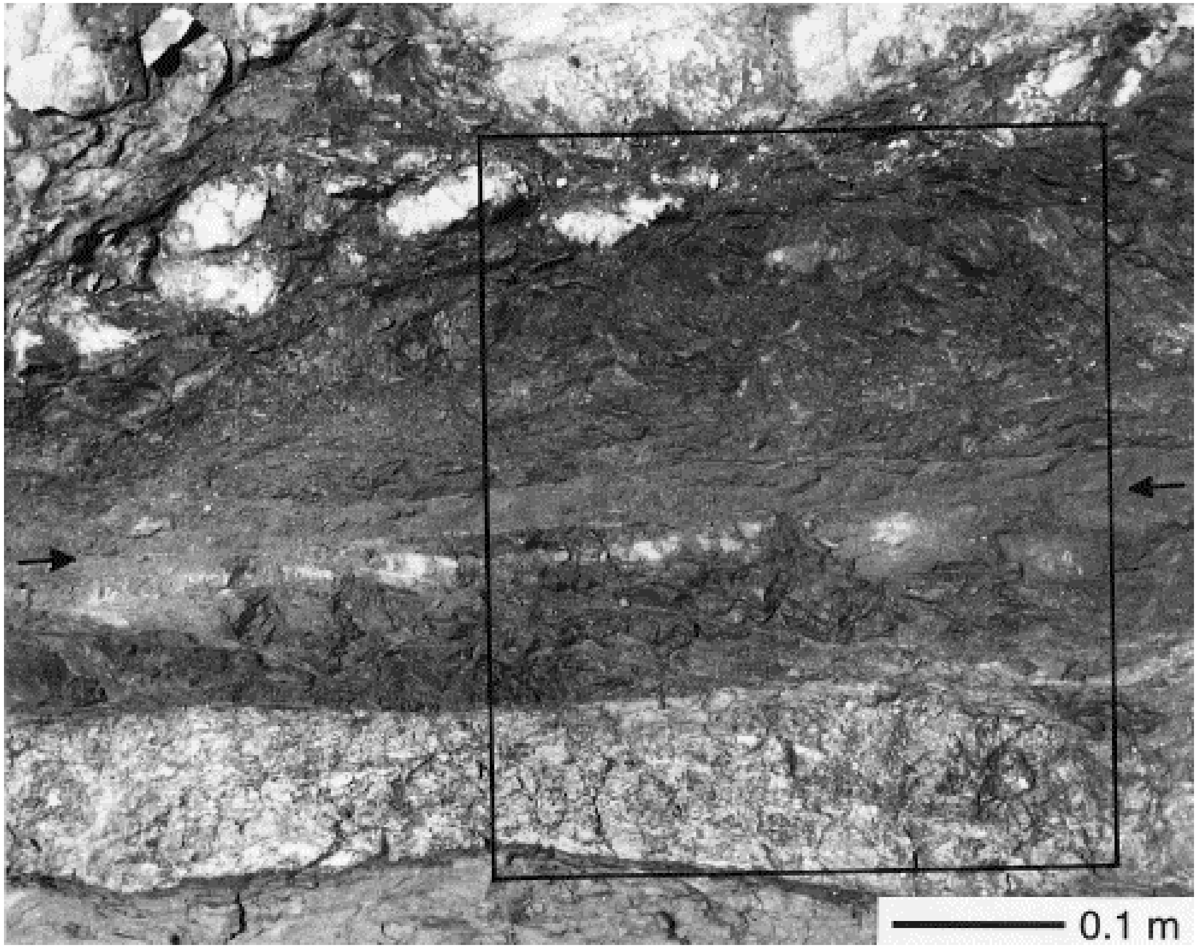
2D crustal plane model
Variations with x and y only,
depth-averaged in z
(similar to Myers et al., 1996)

u	displacement in x direction
$\delta = u_{\vert y=0^+} - u_{\vert y=0^-}$	slip on the interface y = 0
$V = \partial\delta / \partial t$	slip velocity (or slip rate)
$\sigma = -\sigma_{yy} > 0$	compressive normal stress
$\bar{\sigma} = \sigma - p > 0$	effective normal stress
$\tau = \bar{\sigma} f(\delta, V, \theta, ...)$	friction law on the interface

How faults look at depth: Studies of exhumed faults

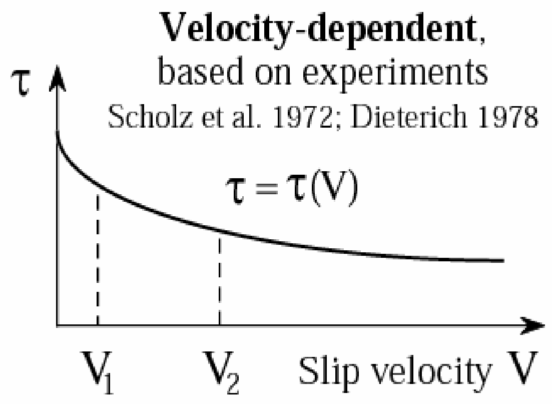


Chester et al., 1993



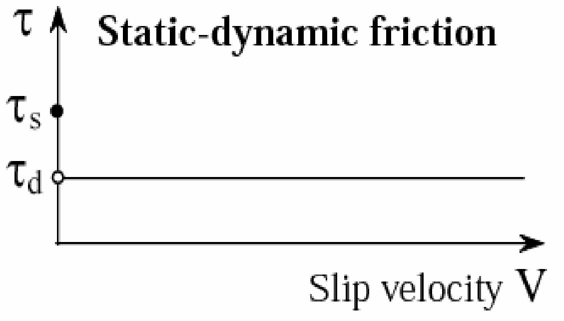
Chester and Chester, 1998: **cm-wide** highly sheared zone;

prominent slip surface of < 1 - 5 mm,
composed of ***micron***- and ***nanometer***-sized particles
(highly compressed granular material or fault gouge)

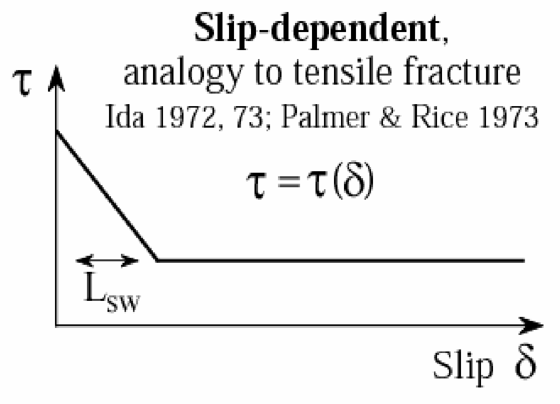


Velocity-dependent, based on experiments

Scholz et al. 1972; Dieterich 1978



Static-dynamic friction

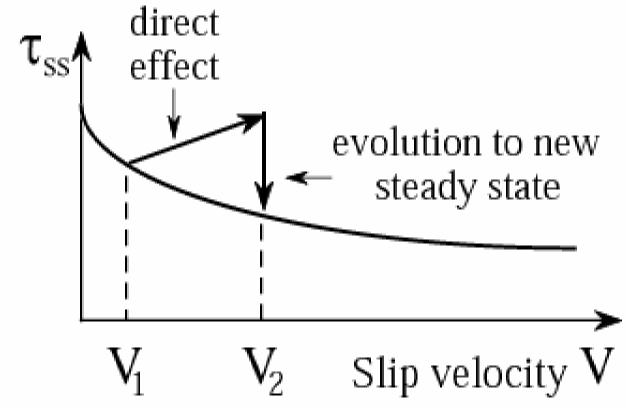


Slip-dependent, analogy to tensile fracture

Ida 1972, 73; Palmer & Rice 1973

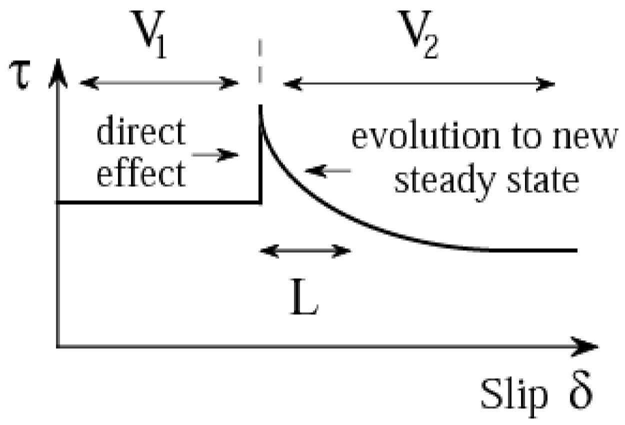
Rate- and state-dependent friction

Velocity-dependent in steady state



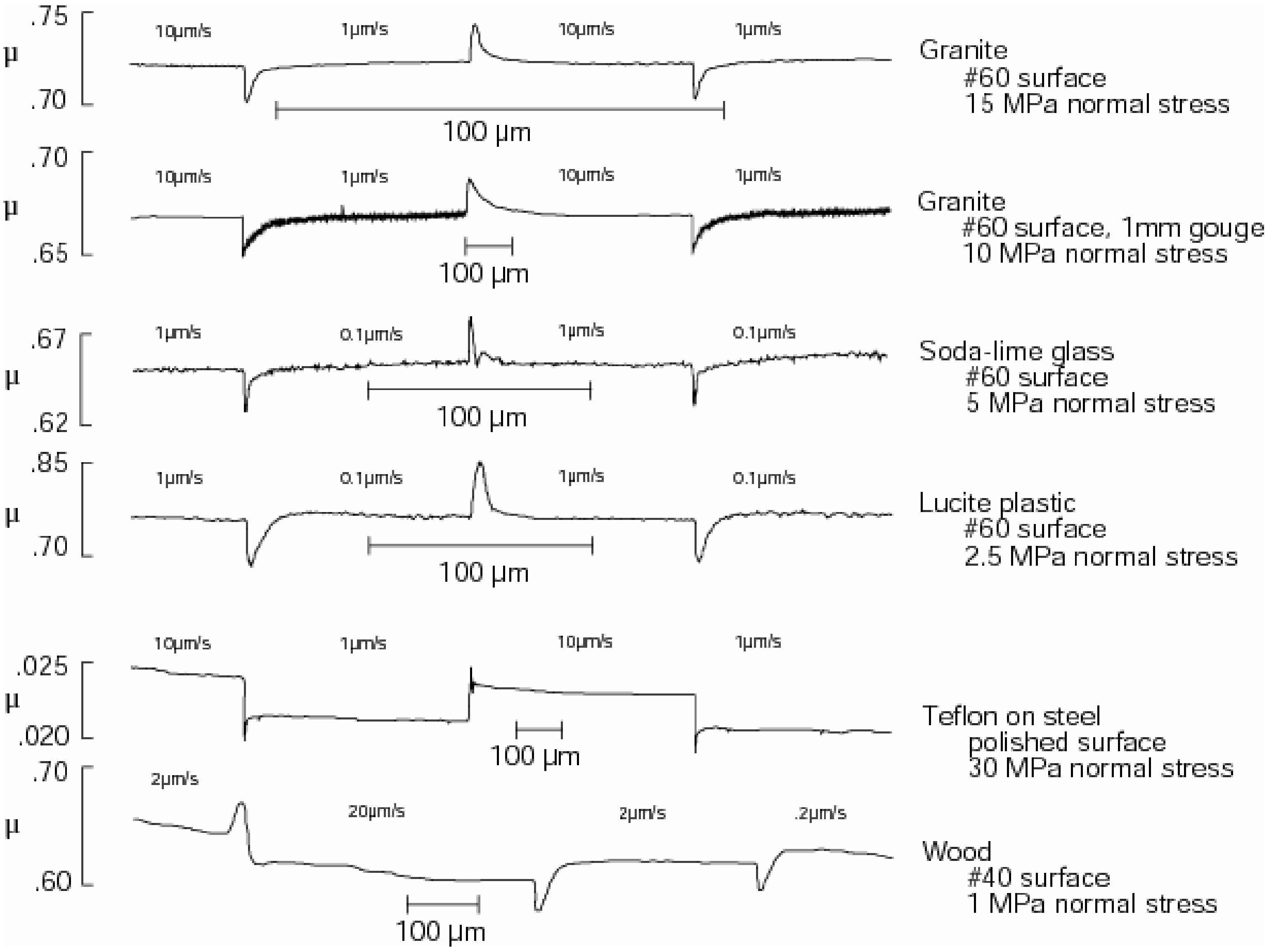
More detailed experiments

Dieterich 1979, 81; Ruina 1980, 83



$$\tau = \bar{\sigma} f = \bar{\sigma}\left(f_o + a \ln \frac{V}{V_o} + b \ln \frac{V_o \theta}{L} \right); \quad \frac{\mathrm{d}\theta}{\mathrm{d}t} = 1 - \frac{V\theta}{L}$$

Rate and state features are observed in sliding of many different materials (Figure from Dieterich and Kilgore, 1994)



Rate and state friction, Dieterich-Ruina formulation

$$\tau = \bar{\sigma} f = \bar{\sigma}\left(f_o + a \ln\frac{V}{V_o} + b \ln\frac{V_o \theta}{L} \right); \quad \frac{\mathrm{d}\theta}{\mathrm{d}t} = 1 - \frac{V\theta}{L}$$

State variable θ measures the lifetime and maturity of frictional contacts;

L is the **characteristic slip distance** for evolution of θ.

Restrengthening in stationary contact

Lab: When there is no sliding,
strength grows with ln t.

Law: $\mathrm{d}\theta/\mathrm{d}t = 1$ for $V = 0$.

Steady state (V = constant)

$\theta \rightarrow \theta_{\mathrm{ss}} = L / V$

$\tau \rightarrow \tau_{\mathrm{ss}} = \bar{\sigma}[f_o + (a-b)\ln(V/V_o)]$

$a - b < 0 \ \Rightarrow$ steady-state vel. weakening

$a - b > 0 \ \Rightarrow$ steady-state vel. strengthening

⇒ Connection to a rate-dependent law!

Behavior at the crack tip

Tensile fracture: opening-dependent cohesion law, notion of fracture energy.

dV/dt large, $V\theta/L \gg 1$, $\theta = Ce^{-\delta/L}$

$\tau = \bar{\sigma}\left(\mathrm{constant} + a \ln V/V_o - b\delta/L\right)$

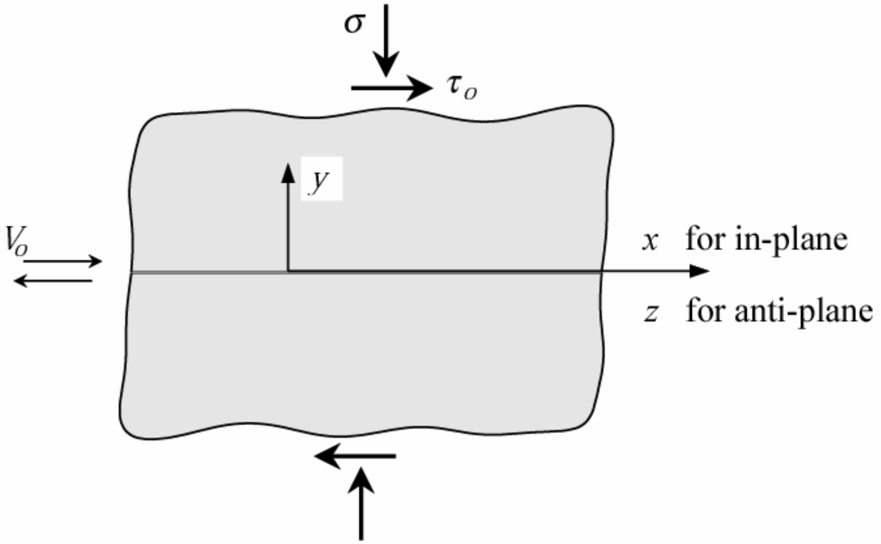
⇒ Connection to a slip-weakening law!

Lab values: Base friction $f_o = 0.6$ at $V_o = 1$ μm/s

Variations for ***small*** slip velocities $a = 0.015$, $b = 0.019$, $L = 1\text{-}100$ μm

Linearized Stability Analysis of Steady Frictional Sliding

(Rice and Ruina, 1983; Rice, Lapusta, Ranjith, 2001)



Look for elastodynamic solutions in the Fourier mode form:

$$u(x,y,t) = 1/2\,\mathrm{sgn}(y)V_o t + U(y)\exp[ikx + pt]$$

↑ steady sliding ↑ perturbation

Equations to find values of ***p*** **(behavior of the perturbation in time)**

Shear stress on the interface y = 0 = **Frictional strength** on the interface y = 0

(Solve elastodynamic eqns in the half-spaces)

(Get from the *linearized* friction law)

Stability Properties of Rate and State Friction

Steady-state velocity strengthening $a - b > 0$

⇒ **Sliding is stable** to perturbations of any wavelengths

Steady-state velocity weakening $a - b < 0$

$\lambda < \lambda_{cr}$	$\lambda = \lambda_{cr}$	$\lambda > \lambda_{cr}$
Stable sliding	Neutrally stable sliding	**Unstable sliding**

Anti-plane elasticity

$$\lambda_{\mathrm{cr}} = \pi \frac{\mu L}{\bar{\sigma}(b-a)} \frac{1}{\sqrt{1+q^2}}, \quad q = \frac{\mu V_o}{2\bar{\sigma} c_s \sqrt{a(b-a)}}$$

Quasi-static estimate

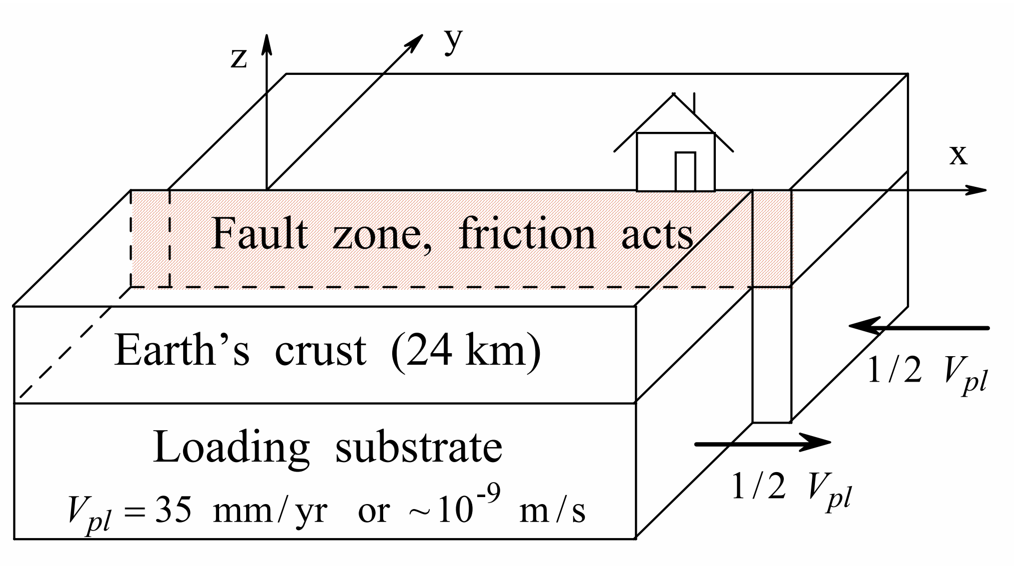
$$L = 1 - 10\ \mu\mathrm{m} \Rightarrow \lambda_{cr} = 0.25 - 2.5\ \mathrm{m}$$

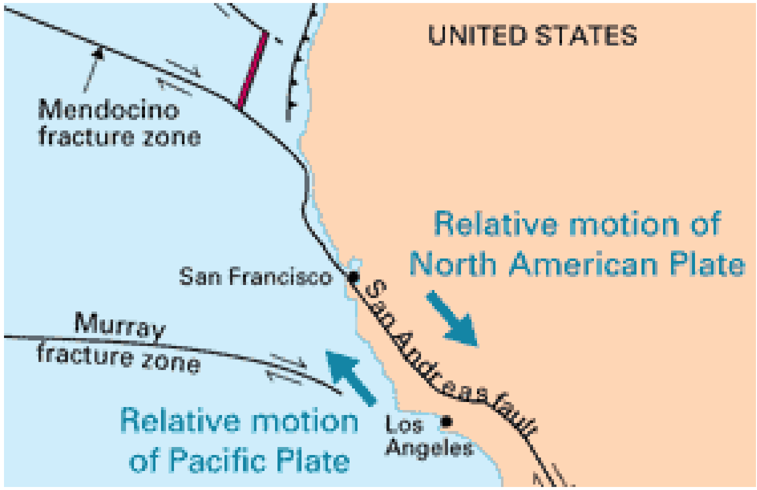
$$(\text{for } \bar{\sigma} = 100\ \mathrm{MPa},\ b - a = 0.004,\ \mu = 30\ \mathrm{GPa})$$

Sizes of smallest earthquakes (*instabilities*) observed are of order 1 m.

Lab-derived values of *L of order microns*
seem to be relevant to real faults *during nucleation*.

Example of modeling with rate and state law





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2D depth-variable model

Variations with z and y only,
no variation with x (Rice, 1993)

Goal: To simulate ***spontaneous*** slip accumulation on the interface by solving the system

Shear traction on the interface =

Friction strength of the interface

Single planar interface,

Inertial effects in surrounding elastic media,

Slow tectonic-type loading (35 mm/year),

Non-linear friction laws.

Main challenge in simulations of earthquake sequences: Even this simplified problem is *multiscale* in nature

Multiple scales in time (dynamic cracks + slow loading)

Loading time	100-1000 years or 10^9-10^{10} seconds
Duration of dynamic event	10-100 seconds
Rapid changes of variables at the crack tip	fraction of a second

Multiple scales in space

Fault dimensions	100 km = 10^5 meters
Nucleation size on faults	1-10 meters (L ~ 10-100 μm)
Rapid changes of variables at the rupture tip	fraction of a meter

Modeling Methodology

(Rice and Ben-Zion, 1996, ..., Lapusta et al., 2000)

Boundary integral method

$$\tau(x,t) = \tau^o(x,t) + f(x,t) - \mu V/(2c_s)$$

Stress on the interface	=	Stress in the absence of the interface	+	**Stress transfer functional**	–	Radiation damping term

Spectral form of the stress transfer functional = static + dynamic part

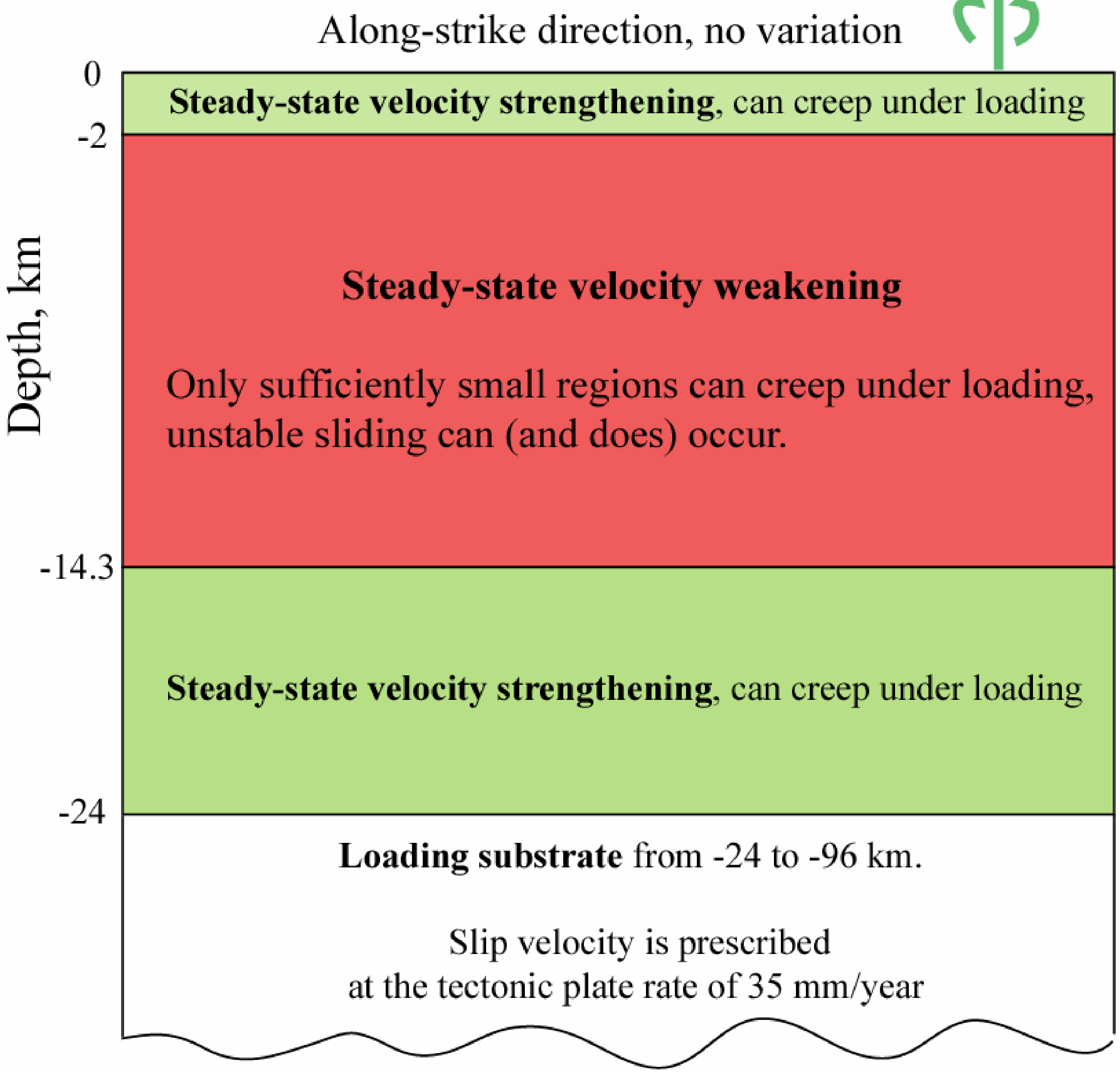
Time convolutions in the dynamic part are truncated

$$\int_0^t \Rightarrow \int_{t-T_w}^t$$

Variable time stepping

$$\text{Time step} = \frac{\text{Coefficients dependent on frictional parameters and grid size}}{\text{Slip velocity}}$$

Frictional properties on the fault



Along-strike direction, no variation

Steady-state velocity strengthening, can creep under loading

Steady-state velocity weakening

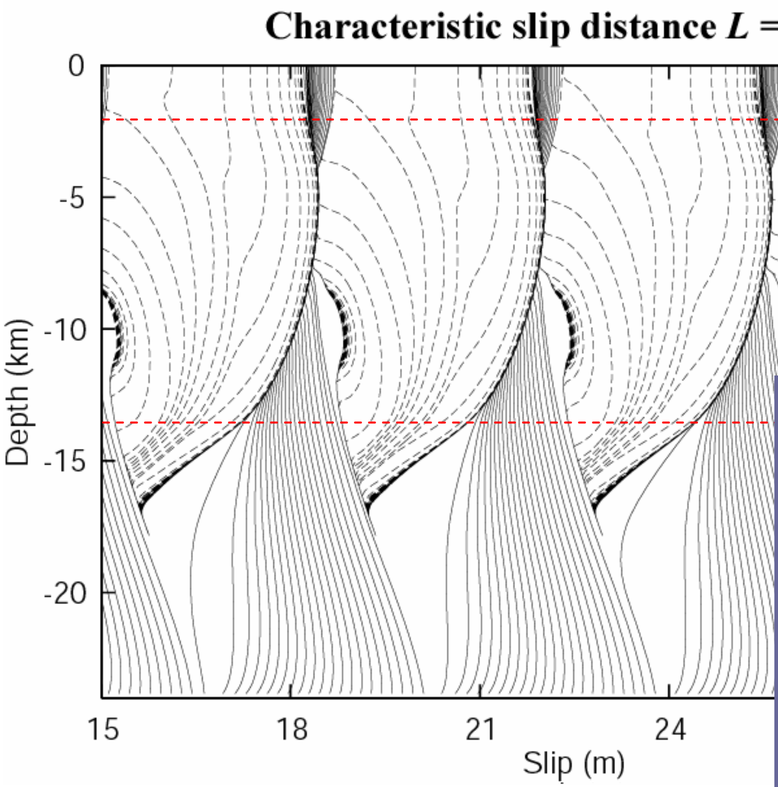
Only sufficiently small regions can creep under loading, unstable sliding can (and does) occur.

Steady-state velocity strengthening, can creep under loading

Loading substrate from -24 to -96 km.

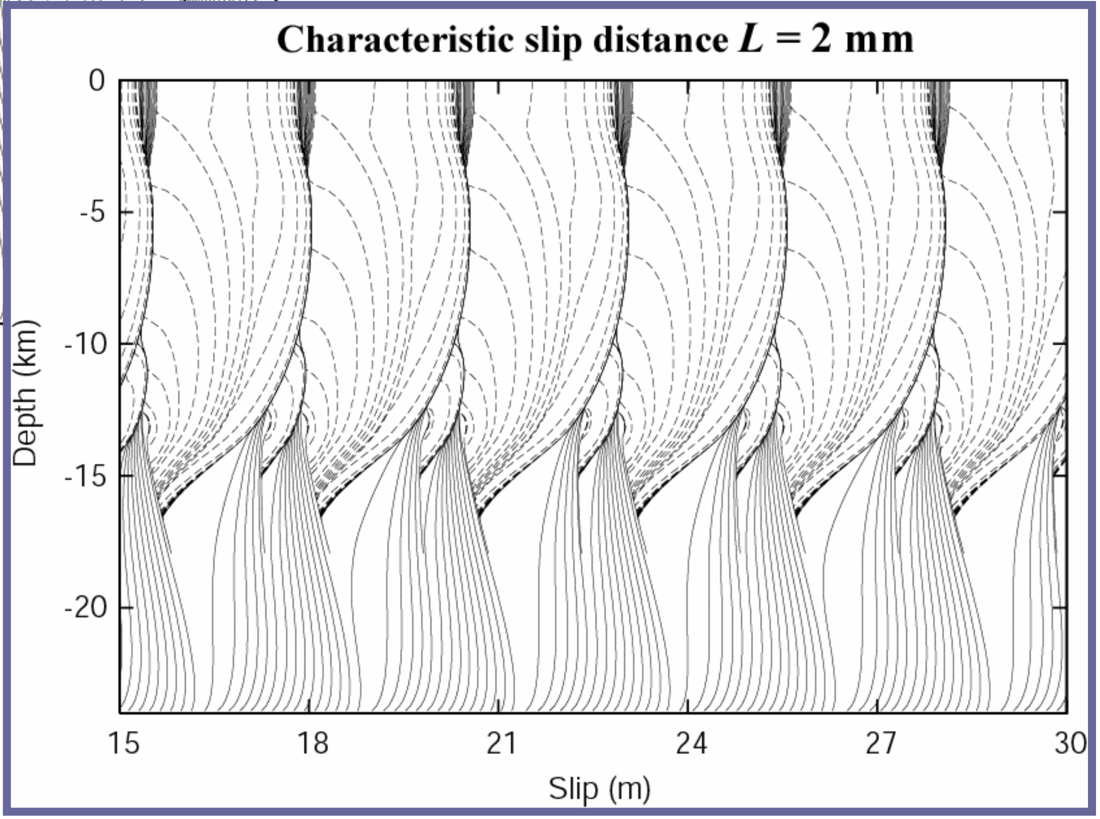
Slip velocity is prescribed
at the tectonic plate rate of 35 mm/year

Spontaneous accumulation of slip, long-term simulations



Solid lines are plotted
every 5 years.

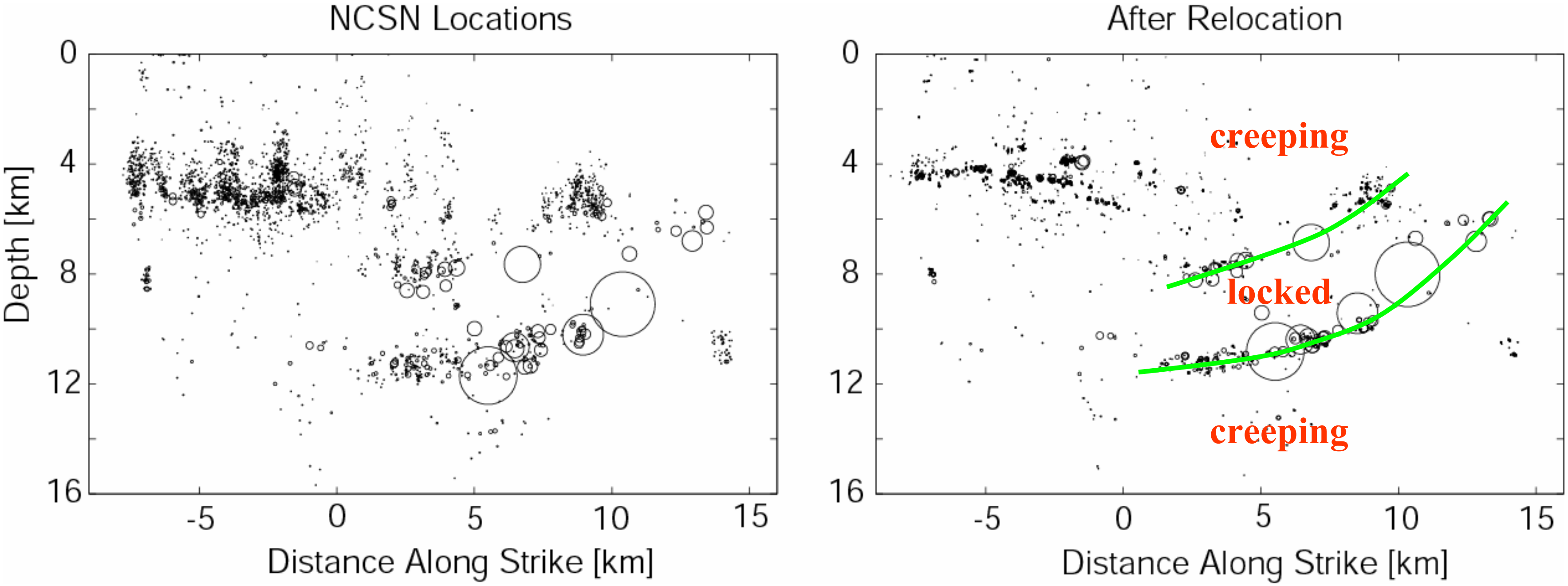
Dashed lines are plotted
every second
when slip velocities > 0.001 m/s.



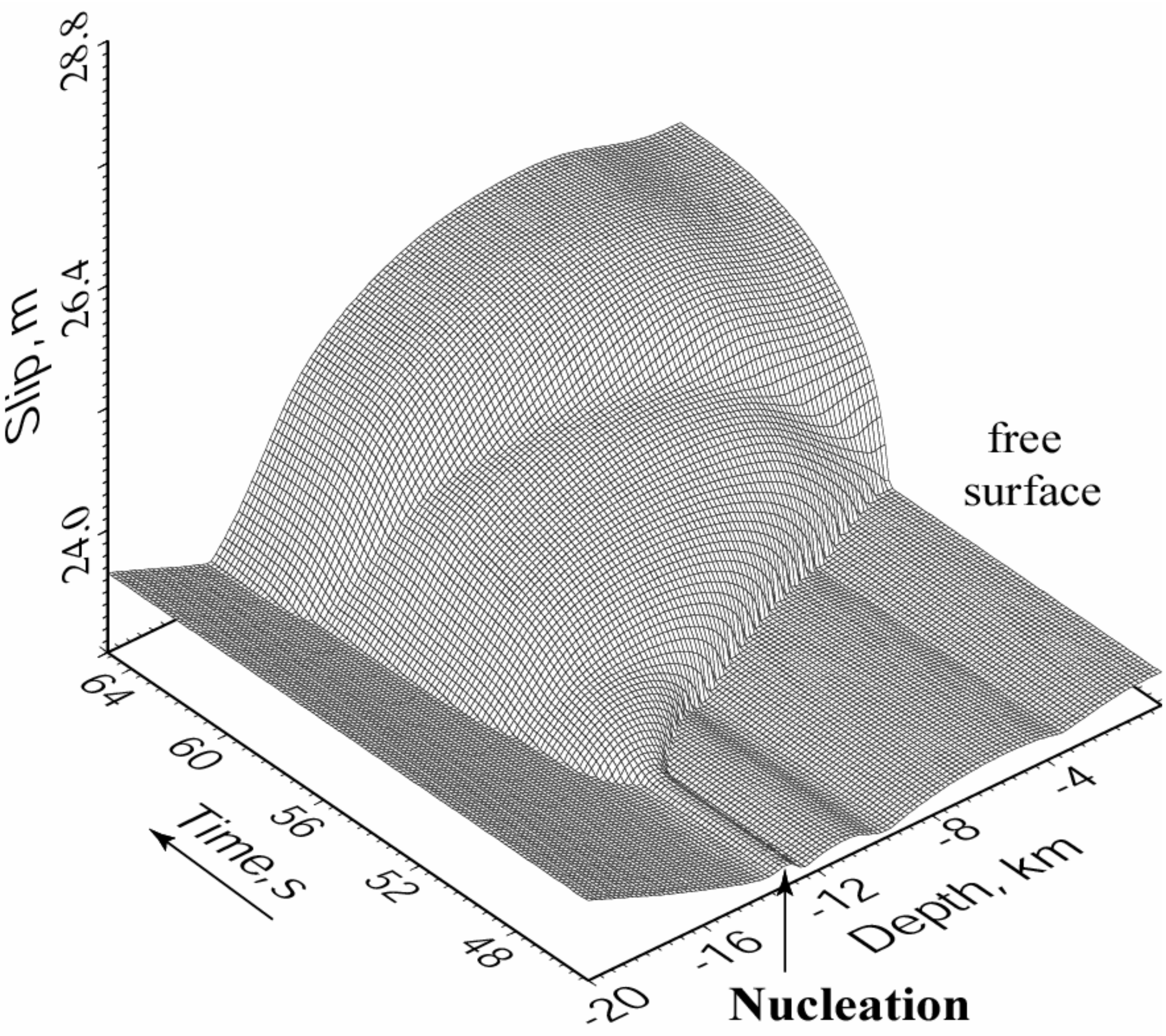
For smaller, more realistic *L*, smaller events appear close to transition between creeping and locked behavior.

Seismicity in the Parkfield region (1984-1999), small events cluster at transitions

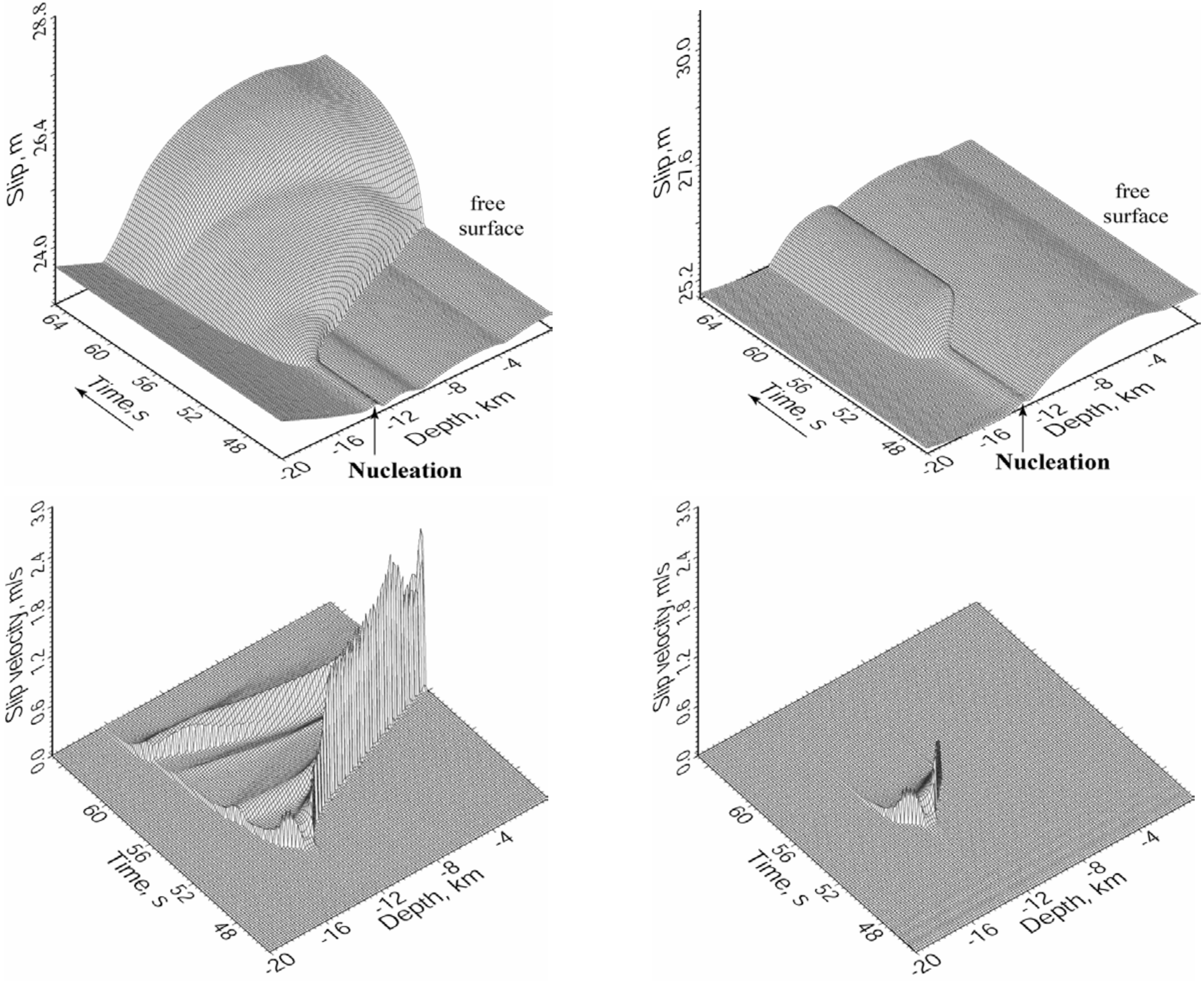
Ellsworth et al., 2000



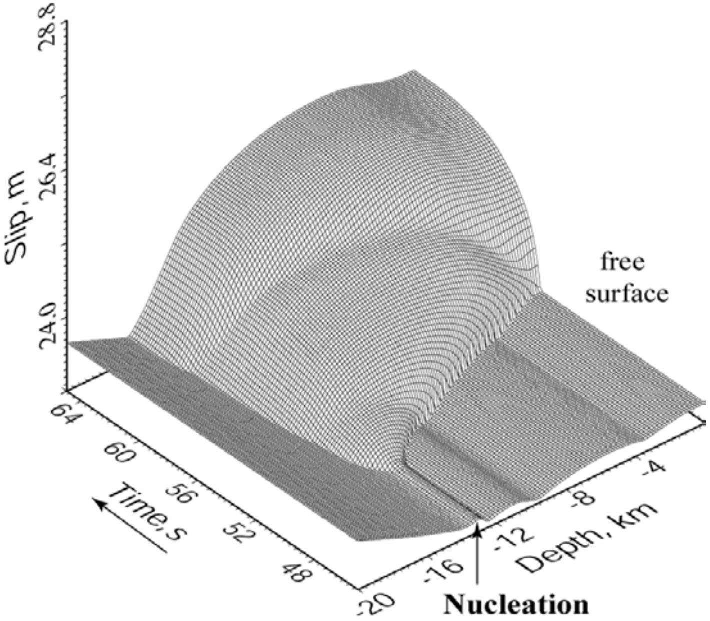
Slip in one “large” event

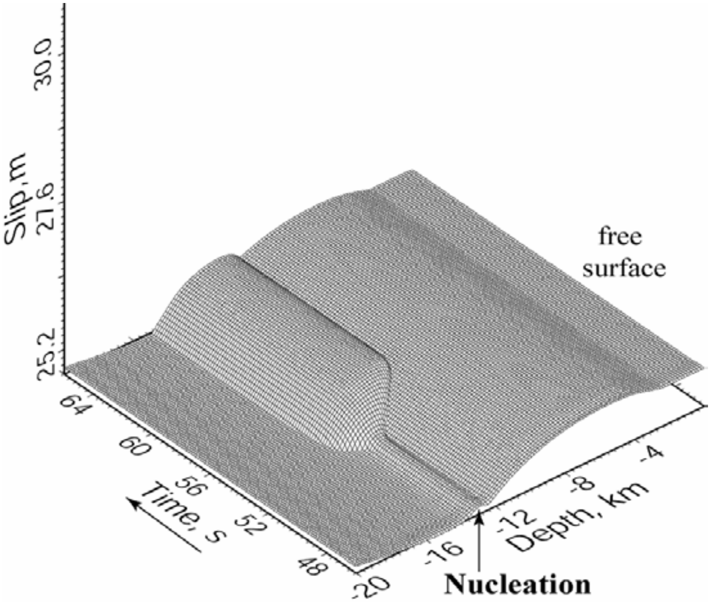


Slip and slip velocity in a “large” event and a “small” event



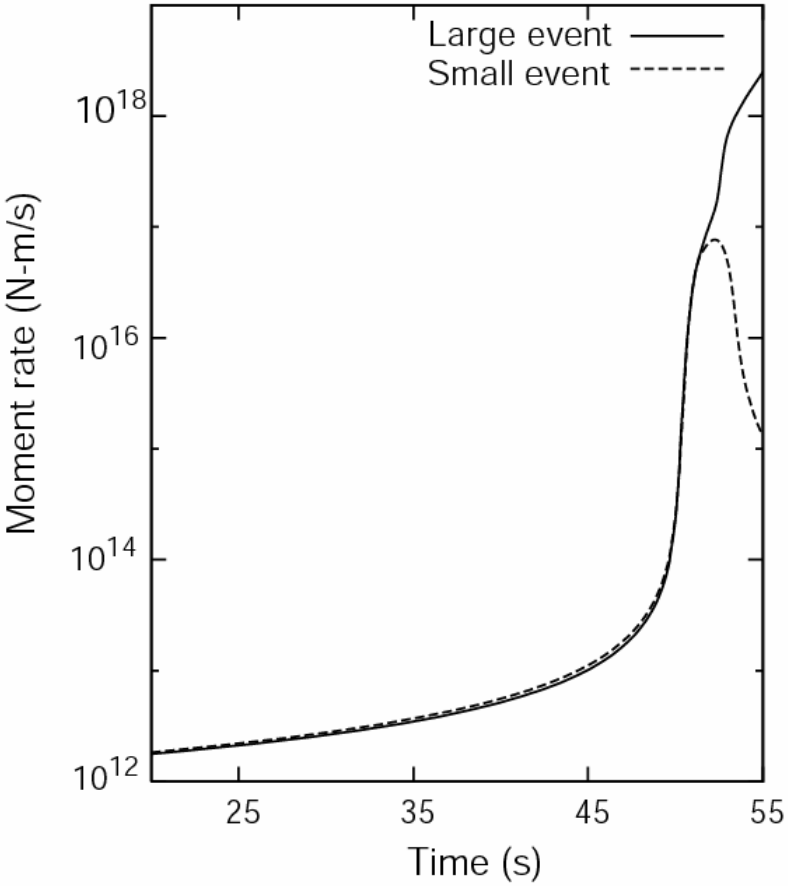
Is nucleation of small and large events different?





Identical signal during nucleation and beginning of seismic propagation

Moment rate $\dot{M}_0(t) = \mu \iint V(x,z,t)\,\mathrm{d}x\,\mathrm{d}z$



Rate and state friction is the backbone of our constitutive relation

Laboratory-derived (Dieterich, 1979, 1981; Ruina, 1980, 1983; ...) **for slip velocities small (~ 10^{-6} – 10^{-3} m/s) compared to the seismic range**.

Unique tool for simulating earthquake sequences in their entirety,

from accelerating slip in slowly expanding nucleation zones

to rapid dynamic propagation of earthquake rupture

to post-seismic slip and interseismic creep

to fault restrengthening between seismic events.

$$\tau = \bar{\sigma} f = (\sigma - p) f = \bar{\sigma}\left(f_o + a \ln \frac{V}{V_o} + b \ln \frac{V_o \theta}{L} \right); \quad \frac{d\theta}{dt} = 1 - \frac{V\theta}{L}$$

Base friction f_o = 0.6 at V_o = 1 mm/s

Variations a = 0.015, b = 0.019, L = 1-100 mm (lab values), L = 0.14-40 mm here

Actual constitutive laws we need to use:

Rate and state friction

combined with dynamic weakening mechanisms

Why would faults be dynamically much weaker?

Several mechanisms, most of them due to ***shear heating***

Flash heating of contact asperities at small slips (Bowden and Thomas, 1954, Lim and Ashby, 1987, Molinary et al., 1999, Rice, 1999; Beeler and Tullis, 2003)

Behavior of partially drained, thermally pressurized fault gouge, and perhaps partially melted liquefied gouge, at larger slips

(Jacques, Rempel, Rice, 2002-2004)

$\Rightarrow$ **LAW 1: Strong weakening with seismic slip velocities *V***

$$\tau_{ss} = (\sigma - p_{amb}) \frac{\text{friction coef-t from rate and state}}{\mathbf{1} + \boldsymbol{V/V_w}}$$

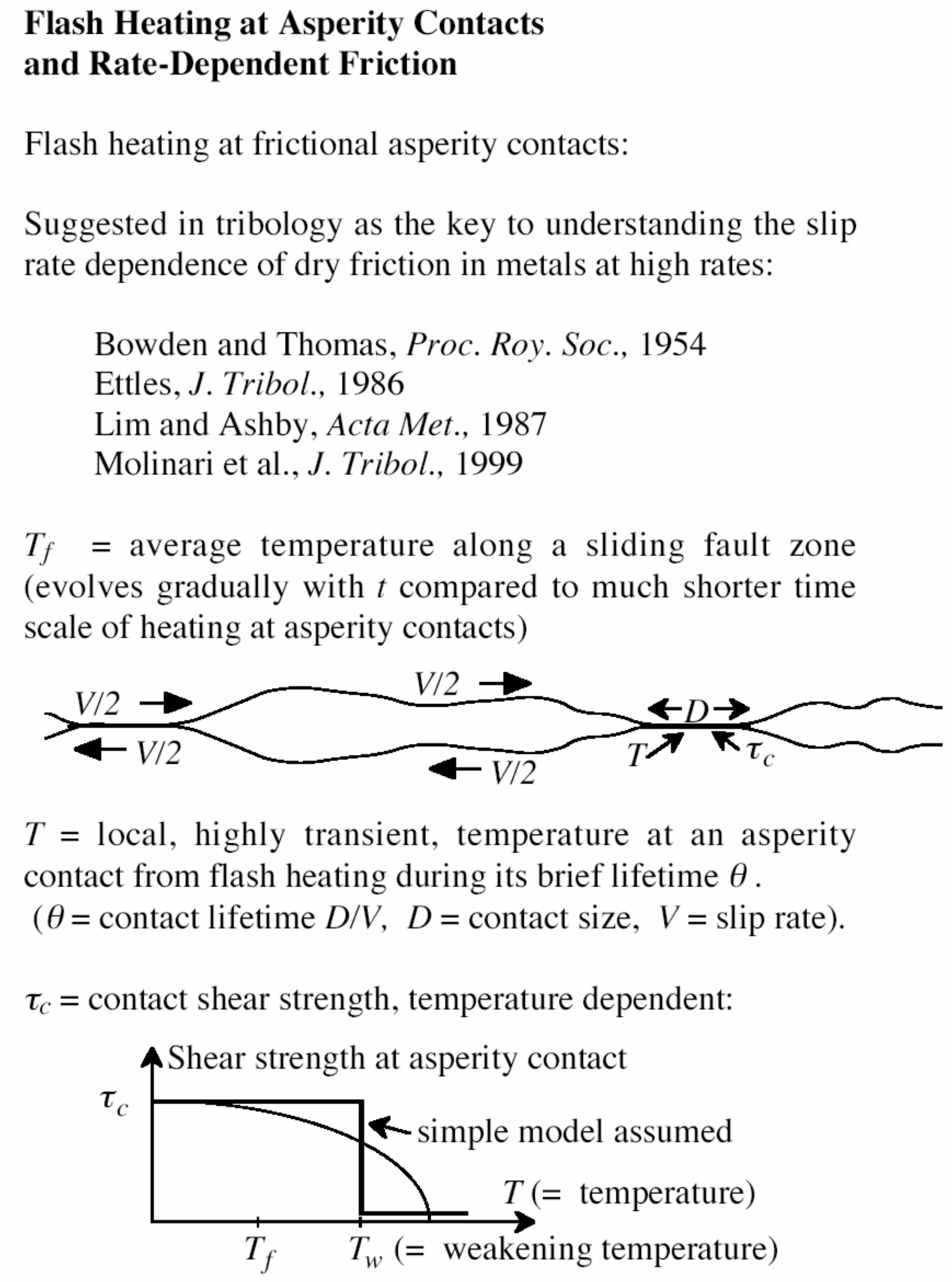
$$V \ll V_w \;\Rightarrow\; 1 + V/V_w \approx 1 \;\Rightarrow\; \text{rate and state friction}$$

$$V \gg V_w \;\Rightarrow\; 1 + V/V_w \approx V/V_w \Rightarrow\; \text{strong dynamic weakening}$$

From theory and experiments: $V_w \sim 0.1 - 1$ m/s

Frictional weakening by flash heating

(Rice, 1999)



Flash Heating at Asperity Contacts and Rate-Dependent Friction

Flash heating at frictional asperity contacts:

Suggested in tribology as the key to understanding the slip rate dependence of dry friction in metals at high rates:

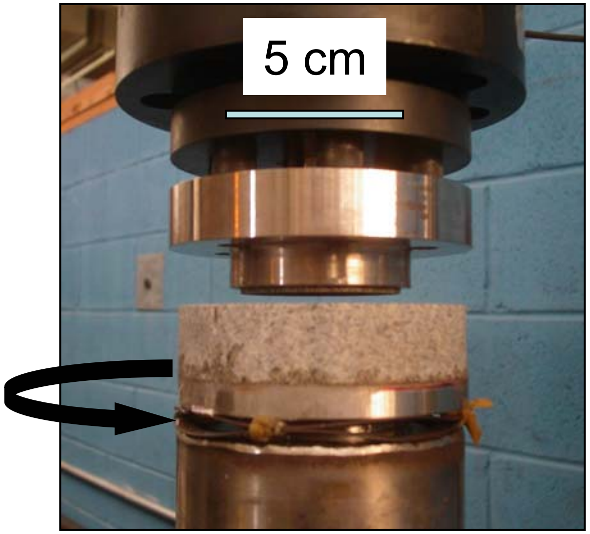
Bowden and Thomas, *Proc. Roy. Soc.*, 1954
Ettles, *J. Tribol.*, 1986
Lim and Ashby, *Acta Met.*, 1987
Molinari et al., *J. Tribol.*, 1999

T_f = average temperature along a sliding fault zone (evolves gradually with t compared to much shorter time scale of heating at asperity contacts)

T = local, highly transient, temperature at an asperity contact from flash heating during its brief lifetime θ.
(θ = contact lifetime D/V, D = contact size, V = slip rate).

τ_c = contact shear strength, temperature dependent:

Rotary Shear Apparatus

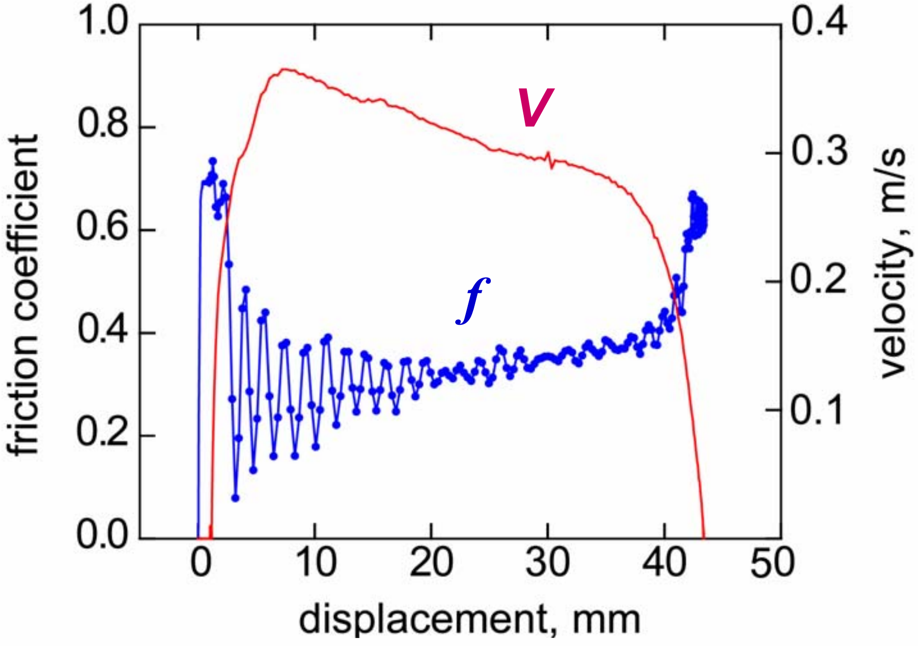


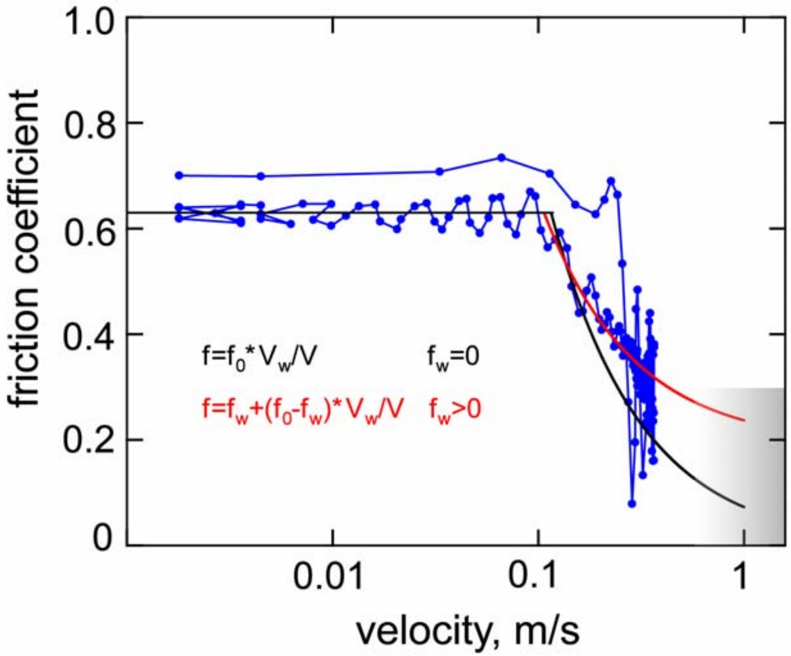
High speed *V* ≤ 0.38 m/s

$\sigma_n = 5$ MPa

Quartz

From Tullis and Goldsby, 2003

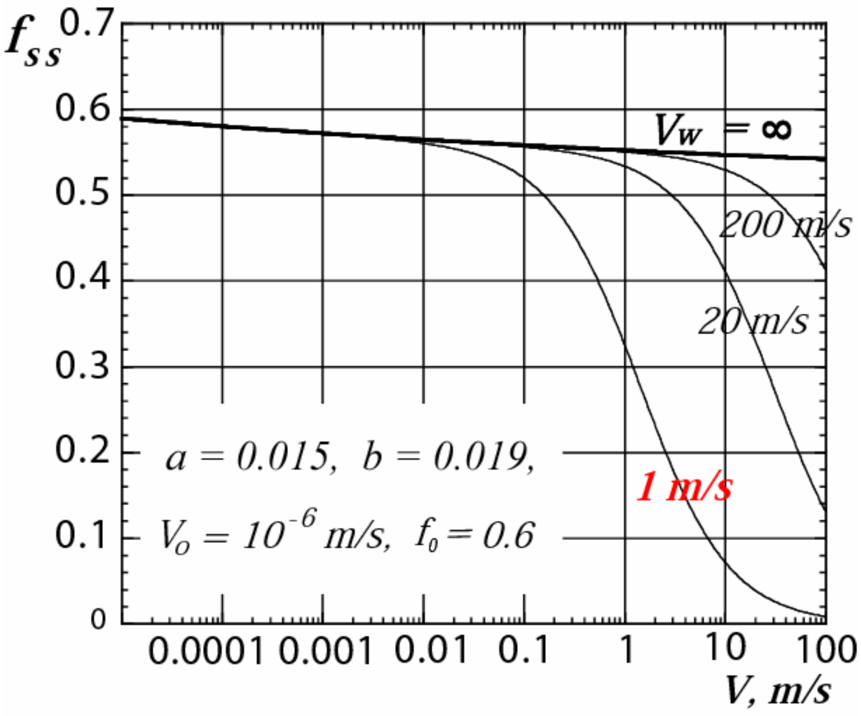


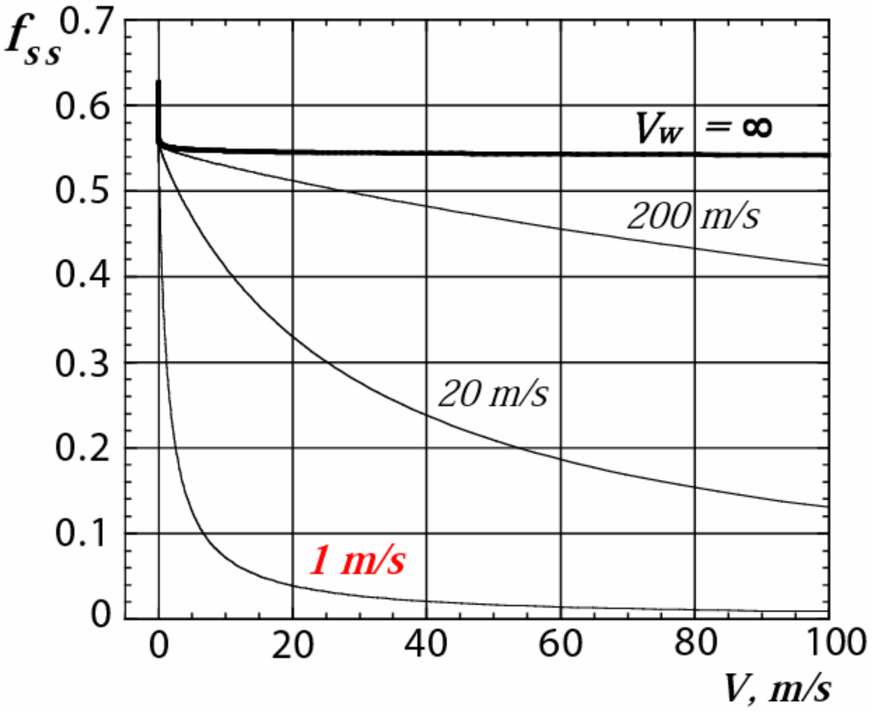


LAW 1: Strong weakening of friction with seismic slip velocities *V*

Our law in steady state $$\tau_{ss} = \bar{\sigma} f_{ss} = \bar{\sigma} \frac{f_o + (a-b)\ln(V/V_o)}{\mathbf{1+V/V_w}}; \quad \bar{\sigma} = (\sigma - p_{amb})$$

Actual law $$\tau = \bar{\sigma} f = \bar{\sigma} \frac{f_o + a\ln(V/V_o) + b\ln(\theta V_o/L)}{1 + L/(\theta V_w)} \qquad \frac{d\theta}{dt} = 1 - \frac{\theta V}{L}$$





Why would faults be dynamically weak? Another possibility

Undrained thermal pressurization of fault gouge (primarily depends on slip)

(Sibson, 1973; Lachenbruch, 1980; Mase and Smith, 1985, 1987;
Andrews, 2003; Jacques and Rice, 2002, 2003; and others).

$\Rightarrow$ **LAW 2: Strong weakening mostly with fast, seismic slip** δ

$$\tau = (\sigma - p)[\text{friction coef-t from rate and state}], \quad \frac{dp}{dt} = \frac{\tau V}{f_o L_p} - \frac{p - p_{amb}}{T_p}$$

This assumes adiabatic undrained shear zone with very low permeability outside, so that expanding pore fluid cannot escape during rapid shearing, but on a longer time scale pressure re-equilibration occurs (we take $T_p = 0.25$ years).

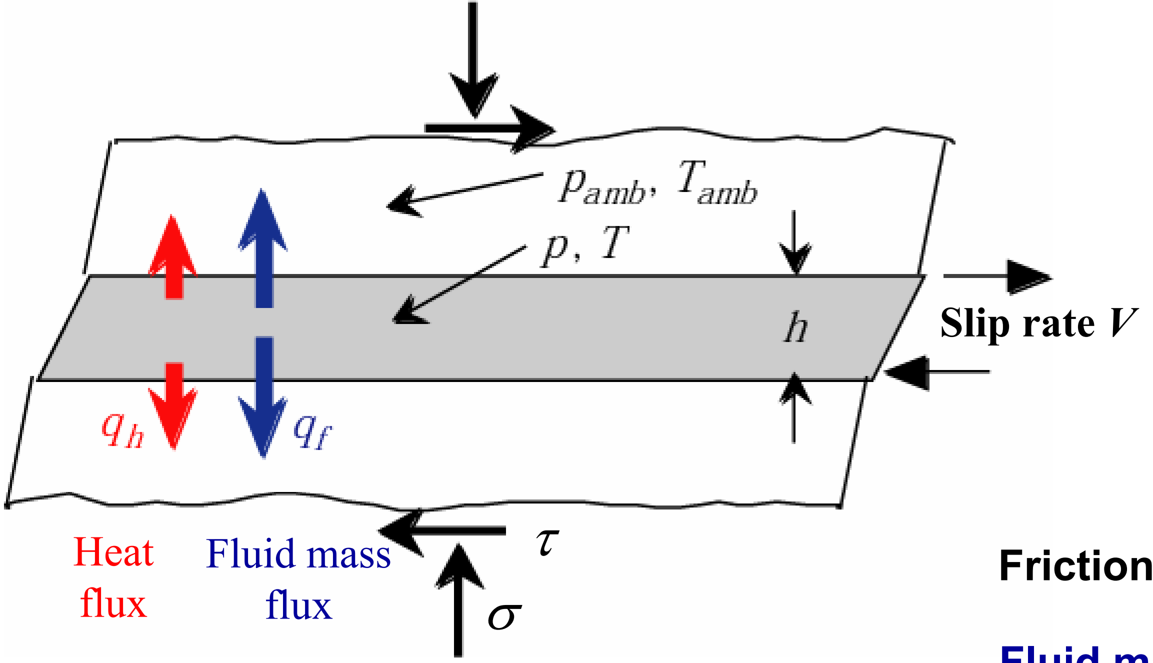
If we consider *short, seismic time scales* by ignoring $(p - p_{amb}) / T_p$
and assume that $f = f_o$ = constant,
then the resulting weakening process depends on slip:

$$\tau = f_o(\sigma - p_{amb})\exp(-\delta / L_p)$$

L_p ~ 1 mm -10 cm from seismic estimates of fracture energy; We use 25 cm – 17 m.

Simple models of shear heating processes at larger slips

Shear zone of fixed thickness h
Pore pressure p and temperature T equated to average values within shear layer



[Rice 2003-5,
building on Sibson (1973),
Lachenbruch (1980),
Mase & Smith (1985, 1987),
Rudnicki & Chen (1988),
Segall & Rice (1995a,b),
Andrews (2001),
Garagash & Rudnicki (2003a,b)]

Friction $\tau = f(\sigma - p)$

Fluid mass conservation

+ Some thermo-poro-elastic calculations

Energy conservation

$$\frac{\tau V}{h} = \rho^o c_{sp.ht.} \frac{dT}{dt} + 2\frac{q_h}{h}$$

$$\frac{dp}{dt} - \Lambda \frac{dT}{dt} + \frac{1}{\beta}\frac{dn^{pl}}{dt} = -2\frac{q_f}{\rho_f \beta h}\,; \quad q_f = -\frac{\rho_f k}{\eta_f}\frac{\partial p}{\partial y}$$

Assume adiabatic conditions (q_h = 0)

[Lachenbruch: *We* can neglect q_h if $h > 3.5(c_{th}\delta / V_{avg})^{1/2}$

or $h > 3.5$ mm $(\delta/\text{m})^{1/2}$ using $V_{avg} \approx 1$ m/s]

Observational constraints to satisfy

Static stress drops in the range **1 – 20** MPa

Static stress drop = Difference in stress before and after the earthquake
Relatively well-constrained from seismic observations

Low-heat, low-stress fault operation

Observations for the San Andreas fault suggest that:

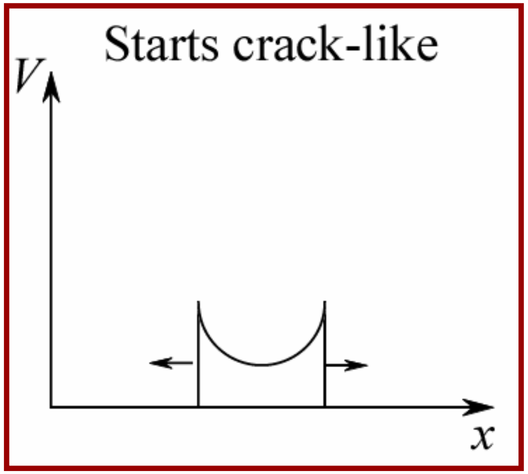
Much less frictional heat is generated that one would predict based on static friction coefficients *f* of 0.6 to 0.8 (lab results for typical rock materials) and effective normal stresses (σ - *p*) of order 150 MPa at typical seismogenic depths (comparable to overburden minus hydrostatic pore pressure);

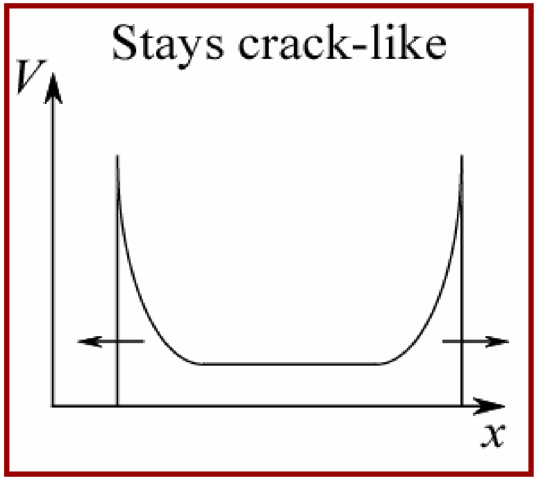
Shear stress resolved onto the fault must be low, as the maximum compressive normal stress makes steep angles to the trace of the SAF.

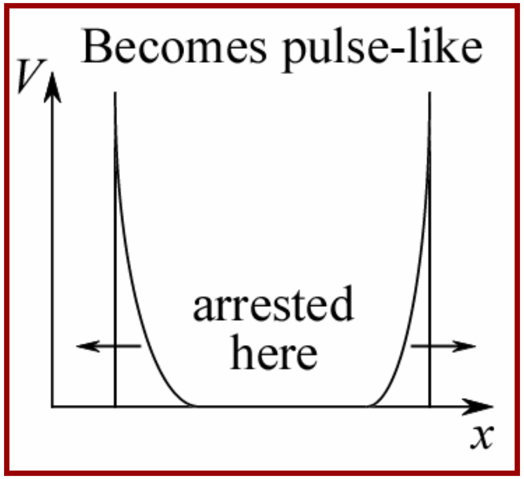
Explanations that are most commonly proposed: $\tau = \bar{\sigma} f = (\sigma - p) f$

(1) Effective normal stress is very low everywhere on the fault, ~10 MPa
OR
(2) Static friction coefficients are very low, < 0.1-0.2

Pulse-like mode of rupture propagation







Earthquakes occur as pulses of slip (e. g., Heaton, 1990)

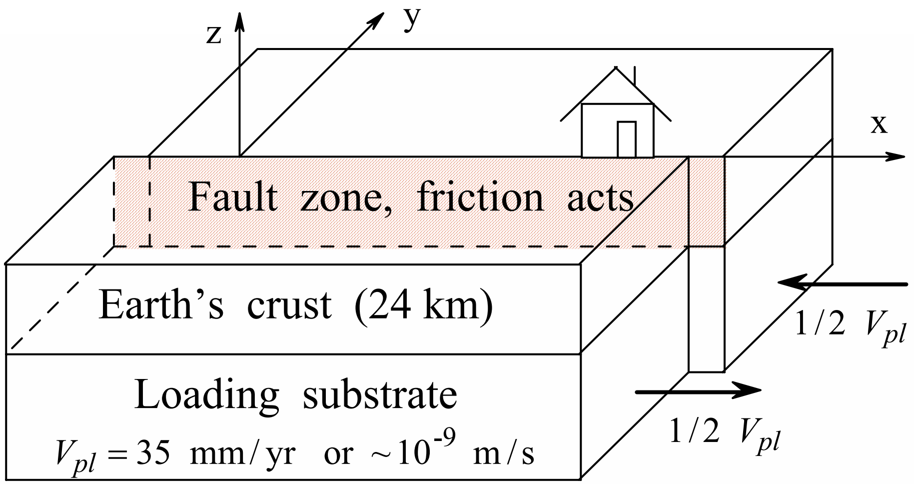
Possible mechanisms that create pulses:

Strong rate weakening (examined in this study)
(e.g., Perrin at al., 1995; Zheng and Rice, 1999)

Strong local heterogeneities in strength
(e.g., Beroza and Mikumo, 1996)

Ruptures on bimaterial interfaces
(e.g., Andrews and Ben-Zion, 1997).

Fault with defect regions to nucleate ruptures



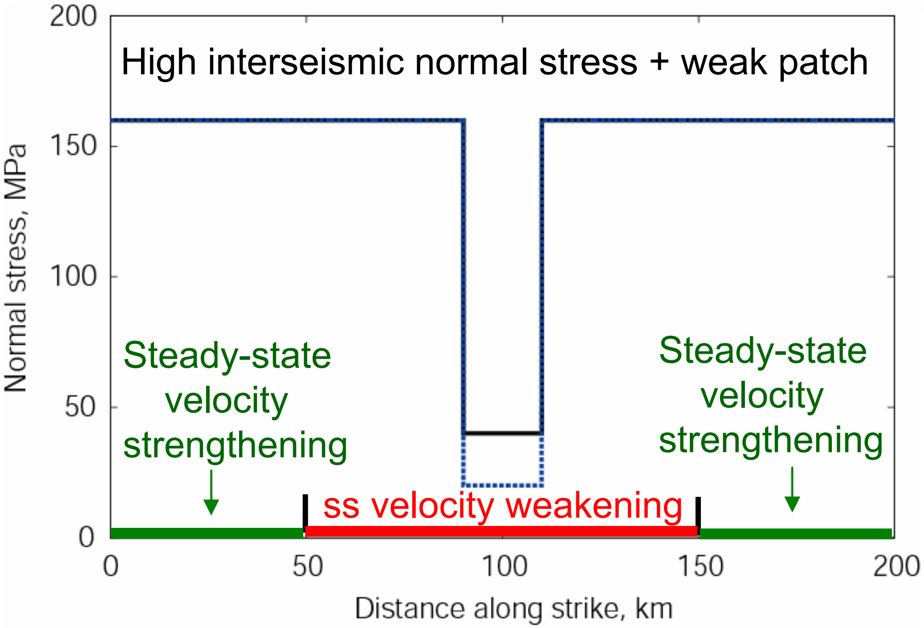
2D crustal plane model

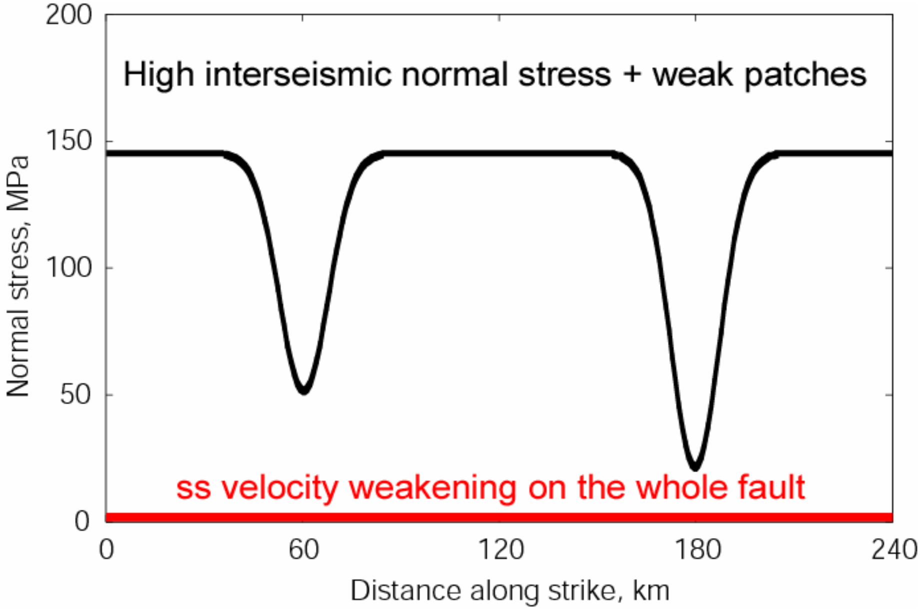
Variations with x and y only,

depth-averaged in z

All earthquake stages are resolved:

Slip nucleation and acceleration;
Dynamic rupture propagation;
Post- and inter-seismic creep





Stress state on the fault through many earthquake cycles

$$\tau_{\text{av}} = \frac{\int_{fault} \tau \, \mathrm{d}A}{\text{fault area}}$$

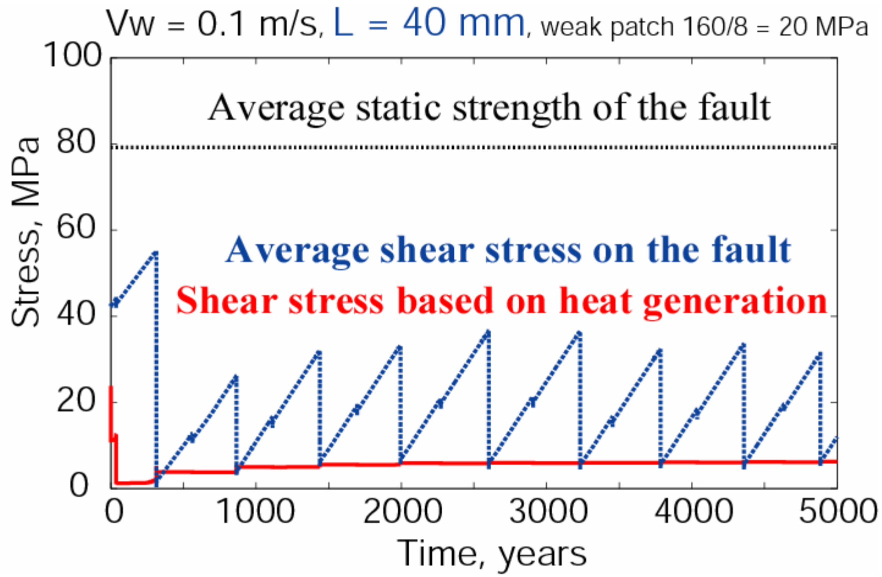
$$\tau_{\text{heat}} = \frac{\int_{time} \int_{fault} \tau V \, \mathrm{d}A\mathrm{d}t}{\int_{time} \int_{fault} V \, \mathrm{d}A\mathrm{d}t}$$

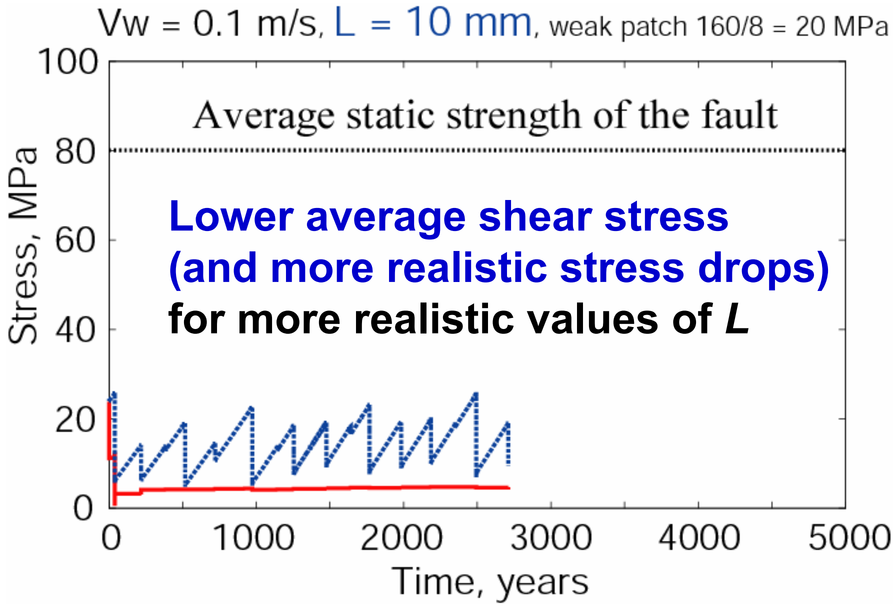
$$\tau_{ss} = (\sigma - p_{amb}) \frac{\text{rate and state friction coef-t}}{1 + V/V_w}$$

V_w = 0.1 m/s

Low-stress fault operation

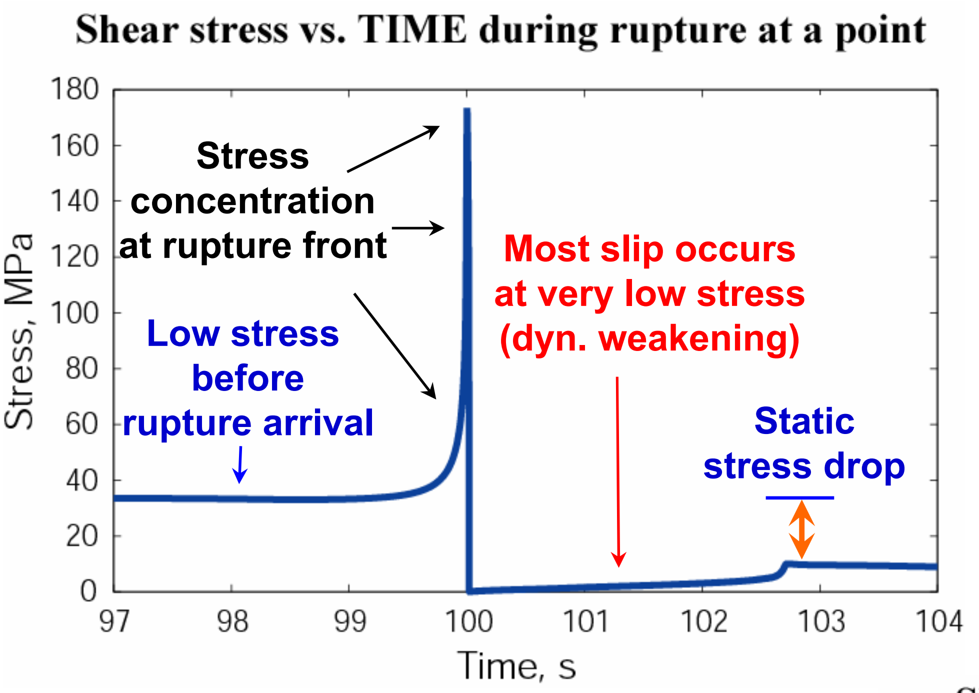
Low heat production





Lower average shear stress (and more realistic stress drops) **for more realistic values of *L***

Shear stress evolution
at a *statically strong* fault point
during dynamic rupture


Shear stress vs. TIME during rupture at a point

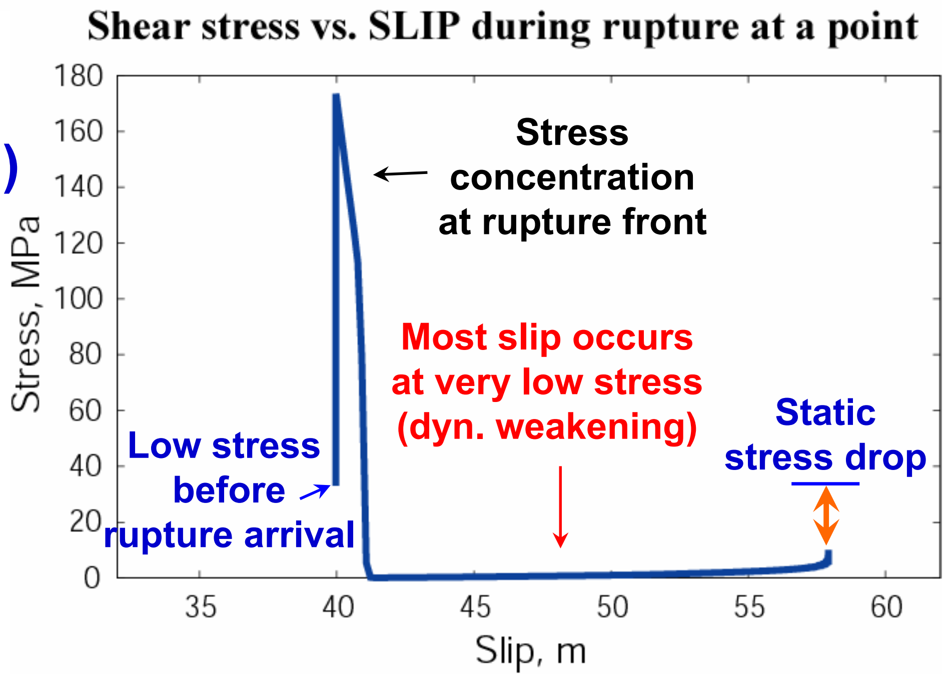
Static stress drop is ***much smaller*** than what one would expect ***because*** shear stress before the earthquake is much smaller than static strength.

Pulse-like rupture (slip duration is about 3 seconds)

Note:

In this case (*L* = 40 mm):
Static stress drop is ~20 MPa;
Slip is ~18 m.

Both should decrease for smaller *L*.


Shear stress vs. SLIP during rupture at a point

Enhanced velocity weakening with NO weak patch

Same statically strong fault

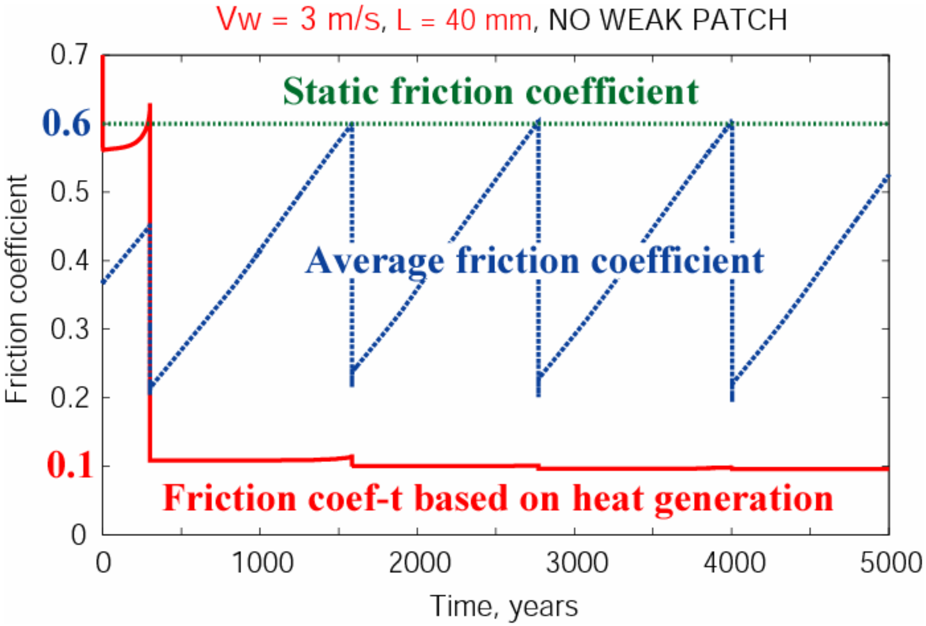
No weak patch ⇒ NO low-stress operation

No place for events to nucleate
under low overall shear stress

STILL very low heat production

Inferred friction coefficient f_{heat} **< 0.125**

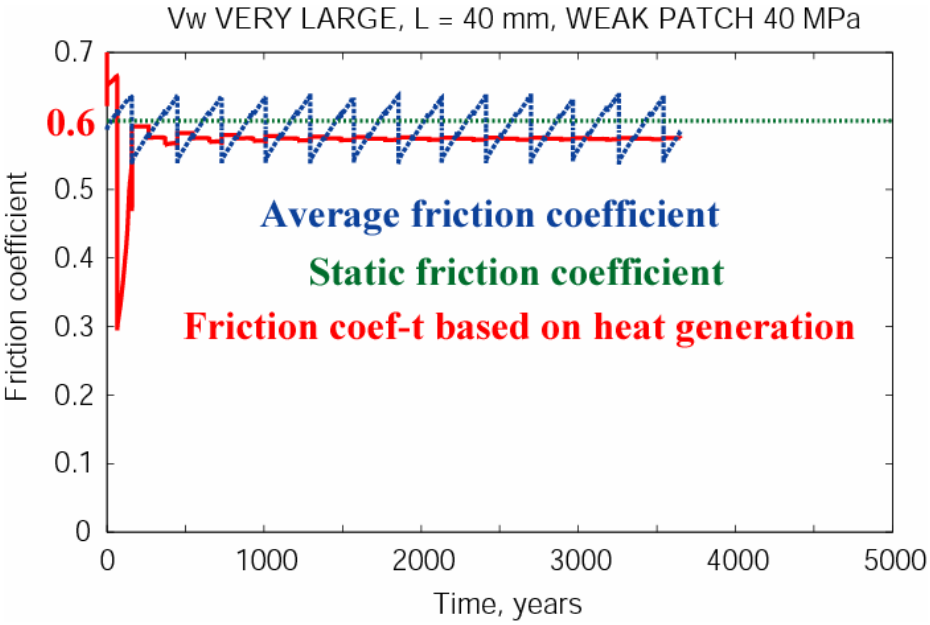
Most slip still occurs at
dynamically reduced shear stresses.



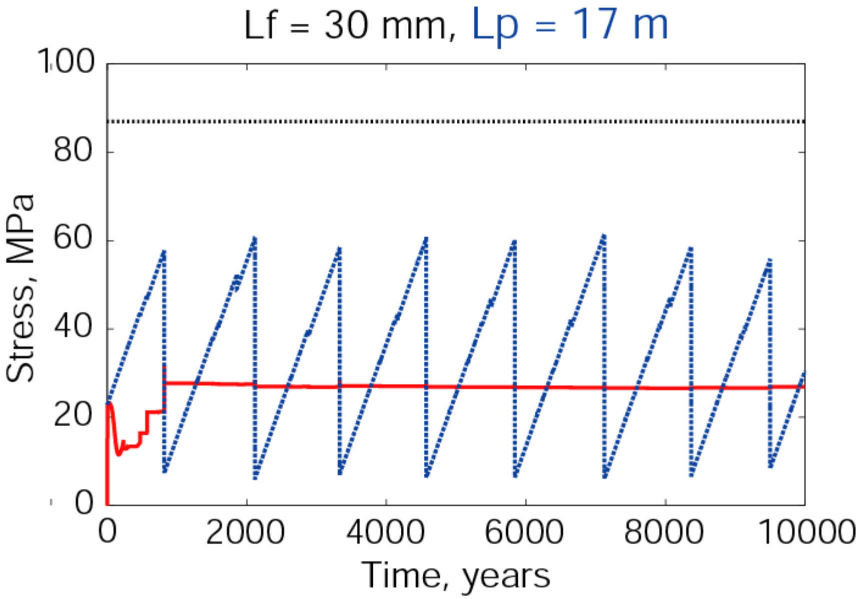
NO enhanced velocity weakening (just rate and state friction)

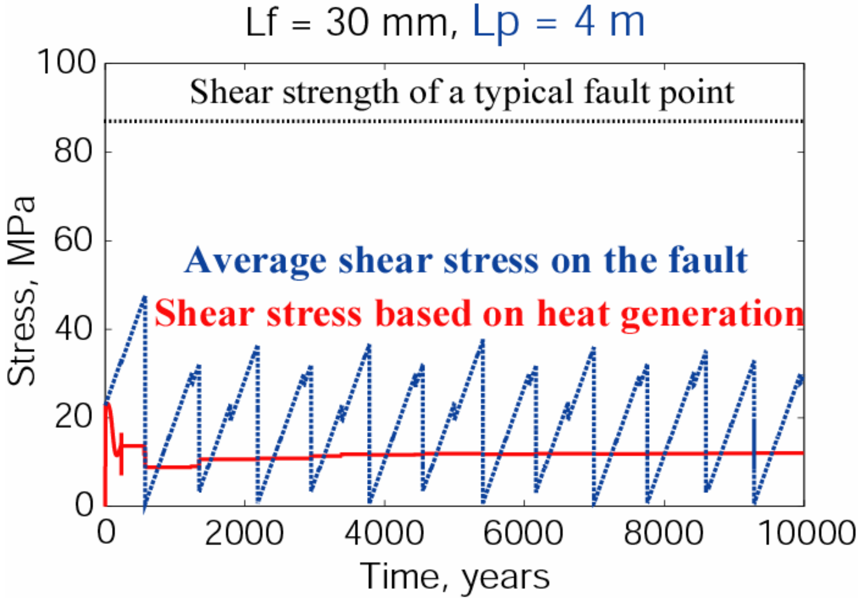
WITH or WITHOUT WEAK PATCH:

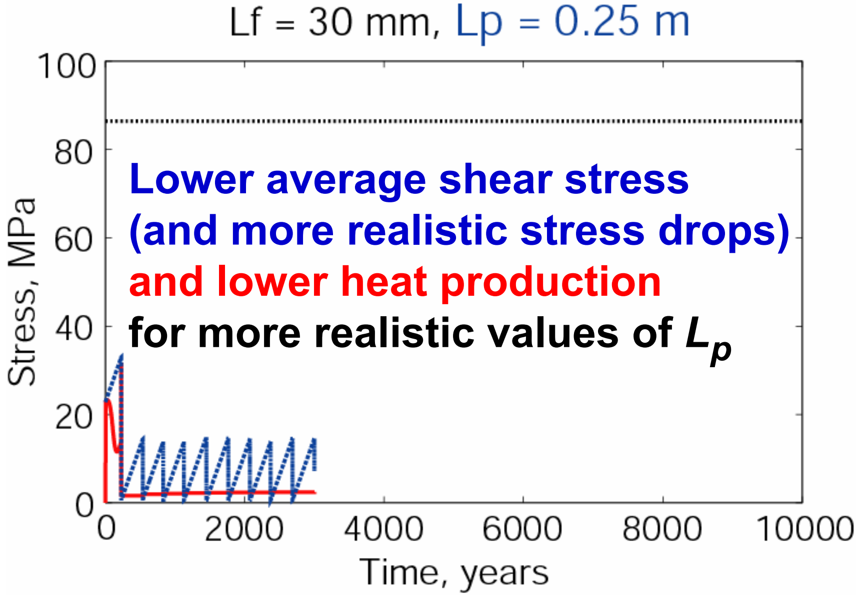
NO low-stress operation
NO low heat production



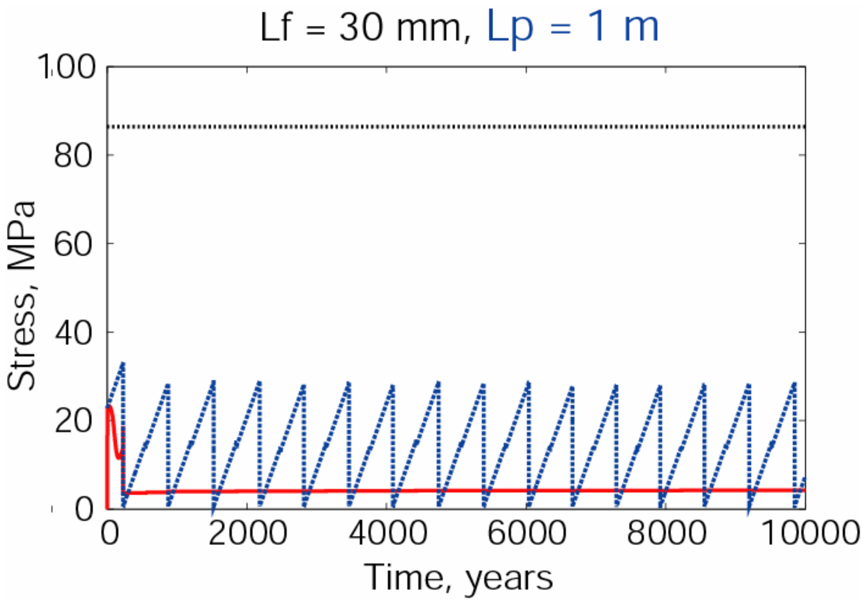
LAW 2: $\tau = (\sigma - p)$[friction coef-t from rate and state], $\dfrac{dp}{dt} = \dfrac{\tau V}{f_o L_p} - \dfrac{p - p_{amb}}{T_p}$







Lower average shear stress
(and more realistic stress drops)
and lower heat production
for more realistic values of L_p



Existence of localized weak regions on faults

Wide spread of stress drops for small events suggests that the effective normal stress $(\sigma - p)$ may be low at many places where ruptures nucleate. (Less scattered stress drops for large events may reflect averaging over weak and strong regions.)

Dieterich, 1994 proposed a ***rate and state-based explanation for the aftershock rates***. To match observations, it requires that $a(\sigma - p)$ is 0.1 to 0.5 MPa. Since lab values of a are of order 0.01, this suggests that most aftershocks happen in places with the (low) effective normal stress of order 10 to 50 MPa.

Borehole studies show over-pressure in at least some shallow-depth locations.

Strong local variations of normal stress σ due to slips off the main fault (fault branches, stepovers, etc.)

Similar fault operation may be possible with stress concentrations instead of weak regions.

Summary

Efficient method for simulating sequences of spontaneous ruptures.

All features of slip accumulation are resolved:

Nucleation, ***dynamic failure***, ***creep-like deformation***

Stability properties of rate and state friction laws
have direct connection to features of spontaneous slip accumulation:
nucleation, stable/unstable sliding, multiple pulses of slip.

Models that combine
rate and state friction with shear-heating weakening mechanisms
produce earthquake sequences that satisfy some basic observational
and lab constraints, such as high static and low dynamic fault strength,
reasonable static stress drops,
low heat and low stress fault operation,
and pulse-like rupture propagation.

Remark about scaling from lab experiments to real faults

Current and future directions

3-D simulations

Aseismic and seismic *nucleation processes* in 3D

Effects of heterogeneities along the interface

Predictive modeling of laboratory experiments

Distinguish between constitutive descriptions by *designing experiments* based on numerical modeling

Dynamic weakening mechanisms

Improved/combined formulations for shear heating effects

Modeling static and dynamic earthquake triggering

Observable effects of triggering, such as aftershock sequences and earthquake clusters, hold clues to the proper constitutive laws

Off-interface processes

Spontaneous *branching*, existing fault geometry/nonplanarity

Off-fault damage, f*ault interaction*