Dense Granular Flows: Modeling Constitutive Relations from Microscopic Considerations

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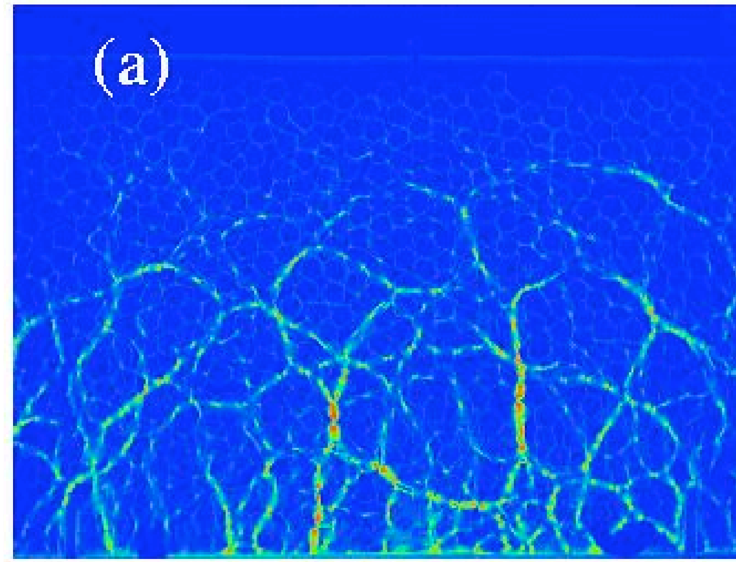
Granular Material

A Granular Material is a collection of non-penetrating objects that dissipate energy upon contact and do not respond to thermal temperature.

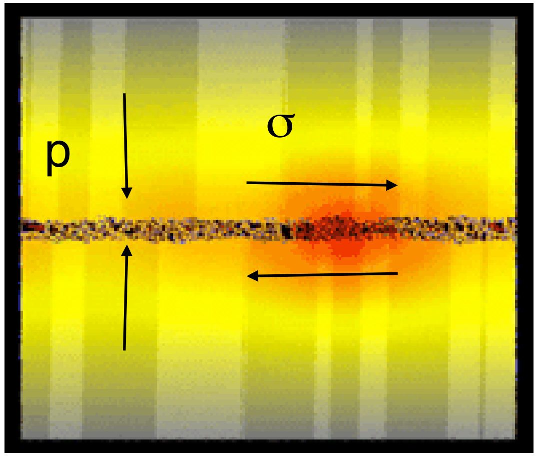


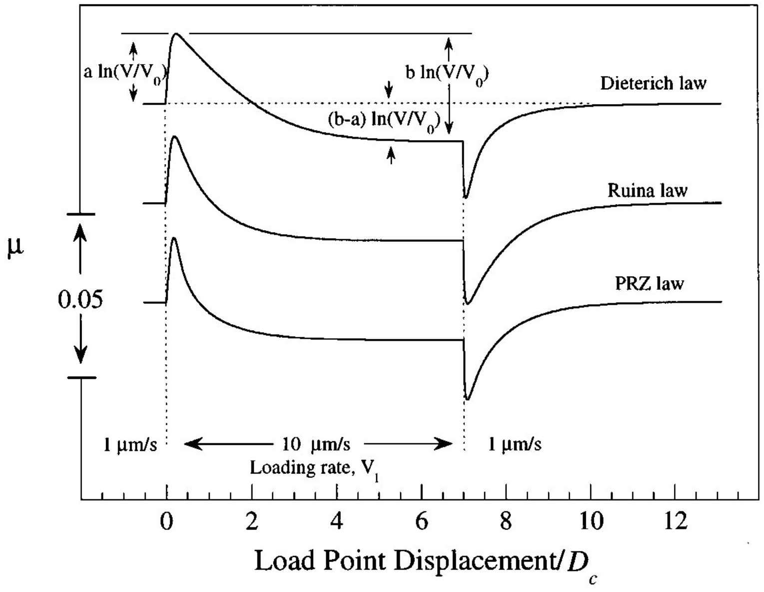
Flowing (from Hermann)

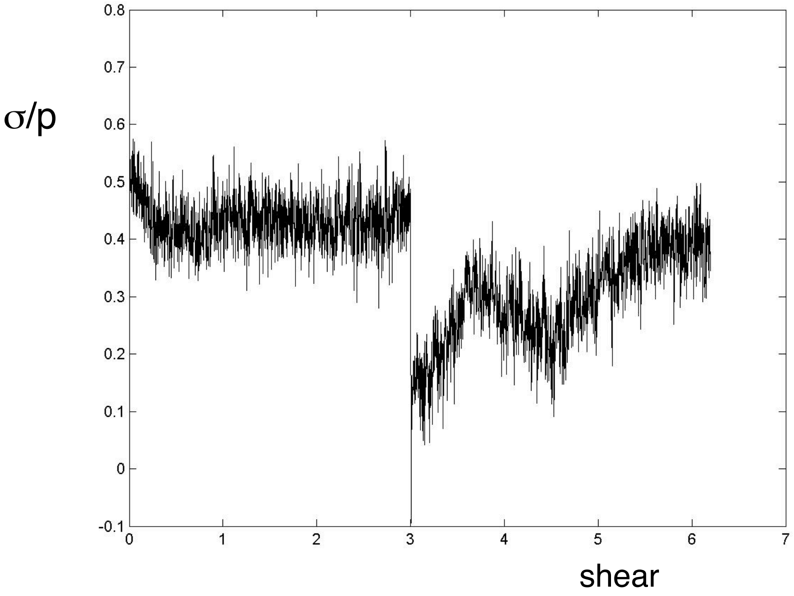
Jammed (from Behringer)

Transition

One Possible Application: Earthquakes

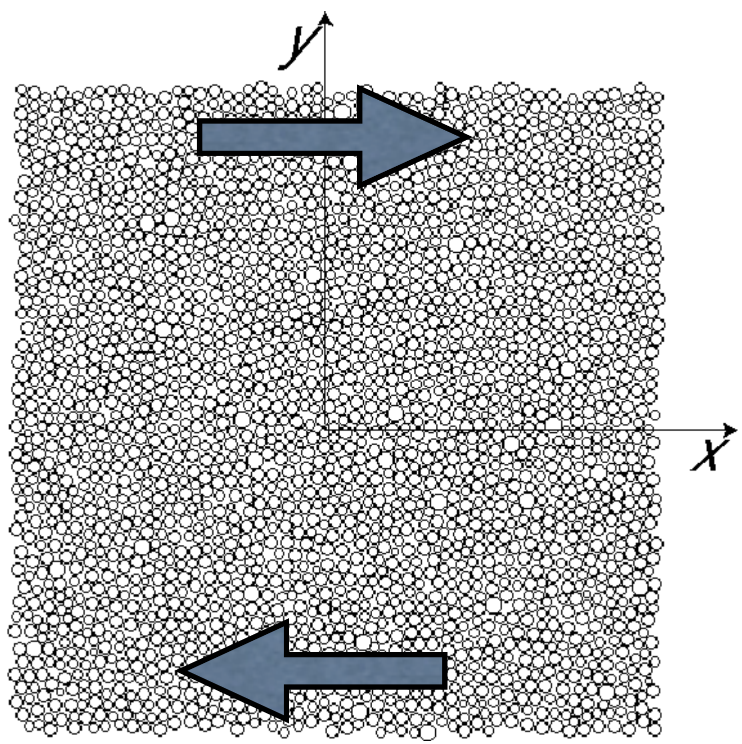




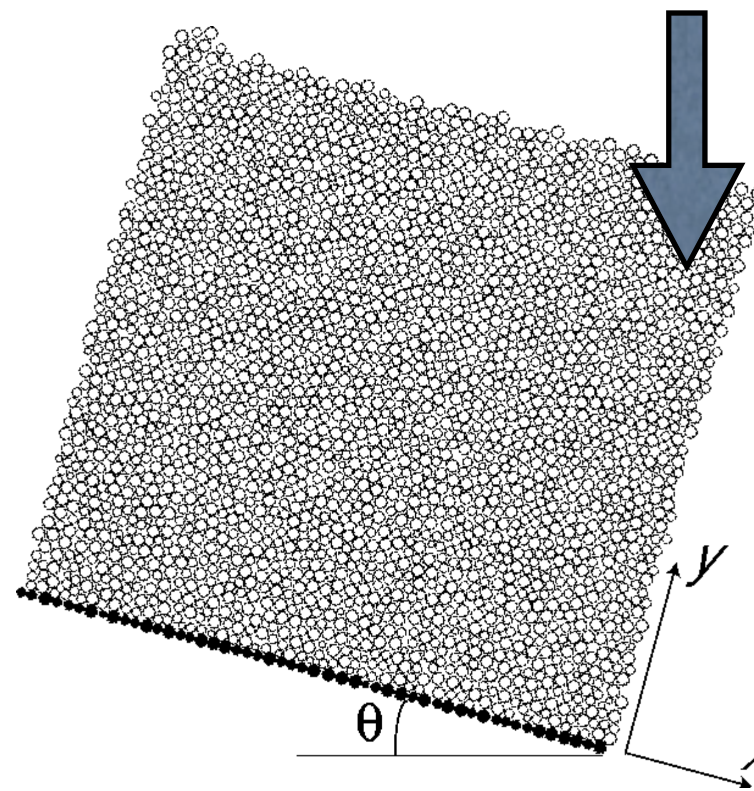


Flows of Granular Materials

We conduct simulations of flowing granular materials, in 2 geometries:



"Simple Shear Flow"

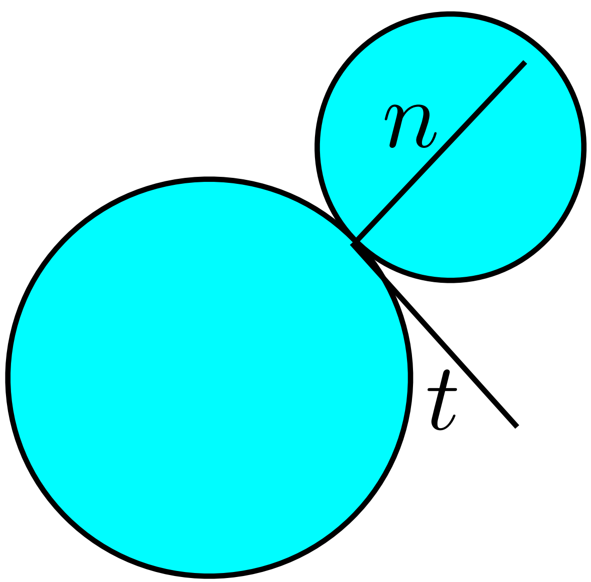


"Incline Flow"

Interaction Between Grains

A common paradigm for the granular interaction is simple restitution resulting from *perfectly hard grains:*

$$v_n' = -ev_n; \quad v_t' = -tv_t; \quad F_t \leq \mu F_n$$



This simple paradigm has been applied to many subjects, including: avalanches, earthquakes, traffic flow, friction, and galaxy formation.

In most of this presentation we concentrate on frictionless materials.

Constitutive Relations

In the geometries we study, we're interested in the following variables

$$\dot{\gamma}_{\alpha\beta} = \partial_\alpha U_\beta = \begin{pmatrix} 0 & \dot{\gamma} \\ 0 & 0 \end{pmatrix}$$ shear rate

$$\Sigma_{\alpha\beta} = \sum_{contacts} D_\alpha F_\beta = \begin{pmatrix} p & -\sigma \\ -\sigma & p \end{pmatrix}$$ pressure and shear stress

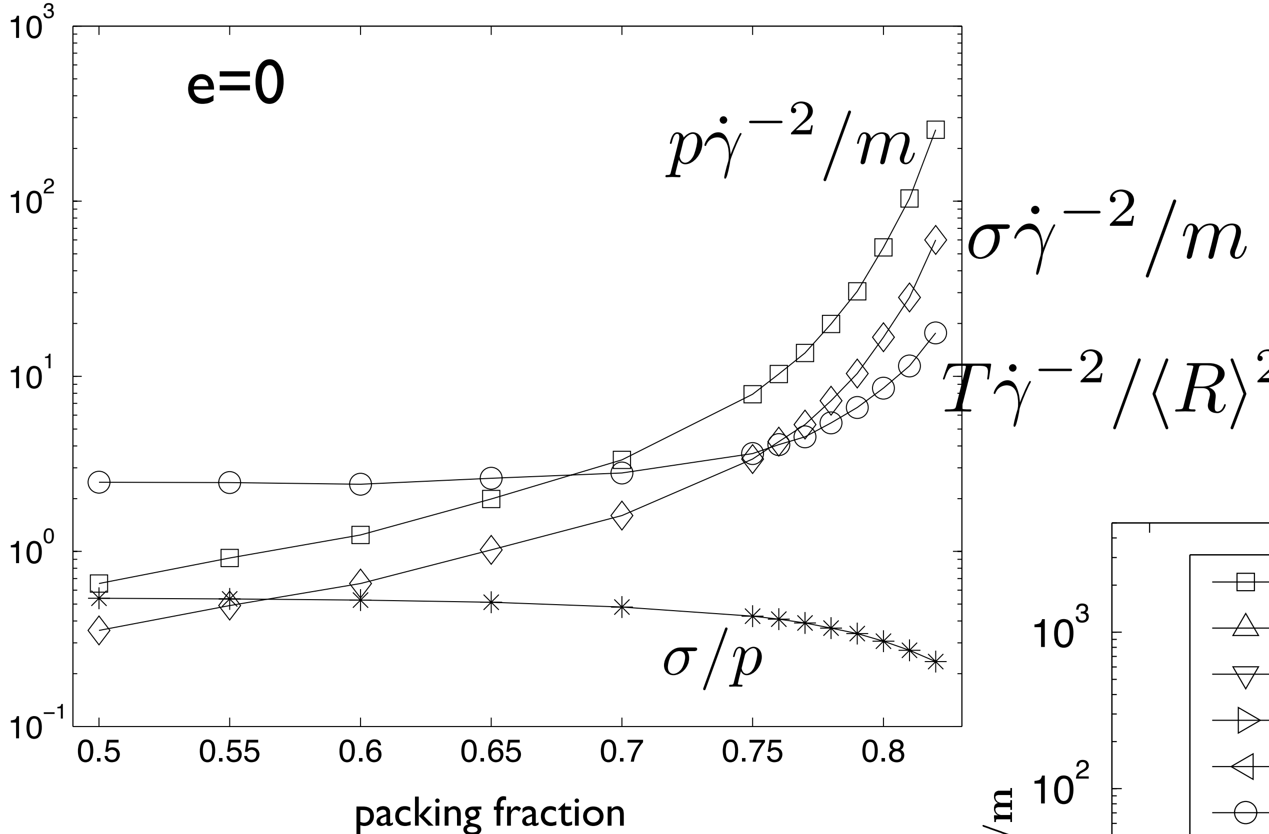
$$T = \sum_{grains} v^2 - \left(\sum_{grains} v \right)^2$$ granular temperature

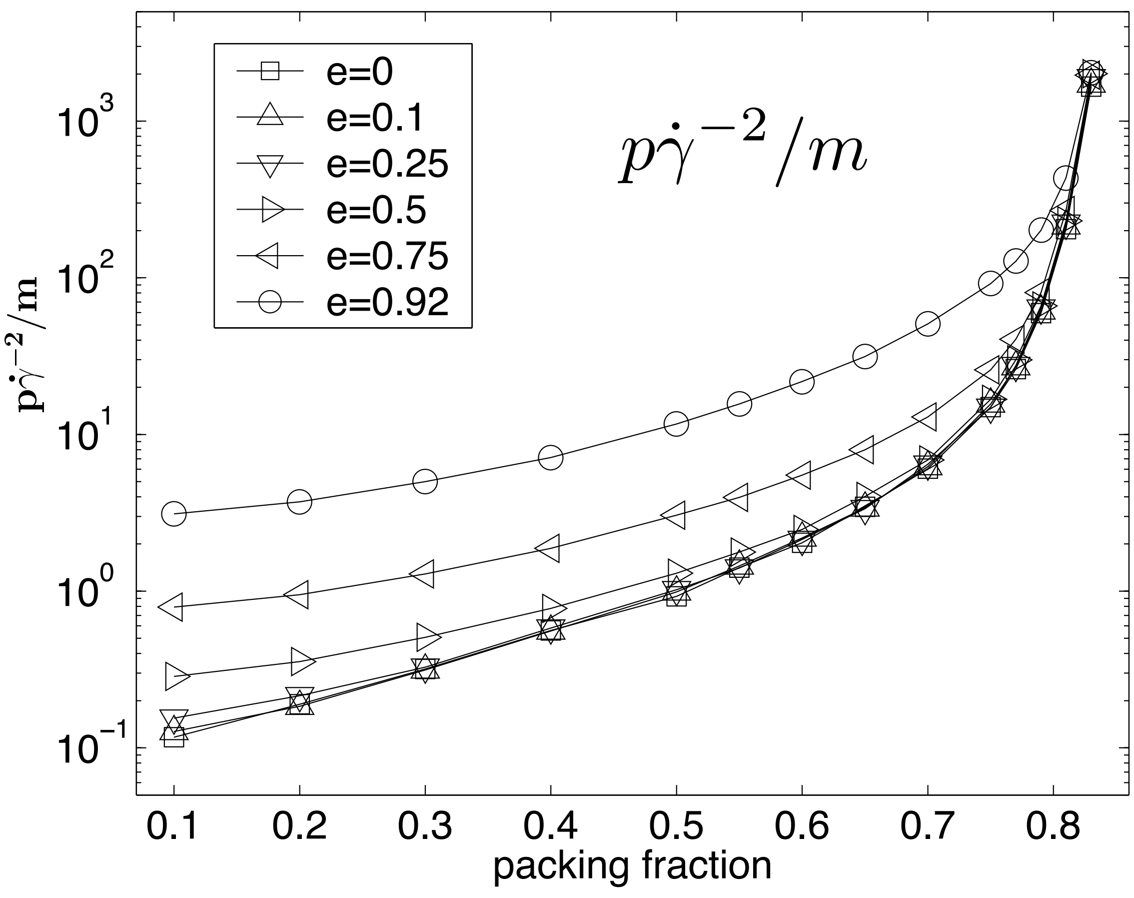
For perfectly hard grains, the only independent time scale is set by $\dot{\gamma}$
Thus,

$$p, |\sigma|, T \propto \dot{\gamma}^2$$

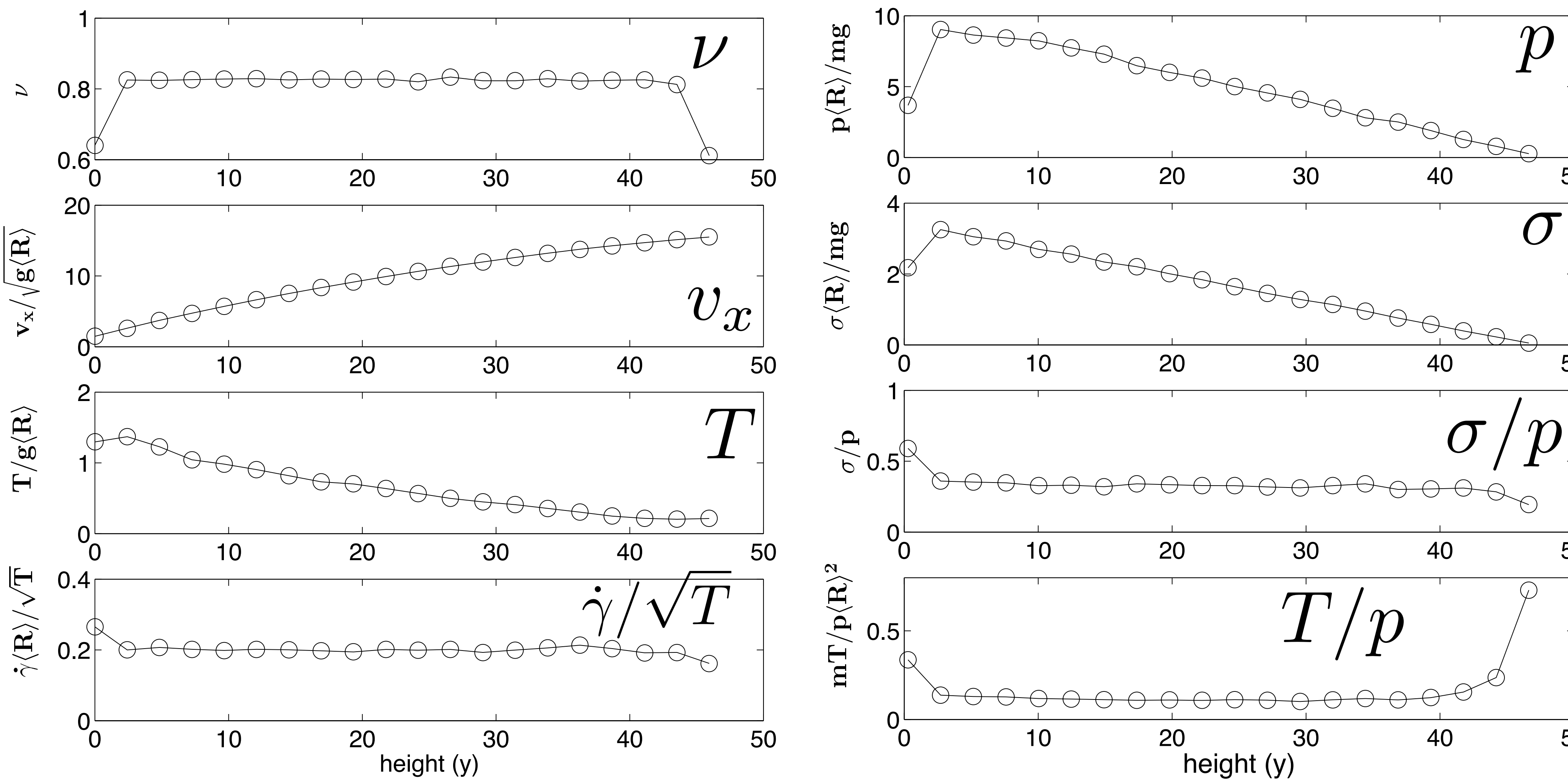
(Observed experimentally in 1954 by Bagnold)

Some Simple Shear Results





Some Incline Flow Results

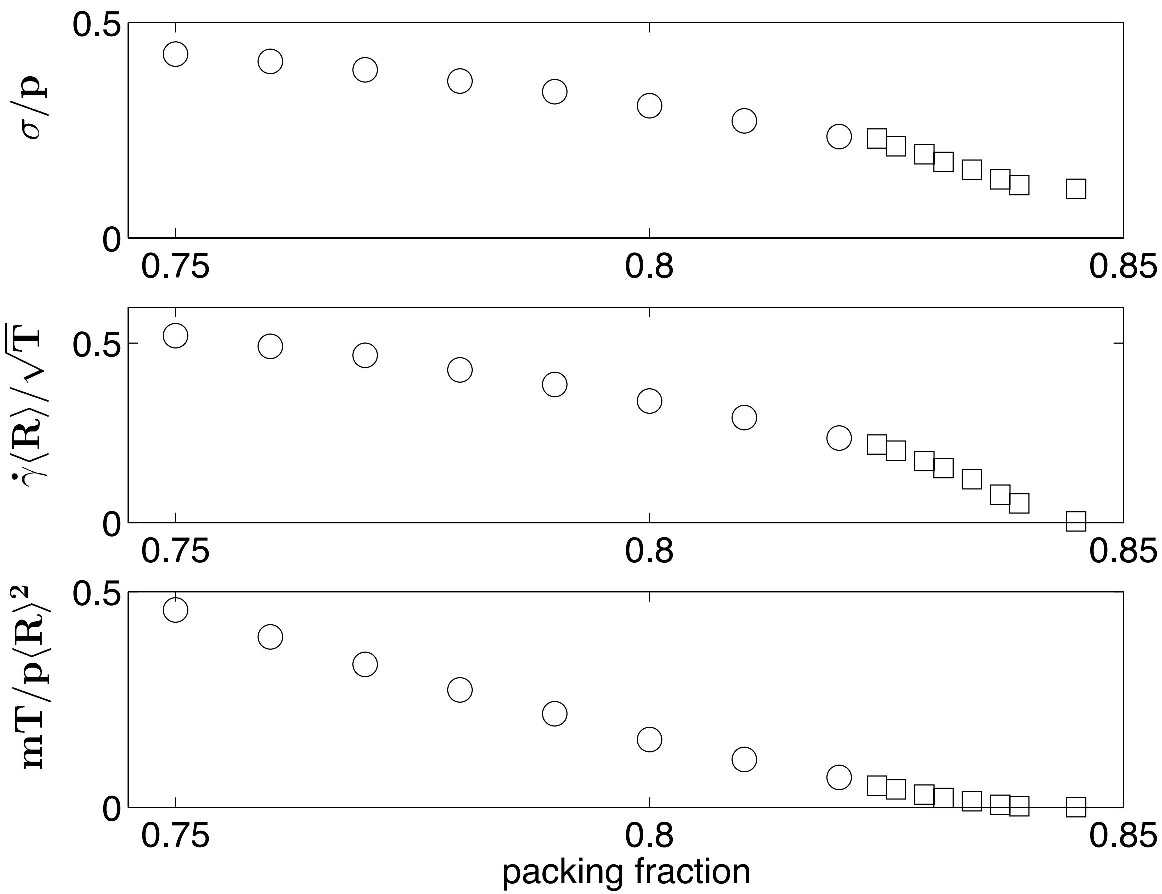


In the bulk of the flow, $T,\ \sigma,\ p \propto \dot{\gamma}^2$

Local Rheology

Given $\sigma,\ p,\ T,\ \dot{\gamma}$ there are 3 invariant quantities

$$\sigma/p,\ \dot{\gamma}\langle R\rangle/\sqrt{T},\ mT/p\langle R\rangle^2$$



How To Model Constitutive Relations

- Kinetic Theory
  Assume no microscopic structure.

- STZ Theory
  Assume a certain type of microscopic structure.

Overview of Kinetic Theory for Granular Materials

Kinetic Theory assumes that particles interact only through binary collisions. This yields an equation for the one-particle distribution function f

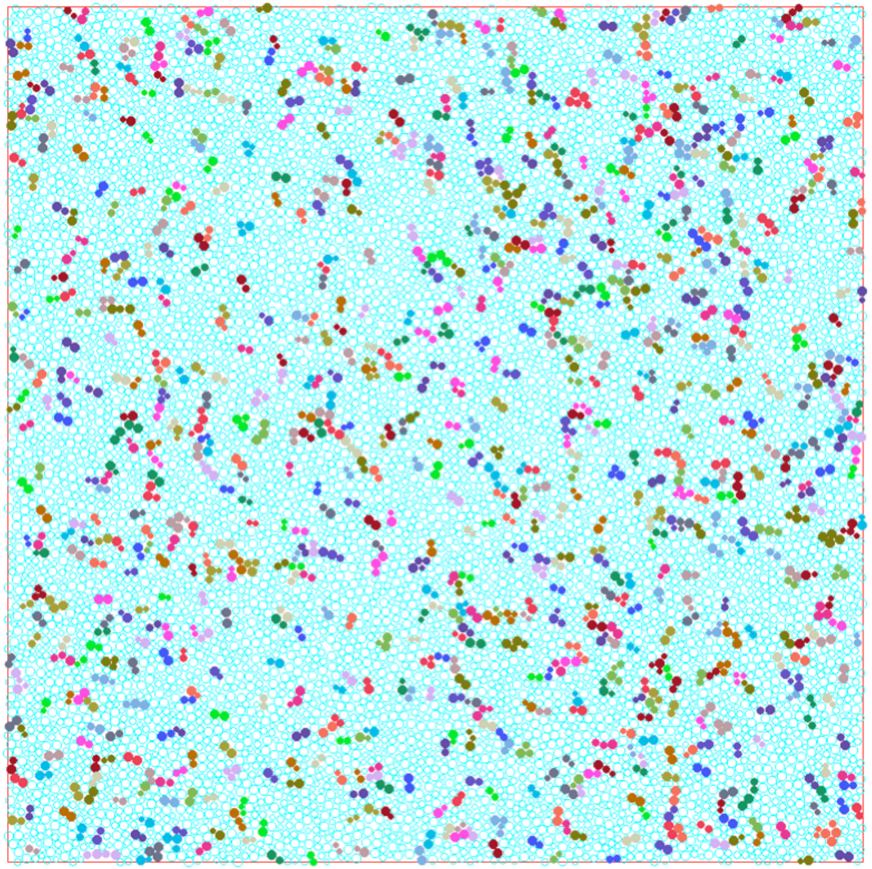
$$(\partial_t + v_\alpha \partial_\alpha) f(v) = \int dv_1 dv_2 \, b(e, v_1, v_2, v) \, f(v_1) f(v_2)$$

The stress tensor is determined, once f is known

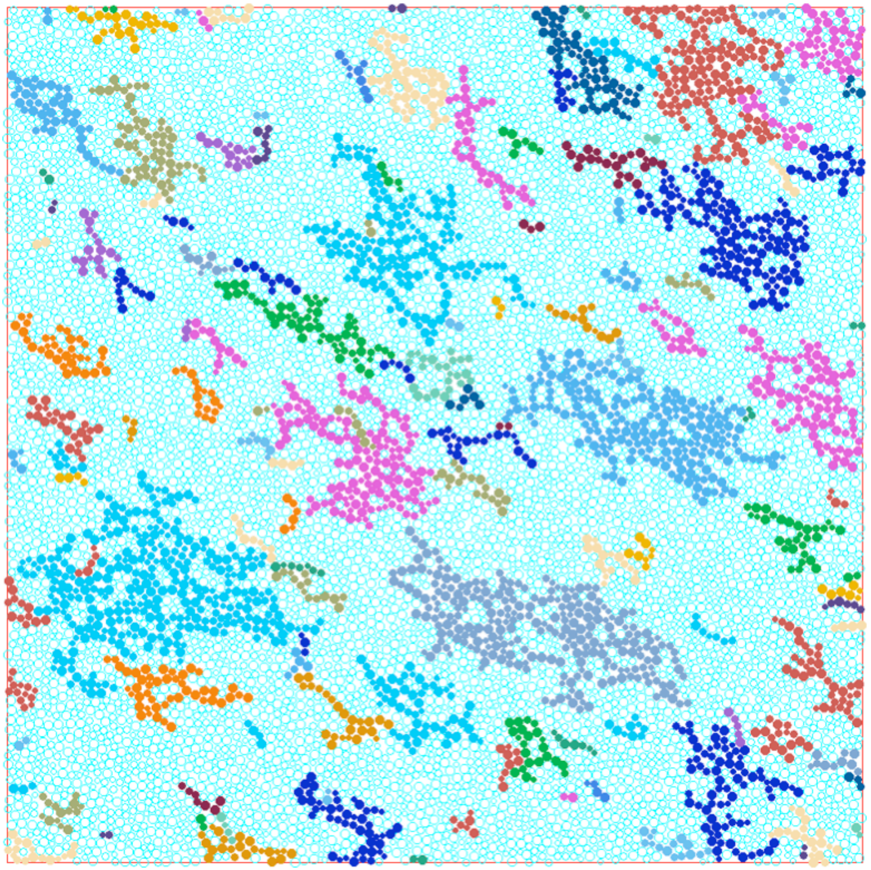
$$\Sigma^{bc}_{\alpha\beta} = J(f)$$

Is the binary collision assumption applicable to granular materials?

Emergence of Clusters



e=0.92: nearly elastic



e=0: totally inelastic

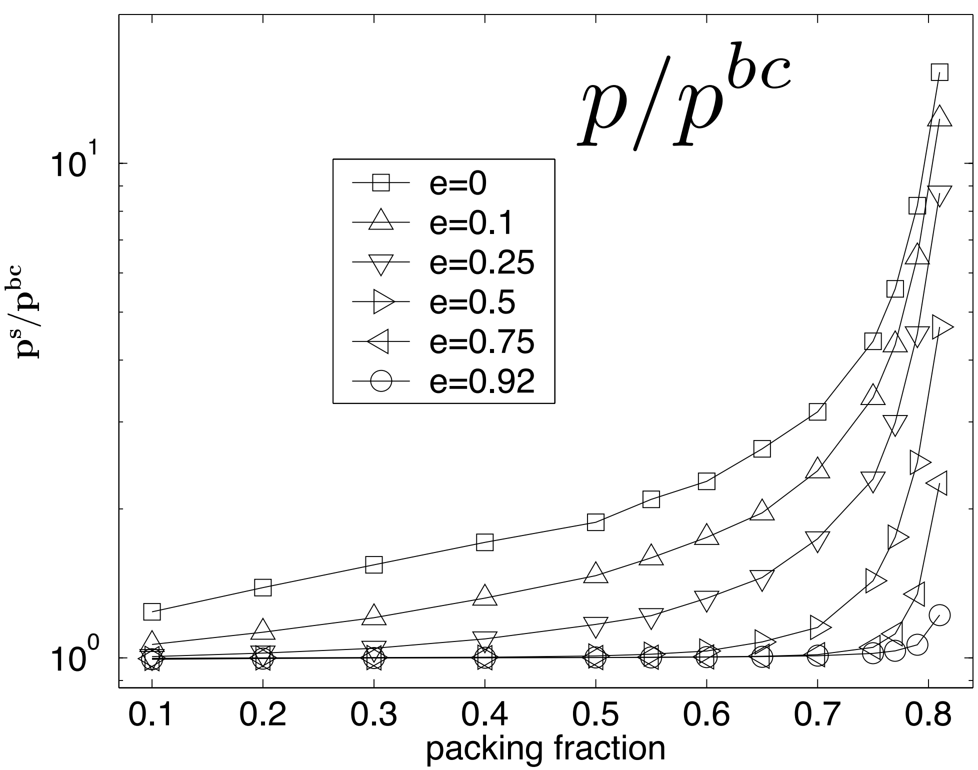
Test of the Binary Collision Assumption

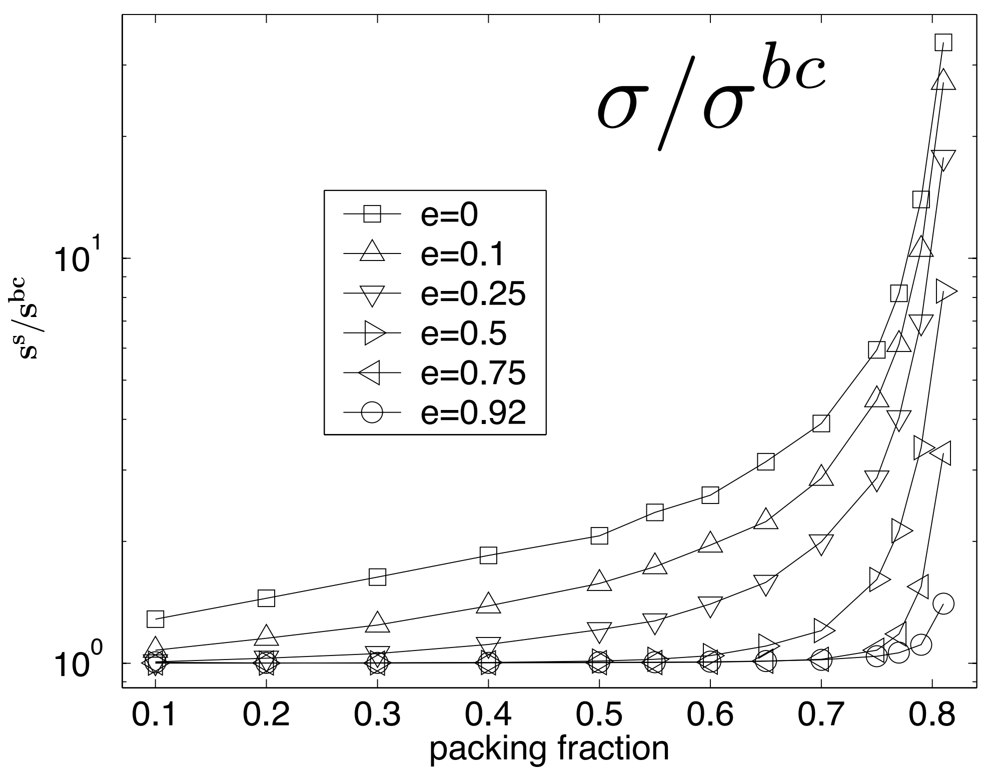
The total stress tensor is measured as

$$\Sigma_{\alpha\beta} = \sum_{\text{contacts}} D_\alpha F_\beta$$

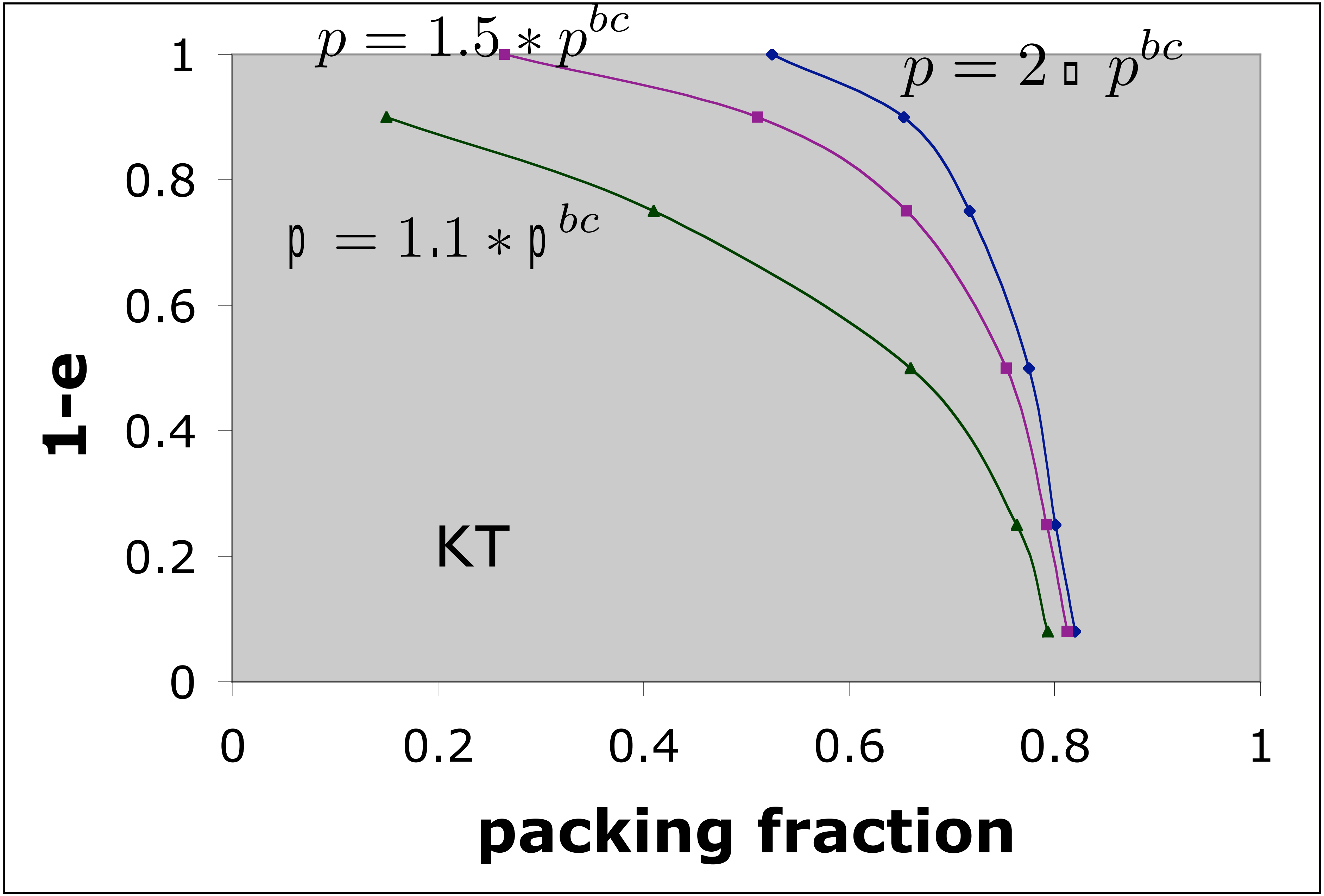
If only binary collisions occur, then this can be written as

$$\Sigma^{bc}_{\alpha\beta} = \frac{1+e}{\Delta t} \sum_{\text{contacts}} \mu D_\alpha v^{\text{rel}}_\beta$$





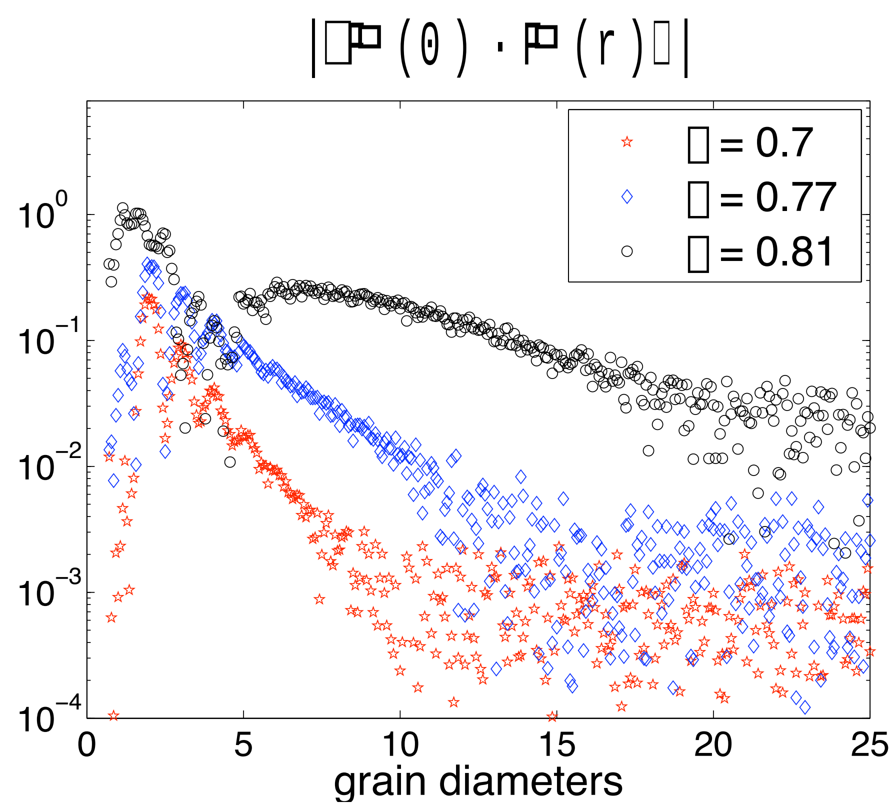
When does KT apply?



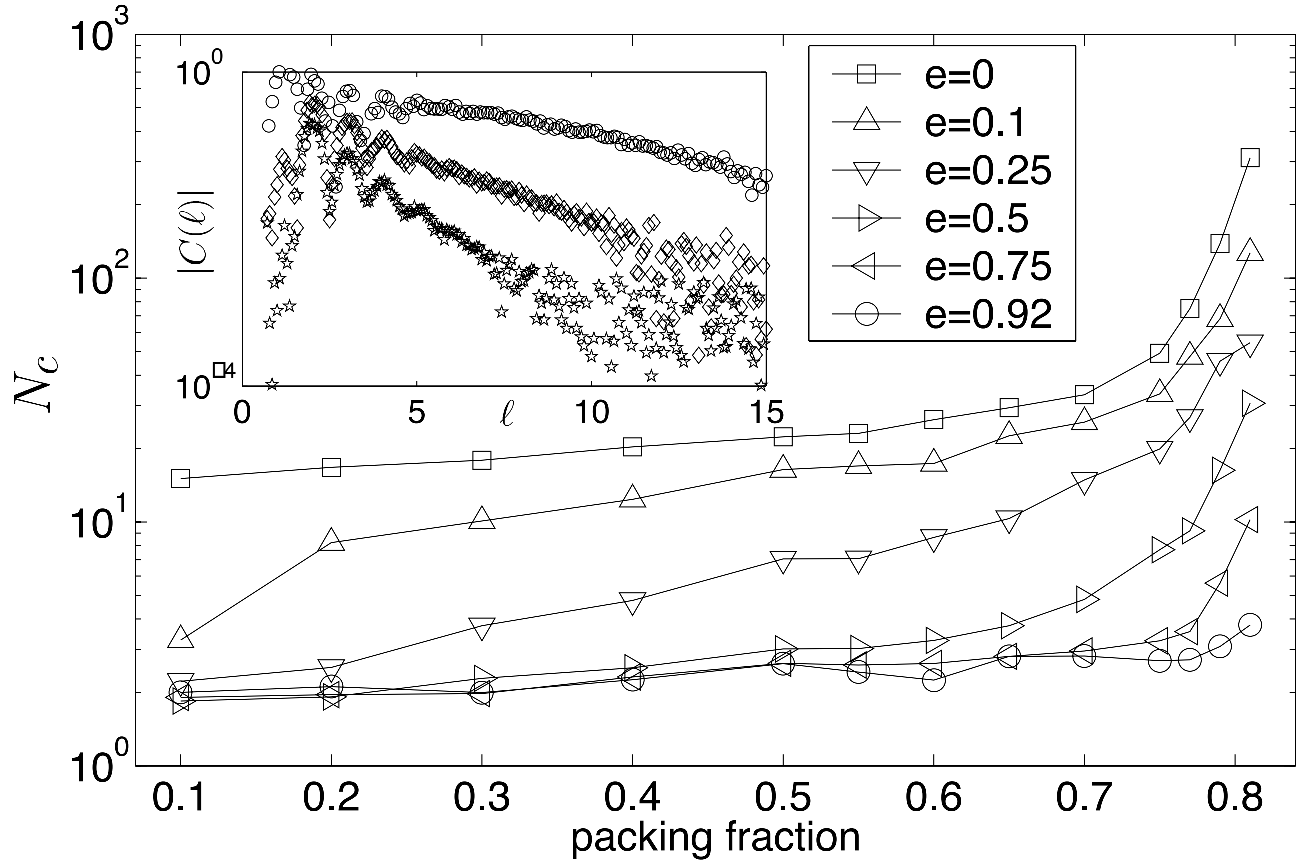
Microscopic Failure of KT

Kinetic Theory breaks down because of the emergence of clusters of grains. Can we measure this?

We measure force-force spatial correlations to find the average cluster size



Growth of Average Cluster Size



The growth of cluster size mirrors the growth of the pressure and shear stress ratios and signals the breakdown of Kinetic Theory.

How To Model Constitutive Relations

- Kinetic Theory

  Assume no microscopic structure.

  Breaks down with the appearance of correlated clusters.

  Not a good theory for geological situations.

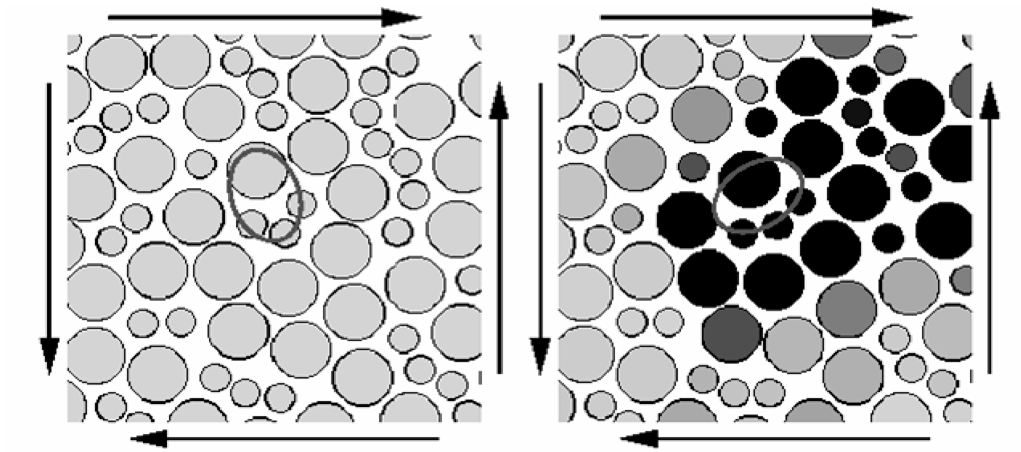
  cond-mat/0507286

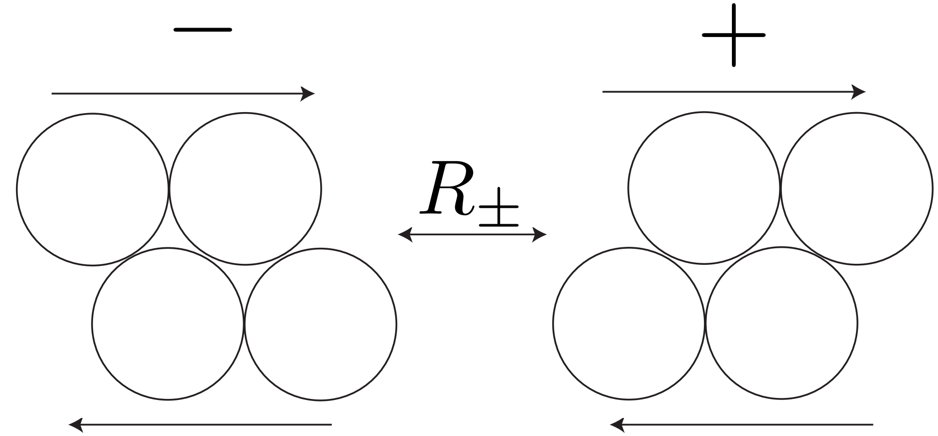
- STZ Theory

  Assume a certain type of microscopic structure.

STZ Theory of Amorphous Solids

(1) Non-affine motion occurs in localized regions
(2) The regions undergoing non-affine motion have orientation





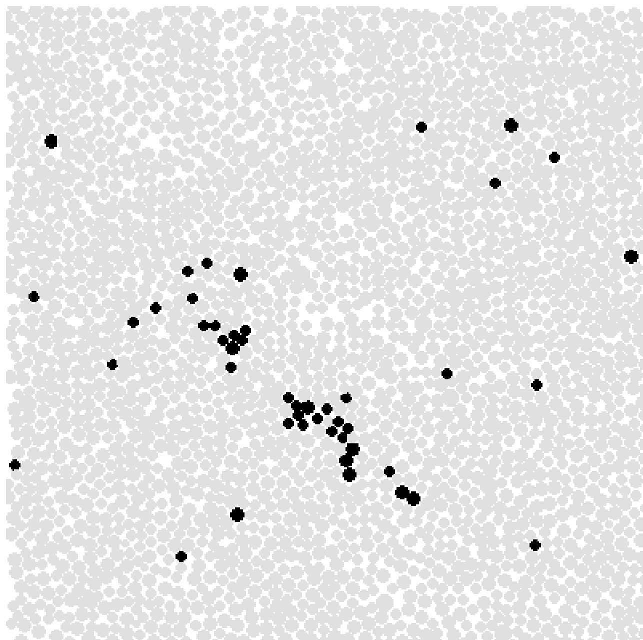
$$\dot{\gamma}^{pl} \square R_- n_- - R_+ n_+$$

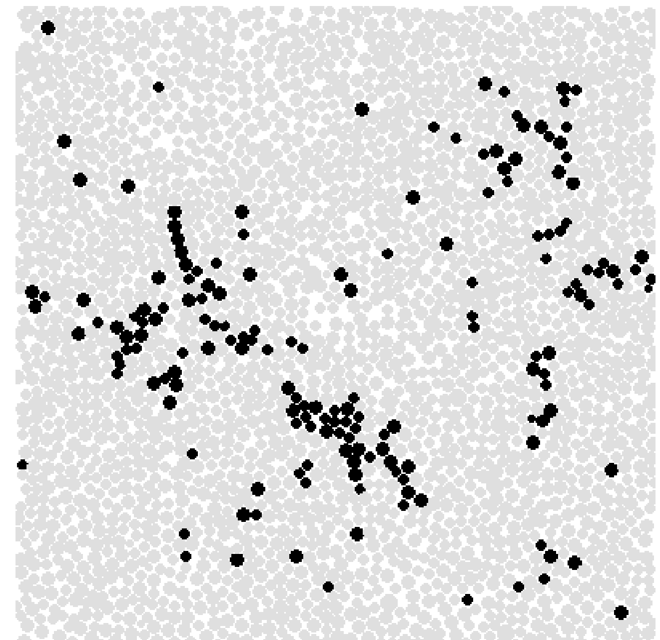
$$\dot{n}_\pm = R_\mp n_\mp - R_\pm n_\pm + w(a - b n_\pm)$$

(Falk & Langer 1997)

Validation of Microscopic Picture

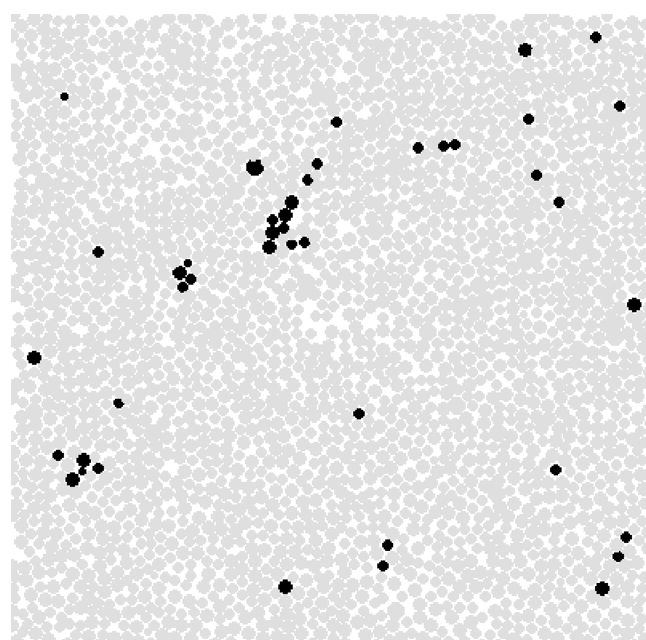
Forward
0-5%





Forward
0-15%

Backward
0-5%



STZ Theory for Granular Materials

$$\dot{\gamma}^{pl} \square R_- n_- - R_+ n_+$$

$$\dot{n}_\pm = R_\mp n_\mp - R_\pm n_\pm + w(a - bn_\pm)$$

(Falk & Langer 1997)

$$\dot{\gamma} = \dot{\gamma}^{pl} \qquad w = \sigma\dot{\gamma}/p \qquad \mathrm{R}_\pm \propto \sqrt{\mathrm{T}}\mathrm{e}^{\pm\kappa\sigma/p}$$

Lemaitre (2002)

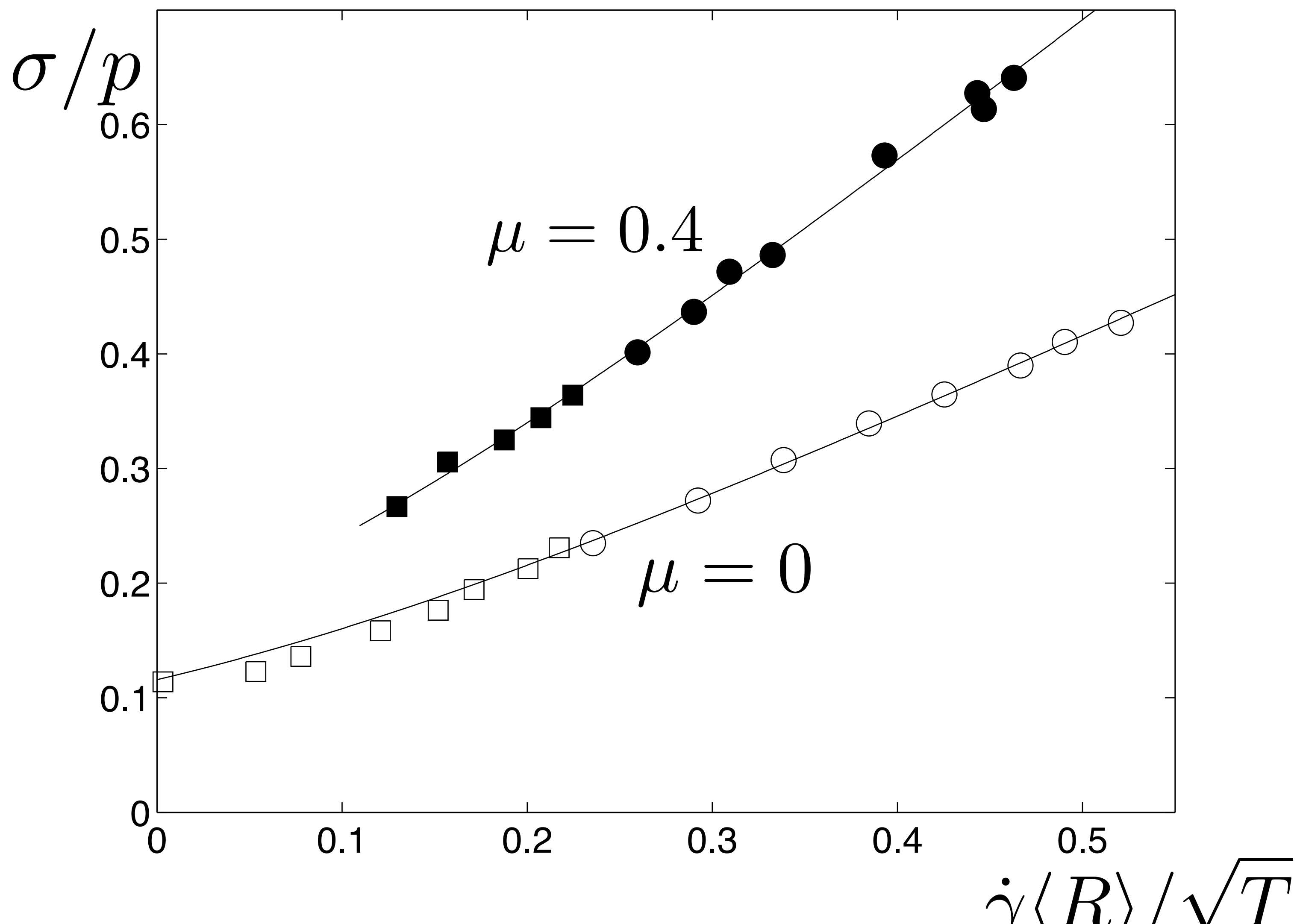
$$\dot{\gamma} \propto \sqrt{T}\,(\Lambda \sinh(\kappa\sigma/p) - \Delta \cosh(\kappa\sigma/p))$$

$$\dot{\Delta} \propto \dot{\gamma}\,(1 - \Delta\zeta\sigma/p)$$

$$\dot{\Lambda} \propto \dot{\gamma}\sigma/p\,(1 - \Lambda)$$

Test of STZ Flowing Steady State

$$\frac{\dot{\gamma}}{\sqrt{T}} \propto \sinh(\kappa\frac{\sigma}{p}) - \frac{p}{\zeta\sigma}\cosh(\kappa\frac{\sigma}{p})$$



How To Model Constitutive Relations

- Kinetic Theory

  Assume no microscopic structure.

- STZ Theory

  Assume a certain type of microscopic structure.

  Works well if the structure exists-- dense granular flows.

  cond-mat/0501535 (or come see my poster)

Where Do We Go From Here?

Given $\sigma,\, p,\, T,\, \dot{\gamma}$ there are 3 invariant quantities

$$\sigma/p,\, \dot{\gamma}\langle R\rangle/\sqrt{T},\, mT/p\langle R\rangle^2$$

To determine these quantities requires 3 relations in steady state. In the dense regime where Kinetic Theory does not apply,

One relation is furnished by STZ Theory.

One relation can be determined through energy balance.

One more relation must be discovered.