Correlation in Granular Flow

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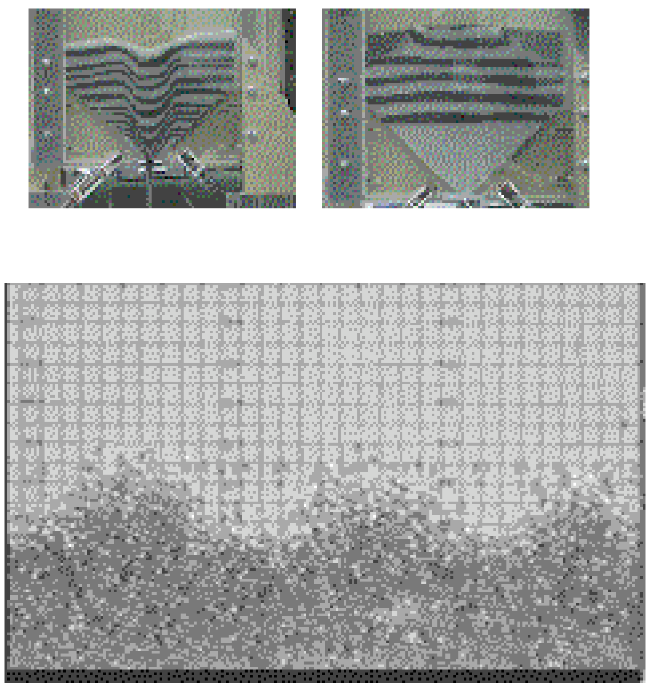
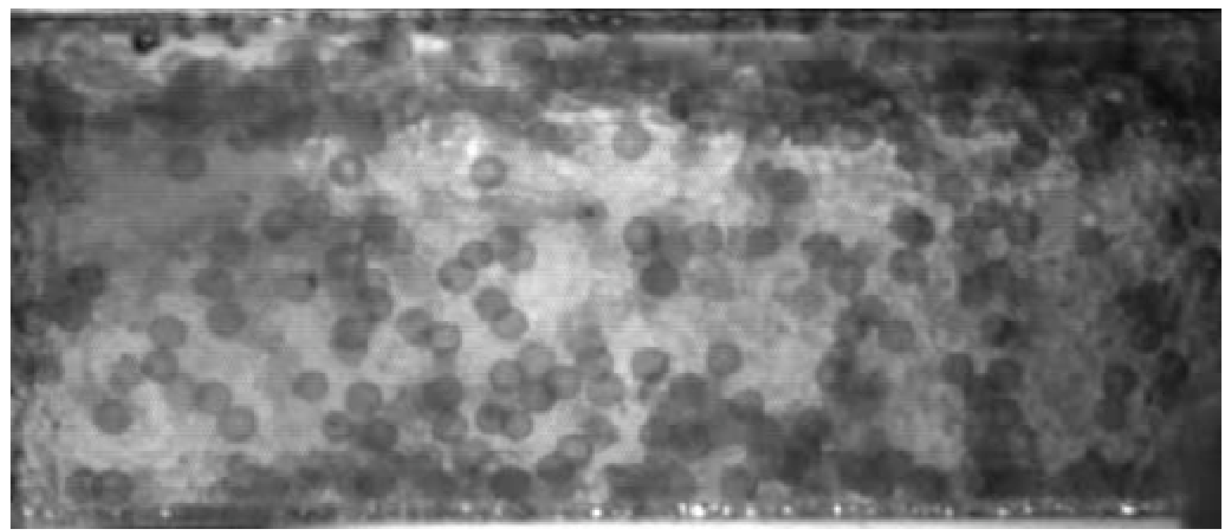
Jean Carlson, Anael Lemaitre

Why Granular Flows?

Big
Prevalent
Simple (Relatively)
Far from Equilibrium



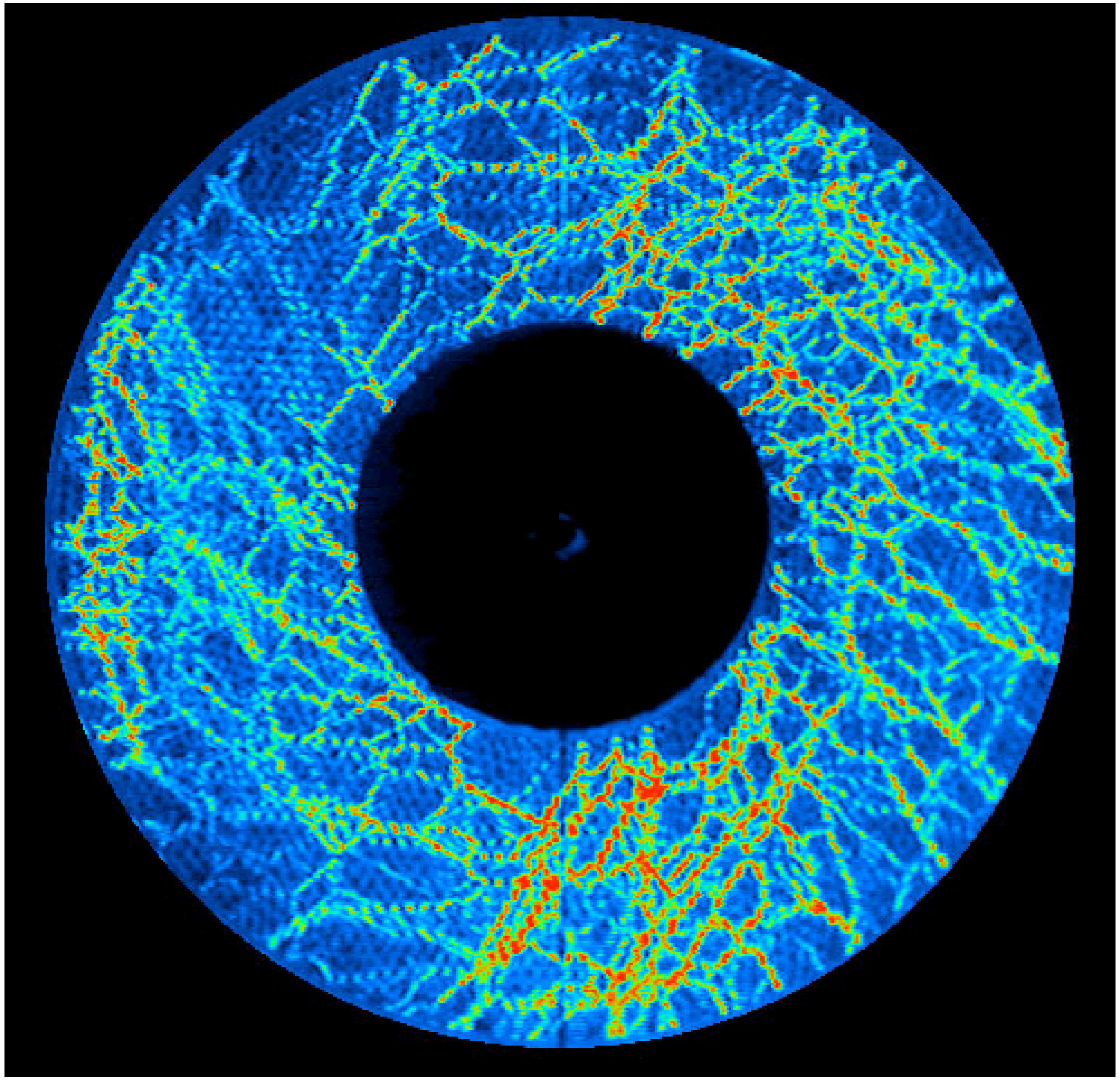
Melany Hunt, Caltech



Jerry Gollub, Haverford

Why Correlation?

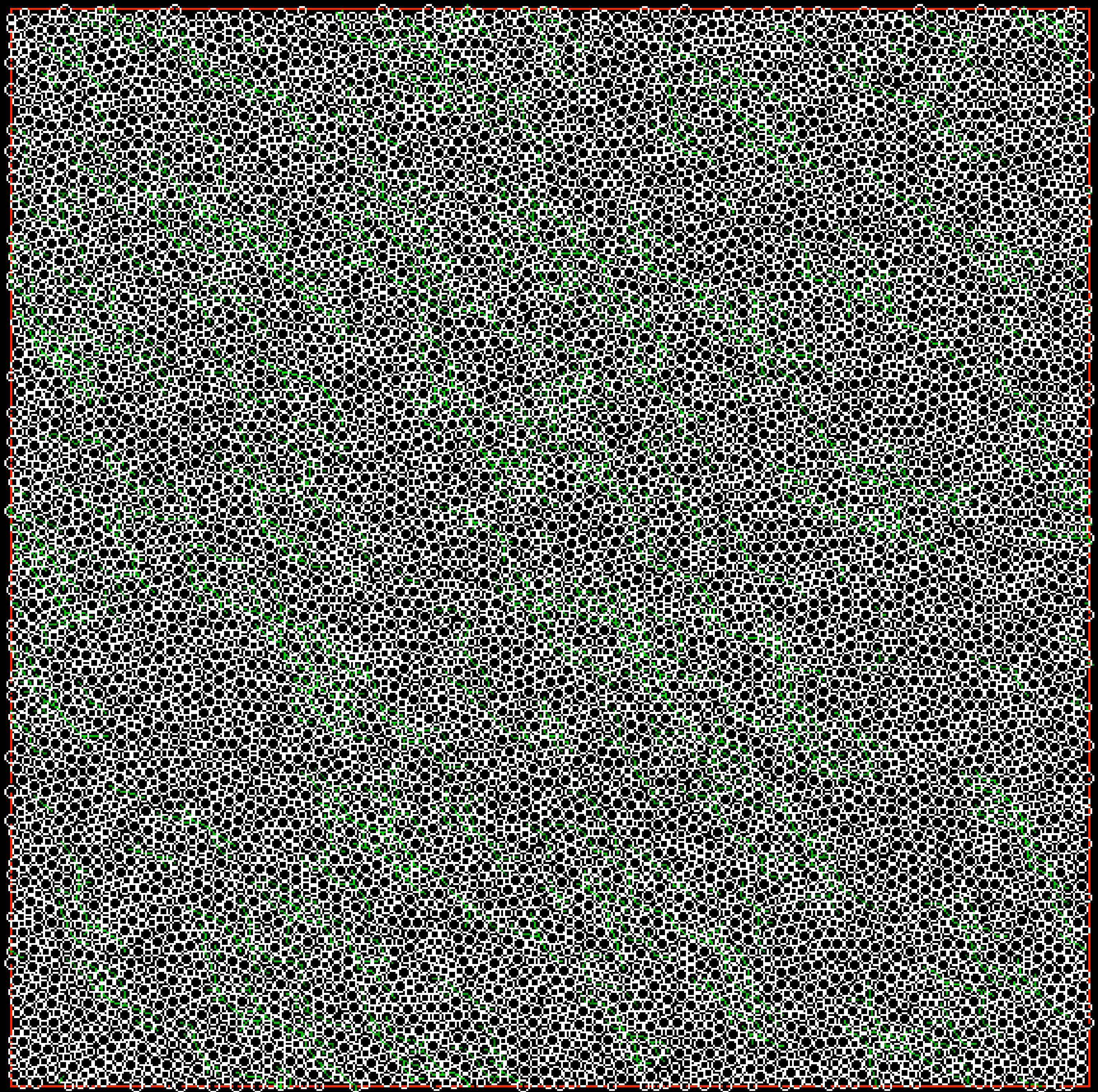
Because we know it should be there!



Bob Behringer, Duke

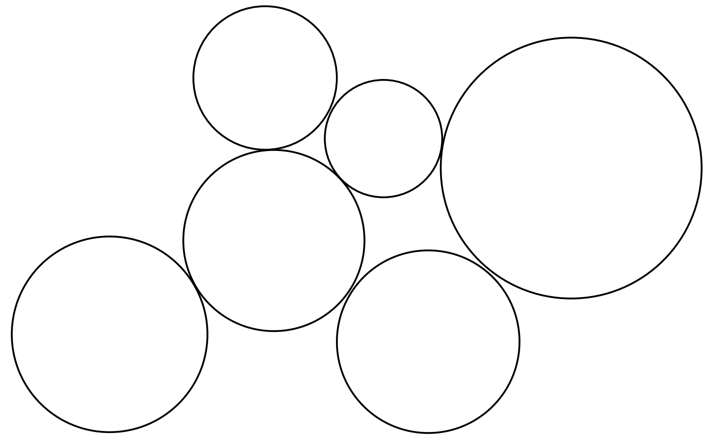
Outline

- Background
  - Numerical Algorithm for Shear Flows
  - Kinetic Theory
- Correlation Length
- Contact Force Distribution Function
- Constitutive Relations from Network Picture
- STZ Theory



Contact Dynamics

$$v_n^f = -e v_n^i$$



$$v_t^f = e_t v_t^i$$

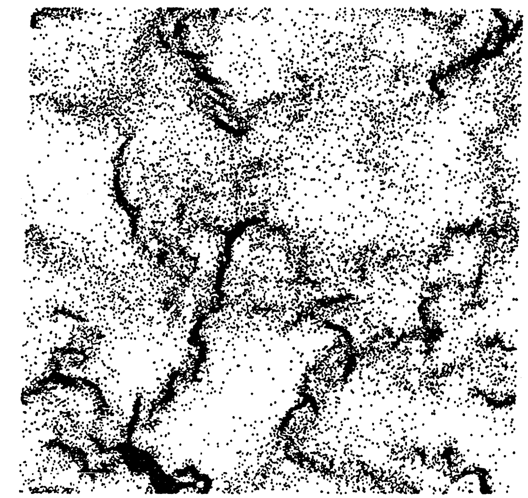
$$F_t \leq \mu F_n$$

No time-scale introduced by the interaction, grains are perfectly rigid

We investigate homogeneous shear flows

Independent variables: $e, e_t, \mu, \nu, \dot{\gamma}$

Simulations of Granular Flow



(Goldhirsch & Zanetti 1993)

Event Driven Simulations:

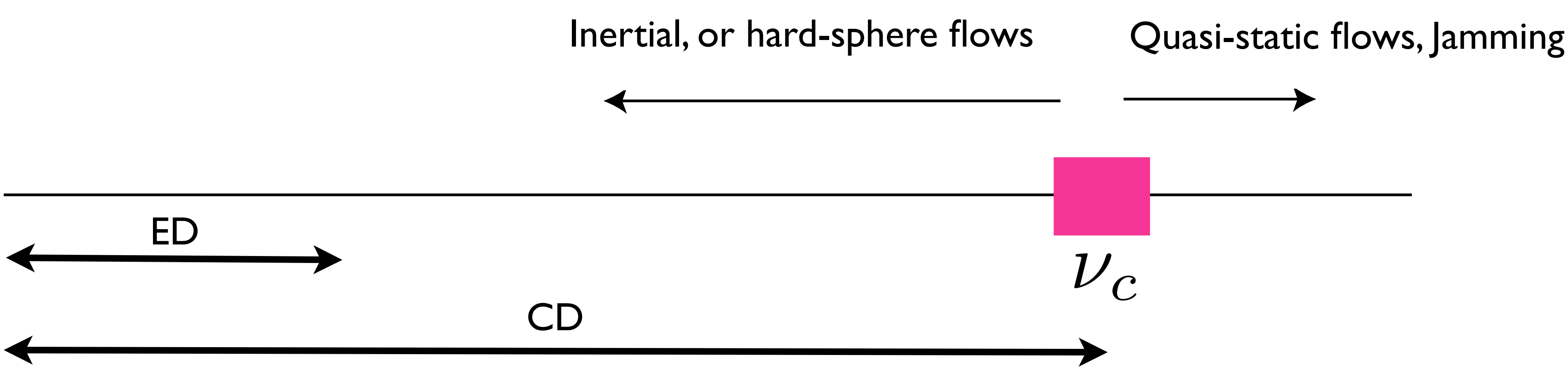
Same rules as CD, but assumes binary collisions

Can be applied to high restitution and low density

Soft-Sphere Simulations:

Characterized by stiffness $\kappa = \dfrac{k_n}{m\dot{\gamma}^2}$

(Campbell 2002, da Cruz 2005)



Kinetic Theory

Well developed for (frictionless) hard-spheres with constant restitution.

Collision Rule + Binary Collisions + Molecular Chaos

$$v_n^f = -e v_n^i$$

Test of Binary Collisions:

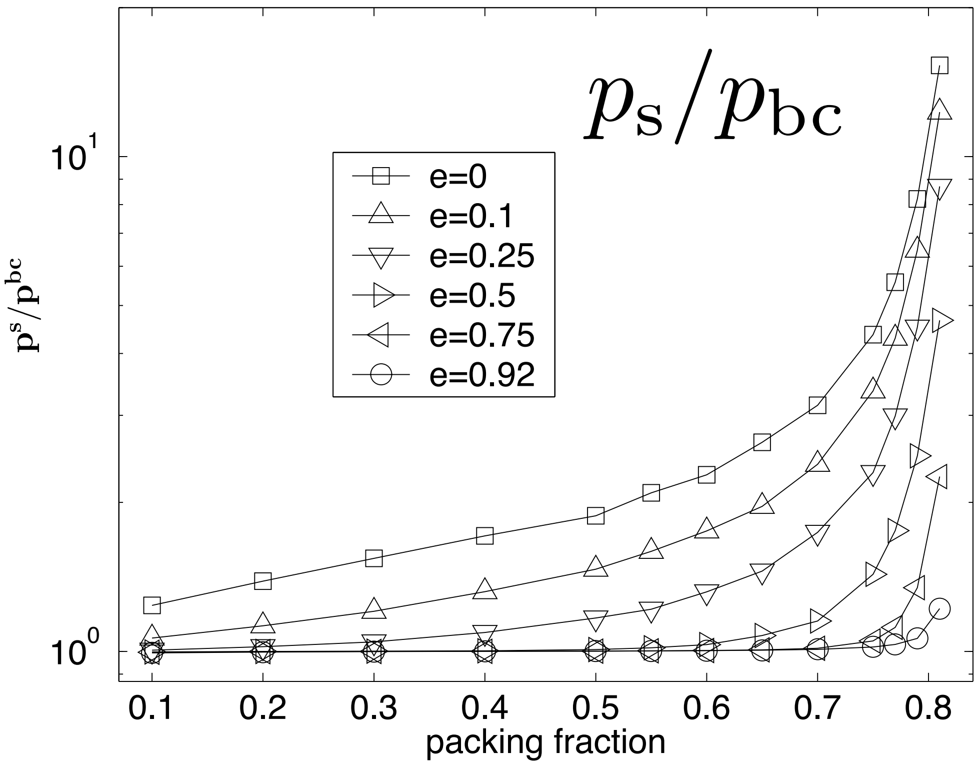
$$\Sigma_s^{\alpha\beta} \sim \sum_{\text{contacts}} \mathbf{R}^\alpha \mathbf{F}^\beta$$

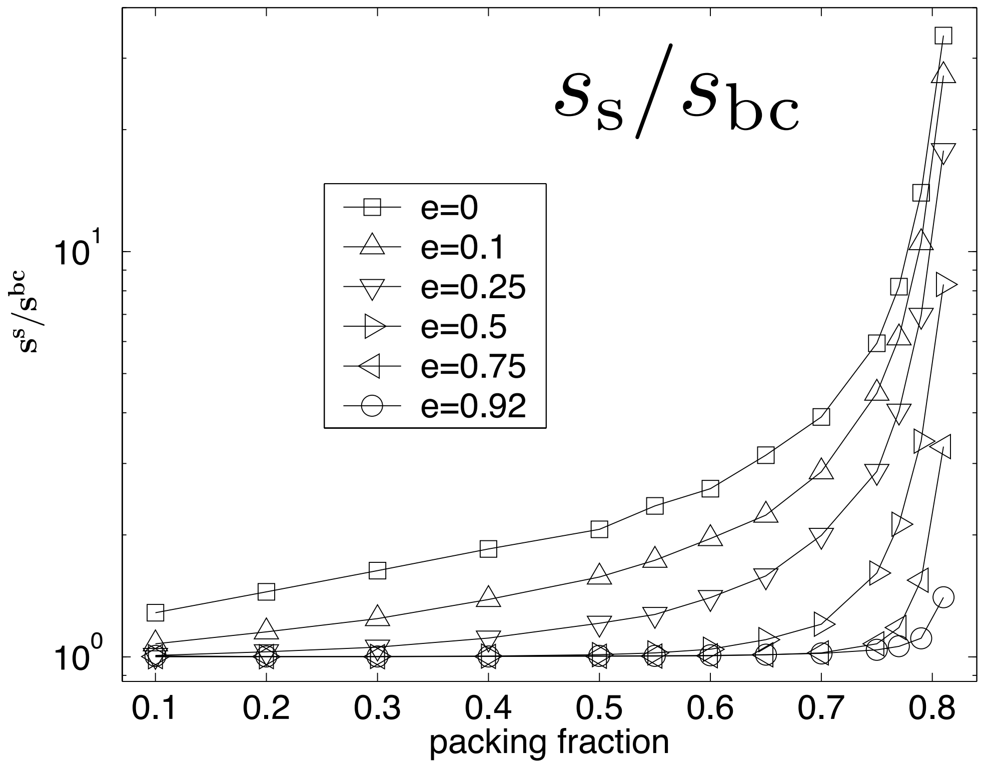
$$\Sigma_{bc}^{\alpha\beta} \sim \frac{1+e_n}{\Delta t} \sum_{\text{contacts}} \mathbf{R}^\alpha \mathbf{v_n}^\beta$$

Test of Molecular Chaos:

Do we see pre-collisional velocity correlations?
Do the predictions of KT match the measured stress?

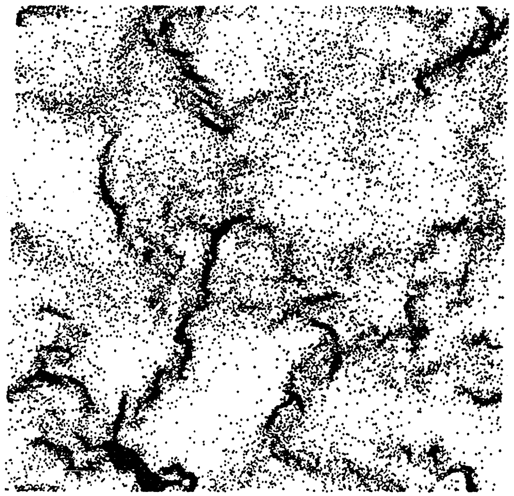
Test of Binary Collisions





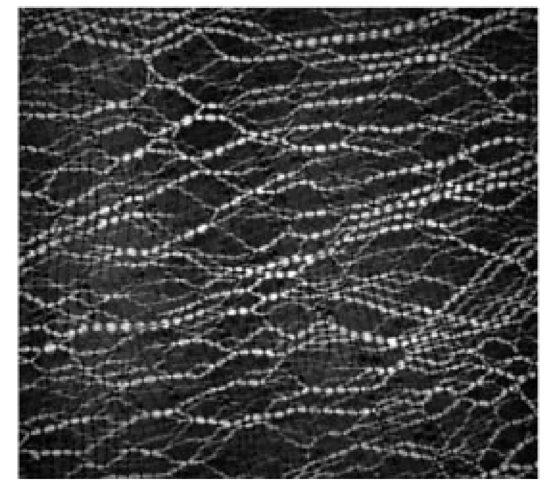
Binary Collision Assumption

There must be a microscopic indicator from some correlation measurement!



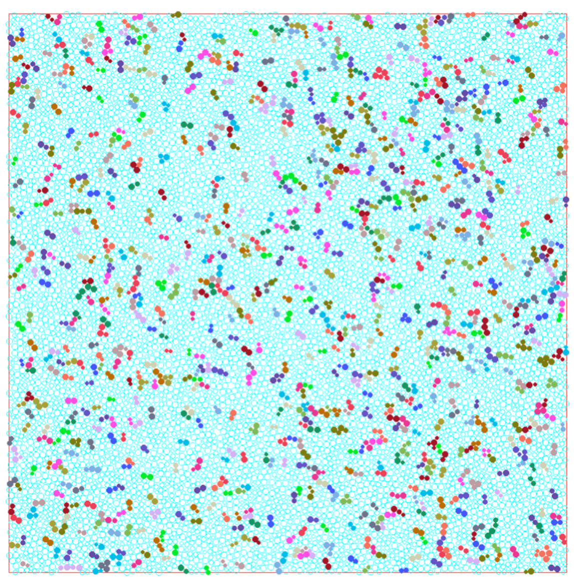
(Goldhirsch & Zanetti 1993)

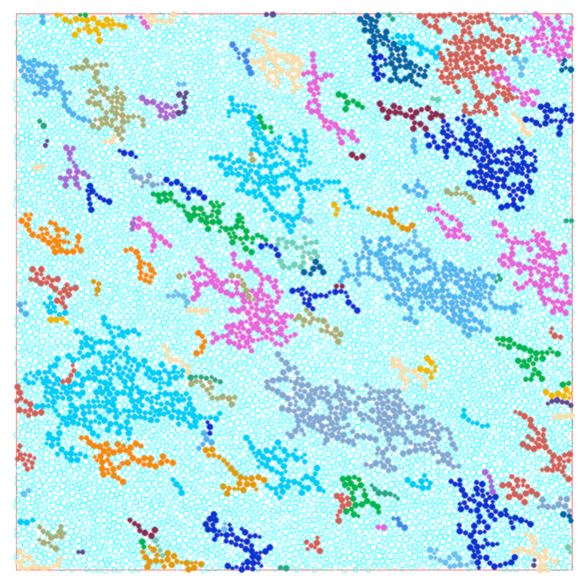
?



(Majmudar & Behringer 2005)

Low ν
High e

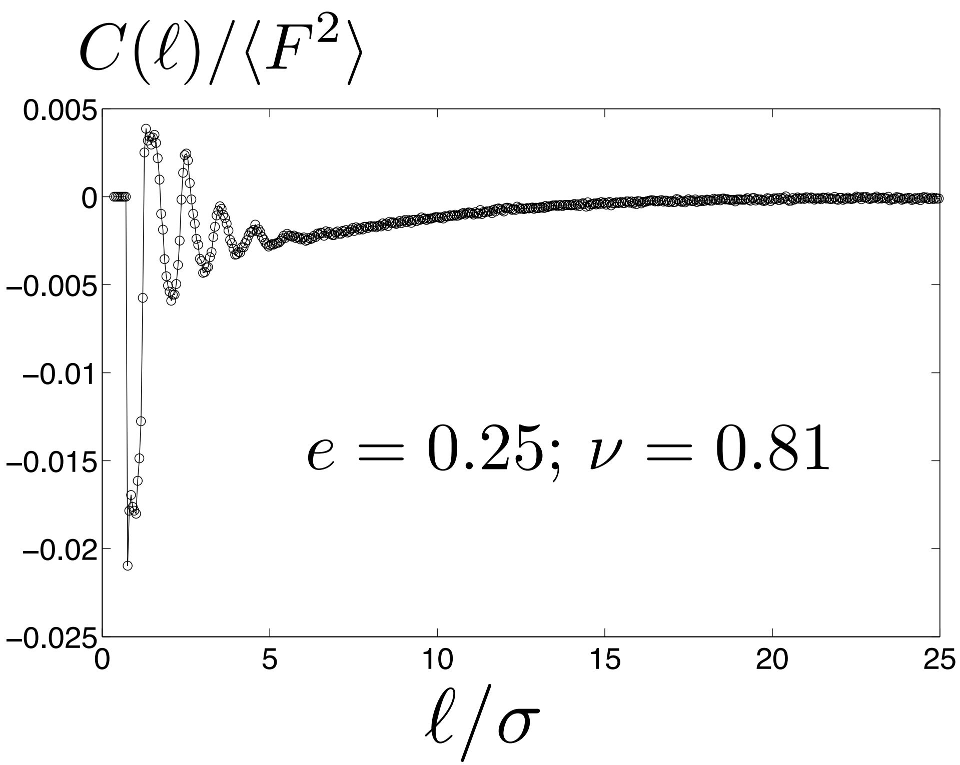


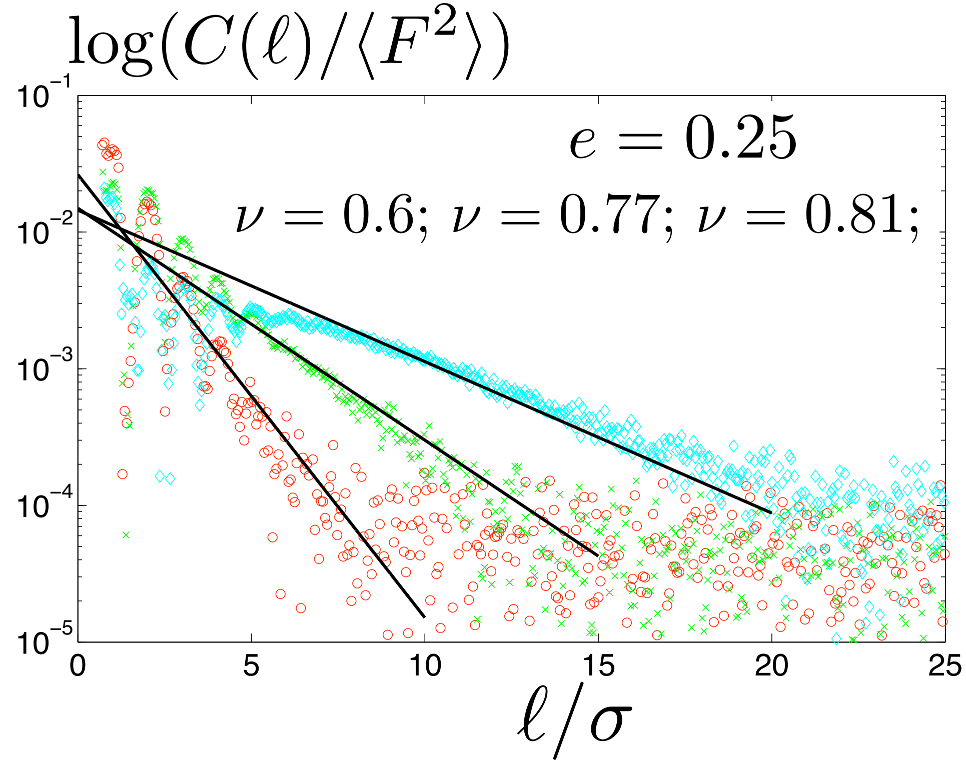


High ν
Low e

Spatial Force Correlations

$$C(\ell) = \sum_{\text{pairs}} \left(\vec{F}(r_i) \cdot \vec{F}(r_j) \right) \delta(\ell - |r_i - r_j|)$$

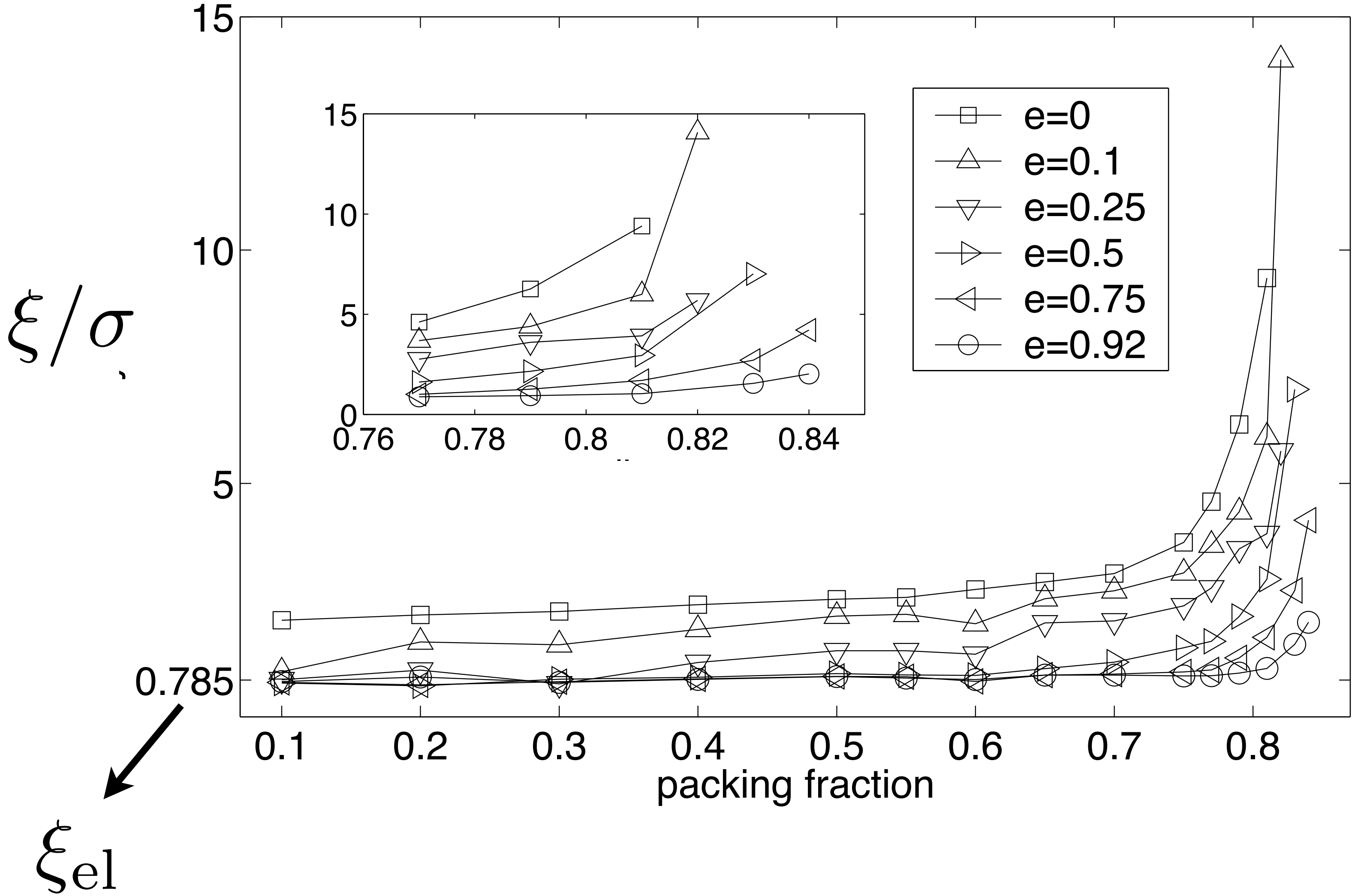




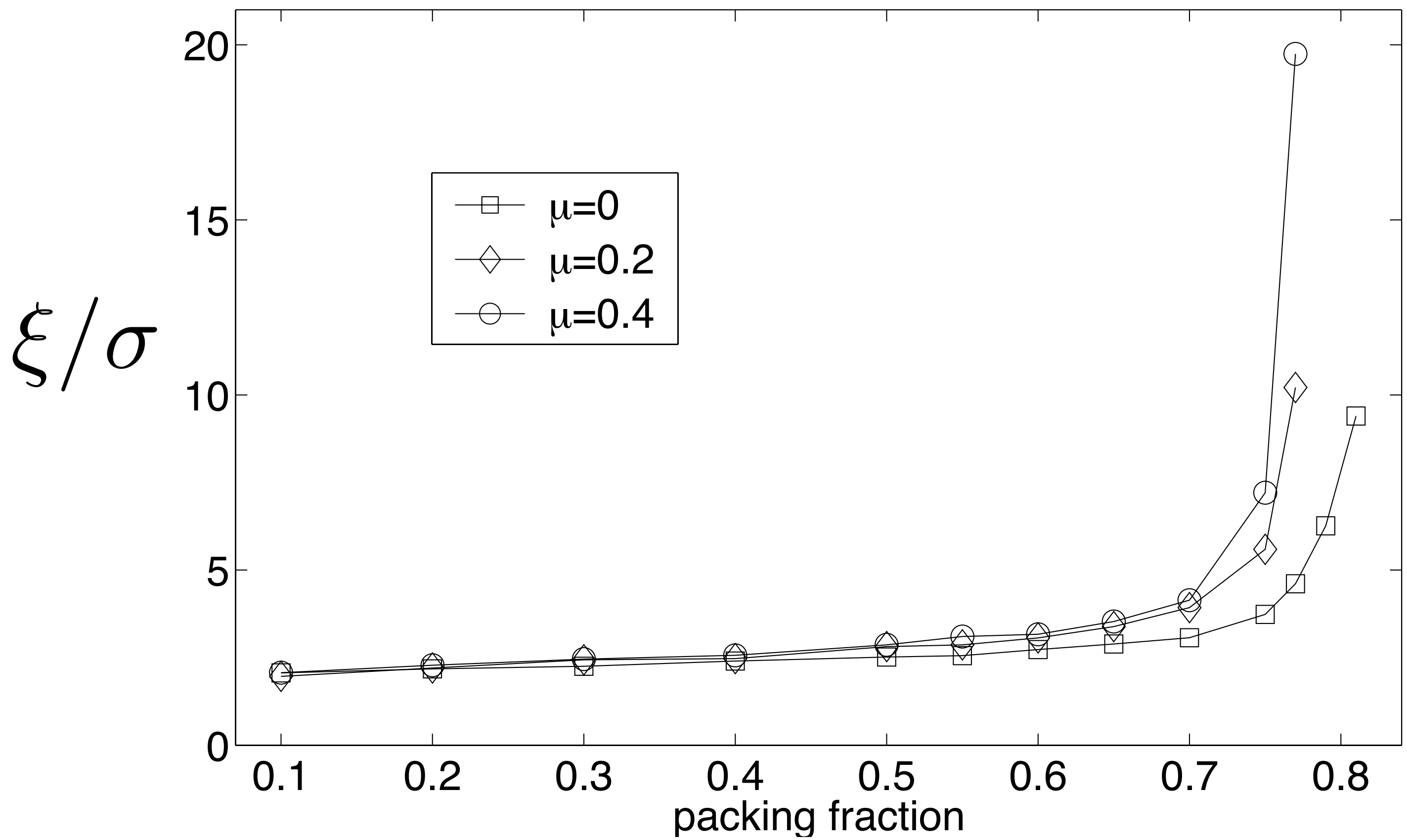
$$C(\ell) \sim e^{-\ell/\xi}$$

$$\xi = \frac{\int d\ell\, \ell C(\ell)}{\int d\ell\, C(\ell)}$$

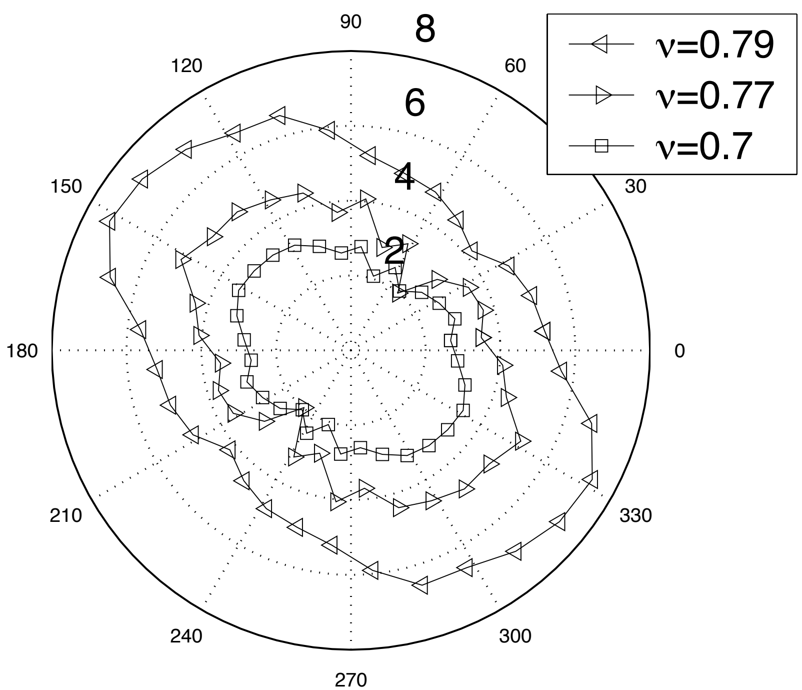
The correlation length

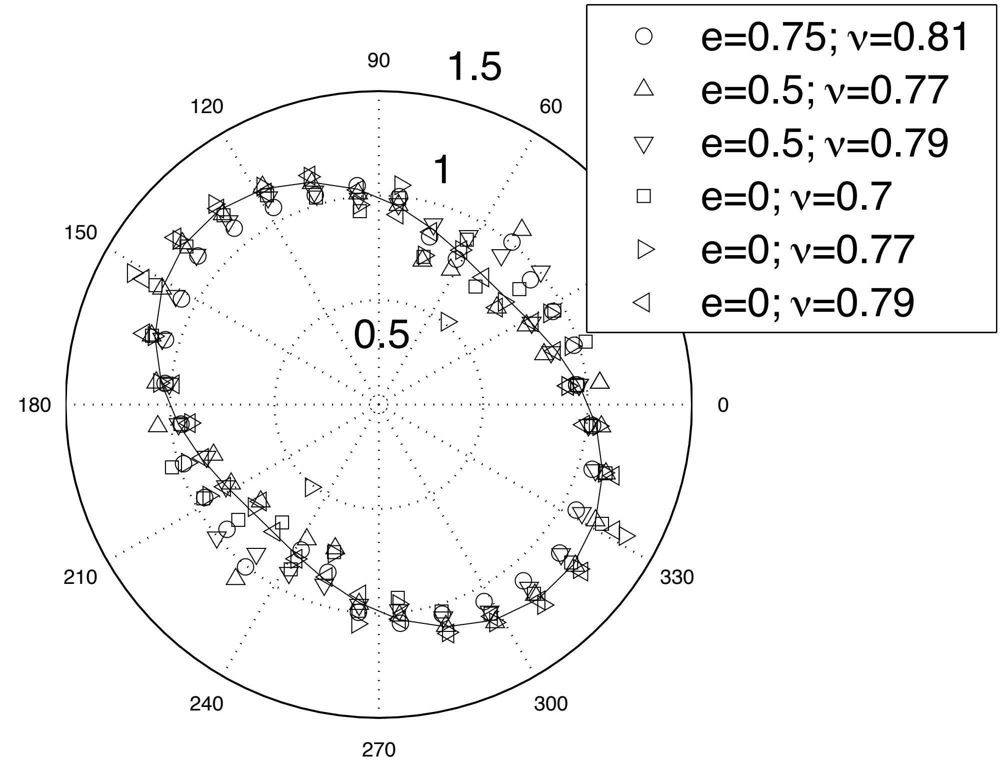


Including Friction



Considering Anisotropy

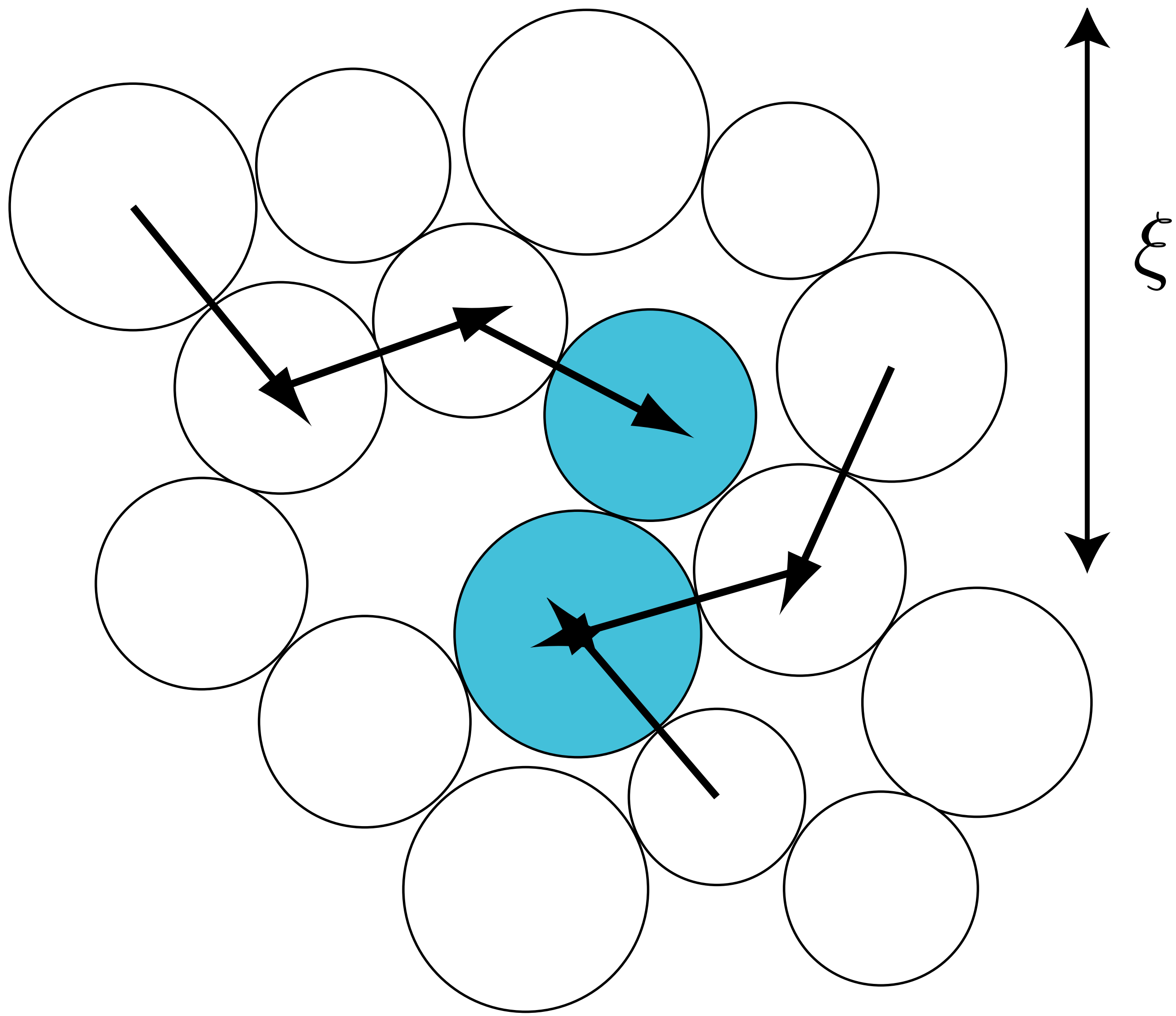




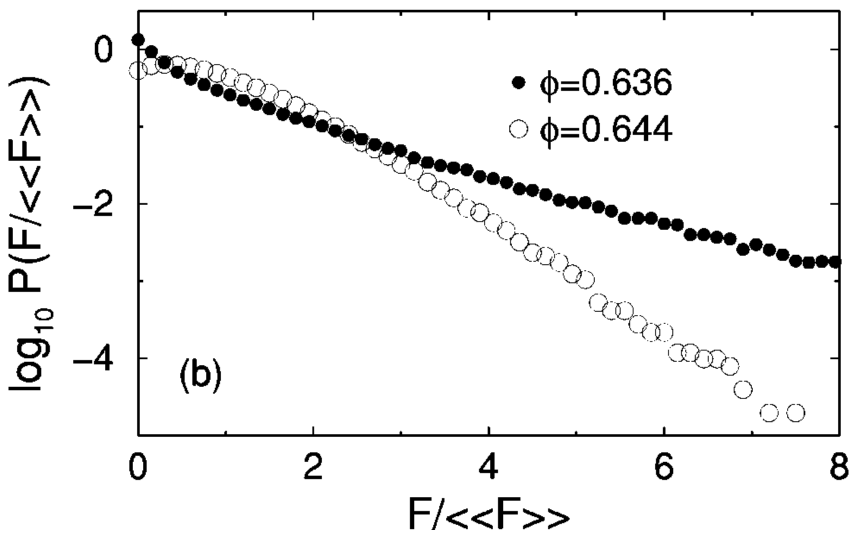
$$\xi(\theta) = \frac{\xi}{2\pi}\left(1 - a_0 \sin\left[2(\theta - \theta_0)\right]\right)$$

$$a_0 = 0.21 \qquad \theta_0 = 0.75^{\circ}$$

So Now What?



Contact Force Distributions

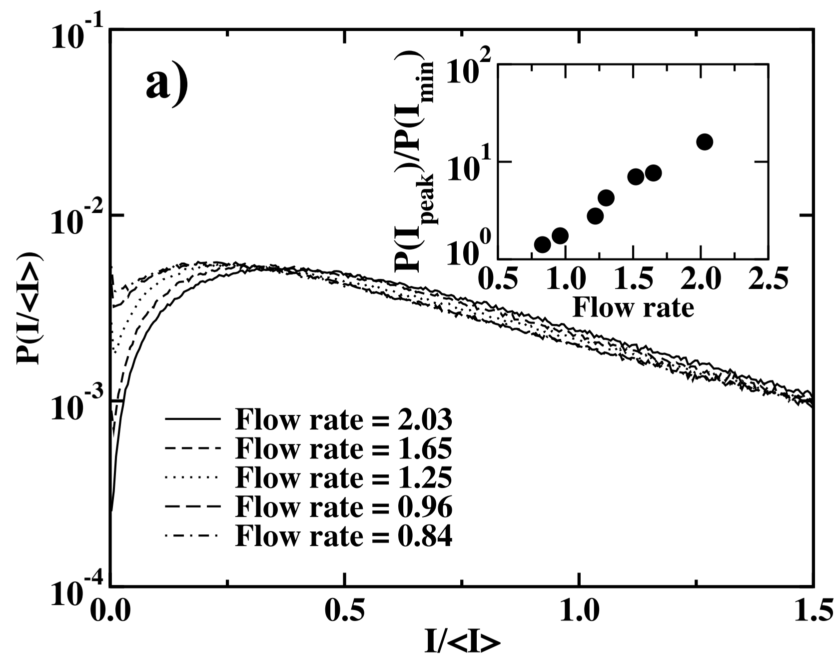


Formation of peak indicates that the system is jammed.

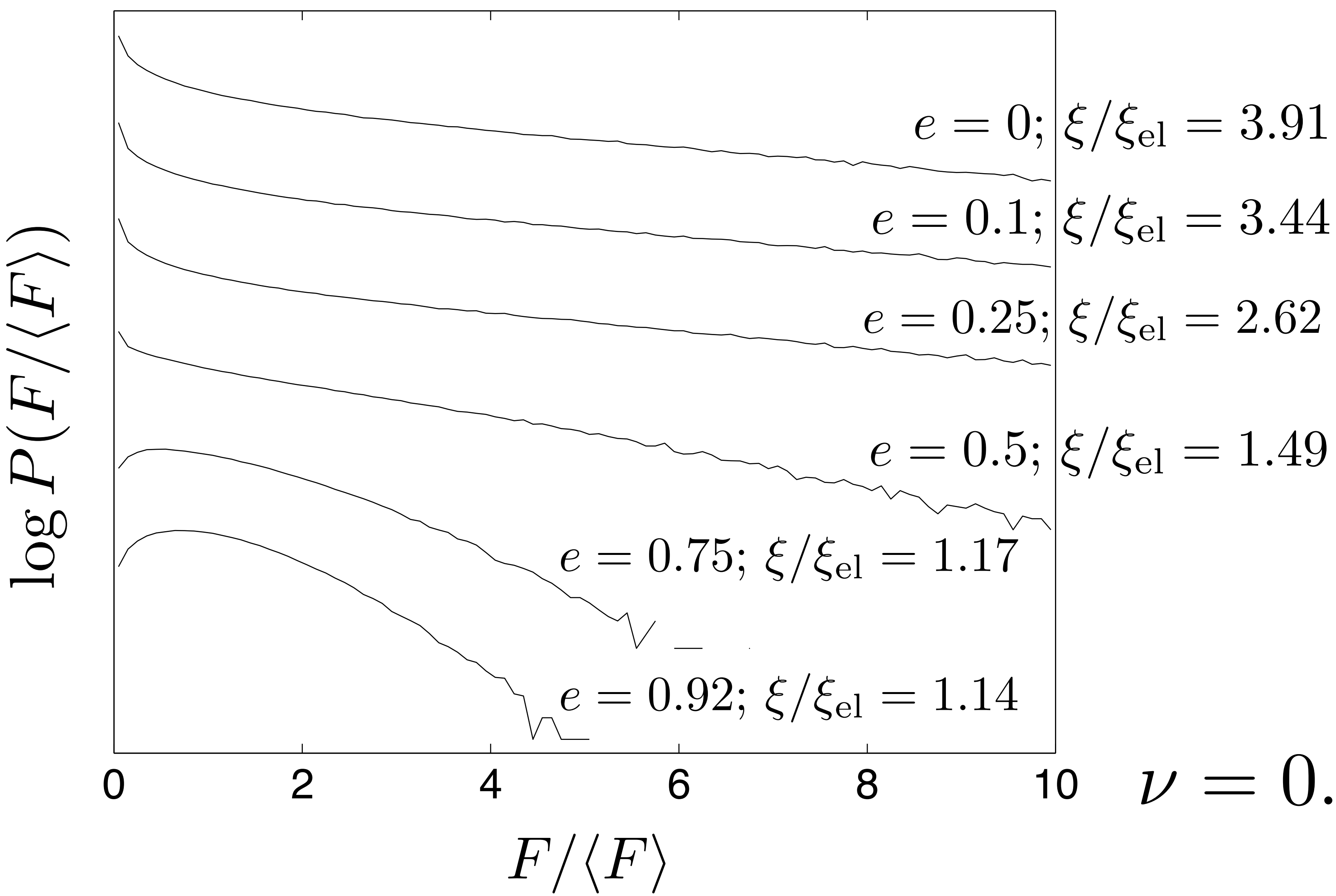
(O'Hern 2001, 2003)

Peak disappears as system approaches jamming.

(Ferguson 2004, Landry 2005)

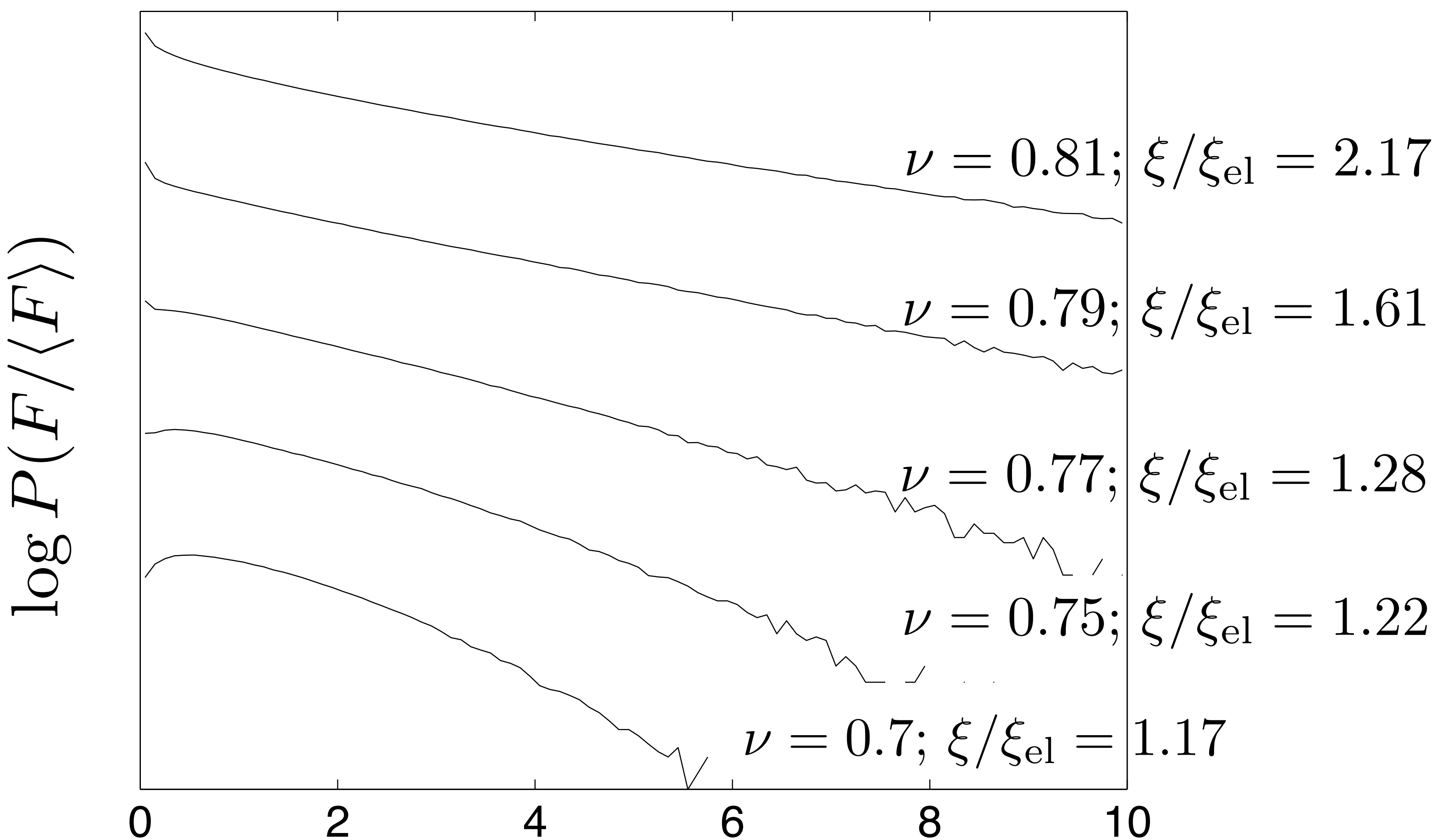


What we find, for constant packing fraction



$\nu = 0.7$

What we find, for constant restitution

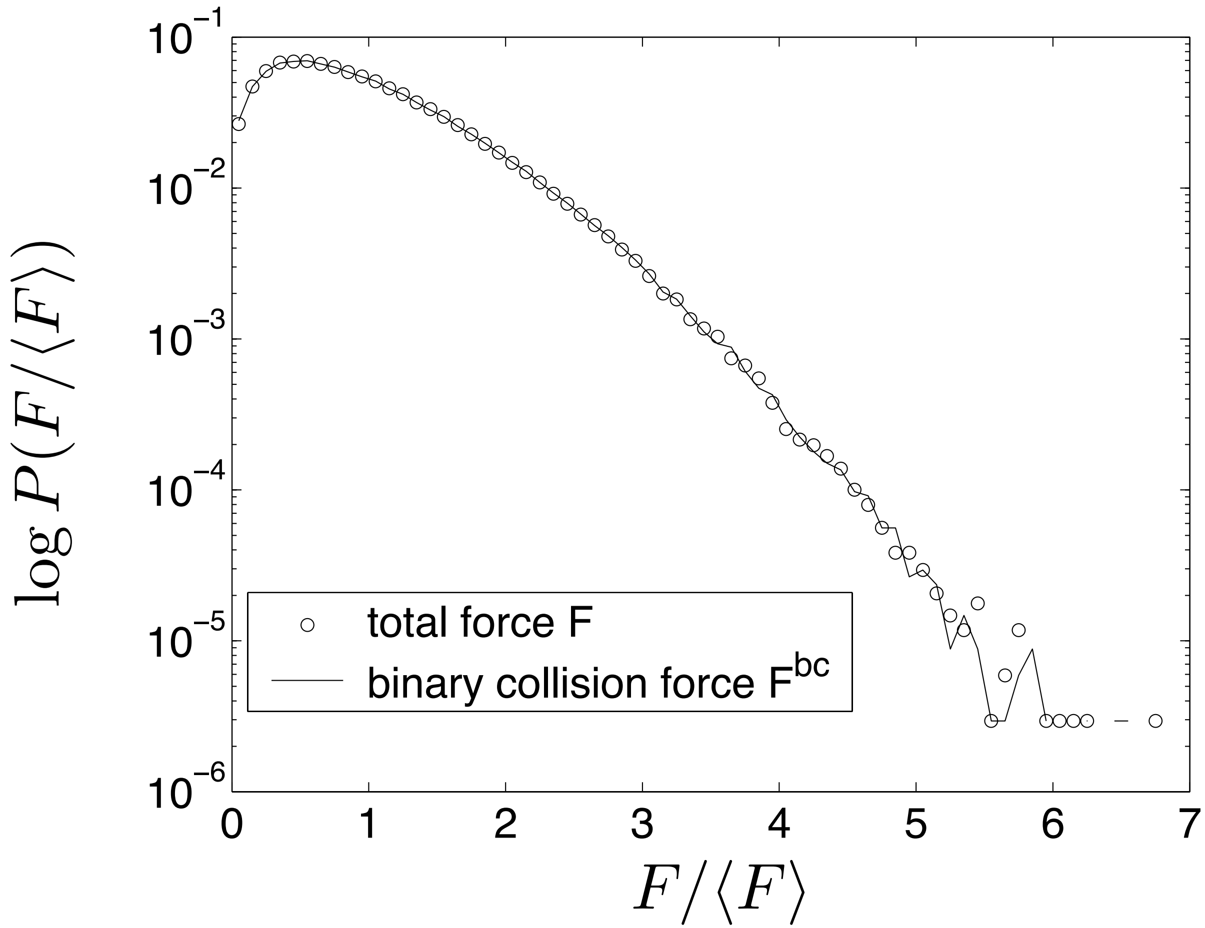


$e = 0.75$

$\xi = 1.25\,\xi_{\mathrm{el}}$

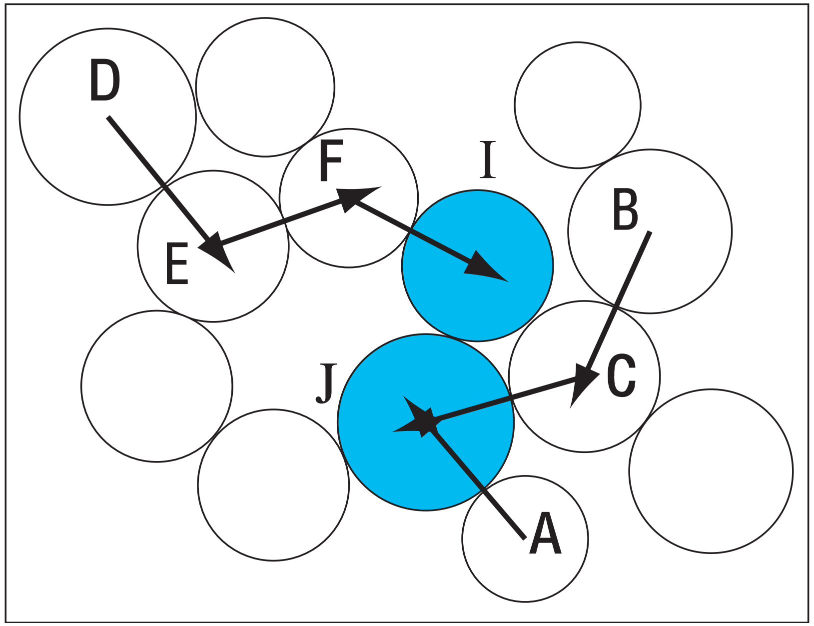
P(F) determined by Kinetic Theory for small ξ

$$e = 0.75;\ \nu = 0.7;\ \xi/\xi_{\mathrm{el}} = 1.17$$



$$F_{ij}^{\mathrm{bc}} = \frac{1+e}{dt}\mu_{ij}(v_i - v_j)\cdot\hat{\sigma}_{ij}$$

In the Dense Regime



$$F_{IJ} = F_{IJ}^{\mathrm{bc}} + \sum_{p=1}^{n} \mathcal{F}_{IJ}(p)$$

$$F_{IJ}^{\mathrm{bc}} = \frac{1+e}{dt} \mu_{IJ} (v_I - v_J) \cdot \hat{\sigma}_{IJ}$$

$$\mathcal{F}_{IJ}(1) = \sum_{m=1;m\neq J}^{z_I} (\hat{\sigma}_{mI} \cdot \hat{\sigma}_{IJ})\, F_{mI}^{\mathrm{bc}} + \sum_{n=1;n\neq I}^{z_J} (\hat{\sigma}_{nJ} \cdot \hat{\sigma}_{IJ})\, F_{nJ}^{\mathrm{bc}}$$

$$\mathcal{F}_{IJ}(2) = \sum_{m=1;m\neq J}^{z_I} (\hat{\sigma}_{mI} \cdot \hat{\sigma}_{IJ}) \sum_{m_2=1;m_2\neq m}^{z_m} (\hat{\sigma}_{mm_2} \cdot \hat{\sigma}_{mI})\, F_{mm_2}^{\mathrm{bc}} + \{I \leftrightarrow J\}$$

Pressure in the Dense Regime

$$p = \frac{1}{2}\mathrm{Tr}\,\Sigma \propto \sum_{ij} \sigma_{ij} F_{ij} \approx \langle \sigma \rangle \langle F \rangle \quad \Rightarrow \quad \frac{p^{\mathrm{s}}}{p^{\mathrm{bc}}} = \frac{\langle F \rangle}{\langle F^{\mathrm{bc}} \rangle}$$

$$\langle F \rangle = \langle F^{\mathrm{bc}} \rangle + \sum_{p=1}^{\xi/\xi_{\mathrm{el}}-1} \langle \mathcal{F}(p) \rangle$$

$$\langle \mathcal{F}(p) \rangle = 2\langle F^{\mathrm{bc}} \rangle [\alpha(z-1)]^p$$

Pressure in the Dense Regime

$$\frac{p^{\mathrm{s}} - p^{\mathrm{bc}}}{p^{\mathrm{bc}}} = 2\frac{G - G^{\xi/\xi_{\mathrm{el}}}}{1 - G} \qquad G = \alpha(z-1)$$

The dilute limit

As $z \longrightarrow 1$ or $\xi \longrightarrow \xi_{\mathrm{el}}$ then $p^{\mathrm{s}} \longrightarrow p^{\mathrm{bc}}$

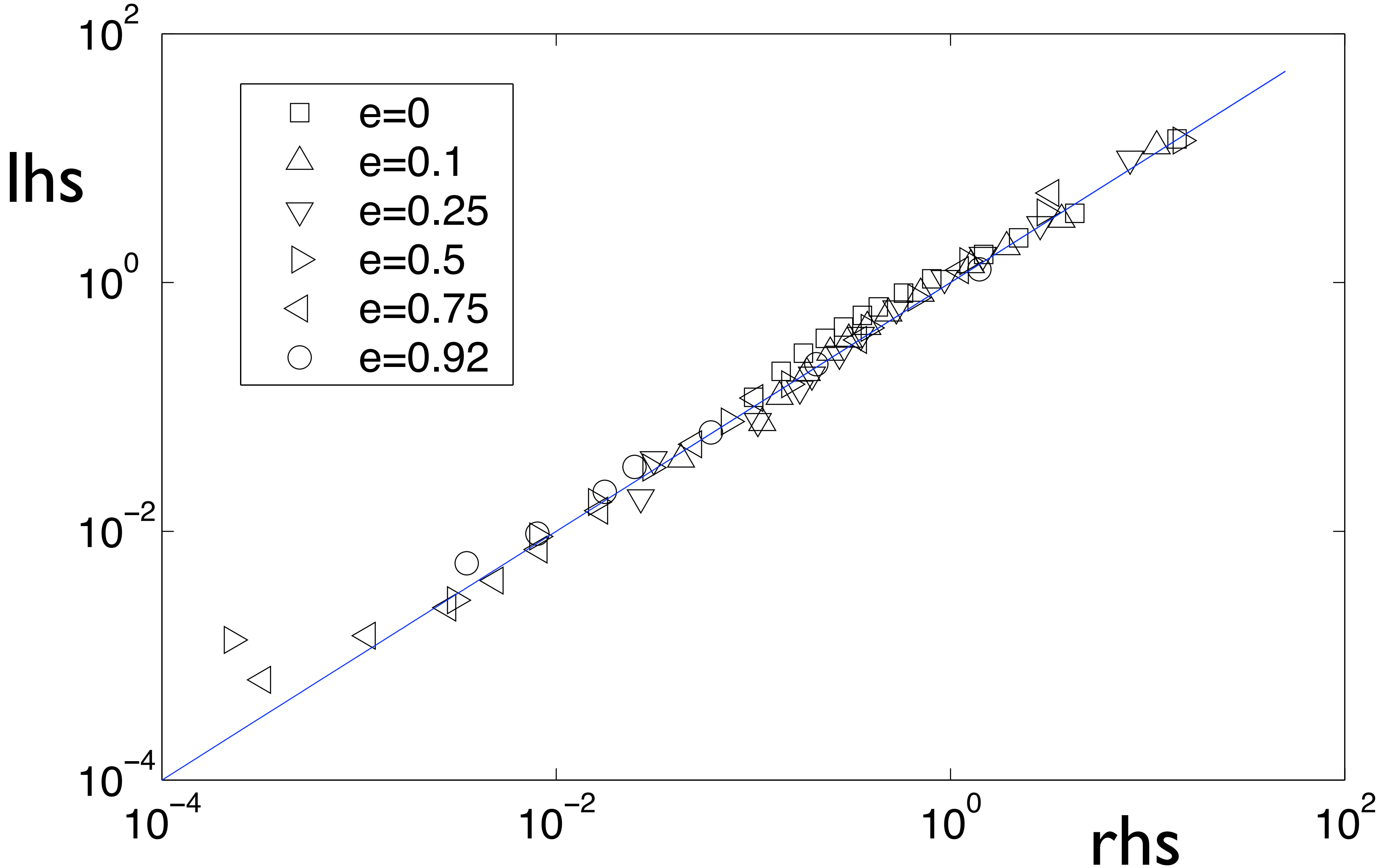
The jamming limit

$p^{\mathrm{bc}} \longrightarrow 0$ which requires that $\xi \longrightarrow \infty$ and $z \geq 1 + \alpha^{-1}$

This sets $\alpha = D^{-1}$

Testing with simulation data

$$\frac{p^{\rm s} - p^{\rm bc}}{p^{\rm bc}} = 2\frac{G - G^{\xi/\xi_{\rm el}}}{1 - G} \qquad G = \alpha(z-1)$$



Shear Stress in the Dense Regime

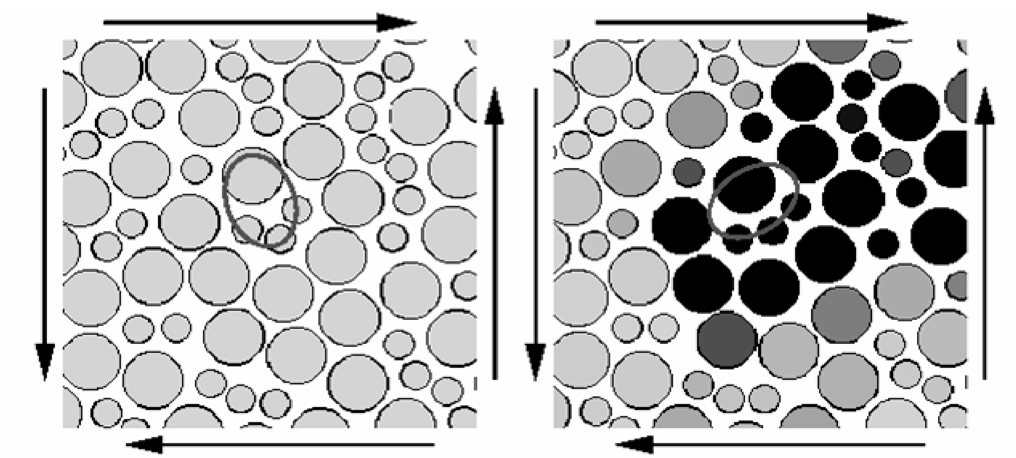
$$s = \frac{1}{2}\left(\Sigma^{xy} + \Sigma^{yx}\right) \propto \sum_{ij} \hat{\sigma}^{x}_{ij} \hat{\sigma}^{y}_{ij} F^{ij}$$

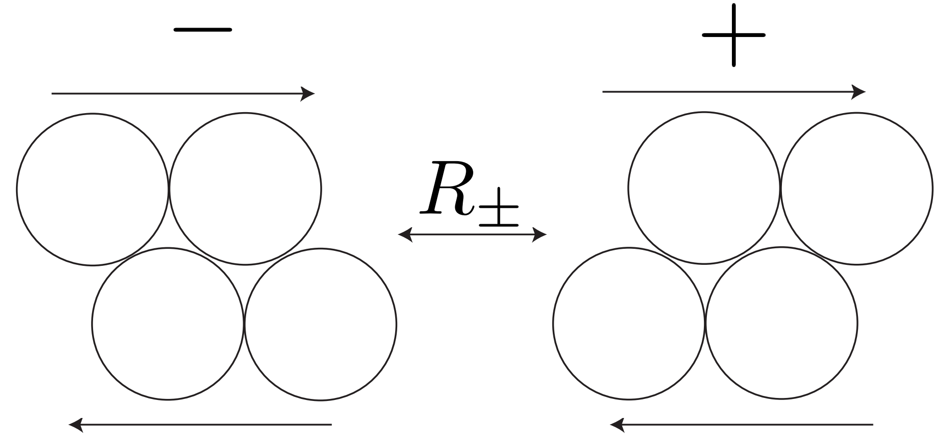
Depends on the orientation of contacts-- the geometry of networks. STZ theory explicitly considers the orientation of contacting zones to determine constitutive relations.

STZ theory has been tested previously (Lois, Lemaitre & Carlson 2005), for frictional and frictionless granular flows with e=0.

STZ Theory of Amorphous Solids

(1) Non-affine (plastic) motion occurs in localized regions
(2) The regions undergoing non-affine motion have orientation





$$\dot{\gamma}^{pl} \propto R_- n_- - R_+ n_+$$

(Falk & Langer 1997)

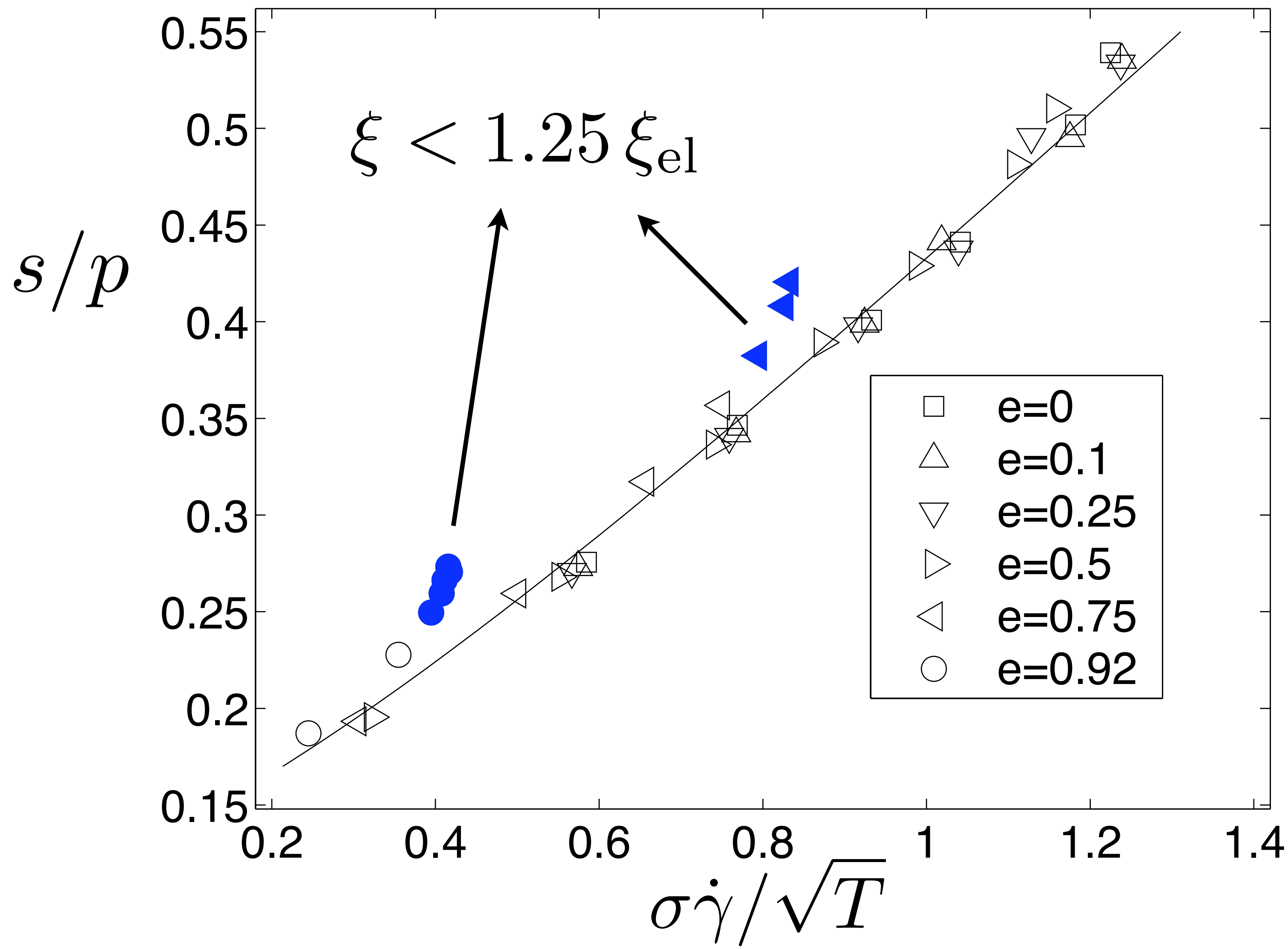
$$\dot{n}_\pm = R_\mp n_\mp - R_\pm n_\pm + w(1 - \zeta n_\pm)$$

$$w \propto s\dot{\gamma}/p \qquad R_\pm \propto \sqrt{T} e^{\pm \kappa\sigma/p}$$

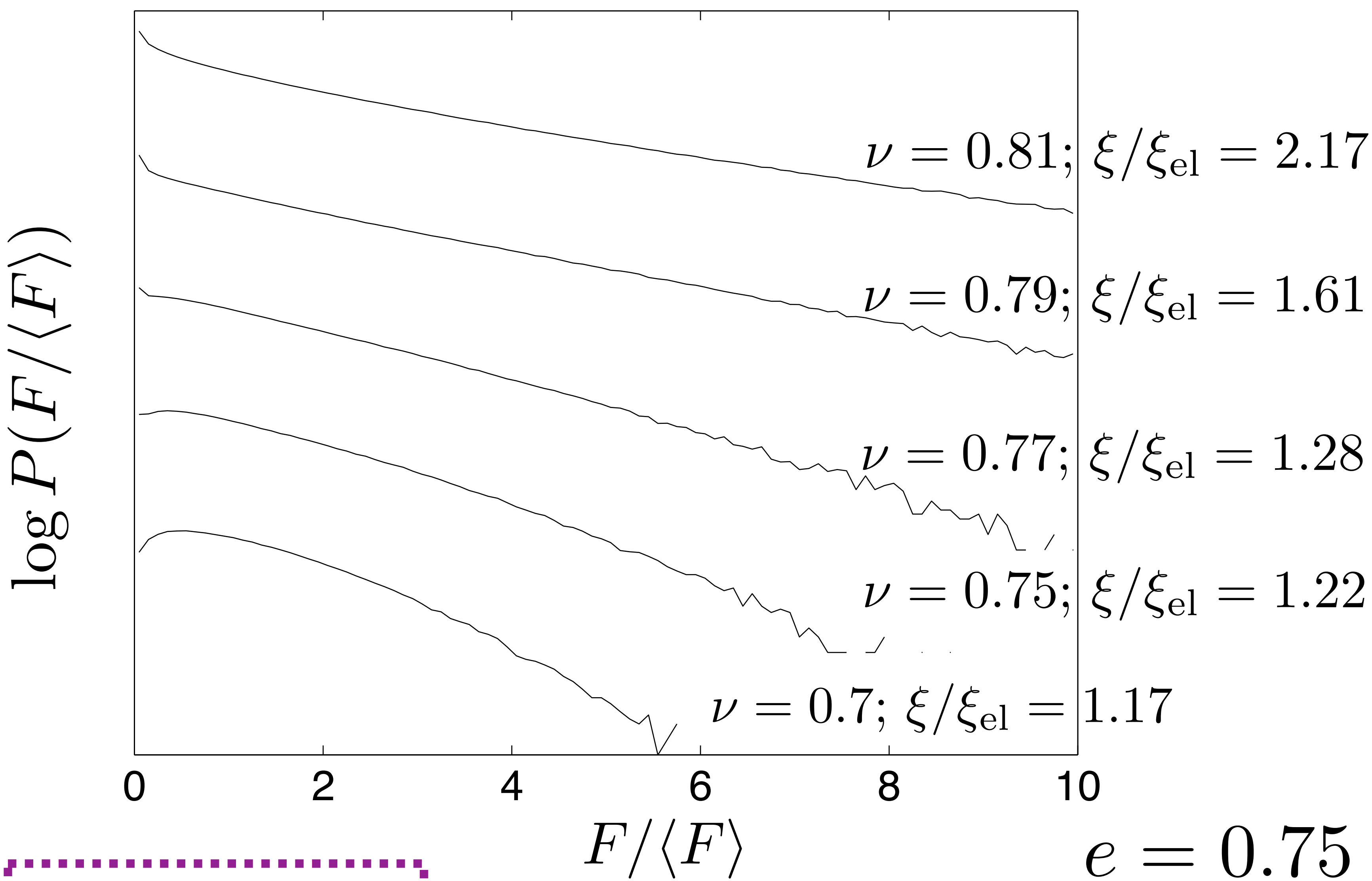
(Lemaitre 2002)

Test of STZ Flowing Steady State

$$\frac{\sigma\dot{\gamma}}{\sqrt{T}} \propto \sinh\left(\kappa\frac{s}{p}\right) - \frac{p}{\zeta s}\cosh\left(\kappa\frac{s}{p}\right)$$



What we find, for constant restitution



$e = 0.75$

$\xi = 1.25\,\xi_{\rm el}$

Conclusions

- Can Measure Correlation to quantify force networks
- Play a key role in constitutive modeling
  - Kinetic Theory
  - Pressure Model
  - STZ Model
- The appearance of correlations can be measured with the contact force distribution function