Depinning of Domain Walls, Contact Lines...and Cracks?

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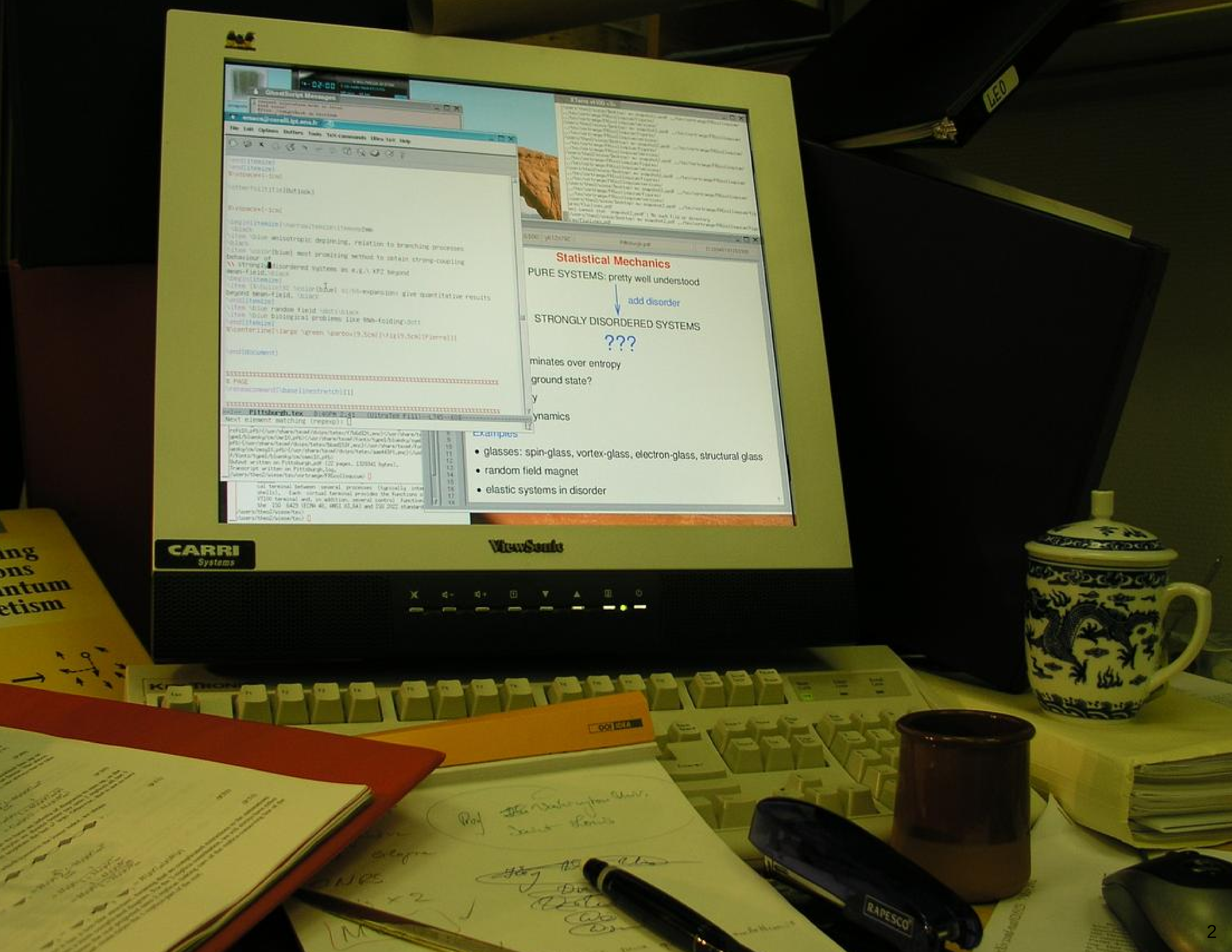
References: PRL 86 (2001) 1785: 2 loop
cond-mat/0302322 : intro + review

⋮

http://www.phys.ens.fr/∼wiese/

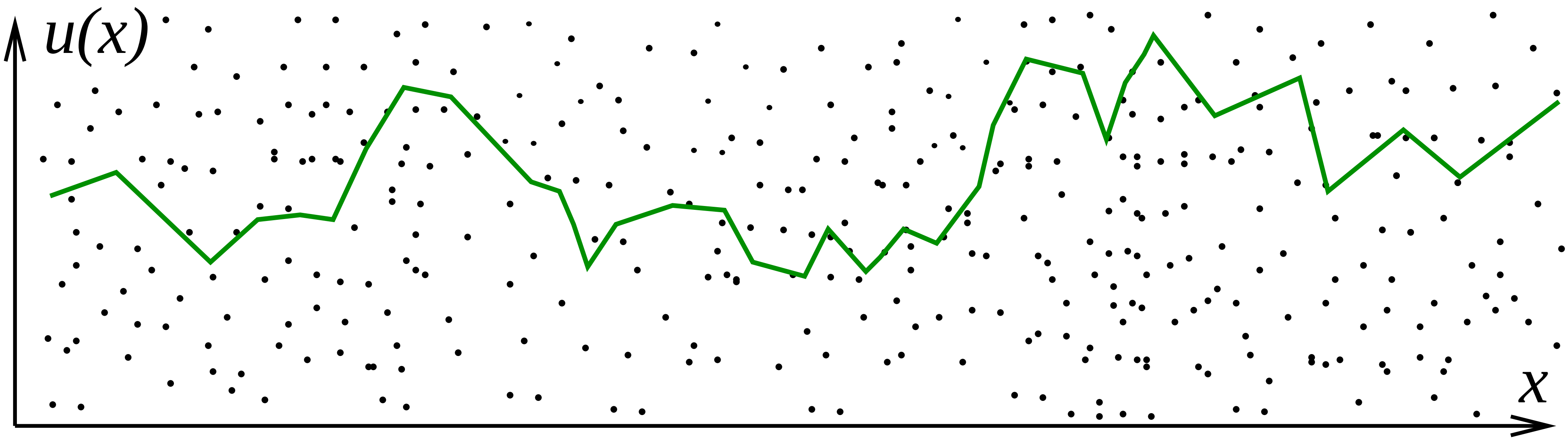
Disorder ?


Statistical Mechanics
PURE SYSTEMS: pretty well understood
add disorder
STRONGLY DISORDERED SYSTEMS
???
minates over entropy
ground state?
ynamics
Examples
• glasses: spin-glass, vortex-glass, electron-glass, structural glass
• random field magnet
• elastic systems in disorder
CARRI Systems
ViewSonic
LEO
RAPESCO

Disorder and dirt are necessary for our survival!

- disorder prevents our harddisk from self-erasing
- disorder makes for so diverse phenomena as quantum Hall effect, glasses, friction, . . .

Elastic Manifolds in Disorder



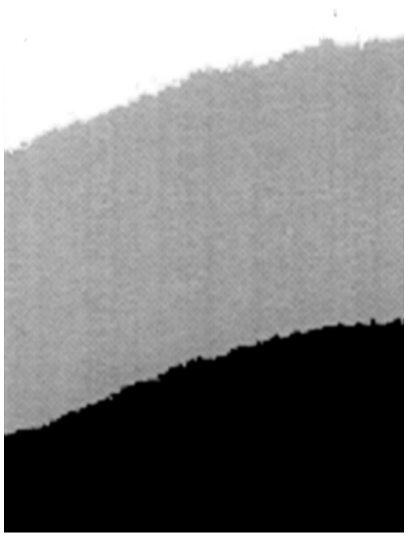
- elastic manifold in a random potential
- disorder dominates over thermal fluctuations: $T = 0$ fixed point
- search for minimum-energy configuration
- attention: multiple minima may exist
- prototype for strongly disordered systems

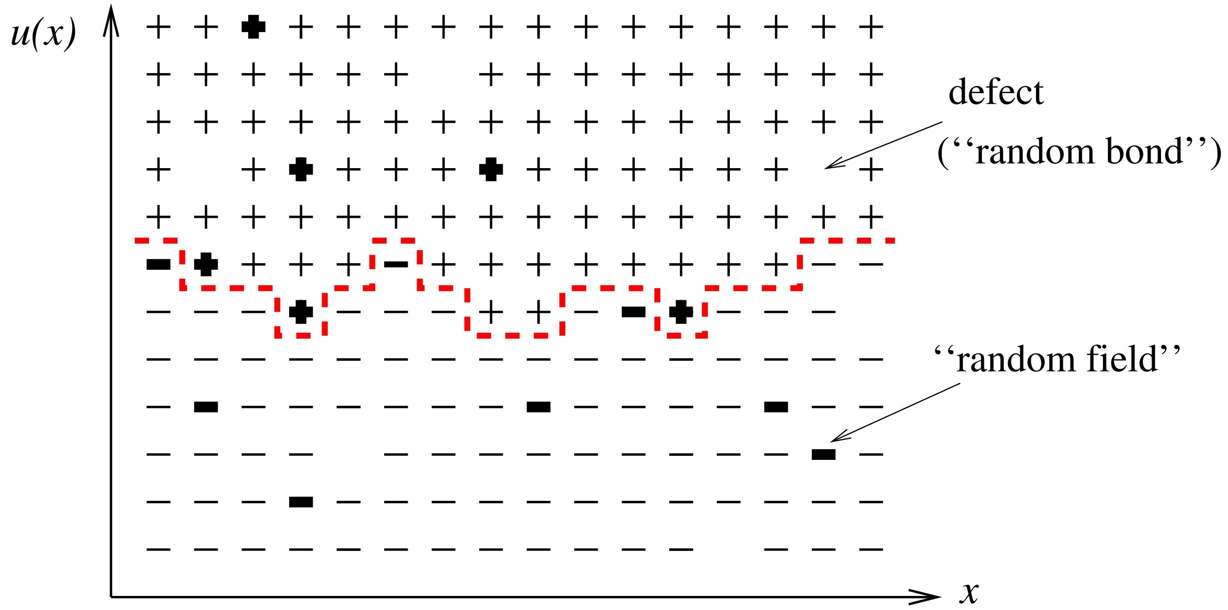
What do we know?

- experiments and simulations available
- phenomenological models (droplet picture)
- mean-field approximation (replica-symmetry breaking)

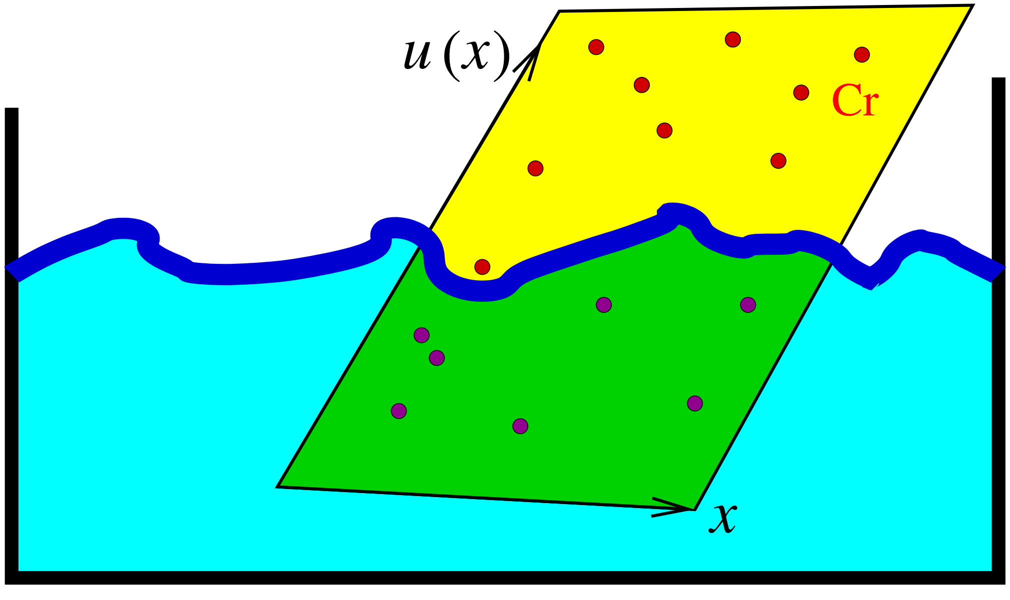
Physical Realizations

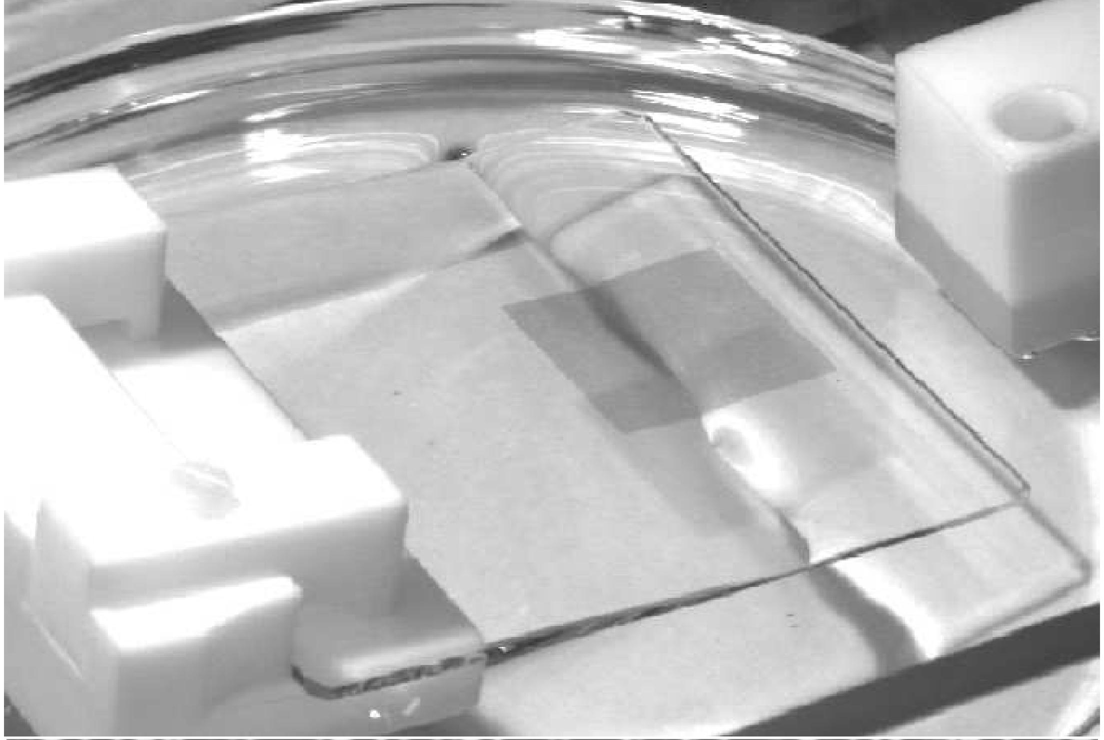
Domain-walls in magnets

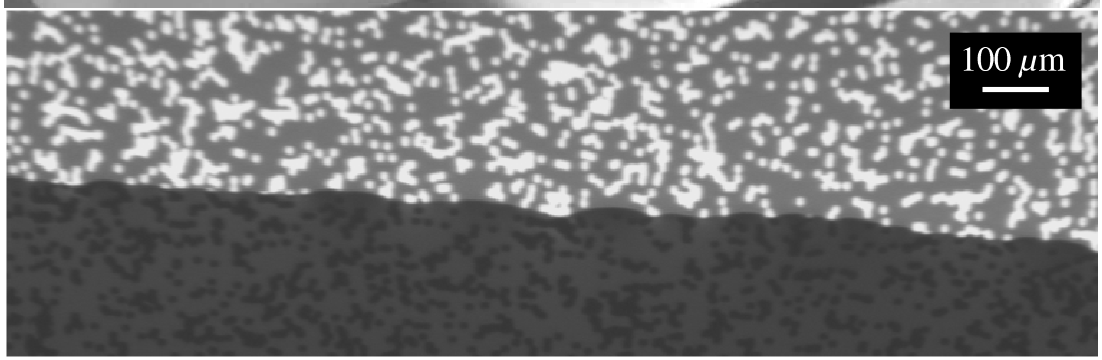


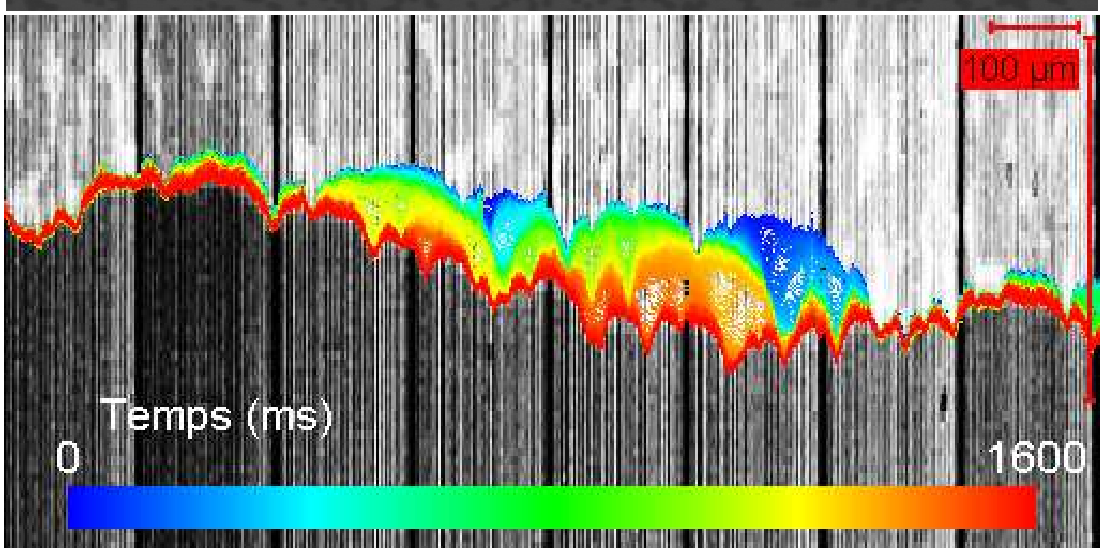


Contact line of liquid Helium/water







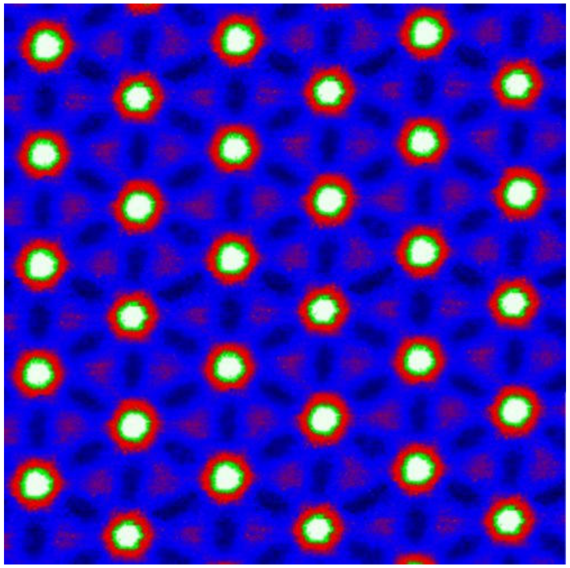


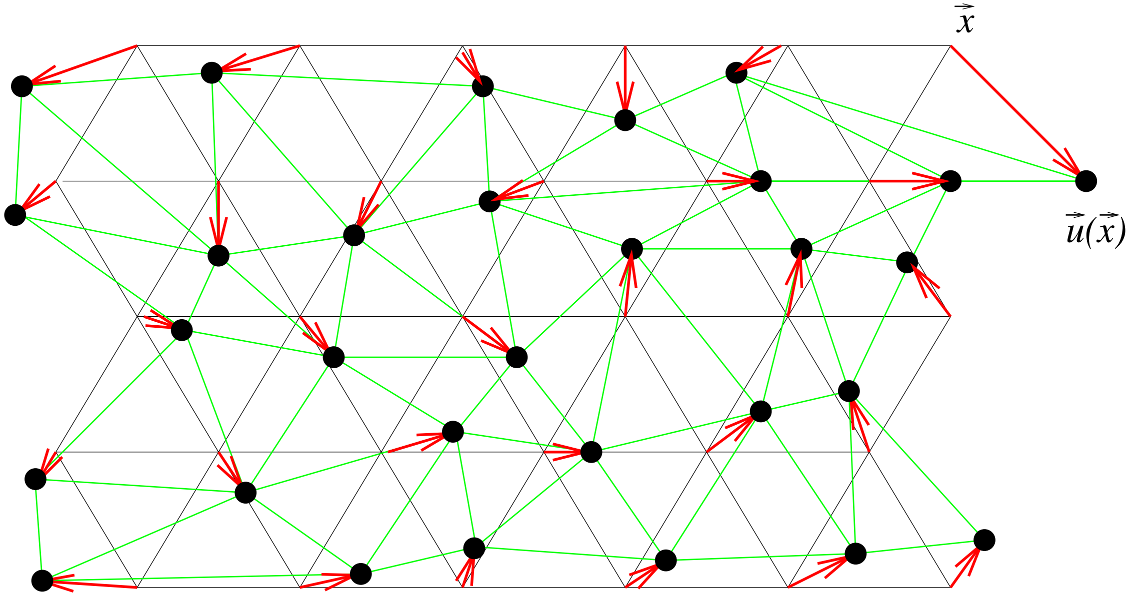
Depinning of contact line

Eur. Phys. J. A 8 (2002) 437

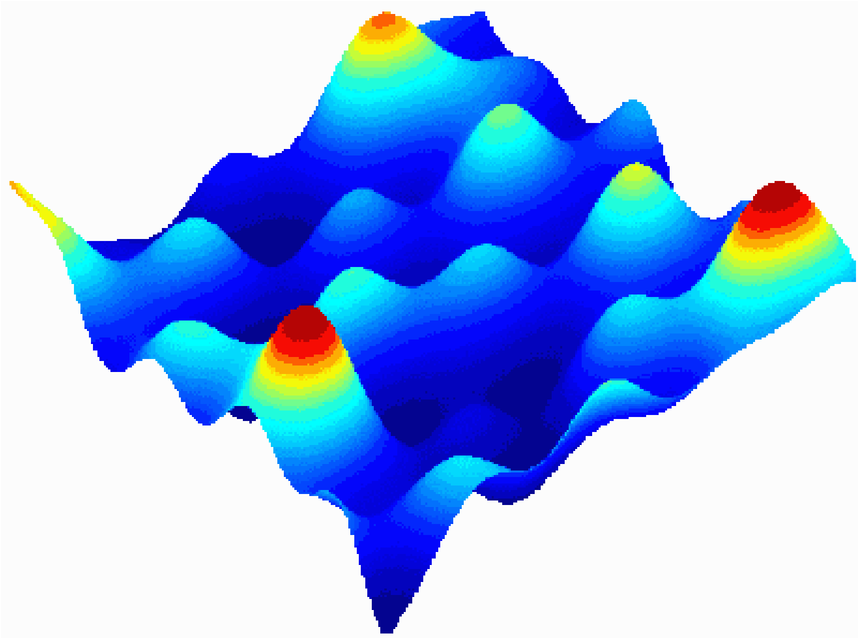
Pictures courtesy of
S. Moulinet, E. Rolley

Vortex-lattice/Bragg glass



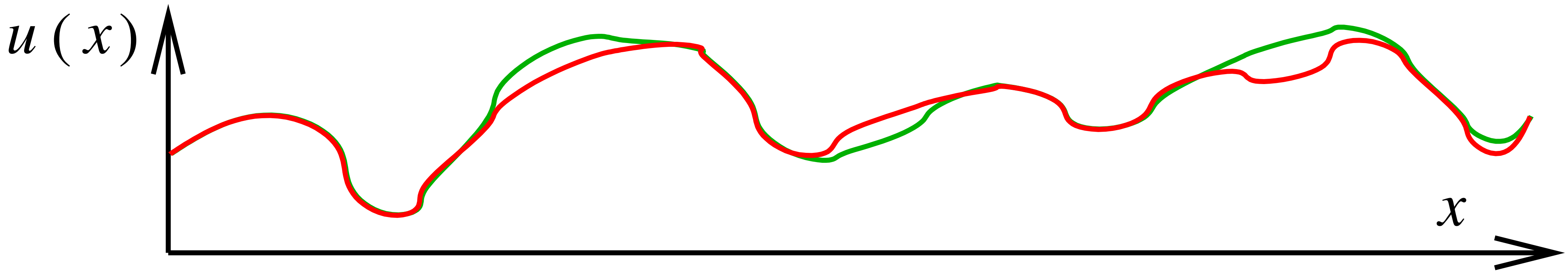


Charge Density wave



Cracks, earthquakes, directed polymer (KPZ), . . .

Model and Observables



Displacement field $$x \in \mathbb{R}^d \longrightarrow u(x) \in \mathbb{R}^N$$

↑ coordinate ↑ displacement-field

Elastic energy: $$\mathscr{H}_{\rm el} = \int {\rm d}^d x \frac{1}{2} [\nabla u(x)]^2$$

Disorder energy $$\mathscr{H}_{\rm DO} = \int {\rm d}^d x V(x, u(x))$$

with correlations ($N = 1$) $$\overline{V(x,u)V(x',u')} = \delta^d(x-x')R(u-u')$$

Observables

roughness ζ $$\overline{[u(x)-u(x')]^2} \sim |x-x'|^{2\zeta}$$

full probability-distribution function

The problem in the treatment of disorder: dimensional reduction

"Theorem" (Efetov, Larkin 1977): A d-dimensional disordered system at zero temperature ($T = 0$) is equivalent to all orders in perturbation theory to a pure system in $d-2$ dimensions at finite temperature. ("Holds" under quite general assumptions.)

Example: Elastic manifolds in disorder

The thermal 2-point function becomes

$$\left\langle [u(x)-u(0)]^2 \right\rangle \sim |x|^{2-d} \quad \longrightarrow \quad \overline{[u(x)-u(0)]^2} \sim x^{4-d}$$

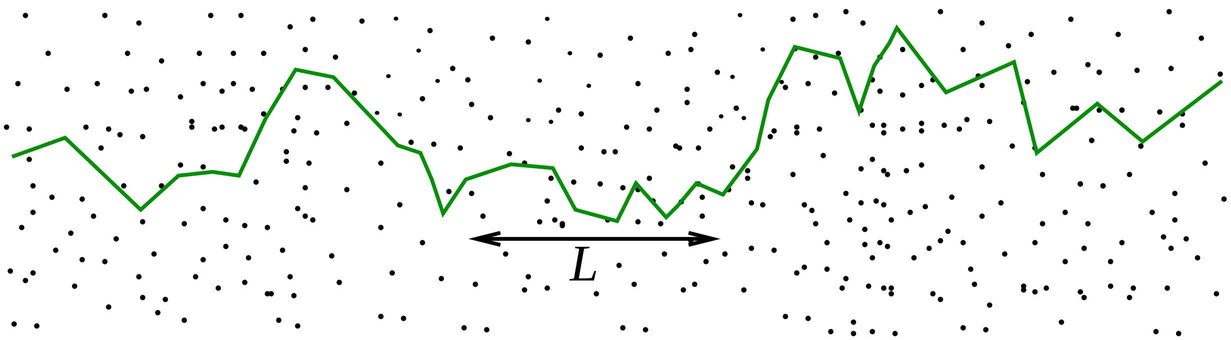
roughness exponent

$$\zeta = \frac{4-d}{2}$$

Counter-example:

3d disordered Ising-model at $T = 0$ is ordered; in contrast to the 1d Ising-model without disorder at $T > 0$.

The Larkin-length



Be the disorder force F_x gaussian, with correlation length r. Typical energy due to disorder on segment

$$\mathscr{E}_{\mathrm{DO}} = \bar{f}\left(\frac{L}{r}\right)^{d/2}$$

Elastic energy $$\mathscr{E}_{\mathrm{el}} = c L^{d-2}$$

Balance energies $\mathscr{E}_{\mathrm{DO}} = \mathscr{E}_{\mathrm{el}}$ at $L = L_c$ (Larkin-length)

$$L_c = \left(\frac{c^2}{\bar{f}^2} r^d\right)^{\frac{1}{4-d}}$$

$d < 4$: Membrane pinned by disorder on scales larger than the L_c:

$d = 4$ is upper critical dimension

Functional renormalization group (FRG)

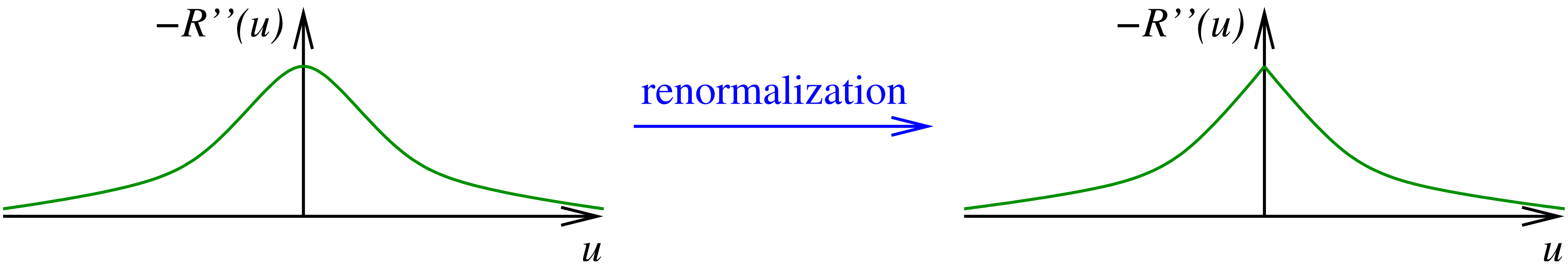
(D. Fisher 1986)

$$\mathscr{H}[u] = \int_x \frac{1}{2T} \sum_{a=1}^{n} [\nabla u_a(x)]^2 - \frac{1}{2T^2} \sum_{a,b=1}^{n} R(u_a(x) - u_b(x))$$

Functional renormalization group equation (FRG) for the disorder correlator $R(u)$ at 1-loop order:

$$\partial_\ell R(u) = (\varepsilon - 4\zeta) R(u) + \zeta u R'(u) + \frac{1}{2} R''(u)^2 - R''(u) R''(0)$$

Solution for force-force correlator $-R''(u)$:



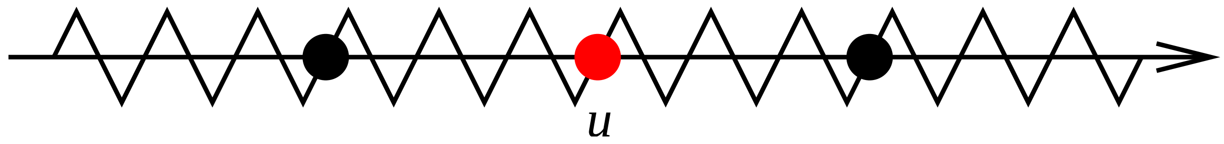
Cusp: $R''''(0) = \infty$ appears after finite RG-time (at Larkin-length)

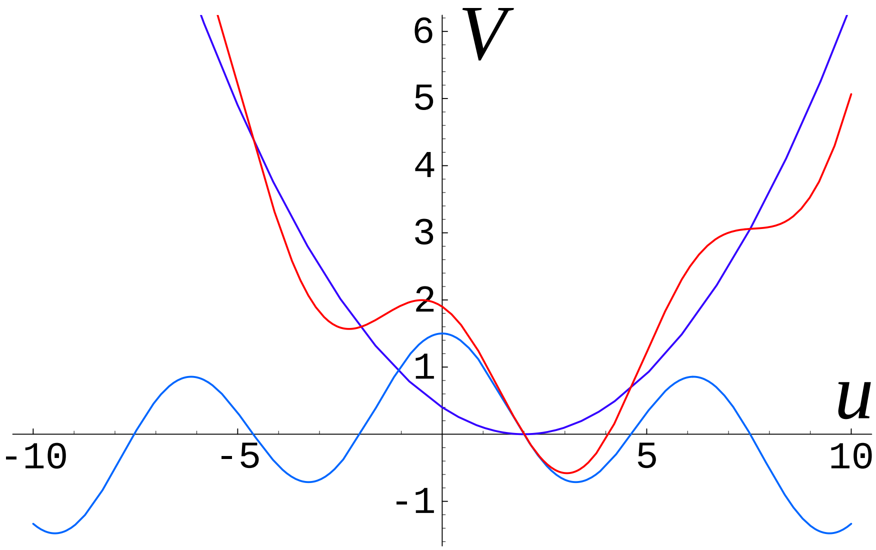
$R''_{L>L_c}(0) \neq$ dim.red.
eventhough formally
$\partial_\ell R''(0) = (\varepsilon - 2\zeta) R''(0)$
($\equiv$ dim.red.)

Renormalization of whole function overcomes dimensional reduction

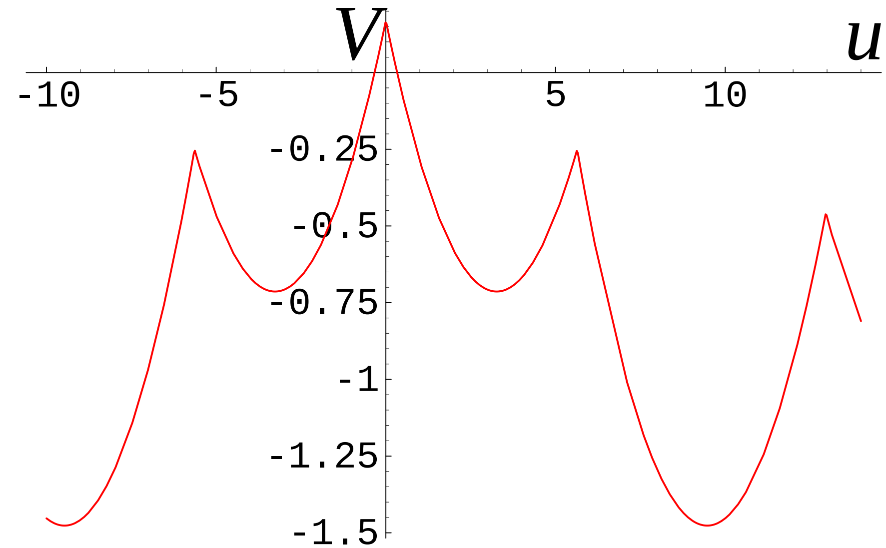
Why is a cusp necessary?

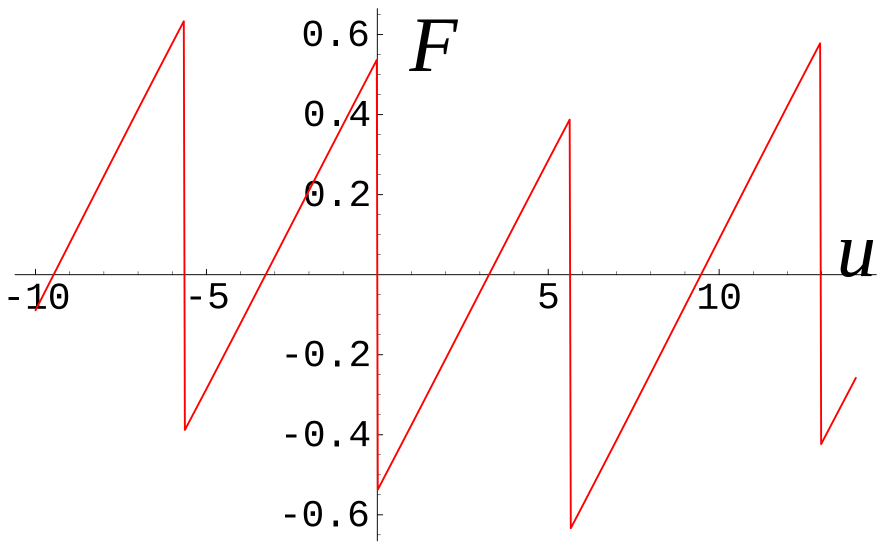
. . . suppose I want to integrate out a single degree of freedom. . .



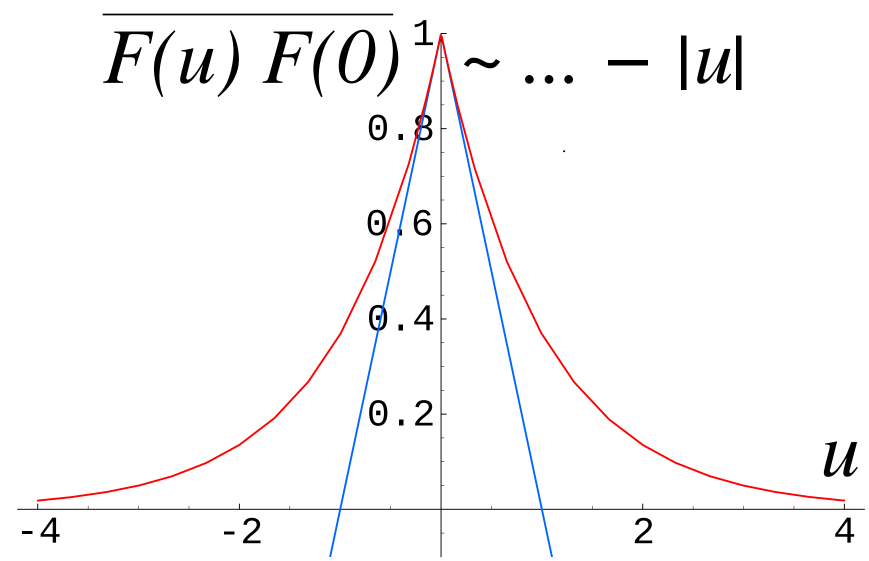


Min $\longrightarrow$



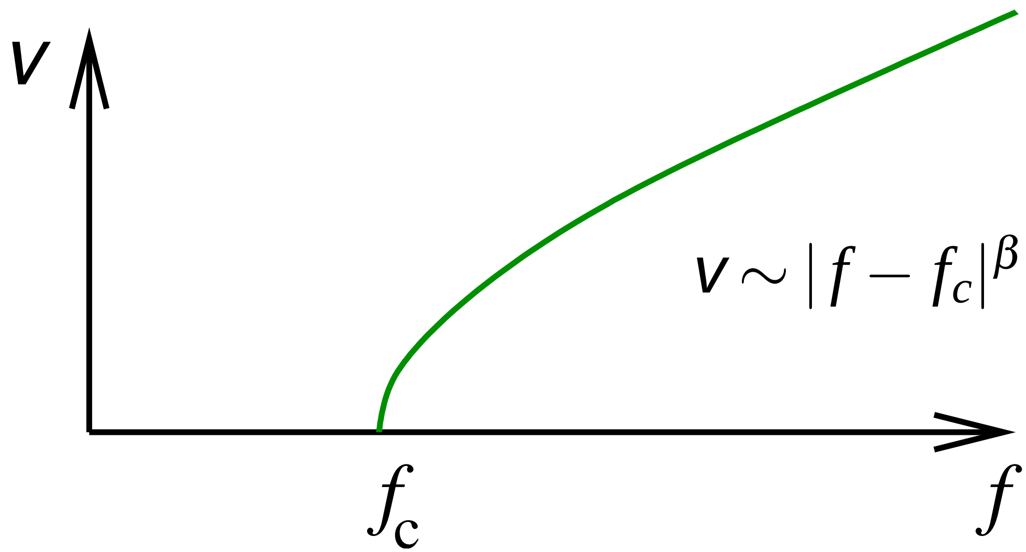


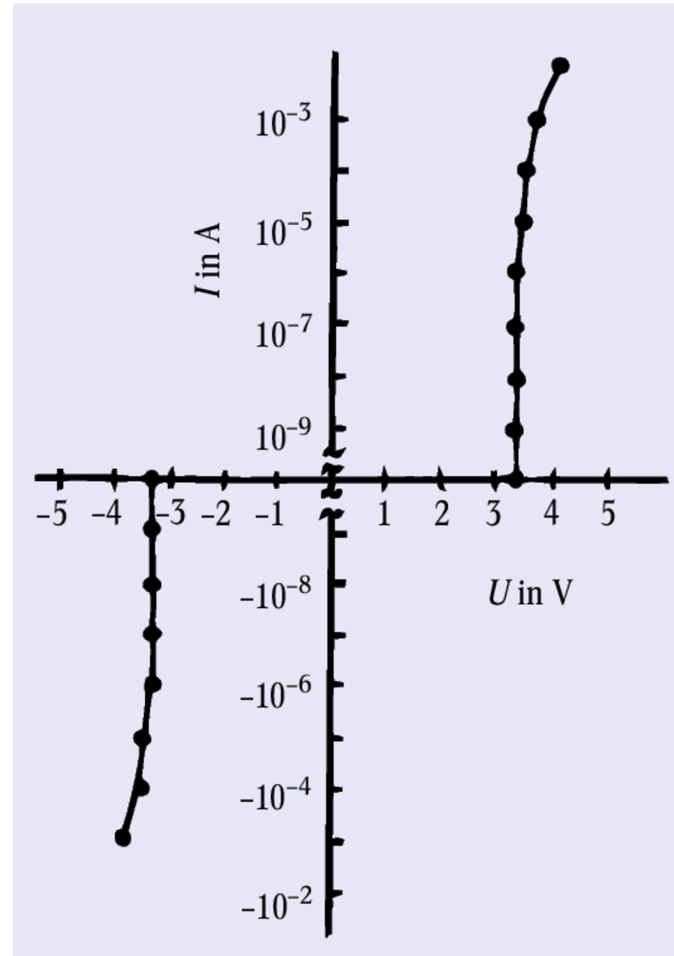
average $\longrightarrow$



Isotropic Depinning

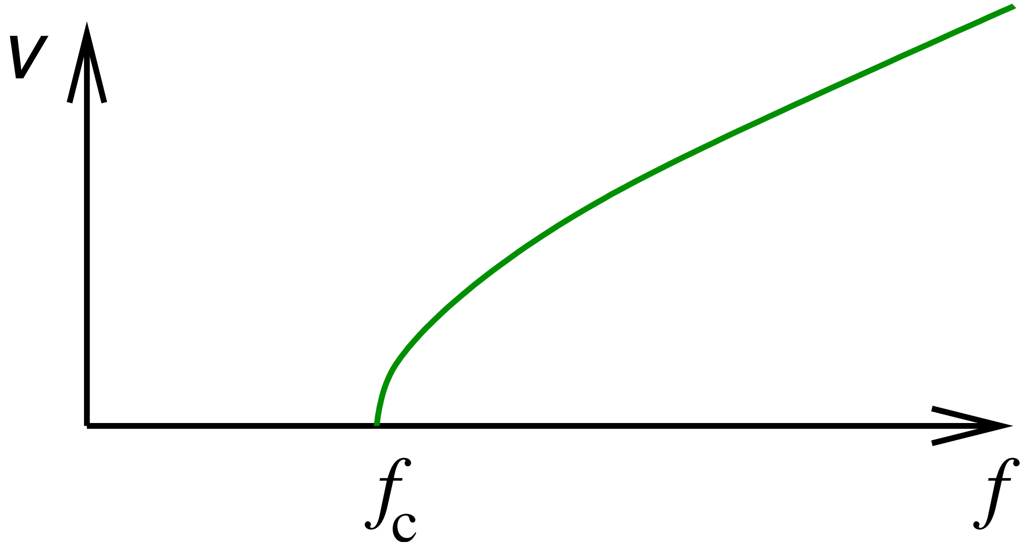
- roughness $\overline{[u(x)-u(0)]^2} \sim |x|^{2\zeta}$
- velocity-force-characteristics, pinning

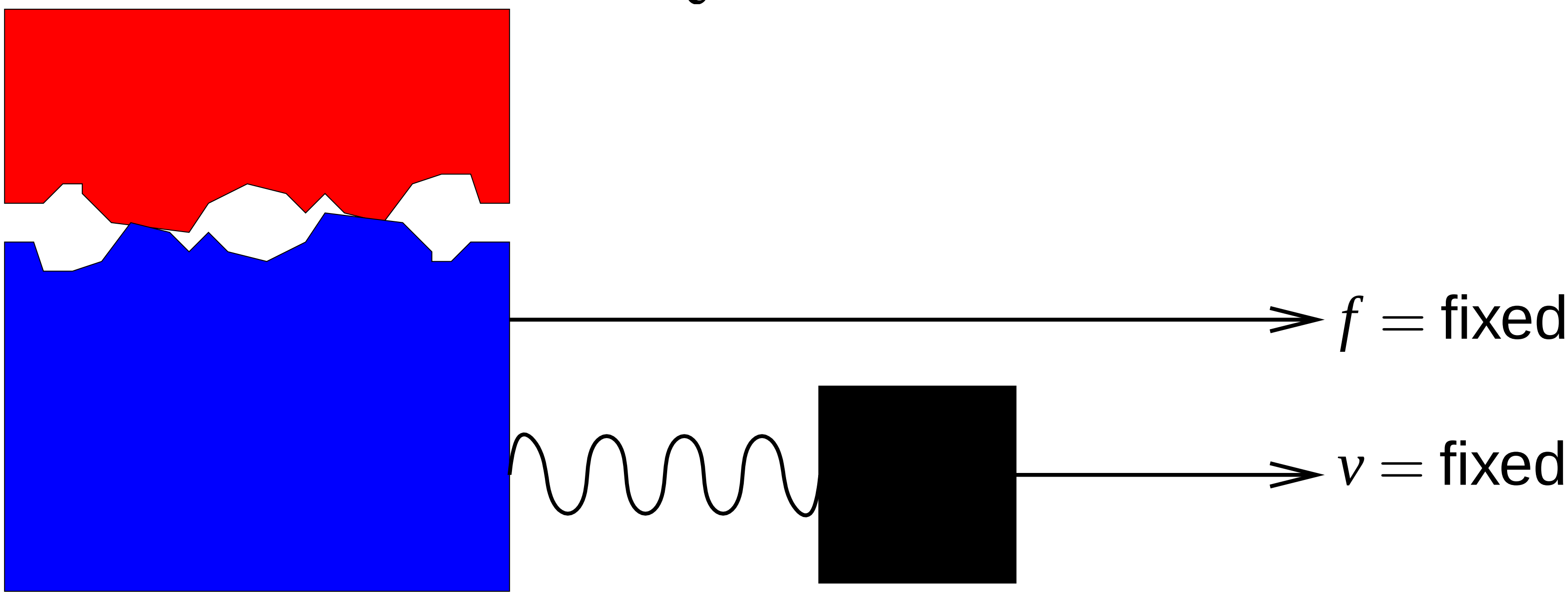




- dynamic exponent z $t \sim x^z$
- correlation length ξ $\xi \sim |f-f_c|^{-\nu}$
- exponent relations $\beta = \nu(z-\zeta)$ $\nu = \frac{1}{2-\zeta}$

Different drivings...





Drive with fixed f (mode 1): tune f to f_c for scale-invariance

Drive with velocity f (mode 1): tune v to 0 for scale-invariance

Avalanches are observed in mode 2.

RG-treatment, 1loop

Nattermann et al., Narayan & Fisher, 1992

- same RG-equation as in the statics, even though physics should be different: Not a consistent theory
- claim: $\zeta = \frac{\varepsilon}{3}$ is exact to all orders; in contradiction with experiments and simulations

2 loop

PRL 86 (2001) 1785, PRB 66 (2002) 174201

- Membrane only jumps ahead ($T = 0$):

$$t > t' \qquad \Longrightarrow \qquad u(x,t) \geq u(x,t')$$

- renders perturbation theory unique

$$\partial_\ell R(u) = (\varepsilon - 4\zeta)R(u) + \zeta u R'(u) + \frac{1}{2}R''(u)^2 - R''(u)R''(0)$$
$$+\frac{1}{2}\left[R''(u) - R''(0)\right]R'''(u)^2 + \frac{1}{2}R'''(0^+)^2 R''(u)$$

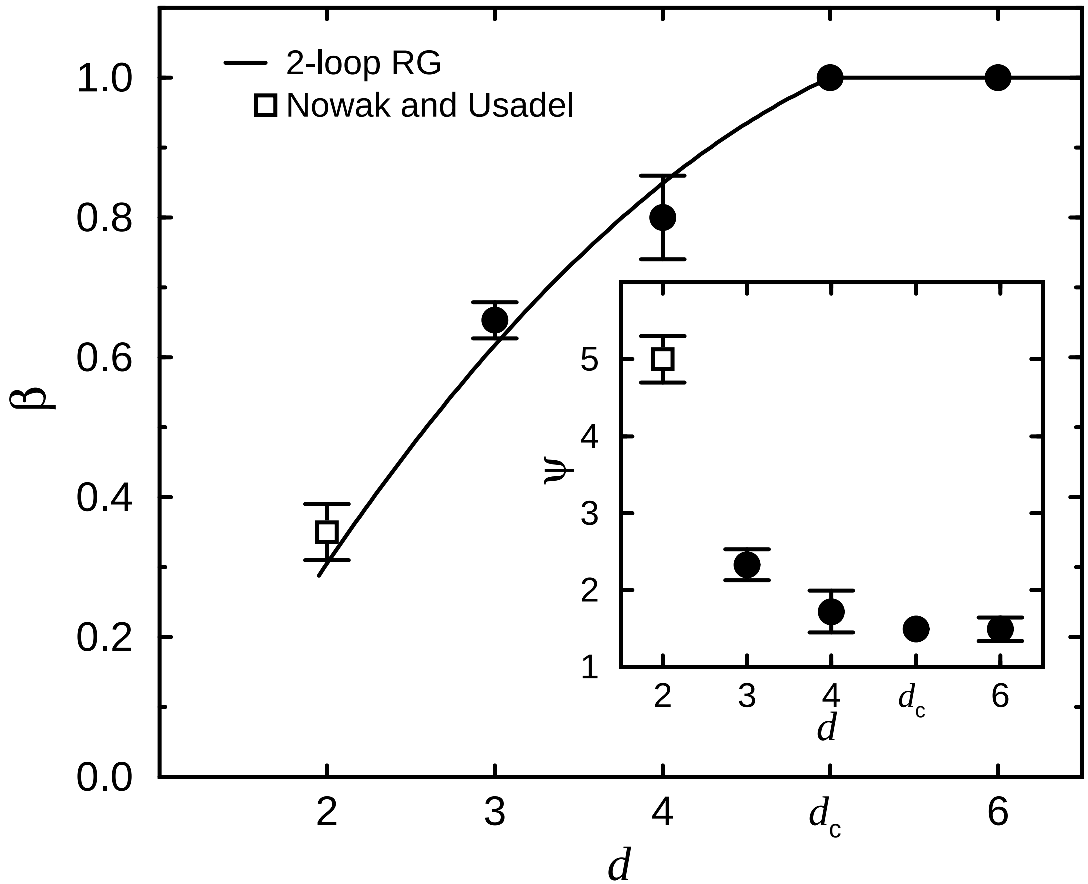
- results for RF: $\zeta = \frac{\varepsilon}{3}(1 + 0.14331\varepsilon + \ldots)$

$$z = 2 - \frac{2}{9}\varepsilon - 0.04321\varepsilon^2 + \ldots$$

Depinning

Rosso, Krauth, et al. (2001)

	d	ε	ε^2	estimate	simulation
	3	0.33	0.38	0.38±0.02	0.34±0.01
ζ	2	0.67	0.86	0.82±0.1	0.75±0.02
	1	1.00	1.43	1.2±0.2	1.25±0.01



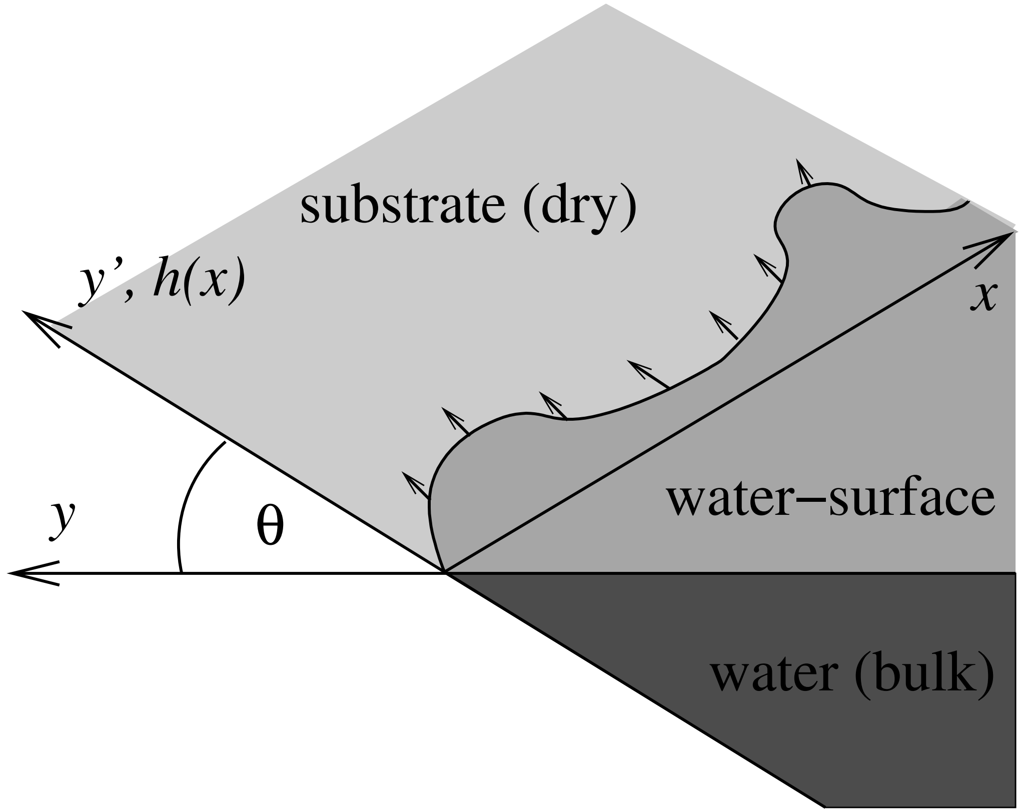
Domain-wall in RF-magnet

Roters et al. PRE 60 (1999) 5202

Depinning, $d = 1$, long-range elasticity

	ε	ε^2	estimate	simulation
ζ	0.33	0.47	0.47±0.1	0.39±0.002

Depinning of contact line – long-range elasticity



Elastic Energy of liquid air interface

$$E_{\mathrm{LA}}[z] \sim \text{area of liquid air interface}$$

It can be written as a function of height $h(x)$ at boundary

$$E_{\mathrm{LA}}[h] = \frac{c_1}{2}\int_q |q| h_q h_{-q} + \frac{\lambda}{2}\int_{q,k} [|q||k| + qk] h_k h_q h_{-q-k} + O(h^4)$$

More . . . long-range elasticity

$$E_{\mathrm{LA}}[h] = \frac{c_1}{2}\int_q |q| h_q h_{-q} + \frac{\lambda}{2}\int_{q,k} [|q||k| + qk] h_k h_q h_{-q-k} + O(h^4)$$

Linear elasticity leads to roughness $\zeta \approx 0.39$ (numerics + FRG)
Experiments on contact line: $\zeta \approx 0.5$

Non-linear terms can not be neglected.

Non-linear terms become relevant under FRG.

This is also relevant to earthquakes, and cracks.

For earthquakes, the elasticity is like above but the dimension is $d = 2$. The system is in its “critical dimension”. Instead of power-laws there are logarithimic dependences on systems size etc.

Avalanches

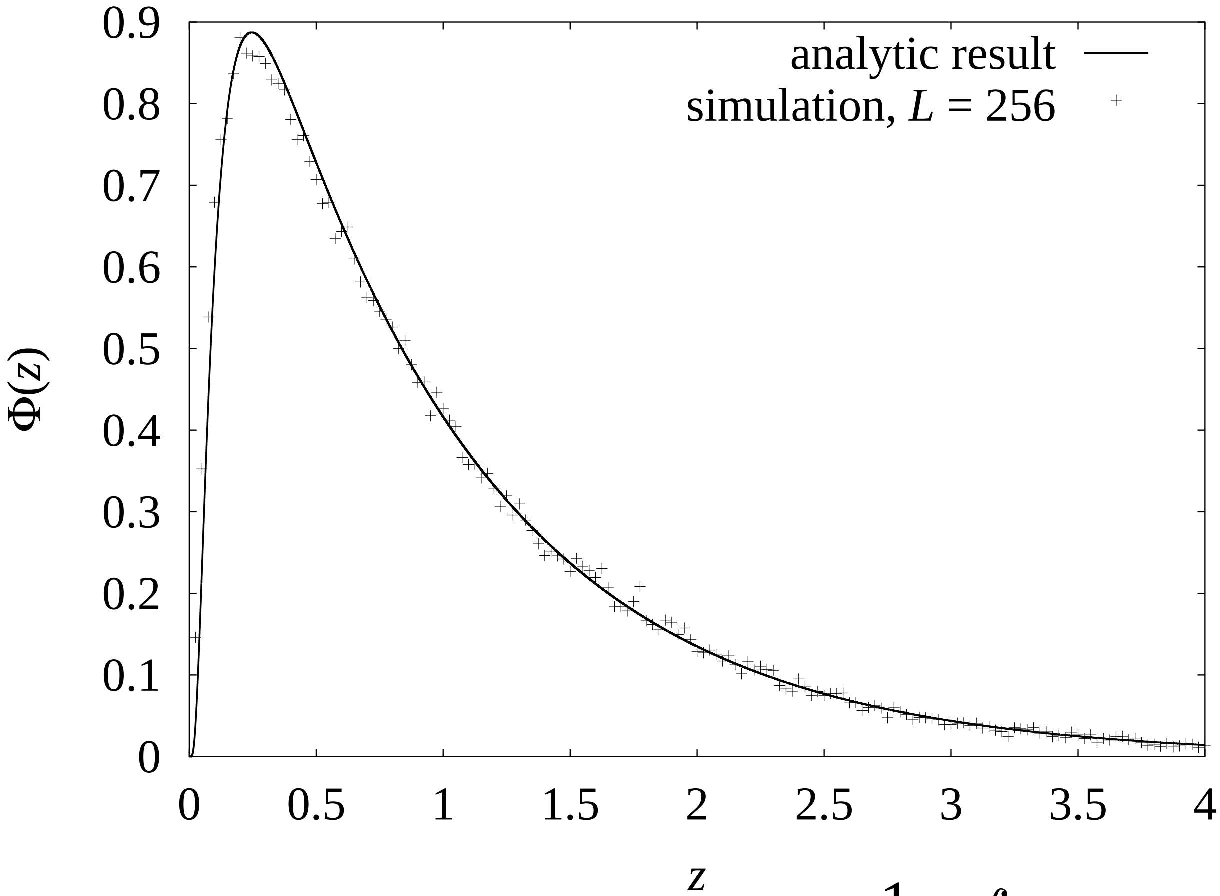
FRG knows about two independent renormalizations: roughness ζ and dynamics exponent z. All other exponents should be related to these by scaling relations, e.g. avalanche-size distribution

$$\mathscr{P}(s) = s^{-\tau} f_s\left(\frac{s}{s_0}\right) , \qquad \tau = 2\left[1 - \frac{1}{d+\zeta}\right]$$

Avalanche-time distribution

$$\mathscr{P}(t) = t^{-\alpha} f_t\left(\frac{t}{t_0}\right) , \qquad \alpha = \frac{d+\zeta+z-2}{z}$$

Probability distribution function for the interface width



A. Rosso, W. Krauth, P. Le Doussal, J. Vannimenus, K.J. Wiese
PRE 68 (2003) 036128

$$w^2 := \frac{1}{\text{Vol}} \int_x \left(u(x) - \langle u \rangle\right)^2$$

Probability distribution function

$$\mathscr{P}(w^2) = \Phi\left(\frac{w^2}{\langle w^2 \rangle}\right)$$

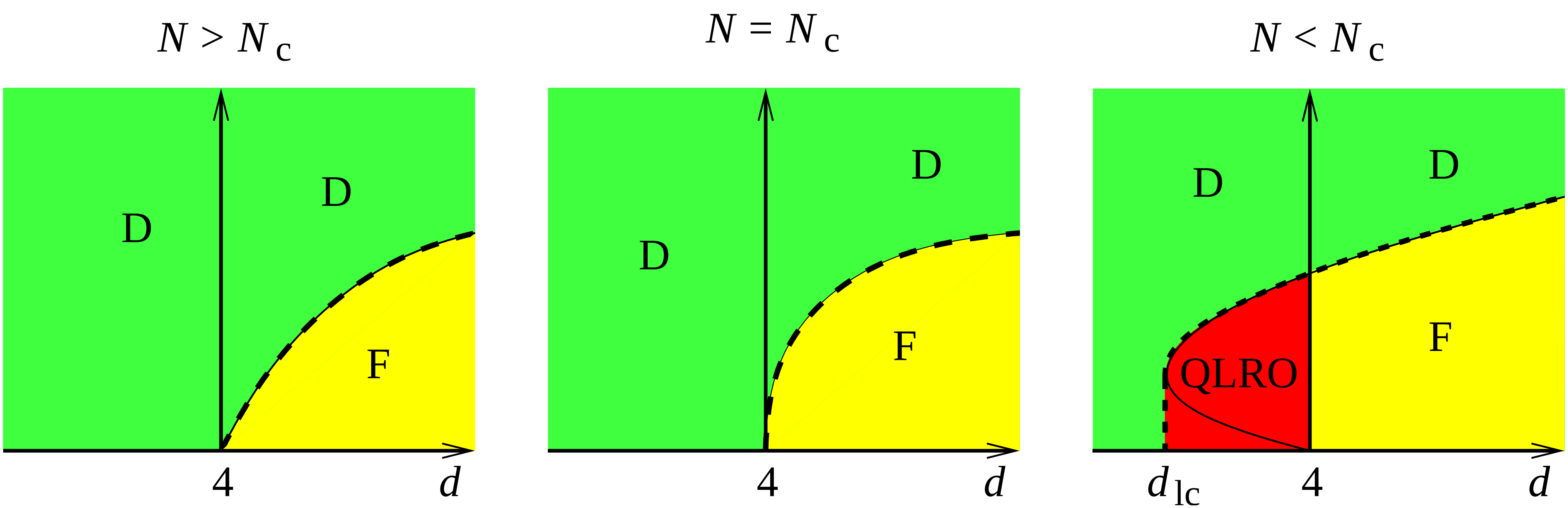
$\Phi(z)$ is universal, depending only on ζ.

Random Field Systems

Expansion about the ordered phase, i.e. with constraint $|\vec{n}| = 1$.

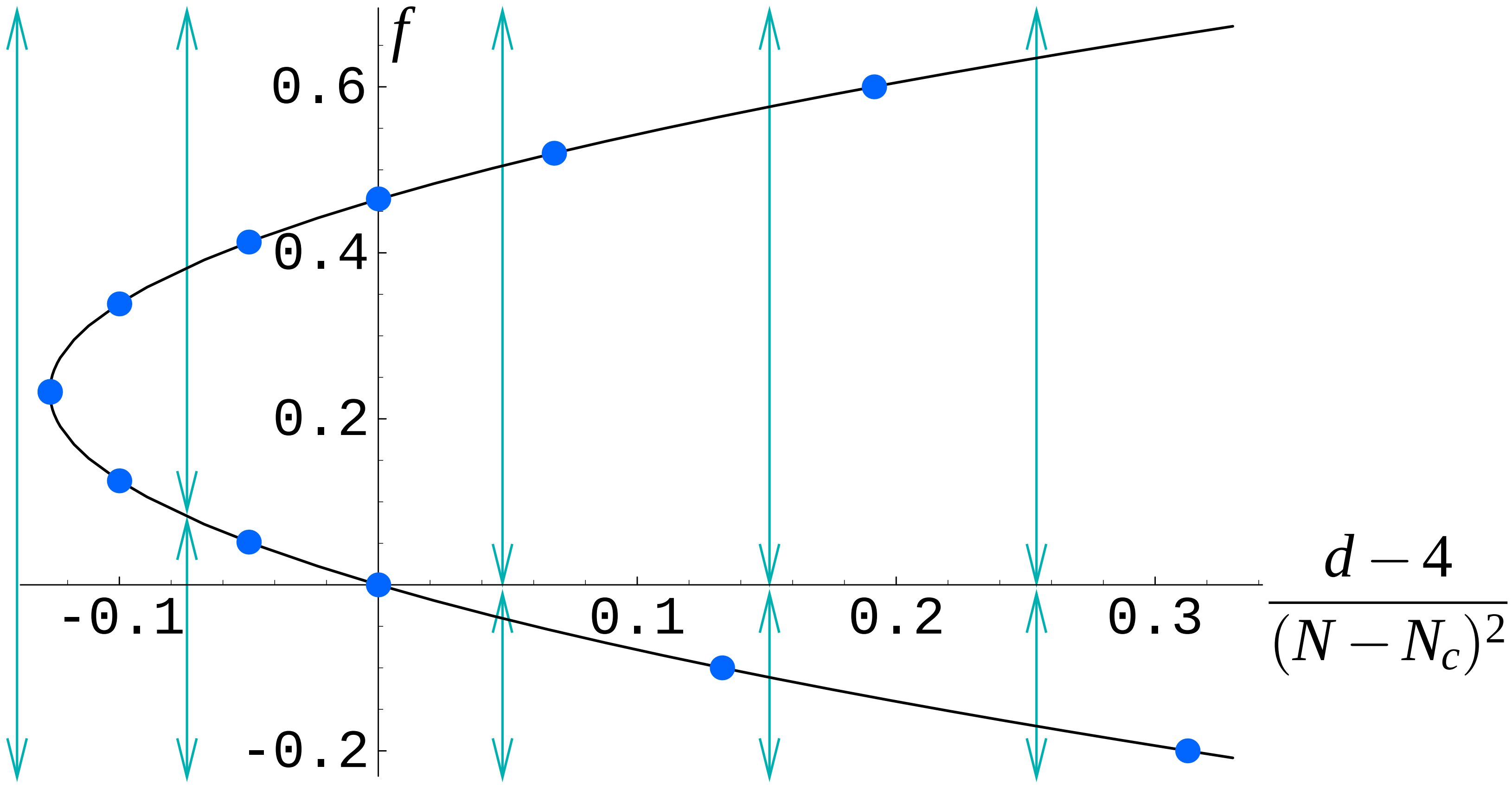
$$\mathscr{H}[\vec{n}] = \int \mathrm{d}^d x \left[\frac{1}{2T} \sum_a (\nabla \vec{n}_a)^2 - \frac{1}{T} \sum_a \vec{M}\vec{n}_a - \frac{1}{2T^2} \sum_{ab} \hat{R}(\vec{n}_a \vec{n}_b) \right]$$

$$\partial_\ell R(\phi) = \varepsilon R(\phi) + \frac{1}{2} R''(\phi)^2 - R''(0) R''(\phi)$$
$$+ (N-2) \left[\frac{1}{2} \frac{R'(\phi)^2}{\sin^2 \phi} - \cot \phi R'(\phi) R''(0) \right] + \text{2 loop terms}$$



Functional RG expansion around N_c

$$R(\phi) = gR_c(\phi) + g^2 \delta R(\phi)\ , \qquad g = f(N_c - N)$$



$N_c = 2.8347$ for Random Field

$N_c = 9.441$ for Random Anisotropy

Summary

- New analytical method to treat strongly disordered systems
  - higher order calculations: cumbersome, but under control
  - exact solution of the large-N limit, precise relation to RSB
  - cusp analytically under control

Outlook

- most promising method to obtain strong-coupling behaviour of strongly disordered systems as e.g. KPZ beyond mean-field.
  - $1/N$-expansion: give quantitative results beyond mean-field.
- random field . . .
- biological problems like RNA-folding. . .
- quantum problems ?