Imperial College London
Claudia de Rham

Causality & Positivity with Gravity

UV Meets the IR
KITP
14th Oct 2020

erc
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Collaborators

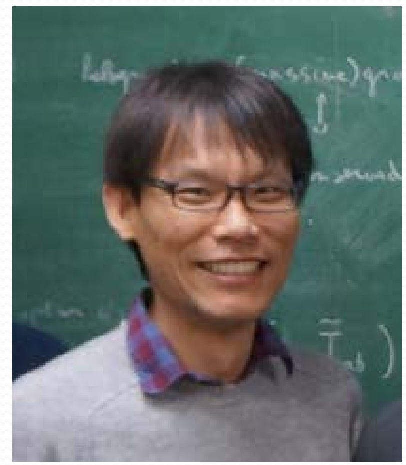
Lasma Alberte
(@ Imperial)

Jeremie Francfort
(@ Geneva)

Sumer Jaitly
(@ Imperial)

Scott Melville
(@ Cambridge)

Andrew Tolley
(@ Imperial)



Jun Zhang
(@ Imperial)

1806.09417, 1909.00881, 2005.13923, 2007.01847, 2007.12667 + to appear

Within low-energy gravitational EFTs,

- Constraints from standard UV completion?
- Constraints from causality?

Non-Gravitational EFT

UV completion

- ✓ Local (Froissart Bound)
- ✓ Unitary (optical theorem)
- ✓ Lorentz invariant (crossing symmetry)
- ✓ CAUSAL (analyticity)

In low-energy Wilsonian EFT

(sub)luminal sound speed

positivity bounds

$$\left.\frac{\mathrm{d}^2\mathcal{A}(s,t)}{\mathrm{d}s^2}\right|_{t=0} > 0$$

$\mathcal{A}$: 2 – 2 elastic scattering amplitude

Adams et. al. 2006

Adding Gravity?

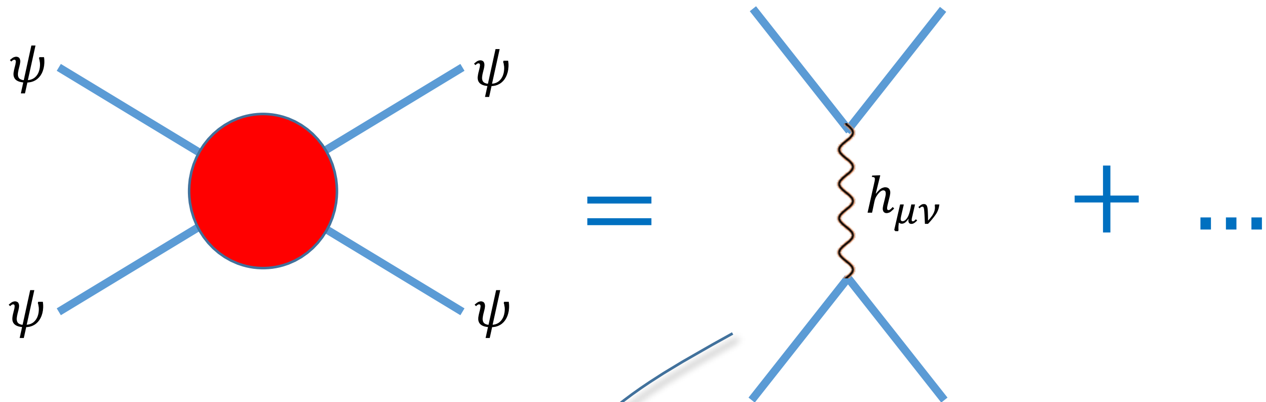
Both of these requirements are more subtle for gravitational EFTs

(sub)luminal sound speed

positivity bounds

Justified for completions of string/Regge higher spin type
Hamada, Noumi & Shiu, 1810.03637

Positivity Bounds in Gravitational LEEFT

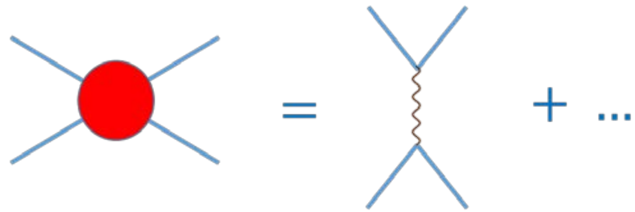


$$\mathcal{A}_{\mathrm{t-channel}} = \# \frac{s^2}{M_{\mathrm{Pl}}^2 t} + \cdots$$

t-channel pole from gravity exchange
compromises positivity bound

$$\left.\frac{\mathrm{d}^2 \mathcal{A}(s,t)}{\mathrm{d}s^2}\right|_{t=0} > 0$$

Positivity Bounds in Gravitational LEEFT



Gravity is non-dynamical in 3d,
upon compactifying 4d→3d× S^1,
contribution from t-channel pole should disappear
Are bounds simply applicable to rest of amplitude?

4d → 3d× S^1 → 4d ⟹ 1902.03250

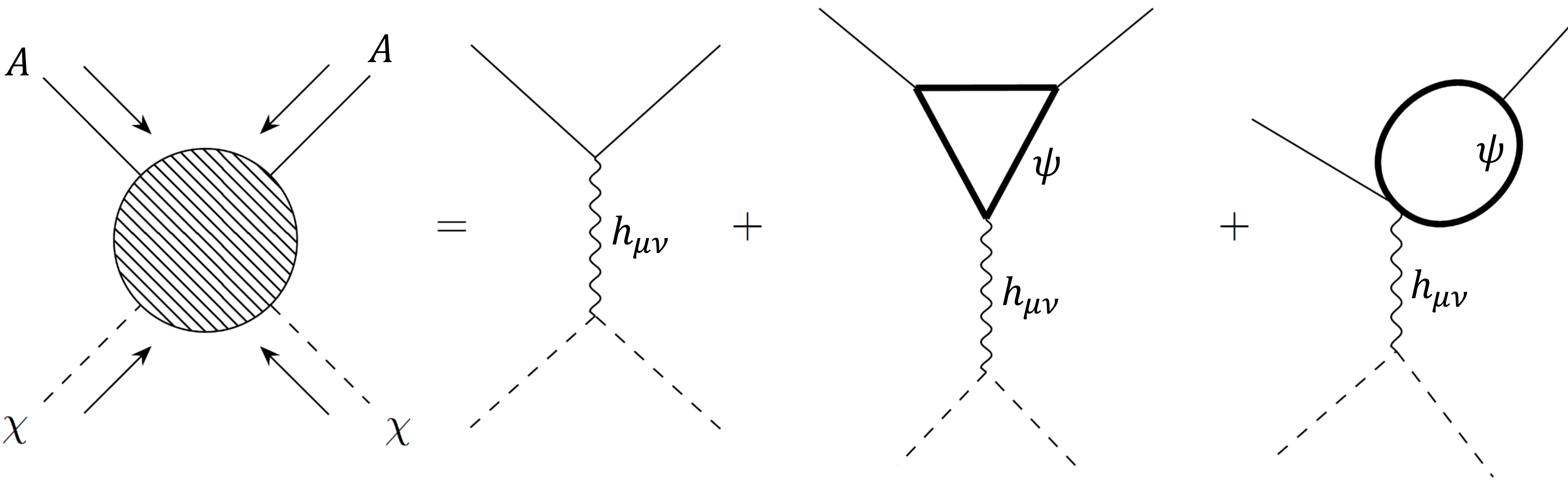
$$\frac{\mathrm{d}^2}{\mathrm{d}s^2}\mathcal{A}^{(4\mathrm{d})}_{\text{t-pole subtracted}}(s,t)\Big|_{t=0} > 0$$

Potential caveats pointed out in Loges, Noumi & Shiu, 1909.01352

Let’s explore the validity of this bound in a specific example with known partial UV completion

Scalar QED with gravity

$$\mathcal{L}_{\rm sQED} = \frac{M_{\rm Pl}^2}{2}R - \frac{1}{2}(\partial A)^2 - \frac{1}{2}(\partial\psi)^2 - \frac{1}{2}M^2\psi^2 - \alpha M A\psi^2$$
$$- \frac{1}{2}(\partial\chi)^2 - \frac{1}{2}m^2\chi^2$$



with Alberte, Jaitly and Tolley 2007.12667

Scalar QED with gravity

$$\mathcal{A}_{\mathrm{sQED}}(s,t) = -\frac{s^2}{M_{\mathrm{Pl}}^2 t} - \frac{\alpha^2 s^2}{90(4\pi)^2 M^2 M_{\mathrm{Pl}}^2} + \mathcal{O}(t^0)$$

1902.03250

$$\boxed{\frac{\mathrm{d}^2}{\mathrm{d}s^2}\mathcal{A}_{\text{t-pole subtracted}}(s,t)\Big|_{t=0} > 0}$$

$$\Rightarrow \quad \frac{\mathrm{d}^2 \mathcal{A}_{\mathrm{sQED,no\,pole}}(s,0)}{\mathrm{d}s^2} = -\frac{2\alpha^2}{90(4\pi)^2 M^2 M_{\mathrm{Pl}}^2} > 0$$

$\longrightarrow$ in contradiction…

Same contradiction for QED minimally coupled to gravity

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Compactified bounds & Scalar QED

1902.03250

$$\frac{d^2}{ds^2}\mathcal{A}_{\text{t-pole subtracted}}(s,t)\Big|_{t=0} > 0$$

in contradiction

(Scalar) QED minimally coupled with QED

- **Either** QED minimally coupled with gravity is not consistent…

would require new interactions between
any massive particles (eg. DM)
and the photon at the scale

$$\Lambda \leq (M_{\text{Pl}} M)^{1/2}$$

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Compactified bounds & Scalar QED

1902.03250

$$\frac{d^2}{ds^2}\mathcal{A}_{\text{t-pole subtracted}}(s,t)\Big|_{t=0} > 0$$

in contradiction

(Scalar) QED minimally coupled with QED

- **Or** 3d compactified bounds are **not** justified

Even though gravity is not dynamical in 3d, the t-channel pole only disappears after Eikonal resummation $\longrightarrow$ leading to an overall delta function

The delta function is the 3d manifestation of 4d pole albeit in a different form

Removing delta function leads to a resulting amplitude $\tilde{\mathcal{A}}$ with $\mathrm{Im}\tilde{\mathcal{A}} \not> 0$

Ciafaloni (1992)

Alternatively amplitude $\tilde{\tilde{\mathcal{A}}}$ can be defined with gravity-redressed states

$\longrightarrow$ compromises crossing symmetry

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Compactified bounds & Scalar QED

1902.03250

$$\left.\frac{\mathrm{d}^2}{\mathrm{d}s^2}\mathcal{A}_{\text{t-pole subtracted}}(s,t)\right|_{t=0} > 0$$

in contradiction

(Scalar) QED minimally coupled with QED

- **Either** QED minimally coupled with gravity is not consistent…
- **Or** 3d compactified bounds are **not** justified

There is no properly defined 3d amplitude which is simultaneously:

- Finite and Analytic
- Has positive Imaginary part
- Enjoys manifest crossing symmetry

Essential for the derivation of the positivity bounds

$\longrightarrow$ t-channel pole affects positivity bounds

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Approximate Positivity

- **Or** 3d compactified bounds are **not** justified

The best we can then argue is that the Positivity bounds ought to be satisfied in a limit $M_{\rm Pl} \to \infty$ where gravity decouples

More precisely, if a 2-2 low-energy elastic scattering amplitude is of the form:

$$\mathcal{A}(s,t) \sim -\frac{s^2}{M_{\rm Pl}^2 t} + \frac{c}{M^4} s^2 + \cdots$$

Then the coupling constant needs not be positive but rather

$$c > -\frac{M^2}{M_{\rm Pl}^2} \times \mathcal{O}(1)$$

Not assuming specific UV behavior

EFT for Gravity

$$\mathcal{L}_{\mathrm{EFT\ GR}} = \sqrt{-g}\left[\frac{M_{\mathrm{Pl}}^2}{2}R + \mathcal{L}^{(\mathrm{light})}(g,\psi) + C_{R^2}R^2 + C_{W^2}W_{\mu\nu\alpha\beta}^2 + \dots\right]$$

EFT for Gravity

$$\mathcal{L}_{\text{EFT GR}} = \sqrt{-g}\left[\frac{M_{\text{Pl}}^2}{2}R + \mathcal{L}^{(\text{light})}(g,\psi) + C_{R^2}R^2 + C_{W^2}W^2_{\mu\nu\alpha\beta} + \dots\right]$$

All the light fields at low-energy
(e.g. including photon)

Consider these fields to be minimally coupled
In this frame, light travels at the speed of light $c = 1$ in the vacuum

Curvature-square operators can be removed by field redefinition
at the price of including non-minimal couplings to light fields

Respective causal structure remains the same,
just shifts the question somewhere else

EFT for Gravity

$$\mathcal{L}_{\mathrm{EFT\ GR}} = \sqrt{-g}\left[\frac{M_{\mathrm{Pl}}^2}{2}R + \mathcal{L}^{(\mathrm{light})}(g,\psi) + C_{R^2}R^2 + C_{W^2}W^2_{\mu\nu\alpha\beta} + \dots\right]$$

Consider tensor fluctuations on FLRW,

$$\mathrm{d}s^2 = a^2(\eta)\eta_{\mu\nu}\mathrm{d}x^\mu\mathrm{d}x^\nu + a\,h_{ij}\mathrm{d}x^i\mathrm{d}x^j$$

$$\left[-\partial_\eta^2 + \left(1 - \frac{16C_{W^2}\dot{H}}{M_{\mathrm{Pl}}^2}\right)\nabla^2\right]\tilde{h} = m_0^2\tilde{h}$$

with Tolley 1909.00881

EFT for Gravity

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$$c_s^2 = 1 + \frac{16 C_{W^2}(-\dot{H})}{M_{\text{Pl}}^2} + \mathcal{O}\left(\frac{H^4}{M_{\text{Pl}}^4}, \frac{k^4 H^2}{M_{\text{Pl}}^6}\right)$$

with Tolley 1909.00881

Speed of Gravity

Within the regime of validity of the EFT,

$$c_s^2 = 1 + \frac{16C_{W^2}(-\dot{H})}{M_{\mathrm{Pl}}^2} + \mathcal{O}\left(\frac{H^4}{M_{\mathrm{Pl}}^4}, \frac{k^4 H^2}{M_{\mathrm{Pl}}^6}\right)$$

- For a maximally symmetric spacetime $\dot{H} = 0$, modes are luminal

- We expect $C_{W^2} \sim \mathcal{O}(1) \quad \Rightarrow \quad |\Delta c_S| \ll \frac{H^2}{M_{\mathrm{Pl}}^2} \sim 10^{-120}$

Speed of Gravity

Within the regime of validity of the EFT,

$$c_s^2 = 1 + \frac{16 C_{W^2}(-\dot{H})}{M_{\rm Pl}^2} + \mathcal{O}\left(\frac{H^4}{M_{\rm Pl}^4}, \frac{k^4 H^2}{M_{\rm Pl}^6}\right)$$

If $\dot{H} \neq 0$ and NEC is satisfied, $\dot{H} < 0$ modes are

$$\begin{array}{rcl} \text{subluminal} & \Leftrightarrow & C_{W^2} < 0 \\ \textit{super}\text{luminal} & \Leftrightarrow & C_{W^2} > 0 \end{array}$$

Does it mean that the low-energy EFT is only consistent if $C_{W^2} < 0$??

Speed of Gravity

$$\mathcal{L}_{\text{EFT GR}} = \sqrt{-g}\left[\frac{M_{\rm Pl}^2}{2}R + \mathcal{L}^{(\text{light})}(g,\psi) + C_{R^2}R^2 + C_{W^2}W^2_{\mu\nu\alpha\beta} + \dots\right]$$

$$\begin{aligned} \text{subluminal} \quad &\Leftrightarrow \quad C_{W^2} < 0 \\ \textit{super}\text{luminal} \quad &\Leftrightarrow \quad C_{W^2} > 0 \end{aligned}$$

From a field theory perspective the constraints on enjoying a standard causal high energy completion are (so far) simply

$$C_{W^2} > -\mathcal{O}\left(\frac{M^2}{M_{\rm Pl}^2}\right)$$

How is this consistent with causality within the low-energy EFT???

Causality

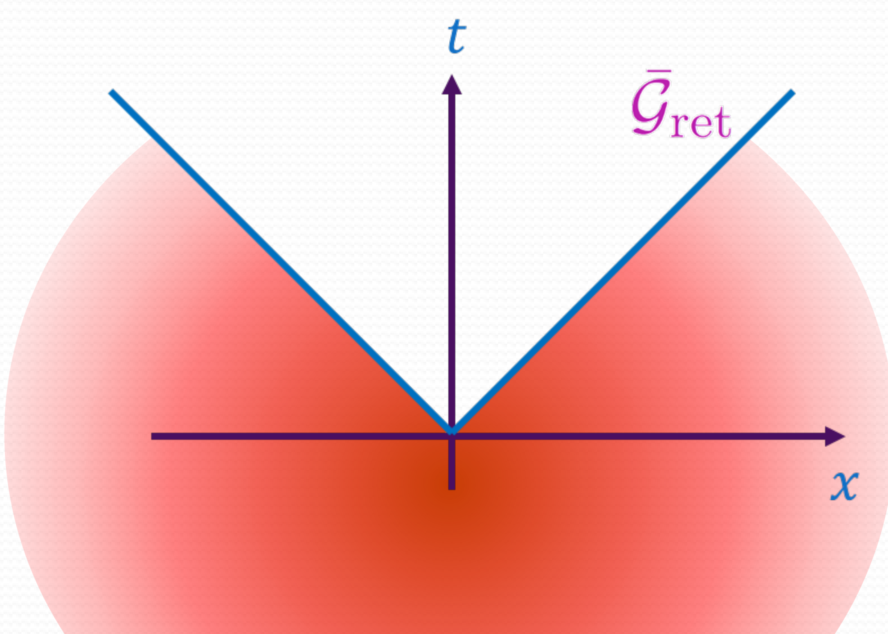
The physical speed of propagation is given by the front velocity:

$$v_{\text{front}} = \lim_{k\to\infty} v_{\text{phase}}(k)$$

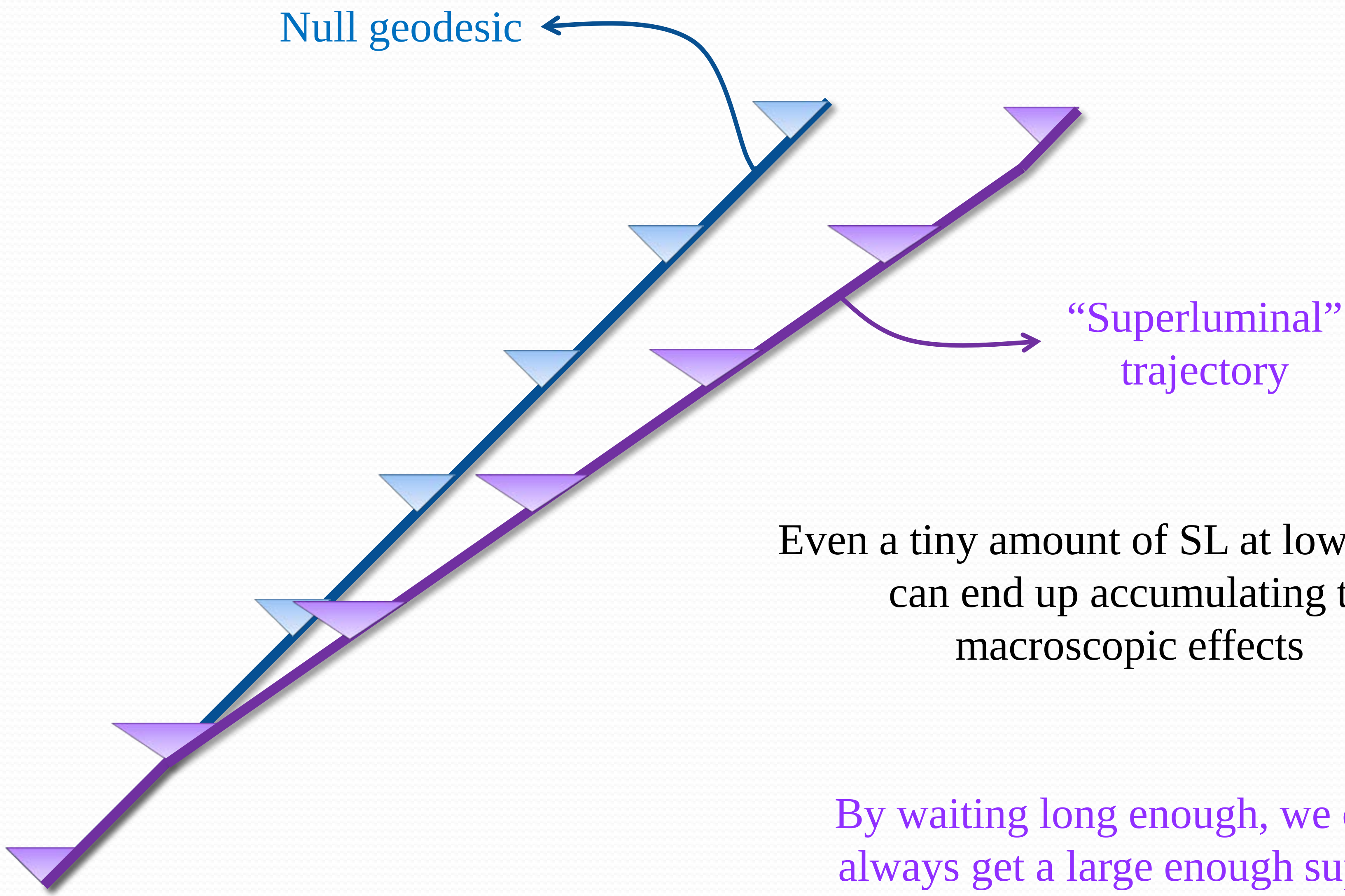
But causality itself requires that the

retarded propagator vanishes outside the light-cone

which typically requires (sub)luminality even at low-energy



Support Outside Light-Cone



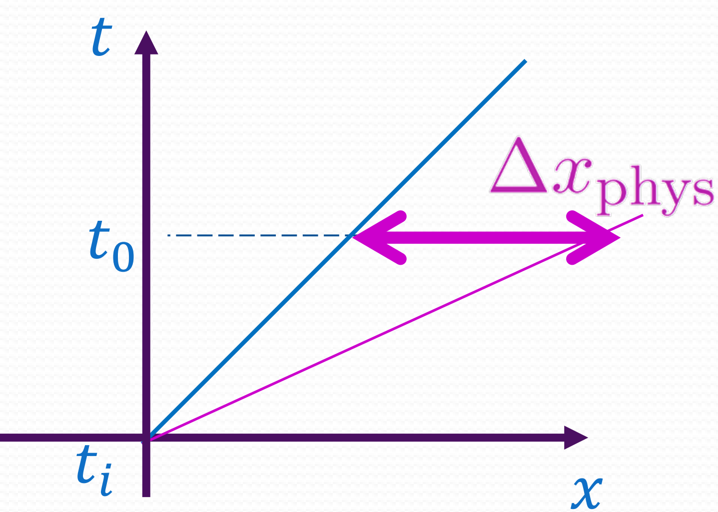
Even a tiny amount of SL at low-energy can end up accumulating to macroscopic effects

By waiting long enough, we could always get a large enough support outside the light-cone…

Support Outside Light-Cone

EFT has a cutoff $M \leq M_{\rm Pl}$

For any mode, with physical frequency k, one can only trust EFT so long as $\Box_{\rm FLRW} \sim \Box_{\rm Minkowski} + \frac{kH}{a} \ll M_{\rm Pl}^2$



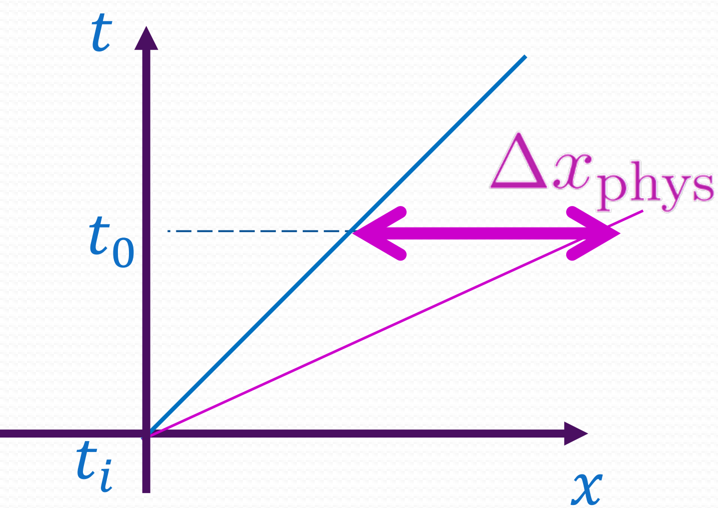
Cannot send a mode with arbitrarily small wavelength

$$\lambda_i \gg \frac{H_i}{M_{\rm Pl}^2}$$

$$\Delta x_{\rm phys} = a_0 \int_{\eta_i}^{\eta_0} \Delta c_S {\rm d}\eta \, = \, a_0 \int_{t_i}^{t_0} \frac{-\dot{H}}{a M_{\rm Pl}^2} {\rm d}t < \frac{a_0}{a_i} \frac{H_i}{M_{\rm Pl}^2} \ll \lambda_{\rm phys}$$

Support Outside Light-Cone

$$\Delta x_{\rm phys} \ll \lambda_{\rm phys}(t_0)$$



There is never support outside the light cone by a resolvable amount within the regime of validity of the EFT

$\longrightarrow$ No violation of causality

The amount of superluminality is so small that it can never build up to lead to macroscopic violation of causality.

with Tolley 2007.01847

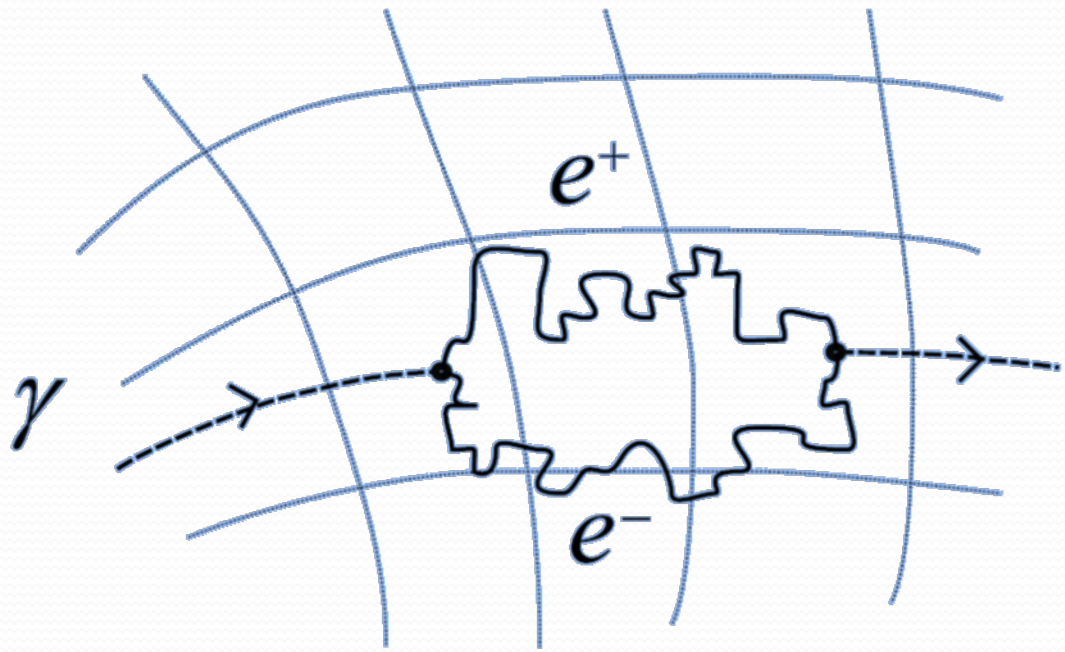
QED on curved spacetime

Drummond & Hathrell, PRD 1980
Hollowood & Shore 0707.2302, 0707.2303, 0806.1019, 0905.0771, 1006.0145, 1006.1238, 1111.3174, 1205.3291, 1512.04952
Goon & Hinterbichler, 1609.00723

M: electron mass

$$\mathcal{L} = \sqrt{-g}\left(-\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{\alpha}{M^2}R_{abcd}F^{ab}F^{cd} + \ldots\right)$$

As the photon propagates, it interacts with virtual electron pairs
$\longrightarrow$ feels the curvature in region around its geodesic



From Hollowood & Shore

QED on curved spacetime

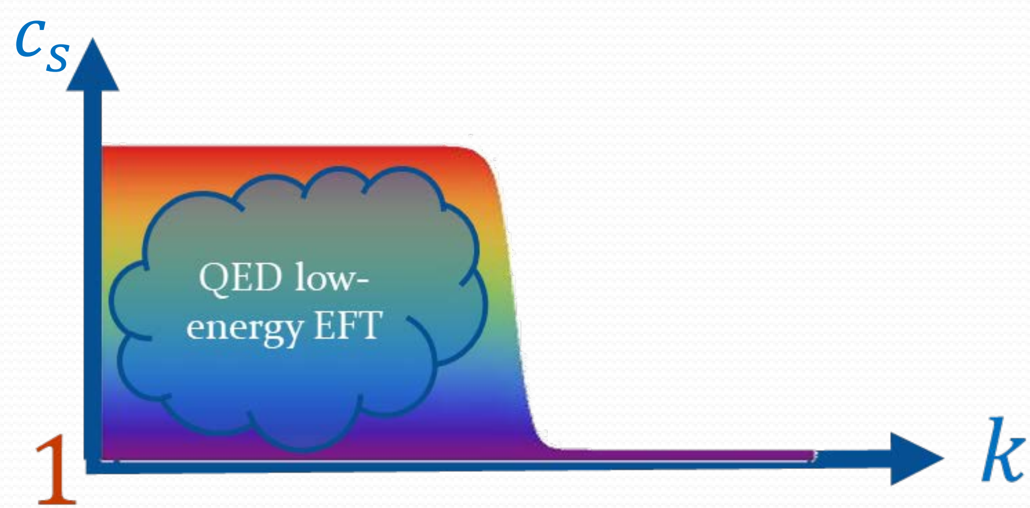
There are many space-time backgrounds for which the low-energy group velocity is superluminal. Eg. Schwarzschild, Type I & II conformally flat backgrounds, …

E.g. on Schwarzschild, $$c_s^2 = 1 + \frac{\beta_P}{M^2}\frac{r_g}{r^3} + \mathcal{O}\left(\frac{r_g^2}{M^4 r^6}\right) + \mathcal{O}\left(\frac{k^4}{M^4}\right)$$

M: electron mass

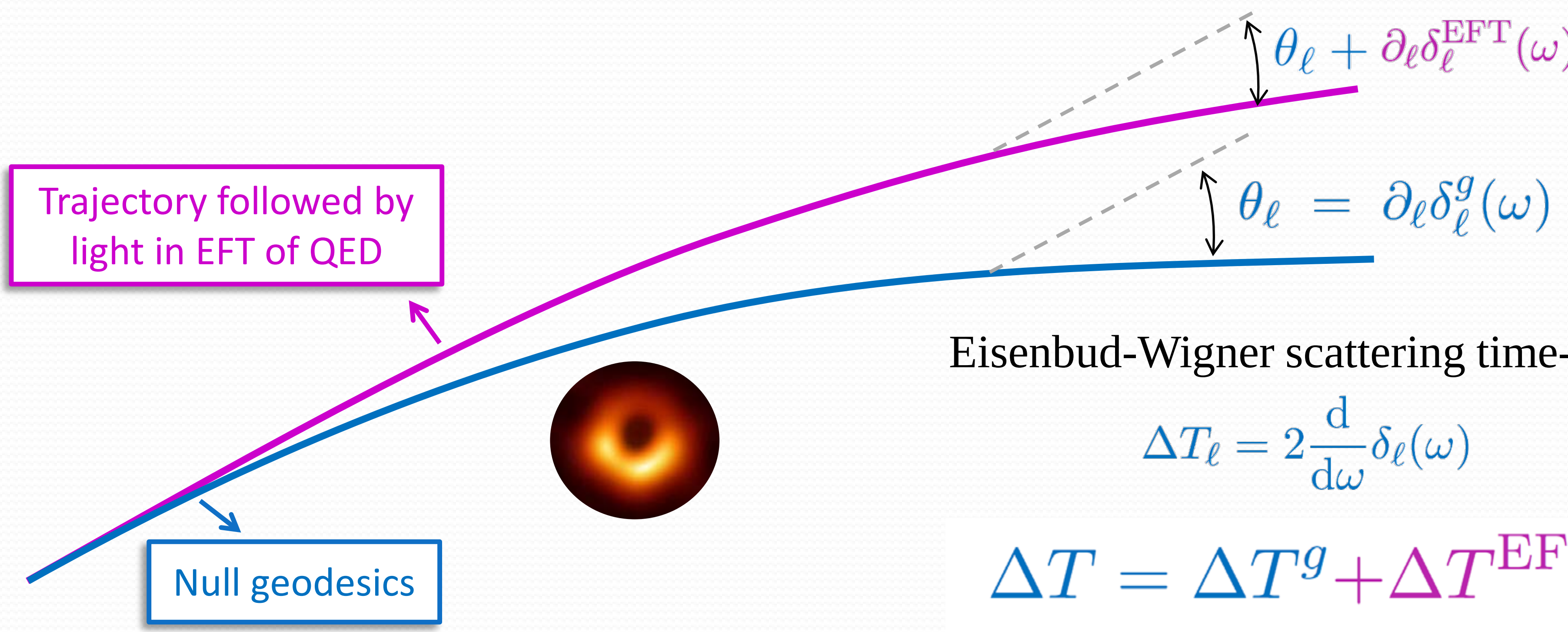
$\beta_P \sim O(1)$ - polarization dependent constant

$\beta_P > 0$ for radially polarized light



Low-energy `superluminality' is precisely related to (non)-positivity bounds

Time-delay/advance



Eisenbud-Wigner scattering time-delay:

$$\Delta T_\ell = 2\frac{\mathrm{d}}{\mathrm{d}\omega}\delta_\ell(\omega)$$

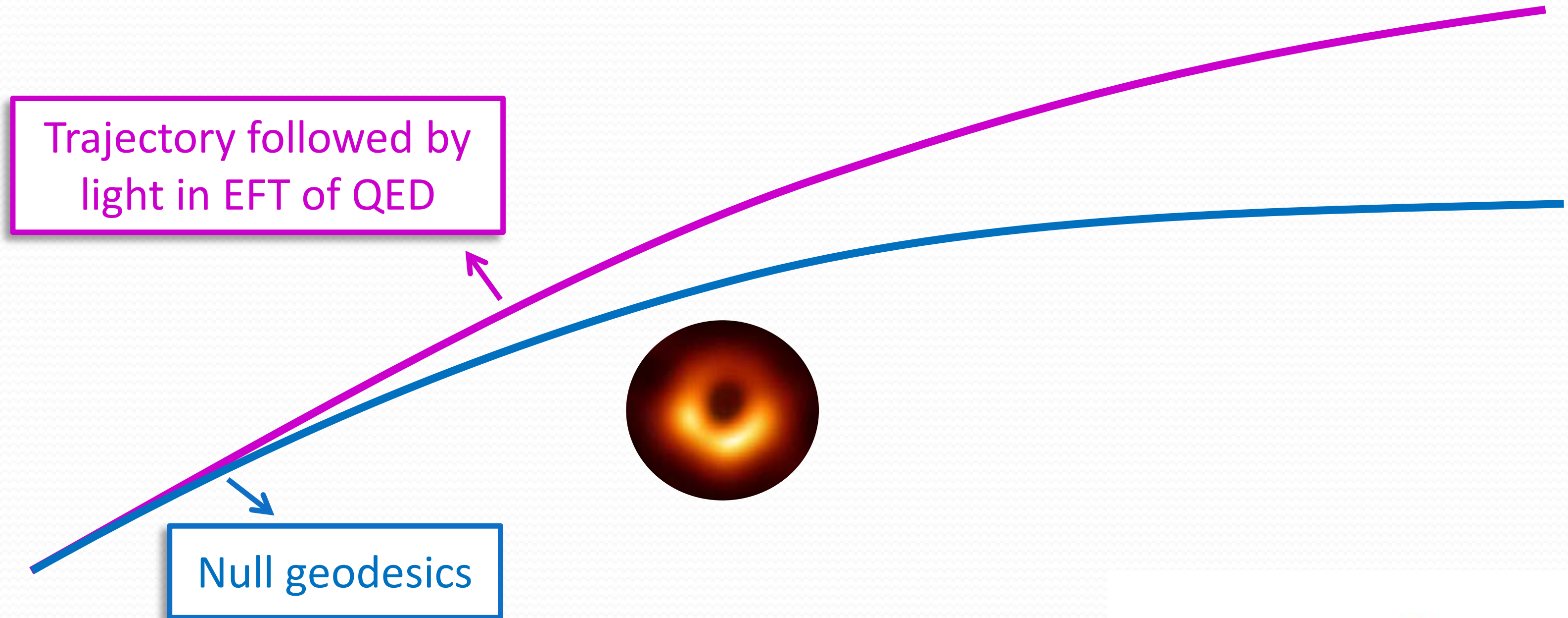
$$\Delta T = \Delta T^g + \Delta T^{\mathrm{EFT}}$$

It was previously argued that while
$\Delta T^{\mathrm{EFT}} < 0$ we still have $\Delta T > 0$
in the regime of validity of the EFT

but that's not enough…

eg see Camanho, Edelstein, Maldacena & Zhiboedov, 1407.5597

Time-delay/advance



in extreme scenario $\Delta c_s^2 \sim \dfrac{\beta_P}{M^2 r_g^2}$

$$\Delta T^{\rm EFT} \sim -\frac{\beta_P}{M^2 r_g} < 0 \quad \text{for} \quad \beta_P > 0$$

Regime of Validity of EFT

The low-energy EFT is only valid below the scale M
Above that scale one should go back to the microscopic description

wave of null momentum $k_\mu = (\omega, \dots)$
in a curved background with Weyl tensor $W_{\mu\nu\alpha\beta}$

$$|W^p| \ll M^{2p}$$

For null momenta this cannot put any direct bound on k_μ nor on ω (ω is not a Lorentz scalar)

However to be within the regime of validity of the EFT other invariants ought to be bounded
E.g. any invariant of the form

$$\left|\left(W^a{}_{bcd}k^c k^d\right)^p\right| \ll M^{4p}$$

Regime of Validity of EFT

The low-energy EFT is only valid below the scale *M*

Above that scale one should go back to the microscopic description

$$\left|\left(W^a{}_{bcd}k^c k^d\right)^p\right| \ll M^{4p}$$

In the extreme scenario $\boxed{\frac{\omega^2}{r_g^2} \ll M^4 \quad \Rightarrow \quad \frac{1}{M^2 r_g} \ll \lambda}$

ω: asymptotic energy of the scattering particle

with Tolley 2007.01847

Causality in Gravitational Theories

$$\Delta T^{\mathrm{EFT}} \sim -\frac{\beta_P}{M^2 r_g} < 0 \quad \text{for} \quad \beta_P > 0$$

$$\boxed{\frac{\omega^2}{r_g^2} \ll M^4 \quad \Rightarrow \quad \frac{1}{M^2 r_g} \ll \lambda}$$

$$\left|\Delta T^{\mathrm{EFT}}\right| \sim \frac{\mathcal{O}(1)}{M^2 r_g} \ll \lambda$$

Amount of SL is small enough not to lead to
any resolvable time advance
(as it should be)

with Tolley 2007.01847

Causality in Gravitational Theories

Conjecture: In a frame where gravity can be decoupled,
a small amount of SL at low-energy
is still consistent with causality so long as

$$\lim_{M_{\mathrm{Pl}}\to\infty} |c_s^2 - 1| \sim M_{\mathrm{Pl}}^{-\alpha} \quad \text{with} \quad \alpha \geq 2$$

with Tolley 2007.01847

Time-advance

In the EFT of gravity or QED, the time advance due to SL is always unresolvable

$$\left|\Delta T_\ell^{\mathrm{EFT}}\right| \ll \omega^{-1}$$

The time advance is smaller than the geometric optics resolution scale
it is not resolvable

This is a very different statement than

$$\left|\Delta T_\ell^{\mathrm{EFT}}\right| \ll M^{-1}$$

While this relation is also true it is not relevant:

1. The low-energy EFT is only used to determine the trajectory, Nothing demands that the time advance should be measured with apparatus that live in the low-energy EFT

2. The time delay is not a Lorentz invariant quantity so one cannot use M^{-1} as its cutoff

No support outside the light-cone

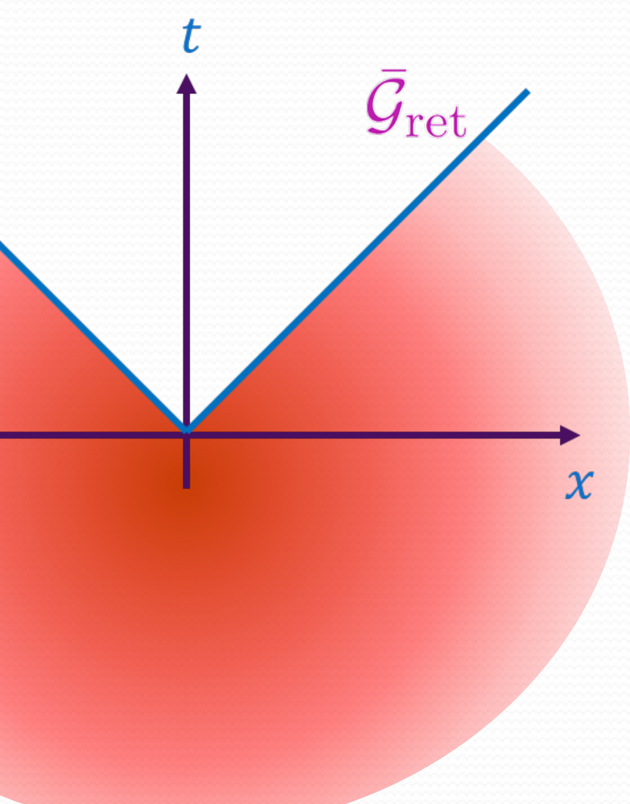
If $|\Delta T^{\rm EFT}| \ll \omega^{-1}$, its sign cannot be directly linked with causality

No support outside the light-cone

If $|\Delta T^{\rm EFT}| \ll \omega^{-1}$, there are is no dangerous growth of secular effects

Retarded Green's function can be computed perturbatively

There can be no support outside the light-cone



$$
\begin{aligned}
\bar{\Box}\bar{\mathcal{G}}_{\rm ret} &= \delta \\
\left(\bar{\Box} + \Delta\mathcal{O}_{\rm EFT}\right)\mathcal{G}_{\rm EFT} &= \delta
\end{aligned}
$$

A perturbative approach $\mathcal{G}_{\rm EFT} = \bar{\mathcal{G}}_{\rm ret}\left(1 + \Delta\mathcal{G} + \cdots\right)$ is justified if the secular effects are bounded

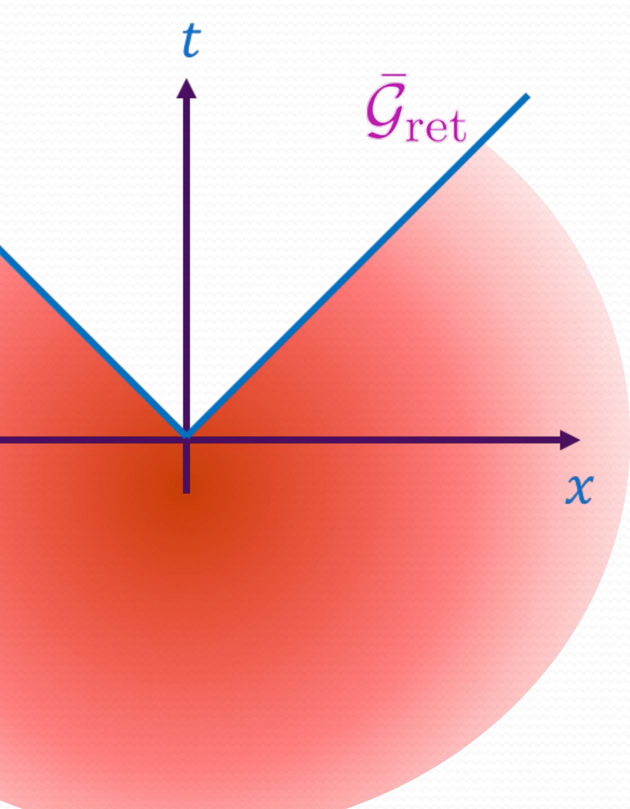
$$|\Delta\mathcal{G}| \ll 1 \text{ with } \Delta\mathcal{G} \sim \int \Delta\mathcal{O}_{\rm EFT}\bar{\mathcal{G}}_{\rm ret}$$

No support outside the light-cone

If $|\Delta T^{\mathrm{EFT}}| \ll \omega^{-1}$, there are is no dangerous growth of secular effects

Retarded Green's function can be computed perturbatively

There can be no support outside the light-cone



Note: in practice we can replace

$$\Delta T^{\mathrm{EFT}} \longleftrightarrow \partial_\omega \delta^{\mathrm{EFT}}(\omega)$$

$$|\Delta T^{\mathrm{EFT}}| \ll \omega^{-1} \longleftrightarrow |\delta^{\mathrm{EFT}}| \ll 1$$

Living with Superluminality

- Gravitational Waves are luminal to a (VERY) good accuracy at LIGO frequencies $-\mathcal{O}\left(10^{-15}\right) < c_T - 1 < \mathcal{O}\left(10^{-16}\right)$

- Within the standard EFT of gravity, GWs are no longer perfectly luminal on backgrounds that spontaneously break Lorentz invariance (eg Schwarzschild, FLRW, the real world,…)

Lesson 1:

- In an arbitrary frame, GWs may be superluminal
- Imposing subluminality priors only makes sense in a frame where gravity can be decoupled
- In the original frame this may correspond to GWs being superluminal by a 'large' amount (not suppressed by M_{pl}^{-2})

Living with Superluminality

Lesson 2:

- Even in the frame where matter and gravity can decouple, a tiny amount of SL *or a negative phase shift* – be it for GWs or other fields – is not in conflict with causality. It may even follow from consistent causal and Lorentz invariant UV completions.

- In the frame where matter and gravity can decouple, superluminality is consistent with causality so long as

$$\boxed{\lim_{M_{\rm Pl}\to\infty} |c_s^2 - 1| \sim M_{\rm Pl}^{-\alpha} \quad \text{with} \quad \alpha \geq 2}$$

Living with Negativity

Lesson 3: Conjecture

- For a 2 – 2 scattering amplitude of the form

$$\mathcal{A}(s,t) \sim -\frac{s^2}{M_{\rm Pl}^2 t} + \frac{c}{M^4}s^2 + \ldots$$

- c needs not be positive so long as

$$\boxed{c > -\frac{M^2}{M_{\rm Pl}^2} \times \mathcal{O}(1)}$$

Not assuming specific UV behavior

Amount of “positivity”-violation directly connected to “allowed” amount of SL