Avalanches in Complex Systems:

From Sandpiles to Internet Storms

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Summary

- Avalanche behavior in nature
- Explaining avalanches: the Self-Organized Criticality (SOC) concept
- A SOC example: sandpiles
- Boundary conditions: Sandpiles are not SOC …
- Avalanches without SOC ?
  - Two not so typical examples
    - Turbidite deposition
    - Internet storms
- Conclusions

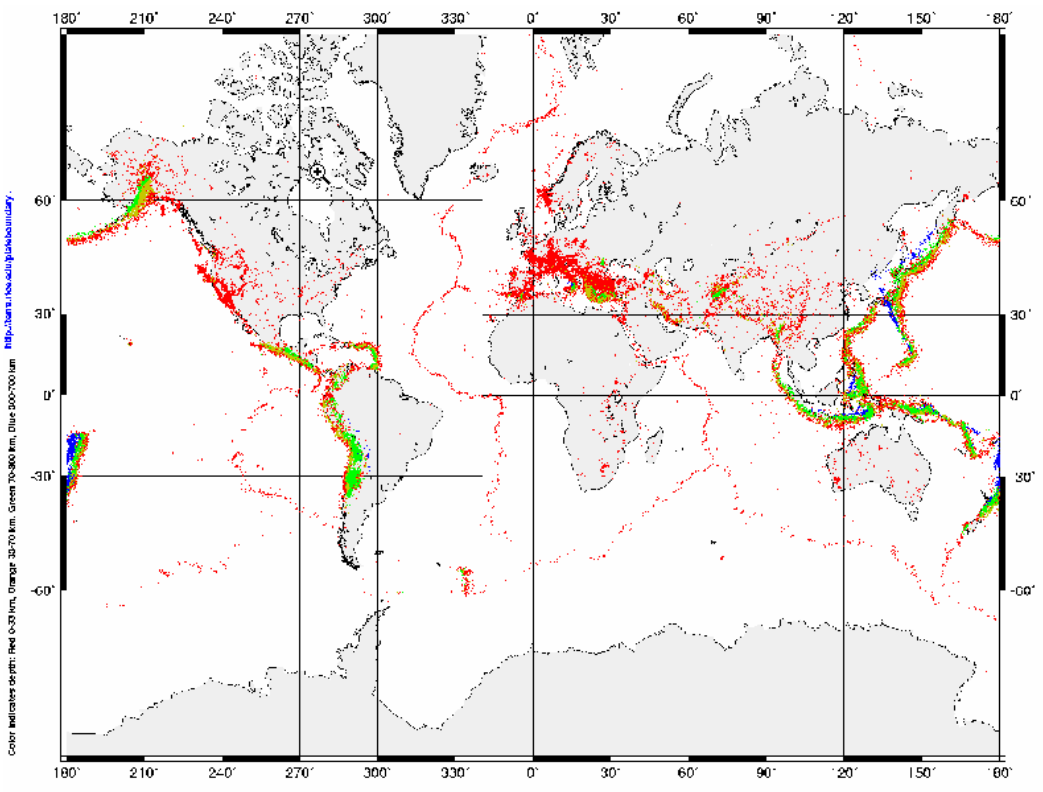
Avalanches in nature

- The activity of many complex systems, composed by many interacting units, is characterized by an avalanche behavior
  - Long times of quiescence, interspersed by bursts of sudden and very strong activity, that last a relatively short time
- Typical characteristic of avalanche activity
  - Avalanche size distributed according to a power law

$$P(s) \sim s^{-\tau}$$

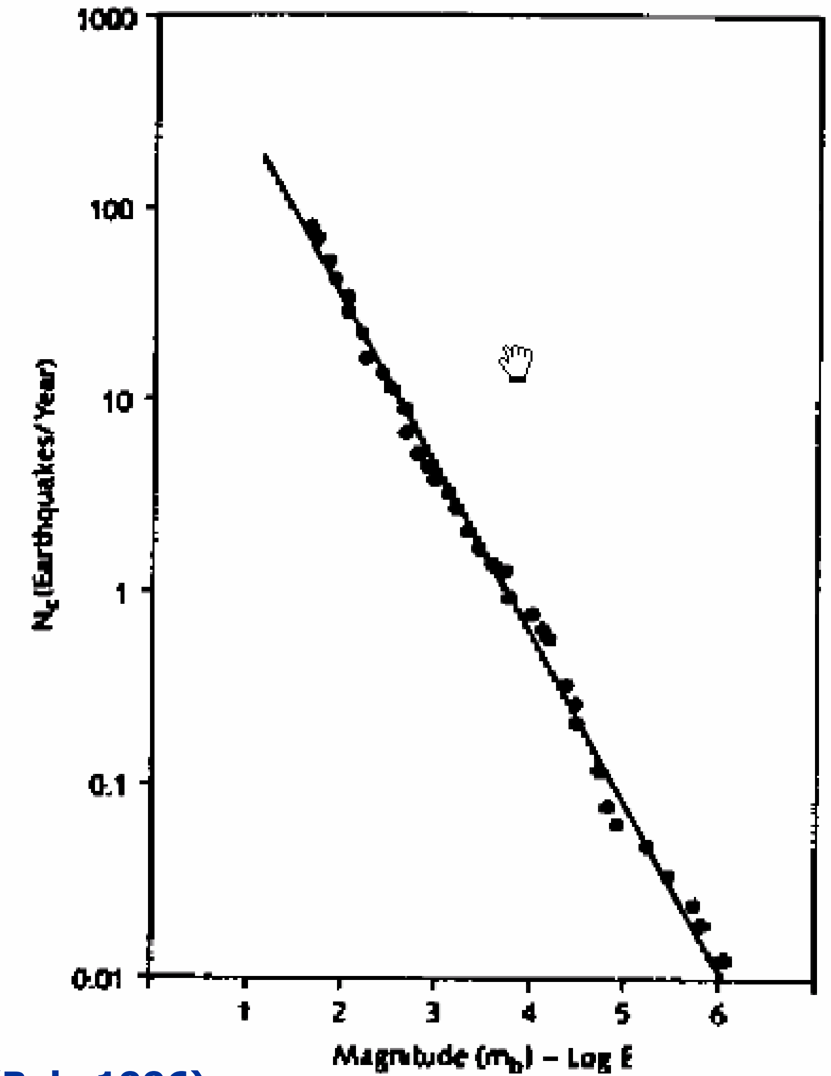
- Power law activity has been associated with natural catastrophes
  - Many physical systems show however this kind of behavior
- Examples …

Earthquakes



- Earthquakes typically result from the movement of geological faults
- After long time accumulating stress, it is released in a sudden (and catastrophic) way

Earthquakes



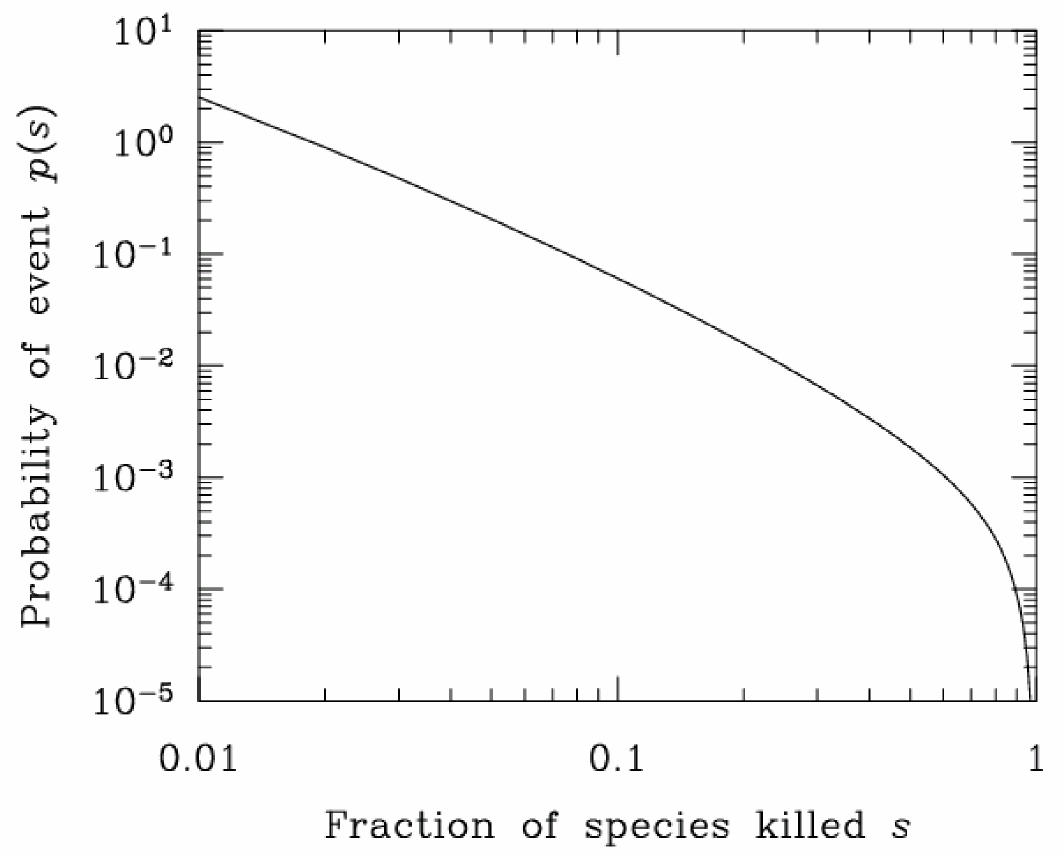
(Bak, 1996)

- Gutenberg-Richter law
  - The probability of observing an earthquake of a given size (magnitude) decreases as a power law

Extinctions

- Extinction of species takes place in short episodes (extinction events) that can involve a large number of species
- Some extinction events have been explained as real catastrophes (Alvarez at al., 1980)
- Many extinctions cannot be explained this way
  - Competition stress between species

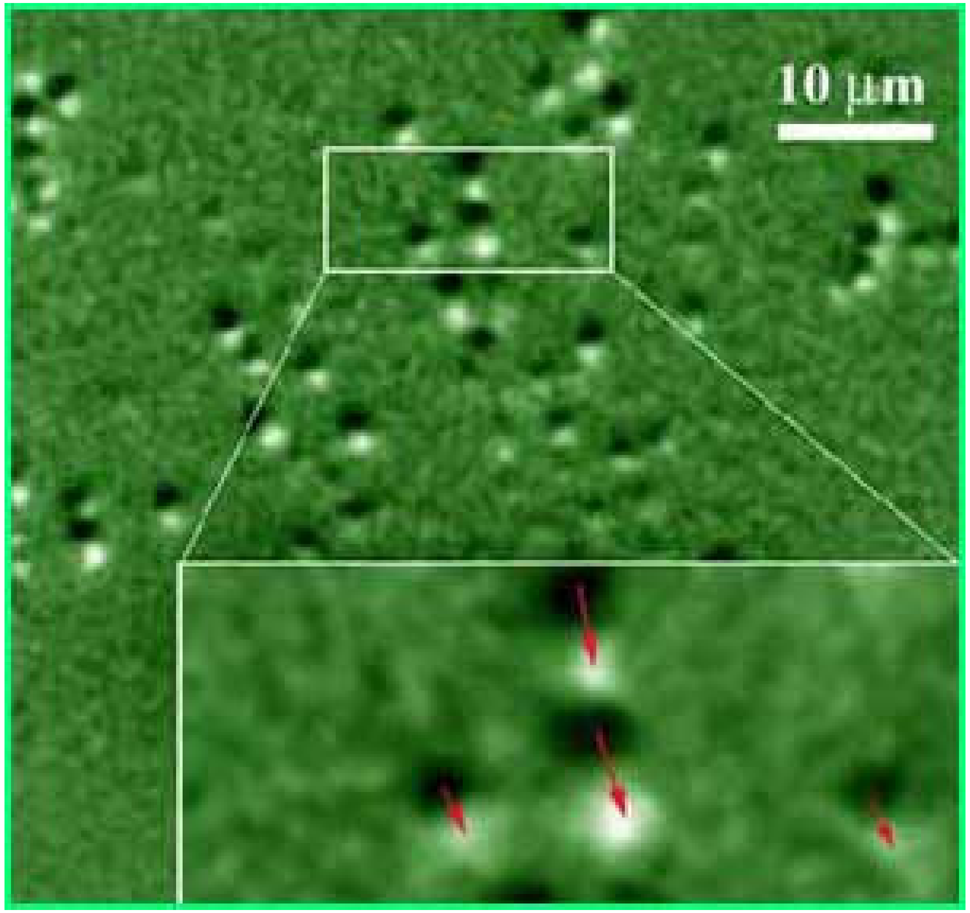
Extinctions



(Newman, 1997)

- The distribution of the size (fraction of species involved) in extinction events is approximately given by a power law

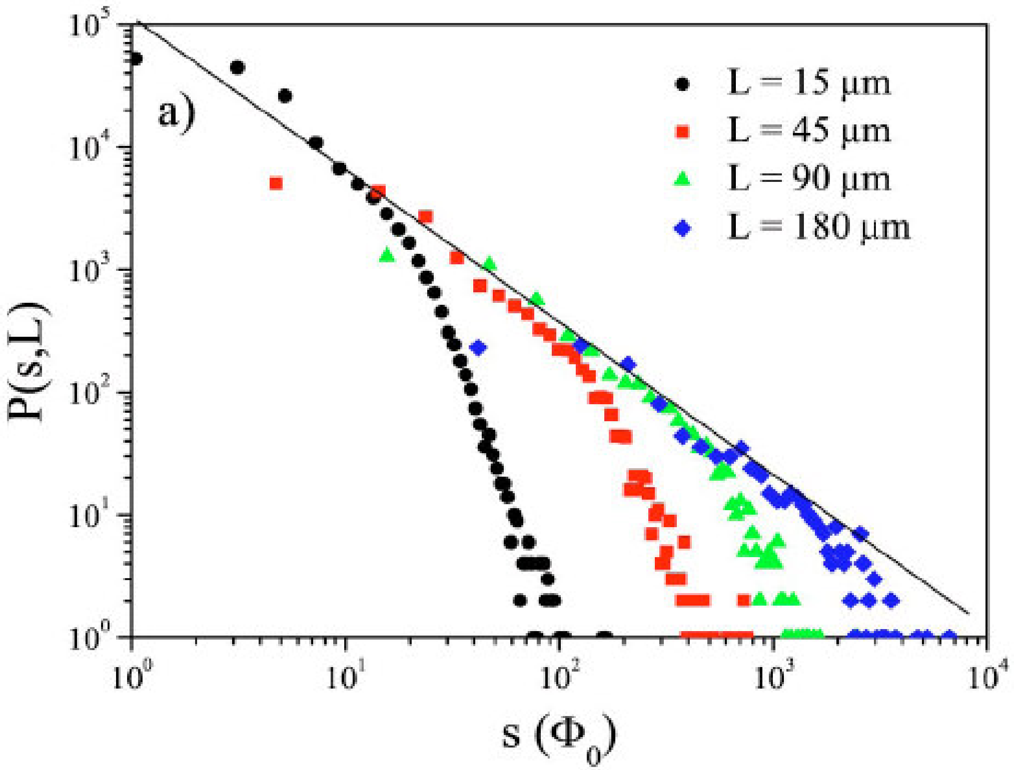
Superconductor vortices



(Altshuler at al., 2004)

- When a type II superconductor is driven by a slowly increasing magnetic field, vortex lines move inside the sample in a collective avalanche-like way
- This motion is due to the interplay among vortex interactions, quenched disorder, and field driving

Superconductor vortices



(Aegerter at al., 2003)

- The size of the avalanches, measured as the number of vortices involved in the movement, scales as a power law

Other avalanche examples

- Physics
  - Dislocation motion
  - Barkhausen effect
  - Charge density waves …
- Complex systems
  - Forest fires
  - Landslides
  - Traffic jams
  - Snow avalanches …

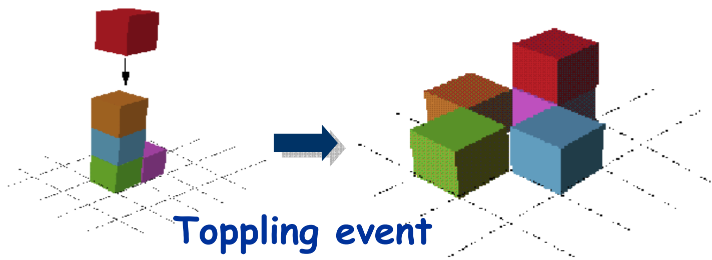
An explanation for avalanche behavior?

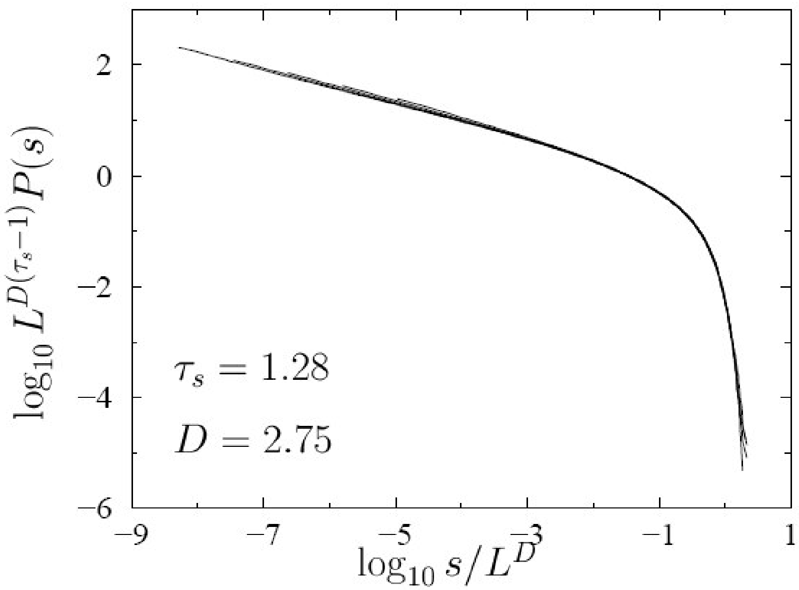
- The wide observation of avalanche behavior with a power law distribution calls for some mechanism to explain it
- Per Bak et al. argument (1987):
  - Power law behavior is common in standard critical points
  - Systems with avalanches could be placed at a critical point?
  - Standard critical points are reached by adjusting tuning parameters …

- Is it possible to have systems that adjust themselves, without any external control, to a critical point?
- The existence of such systems can explain the presence of power laws in nature
- Concept of Self-Organized Criticality (SOC) !!

An example of SOC

- Bak-Tang-Wiesenfeld sandpile model
  - Grains of sand (energy) are injected on a lattice
  - Conserved threshold dynamics of energy
  - Energy is lost at the boundary
  - The addition of a grain of energy can lead to an avalanche of activity





- The size of the avalanches (number of toppling events) scales as a power law.
- Moreover, it shows critical behavior: finite-size scaling

$$P(s) = s^{-\tau} f(s/L^D)$$

A closer look at boundary conditions …

- So sandpile models seem to exhibit SOC (we have tuned no parameter)
- Let's have a closer look at the boundary conditions …
  - Driving: We are adding energy when there is no activity (no topplings)
  - Dissipation: Energy dissipates at the boundary when there is activity
- Look at the energy E in the system …
  - If E is large, there will be activity
  - If E is small, there will be no activity
- Balance of the energy …
  - E large $\rightarrow$ There is dissipation $\rightarrow$ $dE/dt < 0$
  - E small $\rightarrow$ There is driving $\rightarrow$ $dE/dt > 0$
- Boundary conditions are driving the system to a particular value of the energy E_c,
- If this energy corresponds to a critical point:
  - Sandpiles would have a built-in "baby-sitter" driving them to a conventional critical point (Dickman et al. 2000) !!

Firing the baby-sitter …

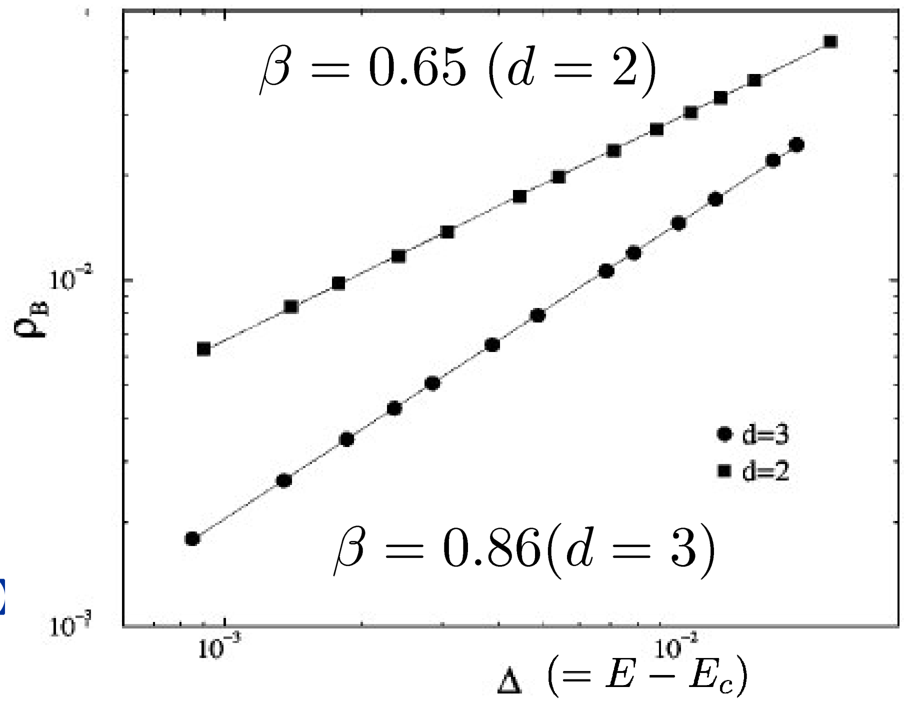
- Consider a sandpile model in which we have fired the baby-sitter
  - No driving (no particles added)
  - No dissipation (periodic boundary conditions)
- Fixed energy sandpile (FES)
- Control parameter: energy of the system E
  - E small $\rightarrow$ no sites above threshold $\rightarrow$ the system is frozen
    - Absorbing phase
  - E large $\rightarrow$ some sites above threshold $\rightarrow$ the system is active
    - Active phase
- The transition between active and absorbing phases is a standard dynamic critical point (absorbing-state phase transition) taking place at a critical value of the energy E_c

Absorbing-state phase transition in FES

- General absorbing-state phase transitions can be studied by the methods used for general critical points
  - Definition in terms of a set of critical exponents (β, $\nu_\perp$, $\nu_\parallel$)

$$\rho \sim (E - E_c)^{\beta}, \;\; \xi \sim (E - E_c)^{-\nu_\perp}, \;\; \tau \sim (E - E_c)^{-\nu_\parallel}$$

- Computer simulations show that indeed FES undergo a completely standard absorbing-state phase, characterized by a set of exponents that define the universality class of sandpiles



(RPS & Vespignani, 2000)

Sandpiles are not self-organized …

- Further observations:
  - The critical energy of the FES E_c and the average energy of the sandpile coincide
  - The sandpile critical exponents (τ, D) can be related to the FES exponents (β, $\nu_\perp$, $\nu_\parallel$)
- Conclusion: Sandpiles are the other side of the coin of a standard absorbing-state transition critical point
  - Instead of adjusting the value of the control parameter, we adjust the boundary conditions so that the system is driven to the critical point
- From a more technical point of view:
  - Sandpiles are spreading experiments in FES
    - Measure of activity created by a single active site at the critical point

Field-theory for sandpiles

- If sandpiles are systems with an absorbing-state phase transition, it should be possible to characterize them by a field-theory (Reggeon field-theory for directed percolation)
- Too difficult for sandpiles (threshold dynamics)
- Universality conjecture (Rossi, RPS, & Vespiganani, 2000):
  - All systems with the same symmetries as sandpiles belong to the same universality class
- Checking the conjecture with non-sandpile models
  - Conserved lattice gas (Rossi, RPS & Vespignani, 2000)
  - Reaction-diffusion system (RPS & Vespignani, 2000)
- Look for a model in the same universality class that is easier to treat analytically

Field-theory for sandpiles

- Consider a reaction-diffusion process with the same symmetries (in the same universality class)

$$B \to A \quad \text{with rate } k_1,$$

$$B + A \to 2B \quad \text{with rate } k_2.$$

- A field-theory can be constructed a using standard formalism (Doi, Peliti, …):

$$\partial_t \psi = D\nabla^2 \psi - r\psi - u_1 \psi^2 - u_2 \psi\phi + \eta_\psi ,$$
$$\partial_t \phi = \lambda \nabla^2 \psi + \eta_\phi .$$

$$\langle \eta_\psi(x,t)\, \eta_\psi(x',t') \rangle = 2u_1 \psi(x,t)\, \delta(x-x')\, \delta(t-t')$$
$$\langle \eta_\psi(x,t)\, \eta_\phi(x',t') \rangle = -u_2 \psi(x,t)\, \delta(x-x')\, \delta(t-t')$$
$$\langle \eta_\phi(x,t)\, \eta_\phi(x',t') \rangle = 0$$

(RPS & Vespignani, 2000)

Can we explain avalanches without SOC?

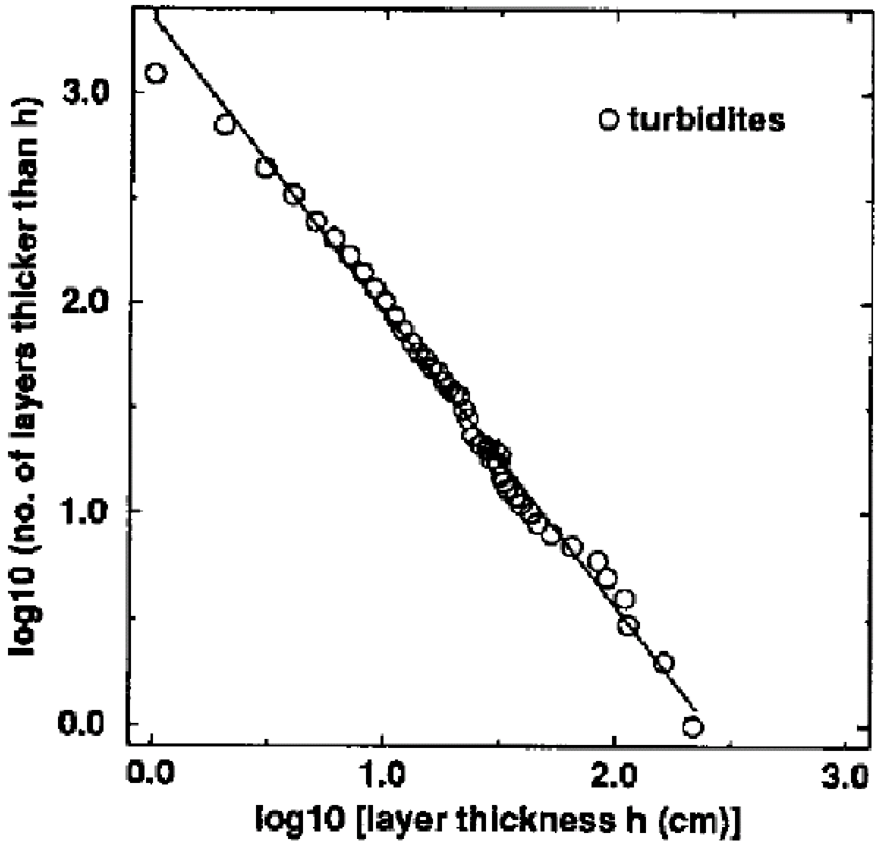
- SOC (or at least sandpiles) does not seem to be a robust paradigm to explain in general the wide presence of power law avalanche behavior in complex systems
  - They are implicitly tuned to a standard critical point by boundary conditions

- Can we say something otherwise about the avalanche behavior observed in nature?
- Answer: Yes, at least in some particular systems
- Examples:
  - Turbidite deposition
  - Internet blackouts (storms)

Turbidite deposition

- Sloping topographies erode in infrequent events (avalanches)
- In submarine topographies, avalanches create gravity-driven flows on material that, after sedimentation, form sedimentary rocks (turbidites)
- Turbidites are formed by different layers, corresponding to different avalanches
- Thickness of layers is related to the size of avalanches

Turbidite deposition



- The thickness distribution of turbidite layers follows in some cases a power law distribution

$$P(\Delta) \sim \Delta^{-\gamma}$$

- Characteristic exponent

$$\gamma \sim 2 - 2.4$$

(Rothman at al., 1995)

One step back …

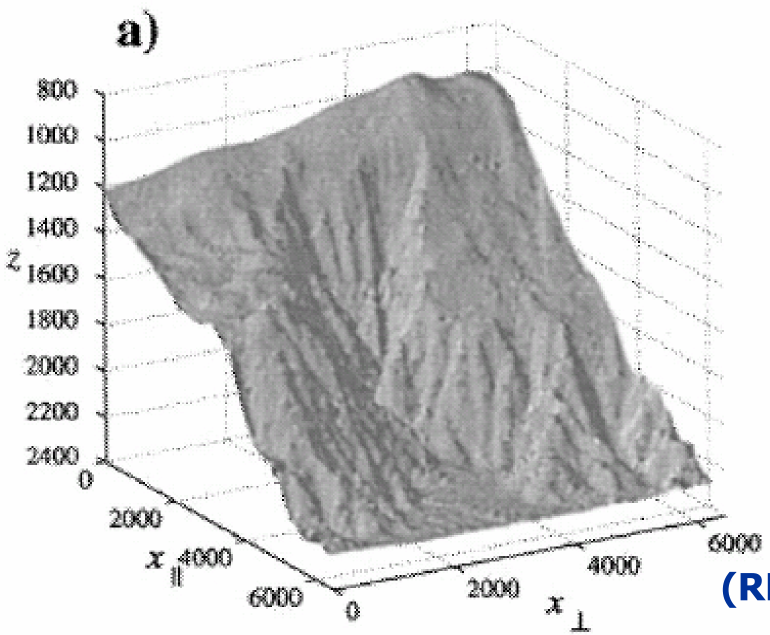
- To understand the scaling of turbidite deposition we must go one step back: consider the topography that generates the avalanches
- Topographical surfaces are rough
  - Described in terms of self-affine surfaces
  - Characterized by means of the height-height correlation function

$$C(r) = \langle |h(x+r) - h(x)|^2 \rangle_x^{1/2} \sim r^\alpha$$

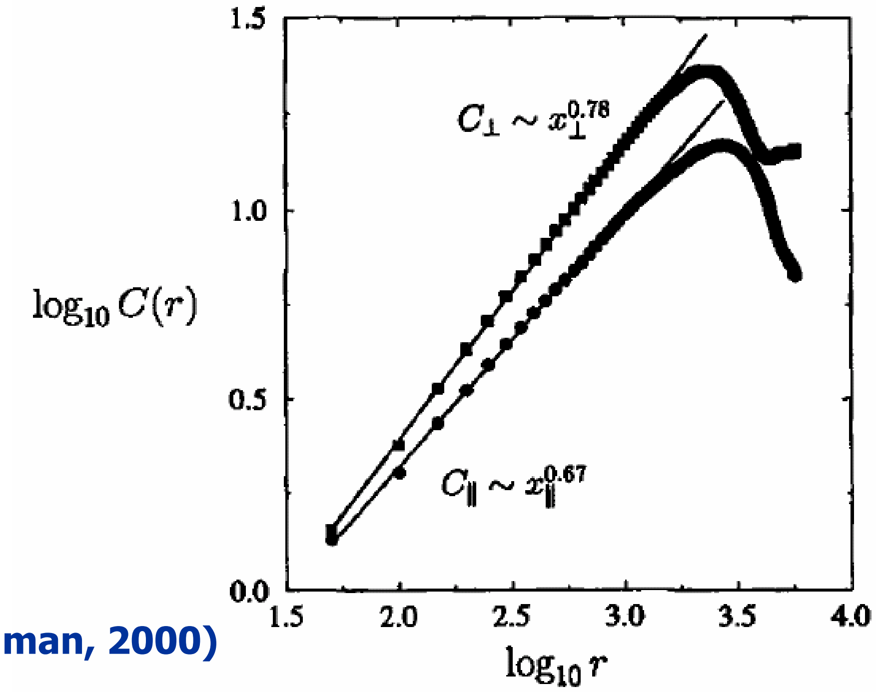
  - α : characteristic roughness exponent
  - Measured values ranging from 0.4 to 0.8 over the world

Tilted topographies

- Many topographies show in addition a average slope
  - Induces a preferred direction (downwards)
- This anisotropy generally induces the presence of two roughness exponents, when measuring correlation in the downwards direction or $x_{\parallel}$ or in the perpendicular direction $x_{\perp}$



(RPS & Rothman, 2000)



A theory for tilted topographies

- Tilted topographies can be analytically studied using the techniques of self-affine growing surfaces:
- Construction of a stochastic growth equation for the landscape height h(x,t), following symmetry arguments:
  - Anisotropy
  - Preferred transport along downwards $x_{\|}$ direction
  - Conservation of material
- Lowest order equation

Source of random noise

$$\frac{\partial h}{\partial t} = \nu_{\|} \partial_{\|}^2 h + \nu_{\perp} \nabla_{\perp}^2 h + \frac{\lambda}{3} \partial_{\|}^2 (h^3) + \eta$$

(RPS & Rothman, 1998)

RG analysis

- Applying the dynamic renormalization group in a one-loop ε-expansion

$$\alpha_\perp = \frac{5\varepsilon}{12}, \qquad \zeta_\perp = 1 + \frac{\varepsilon}{6}, \qquad \varepsilon = 4 - d \quad \alpha_\parallel = \frac{\alpha_\perp}{\zeta_\perp}$$

(RPS & Rothman, 1998)

- At the physical dimension d=2

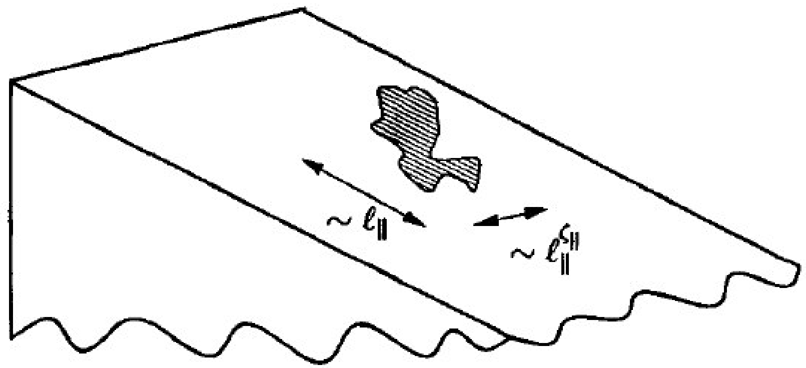
$$\alpha_\perp = \frac{5}{6} \simeq 0.83, \qquad \zeta_\perp = \frac{1}{\zeta_\parallel} = \frac{4}{3}, \qquad \alpha_\parallel = \frac{\alpha_\perp}{\zeta_\perp} = \frac{5}{8} \simeq 0.63$$

  - Reasonable agreement with field measurements ($\alpha_\parallel$ = 0.67, $\alpha_\perp$ = 0.78)

Back to turbidites …

- Assume tubidite layers come from unstable patches of terrain that fall from a tilted submarine landscape
- Size of patches scales scales as the surface

$$s \sim \ell_{||}\ell_{\perp} \sim \ell_{||}^{1+\zeta_{||}}$$



- Assuming a power law distribution of patches sizes

$$P(s) = s^{-\tau} f\left(\frac{s}{L^{1+\zeta_{||}}}\right)$$

Back to turbidites …

- Imposing the condition $\langle s \rangle \sim L$ (patches are very elongated)

$$\tau = 2 - \frac{1}{1+\zeta_{\|}}$$

- Relating thickness with size $\Delta \sim s^{1/3}$ we obtain the thickness distribution

$$P(\Delta) \sim \Delta^{-\gamma} \qquad \gamma = 1 + \frac{3\zeta_{\|}}{1+\zeta_{\|}}$$

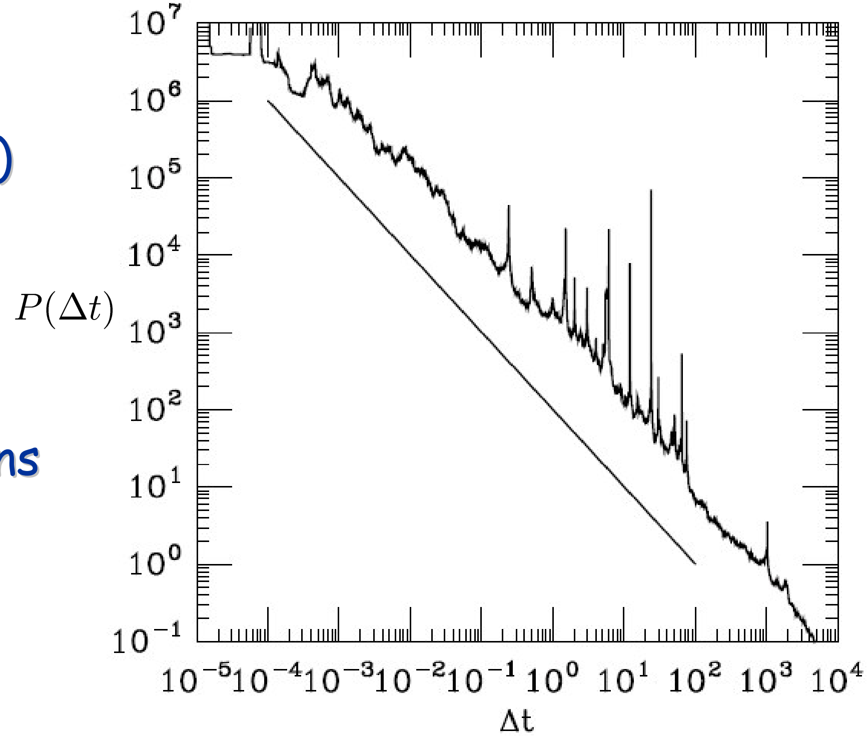
- Inserting values from RG: $\gamma \approx 2.3$
  - Good agreement with field values

Internet storms: Instabilities and congestions

- Everybody has experienced short-term Internet outages:
  - Clicking on a link that does not respond, but responds a few seconds later
- Internet outages usually come from instabilities propagating through the network
  - Configuration errors
  - Traffic congestions
  - Software bugs …
- The propagation of instabilities and congestions can lead to failure avalanches that can collapse regions of the Internet, blocking its access

Internet storms: Instabilities and congestions

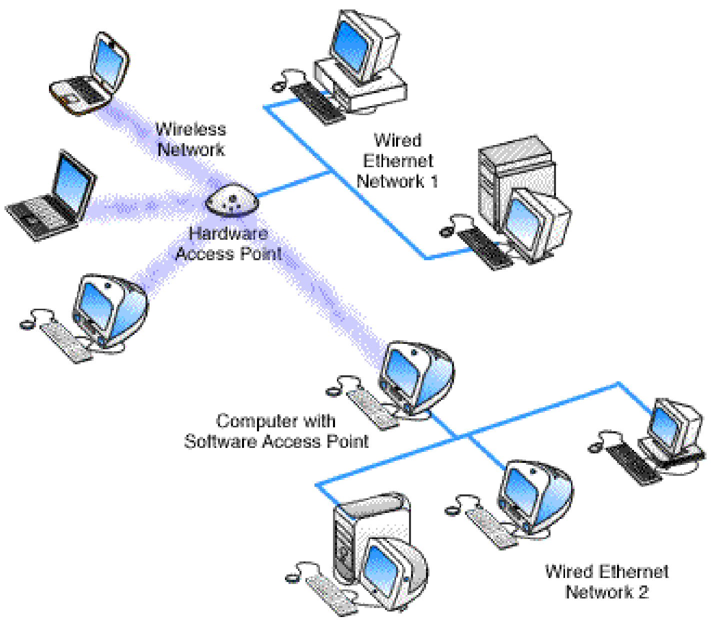
- Empirical measurements show that Internet congestions have a power law signatures
  - Distribution of interarrival error messages (signature of congestion) are distributed according to a power law of exponent -1
  - Measures of the size of congestions ore more difficult but seem to hint towards the same power law trend



(Magnasco, 2000)

One step back …

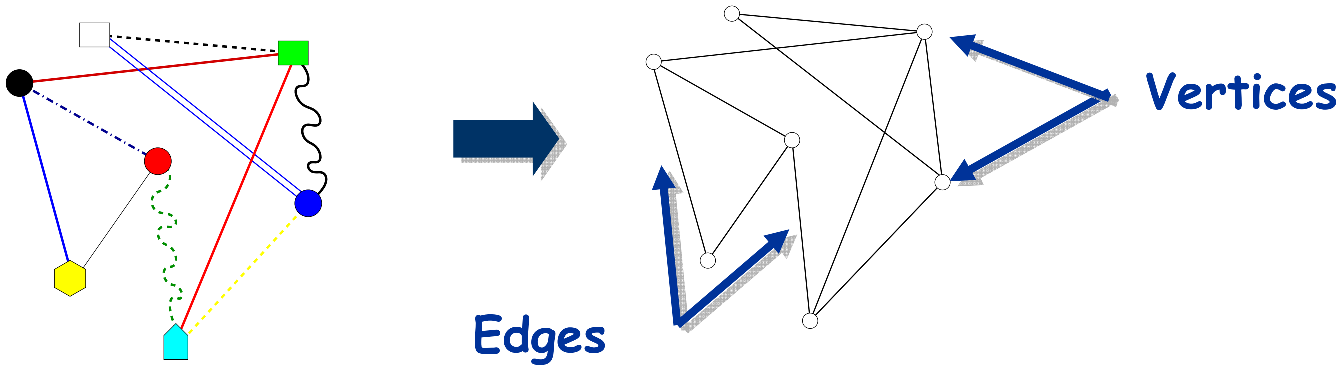
- We can understand Internet storms by understanding the Internet topology
- Key concept: Computer network
  - Set of interconnected computers that can communicate among each other



- Internet: network that interconnects many different computer networks on a world-wide scale
- Main characteristic:
  - Heterogeneity both in design and components

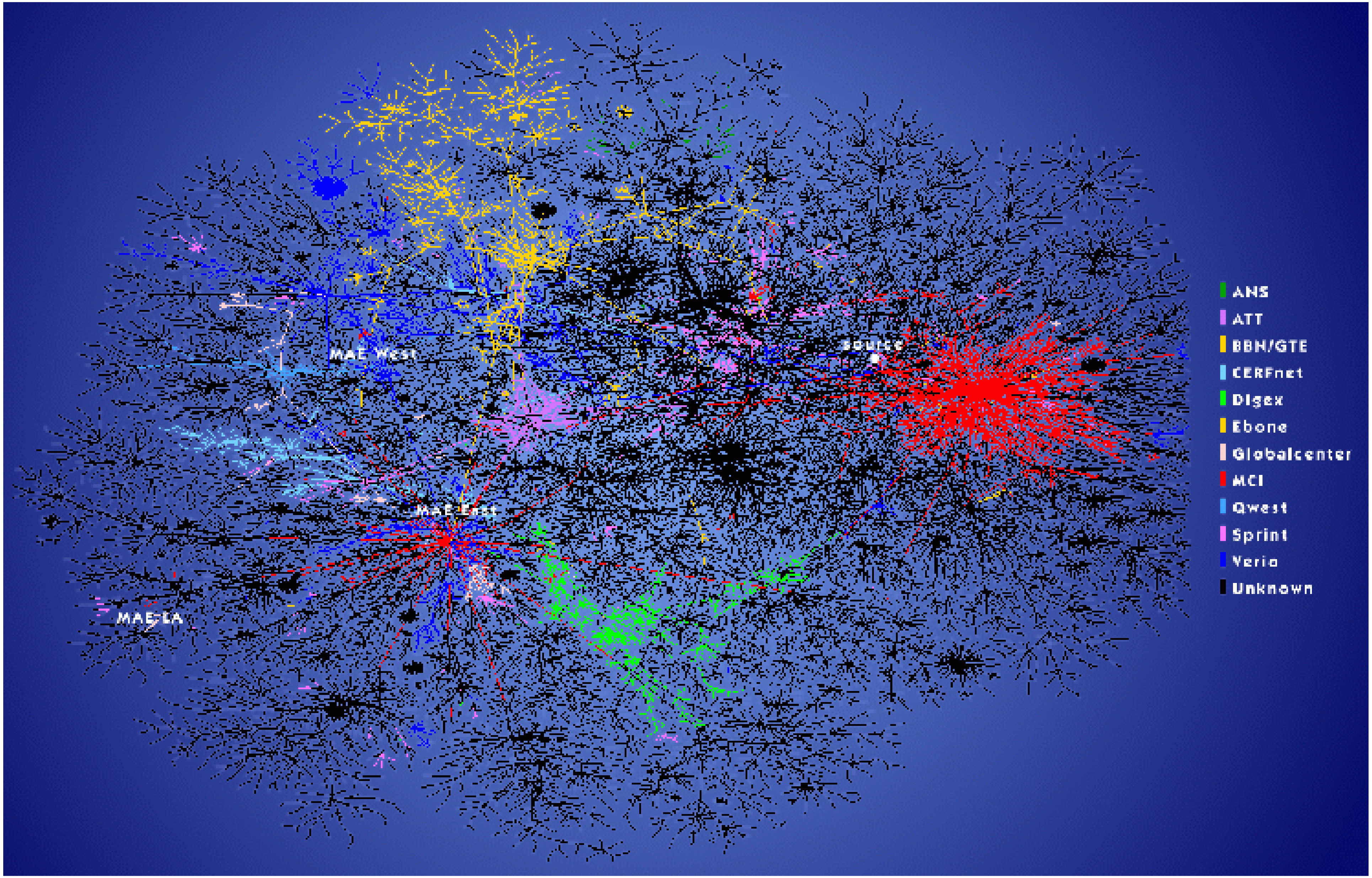
Topology of the Internet

- The Internet belongs to a general class of complex heterogeneous systems
  - Large number of diverse elementary components
  - Interactions between components can also diverse and can be non-local
  - Number of elements $10^3 - 10^6$
- New perspective for the study: representation as a graph or network



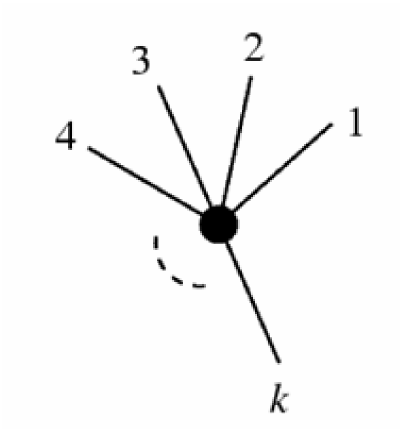
  - Study of the topological properties of the representative network

Representations of the Internet



Topological analysis of networks

- The most important topological characterization of a network is the degree of the vertices
    - Degree k_i



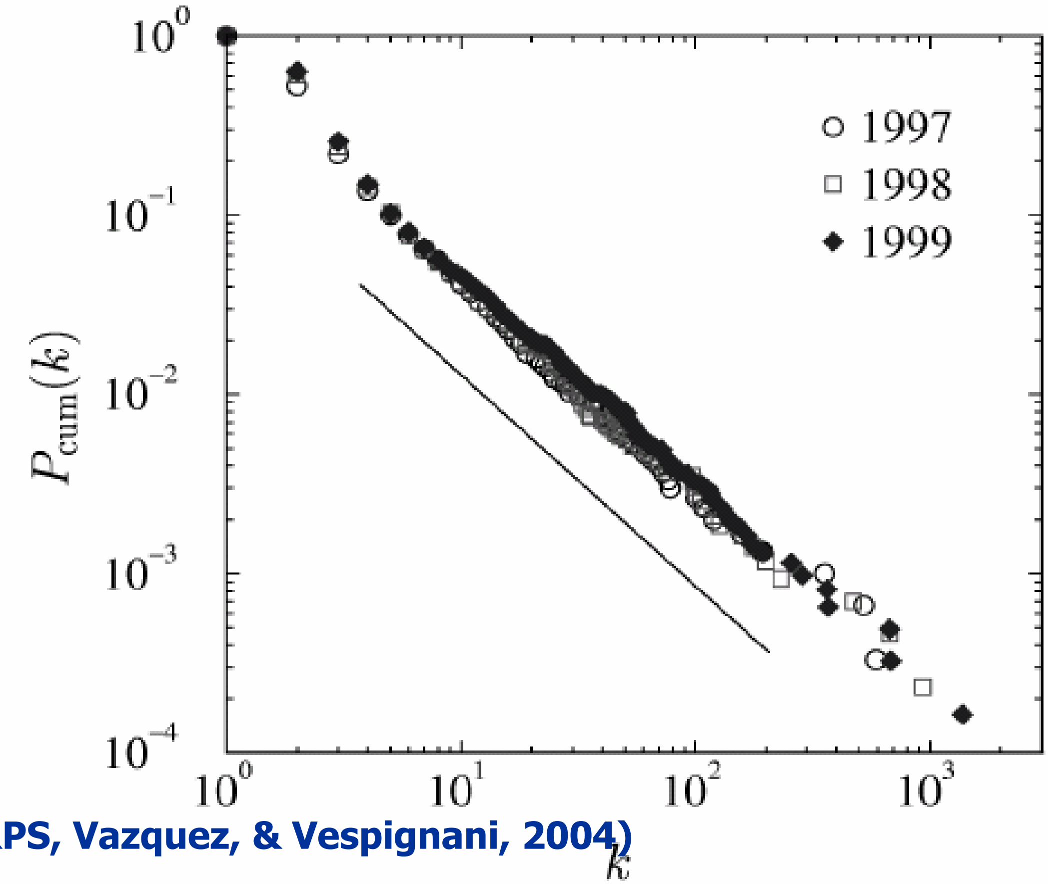
- For large networks: Statistical characterization by the degree distribution

$$P(k)$$

Scale-free Internet

- Empirical investigation shows that the Internet is a scale-free network
  - Degree distribution given by a power law

$$P(k) \sim k^{-\gamma}$$



  - Degree exponent

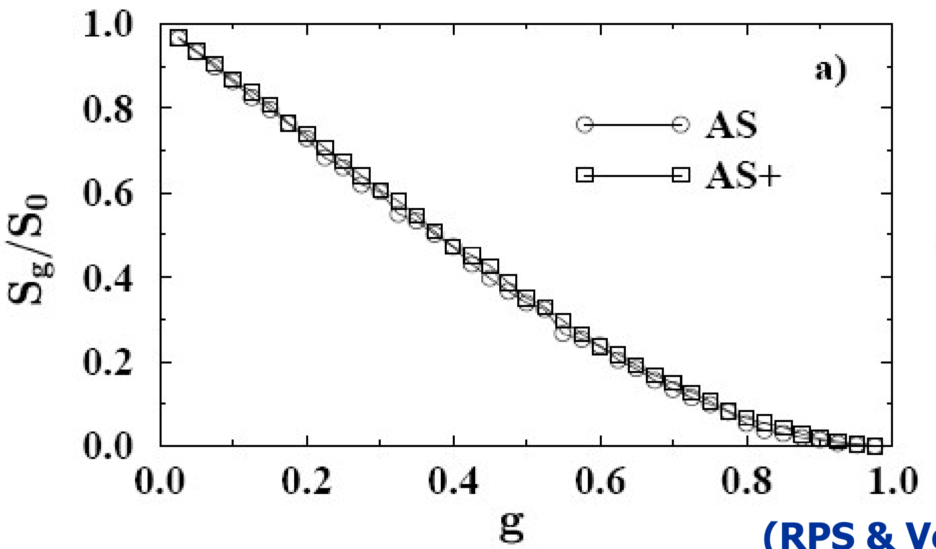
$$\gamma \sim 2.2$$

(RPS, Vazquez, & Vespignani, 2004)

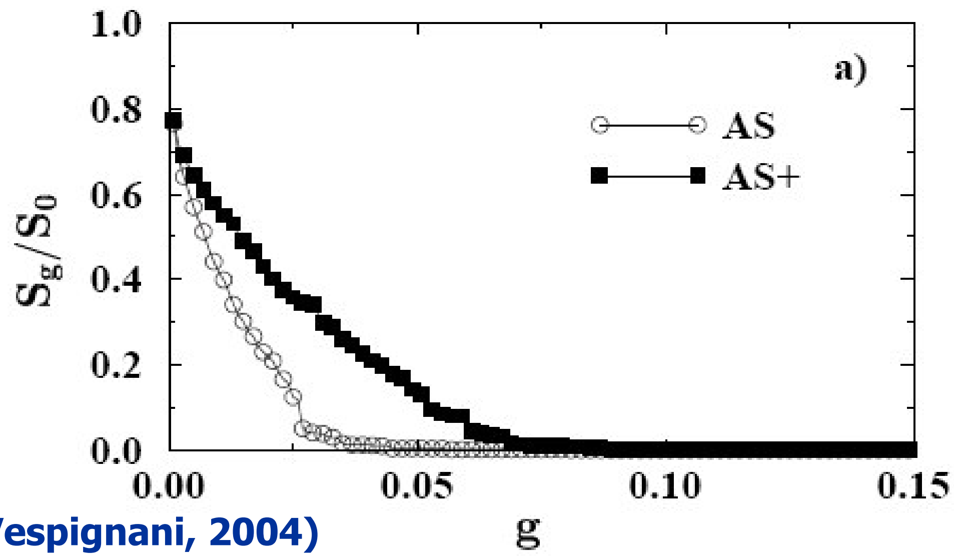
Effects of an scale-free topology

- A scale-free topology can have a very strong impact on the properties of the system
  - The Internet is very strong against random removal of elements
  - On the other hand, it is very weak against the targeted removal of the most connected elements

Random removal of elements



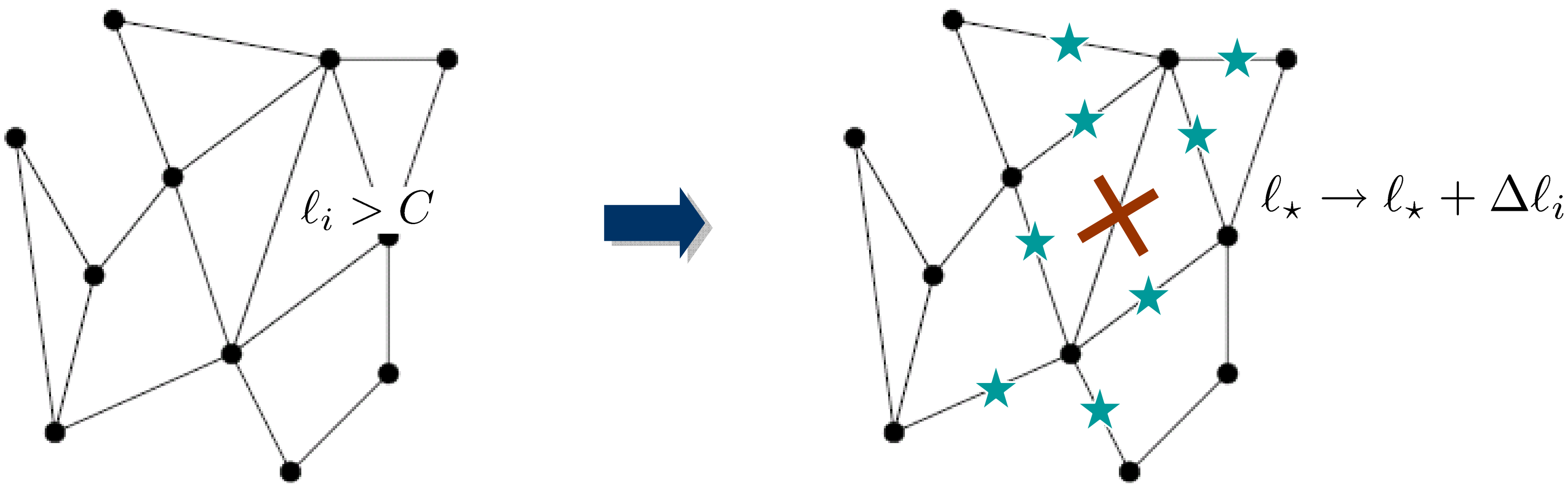
Targeted removal of elements



(RPS & Vespignani, 2004)

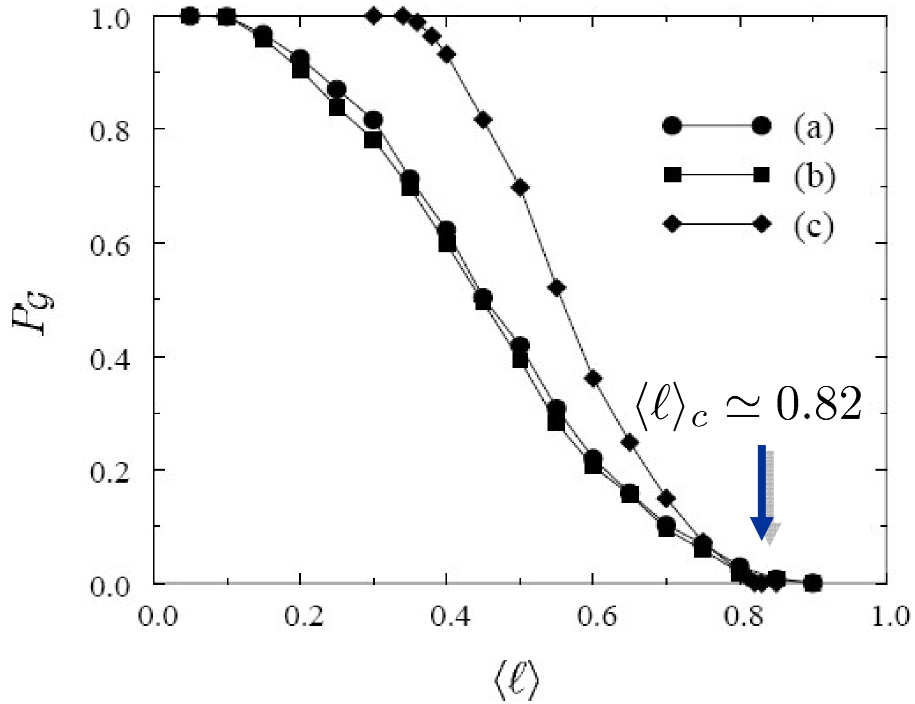
Back to Internet storms …

- Can the topology of Internet explain the distribution of outages?
- Simple model of the static failure of a transport network
  - Scale-free network
  - Each connection (edge) carries load ℓ_i at random from a uniform distribution with average $\langle \ell \rangle$
  - Threshold dynamics: when $\ell_i > C$, the connection breaks and its load is redistributed among nearest connections
  - The redistribution can produce an avalanche of connection failures



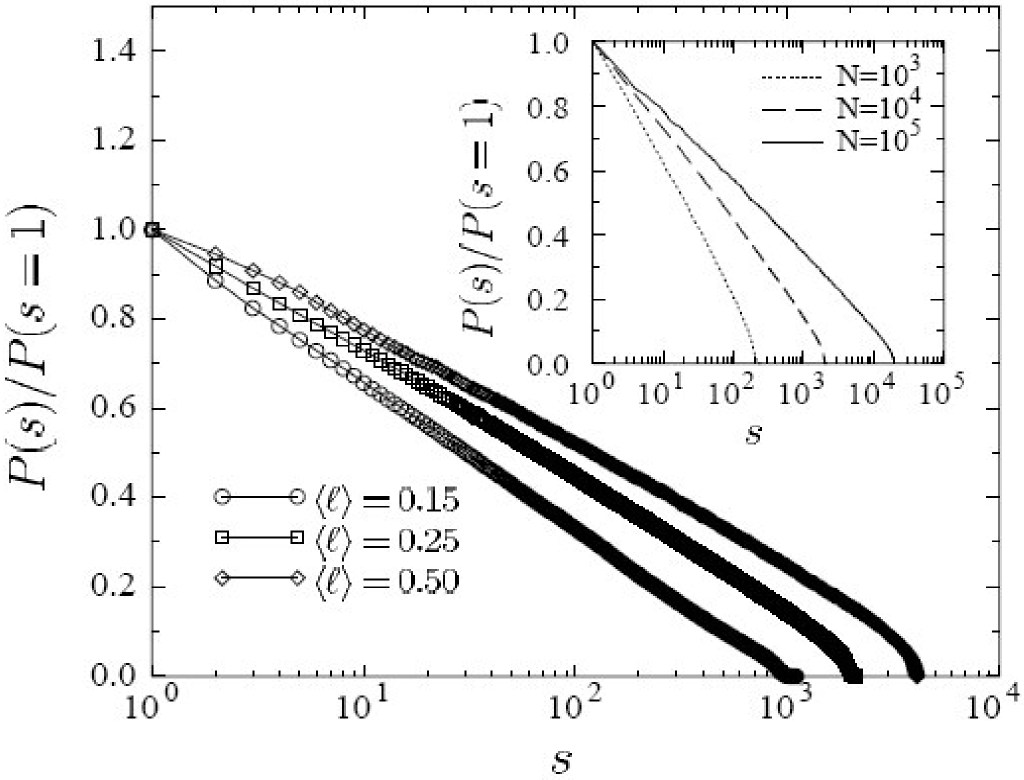
Back to Internet storms …

- When increasing $\langle \ell \rangle$, connections break and the network becomes disconnected
- Study of the size of the largest piece of connected network as a function of the average load
  - There is a phase transition at a finite value $\langle \ell \rangle_c$, separating a disconnected network (with no communication capabilities) from a connected network
- Note: We need to adjust $\langle \ell \rangle_c$
  - Usual phase transition



(Moreno, RPS & Vespignani, 2003)

Back to Internet storms …



(Moreno, RPS & Vespignani, 2003)

- Numerical study of failure avalanches: distribution of broken connections
- Distribution size scales as a power law with exponent -1, for q wide range of values of $\langle\ell\rangle$
  - No need of fine-tuning
- Preliminary results on a dynamic model indicate distribution of times between avalanches as a power law with exponent -1

Conclusions

- Sandpile models are not a good paradigm for possible SOC
  - The boundary conditions drive implicitly the system towards a critical point
  - The critical point in sandpiles is a standard one (absorbing-state phase transition), where the tuning parameter is the energy density
- Without recurring to the SOC concept, we can still say things about avalanche behavior:
  - Avalanche properties can be connected to the geometrical properties of the system (tilted landscapes and turbidites)
  - The very topology of a system can lead to avalanche behavior without any fine-tuning (Internet storms)

Acknowledgements

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