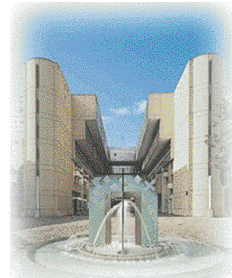
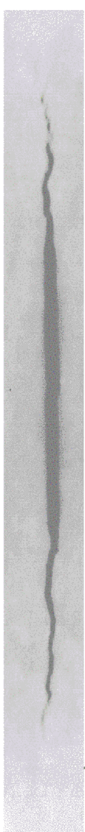


Sub-critical rupture in heterogenous and/or plastic materials : experiments and models

S. Ciliberto, P.-P. Cortet, S. Santucci, L. Vanel, *Physics Laboratory, ENS Lyon*
S. Deschanel, N. Godin, G. Vigier, *GEMPPM, INSA, Lyon*
A. Guarino, *University of French Polynesia (Tahiti)*
A. Garcimartin, *University of Pamplona (Spain)*
A. Politi, *INOA, Florence (Italy)*
R. Scorretti, *University Lyon I*

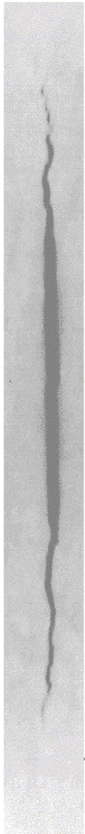


What do we call sub-critical rupture?

Slow rupture of a material when
constant applied stress σ < rupture threshold σ_c

Motivations :

1. Is the lifetime of a sample under constant stress predictable ?
2. May simple models describe sub-critical crack growth ?
3. Effect of the heterogenous structure ?
4. Dependance on material properties ?
   elastic, elastoplastic, viscoplastic, etc.



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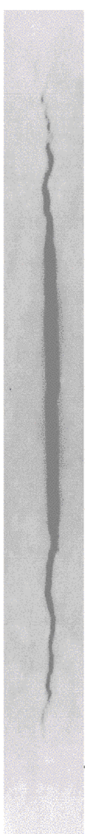

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Main observations about sub-critical rupture : 1

- time to rupture or « lifetime » follows an Arrhenius law
  (Zhurkov, Int. J. Fract. Mech. **1**, 311, 1965)

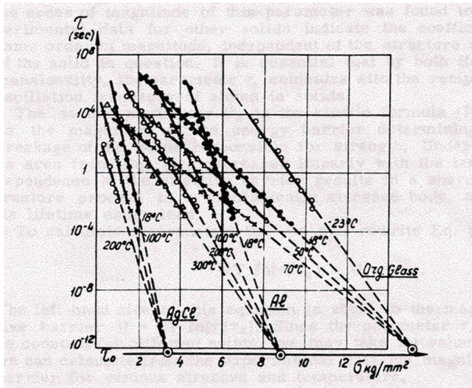


Fig. 5. Time and temperature dependence of the lifetime of solids on stress.
1. Silver chloride (Reference 4)
2. Aluminum (Reference 5)
3. Plexiglas (Reference 6)

$$\tau = \tau_o \exp\left(\frac{U}{k_B T}\right)$$

Empirical energy barrier

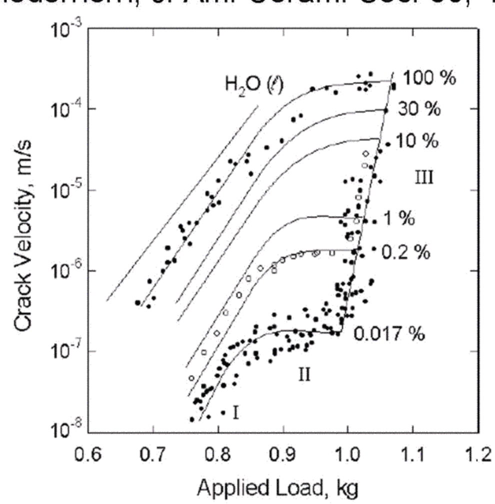
$$U = U_o - \alpha\sigma$$

with $U_o \sim$ sublimation energy

Sub-critical rupture is a thermally activated process ?

Main observations about sub-critical rupture : 2

- environment dependent : effect of humidity
  (S. M. Wiederhorn, J. Am. Ceram. Soc. **50**, 407, 1967)



velocity $\propto$ concentration in H_2O

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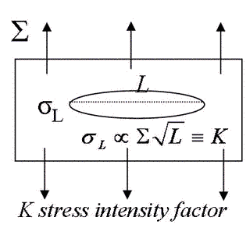
Simple model of thermally activated rupture

crack nucleation in homogenous elastic material

Golubovic (1991), Pomeau (1992), Sethna (1996)

Ideal crack



Potential energy (Griffith 1920) :
mechanical energy + surface energy

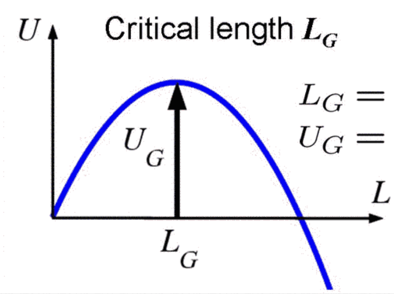
$$U = U_o - \frac{\pi L^2 \Sigma^2}{4Y} + 2\gamma L$$

Σ stress, Y Young modulus, γ surface energy, U_0 elastic energy without crack

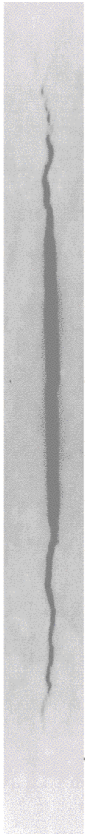
Lifetime prediction :
thermal activation

$$\tau \propto \exp\left[\frac{U_G}{k_B T}\right]$$

Critical length L_G



$$L_G = \frac{4\gamma Y}{\pi \Sigma^2}$$
$$U_G = \gamma L_G$$



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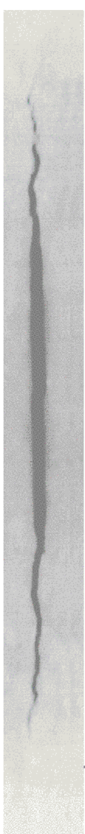


- **Experimental tests**

  Pauchard (93) (quasi 2d cristals) $U \sim \Sigma^{-2}$

  Guarino + Ciliberto (99) (quasi 2d fiberglass, woodchip) $U \sim \Sigma^{-4}$

  corresponds to 3d prediction!

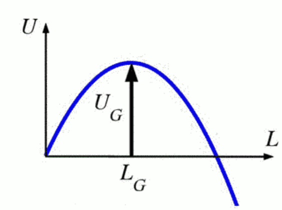
- **Limits of the model**
  - Energy barrier far too high

    Estimate at ambiant T :

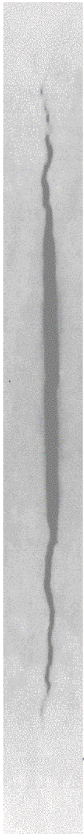
    $\gamma \sim 1$ J.m^{-2}, $\Sigma \sim Y \sim 10^{11}$ Pa : $\frac{U_G}{k_B T} \sim 10^{10}$!!
  - Reversible process up to L_G

    Consequence:

    no mechanisms of progressive damage

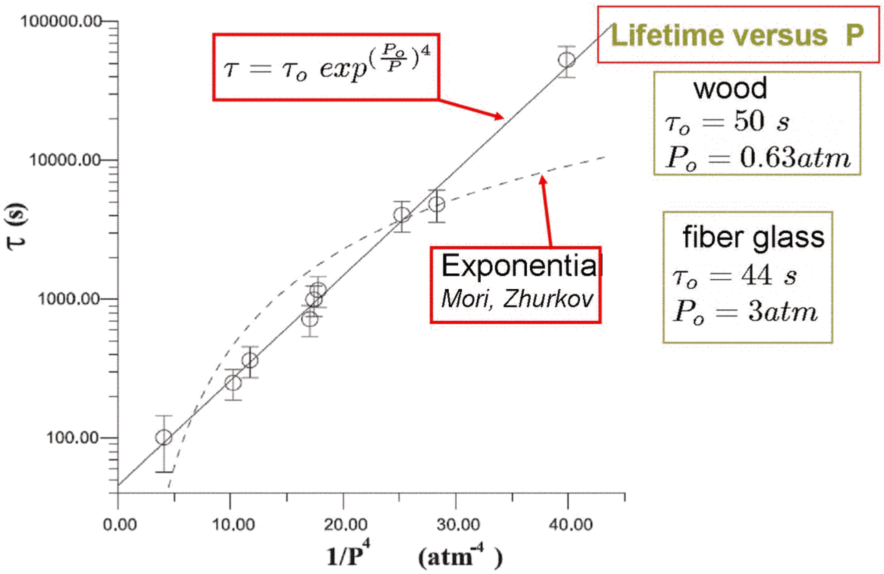


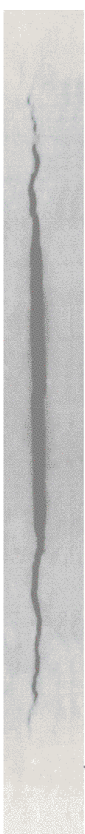
- **What is missing in the model**
  - Existence of an initial crack : $\delta U = U(L_G) - U(L_i) = \gamma L_i \left(\frac{K_i}{K_G} - \frac{K_G}{K_i}\right)^2$
  - **Irreversibility of the rupture process** : steps $\delta << L_G$



Lifetime in composites samples (circular plates)

Guarino et al, European Physical Journal B 2002





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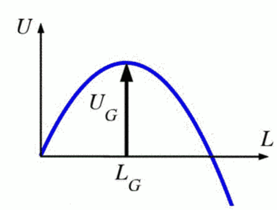

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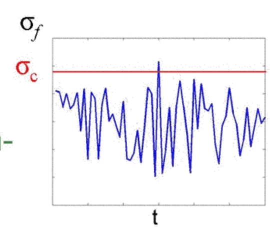
2d model for irreversible crack growth

S.Santucci et al, Europhysics Letters (2003)



a) Driving mechanism :

at a volumic scale V, thermodynamical stress fluctuations σ_f are strong enough to have a non-zero probability to reach rupture threshold σ_c

b) Irreversible process and probability of rupture *P*

average growth velocity : $v(L) \propto P(\sigma_f > \sigma_c)$

$$t = \tau \left[1 - \exp\left(\frac{L - L_i}{\zeta} \right) \right]$$

rupture time

$$\tau = \tau_0 \exp\left[\frac{(\sigma_c - \sigma_i)^2 V}{2 Y k_B T} \right]$$

characteristic length

$$\zeta = \frac{2 Y k_B T}{V} \frac{L_i}{\sigma_i(\sigma_c - \sigma_i)}$$

V volumic scale at which damage occurs unknown !!!

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Single crack growth

Experimental test of Santucci *et al* model on paper sheets

(Santucci et al EPL 2003)

Problems :

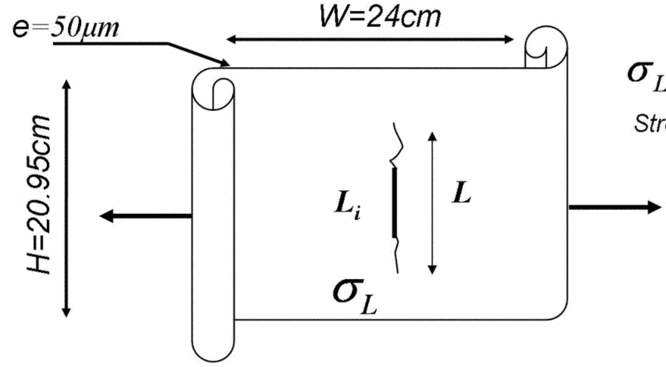
- paper is inhomogenous ⇒ make a large crack
- paper is sensitive to humidity ⇒ control of humidity

Advantages :

- paper breaks essentially in a brittle manner
- sub-critical rupture at room temperature!

Creep experiments (constant applied stress Σ)

Initial crack length L_i in pure tension (mode I loading)



$$\sigma_L \propto \Sigma\sqrt{L} \equiv K$$

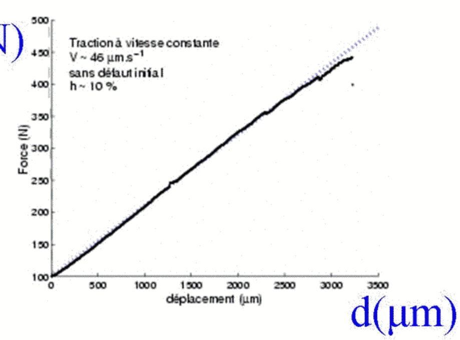
Stress intensity factor K

Sub-critical growth : $\sigma_L < \sigma_C$ (rupture threshold)

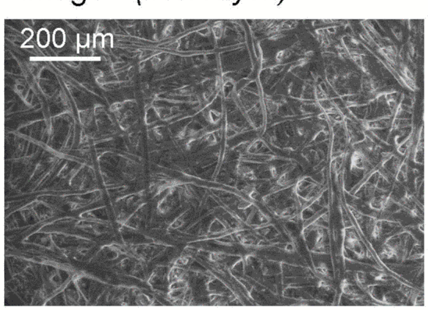
Measurements : Applied load Σ (Force F), strain, crack length L(t)

Paper sample

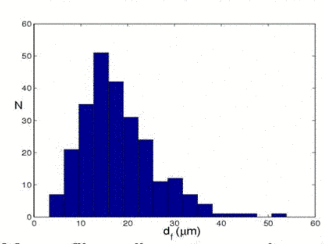
Elastic response



Electronic microscope image *(INSA Lyon)*



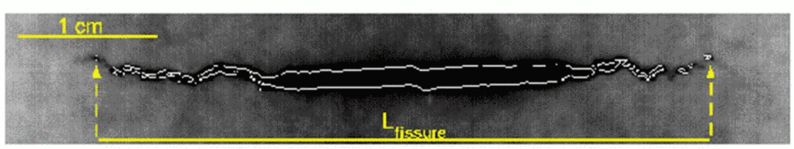
Fiber diameter distribution

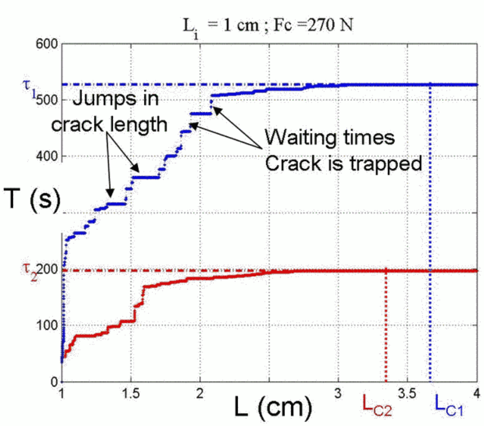


Mean fiber diameter : $\langle d_f \rangle \sim 20$ µm

Crack growth

Crack growth





Complex dynamics

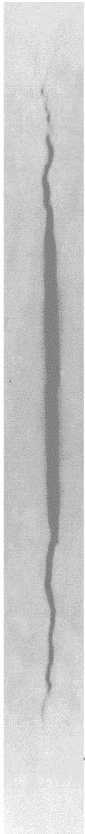
Statistical analysis

- average dynamics
- jump in crack length dynamics

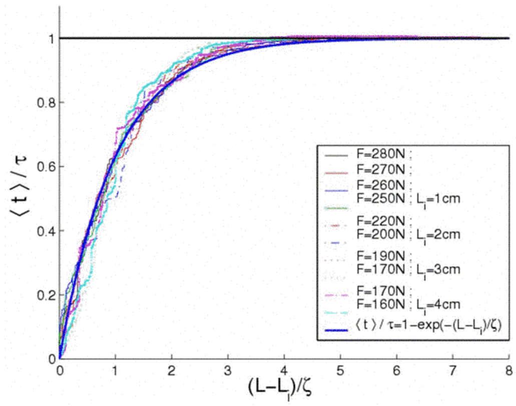
parameter range :

1 cm < L_i < 4 cm

140 N < F < 280 N



Average growth dynamics

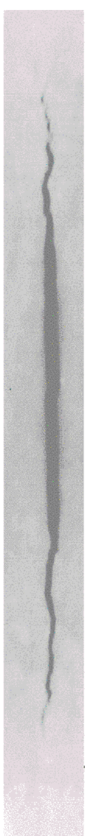


$$\frac{\langle t \rangle}{\tau} = \left[1 - \exp\left(-\frac{L - L_i}{\zeta}\right)\right]$$

- τ rupture time
- ζ characteristic length

(Santucci et al EPL 2003)

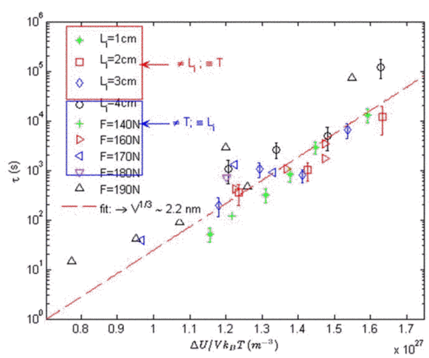
Only one free parameter ζ !

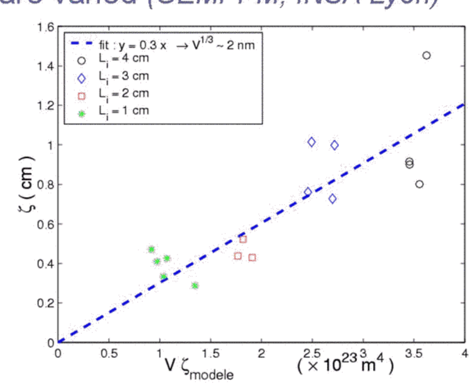


Predictions of lifetime τ and characteristic length ζ

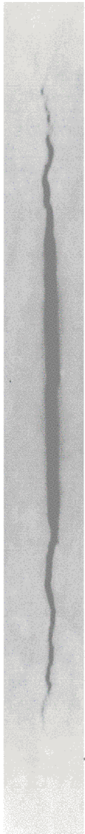
$$\tau = \tau_0 \exp\left[\frac{(\sigma_c - \sigma_i)^2 V}{2 Y k_B T}\right] \qquad \zeta = \frac{2 Y k_B T}{V} \frac{L_i}{\sigma_i(\sigma_c - \sigma_i)}$$

Experimental test : F, L_i, and T are varied *(GEMPPM, INSA Lyon)*



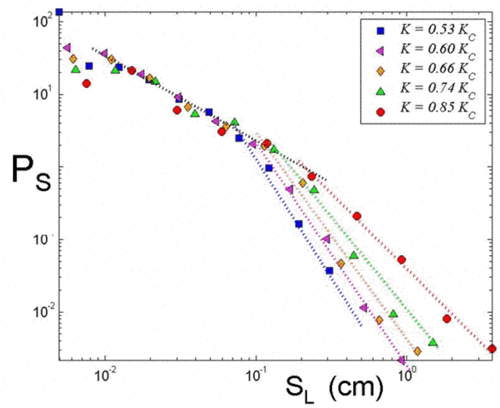


→ $V^{1/3} \sim$ nm → $V^{1/3} \sim$ nm

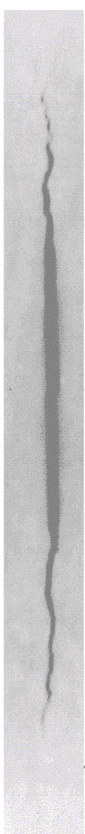


Stepwise growth dynamics

- Crack length jumps S_L
- Distribution $P_S(S_L)$ at constant values of K



→ trap model with an arrest mechanism ?



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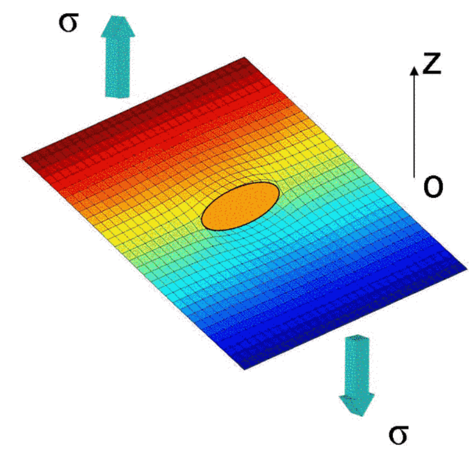
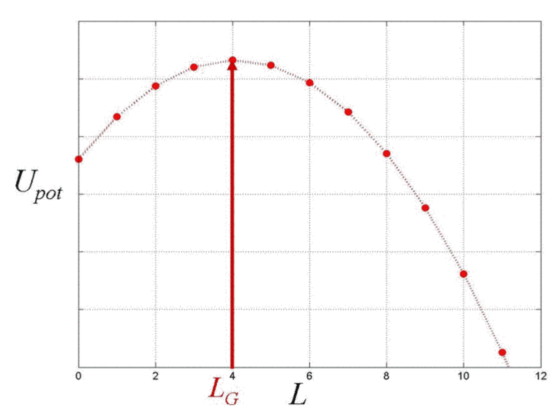
Lattice model : 2-d network of elastic springs

Square network in antiplane deformation (same as fuse network)
Shearing load σ (mode III)
No disorder : constant spring breaking threshold $\boldsymbol{\sigma_c}$

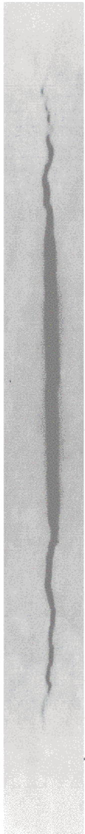


Potential energy U_{pot} in the lattice model



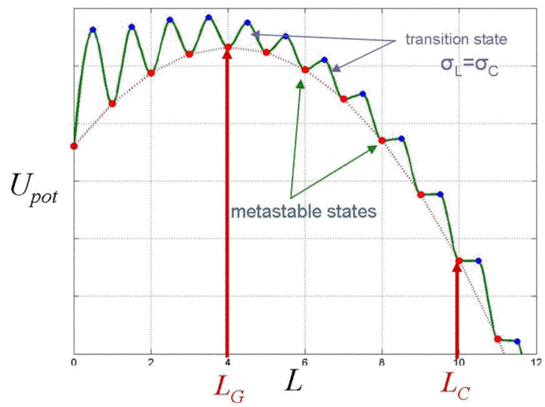
in agreement with expected Griffith energy !

Minimum energy cost : L → L+1 ?



Potential energy U_{pot} in the lattice model

"LATTICE TRAPPING" : R. Thomson (1986), M. Marder (1996)

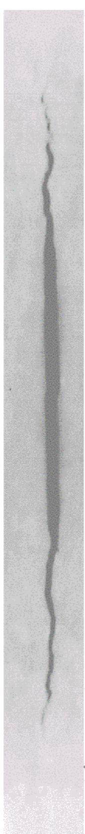


Dynamics controlled by thermal noise :

$L < L_G$: crack closes

$L_G < L < L_C$: ***irreversible*** crack growth

$L_C < L$: fast rupture

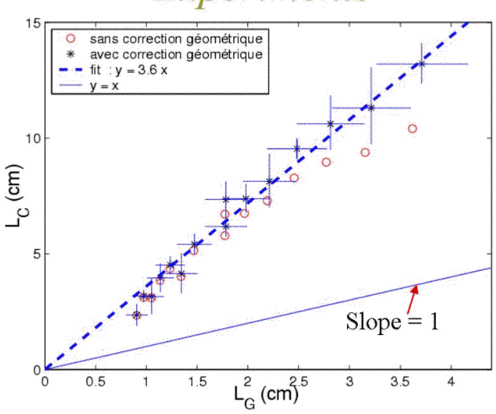


Comparison L_C vs L_G

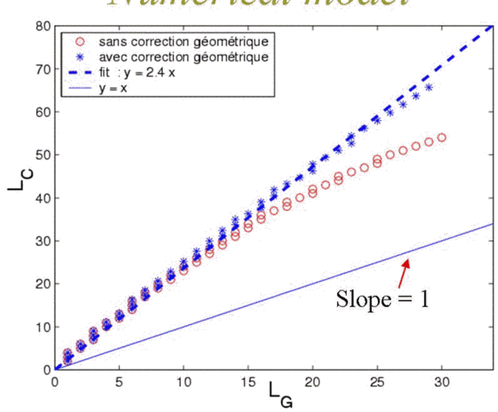
$< \gamma > = 1.5\ 10^3\ N.m^{-1}$

$< Y > = 3.3\ 10^9\ N.m^{-2}$ ....→ $\langle L_G \rangle = \frac{4\langle\gamma\rangle Y}{\pi\Sigma^2}$

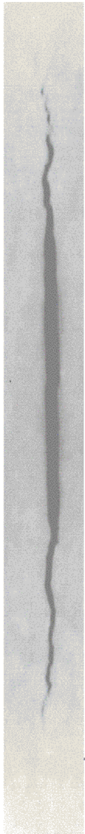
Experiments *Numerical model*



$\langle L_C \rangle = 3.6 \langle L_G \rangle$



$L_C^{sim} = 2.4 L_G^{sim}$



Probability distribution P_S of crack jump sizes S

Santucci et al, Physical Review Letters 2004

Jump distribution P_S ⟷ Distribution of stress fluctuations $G(\sigma_f)$

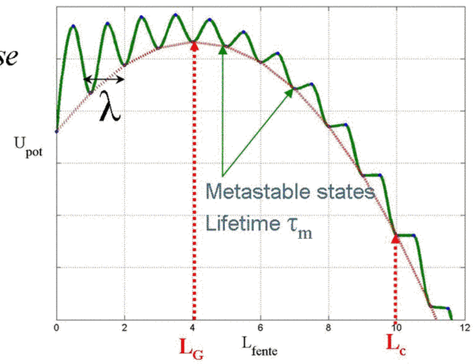
- Crack arrest mechanism

  dissipation : barrier + elastic release

$$s = \frac{\delta U_f}{\delta U_c}\lambda = \frac{(\sigma_f - \sigma_m)^2}{(\sigma_c - \sigma_m)^2}\lambda$$

- Mean growth velocity $V(L)$

$$\begin{cases} V(L) \propto P(\sigma_f > \sigma_c) \\ V(L) = \frac{\langle s \rangle}{\tau_m} = \frac{\int s P_s ds}{\tau_m} \end{cases}$$



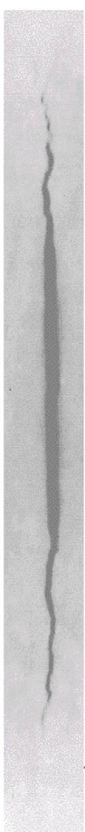
distribution of crack jump sizes

$$P_s = N(V,\lambda)\ s^{-3/2} e^{-s/\xi}$$

Cut-off length $\xi = \lambda \frac{2 Y k_B T}{(\sigma_c - \sigma_m)^2 V}$

V volumic scale of damage
λ discretization scale

Similar to « critical point theory in percolation»

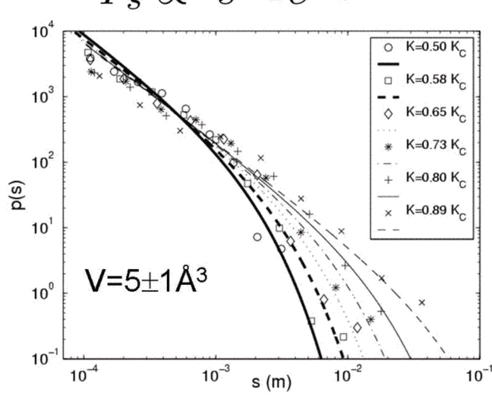


Distribution P_S of crack length jump sizes S_L

as a function of stress intensity K

Distribution

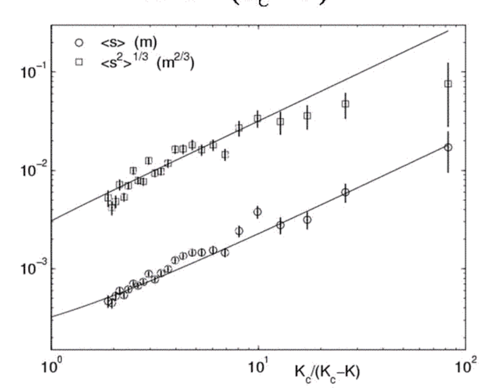
$$P_S \propto s^{-\frac{3}{2}} e^{-\frac{s}{\xi}}$$



λ = 50 μm fiber diameter
V only free parameter $V^{1/3} \sim$ Å

Asymptotic Behavior

$$< s > \propto (\sigma_C - \sigma)^{-1}$$
$$< s^2 > \propto (\sigma_C - \sigma)^{-3}$$



$$\frac{\sigma_C}{\sigma_C - \sigma}$$

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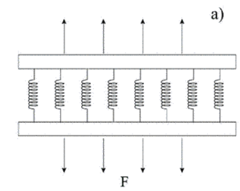
Effect of disorder ?

Experimental approach : Polymer foams (S. Deschanel et al)
work in progress…

Models : Thermally activated fiber bundle model with disorder
1. 1D (Ciliberto et al. Physica D 2001)
2. 2D

Fiber Bundle Model 1d with disorder

(Ciliberto et al. Physica D 2001)



Ingredients:

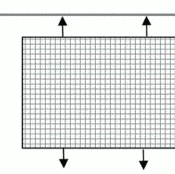
- N parallel linear springs, elastic constant E
- total force F is constant
- n : number of broken springs
- equal force on each spring (democratic): $\sigma(n) = \frac{F}{N-n}$
- initial force on each spring $\sigma(0) = \sigma_0$
- + thermal noise on fiber i : $\sigma_T(i) \propto \sqrt{kT}$
- + disorder : distribution of rupture threshold $\sigma_c(i)$
(standard deviation σ_d and average $< \sigma_c >$)

spring i breaks if : $\sigma(n) + \sigma_T(i) > \sigma_c(i)$

Lifetime prediction :

$$\tau \approx \tau_0 \exp\left[\frac{(\sigma_c^{eq} - \sigma_0)^2}{2EkT}\right] \text{ with } \sigma_c^{eq} = < \sigma_c > - \sqrt{\frac{\pi}{2}}\sigma_d$$

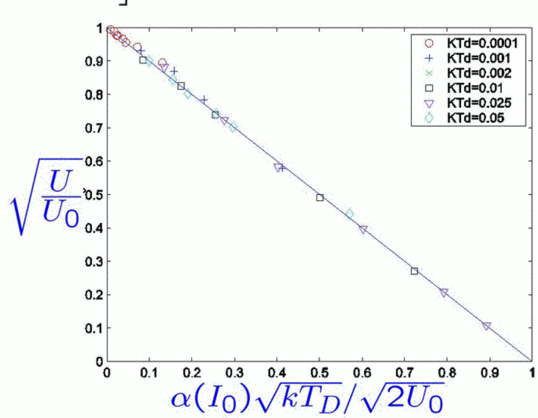
Fiber Bundle Model 2d with disorder



No initial crack

Difference with 1 d : stress concentration effects

$$\tau \approx \tau_0 \exp\left[\frac{(\sigma_c^{eq} - \sigma_0)^2}{2EkT}\right] \text{ with } \sigma_c^{eq} = \sigma_c(\sigma_d = 0) - \alpha(\sigma_0)\sigma_d$$

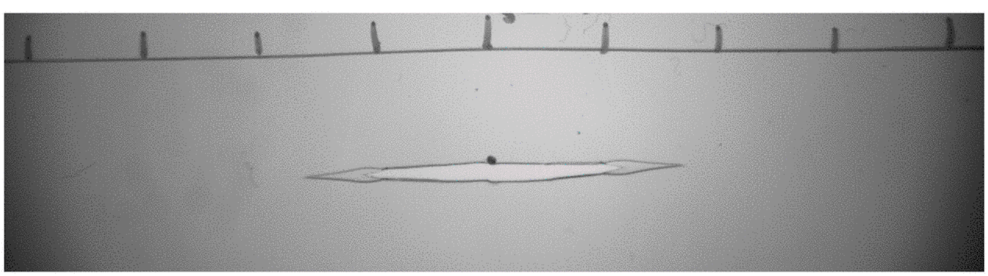


Effect of disorder amplified by applied stress !!!

Outline

1. Introduction
   a. Historical results
   b. A simple model
   c. Problems
2. Model of thermally activated crack growth
3. Experiments on slow crack growth in a paper sheet
4. A revised model
5. Effect of the heterogenous structure ?
6. Experiments on plastic materials

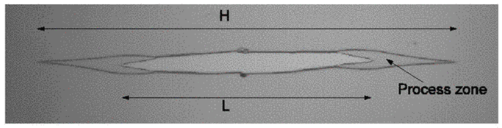
Other materials : polycarbonate

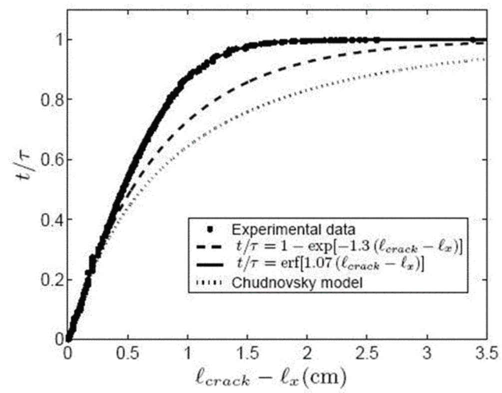


More regular dynamics

Very pronounced plastic zone (black external countour)

Average growth dynamics in polycarbonate





$$\frac{\langle t \rangle}{\tau} \neq \left[1 - \exp\left(-\frac{L-L_i}{\zeta}\right)\right]$$

(Cortet et al, EPL 2005)

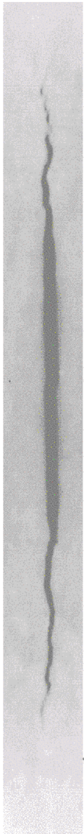
Summary

Thermally activated and irreversible elastic rupture:

1. Average crack growth dynamics
2. Prediction of energy barrier ~ $(\sigma_c - \sigma_i)^2$
3. Statistics of crack jumps ~critical point theory

Effect of disorder in rupture threshold
in 1d :
equivalent threshold ~ standard deviation of disorder
in 2d : x applied stress
(stress concentration effect on disorder)

Influence of visco-plastic properties
crack growth dynamics in polycarbonate?



Open issues

link between scales : $V^{1/3}\ nm \ \ \& \ \ \lambda\ \mu m$?

- more realistic dissipative mechanism
- disorder
- fiber rupture mechanism

uncomplete description of dynamics

- sinuous crack → disorder

Influence of visco-elastic properties

Is the shape of the energy barrier ΔU always the same ?

Experimental results : Pauchard, Guarino or Zhurkov