Velocity Gap and Interface Cracks

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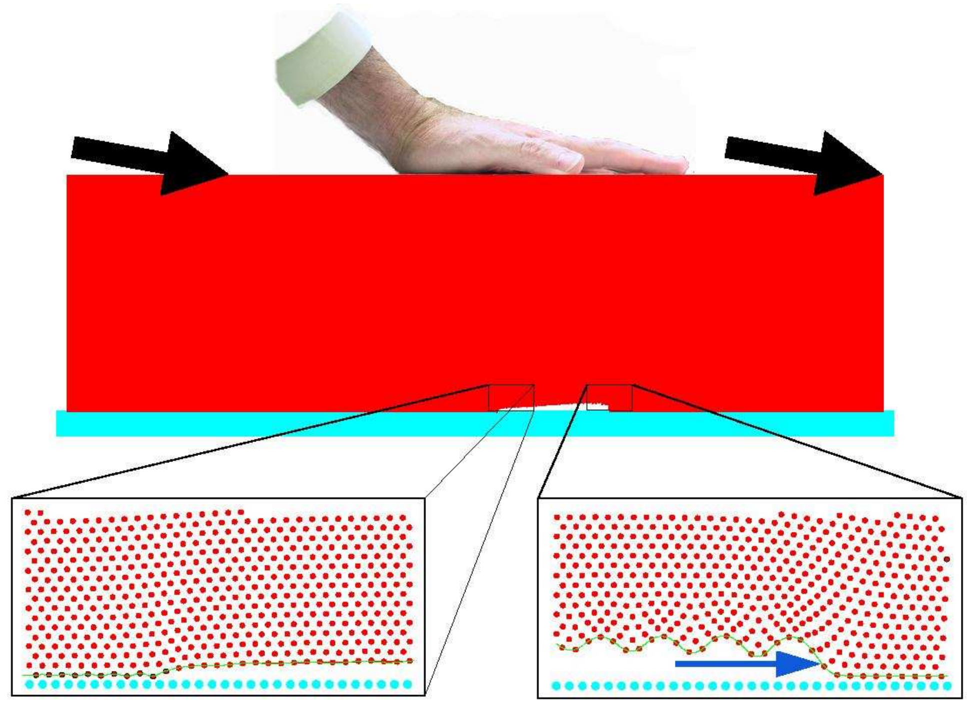
Center for Nonlinear Dynamics



The University of Texas at Austin

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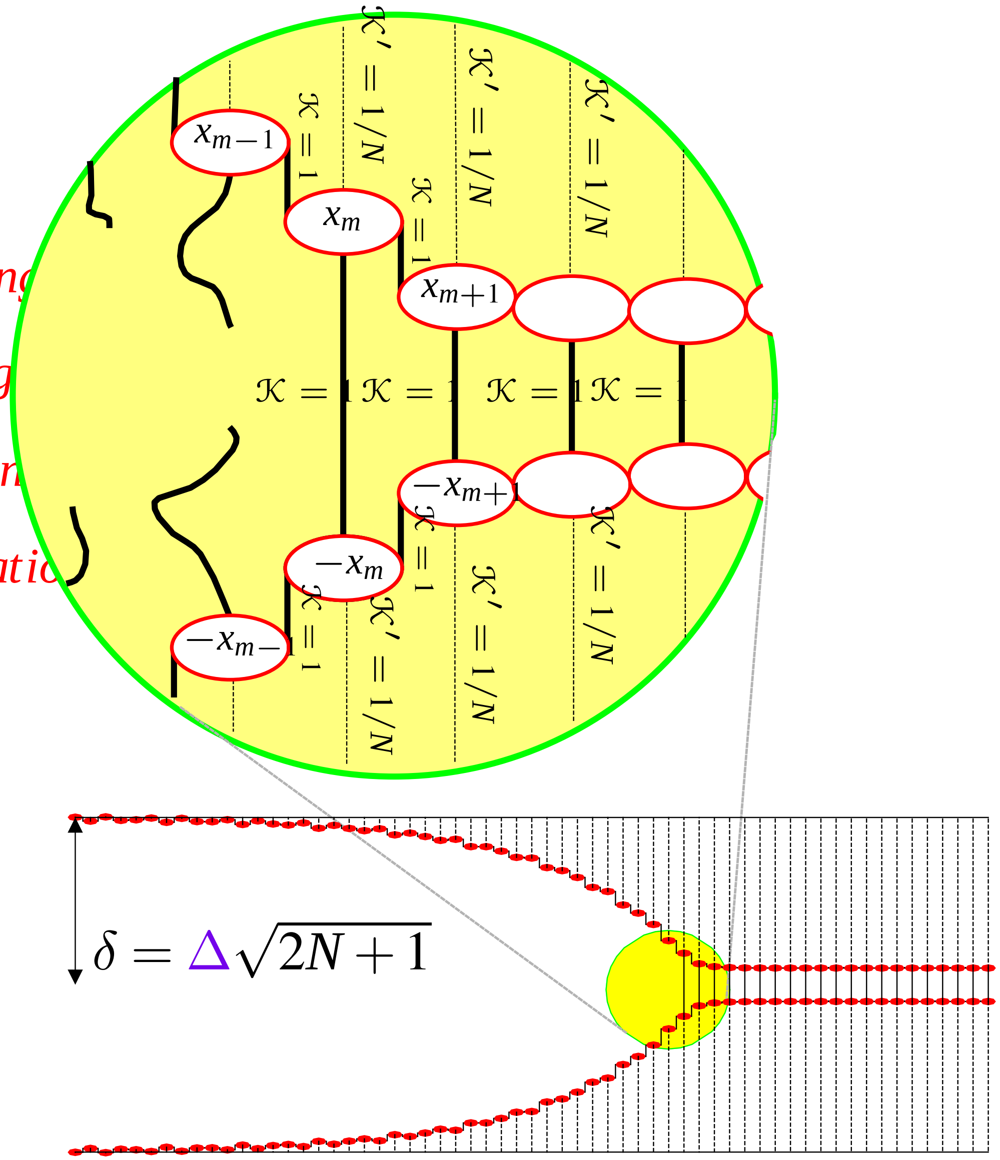
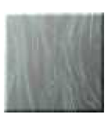
Atomic Solutions

$$F_m = \begin{cases} x_{m+1} - 2x_m + x_{m-1} & \textit{Couplin} \\ +\frac{1}{N}(\Delta\sqrt{2N+1} - x_m) & \textit{Drivin} \\ -2x_m\theta(1-x_m) & \textit{Snappin} \\ -b\dot{x}_m & \textit{Dissipati} \end{cases}$$

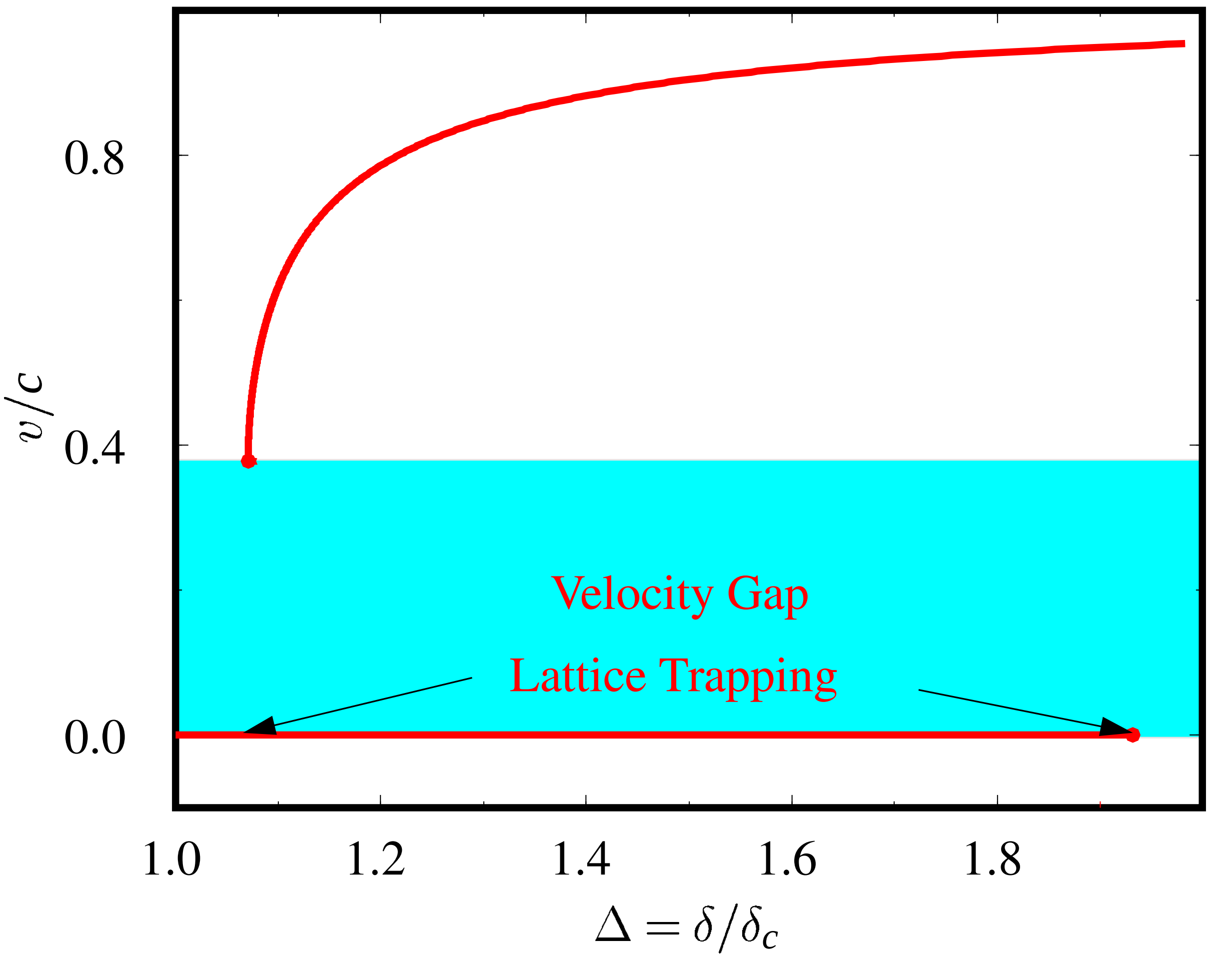


Δ is scaled boundary displacement;

Δ equals 1 when bonds can **first break.**



Atomic Solutions



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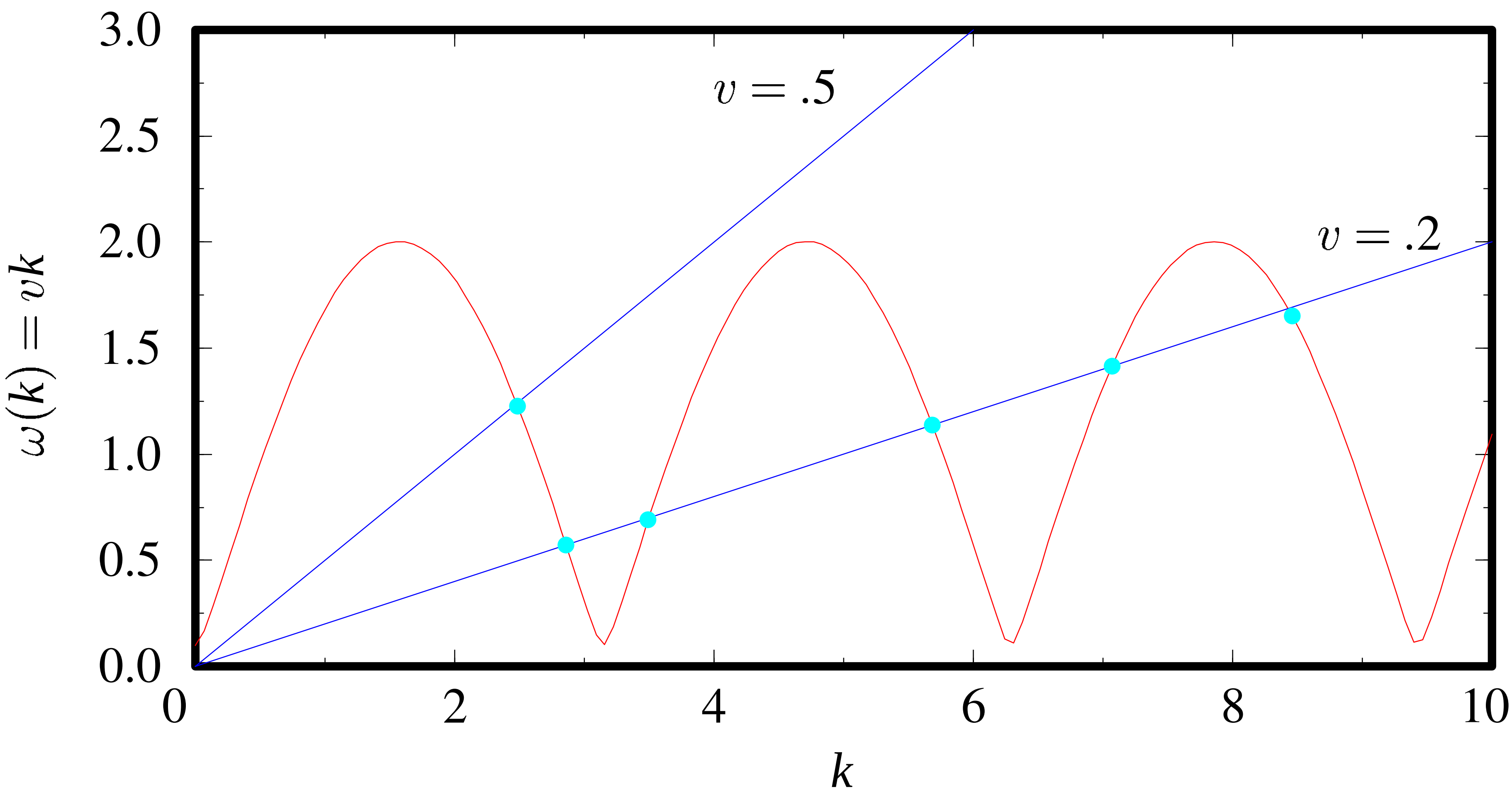
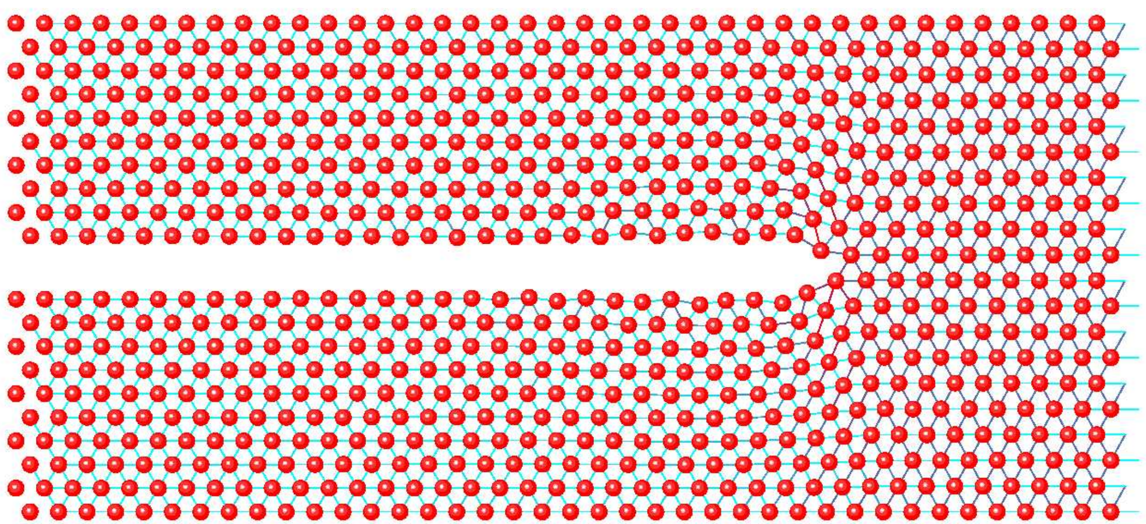


$$\vec{k} \cdot \vec{v} = \omega$$

Two-dimensional models

(Slepyan, 1980; Kulakhmetova, Saraikin and Slepyan 1984, Marder and Gross 1995, Kessler and Levine 1999)

- Steady-state solutions are analytical
- Treat infinite numbers of atoms in all directions
- Connect macroscopic and microscopic scales
- Develop scaling laws



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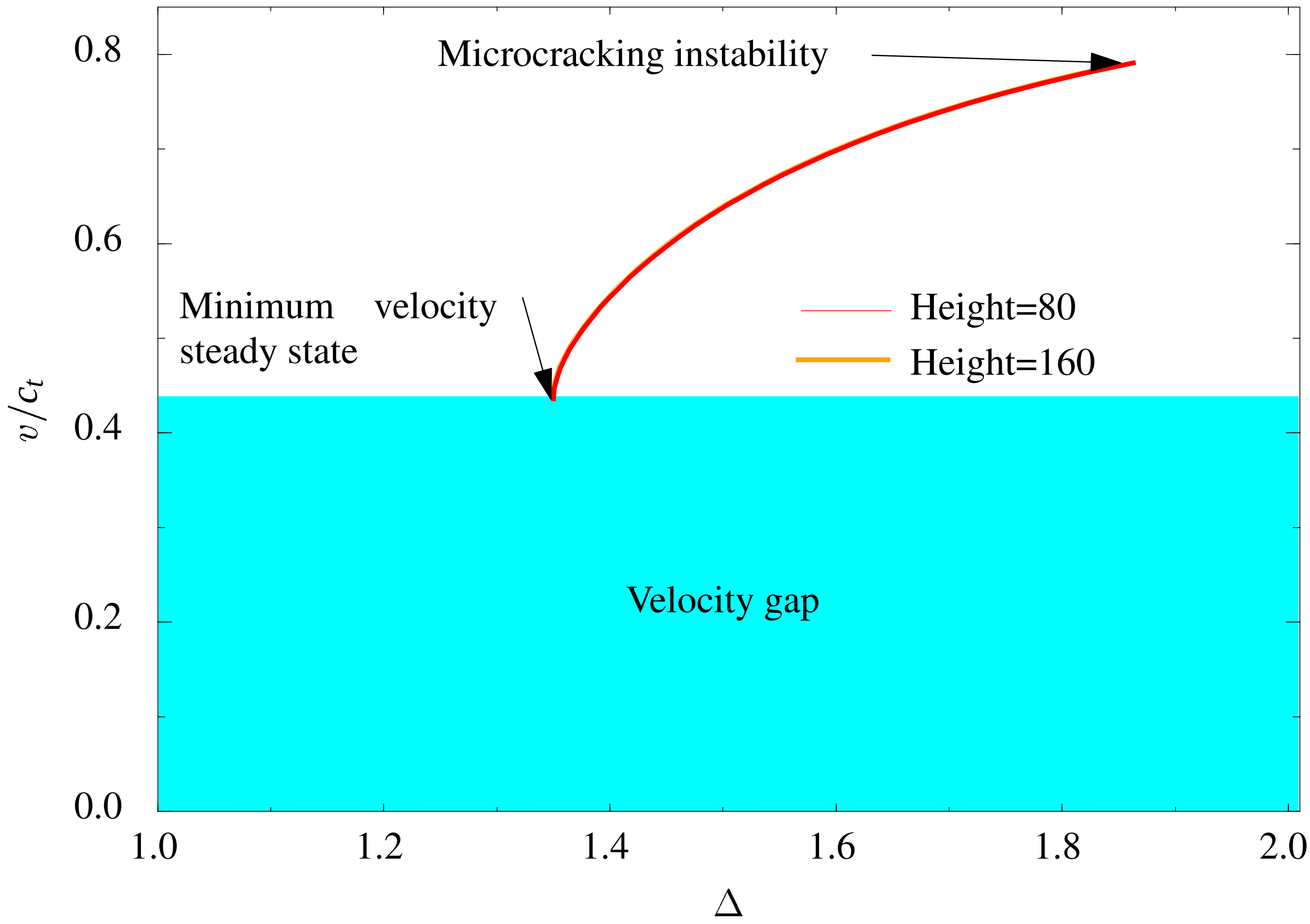
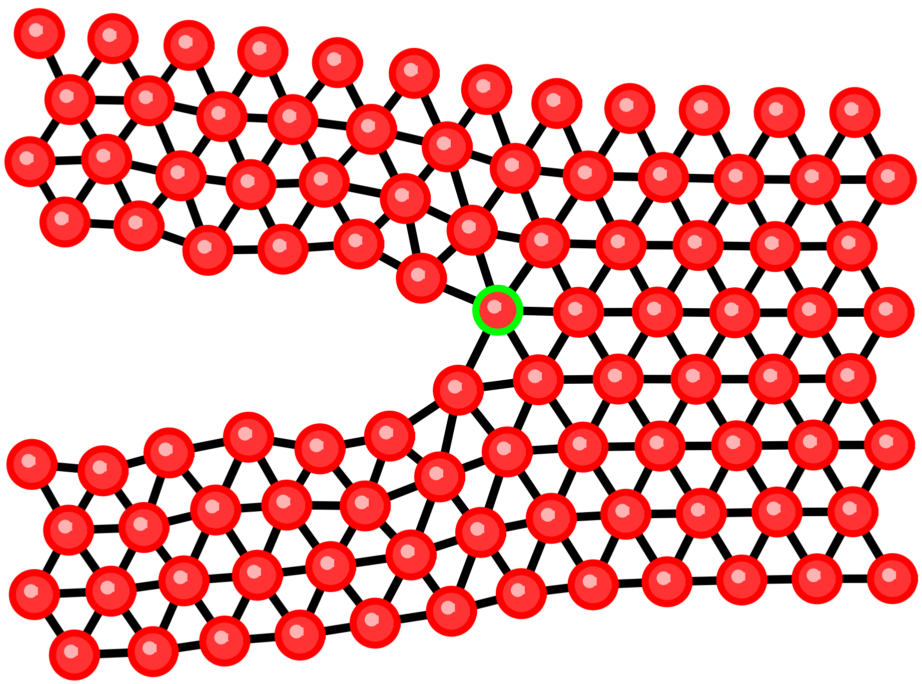


Physical reason for velocity gap



Because lattice can trap crack, next bond must break in no less than vibrational period T, so $v > a/T$

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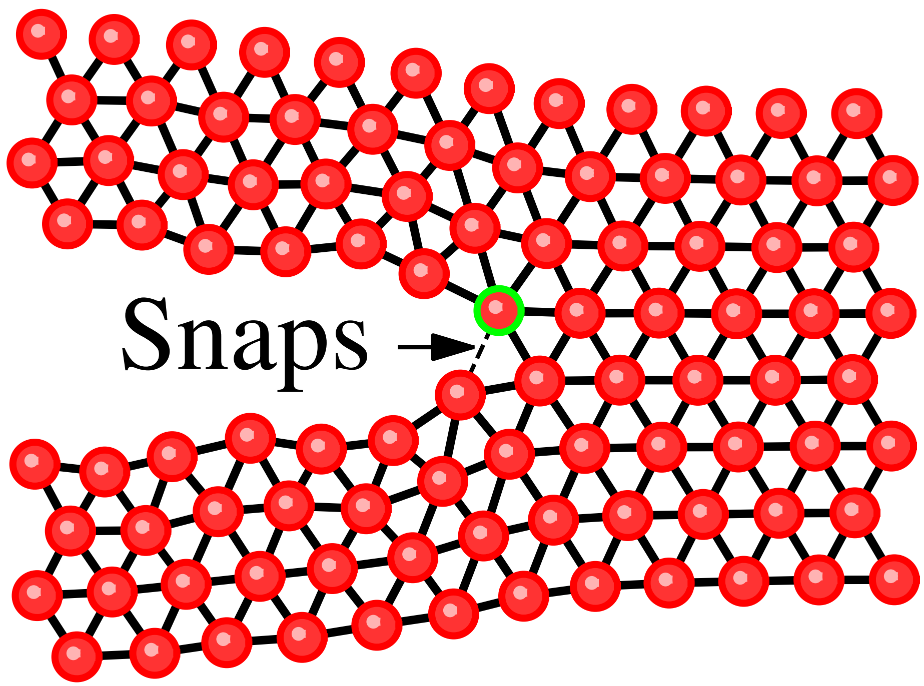

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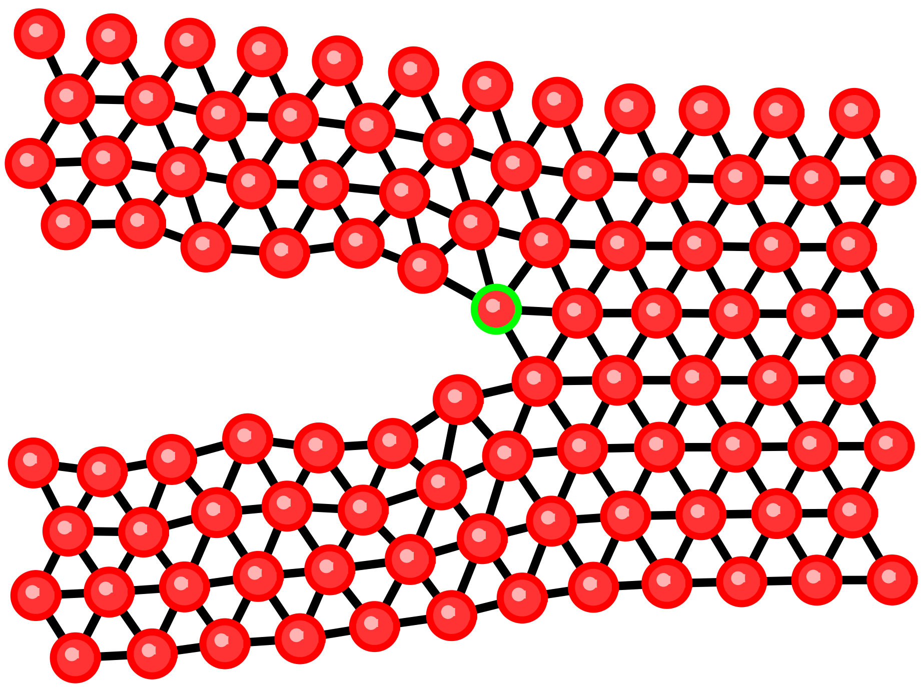

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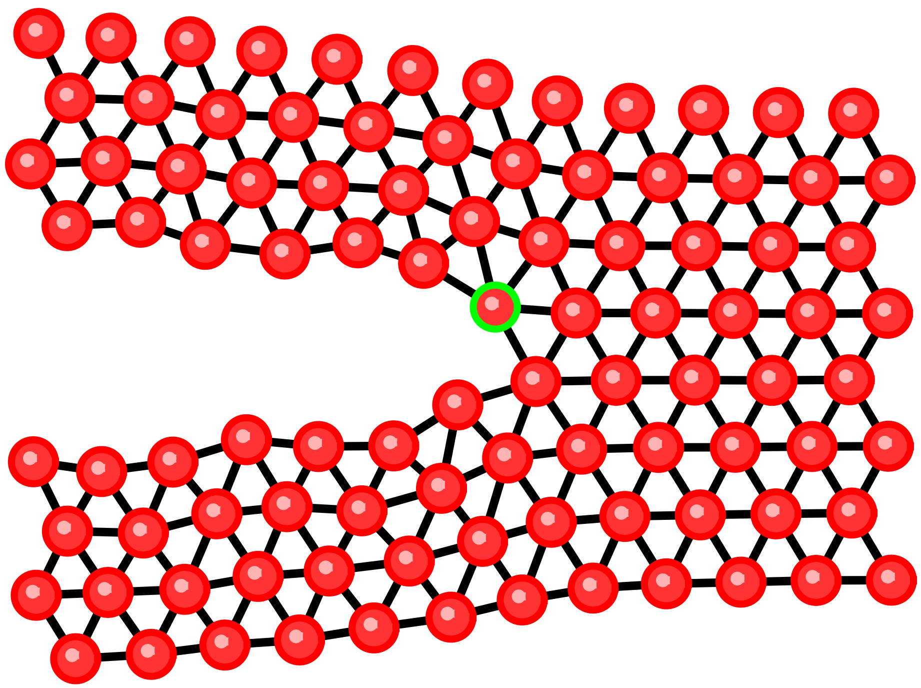

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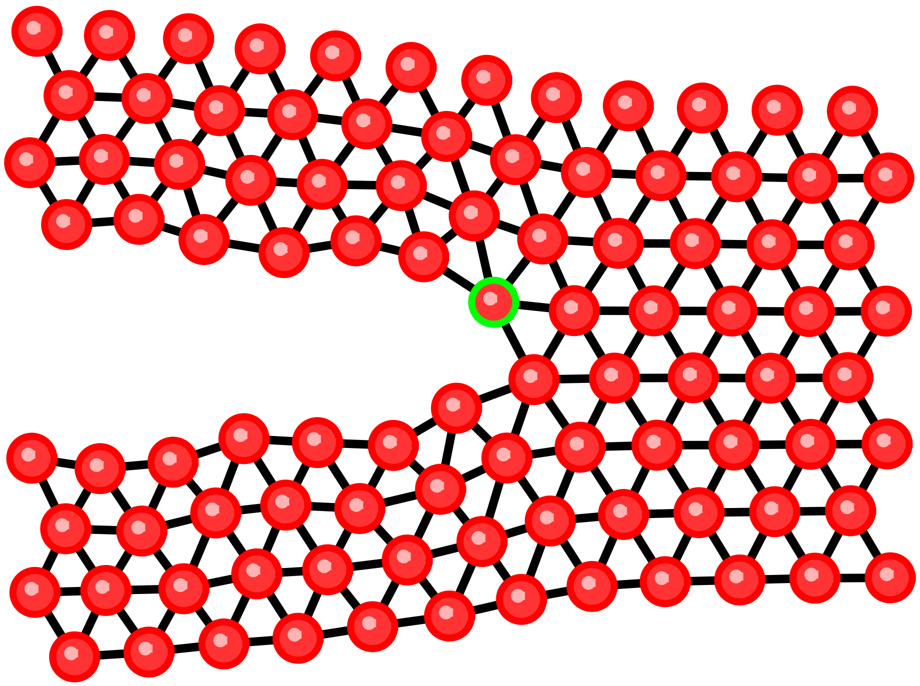

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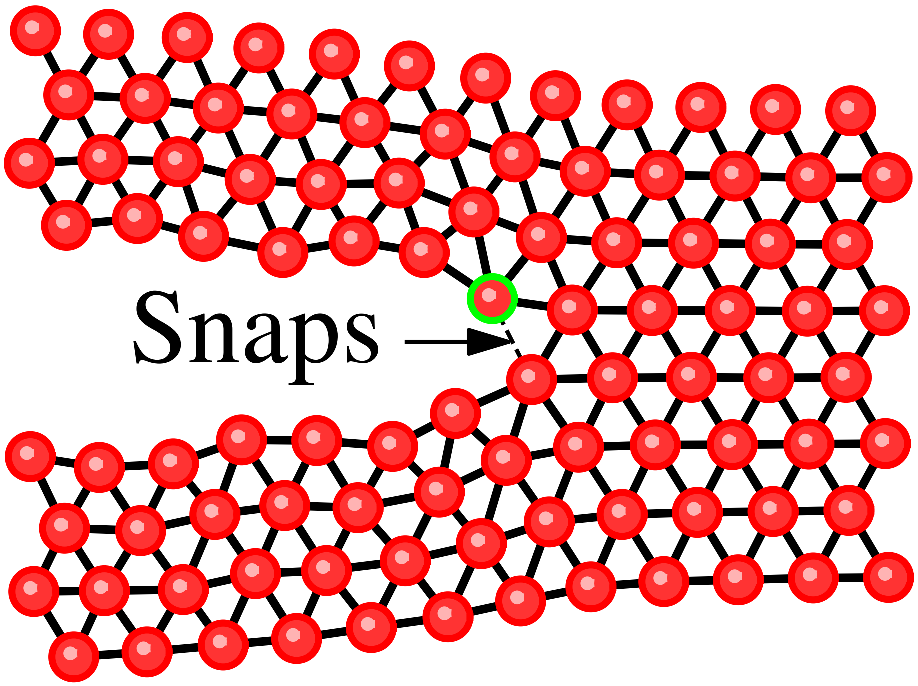

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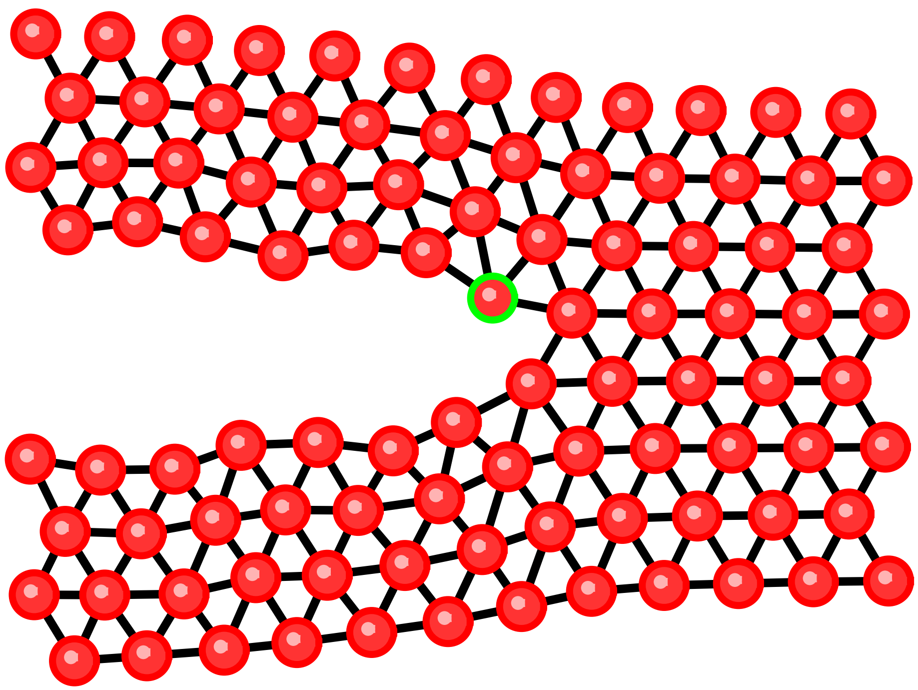

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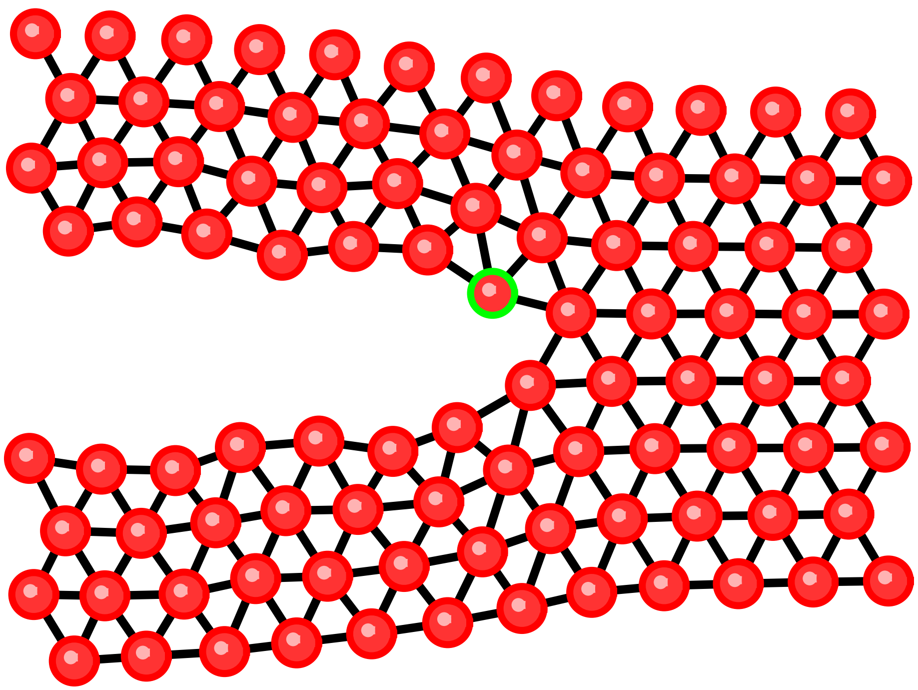
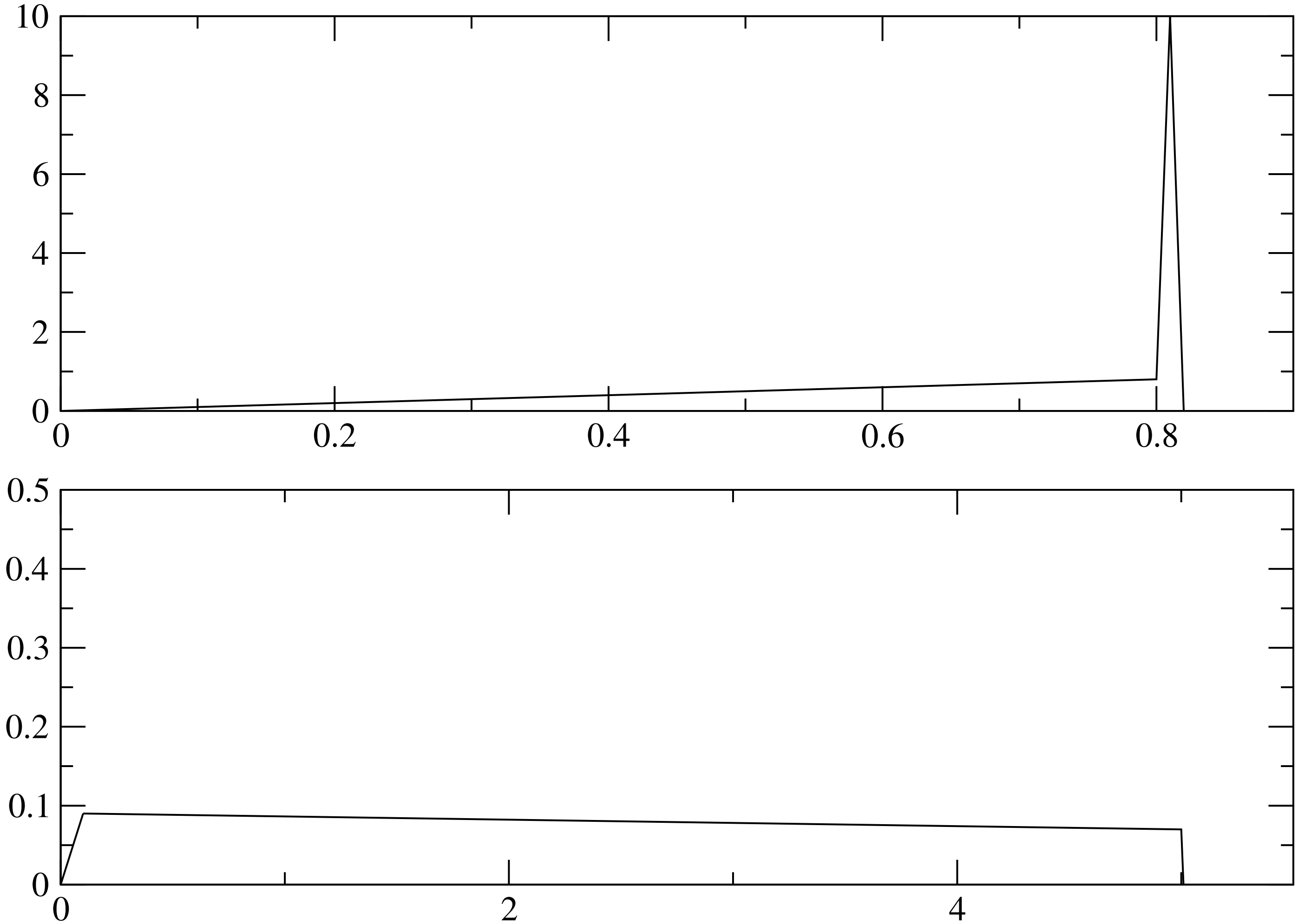


Because lattice can trap crack, next bond must break in no less than vibrational period T, so $v > a/T$

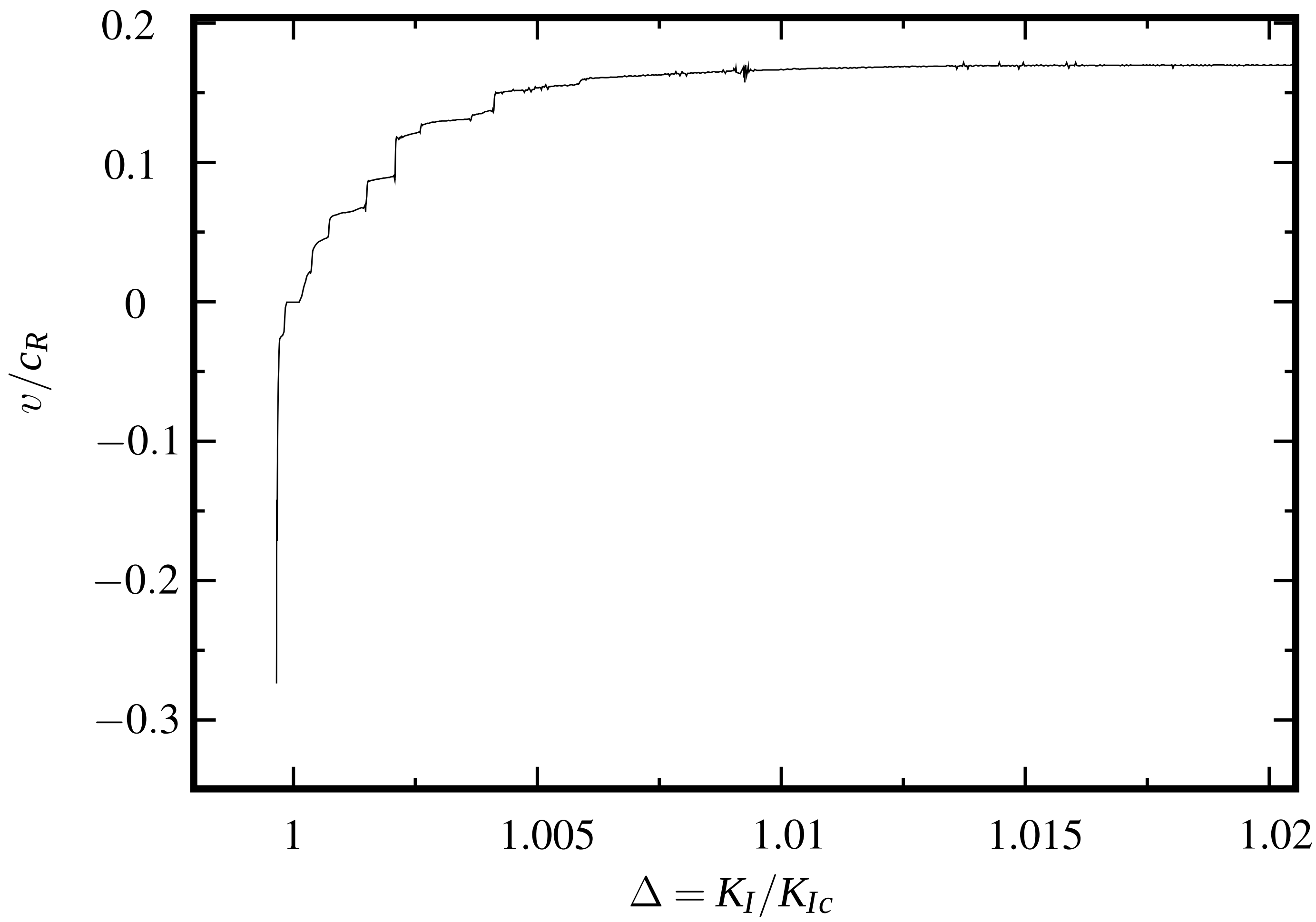
Making lattice trapping large and small

Can make lattice trapping as large or small as desired by changing shape of potential.



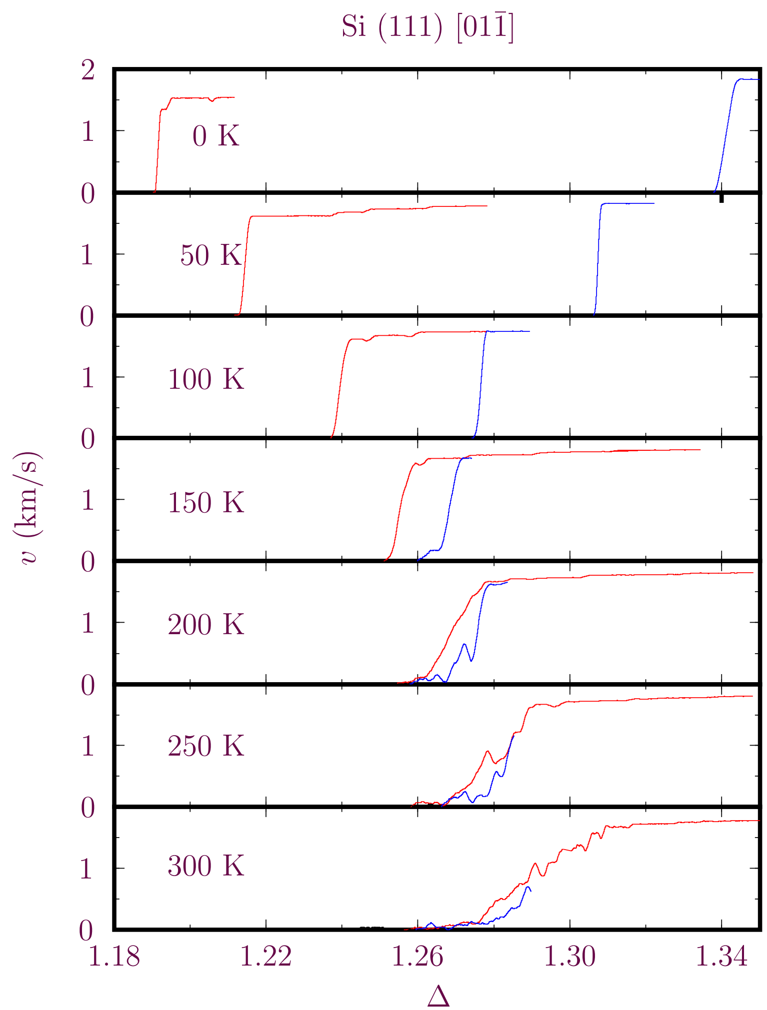
Making lattice trapping large and small

Example where lattice trapping is small:Lennard–Jones potential

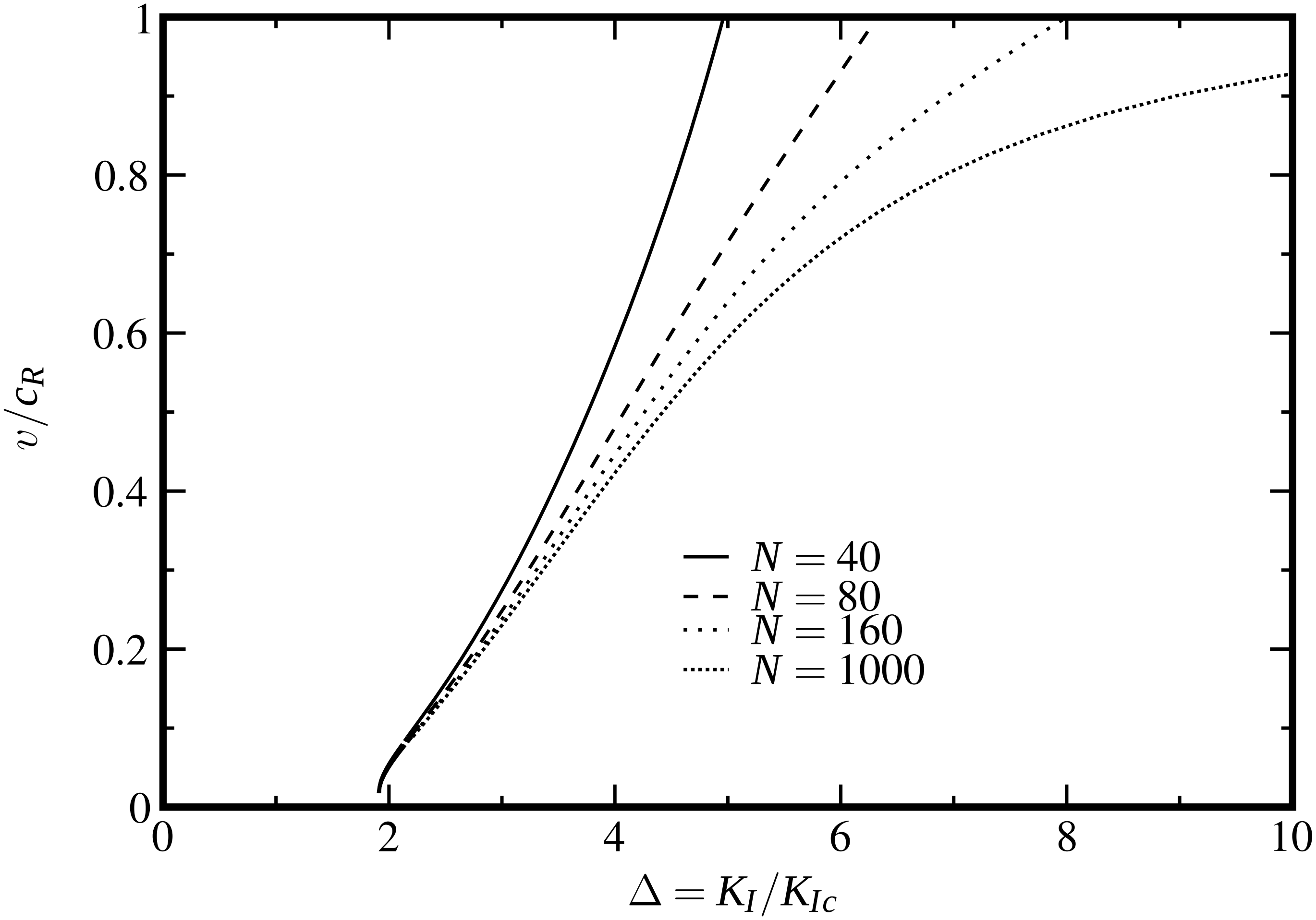


Making lattice trapping large and small

Can make lattice trapping vanish by increasing temperature



Can make lattice trapping vanish by increasing dissipation



Self-healing cracks on interfaces

[Williams, Comninou and Dundurs, Freund, Rice]

Existence of cracks solutions of this type has been doubted

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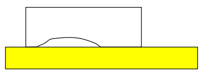
Solutions are

- ☞ mathematically troublesome
- ☞ Crack surfaces oscillate and cross one another infinitely often.

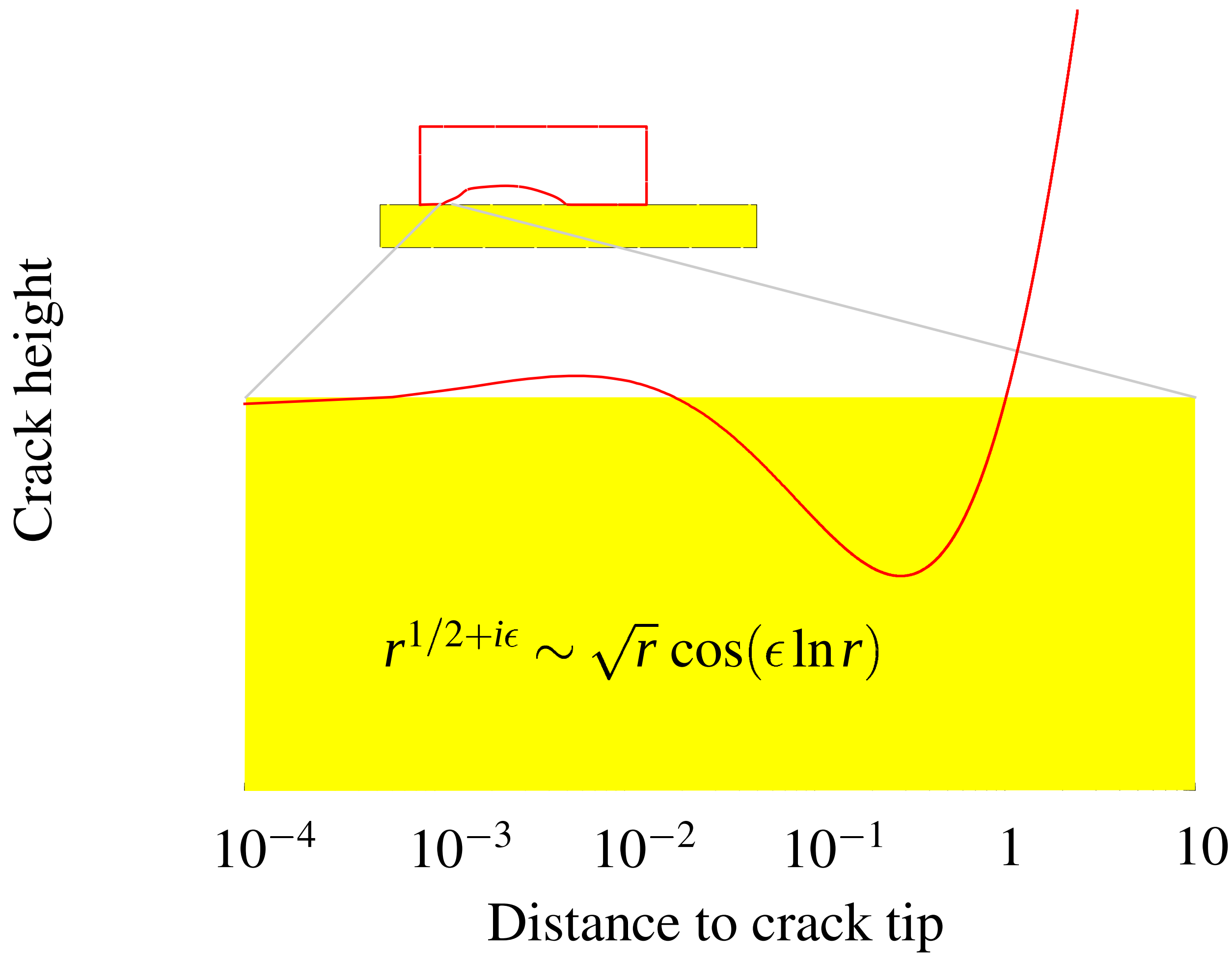
Self-healing cracks on interfaces

Crack surfaces oscillate and cross one another infinitely often.



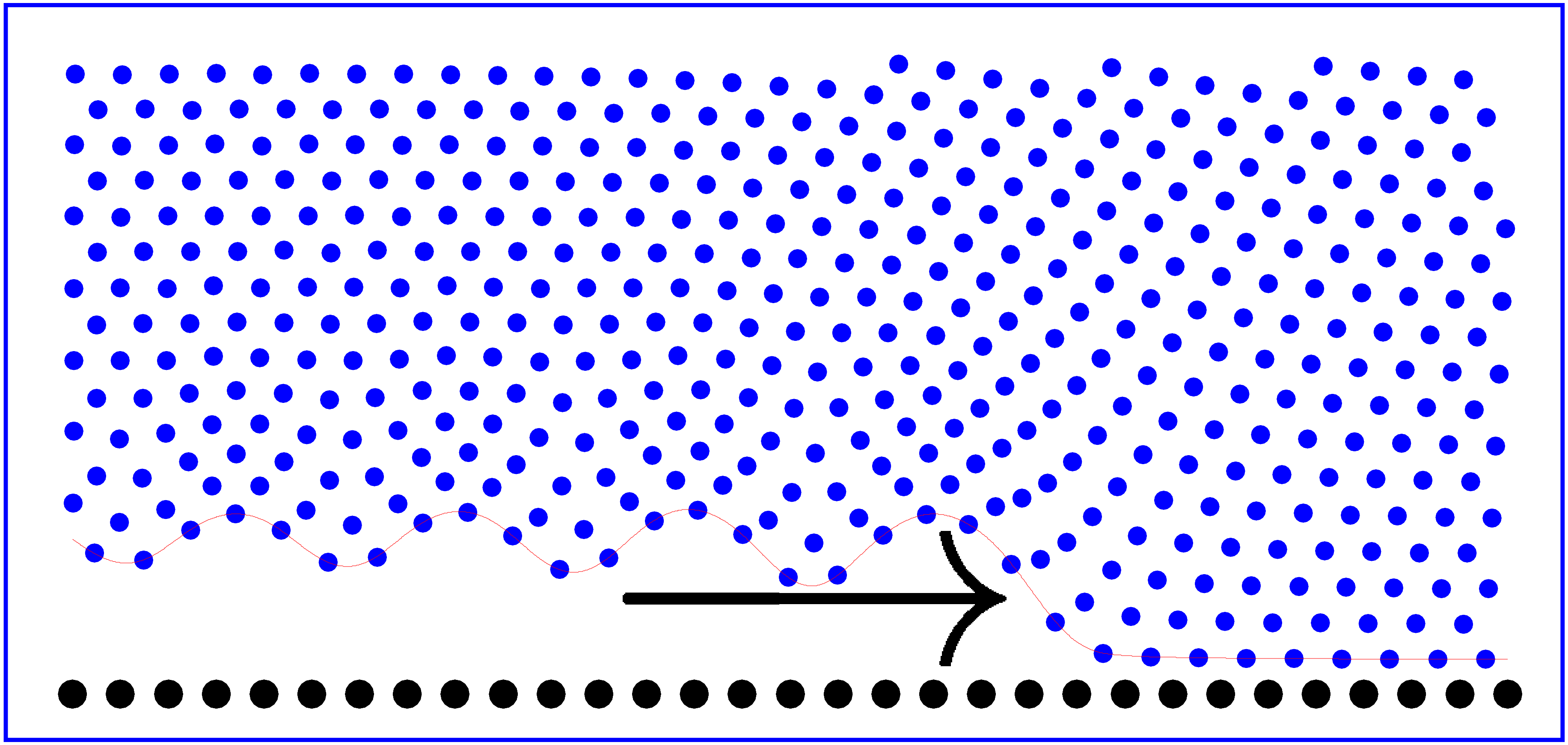
Paradoxes

Crack surfaces oscillate and cross one another infinitely often.



Do these cracks exist?

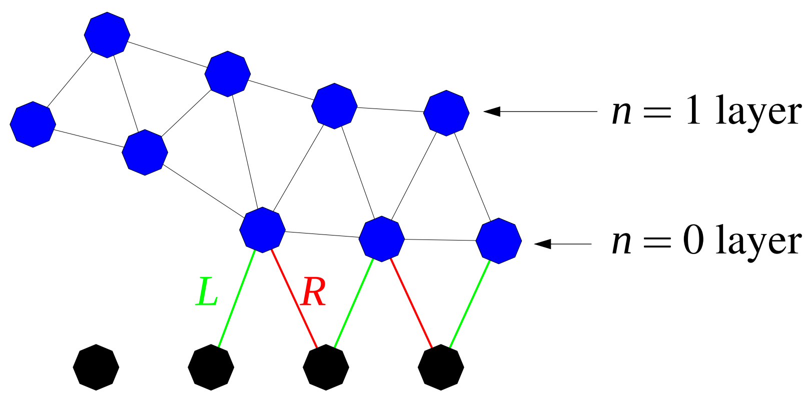
Interface solutions



Steady states

Steady-state hypothesis:

$$\mathbf{u}_{mn}(t) = \mathbf{u}_{0n}\left(t - \frac{m}{v}\right) \tag{1}$$



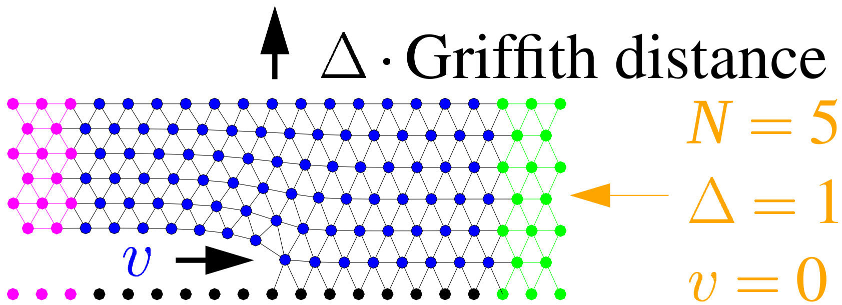
v is the crack velocity, and s is the syncopation.

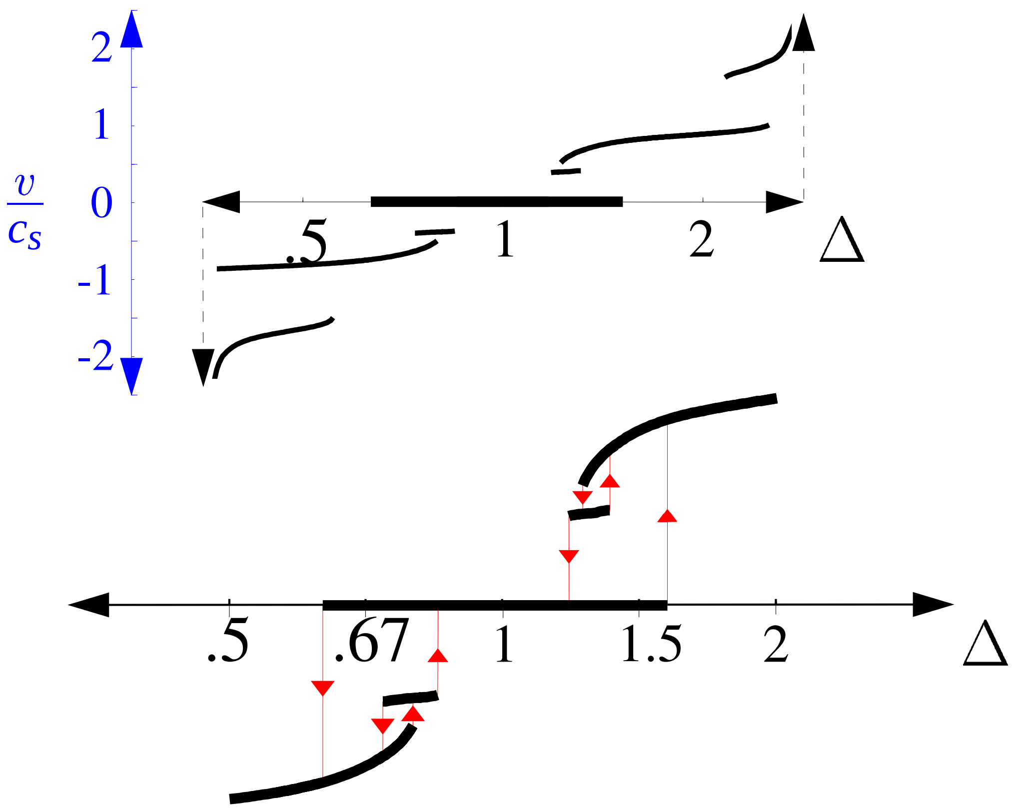
Left bonds break at times

$$t = \ldots, \frac{-2}{v}, \frac{-1}{v}, 0, \frac{1}{v}, \frac{2}{v}, \ldots$$

$$t = \ldots, \frac{-2}{v} + s, \frac{-1}{v} + s, s, \frac{1}{v} + s, \frac{2}{v} + s, \ldots$$

Interface solutions





Atomic methods cannot describe self-healing cracks

Multiscale Analysis

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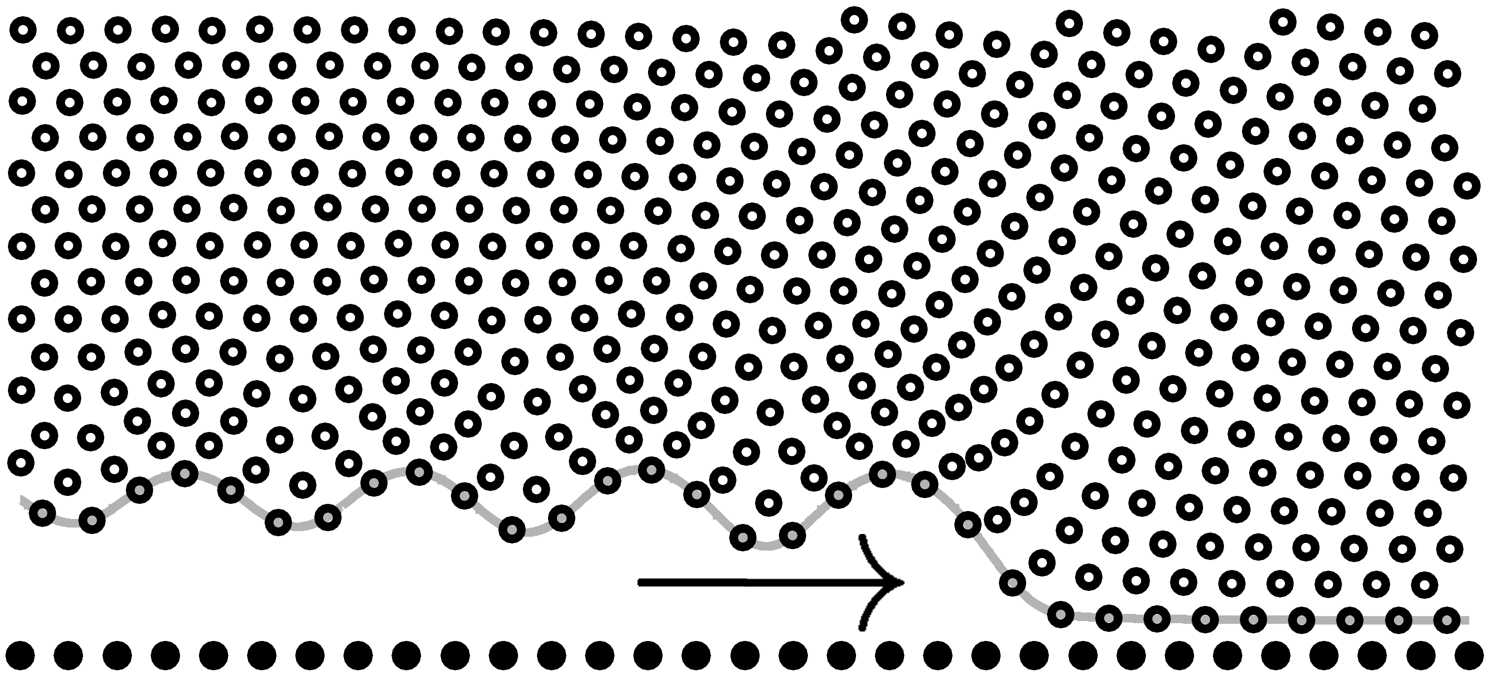
Match near-tip continuum fields to far-field atomic solution

Scaling of Atomic Solution



Time traces for atoms near the tip have a universal form dictated by the velocity v and syncopation s.

$$\frac{c_s D'_x + i c_d D'_y}{c_s D_x + i c_d D_y} = \left(\frac{N'}{N}\right)^{\frac{1}{2} - i\epsilon(v)}$$

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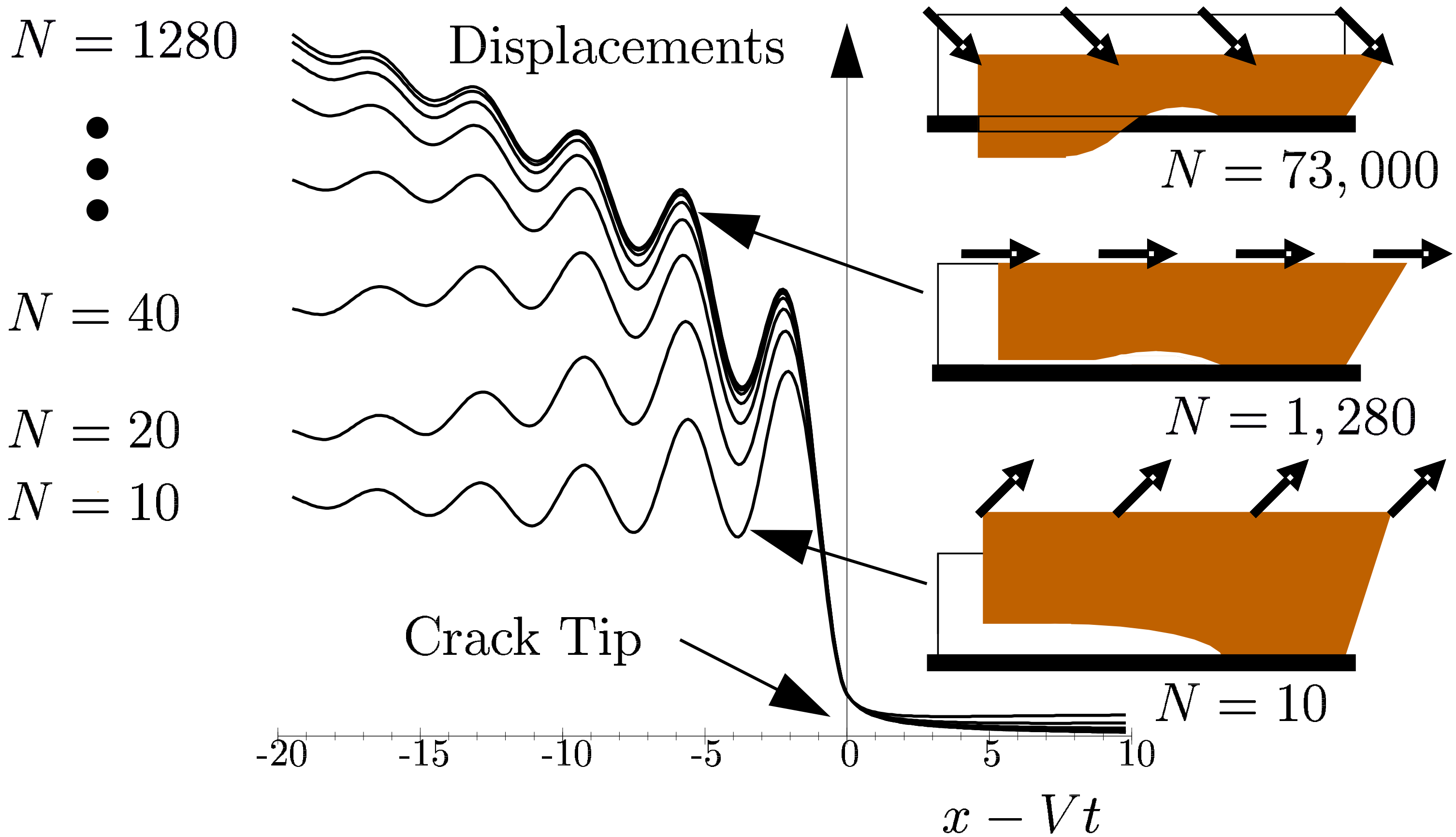
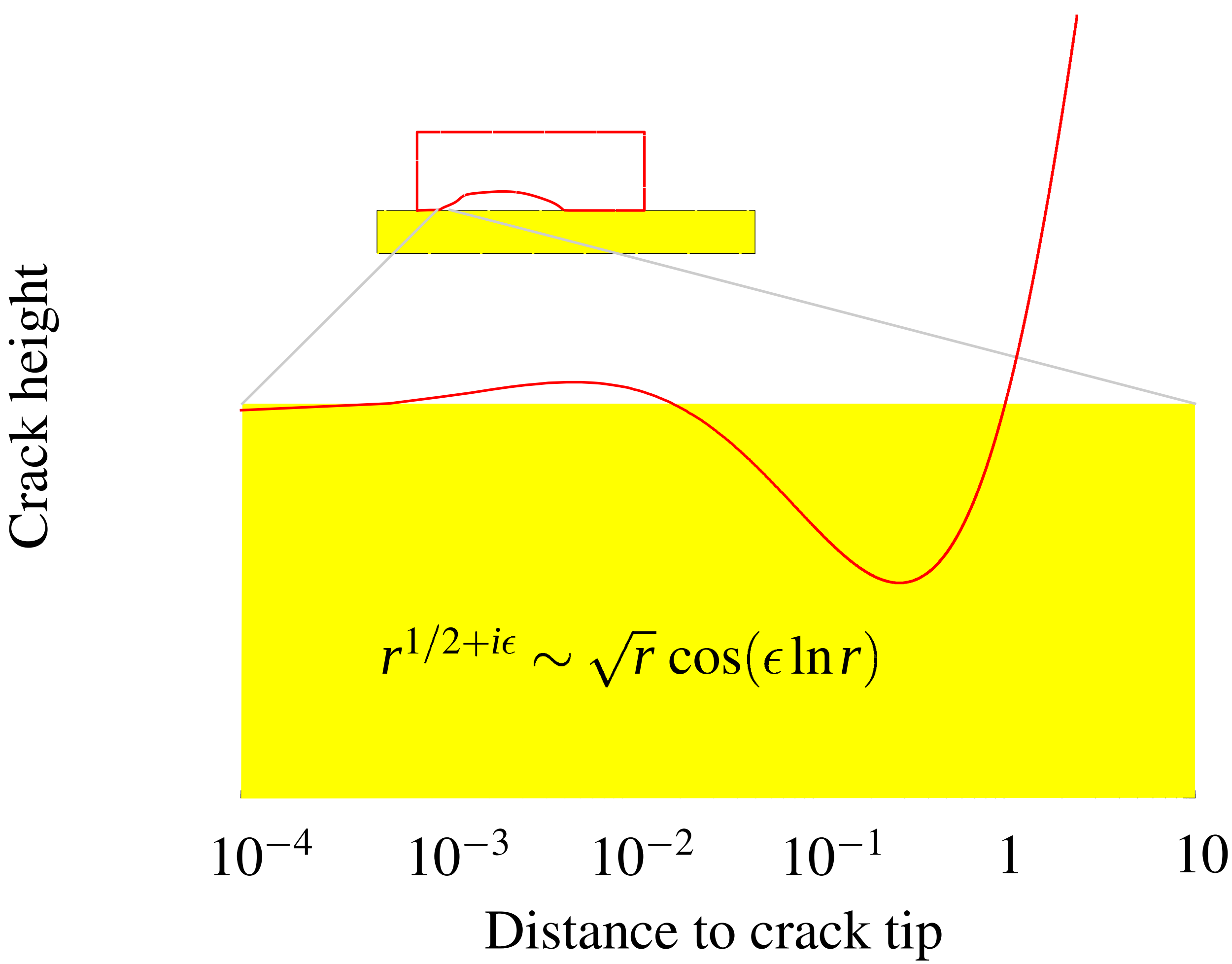


D_x and D_y are displacements of upper boundary. The loading must rotate with system size to reveal the universality.

Crack tip can be in tension while boundary is in compression.

Stress Rotation



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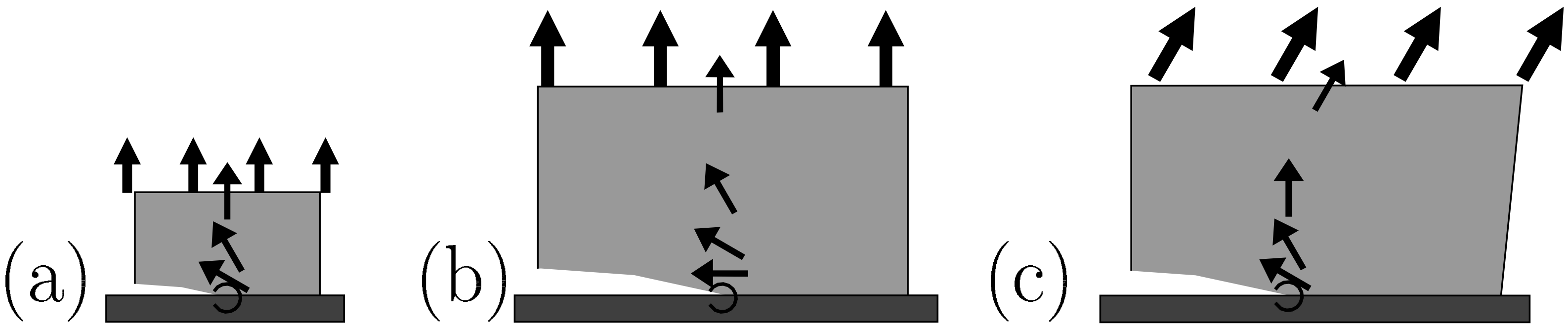
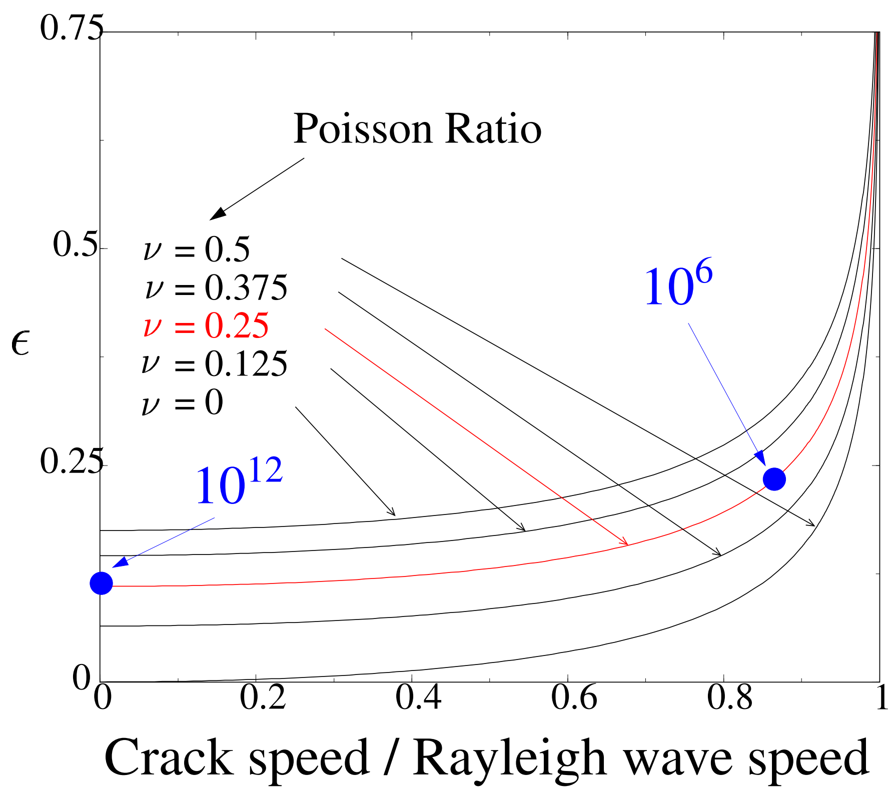


Identical fields at crack tip requires rotation of external loads as sample size changes.

Single tip

Crack-tip singularity has form

Single tip



$$\sigma \propto r^{\lambda} \equiv r^{-1/2+i\epsilon} \quad \text{where } \epsilon \text{ is real.}$$

$$q^2 + 2pq + 1 = 0$$

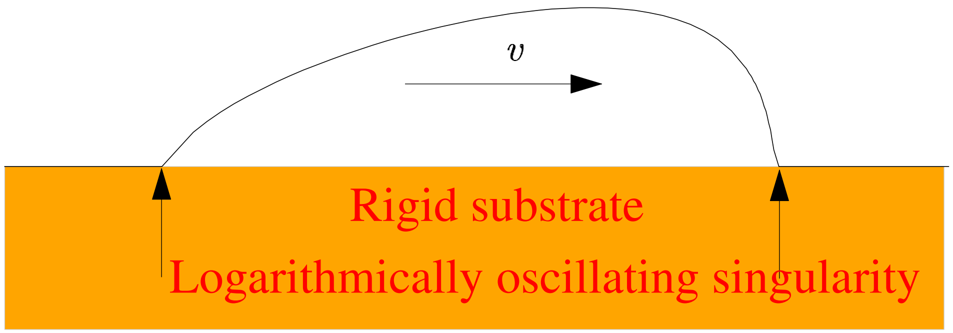
$$q = e^{2\pi i \lambda},$$

$$q = -p \pm \sqrt{p^2 - 1},$$

$$p = 1 + \frac{2\left((1+\beta^2) - 2\alpha\beta\right)^2}{(\alpha\beta - 1)\left((1+\beta^2)^2 - 4\alpha\beta\right)}$$

$$\alpha = \sqrt{1 - v^2/c_l^2} \quad \beta = \sqrt{1 - v^2/c_t^2}.$$

Two tips



$$\Phi(z) = A_1(z-t)^{\lambda}(z+t)^{\bar{\lambda}} + A_2(z-t)^{\lambda}(z+t)^{\bar{\lambda}+1} + \text{c.c.}$$

Continuum: Self–healing crack of length l and velocity v is described by four constants, corresponding to

- Stresses σ_{xy} and σ_{yy} at infinity.
- Slip in x and y direction produced by crack.

or

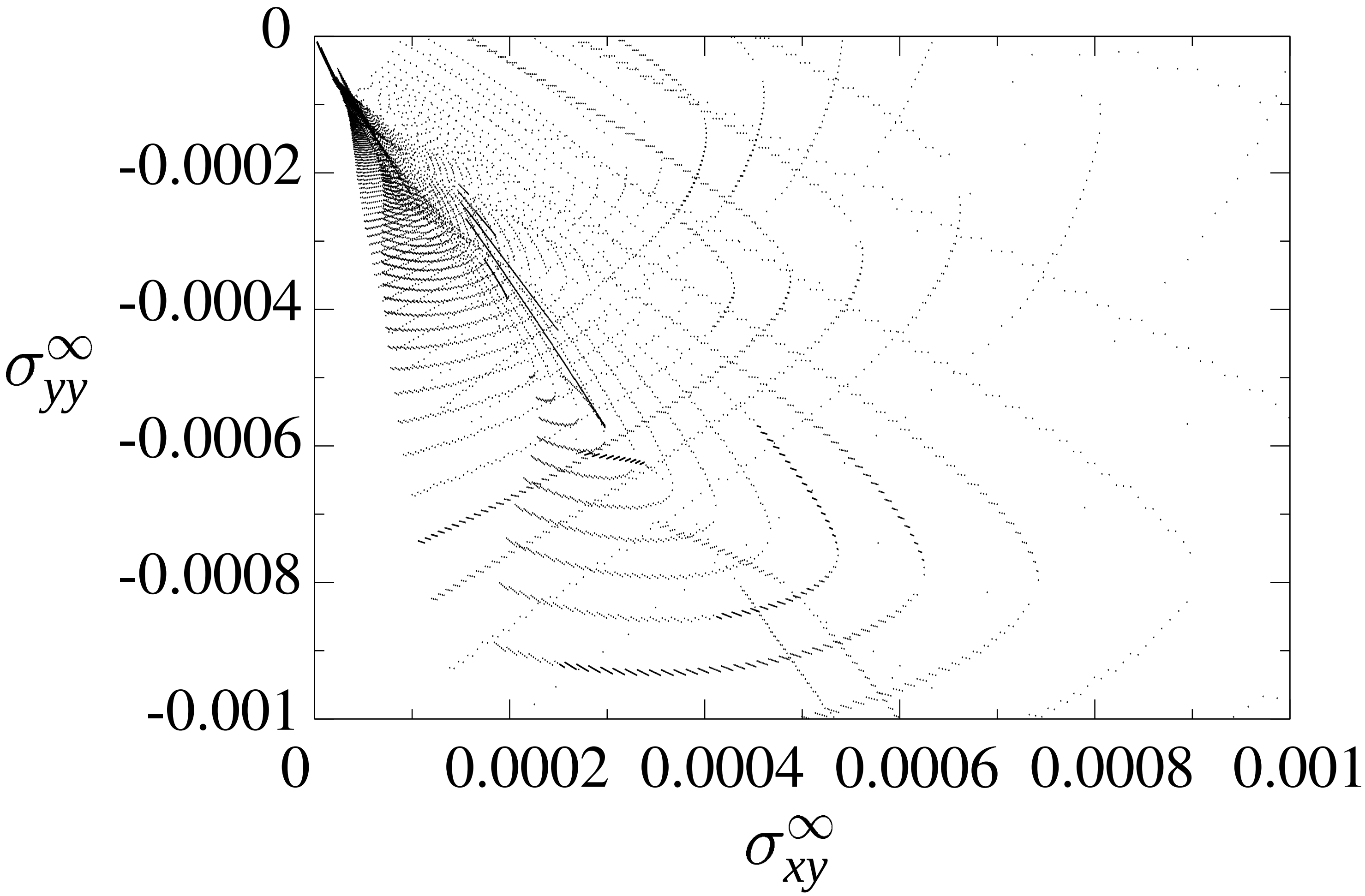
- Real and imaginary parts of stress intensity factors at two crack tips.

Microscopic: Self–healing crack of length of l and velocity v is described by two constants, corresponding to syncopations s_f and s_b front and back.

Assemble results

Predicting Friction

Catalog of 100,000 self-healing crack states.



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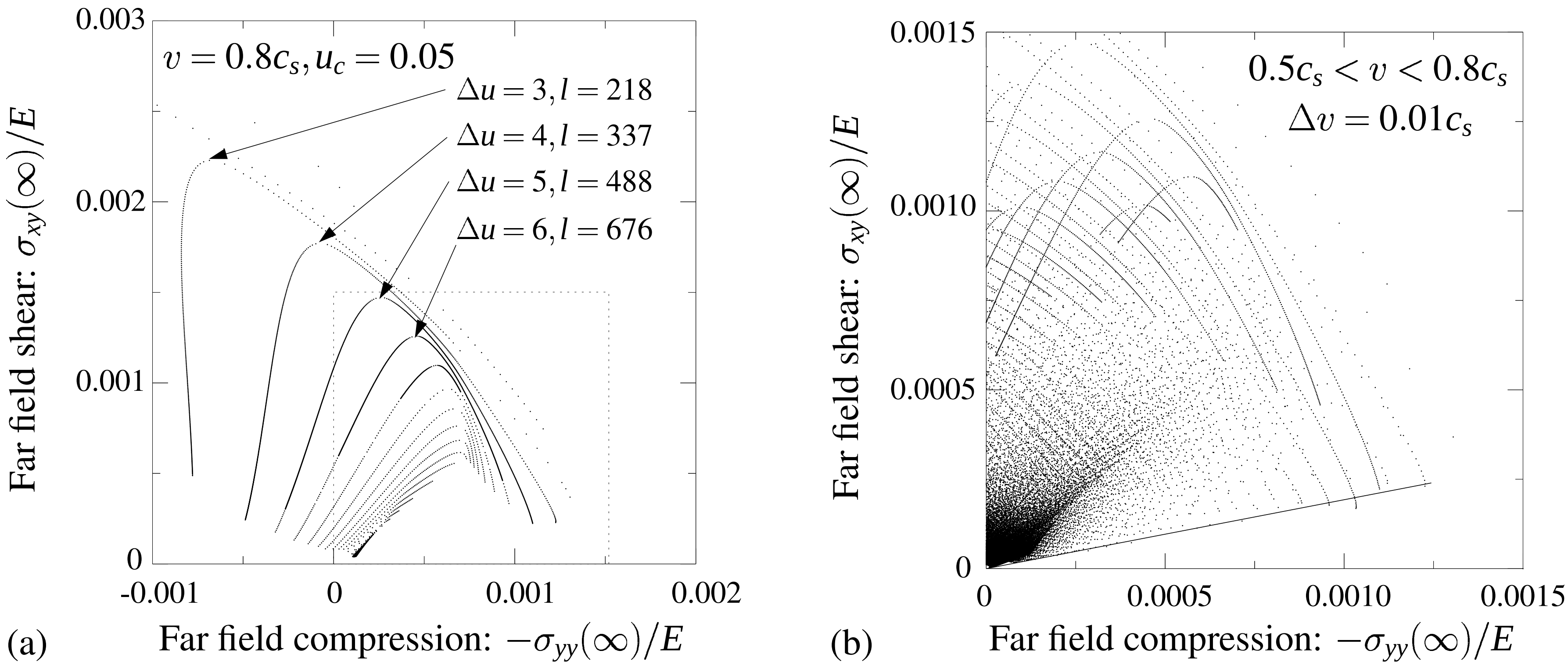

Scenario and questions

Questions:

- Nucleation?
- Interaction?
- Temperature?
- Do results depend upon crystallinity?
- What is effect of surface roughness?
- Can these states coexist with asperities?
- How can they be detected experimentally?
- In what systems?

Scenario and questions