Obstacles to Constructing De Sitter Space in String Theory

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Michael Dine, with Jamie Law-Smith, Shijun Sun, Yan Yu, Duncan Wood

Department of Physics
University of California, Santa Cruz.

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While string theory has provided tools to think about many questions in quantum gravity, cosmologies resembling our own remain inaccessible to controlled approximations in the theory. Conceivably the observed big bang is not described by a quantum theory of gravity or requires some still larger structure, but it would seem more likely that this simply represents a failure of our present collection of theoretical tools.

Our universe appears to have entered a stage of exponential expansion, well-described as a de Sitter solution of Einstein's equations. At a time shortly after the big bang, there is good reason to think that the universe also went through a period of exponential expansion. So de Sitter space seems likely to play an important role in any understanding of our present and past universe. The inflationary period lasted only for a brief moment; our limited understanding of how de Sitter space might arise in string theory would suggest that even our present de Sitter universe is metastable.

The notion of a *cosmic landscape* introduces another role for spaces of positive cosmological constant (c.c.). In particular, such a landscape might allow a realization of anthropic selection of the c.c. , but would seem to require the existence of a vast set of metastable, positive c.c. vacua.

The De Sitter Swampland Conjecture

It has proven difficult to find explicit constructions of metastable de Sitter space in string theory, and this led Obied, Ooguri, Hirosi and Spodyneiko and Vafa to conjecture that that metastable de Sitter space lies in the swampland of quantum gravity. If true, this has potentially dramatic implications, the nature of the currently observed dark energy, and implementing the anthropic explanation of the c.c. Rather than address the implications, though, we'll look at the starting point for the conjecture.

One should first ask: what would it mean to construct de Sitter space in string theory? In most constructions, one starts with some classical solution of the equations of critical string theory. These solutions invariably have moduli or pseudomoduli. Then one adds features, such as fluxes, branes, and orientifold planes which give rise to a potential for these moduli, and looks for a local minimum with positive four-dimensional c.c.

These attempts to construct de Sitter space generally raise two questions. First, what is the approximation scheme that might justify any such construction? Second, any would-be de Sitter space found in this way is necessarily, at best, metastable: inevitably there is a lower energy density in asymptotic regions of the original moduli space. Quantum mechanically, the purported de Sitter state cannot be eternal. It has a history; it will decay in the future and must have been created by some mechanism in the past. The quantum mechanics of this process is challenging to pin down.

In this talk, we will argue that already classically, the notion of an eternal de Sitter space in string theory is problematic; small perturbations near the de Sitter stationary point of the effective action evolve to singular cosmologies.

Two challenges to the search for metastable de Sitter space in string theory

(1) One requires a small parameter(s) allowing a controlled approximation to finding stationary points of an effective action. Here one runs into the longstanding problem that without introducing additional, fixed parameters (i.e., introducing parameters not determined by moduli), would-be stationary points in the potential for the moduli lie at strong coupling.

Typically, attacks on this problem (and the question of de Sitter space) exploit large fluxes (I'll discuss KKLT later.). If there is to be a systematic approximation, it is necessary that the string coupling be small and compactification radii large at any would-be stationary point found in this way. If the strategy is to obtain inverse couplings and radii scaled by some power of fluxes, it is also important that these fluxes (and possibly other discrete parameters) can be taken arbitrarily large, without spoiling the effective action treatment. Even allowing uncritically for this latter possibility, we will see that it is quite challenging to realize arbitrarily weak string coupling and large radius, with positive or *negative* c.c.This point has been noted by Junghans, Wrase and others.

(2) If one finds such a stationary point, one must ask about stability, beyond the requirement that the quadratic fluctuation operator have a positive definite spectrum. De Sitter space introduces new elements into the problem.

In string theory, we are used to searching for suitable background geometries and field configurations by requiring that the evolution of excitations about these configurations is described by a unitary S matrix. Classically, at least in a flat background, this is the statement that any initial perturbation of the system has a sensible evolution to some final perturbation. Again, we will see that this requirement is problematic for any would-be classical de Sitter stationary point in such a theory; even if all eigenvalues of the mass-squared matrix (small fluctuation operator) are positive, large classes of small perturbations evolve to singular geometries.

Overall, then, we will argue that we lack theoretical methods to address, in any systematic fashion, the problem of constructing de Sitter space in string theory, much as we lack the tools to understand big bang or big crunch singularities in any controlled approximation. The existence of metastable de Sitter states may be plausible or not, but it is a matter of speculation.The failure to find such states in any controlled analysis appears, at least at present, inevitable.

Searching for Stationary Points of an Effective Action

After introducing branes and fluxes, typically one searches for particular stationary points of the action with positive cosmological constant, and asks whether the string coupling is small and the compactification radii large at these points (e.g. work of Andriot, Wrase).

Even if one succeeds in finding particular solutions with numerically small couplings and large compactification radii, this, by itself, does not address the question of whether there is a systematic approximation. The system with branes and fluxes is not a small perturbation of the system without, and the range of validity of the expansion in one is not related to that of the other. If there is to be a systematic approximation of any sort, one requires a sequence of such stationary points as one increases the flux numbers; the would-be small parameters are the inverse of some large flux numbers. For this discussion we will assume that it makes sense to take such numbers arbitrarily large.

The goal is to find stable, stationary points of the action where

1. The string coupling is small.
2. All compactification radii are large.
3. The cosmological constant is small and positive.

Study Type II theories in the presence of an O_p plane (following Andriot), and background geometry with metric

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu + \rho \, g^0_{IJ} \, dy^I dy^J. \tag{1}$$

Here g^0_{IJ} represents a background reference metric for the compactified dimensions. $g_{\mu\nu}$ represents the metric of four dimensional space-time, which we hope to be de Sitter. We also include NS-NS 3-form and R-R q-form fluxes, $H^{(n)}_{IJK}, \; F^{(n)}_q$.

We focus on the two moduli: ρ and $\tau = \rho^{3/2} \, e^{-\phi}$ where ϕ is the dilaton.

The resulting action is:

$$-\tau^{-2}\left(\rho^{-1}R_6(\sigma)-\frac{1}{2}\rho^{-3}\sum_n \sigma^{6n-3(p-3)}|H^{(n)}|^2\right)$$

$$-\tau^{-3}\rho^{\frac{p-6}{2}}\sigma^{\frac{(p-3)(p-9)}{2}}\frac{T_{10}}{p+1}$$

$$+\frac{1}{2}(\tau^{-4}\sum_{q=0}^{4}\rho^{3-q}\sum_n \sigma^{6n-q(p-3)}F_q^{(n)2}+\frac{1}{2}\tau^{-4}\rho^{-2}\sum_n \sigma^{6n-5(p-3)}F_5^{(n)2})$$

Call the fluxes $F_i = n_i$, $H_3 = N_3$. To illustrate the issues, we'll take $n_4 \gg n_2 \gg N_3$. For $3 \leq p \leq 7$, solving for the minimum:

$$\rho^2 = -\frac{1}{3}\left(\frac{n_4}{n_2}\right)^2 ;\ \tau^2 = \frac{2}{3}\frac{n_4^2}{R_6}. \tag{2}$$

Negative ρ^2 is not acceptable. But even if somehow ρ^2 had been positive, we would have had:

$$g^2 = \frac{\rho^3}{\tau^2} \propto R_6 \left(\frac{n_4}{n_2^3}\right); \tag{3}$$

so the string coupling would not have been weak.

In other cases, one finds these and other pathologies—AdS rather than dS stationary points and instabilities. Searches involving broader sets of moduli (Andriot, Wrase) seem to allow at best a few isolated regions of parameter space where such solutions might exist. Whether these might exhibit a sensible perturbation expansion is currently an open question, but our results above suggest that the combination is a tall order.

The Challenge of Cosmological Solutions

String theory has had many dramatic successes in understanding issues in quantum gravity. But one severe limitation is its inability, to date, to describe cosmologies resembling our own, which *appear* to emerge from a big bang singularity or evolve to a big crunch singularity. This could reflect some fundamental limitation; more likely, it reflects the inadequacy of our present theoretical tools to deal with situations of high curvature and strong coupling.

For example, consider a pseudomoduli space where the potential falls to zero for large fields in the positive direction. If one starts the system in the far past with expanding boundary conditions, then further in the past there is a big bang singularity; if one starts with contracting boundary conditions, there is a big crunch in the future (T. Banks and M.D.). These high curvature/strong coupling regions are inevitable, despite the system being seemingly weakly coupled through much of this history.

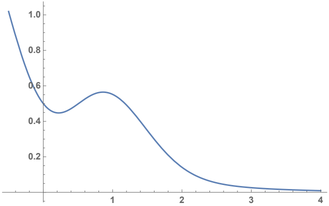

Figure: Metastable potential for a modulus, ϕ

Are things better for an initial configuration at the metastable minimum? If we start the system at the local minimum of the potential, classically, it will stay there eternally. But might there be small disturbances that drive the field to explore the region on the other side of the barrier, exhibiting the pathologies described above?

In one presentation of de Sitter space:

$$ds^2 = -d\tau^2 + \cosh^2(H\tau)\left[d\chi^2 + \sin^2\chi d\Omega_2^2\right]. \quad (4)$$

A homogeneous scalar field in this space, $\phi(\tau)$, obeys

$$\ddot{\phi} + 3H\frac{\sinh(H\tau)}{\cosh(H\tau)}\dot{\phi} + V'(\phi) = 0. \quad (5)$$

Consider, first, a potential which rises in all directions (no metastability). For large positive τ, any perturbation of ϕ about a local minimum damps; for large negative τ, the motion is amplified as τ increases (it damps out in the past). Correspondingly, in the far past and the far future, the field approaches the local minimum. We would expect the same to be true allowing for initially inhomogeneous configurations.

Now for a potential that has a local minimum with positive energy density, and that falls to zero for large $|\phi|$, we might expect that if we create a small, localized perturbation at some (r_0, τ_0) this perturbation will damp out if $\tau_0 \gg 0$. But if $\tau_0 \ll 0$, the perturbation will grow, possibly crossing over the barrier while $\tau \ll 0$. In this case, the emergent universe on the other side of the barrier is contracting, and we might expect the system to run off towards $\phi = \infty$, until the universe undergoes gravitational collapse.

Passing Over the Barrier Vs. Through the Barrier

We are interested in disturbances which lead to motion over a barrier, rather than tunneling. We might expect, however, that once the system passes over the barrier, its subsequent evolution is not particularly sensitive to whether it passed over the barrier or tunneled through it. The bubble, in either case, quickly becomes relativistic, energy is proportional to t^3, dwarfing any difference in the energy of order the barrier height at the time of bubble formation.

We can develop an intuitive picture by treating a thin-wall bubble and the radius of the bubble as a collective coordinate: With

$$V(\phi) = -\frac{1}{2}\mu^2\phi^2 + \frac{1}{4}\lambda\phi^4 + \epsilon\phi + V_0.$$

For small ϵ, the minima of the potential lie at

$$\phi_\pm \approx \pm\sqrt{\frac{\mu^2}{\lambda}}. \tag{6}$$

We can define our bubble configuration, with radius R large compared μ^{-1}, as the kink solution of the one dimensional problem,

$$\phi_B(r;R) = \frac{\phi_+ - \phi_-}{2}\tanh\left(\frac{\mu(r-R)}{\sqrt{2}}\right) + \frac{\phi_+ + \phi_-}{2}. \tag{7}$$

If $R_0(t)$ is slowly varying in time (compared to μ^{-1}), then we can write an action for R,

$$S = \int dt \int r^2 dr d\Omega \left(\frac{1}{2}(\partial_t \phi_B(r;R))^2 - (\vec{\nabla}\phi_B(r,R))^2 - V(\phi_B(r,R)) \right)$$

$$= 4\pi \int dt \left(\sqrt{\frac{2}{3}} \mu^3 (R^2 \dot{R}^2 - 2R^2) + \frac{\epsilon}{3} R^3 \right).$$

Correspondingly, the energy of the configuration is:

$$E(R,\dot{R}) = 4\pi \left(\sqrt{\frac{2}{3}} (R^2 \dot{R}^2 + 2R^2 - \frac{1}{3}\epsilon R^3) \right) \equiv \frac{M(R)}{2} \dot{R}^2 + V(R). \quad (9)$$

We have checked, numerically, that starting with a field configuration corresponding to $\phi(x, t = 0) = \phi_B(r; R),\ \dot{\phi}(x, t = 0) = 0$, to the left of the barrier, the bubble collapses. Starting slightly to the right, the wall quickly becomes relativistic and expands.

For $G_N = 0$, the system quickly evolves to resemble the critical, Coleman-DeLuccia bubble. This is consistent with an intuition that the energy of conversion of false vacuum to true is largely converted into the energy of the wall. Indeed the solution coincides with the critical bubble at large times.

So for G_N sufficiently small, after a time the system will evolve like the critical CDL bubble. So we can ask how the CDL bubble behaves for a potential which falls asymptotically to zero.

Consider the bubble evolution in the timelike region.

$$ds^2 = -d\tau^2 + \rho(\tau)^2 \left(d\sigma^2 + \sinh^2(\sigma) d\Omega_2^2 \right), \tag{10}$$

the equations for ρ and ϕ are:

$$\frac{d^2\phi}{d\tau^2} + 3\frac{\dot{\rho}}{\rho}\frac{d\phi}{d\tau} + V'(\phi) = 0 \tag{11}$$

and

$$\dot{\rho}^2 = 1 + \frac{\kappa}{3}\left(\frac{1}{2}\frac{d\phi}{d\tau}^2 + V(\phi) \right) a^2. \tag{12}$$

We might expect, for an exponentially falling potential, that the potential is not important aymptotically. Then

$$\rho \propto (\tau - \tau_0)^{1/3}, \ \ \tau > \tau_0; \qquad \rho \propto (\tau_0 - \tau)^{1/3}, \ \ \tau < \tau_0. \tag{13}$$

Also:

$$\frac{\dot{\rho}}{\rho} = \pm\sqrt{\frac{\kappa}{6}}\dot{\phi}. \tag{14}$$

So

$$\frac{d^2\phi}{d\tau^2} \pm \sqrt{\frac{3\kappa}{2}}\dot{\phi}^2 = 0. \tag{15}$$

We look for a solution of the form

$$\dot{\phi} = \alpha(\tau - \tau_0)^{-1}, \tag{16}$$

$$\alpha = \sqrt{\frac{2}{3\kappa}}. \tag{17}$$

Plugging this back into the $\dot{\rho}$ equation gives

$$\frac{\dot{\rho}}{\rho} = \pm\frac{1}{3}\frac{1}{\tau - \tau_0}, \tag{18}$$

which is consistent with the expected $(\tau - \tau_0)^{1/3}$ behavior. So we have a singularity in the past or the future.

For numerical studies, we designed a potential with a local de Sitter minimum that tends to zero for large ϕ

$$V(\phi) = \frac{1}{2} e^{-\phi} + \phi^2 e^{-\phi^2}. \tag{19}$$

We solve the equations for ϕ and ρ with ϕ_0 taken to be not too far from the local minimum, with $d\phi/d\tau = 0$, and with the negative sign in the root of the ρ equation.

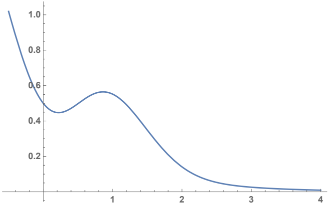

Figure: ϕ potential.

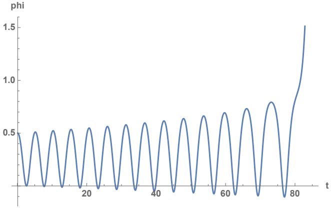

Figure: ϕ crosses the barrier.

Implications of the singularity

Our main concern with the singularity is whether it is an obstruction to any sort of systematic analysis. If we have a weak coupling, small curvature description of the system, allowing a perturbative analysis, we expect to be able to write an effective Lagrangian including terms of successively higher dimension—higher numbers of derivatives—such as:

$$\mathcal{L} = \sqrt{g}\left(\frac{1}{G_N}\mathcal{R} + \mathcal{R}^2 + \frac{1}{M^2}\mathcal{R}^4 + \dots \right.$$
$$\left. + (\partial_\mu \phi)^2 + \frac{1}{M^4}(\partial_\mu \phi)^4 \right). \quad (20)$$

If one tries to analyze the resulting classical equations perturbatively, in the presence of $\dot{\phi} \sim 1/(t - t_0)$ and $\mathcal{R} \sim 1/(t - t_0)^2$, at low orders, the terms in the expansion diverge and the expansion breaks down. This is similar to the phenomena at a big bang or big crunch singularity.

Conclusions

We have argued, from two points of view, that one cannot construct de Sitter space in any controlled approximation in string theory. First, we have seen that even allowing the possibility of arbitrarily large fluxes, it is very difficult to find stationary points for which both the string coupling is small and compactification radii are large, even before asking whether the corresponding cosmological constant is positive or negative. We have seen that typically when sensible stationary points exist, even if formally radii are large and couplings small, higher order terms in the expansions are not small.

But our second obstacle seems even more difficult to surmount: a set of small perturbations of any would-be metastable de Sitter state, classically, will evolve to uncontrollable singularities.

This is *not* an argument that metastable de Sitter states do not exist in quantum theories of gravity; only that they are not accessible to controlled approximations. The problem is similar to the existence of big bang and big crunch singularities; we have empirical evidence that the former exists in the quantum theory that describes our universe, but we do not currently have the tools to describe these in a quantum theory of gravity.

Observations on the KKLT Construction

KKLT invoke vacua with fluxes, but the small parameter is not provided by taking all fluxes particularly large; rather, it arises from an argument that there are so many possible choices of fluxes that in some cases, purely at random, there is a small superpotential. In other words, there is conjectured to be a vast set of (classically) metastable states of which only a small fraction permit derivation of an approximate four-dimensional, weak coupling effective action. Perhaps this is evidence that if in some cosmology one lands for some interval in such a state, the state can persist for a long period. But a complete description of such a cosmology is beyond our grasp at present.

Approximately Supersymmetric States in a Landscape

In considering the cosmic landscape, the lack of weak coupling suggests thatlong-lived de Sitter vacua will be very rare. This is particularly problematic for the state we currently inhabit. We'd have to be lucky (anthropic? but why not decay tomorrow?) unless protected by some degree of approximate supersymmetry. The breaking of supersymmetry would almost certainly be non-perturbative in nature; searches for concrete realizations of such states (as opposed to statistical arguments for the *existence* of such states, along the lines of KKLT) would be challenging.

Ultimately, at a quantum level, reliably establishing the existence of metastable de Sitter space appears to be a very challenging problem. One needs a cosmic history, and it would be necessary that this history be under theoretical control, both in the past and in the future. As a result, the significance of failing to find stationary points of an effective action describing metastable de Sitter space is not clear. We have seen that even thought of as classical configurations, there are questions of stability and obstacles to understanding the system eternally, once small perturbations are considered. We view the question of the existence of metastable de Sitter space as an open one.

Backup Slides

We can make this latter statement more precise. If we write:

$$\phi(r,t) = \phi_{\rm cr}(t,r) + \chi(t,r), \qquad |\chi| \ll \phi_{\rm cr}, \tag{21}$$

where $\phi_{\rm cr}$ is the critical bubble solution, then

$$(\partial^2 + m^2(r,t))\chi = 0. \tag{22}$$

Here m^2 is essentially a θ function, transitioning between the mass-squared of χ in the false and true vacua. Since the bubble wall moves at essentially the speed of light, and undergoes a length contraction by $t \sim \gamma$, we have that

$$m^2(t,r) \approx m^2(t^2 - r^2) \tag{23}$$

and the χ equation is solved by

$$\chi = \frac{1}{r}\chi(t^2 - r^2). \tag{24}$$

So the amplitude of χ decreases with time, and the energy stored is small compared to that in the bubble wall.

Tunneling with $G_N = 0$

Consider, first, the bounce solution without gravity. We consider a potential, $V(\phi)$, with local minima at $\phi_{\rm true},\ \phi_{\rm false}$, where $V(\phi_{\rm false}) > V(\phi_{\rm true})$. Starting with the field equations,

$$\Box\Phi + V'(\phi) = 0, \tag{25}$$

for points that are space-like separated from the origin (the center of the bubble at the moment of its appearance), we introduce $\xi^2 = r^2 - t^2$, in terms of which

$$\frac{d^2\phi}{d\xi^2} + \frac{3}{\xi}\frac{d\phi}{d\xi} - V'(\phi) = 0. \tag{26}$$

This is the Euclidean equation for the bounce.

For points that are time-like separated, calling $\tau^2 = t^2 - r^2$,

$$\frac{d^2\phi}{d\tau^2} + \frac{3}{\tau}\frac{d\phi}{d\tau} + V'(\phi) = 0. \tag{27}$$

These equations are related by $\xi = i\tau$.
On the light cone, $\xi = \tau = 0$, we have $d\phi/d\tau = d\phi/d\xi = 0$, and we have to match $\phi(0) = \phi_0$. In the tunneling problem [**?**], ϕ_0 is determined by the requirement that $\phi \to \phi_{\rm false}$ as $\xi \to \infty$; this can be thought of as a requirement of finite energy relative to the configuration where $\phi = \phi_{\rm false}$ everywhere.

Classical perturbations of the false vacuum with $G_N = 0$

Without gravity, we might consider starting the system in the false vacuum and giving it a “kick” so that, in a localized region, the system passes over the barrier. On the other side, the system looks like a bubble, but not of the critical size. We might expect that the evolution of the bubble, on macroscopic timescales, is not sensitive to the detailed, microscopic initial conditions. For a thin-walled bubble, for example, we can think of configurations where at time $t = 0$, one has a bubble of radius R_0, inside of which one has true vacuum, outside false vacuum, and a transition region described by the kink solution of the one dimensional field theory problem with nearly degenerate minima.

Behavior of the disturbance with small G_N

Consider the same system, now with a small G_N. Again, our disturbance, after a short period of time, approaches the critical ($G_N = 0$) bubble. At larger time, it will then agree with the Coleman-De Luccia solution, including the small effects of gravity.

As we will see in the next section, for the asymptotically falling potential, with expanding boundary conditions, the evolution of the configuration is non-singular. But with contracting boundary conditions, one encounters, as expected, a curvature singularity.

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Behavior of the Bounce with Asymptotically Falling Potential

We have argued that, independent of the microscopic details of the initial conditions, in the case of a disturbance that connects two metastable minima of a scalar potential, the large time evolution of an initial disturbance that crosses the barrier is that of the critical bubble, in the limit of small G_N. We expect that the same is true for a potential that falls asymptotically to zero. Once more, the underlying intuition is that at late times, the energy released from the change of false to true vacuum overwhelms any slight energy difference in the starting point. So we expect the solution to go over to $\phi(\tau)$. So in this section, we will focus principally on the behavior of the critical bubble, $\phi(\tau)$.

Field evolution with small G_N

For small but finite G_N, there is a long period where $G_N \times T_{00} \times \tau^2 \ll 1$, gravitation is negligible, and the picture of the previous section of the flat-space evolution of the bubble (or disturbance) is unaffected. For a vacuum bubble in de Sitter space, gravitational effects become important, for fixed $r \ll H^{-1}$, for example, only once $t \sim H^{-1}$. Provided the bubble has evolved to a configuration approximately that of the critical bubble, we can take over the critical bubble results (with gravity).

For $3 \leq p \leq 7$ and choosing $T_{10} = 1$, $R_6 \sim 1$, we can drop the T_{10} term because the R_6 term will dominate. We can attempt to find large τ and ρ by turning on $F_2 = n_2$ and $F_4 = n_4$ (other combinations of fluxes give similar results). Relevant terms:

$$-\tau^{-2}\rho^{-1}R_6 + \frac{1}{2}\tau^{-4}\left(n_2^2\rho + n_4^2\rho^{-1}\right). \tag{28}$$

Differentiating with respect to ρ and τ, for $n_4 \gg n_2 \gg 1$

$$\rho^{-2}R_6 + \frac{1}{2}\tau^{-2}\left(n_2^2 - n_4^2\rho^{-2}\right) = 0 \tag{29}$$

and

$$\rho^{-1}R_6 - \tau^{-2}\left(n_2^2\rho + n_4^2\rho^{-1}\right) = 0. \tag{30}$$

Behavior of the equations for large τ

Before describing our numerical results, it is helpful to consider some crude approximations which give insight into the behavior of the system. In the timelike region with $\xi = i\tau$, the equations for ϕ and the scale factor, ρ, are:

$$\frac{d^2\phi}{d\tau^2} + 3\frac{\dot{\rho}}{\rho}\frac{d\phi}{d\tau} + \frac{dU}{d\phi} = 0, \tag{31}$$

$$\dot{\rho} = \pm\sqrt{-1 + \frac{\kappa}{3}\rho^2\left(\frac{1}{2}\dot{\phi}^2 + U(\phi)\right)}. \tag{32}$$