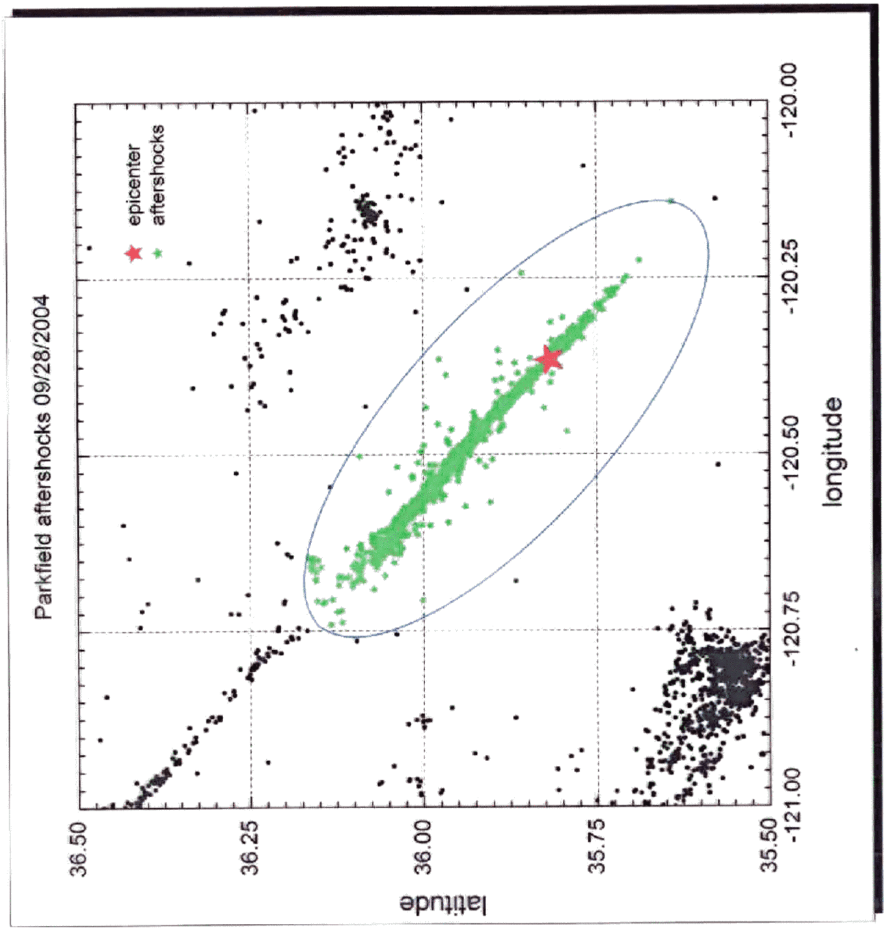


The Modified Form of Båth's Law

▶ The Gutenberg-Richter scaling for aftershocks

$$\log N(> m) = a - b\,m\,.$$

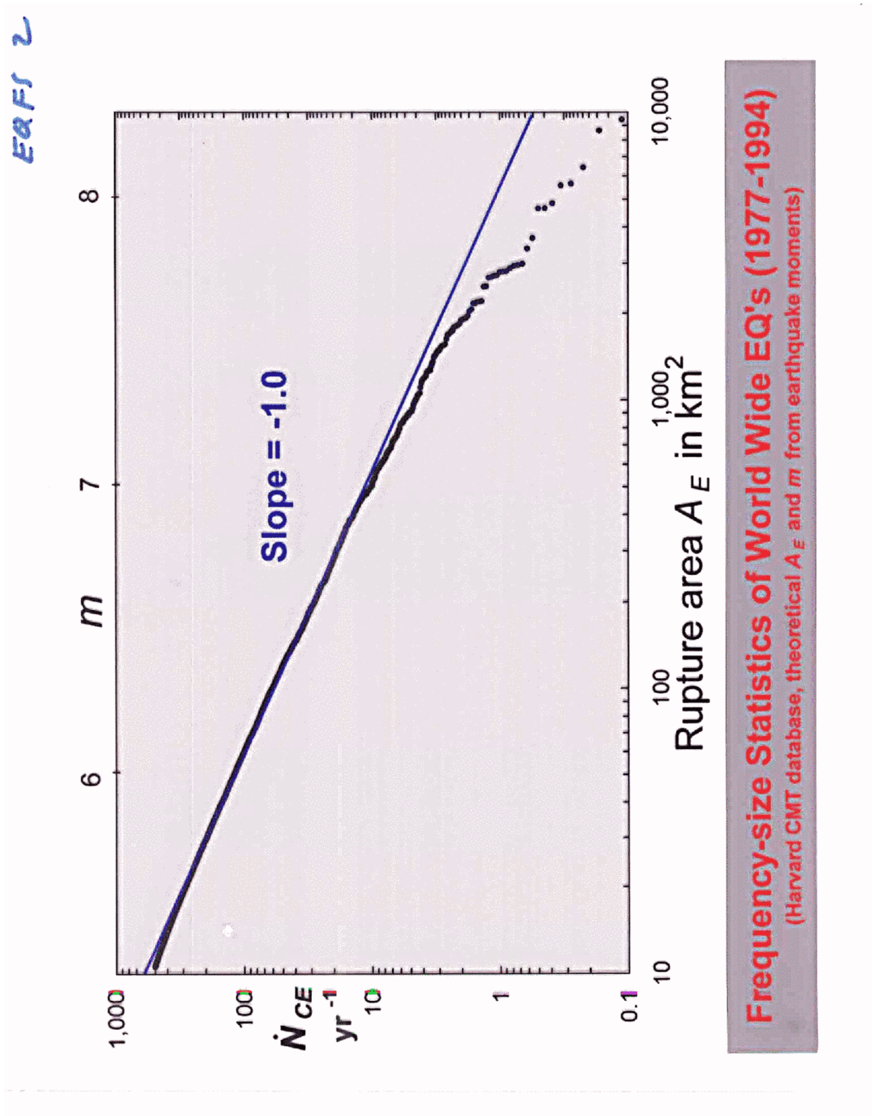
▶ Infer the magnitude m^* of the greatest aftershock from an extrapolation of the Gutenberg-Richter scaling applied to the aftershock sequence:

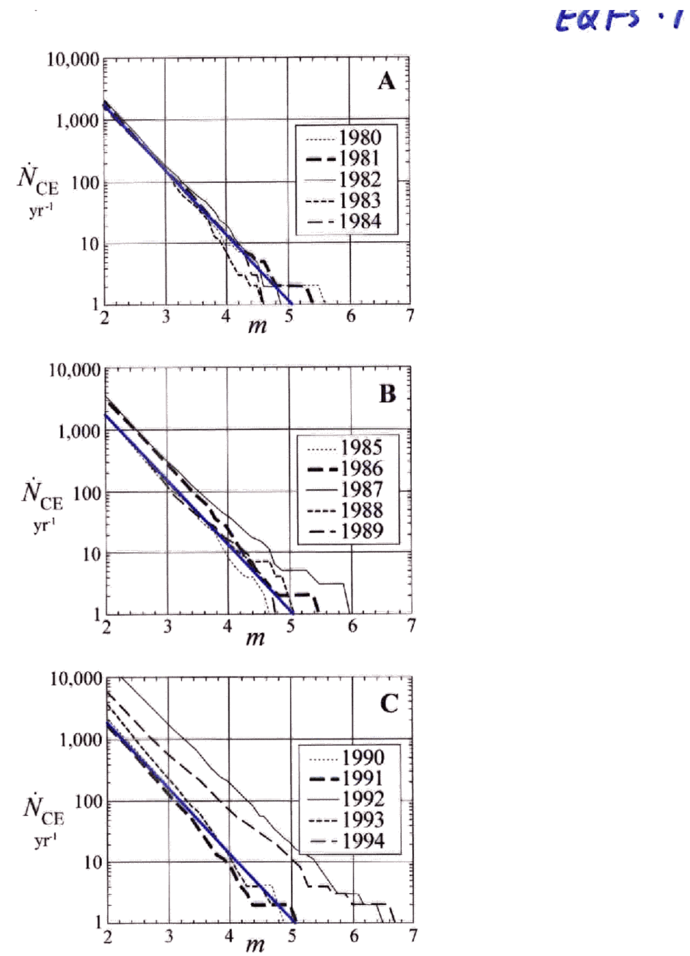
$$m^\star = m_{\rm ms} - \Delta m^\star.$$

▶ The constant a is obtained by setting $N(> m^\star) = 1$ so that $a = b\,m^\star$. Finally we obtain

$$\log N(> m) = b\,(m^\star - m)\,,$$

$$\log N(> m) = b\,(m_{ms} - \Delta m^* - m)$$





Southern California Earthquakes, cumulative frequency-size distributions for 15 individual years.

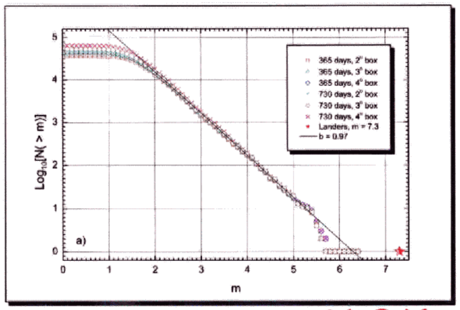
The solid blue straight line represents best-fit to all data 1980-1994, $\log \dot{N}_{CE} = -1.05\, m + 5.3$.

DAB-03

Application to Southern California

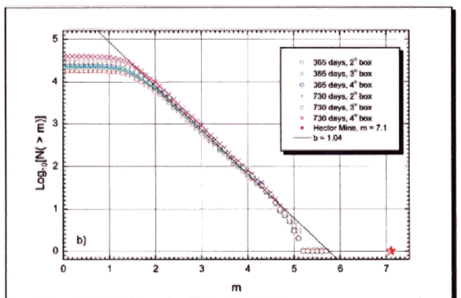
▶ Consider eight earthquakes with magnitude greater than 5 in southern and Baja California that occurred between 1987 and 2002.

$m_{ms} = 7.3$
$m_{aft\,max} = 6.3$
$m^* = 8.2$
$\Delta m^* = 1.1$
$b = 0.97$

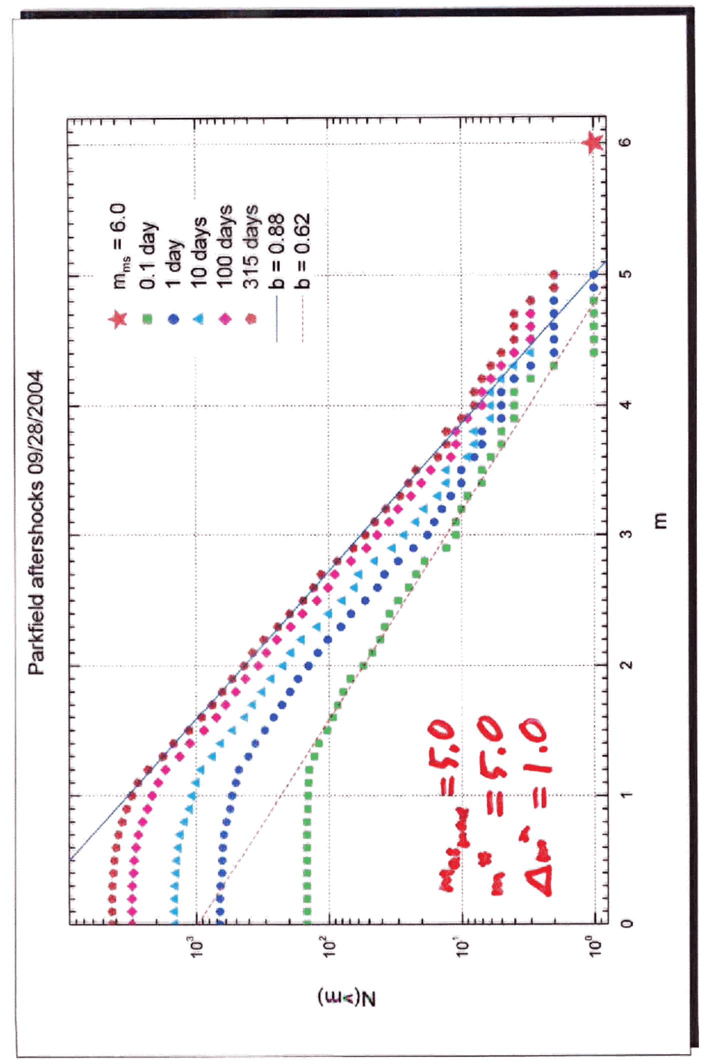


Landers 6/28/92

$m_{ms} = 7.1$
$m_{aft\,max} = 5.8$
$m^* = 5.75$
$\Delta m^* = 1.35$
$b = 1.00$



Hector Mine 12/22/03



DAB-10

Partitioning of Energy

▶ The energy E radiated in an earthquake with moment magnitude m

$$\log_{10} E(m) = \frac{3}{2} m + \log_{10} E_0$$

with $E_0 = 6.3 \times 10^4$ Joules.

▶ The moment magnitude of the main shock m_{ms}

$$E_{ms} = E_0 \, 10^{\frac{3}{2} m_{ms}} .$$

▶ The total radiated energy in the aftershock sequence

$$E_{as} = \int_{-\infty}^{m^\star} E(m) \left(-\frac{dN}{dm}\right) dm .$$

DAB-11

▶ The aftershock rate

$$dN = -b\,(\ln 10)\, 10^{b\,(m_{ms} - \Delta m - m)} dm .$$

▶ Carrying out the integration we find

$$E_{as} = \frac{b}{\left(\frac{3}{2} - b\right)} E_0 \, 10^{\frac{3}{2}(m_{ms} - \Delta m)} .$$

▶ The ratio of the total radiated energy in aftershocks E_{as} to the radiated energy in the main shock E_{ms}

$$\frac{E_{as}}{E_{ms}} = \frac{b}{\frac{3}{2} - b} 10^{-\frac{3}{2}\Delta m} .$$

▶ The fraction of the total energy associated with aftershocks

$$\frac{E_{as}}{E_{ms} + E_{as}} = \frac{\frac{b}{\frac{3}{2}-b} 10^{-\frac{3}{2}\Delta m}}{1 + \frac{b}{\frac{3}{2}-b} 10^{-\frac{3}{2}\Delta m}} .$$

▶ Combined scaling gives Omori's law

$$\frac{dN}{dt} = \frac{(p-1)\,10^{b\,(m_{\rm ms}-\Delta m - m)}}{c\,(1+t/c)^p}$$

or

$$\frac{1}{N_T}\frac{dN}{dt} = \frac{(p-1)}{c\,(1+t/c)^p},$$

Identical to the form used by Reasenberg and Jones (1989) and by Yamanaka and Shimazaki (1990)

$$\frac{dN}{dt} = \frac{10^{a+b\,(m_{\rm ms}-m)}}{(c+t)^p}$$

if

$$\Delta m = \frac{1}{b}\left[\log_{10}(p-1) + (p-1)\,\log_{10}(c) - a\right]$$

For 62 California aftershock sequences RJ give $c = 0.04$ days, $a = -1.67$, $b = 0.9$, $p = 1.08 \Rightarrow \Delta m = 0.52$.

For 27 Japanese aftershock sequences YS give $c = 0.3$ days, $a = -1.83$, $b = 0.85$, $p = 1.3 \Rightarrow \Delta m = 1.35$

DAO-2

DAO 12

▶ Modified form of Omori's law

$$\frac{dN_{as}}{dt} = \frac{1}{\tau}\,\frac{1}{\left(1+\frac{t}{c}\right)^p}$$

▶ Gutenberg-Richter law for aftershocks

$$N_{as}(\geq m) = 10^{a-b\,m}$$

▶ Modified form of Båth's law for aftershocks

$\Delta m^* = m_{ms} - m^* \approx$ constant ≈ 1.2

m_{ms} – main shock magnitude

m^* – "largest" aftershock (from GR)

$1 = 10^{a-b\,m^*}$ or $a = b\,m^*$

$$N_{as}(\geq m) = 10^{b\,(m_{ms}-\Delta m^* - m)}$$

$$N_{as}(\geq m) = \int_0^\infty \frac{dN_{as}}{dt}\,dt = \frac{c}{(p-1)\,\tau}$$

$$\frac{c}{\tau} = (p-1)\,10^{b\,(m_{ms}-\Delta m^* - m)}$$

DAO 13

- Hypothesis I:

$c = c_0 =$ constant

$\tau(m) = \frac{c_0}{(p-1)} 10^{-b\,(m_{ms} - \Delta m^* - m)}$

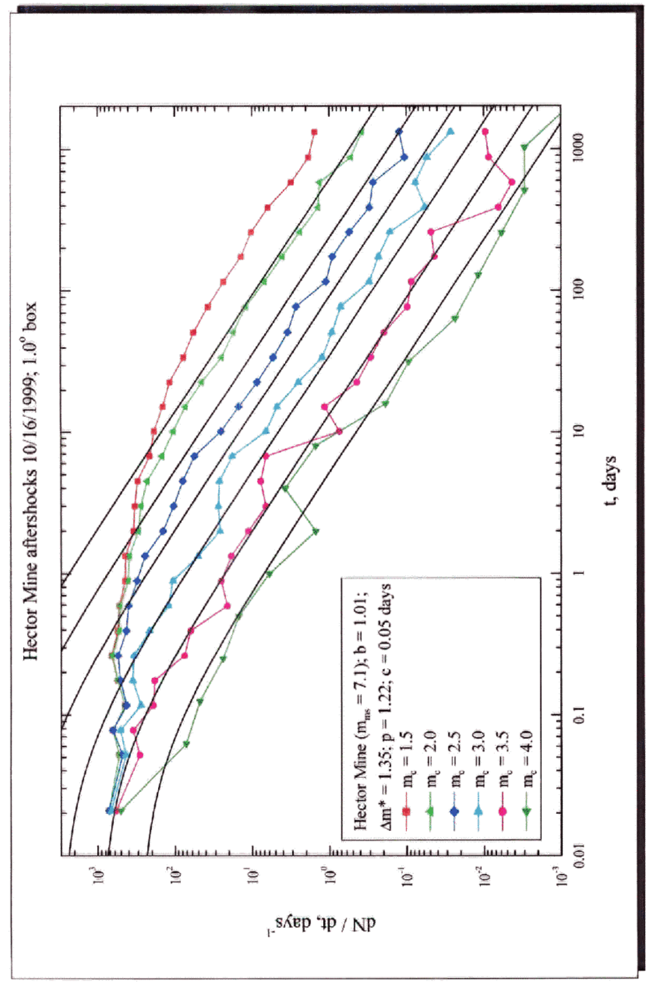
$$\frac{dN_{as}}{dt} = \frac{(p-1)\,10^{b(m_{ms} - \Delta m^* - m)}}{c_0} \frac{1}{\left(1 + \frac{t}{c_0}\right)^p}$$

- Hypothesis II:

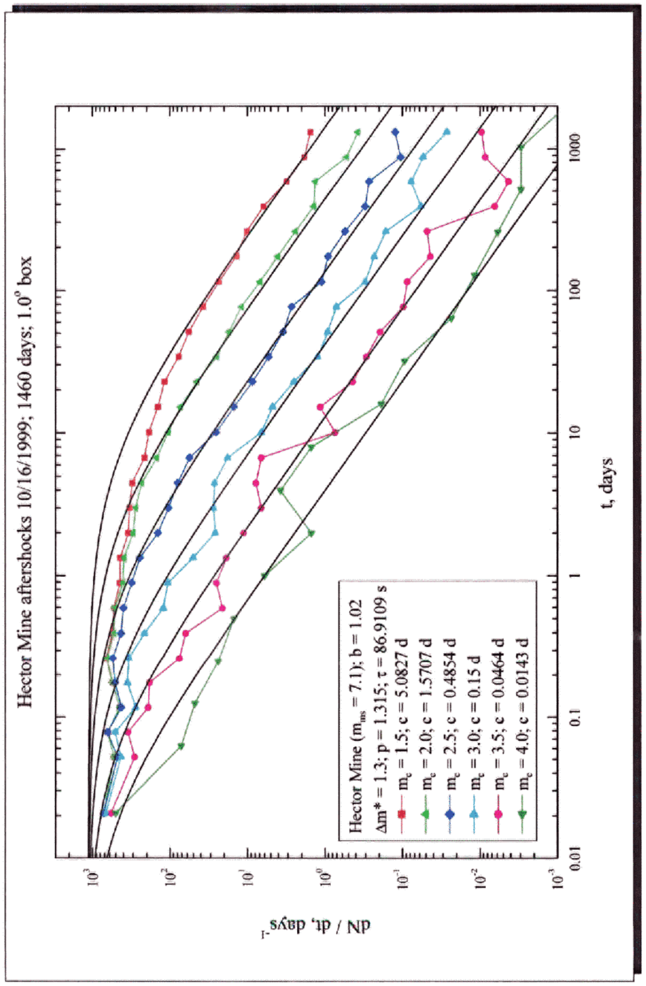
$\tau = \tau_0 =$ constant

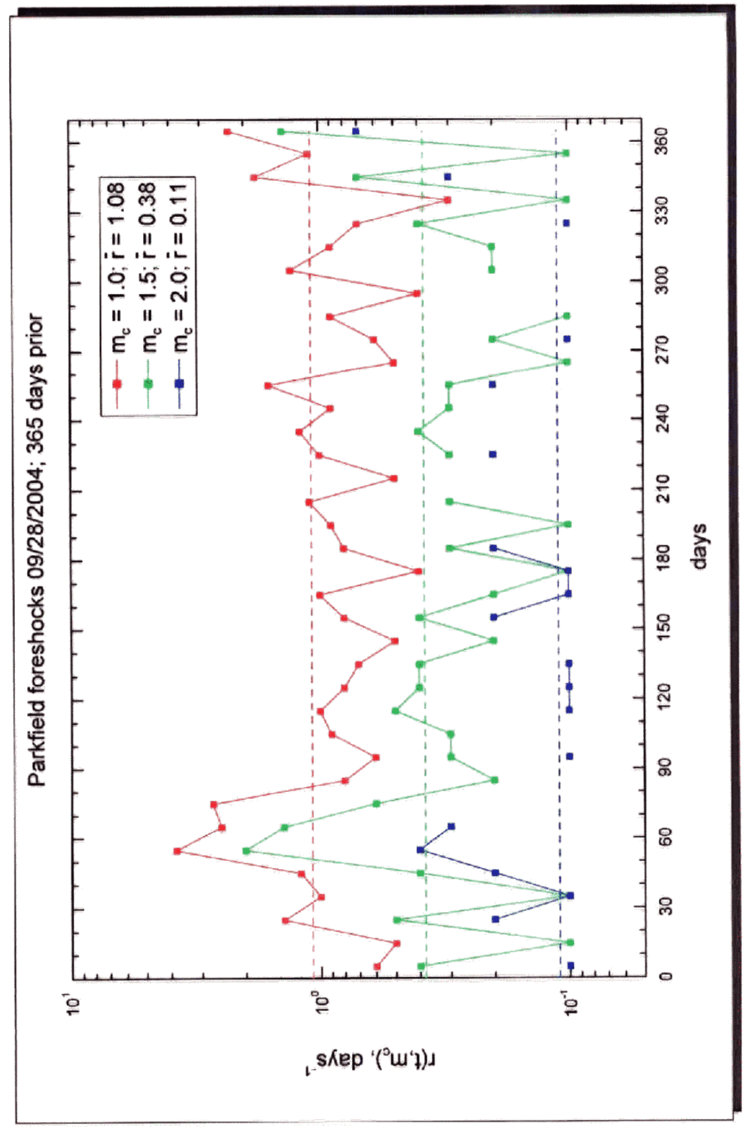
$c(m) = \tau_0\,(p-1)\,10^{b\,(m_{ms} - \Delta m^* - m)}$

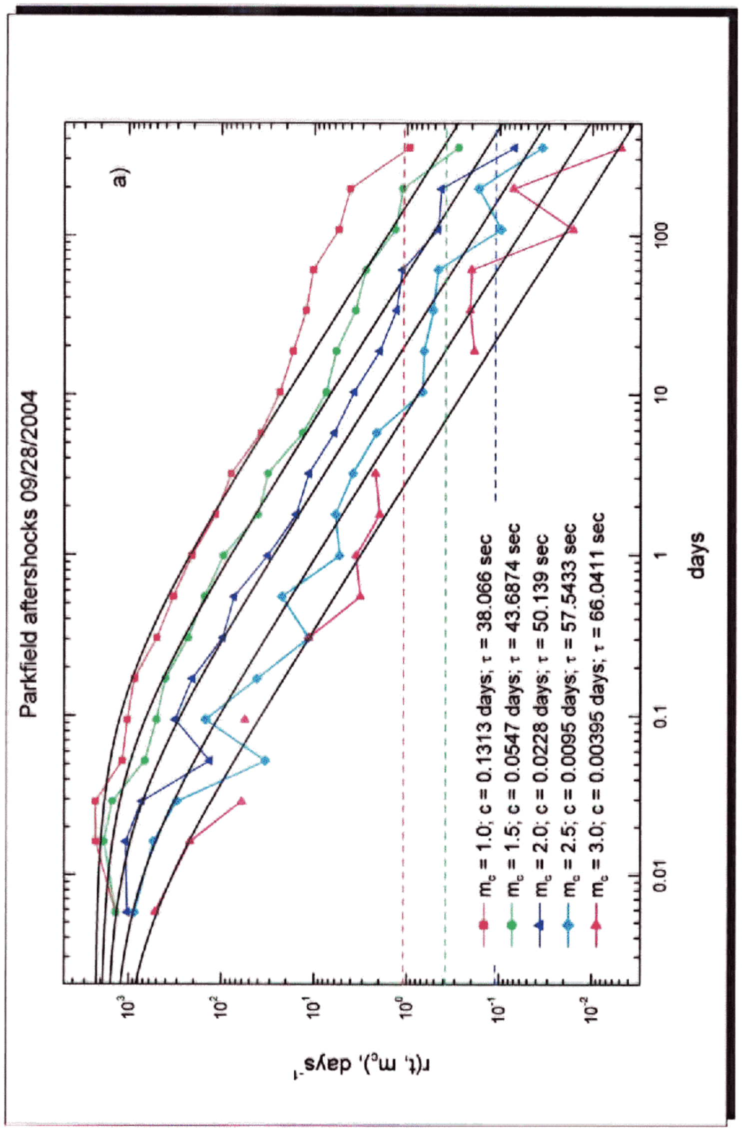
$$\frac{dN_{as}}{dt} = \frac{1}{\tau_0} \frac{1}{\left(1 + \frac{t}{c(m)}\right)^p}$$

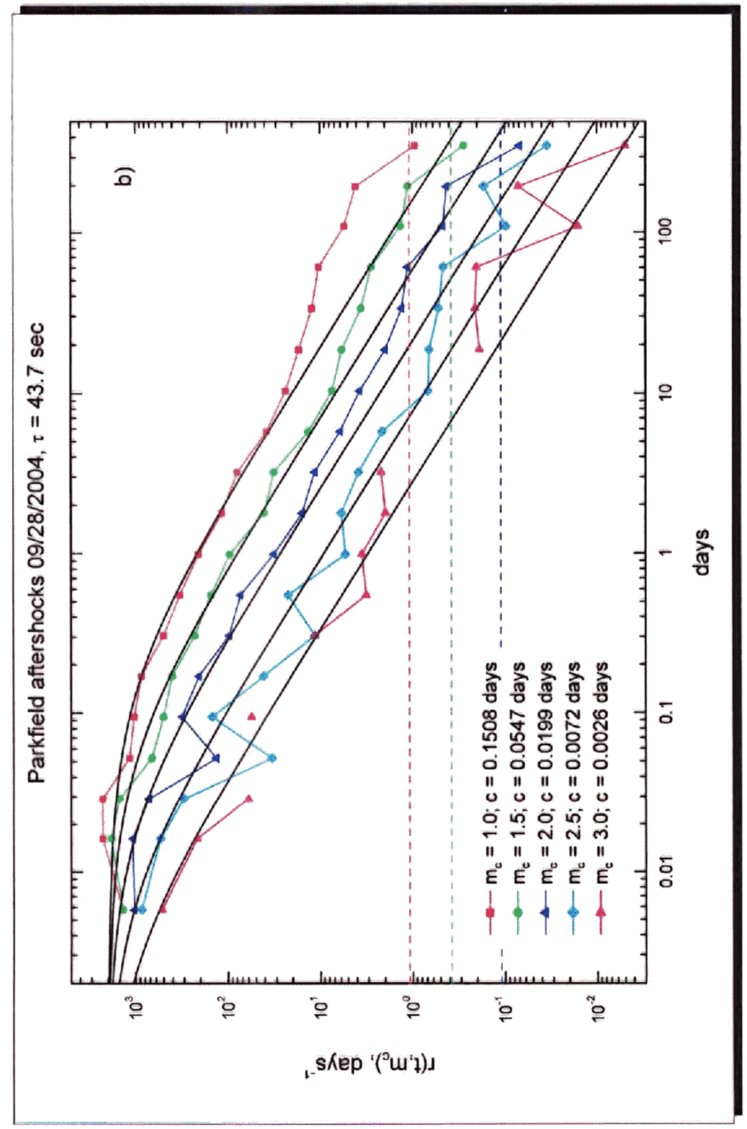


DAO 14


Hector Mine aftershocks 10/16/1999; 1460 days; 1.0° box
Hector Mine (m_ms = 7.1); b = 1.02
Δm* = 1.3; p = 1.315; τ = 86.9109 s
m_c = 1.5; c = 5.0827 d
m_c = 2.0; c = 1.5707 d
m_c = 2.5; c = 0.4854 d
m_c = 3.0; c = 0.15 d
m_c = 3.5; c = 0.0464 d
m_c = 4.0; c = 0.0143 d
dN / dt, days^-1
t, days


Parkfield foreshocks 09/28/2004; 365 days prior
m_c = 1.0; r̄ = 1.08
m_c = 1.5; r̄ = 0.38
m_c = 2.0; r̄ = 0.11
r(t,m_c), days^-1
days


Parkfield aftershocks 09/28/2004
a)
r(t, m_c), days^-1
days
m_c = 1.0; c = 0.1313 days; τ = 38.066 sec
m_c = 1.5; c = 0.0547 days; τ = 43.6874 sec
m_c = 2.0; c = 0.0228 days; τ = 50.139 sec
m_c = 2.5; c = 0.0095 days; τ = 57.5433 sec
m_c = 3.0; c = 0.00395 days; τ = 66.0411 sec


Parkfield aftershocks 09/28/2004, τ = 43.7 sec
b)
r(t,m_c), days^-1
days
m_c = 1.0; c = 0.1508 days
m_c = 1.5; c = 0.0547 days
m_c = 2.0; c = 0.0199 days
m_c = 2.5; c = 0.0072 days
m_c = 3.0; c = 0.0026 days

Earthquake	m_{ms}	b	$\Delta m^\star$	p	R	β	bp	$c(m^\star)$, s	τ_0, s
Landers	7.3	0.98	1.10	1.22	4.0	1.20	1.19	33	107.3
Northridge	6.7	0.91	0.75	1.17	3.4	1.06	1.06	23	53.1
Hector Mine	7.1	1.02	1.35	1.22	4.1	1.23	1.24	32	83.7
San Simeon	6.5	1.0	1.10	1.12	3.5	1.09	1.12	55	50.2
Parkfield	6.0	0.88	1.0	1.09	3.16	0.96	0.96	5.2	43.7
Sumatra	9.3	1.26	1.8	1.11	3.46	1.60	1.4	50	172.8

$$\frac{r(0, m_c)}{r(0, m_c + dm)} = 10^{\alpha\, dm}, \qquad \frac{r(t \gg 1, m_c)}{r(t \gg 1, m_c + dm)} = 10^{\beta\, dm}$$

$$\beta = bp \quad \text{if} \quad \alpha = 0$$

DAO 16

▶ Definition of the equilibration time $t_e(m)$

• It is the time after a main shock required to establish the validity of GR scaling for aftershocks with magnitudes m

• It is the time after a main shock when the aftershock rate given by the modified Omori's law

$$\frac{dN_{as}}{dt} = \frac{1}{\tau} \frac{1}{(1 + \frac{t}{c})^p}$$

deviates from power-law scaling

$$\left(\frac{dN_{as}}{dt}\right)_{ps} = \frac{1}{\tau} \left(\frac{c}{t}\right)^p$$

by 10%

$$\frac{(dN_{as}/dt)}{(dN_{as}/dt)_{ps}} = \frac{(t_e/c)^p}{(1 + t_e/c)^p} = 0.90$$

$$t_e = \frac{c(m)}{\left[\frac{1}{0.9^{1/p}} - 1\right]}$$

DAO 17

- Assume $p = 1.2$

  Equilibration time: $t_e = 10.9\, c(m)$

- Hypothesis II: $c(m) = 0.2\, \tau_0\, 10^{b(m_{ms} - \Delta m^* - m)}$

- Assume $b = 1$, "largest" aftershock $m^* = m_{ms} - \Delta m^*$

  but $10^{m^* - m} = \frac{A^*}{A}$

  where A^* rupture area of m^* earthquake and
  A rupture area of m earthquake

  therefore

$$t_e = 2.18\, \tau_0 \frac{A^*}{A}$$

Continuum Damage Mechanics with a Yield Stress

▶ Uniaxial stress

$$\sigma = E_0\, \epsilon \quad \text{if} \quad \sigma \leq \sigma_y$$

E_0 – Young's modulus (undamaged)

$$\sigma - \sigma_y = E_0\, (1 - \alpha)\, (\epsilon - \epsilon_y) \quad \text{if} \quad \sigma > \sigma_y$$

α – damage variable.

▶ Kinetic equation

$$\frac{d\alpha(t)}{dt} = 0 \qquad \text{if} \quad 0 \leq \sigma \leq \sigma_y$$

$$\frac{d\alpha(t)}{dt} = \frac{1}{t_d} \left(\frac{\sigma}{\sigma_y} - 1\right)^{\rho} \left(\frac{\epsilon}{\epsilon_y} - 1\right)^{2} \quad \text{if} \quad \sigma > \sigma_y$$

t_d – characteristic time for damage

DAO-7

- Constant stress $\sigma_0 > \sigma_y$ applied instantaneously at $t = 0$

$$\frac{d\alpha(t)}{dt} = \frac{1}{t_d}\left(\frac{\sigma_0}{\sigma_y} - 1\right)^{\rho}\left(\frac{\epsilon}{\epsilon_y} - 1\right)^2$$

$$(\sigma_0/\sigma_y - 1) = (\epsilon/\epsilon_y - 1)\,(1 - \alpha)$$

with $\alpha = 0$ at $t = 0$.

- Integration gives

$$\alpha = 1 - (1 - t/t_f)^{1/3}\,, \quad t_f = \frac{t_d}{(\sigma_0/\sigma_y - 1)^{\rho+2}}$$

$$\epsilon = \epsilon_y\left[1 + \frac{\sigma_0/\sigma_y - 1}{(1 - t/t_f)^{1/3}}\right]$$

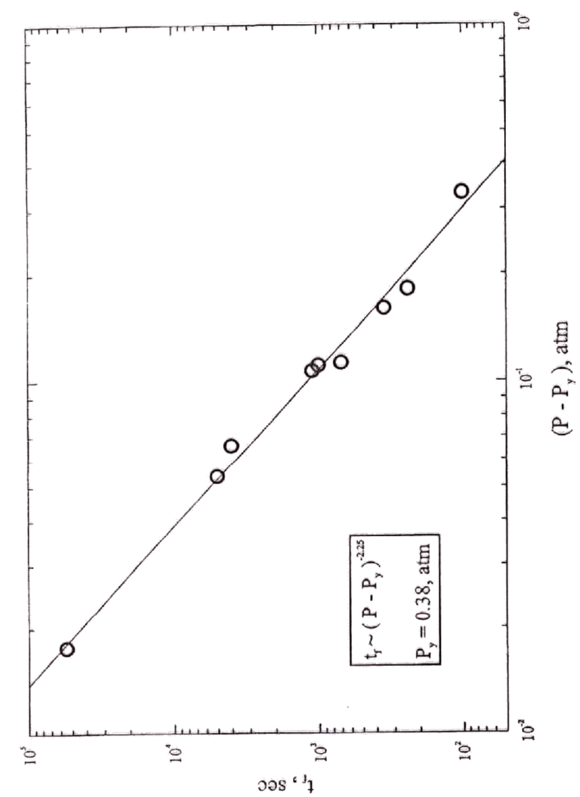
- Energy in acoustic emissions

$$e_a = \frac{1}{2}(\sigma_0 - \sigma_y)(\epsilon - \epsilon_y) = \frac{(\sigma_0 - \sigma_y)^2}{2\,E_0}\left[\frac{1}{(1 - t/t_f)^{1/3}} - 1\right]$$

$$\frac{de_a}{dt} = \frac{(\sigma_0 - \sigma_y)^2}{6\,t_f\,E_0}\,\frac{1}{(1 - t/t_f)^{4/3}}$$

DAO-8

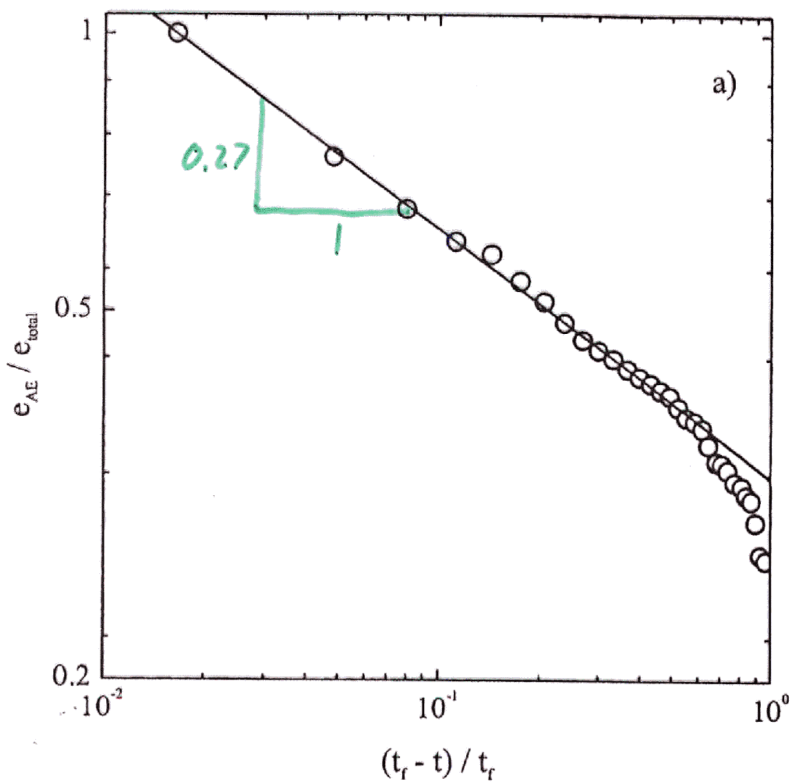
Dig 15'



Time to failure t after the instantaneous application of a differential pressure at $t = 0$.

Guarino et al. (1999)

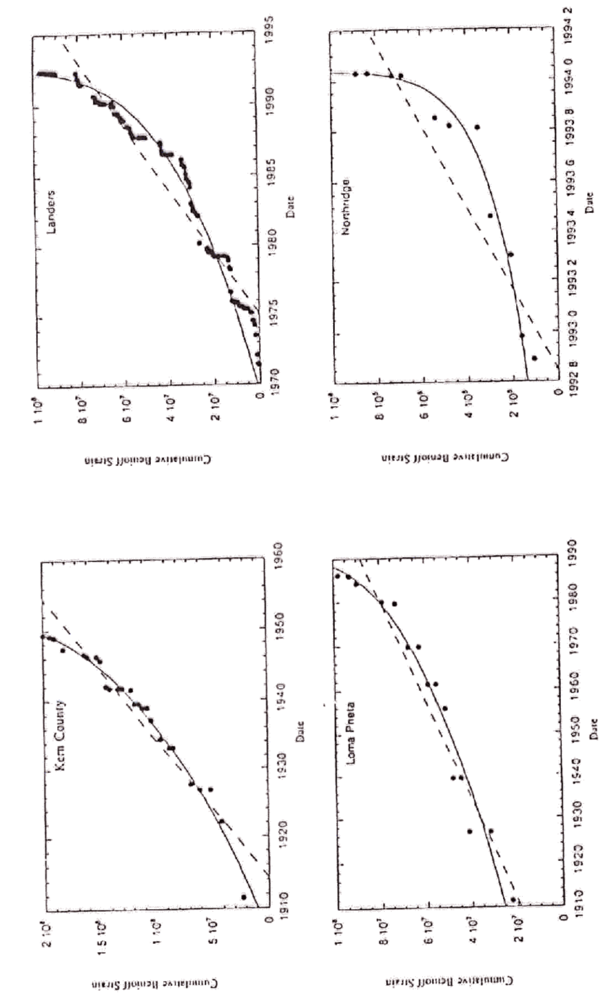
Dis 12



acoustic emissions after instantaneous application of stress σ_0

t_f time to failure after stress application

Guarino et al. (1999)



Power-law activation
Bufe and Varnes JGR 98, 9871 (1993)
Bowman et al. JGR 103, 24359 (1998)

EP 6

Fig. 21

- Constant strain $\epsilon_0 > \epsilon_y$ applied instantaneously at $t = 0$

$$\frac{d\alpha(t)}{dt} = \frac{1}{t_d}\left(\frac{\sigma}{\sigma_y} - 1\right)^{\rho}\left(\frac{\epsilon_0}{\epsilon_y} - 1\right)^2$$

$$(\sigma/\sigma_y - 1) = (\epsilon_0/\epsilon_y - 1)\ (1 - \alpha)$$

with $\alpha = 0$ at $t = 0$.

- Integration gives

$$\alpha = 1 - \left[1 + (\rho - 1)(\epsilon_0/\epsilon_y - 1)^{\rho+2}\, t/t_d\right]^{-1/(\rho-1)}$$

$$\frac{\sigma}{\sigma_y} = 1 + \frac{(\epsilon_0/\epsilon_y - 1)}{[1 + (\rho - 1)(\epsilon_0/\epsilon_y - 1)^{\rho+2}\, t/t_d]^{1/(\rho-1)}}$$

- Energy in acoustic emissions

$$e_{ae} = \frac{E_0}{2}(\epsilon_0 - \epsilon_y)^2\left[1 - \left[1 + (\rho - 1)(\epsilon_0/\epsilon_y - 1)^{\rho+2} t/t_d\right]^{-1/(\rho-1)}\right]$$

$$\frac{de_{ae}}{dt} = \frac{\frac{E_0\epsilon_y^2}{2t_d}(\epsilon_0/\epsilon_y - 1)^{\rho+4}}{[1 + (\rho - 1)(\epsilon_0/\epsilon_y - 1)^{\rho+2} t/t_d]^{\rho/(\rho-1)}}$$

DAO–10

- Damage mechanics applied to aftershocks

$$e_{ae} = \frac{E_0}{2}(\epsilon_0 - \epsilon_y)^2\left[1 - \left[1 + (\rho - 1)(\epsilon_0/\epsilon_y - 1)^{\rho+2} t/t_d\right]^{-1/(\rho-1)}\right]$$

$$t \to \infty,\ e_{aet} = \frac{E_0}{2}(\epsilon_0 - \epsilon_y)^2 \quad \text{(total ae)}$$

$$\frac{1}{e_{aet}}\frac{de_{ae}}{dt} = \frac{\frac{1}{t_d}(\epsilon_0/\epsilon_y - 1)^{\rho+2}}{[1 + (\rho - 1)(\epsilon_0/\epsilon_y - 1)^{\rho+2} t/t_d]^{\rho/(\rho-1)}}$$

Let $\frac{\rho}{\rho-1} = p$ and $c = \frac{t_d}{(\rho-1)(\epsilon_0/\epsilon_y-1)^{\rho+2}}$

$$\frac{1}{e_{aet}}\frac{de_{ae}}{dt} = \frac{p-1}{c}\frac{1}{(1 - t/c)^p}$$

Parkfield
$p = 1.09$
$\rho = 12$

- Omori's law for aftershocks

$$\frac{1}{N_T}\frac{dN}{dt} = \frac{p-1}{c}\frac{1}{(1 - t/c)^p}$$

DAO–11

▶ Time dependent rate for the decay of the aftershock activity

$$r(t) = \frac{1}{\tau\ (1 + t/c)^p}$$

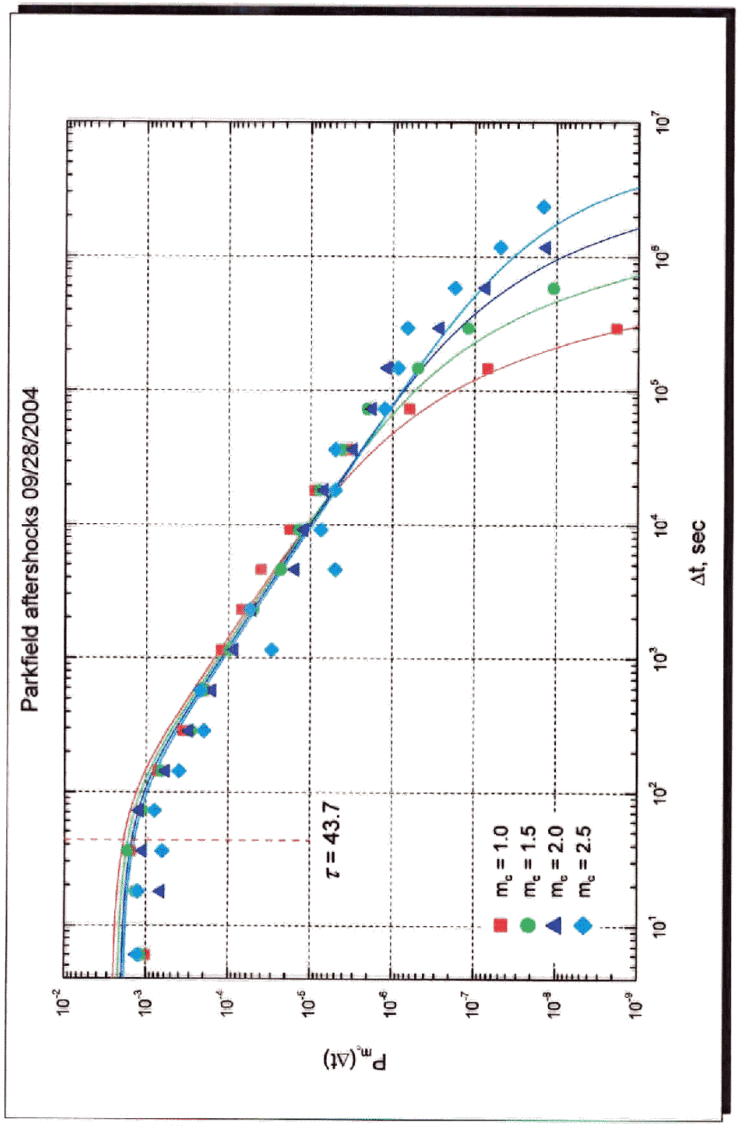
▶ The probability density function of inter-occurrence times over a finite time period T

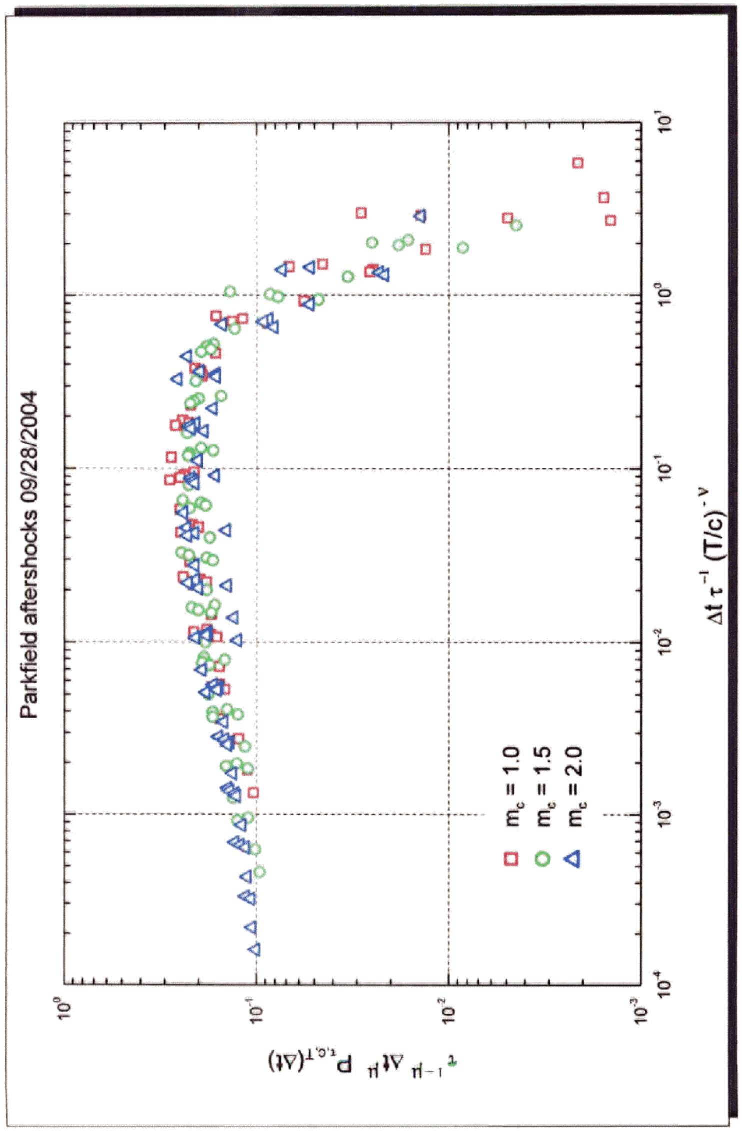
$$P_T(\Delta t) = \frac{1}{N}\left[r(\Delta t)\, e^{-\int_0^{\Delta t} r(u)\,du} + \int_0^{T-\Delta t} r(s)\, r(s+\Delta t)\, e^{-\int_s^{s+\Delta t} r(u)\,du}\, ds \right],$$

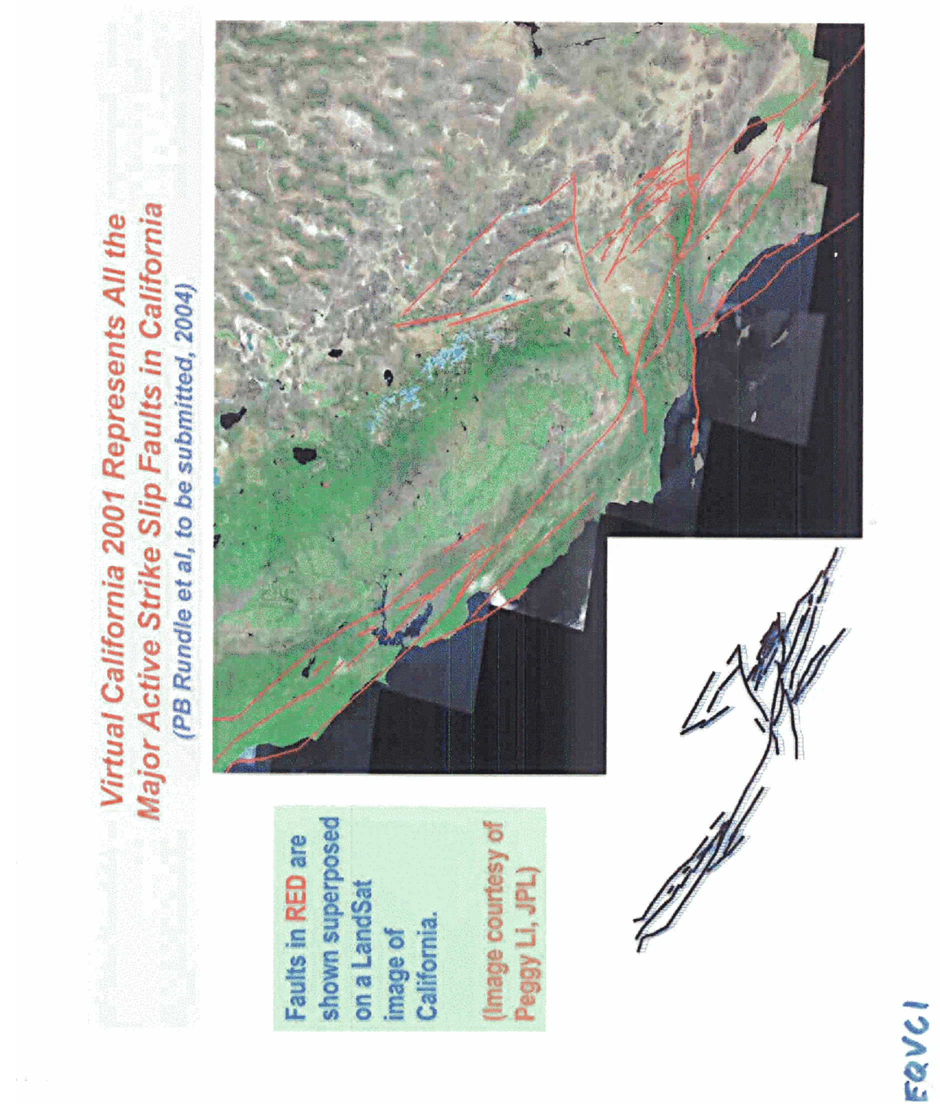
where $N = \int_0^T r(u)\,du$ is the total number of events during a time period T.

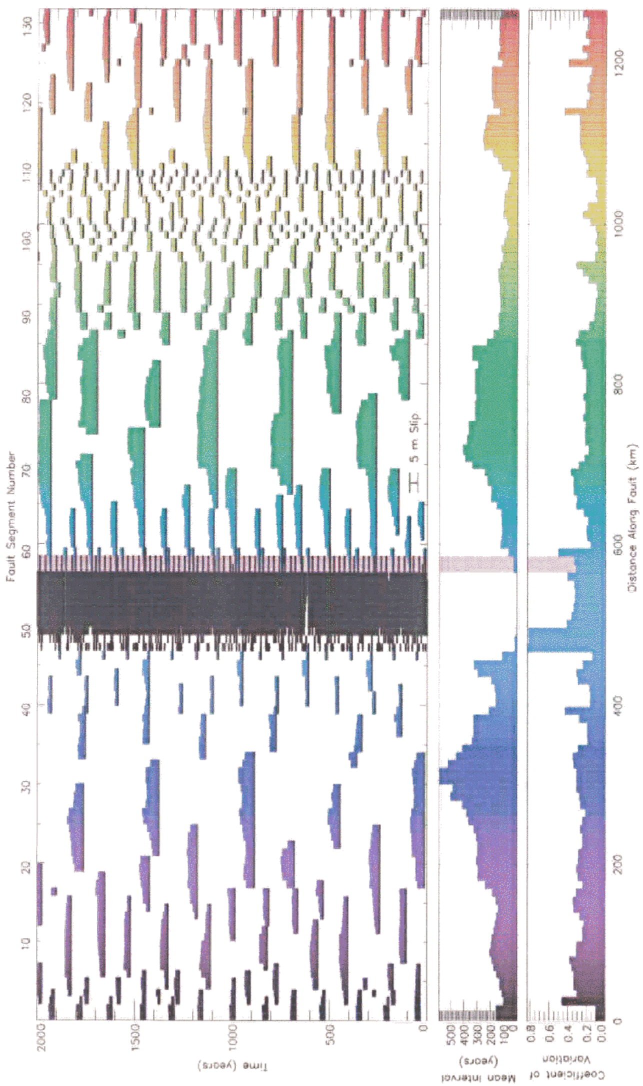
▶ Scaling law for the above distribution

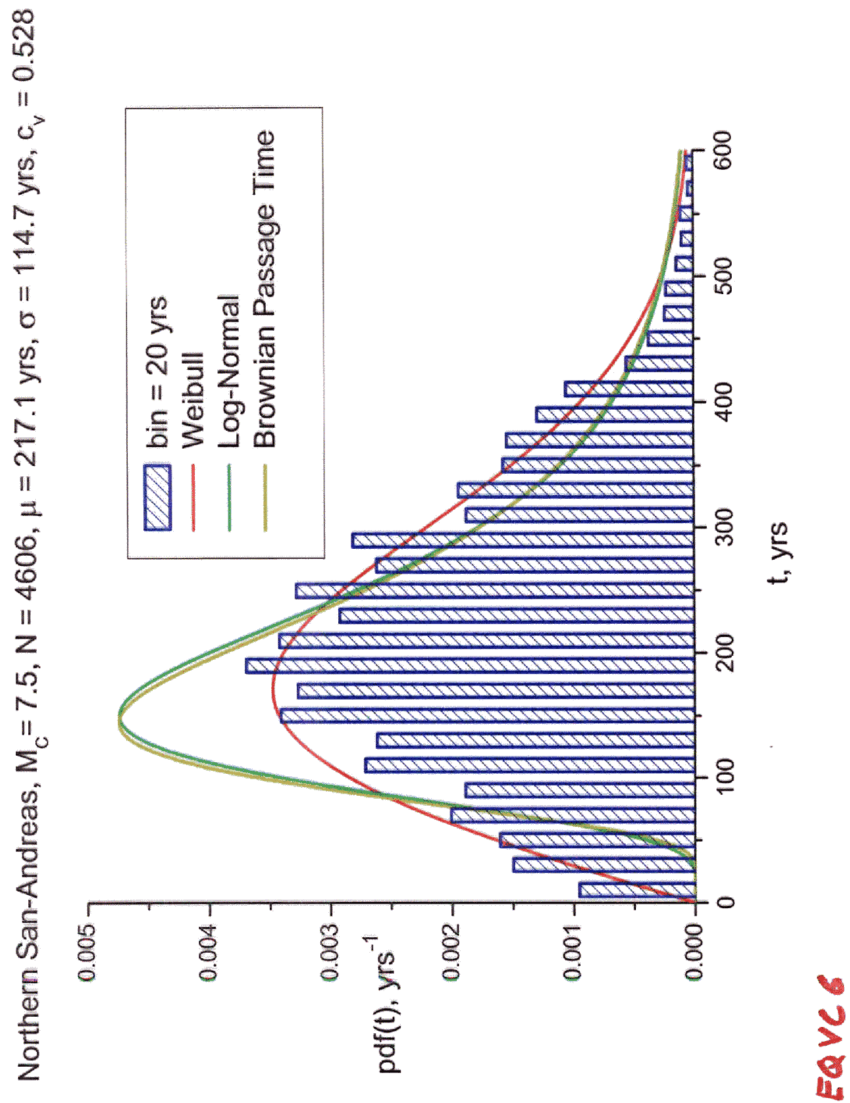
$$P_{\tau,c,T}(\Delta t) = \frac{1}{\tau}\left(\frac{\Delta t}{\tau}\right)^{-\alpha} f\left[\frac{\Delta t}{\tau}\left(\frac{T}{c}\right)^{-\beta}\right]$$




Parkfield aftershocks 09/28/2004
m_c = 1.0
m_c = 1.5
m_c = 2.0
$\Delta t\, \tau^{-1}\, (T/c)^{-\nu}$


Virtual California 2001 Represents All the Major Active Strike Slip Faults in California
(PB Rundle et al, to be submitted, 2004)
Faults in RED are shown superposed on a LandSat image of California.
(Image courtesy of Peggy Li, JPL)


Fault Segment Number
Distance Along Fault (km)
Time (years)
Mean Interval (years)
Coefficient of Variation
5 m Slip


Northern San-Andreas, M_C = 7.5, N = 4606, μ = 217.1 yrs, σ = 114.7 yrs, c_v = 0.528
bin = 20 yrs
Weibull
Log-Normal
Brownian Passage Time
pdf(t), yrs^{-1}
t, yrs
EQVC 6

EQVC 13

▶ Dynamic fiber-bundle model

$$\frac{dN}{dt} = -\nu\left(\frac{\sigma}{\sigma_0}\right)N = -\nu_0\left(\frac{\sigma}{\sigma_0}\right)^{\rho}N$$

N – number of fibers, σ – stress

ν_0 — reference hazard rate

▶ Assume $\frac{\sigma}{\sigma_0} = \frac{t}{\tau}$, let $\rho = \beta - 1$, $\nu_0 = \frac{\beta}{\tau}$

$$\frac{dN}{dt} = -\frac{\beta}{\tau}\left(\frac{t}{\tau}\right)^{\beta-1}N$$

with $N = N_0$ at $t = 0 \Rightarrow \frac{N}{N_0} = \exp\left[-\left(\frac{t}{\tau}\right)^{\beta}\right]$

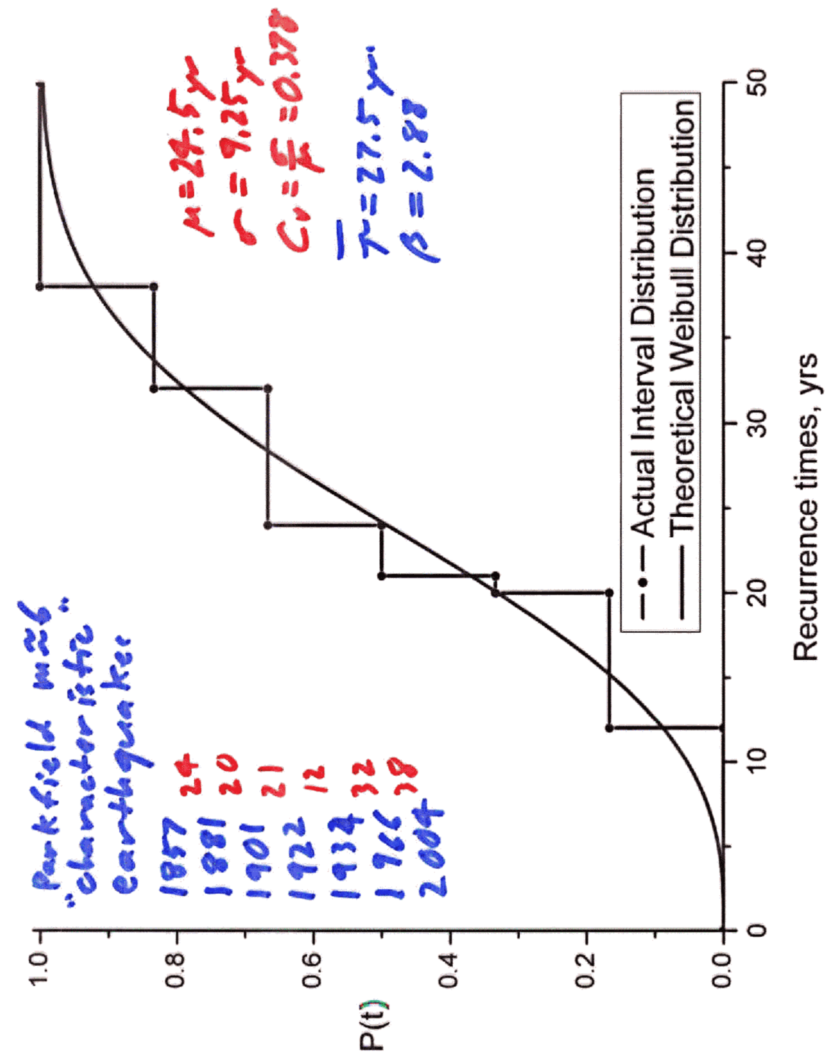
▶ Number of failed fibers $N_f = N_0 - N$

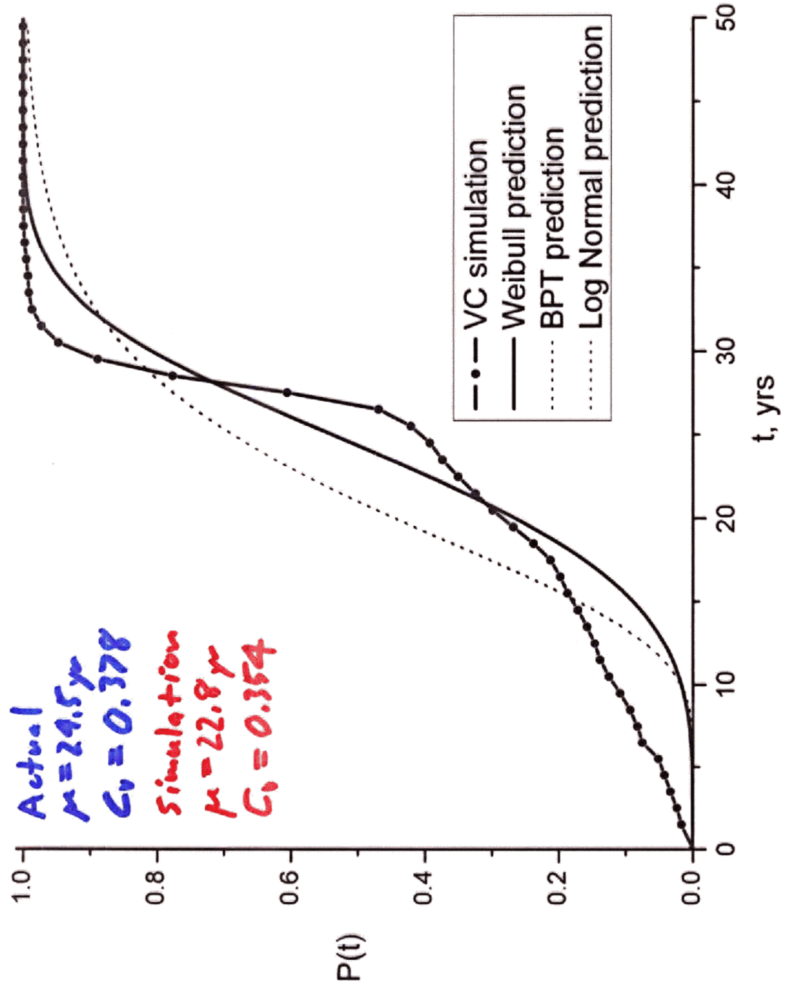
▶ Cumulative probability of failure

$$P(t) = 1 - \exp\left[-\left(\frac{t}{\tau}\right)^{\beta}\right] \quad \text{Weibull dist.}$$

▶ Conditional cumulative probability of failure at time t if no failure at time t_0

$$P(t, t_0) = 1 - \exp\left[\left(\frac{t_0}{\tau}\right)^{\beta} - \left(\frac{t}{\tau}\right)^{\beta}\right]$$




Actual
μ = 24.5 yr
Cv = 0.378
Simulation
μ = 22.8 yr
Cv = 0.354
VC simulation
Weibull prediction
BPT prediction
Log Normal prediction
P(t)
t, yrs