How does disorder affect dynamics and structure?

Mark Robbins, KITP October 5, 2005

Different types of disorder have different effects

Static or Quenched → Fixed impurities, microcracks, grain boundaries

Annealed → Fluctuating – uncorrelated in space and time
Typically thermal or statistical (shot noise, deposition)

Dynamically generated → shear rearranges structure, new STZ's, SOC
inertia on earthquake faults → disorder (Carlson,Langer,Shaw)

Because static disorder persists, tends to have stronger effect than annealed disorder

Static most relevant for one-time failure: crack, static friction

Dynamically generated for steady process: shear in bulk or on fault

Disorder competes with elastic or surface energy

Competition depends on dimensionality, range, ...

Non-elastic response, topological defects, inertia, can make qualitative change.

Examples and Random Early References

Charge-density waves, flux lattice – d-dimensional elastic manifold moving through disorder in d-dimensions (Fisher, PRB32, 1396, (1985), Blatter et al. Rev Mod Phys 66, 1125, (1994))

Flux lines – 1D string moving through 3D solid, long range interactions and elasticity

Domain walls, fluid interfaces – d-1 dim surface through d dim disordered medium (Stokes et al., PRL60, 1386 (1988), Nolle et al. PRL71, 2074 (1993))

Contact lines, cracks – 2D elastic surface interacts along 1D line with disorder on surface or throughout bulk
Can approximate by 1D line with long-range elasticity from second dimension.(Robbins &^Joanny, Europhys. Lett. 3, 729 (1987))

Friction – 3D solid interacting along 2D with disorder (Vollmer & Natterman, Z. Phys. B104, 363, 1977)

General Results – Critical onset of motion

(See accompanying article on: "Growth in Systems with Quenched Disorder", Robbins, Cieplak, Ji and Koiller, 1993)

For $d<d_c$ disorder wins at large scales $\rightarrow$system pinned v=0 for $F<F_c$

As F increase to F_c, fewer stable pinned states
series of "avalanches" as jump from unstable to stable state

Avalanches have power law distribution with large scale cutoff that diverges as $F \rightarrow F_c$ (related to correlation length)

Time for avalanche scales as power of size.

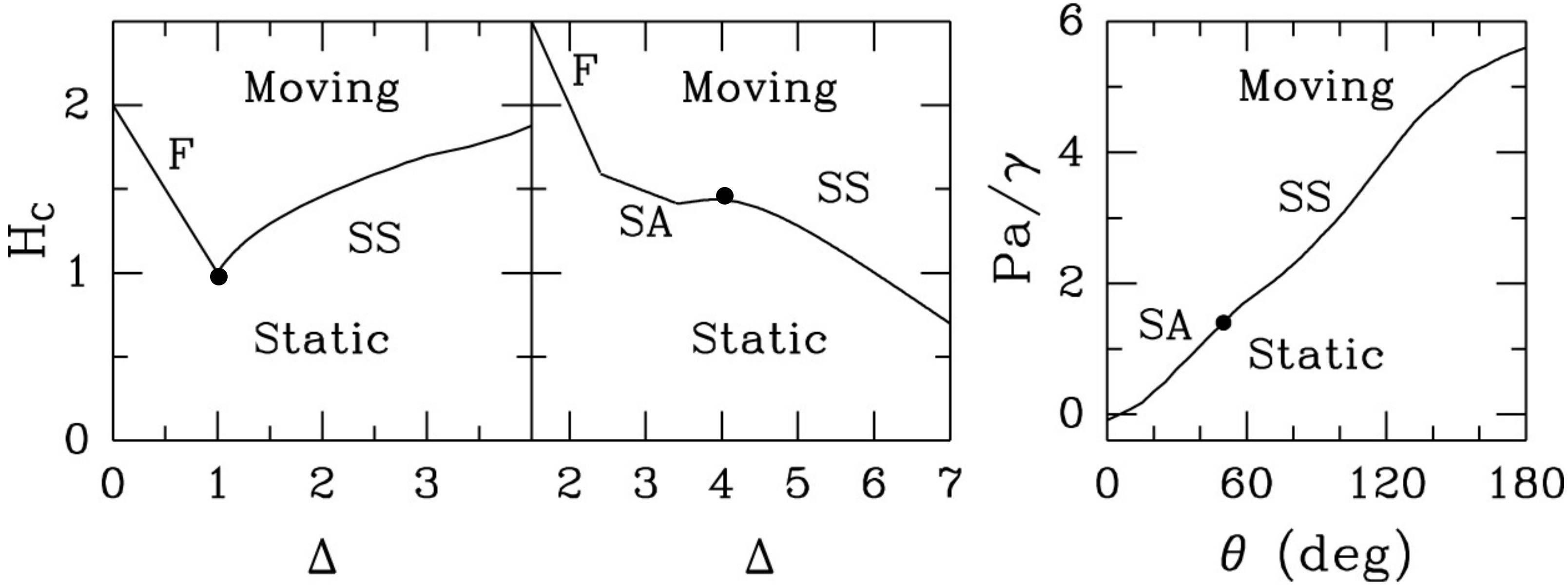
At $F=F_c$ interface has structure on all scales
– self-affine or self-similar if disorder >> elasticity

Above F_c have v rising as power of $F-F_c$
retain critical scaling at small lengths
see structure produced by annealed disorder at large lengths
avalanches replaced by velocity correlations

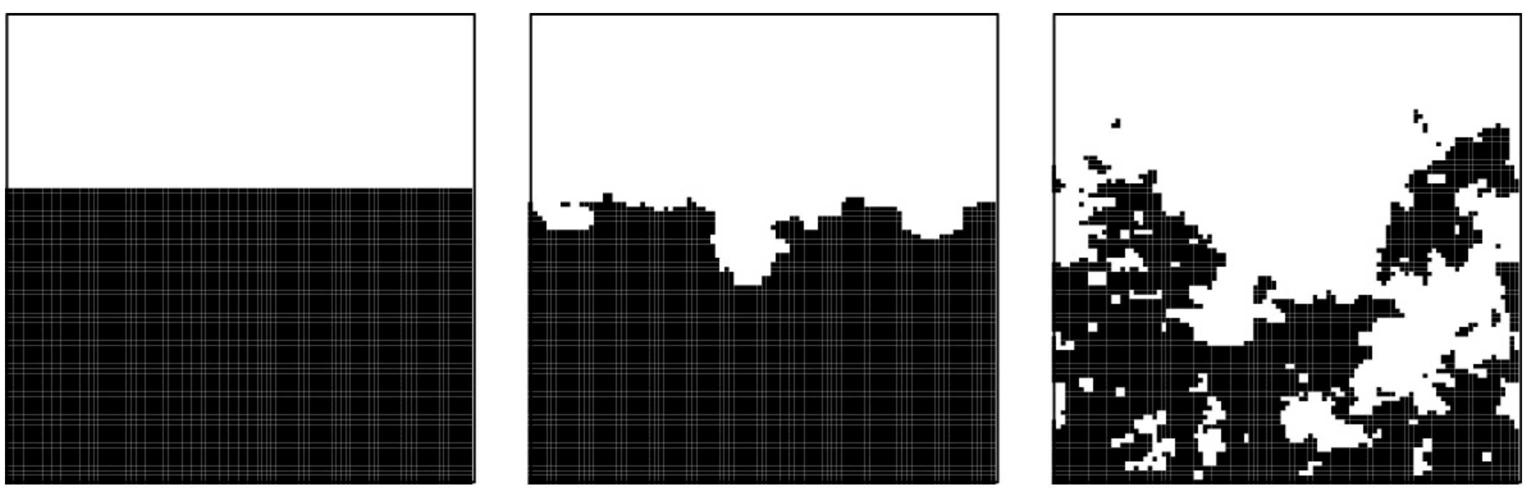
Multicritical point at disorder where change from self-affine to self-similar structure or to faceted if have lattice

Phase Diagrams and Sample Morphologies

2D RFIM 3D RFIM Fluid Invasion



F=faceted SA= Self-Affine SS=Self-Similar



Self-affine growth regime like ferromagnetic phase

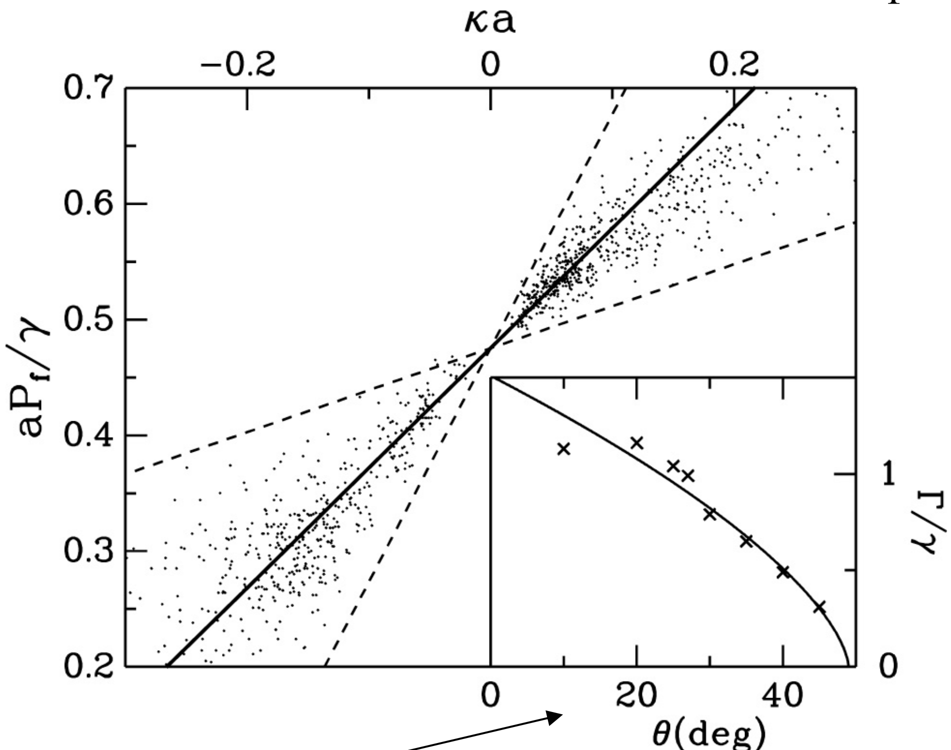
Long-range order in surface orientation like long-range order in spin orientation

Large scale surface tension Γ like spin wave stiffness

Pressure to initiate flow P_f rises with curvature k of interface: $P_f = \Gamma\kappa$

Martys, Cieplak, Robbins, PRL66, 1058 (1991)

γ=microscopic surface tension,
a~pore size



Γ goes to zero like an order parameter as effective disorder increases and growth changes to self-similar (inset)

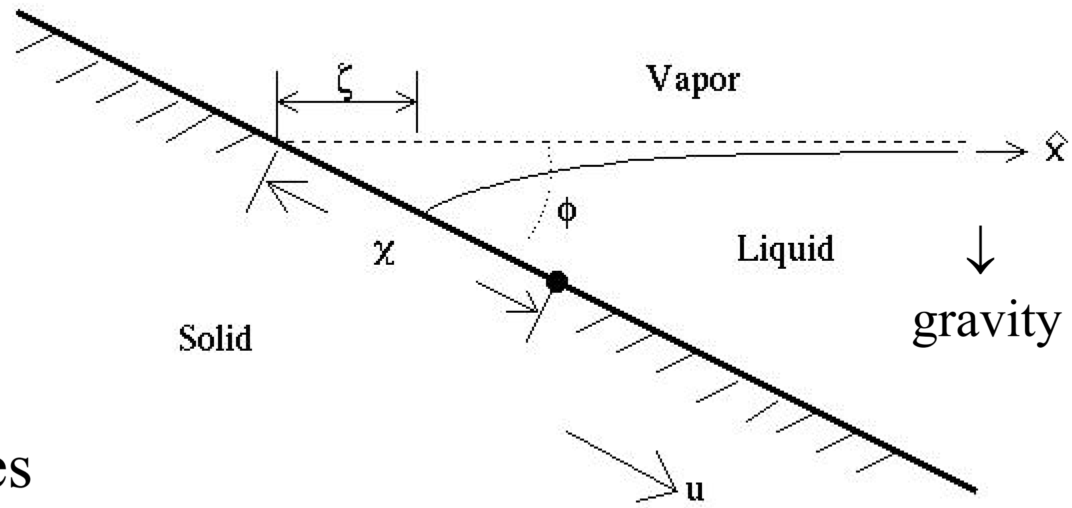
Example: Contact Line Motion on Disordered Surfaces

Collaborators: J. F. Joanny, P. A. Thompson, Y. Zhou (theory)

S. Kumar, A. P. Kushnick, D. H. Reich, J. P. Stokes (exp.)

- Disorder leads to many metastable states
  - ⇒ range of stable θ → contact angle hysteresis
  - ⇒ self-affine fractal contact line
- Onset of motion → critical depinning transition

  (analogous to charge density waves, fluid invasion, …)
  - ⇒series of avalanches between states
  - ⇒power law distribution of avalanche sizes & times
  - ⇒change in contact line morphology
- Evaluate critical exponents for contact line morphology, velocity, avalanches, …



Experimental Geometry:

•Plate advances into fluid at equilibrium contact angle ϕ for average wetting properties

➢Contact line position ζ =0 for static, uniform plate

➢Viscous drag, surface heterogeneity and surface tension produce forces that change ζ

⇒Weak heterogeneity & small velocity u (Joanny & Robbins, 1990), dimensionless equation for contact line position ζ is:

$$d\zeta(y)/dt - d\chi/dt = -H'[\zeta(y)-\chi, y] - \int dy' K(y-y')\zeta(y')$$

viscous drag heterogeneity surface tension

Surface tension is nonlocal! Makes calculation very difficult.

Fourier transform of K, $K(q)=(q^2+\kappa^2)^{1/2}$, $1/\kappa$=capillary length

Only parameter ➢ dimensionless strength of heterogeneity H'.

For interfacial fracture, crack tip →contact line, surface energy similar

Smooth circular defects of peak strength $\pm h_0$ on plate

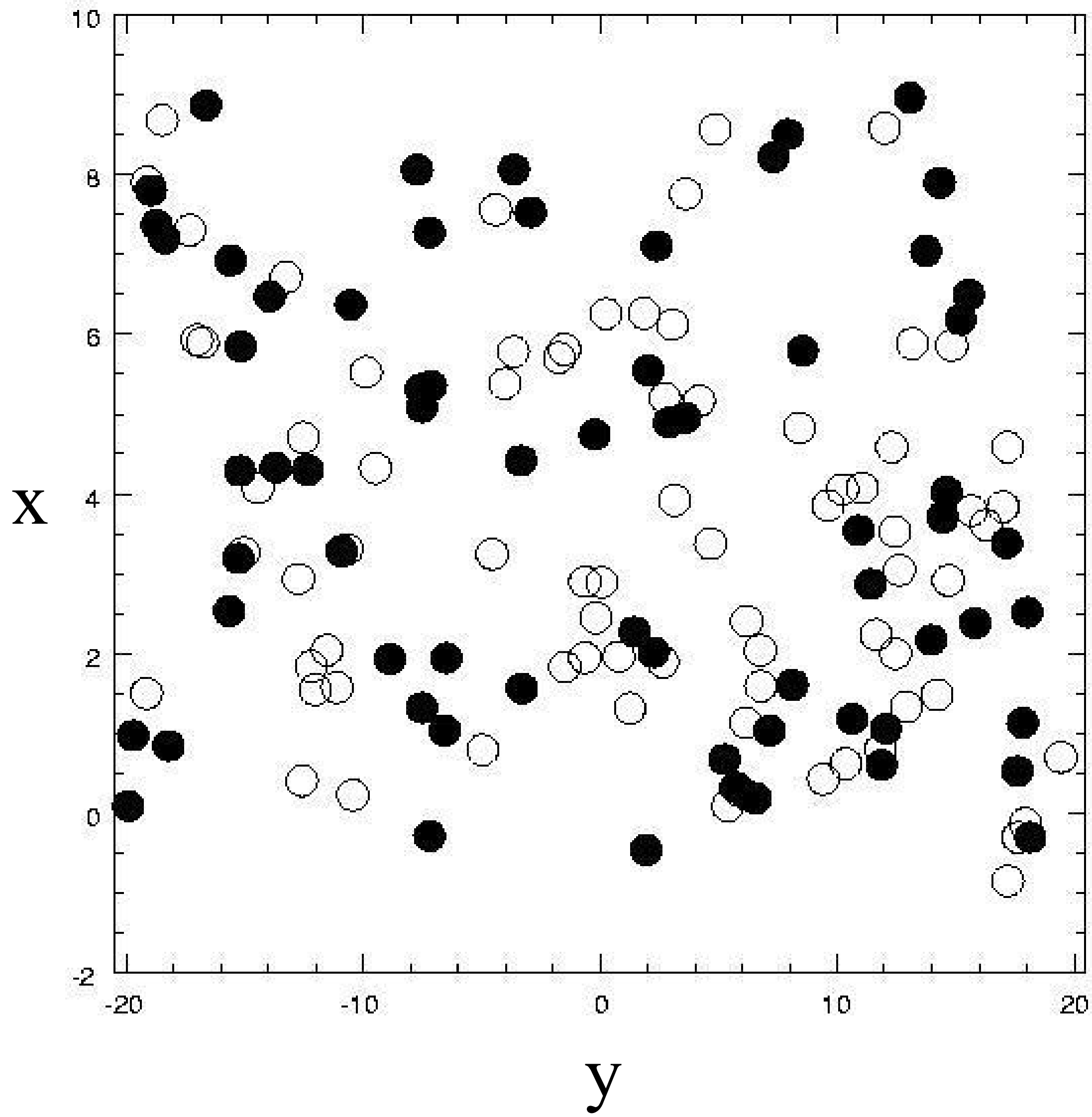
(Allowing or disallowing overlap does not change exponents)

Radius a is length unit

Quartic form of each defect: $h_0(r^2-1)^2$

Mean density = 0.32

Y. Zhou, PhD Thesis 1999



Weak pinning limit – small h_0 (Robbins & Joanny, 1987)

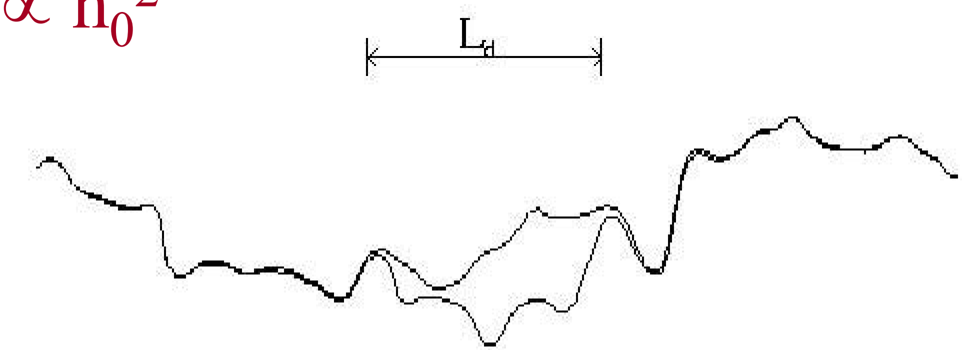
Each defect too weak to produce metastability individually

Instead pin cooperatively on longer length scales

⇒Minimum scale over which two metastable states differ is 'jog length' $L_d \propto 1/h_0^2 >>$ separation between defects

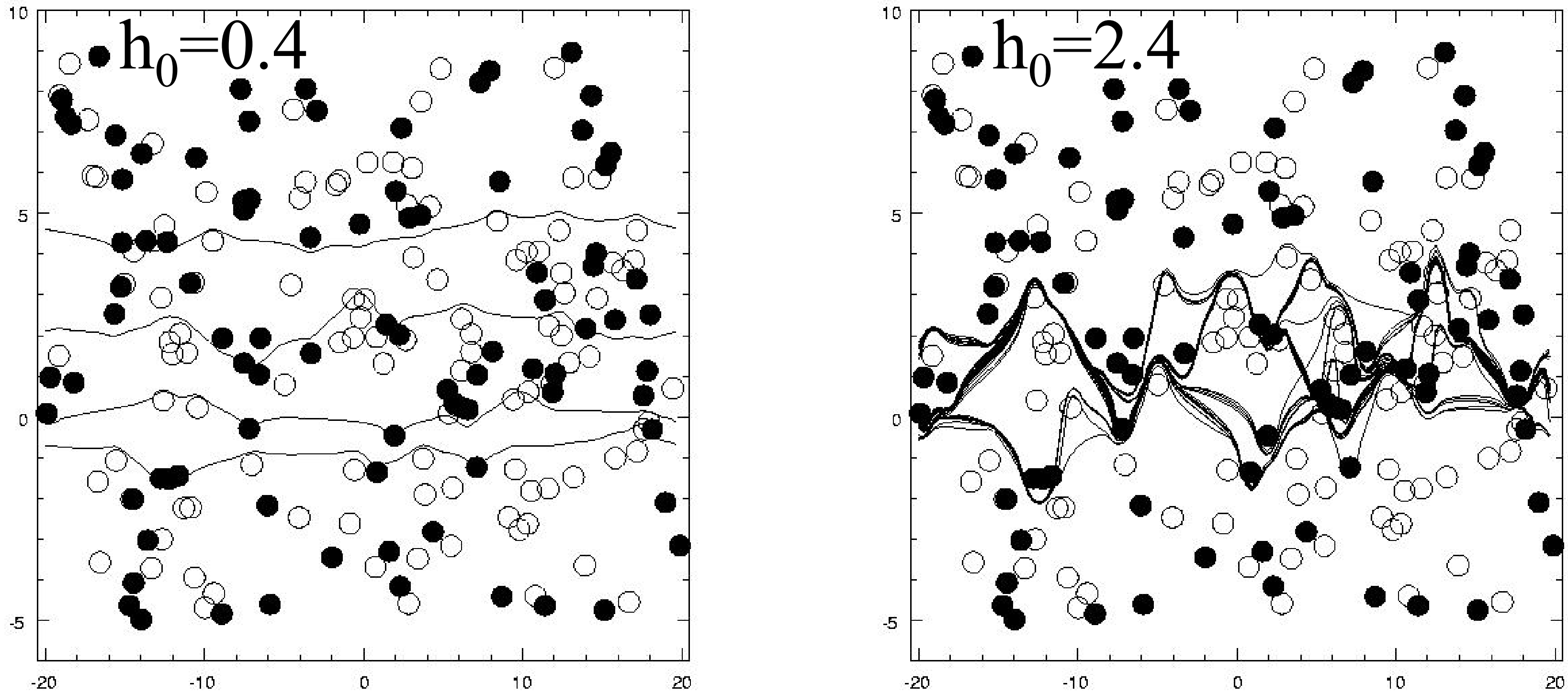
L_d determined by balance between elastic energy cost and energy gained from disordered potential

⇒Contact angle hysteresis $\propto h_0^2$



Want h_0 large so L_d much smaller than system, but not so large that out of weak pinning limit

Finding all metastable states shows L_d bigger than 40 for h_0=0.4 and comparable to the defect spacing for h_0=2.4



Reach same conclusion with other approaches

For h_0<1 find $L_d \propto 1/h_0^2$, hysteresis $\propto h_0^2$, as predicted

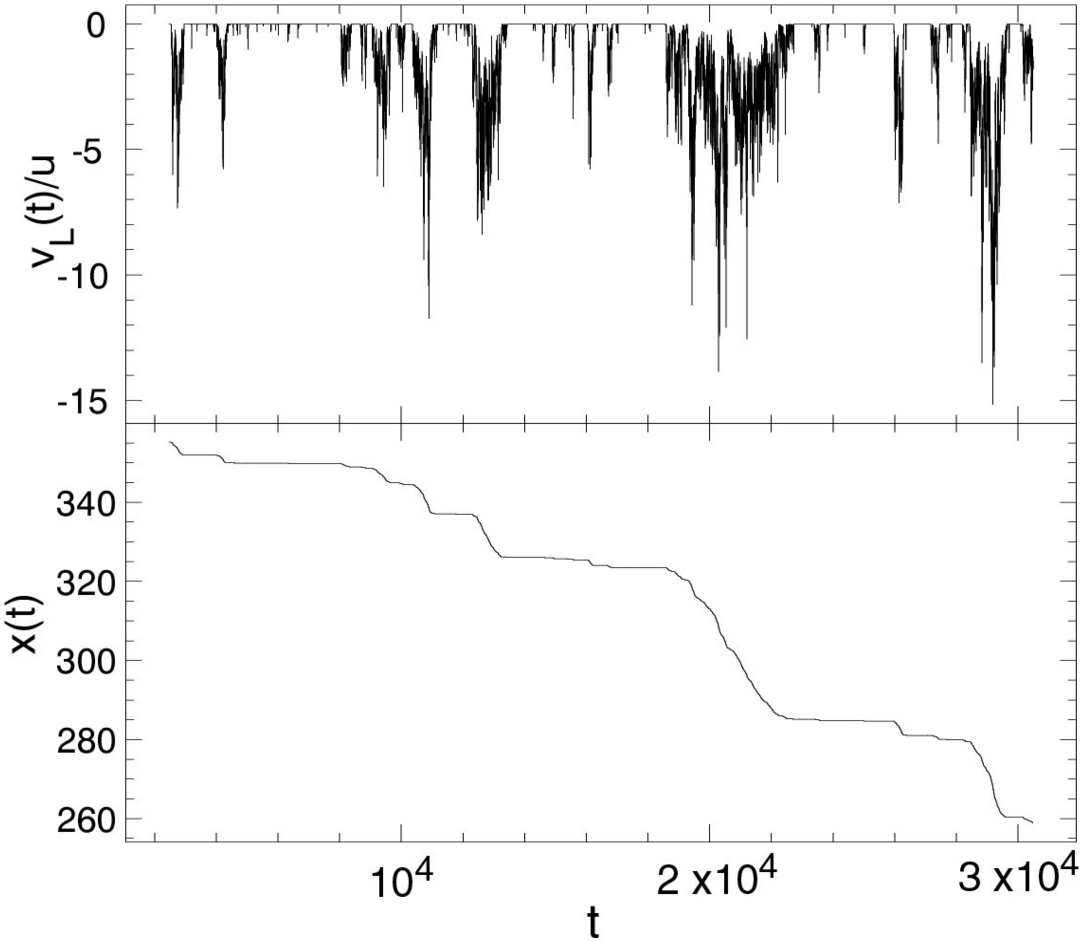
Show range of h_0 in following to test universality

Can analogous scale be defined for crack or STZ?

Avalanches During Quasi-static Motion of Contact Line (CL)

Many systems display power law distribution of "avalanches" when driven slowly, e.g. earthquakes, sandpiles, …

At low velocity CL advances in discrete jumps (shown for $u=2^{-8}$).



To approximate $u \to 0$ limit:

- push the plate at $u=2^{-8}$
- stop pushing when avalanche starts
- allow CL to reach new metastable state
- return plate velocity to $u=2^{-8}$
- repeat

Find avalanches almost stop, then continue as in shear (Maloney et al.)

Describe avalanches by:

ζ_m (max. forward jump),

A (area swept), and

t_p (avalanche time).

All have power law probability distributions

(i.e. $P(A) \propto A^{-1.2}$)

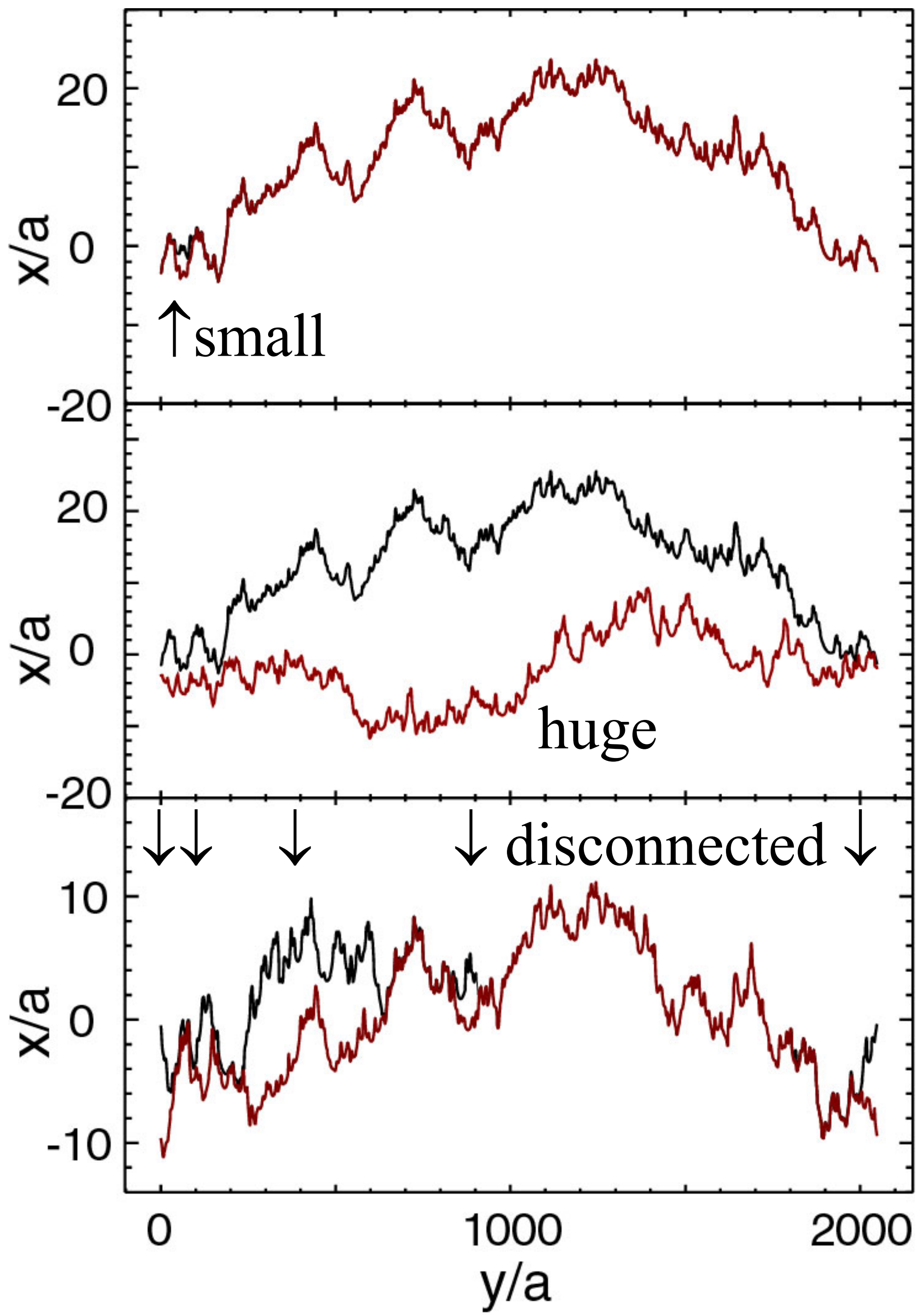
and are related by to each other by power laws

(i.e. $A \propto t^{2.3}$).

Nonlocal elasticity

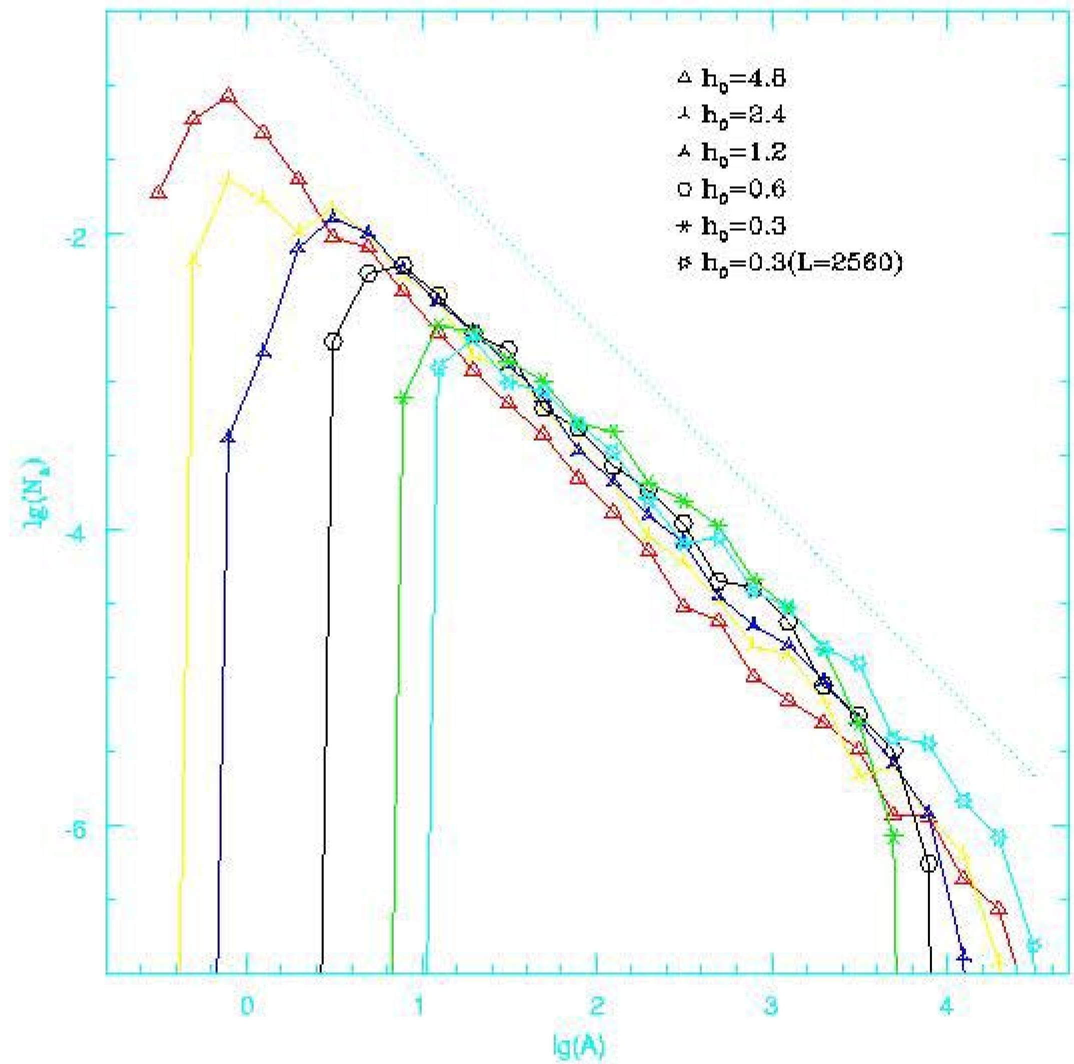
⇒disconnected avalanches

⇒ very unusual



Probability distribution for A

$P(A) \sim A^{-\gamma_A}$, with $\gamma_A = 1.2 \pm 0.1$ for all disorders



Summary of Avalanche Exponents

Y. Zhou PhD Thesis, 1999

- Results are independent of h_0 within error bars (although γ_t seems to shift slightly: 1.4 for h_0=1.4 to 1.6 for h_0=4.8):

Area distribution:	$\gamma_A = 1.2 \pm 0.1$, $P(A) \sim A^{-1.2}$.
Forward jump distribution:	$\gamma_\zeta = 1.5 \pm 0.2$, $P(\zeta_m) \sim \zeta_m^{-1.5}$.
Time distribution:	$\gamma_t = 1.5 \pm 0.1$, $P(t) \sim t^{-1.5}$.
Area vs. time:	$\varphi_{At} = 2.3 \pm 0.2$, $A \sim t^{2.6}$.
Area vs. forward jump:	$\varphi_{A\zeta} = 2.8 \pm 0.2$, $A \sim \zeta_m^{2.8}$.
Forward jump vs. time:	$\varphi_{\zeta t} = 0.8 \pm 0.1$, $\zeta_m \sim t^{0.8}$.

- Consistency:

  Probability transformation requires the identities:

  $\gamma_t - 1 = (\gamma_A - 1)\,\varphi_{At}$; $\gamma_t - 1 = (\gamma_\zeta - 1)\,\varphi_{\zeta t}$; $\gamma_\zeta - 1 = (\gamma_A - 1)\,\varphi_{A\zeta}$

  ✓Consistent with independently determined exponents

Quasistatic interface predicted to be self-affine fractal
$\Rightarrow$Change in height, w, over length, l, scales as $w \propto l^{\alpha}$
(Example: Time-ordered random walk $\alpha=1/2$)

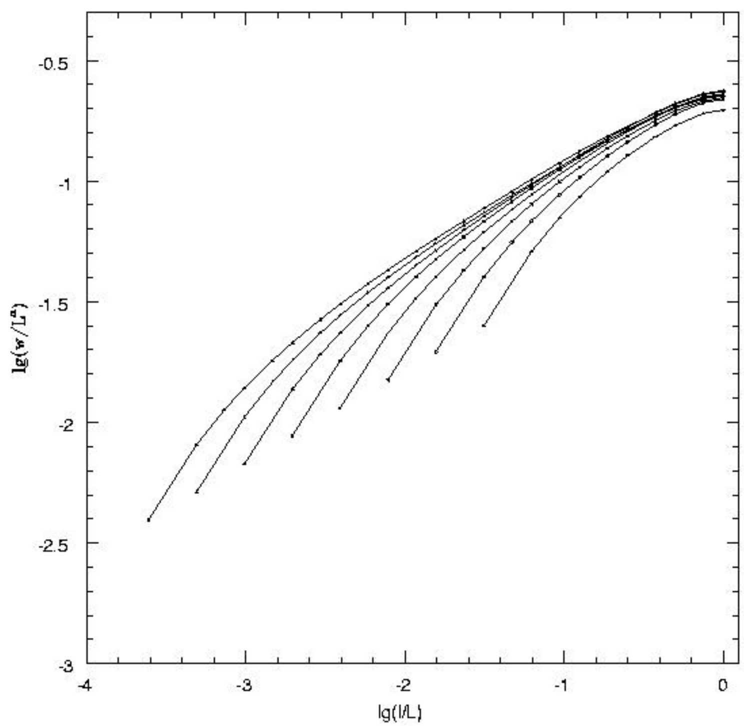
Prediction: $\alpha=1/3$ $\Rightarrow$ Scaling theory (Robbins & Joanny),
$\Rightarrow$ Renormalization Group (Ertas & Kardar)

Most accurate to find $w(l,L)$ at different system sizes L and use finite-size scaling to get α.
Assume L is only intrinsic scale, only l/L matters

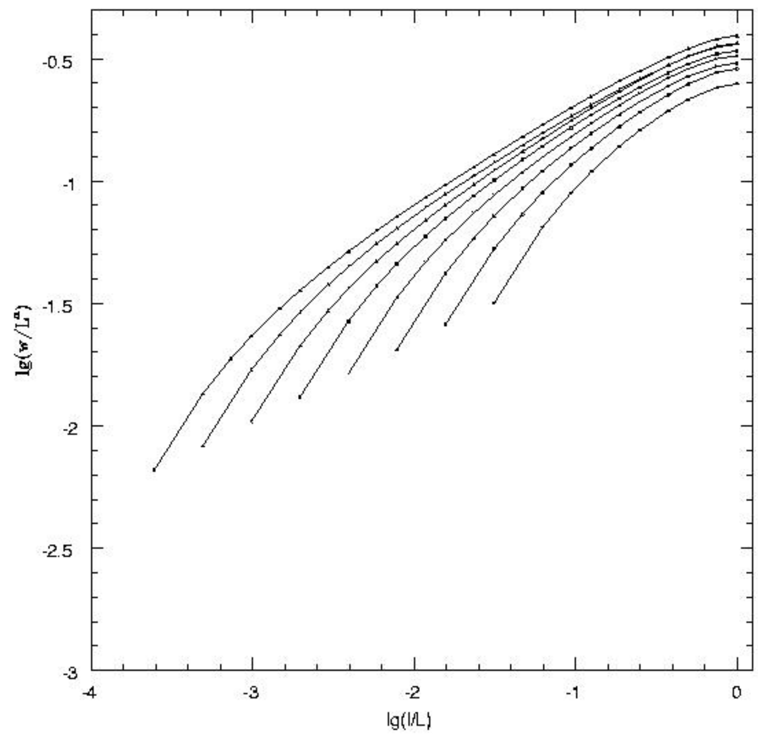
For finite size system L, finite-size scaling ansatz:

$$w(l,L)/L^{\alpha} = f(l/L)$$

where f is a universal function independent of L.

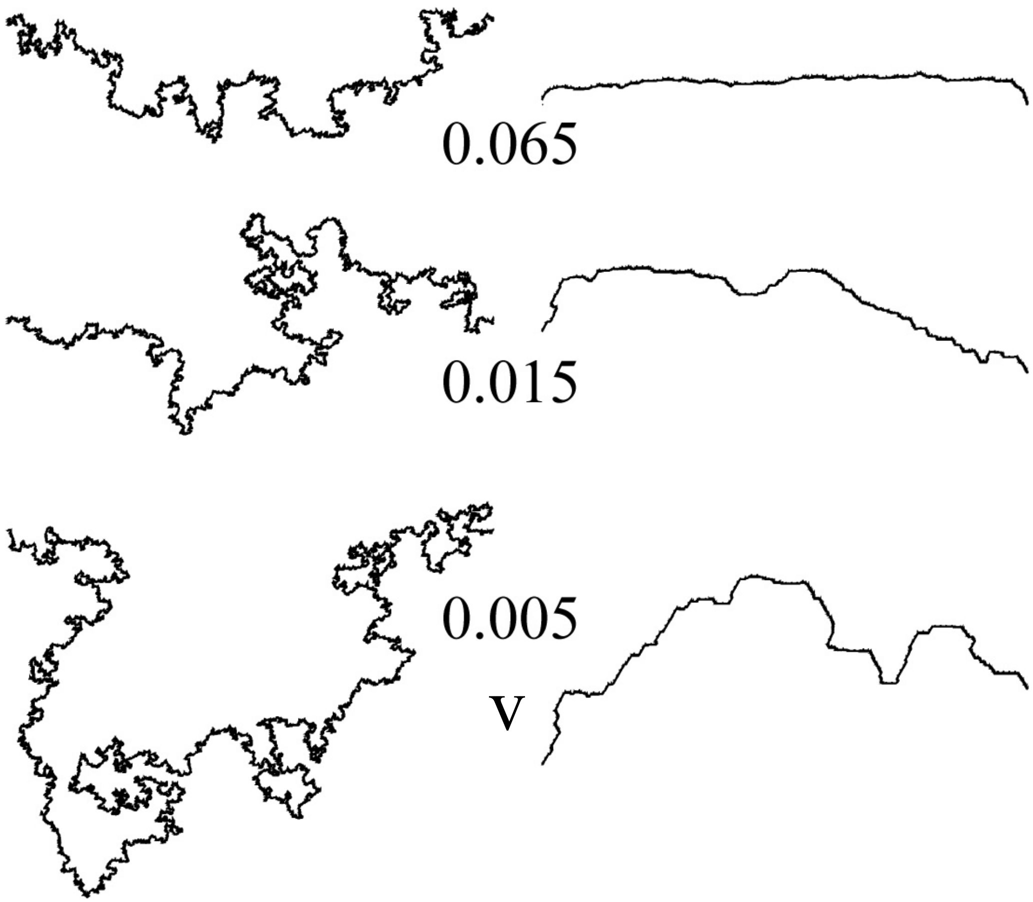


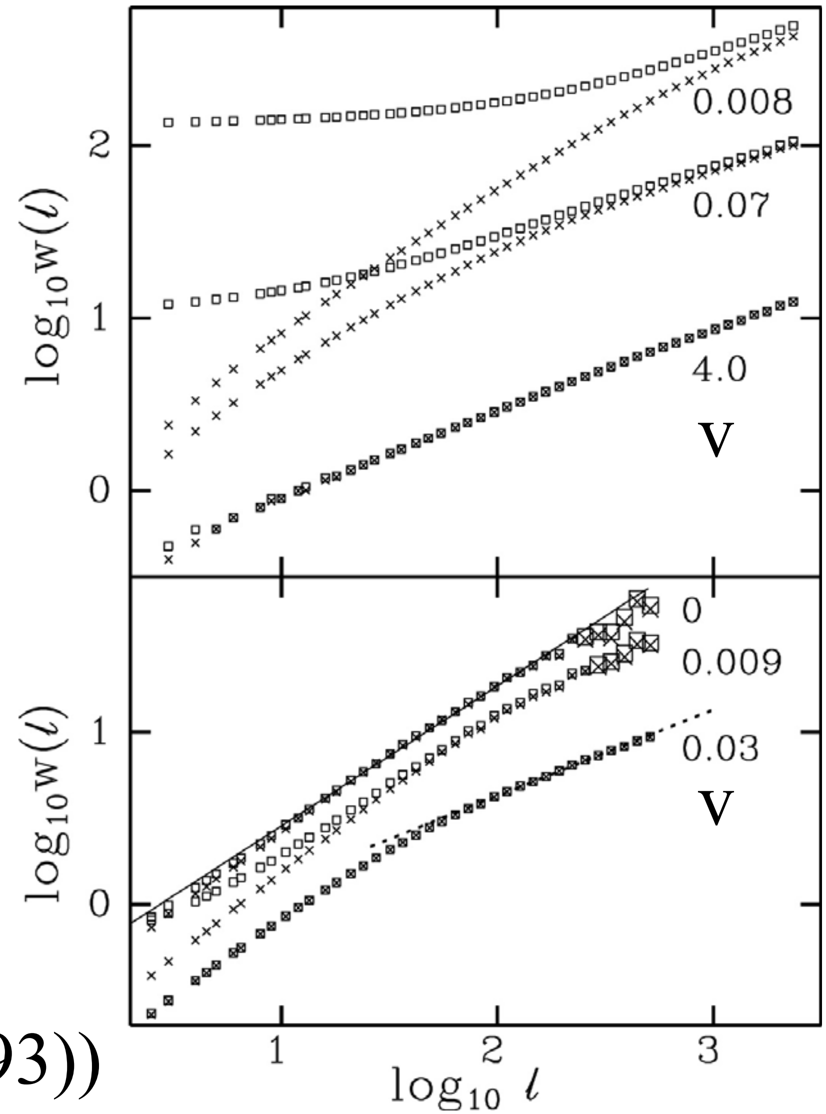
Best fit α=0.39, consistent with the slope.



RG calculation α=0.333, inconsistent with the slope.

As u increases, self-affine scaling is cut off at a dynamic correlation length $\xi_u \sim u^{-\mu}$. At $l > \xi_u$, disorder is no longer "quenched", but behaves like thermal noise.





Nolle et al. PRL71, 2074 (1993))

Force-Velocity Relation
$F - F_T \sim v^{\beta}$

- Experimental Results

 Kumar, et al. PRE 52, R5776 (1995): β=0.20(3)

 Calvo et al. JCIS 141, 384 (1991): 0(ln divergence)

 Stokes, et al. PRL 65, 1885 (1990): β=0.4

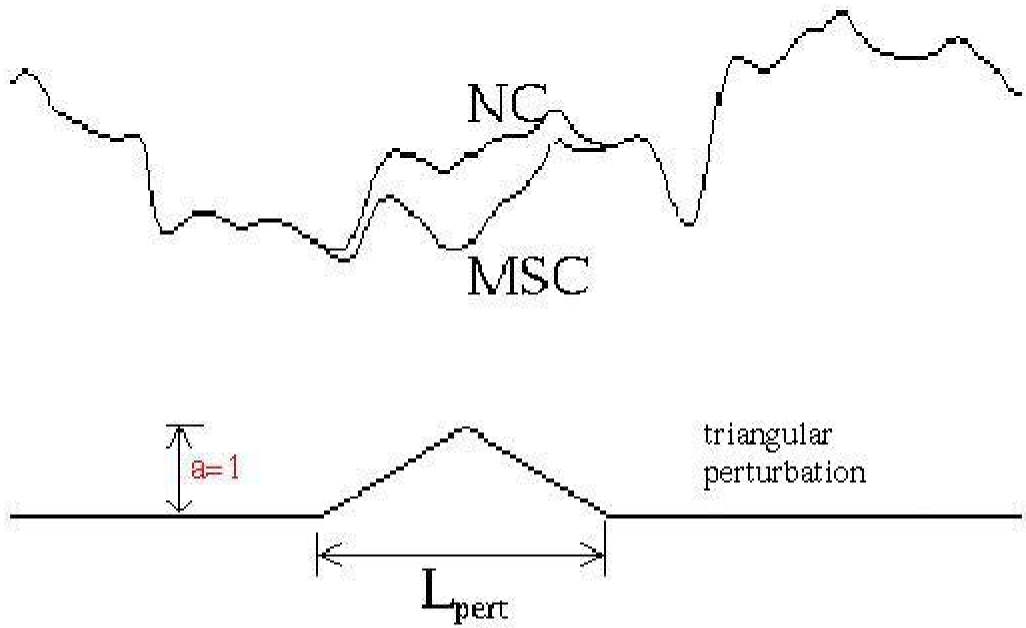
 Schaffer and Wong, PRL80, 3069 (1998): not universal

- Renormalization Group Calculation: 1.29
- We find: 1.04±.05⇒ Consistent with mean field

Adding a triangular perturbation onto a metastable configuration.

If fail to produce new metastable configuration, increase ΔL until get new metastable state.

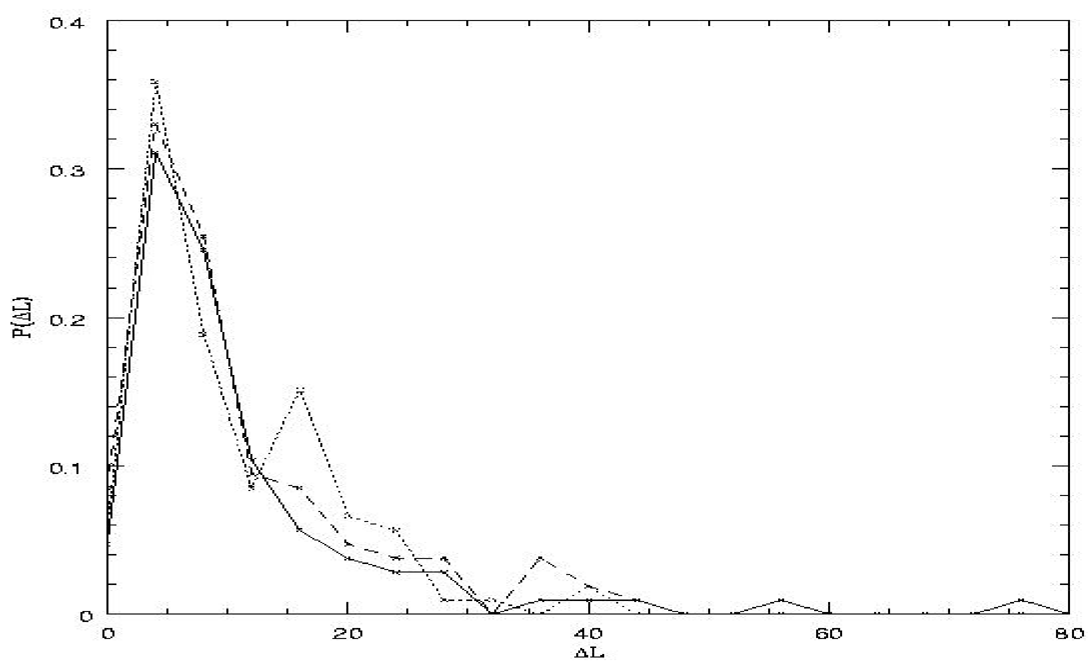
Find probability distribution of ΔL needed to get new state.



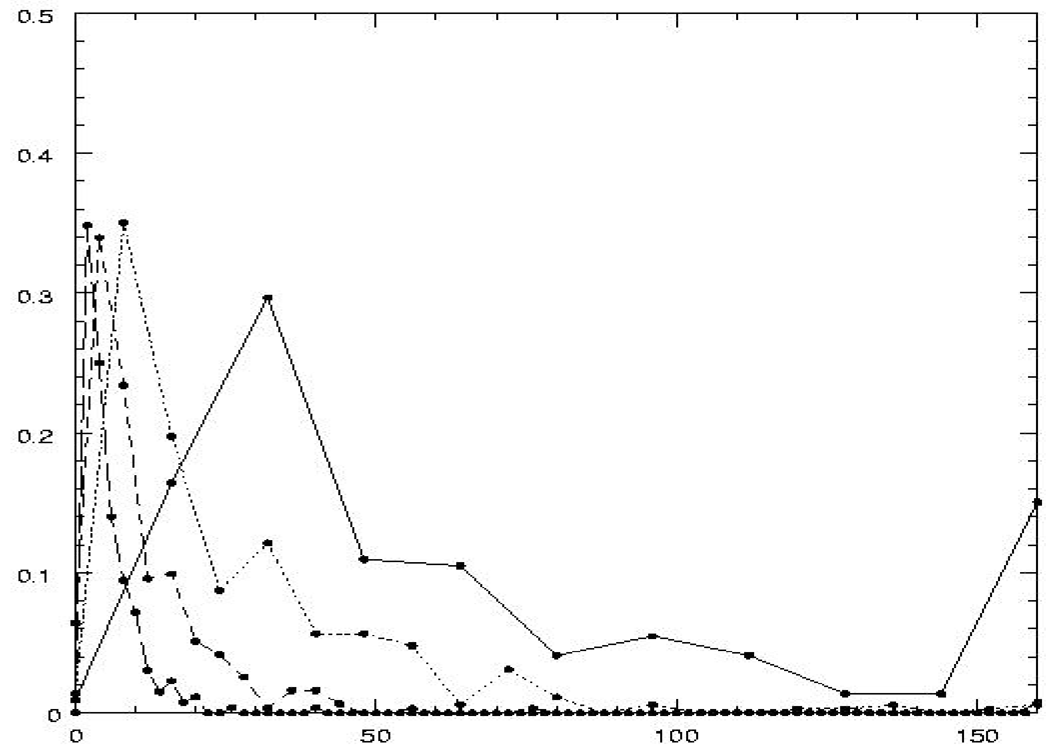
Three different ways to chose the “perturbation center”:
•randomly.
•the weakest pinning site.
•The farthest point behind.

Results are indistinguishable:

Distribution of ΔL for h_0=1.2.



Distribution of ΔL for h_0=0.3, 0.6, 1.2, 2.4.



- Identify L_d with the peak position: typical length of sections that must be moved together in order to get new metastable state.
- $Ld \sim h_0^{-2}$ for $h_0<1.2$.

Questions

- Does spontaneous disorder generated during shear lead to a critical state?
- If not, is it because of inertia, sound propagation time, or breaking of elasticity?
- Do failures of theory to describe contact line structure and dynamics reflect inertia, breakdown of assumed elastic kernel which requires small angles, no overhangs, and no pre-cracks?