Magnitude-dependent Omori law

Multifractal Scaling of Thermally Activated Rupture Processes

The earthquake deformation flow as a multifractal measure / conditional Poisson process

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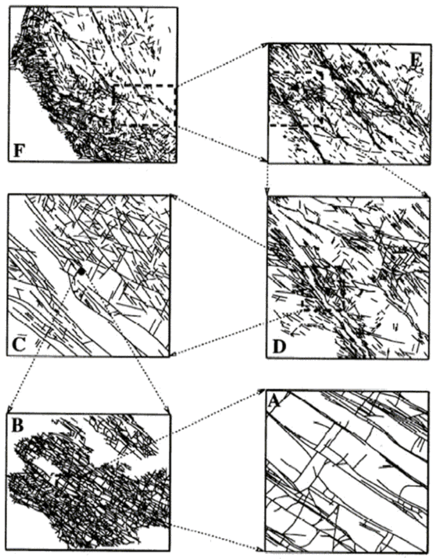
Organization of seismicity

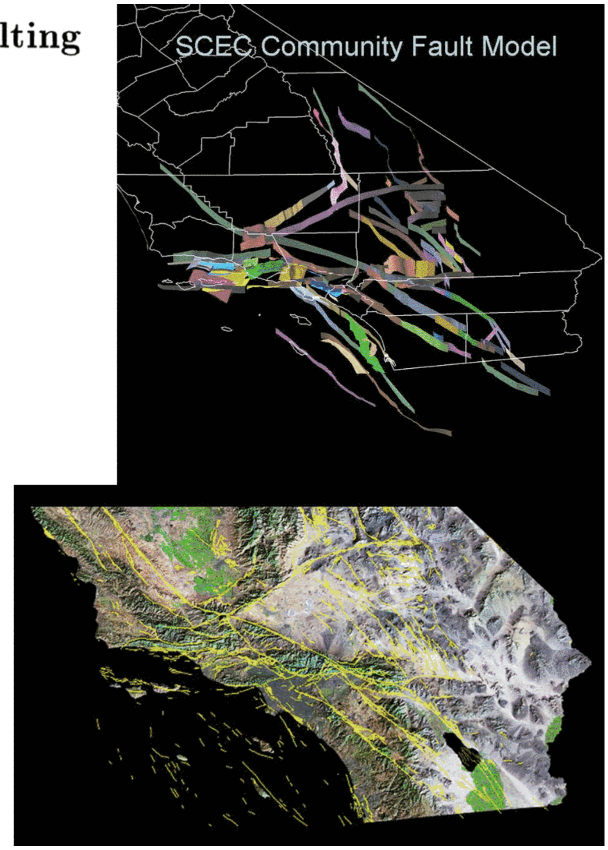
- Gutenberg-Richter law: $\sim 1/E^{1+\beta}$ (with $\beta \approx 2/3$)
- Omori law $\sim 1/t^p$ (with $p \approx 1$ for large earthquakes)
- Productivity law $\sim E^a$ (with $a \approx 2/3$)
- Power law PDF of fault lengths $\sim 1/L^2$
- Fractal/multifractal structure of fault networks $\zeta(q)$, $f(\alpha)$
- Power law PDF of seismic stress sources $\sim 1/s^{2+\delta}$ (with $\delta \geq 0$)

Hierarchical geometry of faulting

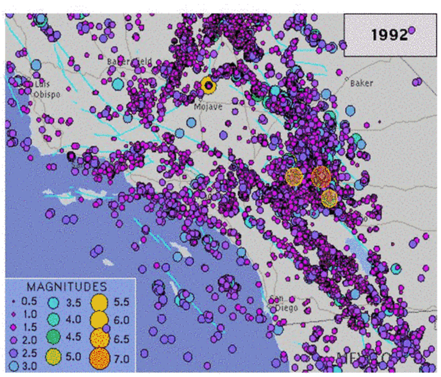
Ouillon, Castaing, Sornette (JGR 1996)

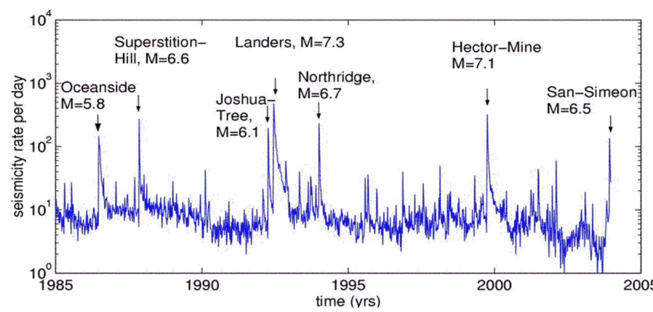


Map A: linear size=10 m, orig. scale=1:1
Map B: linear size=60 m, orig. scale=1:220
Map C: linear size=11 km, orig. scale=1:62,500
Map D: linear size=45 km, orig. scale=1:125,000
Map E: linear size=150 km, orig. scale=1:250,000
Map F: linear size=400 km, orig. scale=1:1,000,000



Spatial and temporal organization of seismicity in Californie



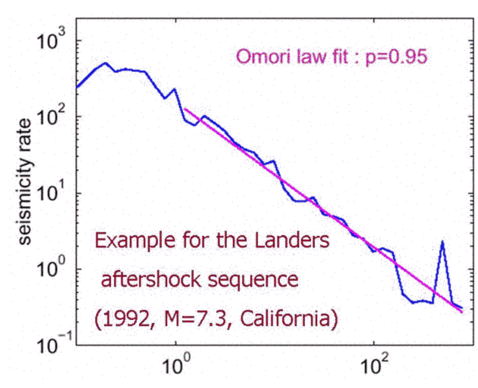


Temporal decay of the rate N(t) of aftershocks after a mainshock at t=0

$N(t) = K/(t+c)^p$

p is in the range [0.3, 2], often close to 1

[Omori, 1894; Utsu, 1960]



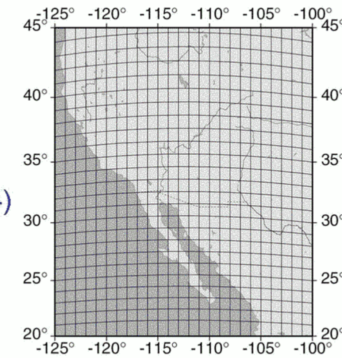
rate of seismic events of magnitude $M > m$ occurring in a cell of size $L \times L$

Monofractal view:

$$\lambda(m,L,T) = a\, 10^{-bm}\, L^{d}\, T^{-p}$$

Unified Scaling Law for Earthquakes

(Bak et al, PRL 2002; Corral, 2003; Baiesi and Paczuski, 2004)



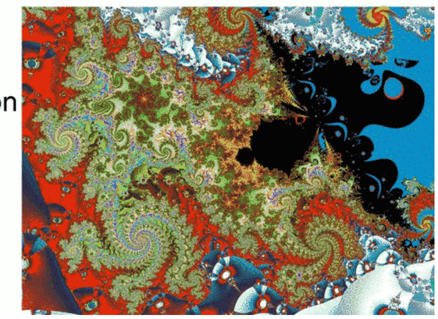
Multifractal view ("metric"):

$$\lambda_i(m,L,T) = a_i\, 10^{-b_i m}\, L^{d_i}\, T^{-p_i}$$

exponents are inter-related

Van Gogh View of turbulence

Multifractal view of fragmentation



(Lee Skinner)

Earthquakes as thermally activated processes

- ❖Thermal activation controls creep rupture [Scholz, 2002]
- ❖Eyring rheology and other thermal-dependent friction laws describe creep failure in many compounds and material interfaces [Liu and Ross, 1996; Vulliet, 2000]
- ❖Stress corrosion with pre-existing cracks in rocks [Atkinson, 1984] and hydrolytic weakening [Griggs et al, 1957]
- ❖Ruina-Dieterich state-and-velocity dependent friction law [Dieterich, 1979; Ruina, 1983; Scholz, 1998]

thermal rupture activation process

Poisson Intensity (average conditional seismicity rate)
At position $\vec{r}$ and time t

$$\lambda(\vec{r}, t) \sim \exp\left[-\beta E(\vec{r}, t)\right]$$

$$E(\vec{r}, t) = E_0(\vec{r}) - V\Sigma(\vec{r}, t)$$ (Zhurkov, 1965)

stress corrosion, damage, state-and-velocity dependent friction
and mechano-chemical effects

$$\Sigma(\vec{r}, t) = \Sigma_{\text{far field}}(\vec{r}, t) + \int_{-\infty}^{t} \int dN[d\vec{r}\,' \times d\tau]\Delta\sigma(\vec{r}\,', \tau) g(\vec{r} - \vec{r}\,', t - \tau)$$

$$\lambda_i(t) = \lambda_{\text{tec}}(t) \exp\left[\beta \sum_j \int_{-\infty}^{t} d\tau\ \Delta\sigma_j(\tau)\ g_{ij}(t-\tau)\right]$$

Generalization of stress release models [Vere-Jones et al.]

approximation

$$\int_{\vec{r}} dN[d\vec{r}\,' \times d\tau]\, \Delta\sigma(\vec{r}\,', \tau)\, g(\vec{r} - \vec{r}\,', t - \tau) \approx dN[\tau]\, s(\tau)\, h(t - \tau)$$

$$\lambda(\vec{r}, t) = \lambda_{\text{tec}}(\vec{r}, t)\ \exp\left[\beta \int_{-\infty}^{t} d\tau\ s(\vec{r}, \tau) h(t - \tau)\right]$$

$$s(\vec{r}, \tau) = \int d\vec{r}\,'\ \Delta\sigma(\vec{r}\,', \tau)\ f(\vec{r} - \vec{r}\,')$$

Effective source at time τ at point r resulting from all events occurring in the spatial domain at that time τ

Physical model

- Rupture of triggered events is a thermally activated processes (creep rupture, subcritical crack growth, state and rate friction…), depending exponentially on stress.
- Bulk rheology displays a slow relaxation of stress, with a long relaxation time τ (much larger than T=1 year). This relaxation takes the form :

$$h(t) = \frac{h_0}{(t + c_0)^{1+\theta}} \quad ,0 < t < \tau$$

- At any place, stress fluctuations due to past events obey a power-law distribution :

$$P(s) \propto \frac{C}{s^{1+\mu}}$$ (Kagan, 1994; Marsan, 2004)

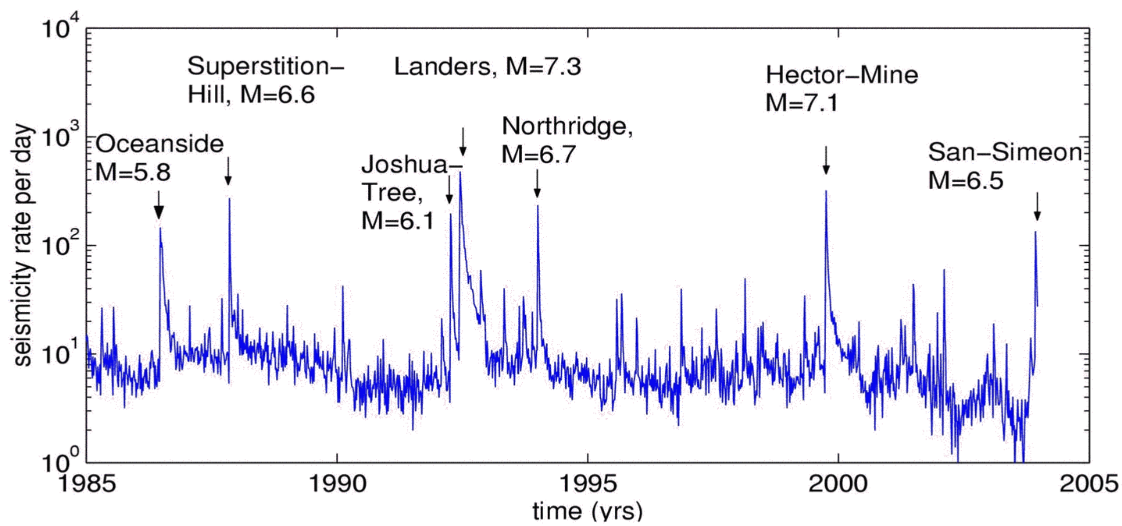
- In continuous form, the seismicity rate can thus be written :

$$\lambda(t) = \lambda_{tec}(t) \exp\left[\beta V \int_{-\infty}^{t} d\tau\, s(\tau) h(t-\tau)\right]$$

where $\lambda_{tec}(t)$ is the average long-term seismicity rate imposed by tectonic loading and β is the inverse temperature. V is the activation volume.

Theoretical predictions using tail covariance concept (Ide-Sornette, 2001)

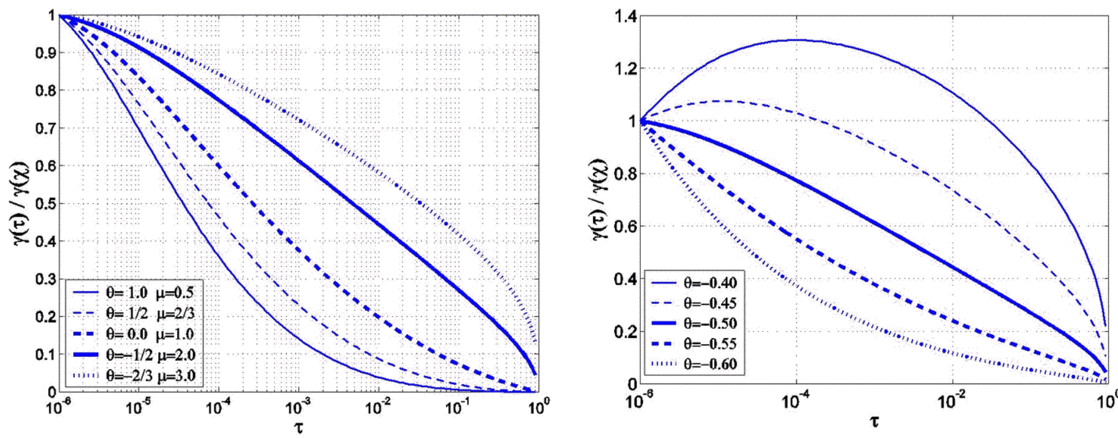
$$\Pr[\lambda(t) > \lambda_q | \lambda_M] = \Pr[e^{\beta\omega(t)} > \frac{\lambda_q}{\lambda_{tec}} | \omega_M] = \Pr[\omega(t) > (1/\beta) \ln\left(\frac{\lambda_q}{\lambda_{tec}}\right) | \omega_M]$$



$$\lambda_q(t) = A_q \ \lambda_{\rm tec} \ e^{\beta \gamma(t) \omega_M}$$

$$\gamma(t) = \frac{h_0^2}{\Delta t^{2/\mu}} \left(\frac{1}{t^{2m-1}} \int_{c/t}^{\frac{T+c}{t}-1} dy \frac{1}{(y+1)^m} \ \frac{1}{y^m} \right)^{\frac{2}{\mu}} \qquad m = (1+\theta)\mu/2.$$

Since $\gamma(t) \sim \ln(t)$ and $\omega_m \sim m$, we obtain $p(m) = a\, m + b$



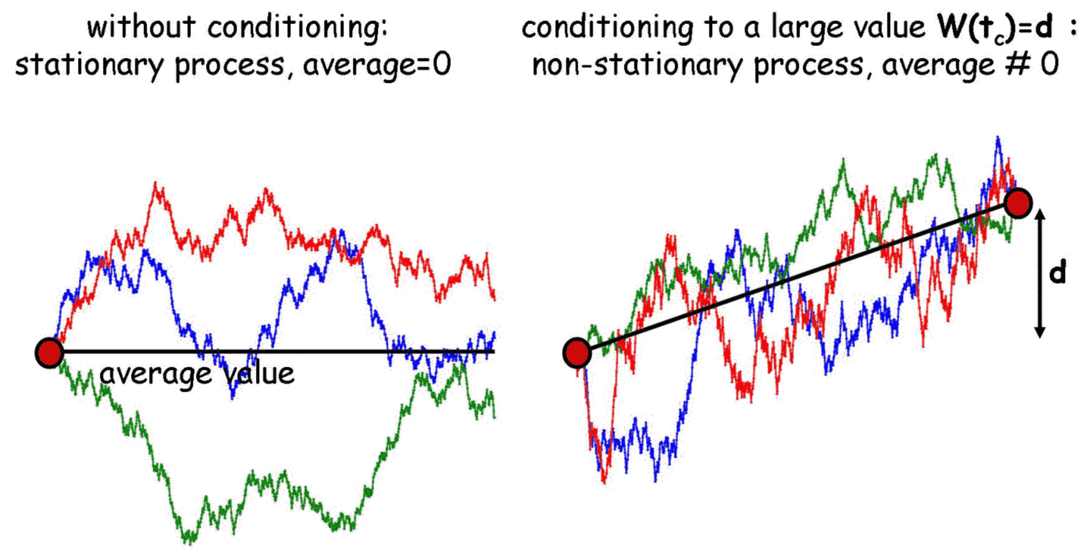
$$A(t) = \int_{-\infty}^{t} d\tau \ \eta(\tau) \ K(t-\tau)$$

Endogeneous shock

$$\mathrm{E}[X(t)|Y = A_0] - \mathrm{E}[X(t)] = (A_0 - \mathrm{E}[Y]) \ \frac{\mathrm{Cov}(X(t), Y)}{\mathrm{E}[Y^2]}$$

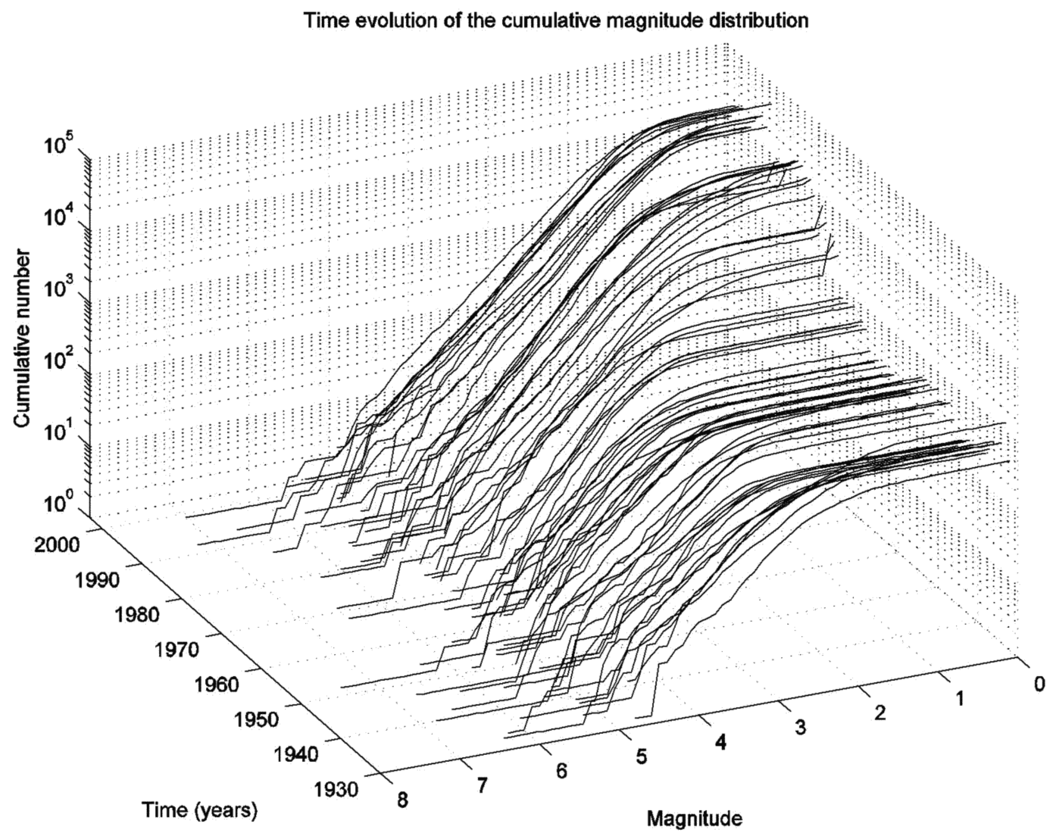
$$\mathrm{Cov}(A(t), A(0)) = \int_{-\infty}^{0} d\tau \ K(t-\tau) \ K(-\tau)$$

$$\mathrm{E}_{\rm endo}[A(t)|A(0) = A_0] \propto A_0 \int_0^{+\infty} du \ K(t+u) \ K(u)$$



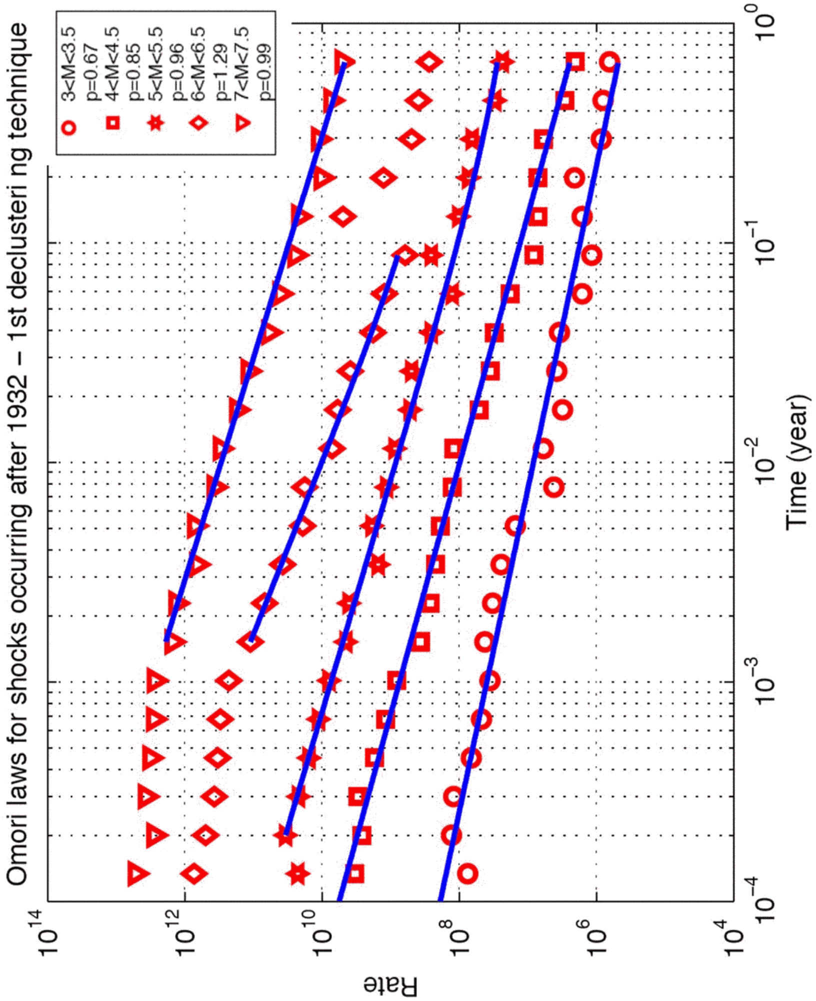
Data used for analysis

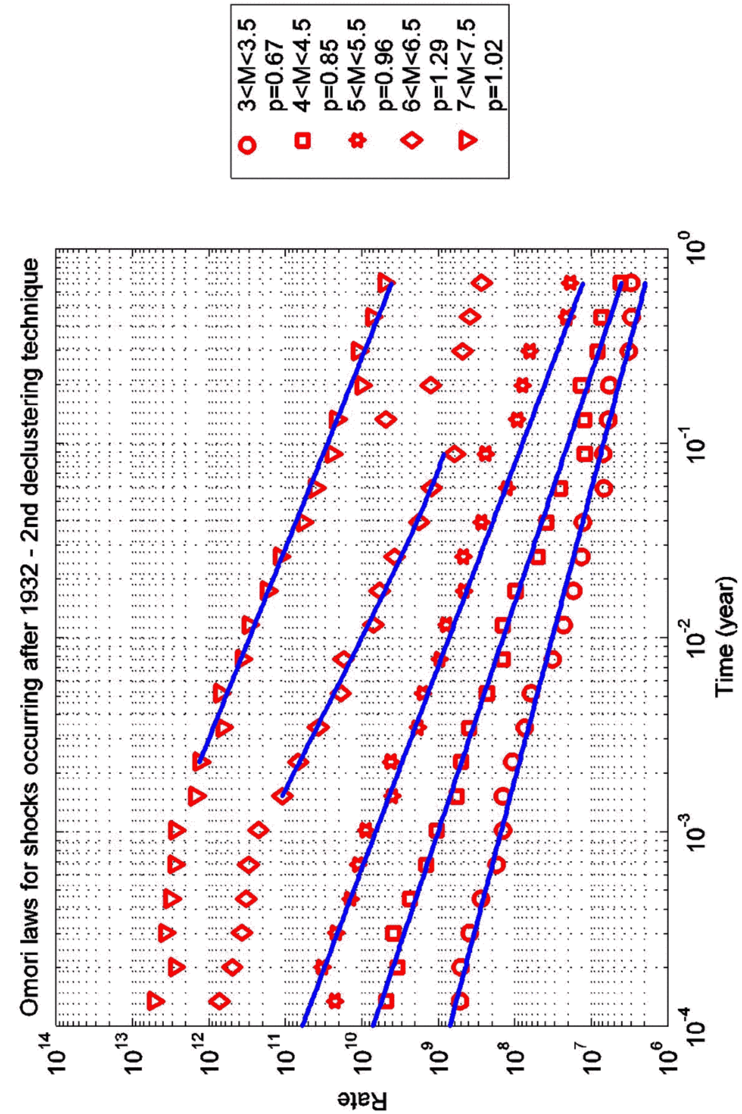
- We use the SCEC catalog (32° to 37°N, -122° to -128°W)
- We define 4 subcatalogs, according to their completeness
  - 1932-2003 for events with M > 3.0
  - 1975-2003 for events with M > 2.5
  - 1992-2003 for events with M > 2.0
  - 1994-2003 for events with M > 1.5
- Each subcatalog will be analyzed separately

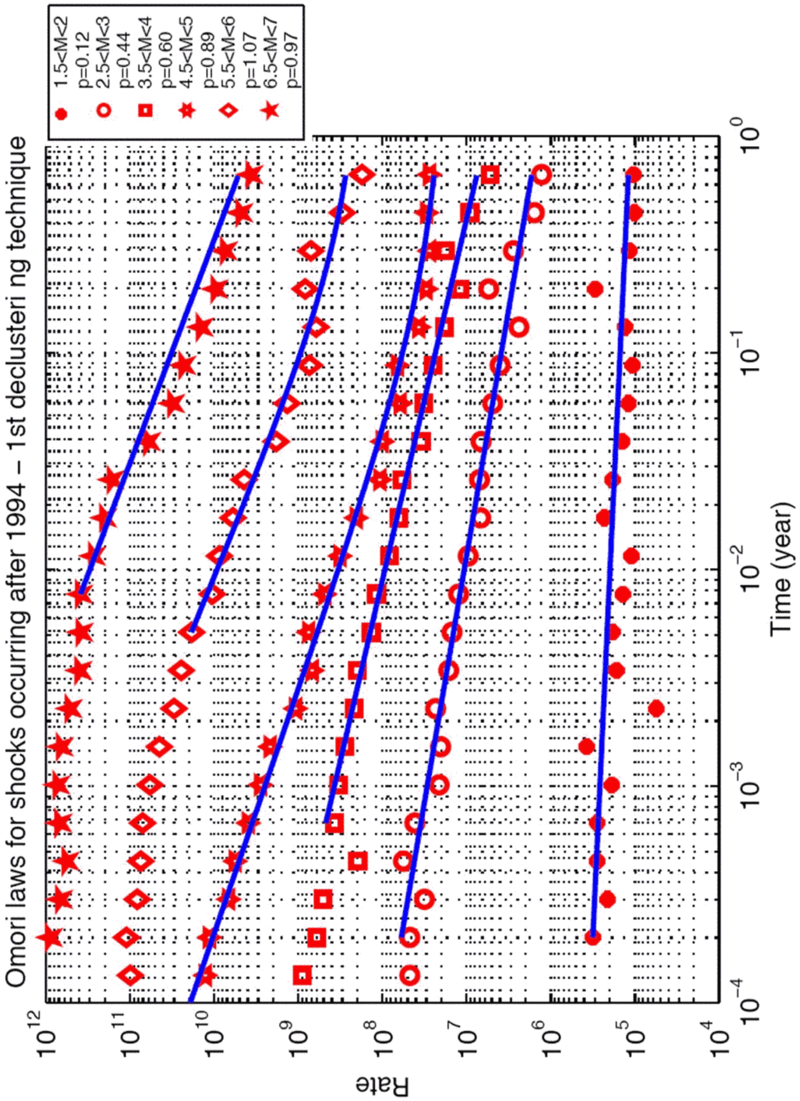


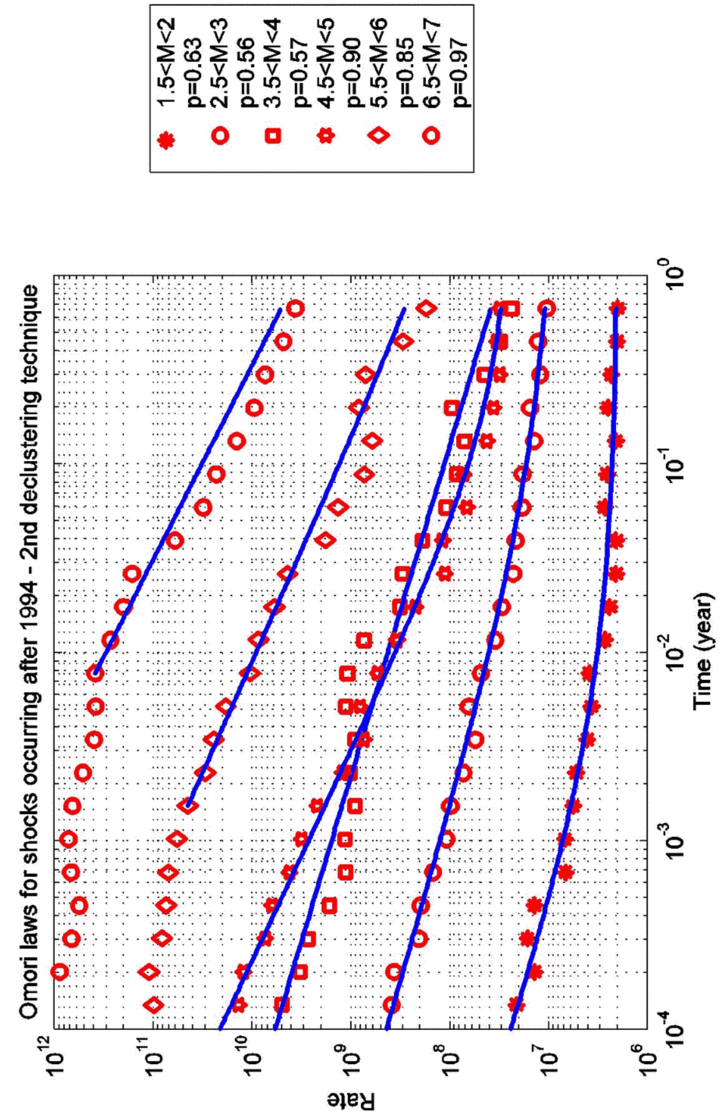
Data processing

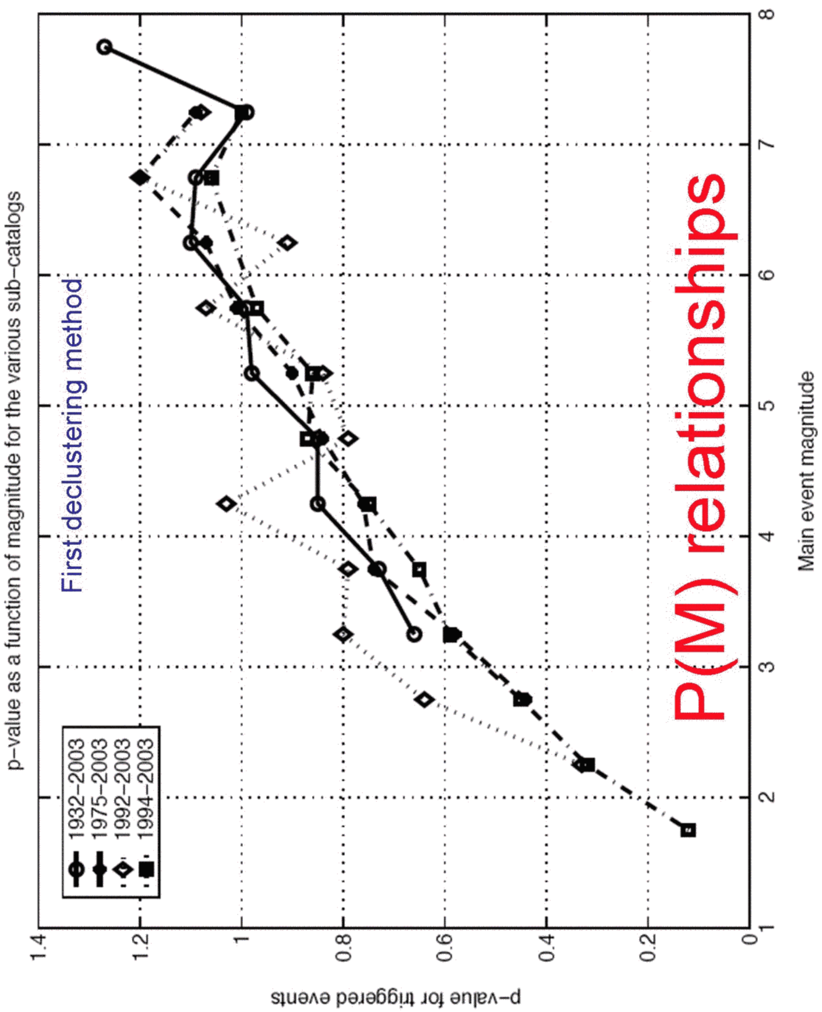
- An event is considered as triggered by another event of magnitude M if it falls within a spatial window of size d or L or $L_{preceding}$ around that event within T=1 year after its occurrence.
- Size L is taken either equal to the estimated main rupture length ($L=10^{-2.57+0.6M}$), or twice that length.
- We bin mainshock magnitudes in consecutive intervals [1.5;2.0], [2.0;2.5],… up to [7.0;7.5]
- In each main event magnitude interval [M_1;M_2], we translate each triggered sequence to a common origin time t=0, and stack all sequences.
- We fit composite sequences by $N(t) = B + a/(t+c)^p$ using linear least-squares or use Maximum Likelihood.
- We can then obtain the average value of p as a function of main event magnitude.
- Use of different definitions of mainshocks and robustness of the results.


Omori laws for shocks occurring after 1932 – 1st declusteri ng technique
3<M<3.5
p=0.67
4<M<4.5
p=0.85
5<M<5.5
p=0.96
6<M<6.5
p=1.29
7<M<7.5
p=0.99
Rate
Time (year)


Omori laws for shocks occurring after 1932 - 2nd declustering technique
3<M<3.5
p=0.67
4<M<4.5
p=0.85
5<M<5.5
p=0.96
6<M<6.5
p=1.29
7<M<7.5
p=1.02
Rate
Time (year)


Omori laws for shocks occurring after 1994 – 1st declusteri ng technique
1.5<M<2 p=0.12
2.5<M<3 p=0.44
3.5<M<4 p=0.60
4.5<M<5 p=0.89
5.5<M<6 p=1.07
6.5<M<7 p=0.97
Rate
Time (year)


Omori laws for shocks occurring after 1994 - 2nd declustering technique
1.5<M<2 p=0.63
2.5<M<3 p=0.56
3.5<M<4 p=0.57
4.5<M<5 p=0.90
5.5<M<6 p=0.85
6.5<M<7 p=0.97
Rate
Time (year)



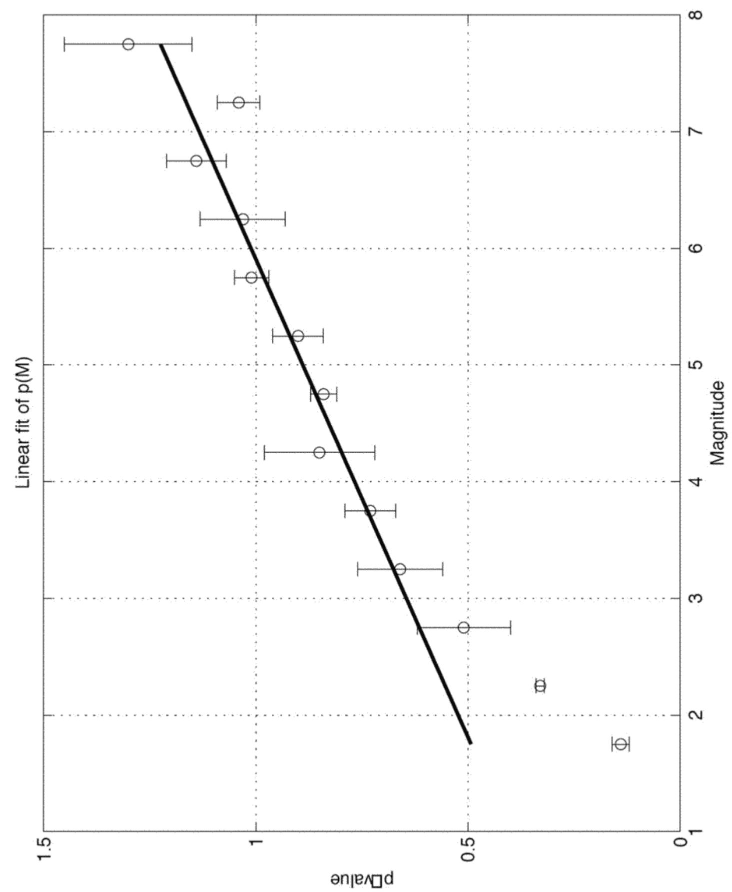
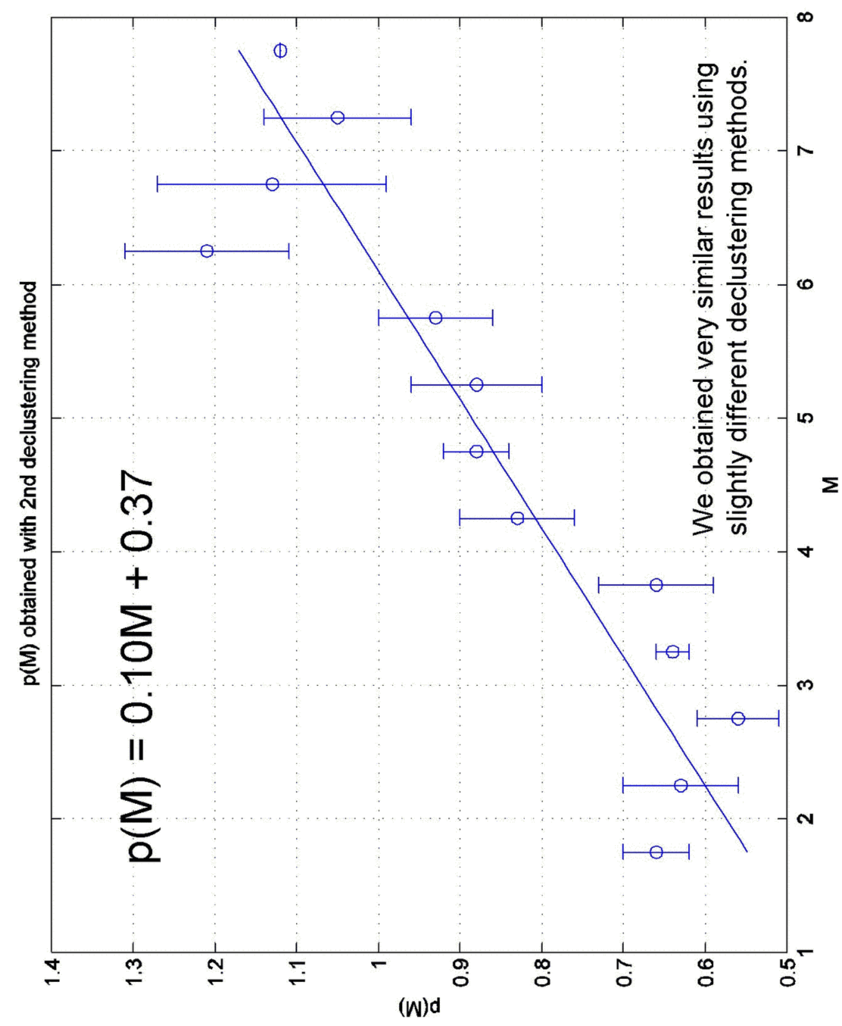
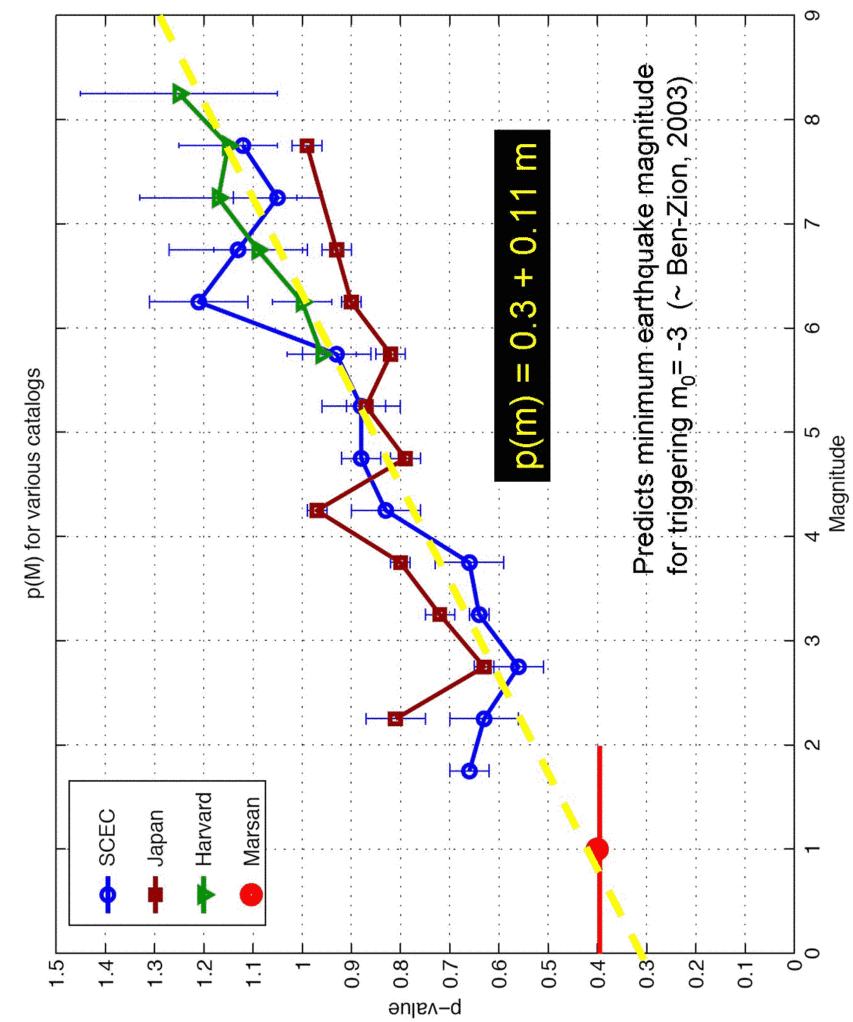


Figure 10. Average p-values and error bars obtained from Figure 9 as described in the text. The straight line is the linear fit with $p(M) = 0.12M_L + 0.28$.

(first declustering method)


p(M) obtained with 2nd declustering method
p(M) = 0.10M + 0.37
We obtained very similar results using
slightly different declustering methods.
M
p(M)


p(M) for various catalogs
p(m) = 0.3 + 0.11 m
Predicts minimum earthquake magnitude
for triggering m_0= -3 (~ Ben-Zion, 2003)
SCEC
Japan
Harvard
Marsan
Magnitude
p−value

Multifractal stress activation (MSA) model:

$$\lambda(\vec{r},t) = \lambda_{\rm tec}(\vec{r},t)\ \exp\left[\beta \int_{-\infty}^{t} d\tau\ s(\vec{r},\tau) h(t-\tau)\right]$$

$$\lambda(t) \ = \ \lambda_{\rm tec} \prod_{i \ | \ t_i<t} \exp\left[\beta s(t_i)\ h(t-t_i)\right]$$

$$\beta s(t_i) h(t-t_i) = \beta s(t_i) h_0\ e^{-t/T}\ .c^{\theta}/(t+c)^{1+\theta}$$

For $\beta s(t_i) h_0$ small, expand the exponential and get

ETAS conditional Poisson intensity:

$$\lambda(t) = \lambda_{\rm tec} + \sum_{i \ | \ t_i<t} \rho_i h(t-t_i)$$

with $\rho_i \equiv \beta s(t_i)$

ETAS = mono-fractal approximation of richer Multifractal model

Epidemic Type Aftershock Sequence (ETAS)

Model proposed by Kagan and Knopoff [1981, 1987] and Ogata [1988]

- each earthquake can be both a mainshock, an aftershock and a foreshock
- each earthquake triggers aftershocks according to the Omori law, that in turn trigger their own aftershocks

$$\phi(t) = \frac{K}{(t+c)^{1+\theta}}$$

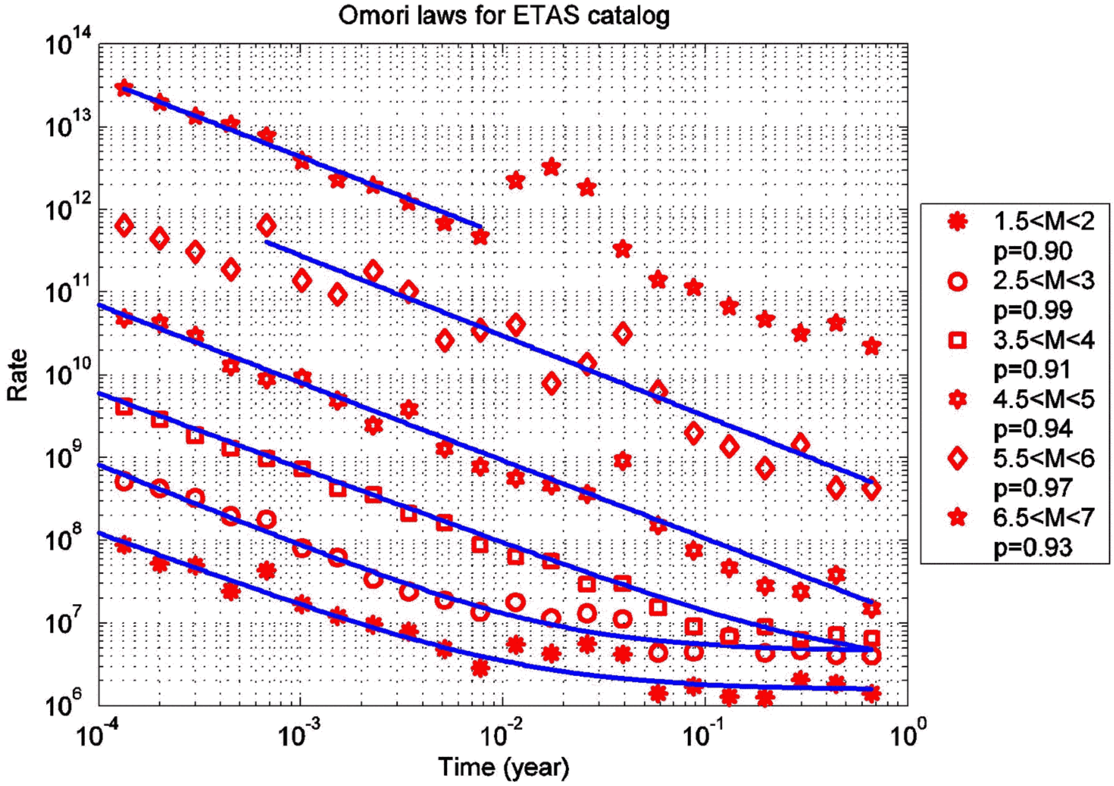
- the number of aftershocks triggered by a mainshock depends on the mainshock magnitude :

$$N(m) \sim 10^{\alpha m}$$

- aftershock magnitudes follow the Gutenberg-Richter distribution, independently of the time and of the mainshock magnitude

$$P(m) \sim 10^{-bm}$$

$$\phi_m(r,t)\ dr\ dt = K\ 10^{\alpha(M-M_0)}\ \frac{\theta\ c^{\theta}\ dt}{(t+c)^{1+\theta}}\ \frac{\mu\ d^{\mu}\ dr}{(r+d)^{1+\mu}}$$



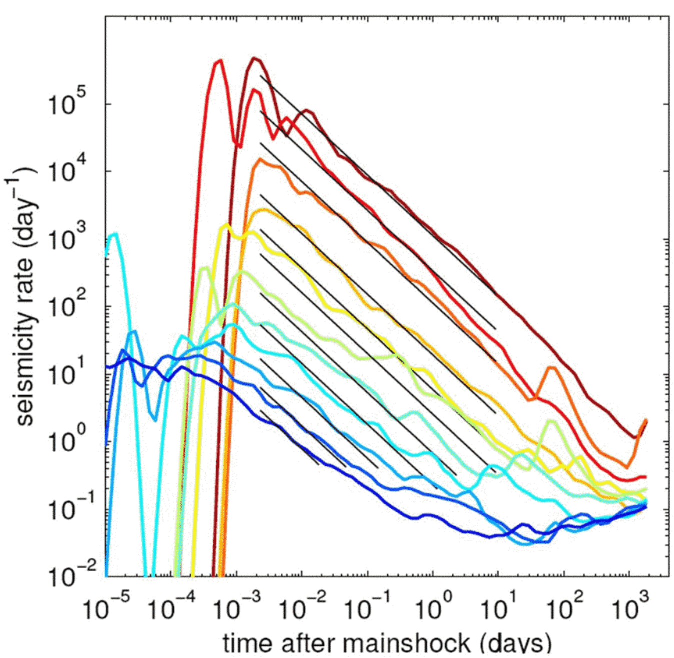


Figure 2. Same as Figure 1 except that we have used $m_d = 2$ and we have corrected the seismicity rate for missing early aftershocks (assuming GR law with $b = 1$). We fit the seismicity rate in the time interval $0.002 < t < 10$ days and for $\lambda(t, m_M) > 0.5$ day^{-1}. The fit of $K(m_M)$ give $K_0 = 0.008$ day^{-1} and $\alpha = 1.01$.

"We found that the rate of triggered events decays with time according to Omori's law 1/(t+c)^p with p=0.9 and c<3 minutes (after correcting for the increase in the magnitude of completeness after a large mainshock). This decay is independent of the mainshock magnitude m for 2<m<7.5."

Helmstetter, Kagan & Jackson, 2005

Arguments supporting our results

- The model predicts that p=aM+b is independent of inverse temperature β. Until now, no clear empirical relationship between p and temperature has ever been presented.
- Bohnenstiehl et al (2003) sum several triggered sequences whatever the magnitude of the mainshock, and note that raising the magnitude threshold of the mainshocks increases the inverted p-value.
- Marsan et al (2003), using all pairs of events in a mine, obtain a global p-value of 0.4 – using our empirical p(M) relationship, this corresponds to a magnitude of 0.3, which is a rather reasonable estimate of the size of mining-induced events.
- Assuming that the mean modulus of stress variation in the area where aftershocks occur is S_0, then the ratio between the number of triggered events in regions of stress increase to the number of triggered events in regions of stress decrease is of order $R=\exp(2\beta S_0 V)$, where V is the activation volume. Considering that R varies from 1.5 (Parsons, 2002) to 10, that S_0 varies from 0.01 to 1MPa and that temperature at seismogenic depth is about 600K, then one can invert for V. We then obtain an activation scale=$V^{1/3}$ of about 1 nanometer, which is in agreement with the microscopic process that is thermal activation.

CONCLUSIONS

•Quantitative generic mechanism for multifractality in geophysics

•Implications for forecasts

$$\mu_{\Delta t}(t,r) = \varepsilon(t,r)\, e^{\omega_{\Delta t}(t,r)}$$

•Spatio-temporal version

$$\omega_{\Delta t}(t) = \mu_{\Delta t} + \int_{-\infty}^{t} d\tau\, \eta(\tau)\, K_{\Delta t}(t-\tau)$$

•Multifractal ETAS model

•Multifractal conditional Poisson model

•Log-gamma multifractal measure: continuous “deformation flow” (deriving GR, Omori and productivity law from multifractality flow)

Towards fulfilling Yan Kagan’s dream:

“IS AN EARTHQUAKE A PHYSICAL ENTITY?”

The Multifractal Randow Walk (MRW) model

$$r_{\Delta t}(t) = \epsilon(t) \cdot \sigma_{\Delta t}(t) = \epsilon(t) \cdot e^{\omega_{\Delta t}(t)}$$

$$\mu_{\Delta t} = \frac{1}{2} \ln(\sigma^2 \Delta t) - C_{\Delta t}(0)$$

$$C_{\Delta t}(\tau) = \mathrm{Cov}[\omega_{\Delta t}(t), \omega_{\Delta t}(t+\tau)] = \lambda^2 \ln\left(\frac{T}{|\tau| + e^{-3/2}\Delta t}\right)$$

$$\omega_{\Delta t}(t) = \mu_{\Delta t} + \int_{-\infty}^{t} d\tau\ \eta(\tau)\ K_{\Delta t}(t-\tau)$$

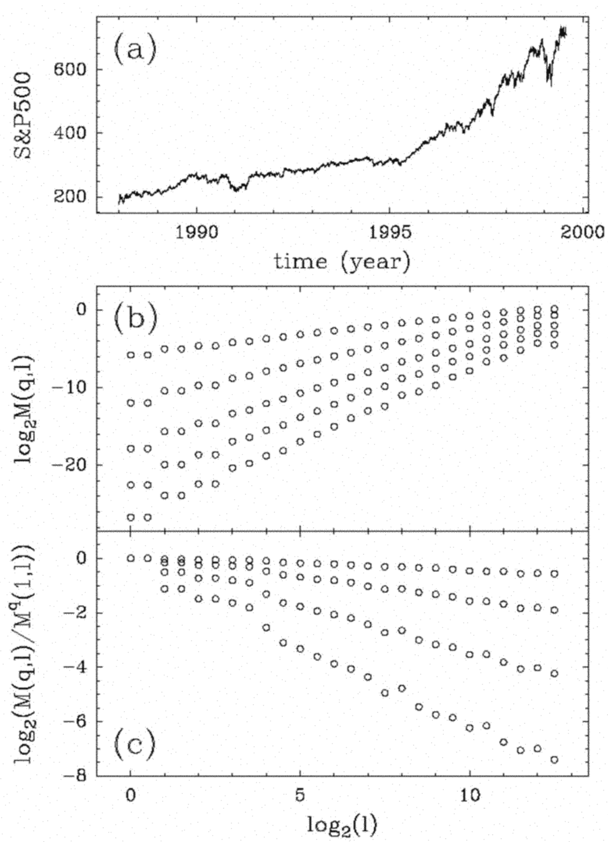
$\omega_{\Delta t}(t)$ is Gaussian with mean $\mu_{\Delta t}$ and variance $V_{\Delta t} = \int_0^\infty d\tau\ K_{\Delta t}^2(\tau) = \lambda^2 \ln\left(\frac{T e^{3/2}}{\Delta t}\right)$

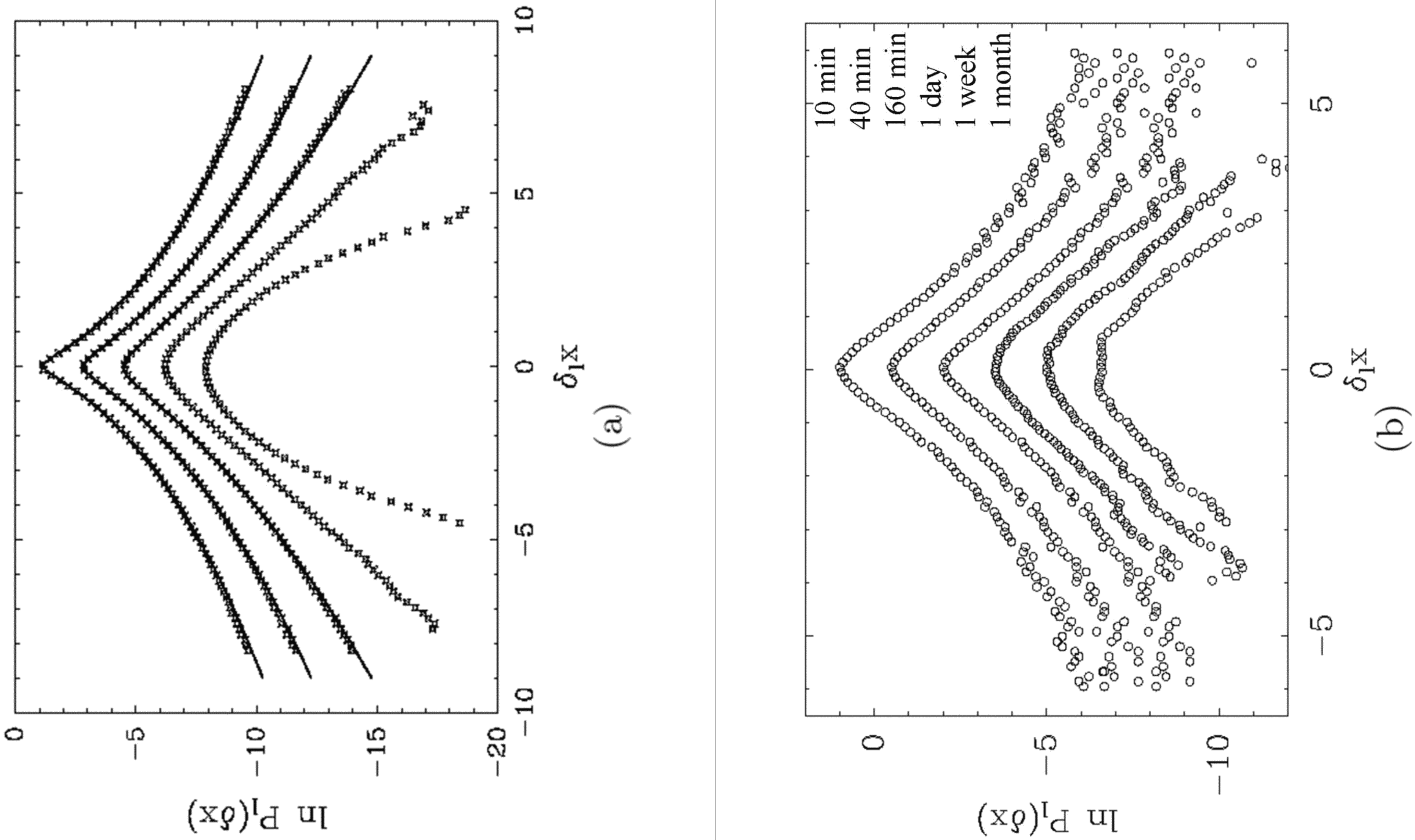
$$C_{\Delta t}(\tau) = \int_0^\infty dt\ K_{\Delta t}(t) K_{\Delta t}(t + |\tau|)$$

$$\hat{K}_{\Delta t}(f)^2 = \hat{C}_{\Delta t}(f) = 2\lambda^2\ f^{-1} \left[\int_0^{Tf} \frac{\sin(t)}{t} dt + O\left(f \Delta t \ln(f \Delta t)\right)\right]$$

$$K_{\Delta t}(\tau) \sim K_0 \sqrt{\frac{\lambda^2 T}{\tau}} \quad \text{for } \Delta t << \tau << T$$

D. Sornette, Y. Malevergne and J.F. Muzy, Volatility fingerprints of large shocks: Endogeneous versus exogeneous, Risk 16 (2), 67-71 (2003) http://arXiv.org/abs/cond-mat/0204626)

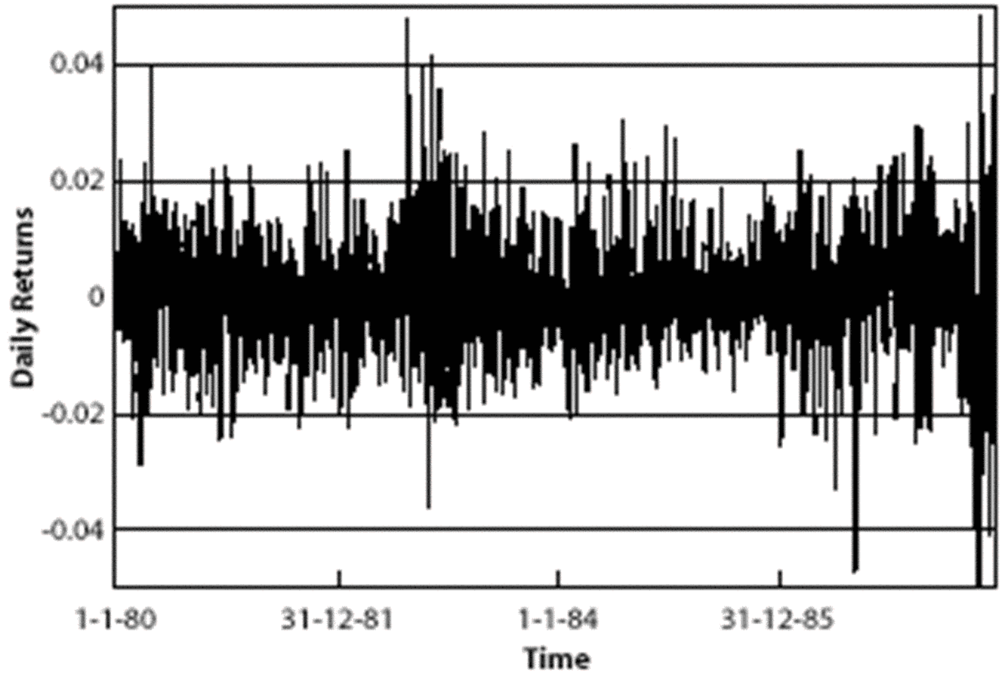




ln P_l(δx)
δ_l x
(a)
10 min
40 min
160 min
1 day
1 week
1 month
ln P_l(δx)
δ_l x
(b)

Volatility

Dow Jones Index Returns Jan. 2nd 1980–Dec.31st 1987



“Conditional response” to an endogeneous shock

$$\mathrm{E}_{\mathrm{endo}}[\sigma^2(t) \mid \omega_0] = \overline{\sigma^2(t)} \exp\left[2(\omega_0-\mu)\cdot\frac{C(t)}{C(0)} - 2\frac{C^2(t)}{C(0)}\right] = \overline{\sigma^2(t)} \left(\frac{T}{t}\right)^{\alpha(s)+\beta(t)}$$

Interplay between
-long memory
-exponential

where

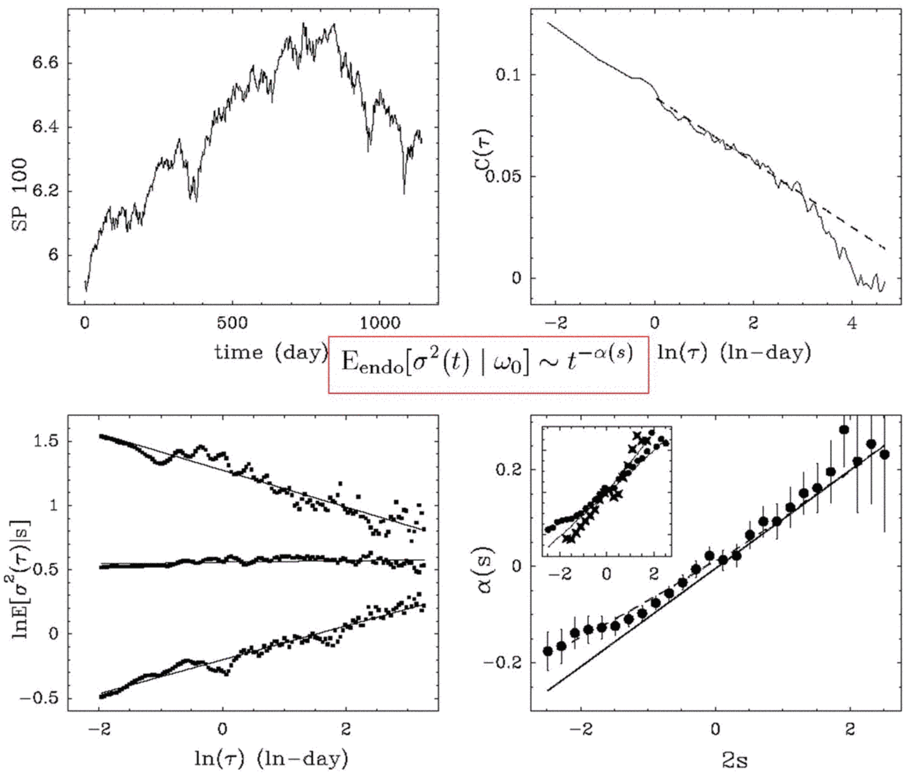
$$\alpha(s) = \frac{2s}{\ln\left(\frac{Te^{3/2}}{\Delta t}\right)},$$

$$\beta(t) = 2\lambda^2 \frac{\ln(t/\Delta t)}{\ln(Te^{3/2}/\Delta t)}$$

Within the range $\Delta t < t << \Delta t e^{\frac{|s|}{\lambda^2}}$, $\beta(t) << \alpha(s)$

$$\mathrm{E}_{\mathrm{endo}}[\sigma^2(t) \mid \omega_0] \sim t^{-\alpha(s)}$$

Real Data and Multifractal Random Walk model



$\mathrm{E_{endo}}[\sigma^2(t) \mid \omega_0] \sim t^{-\alpha(s)}$

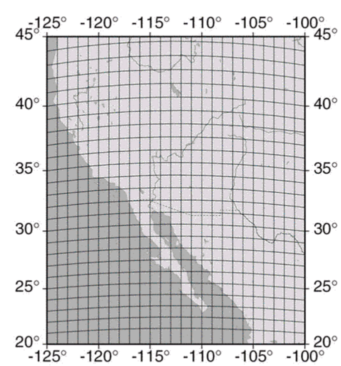
rate of seismic events of magnitude $M > m$ occurring in a cell of size $L \times L$

Monofractal view:

$\lambda(m,L,T) = a\ 10^{-bm}\ L^{c}\ T^{-p}$

Unified Scaling Law for Earthquakes

Bak et al, PRL 2002)



(Lee Skinner)

Multifractal view ("metric"):

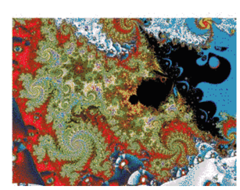
$\lambda_i(m,L,T) = a_i\ 10^{-b_i m}\ L^{c_i}\ T^{-p_i}$

exponents are inter-related

Molchan and Kronrod (2004) have shown that c_i is multifractal

Van Gogh View of turbulence

Multifractal view of fragmentation



Determination of the sources of endogeneous shocks

$W(t) \equiv \int_{-\infty}^{t} d\tau \ \eta(\tau)$, where $\eta(t)$ is a standardized Gaussian white noise

$$\mathrm{E}_{\mathrm{endo}}[W(t) \mid \omega_0] = \frac{\mathrm{Cov}[W(t), \omega_0]}{\mathrm{Var}[\omega_0]} \cdot (\omega_0 - \mathrm{E}[\omega_0]) \propto (\omega_0 - \mathrm{E}[\omega_0]) \int_{-\infty}^{t} d\tau \ K(-\tau)$$

the expected path of the continuous information flow prior to the endogeneous shock (i.e., for $t < 0$) grows like $\Delta W(t) = \eta(t)\Delta t \sim K(-t)\Delta t \sim \Delta t/\sqrt{-t}$

Similar to the expectation of random walk increments conditioned on the knowledge of the fixed values of the two end points