Dynamical Constraints on RG Flows and Cosmology

Based on: 1906.10226 with Baumann and Green

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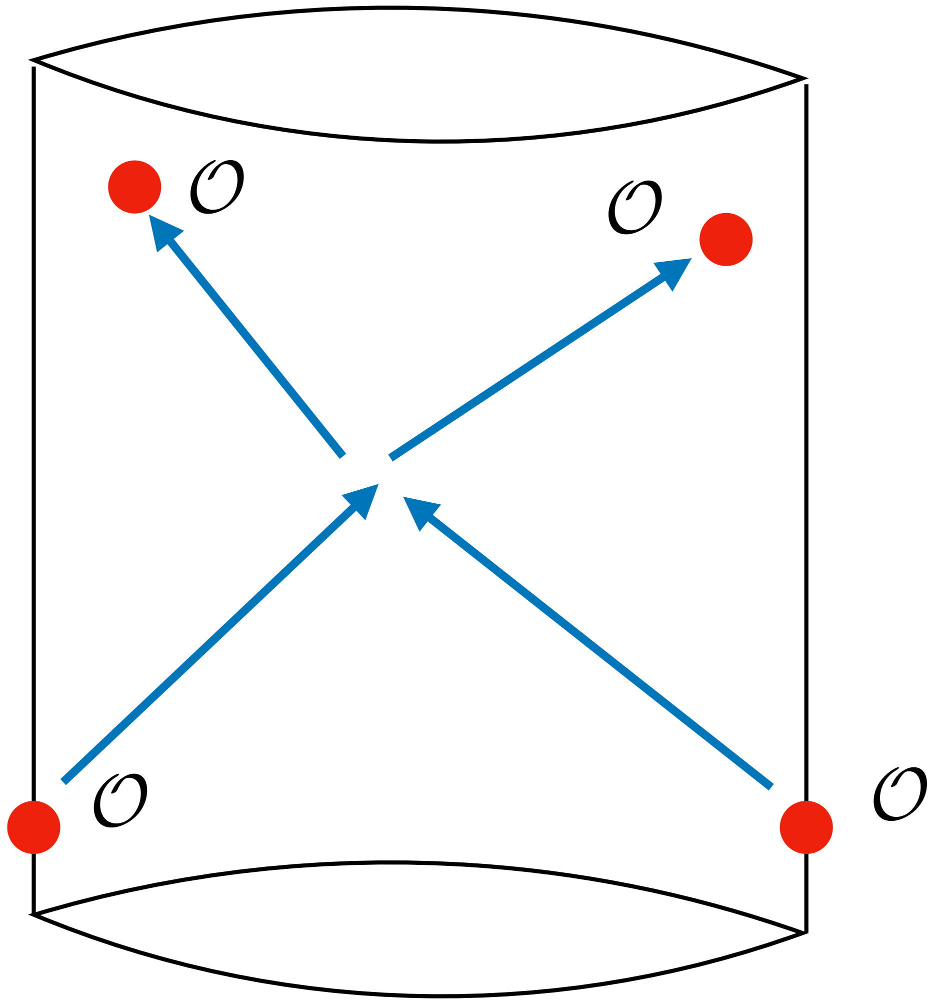
UV Meets the IR **✦ KITP ✦ October 12, 2020**

Some background:

Gravity Constraints from Conformal Bootstrap



AdS

$$\langle \mathcal{O}(x_1)\mathcal{O}(x_2)\mathcal{O}(x_3)\mathcal{O}(x_4)\rangle$$

$$\Updownarrow$$

$$\mathcal{A}(s,t,u)$$

Use conformal bootstrap to carve out the parameter space of quantum gravity

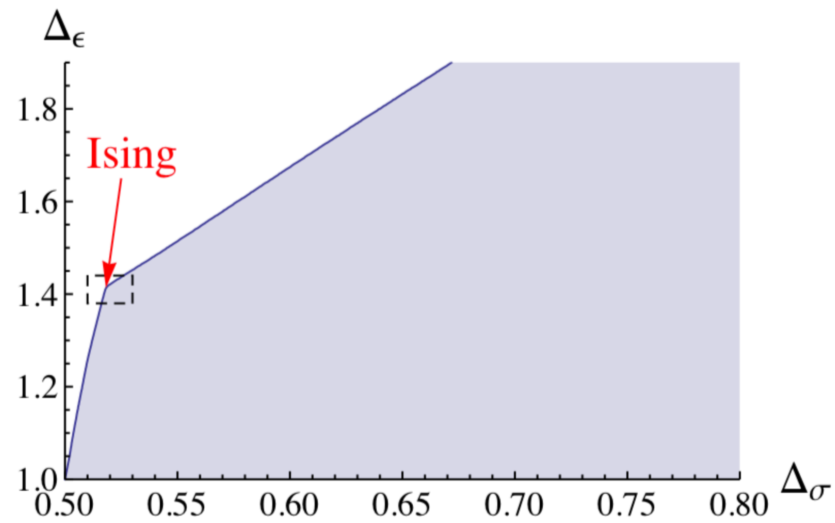
The target

Weak gravity conjecture; swampland

Strings, extra dimensions?

Theories on the edge (*cf*. Ising)

de Sitter?



El-Showk, Paulos, Poland, Rychkov, Simmons-Duffin, Vichi

Current status

Detailed mapping of CFT correlators to bulk amplitudes

Solutions of crossing = Bulk Lagrangians

Bootstrap constraints on bulk couplings on the leading Regge trajectory

de Sitter?

Constraints on the leading Regge trajectory

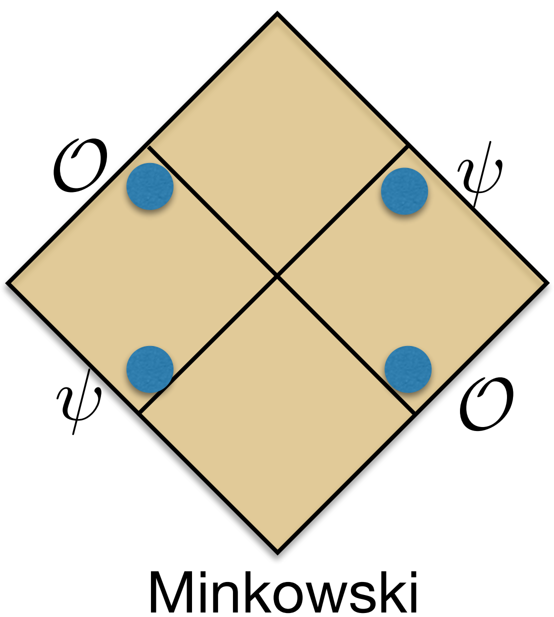
Averaged Null Energy Condition (ANEC)

$$\langle\Psi|\int dx^{+}T_{++}|\Psi\rangle \geq 0$$

Higher spin ANEC

Bootstrap derivation

[TH, Jain, Kundu '15]
[TH, Kundu, Tajdini '16]



$$\mathcal{O}\mathcal{O} \sim 1 + \int T$$

$$\langle\psi \int T\psi\rangle = \int \langle[\mathcal{O},\psi][\mathcal{O},\psi]\rangle$$

$$\geq 0$$

(unitarity in the UV)

Sum rules on all Regge trajectories

The Lorentzian Inversion Formula

[Caron-Huot ’17]

$$[\text{OPE Coeff.}] = \int dz d\bar{z} (\text{Kernel}) \langle [\mathcal{O}, \mathcal{O}], [\mathcal{O}, \mathcal{O}] \rangle$$

Dispersion relations, sum rules, etc…

[Carmi, Caron-Huot ’19]
[Mazac, Rastelli, Zhou ’19]
[Caron-Huot, Mazac, Rastelli, Simmons-Duffin ’20]
[Carmi, Penedones, Silva, Zhiboedov ’20]
etc.

An application to bulk EFTs

$$\mathcal{L} \sim (\nabla\phi)^2 + \lambda(\nabla\phi)^4 + \cdots \qquad \text{(in AdS)}$$

In the boundary CFT, this turns on an anomalous dimension for the spin-2 operator

$$\mathcal{O}\partial_\mu\partial_\nu\mathcal{O}$$

This is the leading contribution to the OPE in Regge kinematics, so

$$\int \langle [\mathcal{O},\mathcal{O}][\mathcal{O},\mathcal{O}]\rangle \sim -\gamma_2 \sim \boxed{\lambda > 0}$$

Momentum space version (Deep inelastic scattering)

[Komargodski, Zhiboedov '12]
[Komargodski, Kulaxizi, Parnachev, Zhiboedov '16]

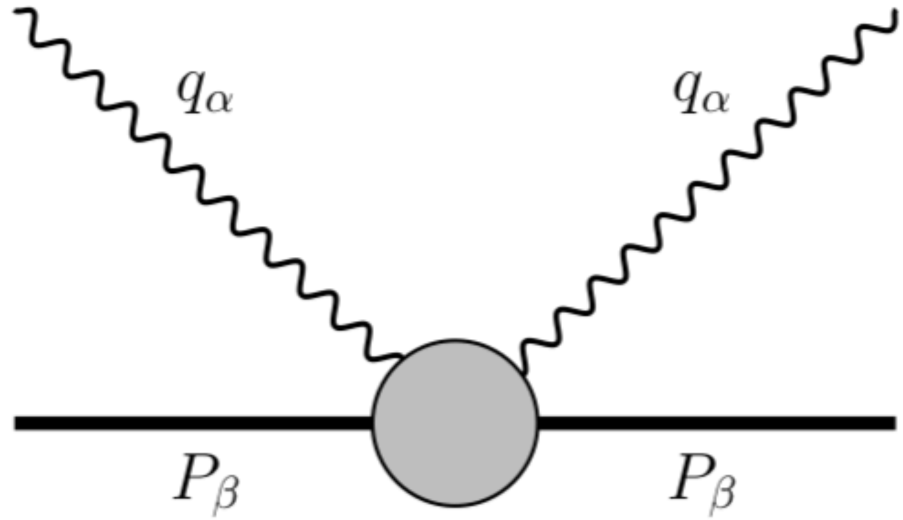
$$\mathcal{A}_{DIS} = \langle P|O(q)O(-q)|P\rangle$$

$\mathcal{O}(P)$

$$x = \frac{q^2}{2P \cdot q}$$

$$-\gamma_2 = \int_0^1 dx\, x \mathrm{Im}\, \mathcal{A}_{DIS} \geq 0$$

IR **UV**



Upshot

Sum rules for boundary correlation functions constrain the sign of the leading contribution at each even spin $s \geq 2$

Can we use this new technology to learn something about inflation?

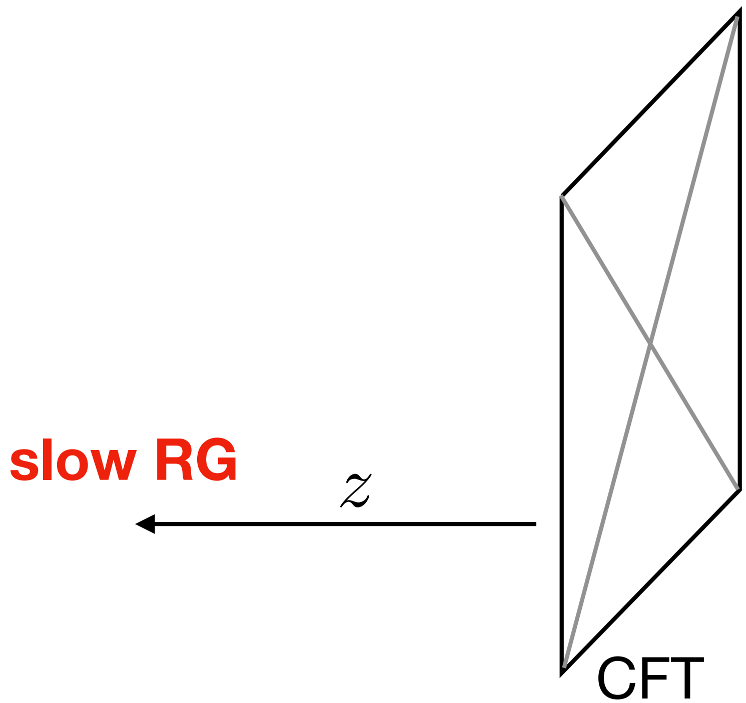
work with Daniel Baumann and Dan Green [2019]

A direct study of scattering amplitudes in the EFT of inflation is also possible, but hampered by the fact that the QFT is non-relativistic.

[Baumann, Green, Lee, Porto '15]

anti-de Sitter

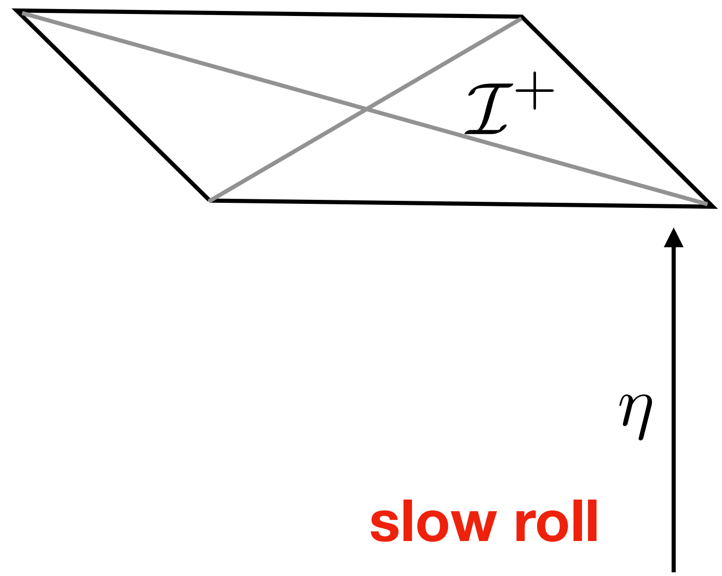
$$ds^2 = \frac{\ell^2}{z^2}\left[dz^2 - dt^2 + dx^2\right]$$



$$SO(d,2)$$

de Sitter

$$ds^2 = \frac{\tilde{\ell}^2}{\eta^2}\left[-d\eta^2 + dy^2\right]$$



$$SO(d+1,1)$$

anti-de Sitter

$$ds^2 = \frac{\ell^2}{z^2}\left[dz^2 - dt^2 + dx^2\right]$$

de Sitter

$$ds^2 = \frac{\tilde{\ell}^2}{\eta^2}\left[-d\eta^2 + dy^2\right]$$

$$t \to i y_1$$

$$z \to i\eta$$

$$\ell \to i\tilde{\ell}$$

$$\Lambda \sim \frac{1}{\ell^2} \to -\Lambda$$

"dS/CFT correspondence" ?

The Wick rotation makes a huge difference.

It's not clear whether there is really any dS/CFT for quantum gravity.

But we can take advantage of the formal correspondence for certain observables at $\mathcal{I}^+$

[Strominger '01, Mazur & Mottola '01, Witten '01, Maldacena '02]

We will derive bootstrap-style sum rules for the effective field theory of inflation, involving the inflationary speed of sound c_s

But first let's study the analogous problem in AdS, which is interesting too:

Sum rules for the EFT of holographic RG flows

EFT of inflation: [Cheung et al '07, Weinberg '08]

EFT of holographic RG: [Kaplan, Wang '15]

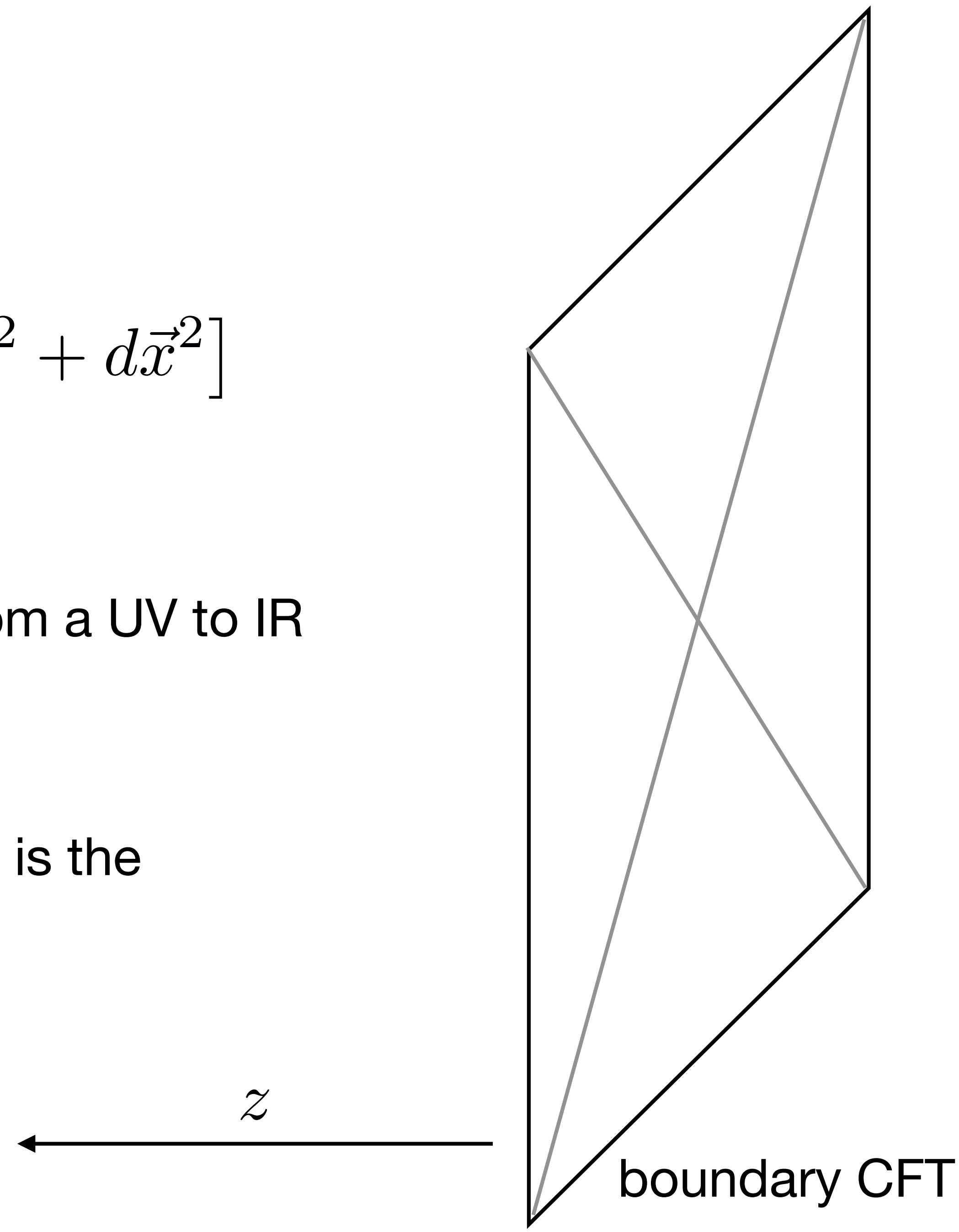
Holographic RG flows

$$ds^2 = \frac{\ell^2}{z^2}\left[C(z)^2 dz^2 - dt^2 + d\vec{x}^2\right]$$

This is dual to a QFT flowing from a UV to IR fixed point.

The analogue of c_s in inflation is the

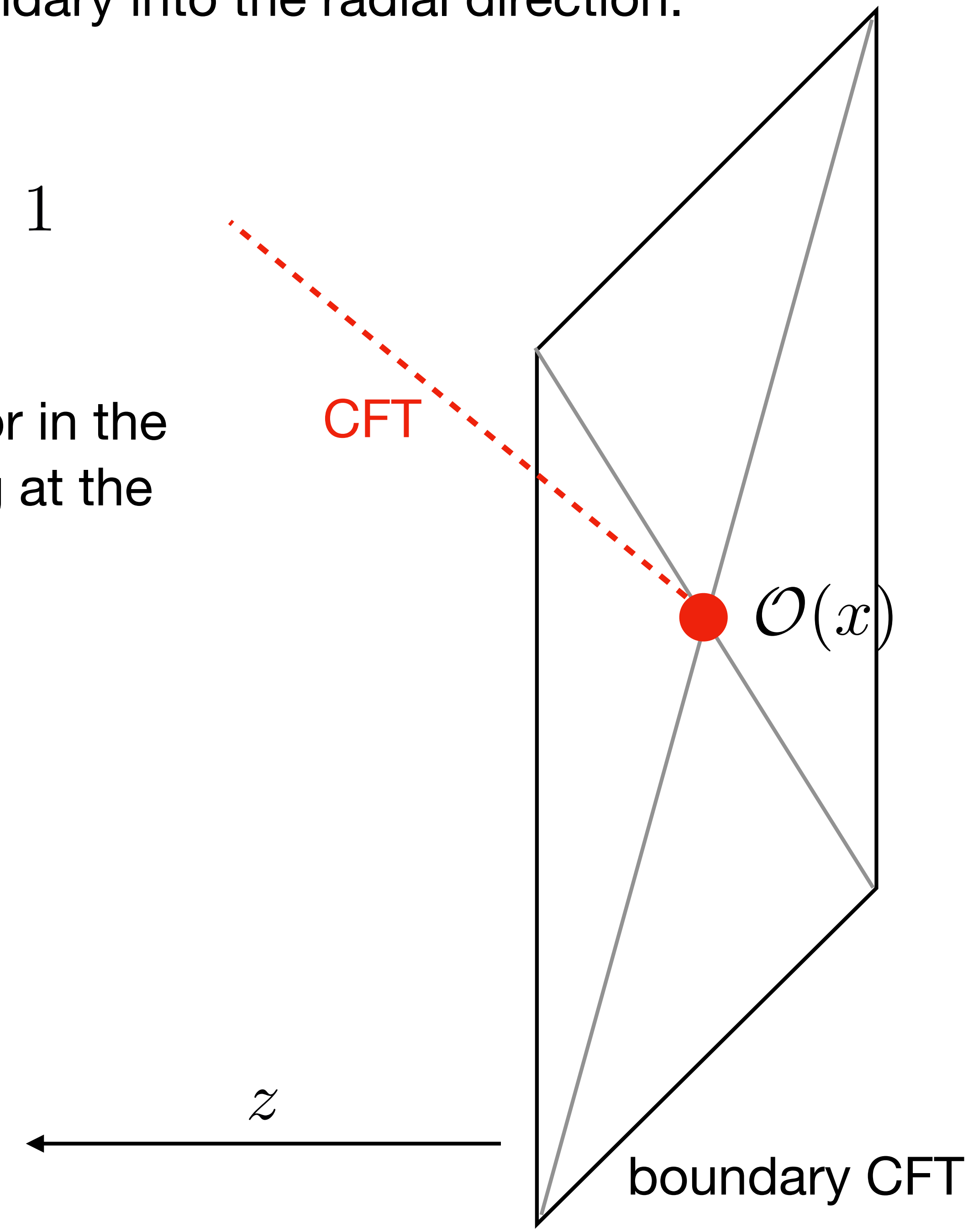
"radial speed" c_z



Drop an object from the boundary into the radial direction.

In the CFT vacuum, $$\frac{dz}{dt} = 1$$

If we reconstruct this operator in the boundary CFT, it is spreading at the speed of light.



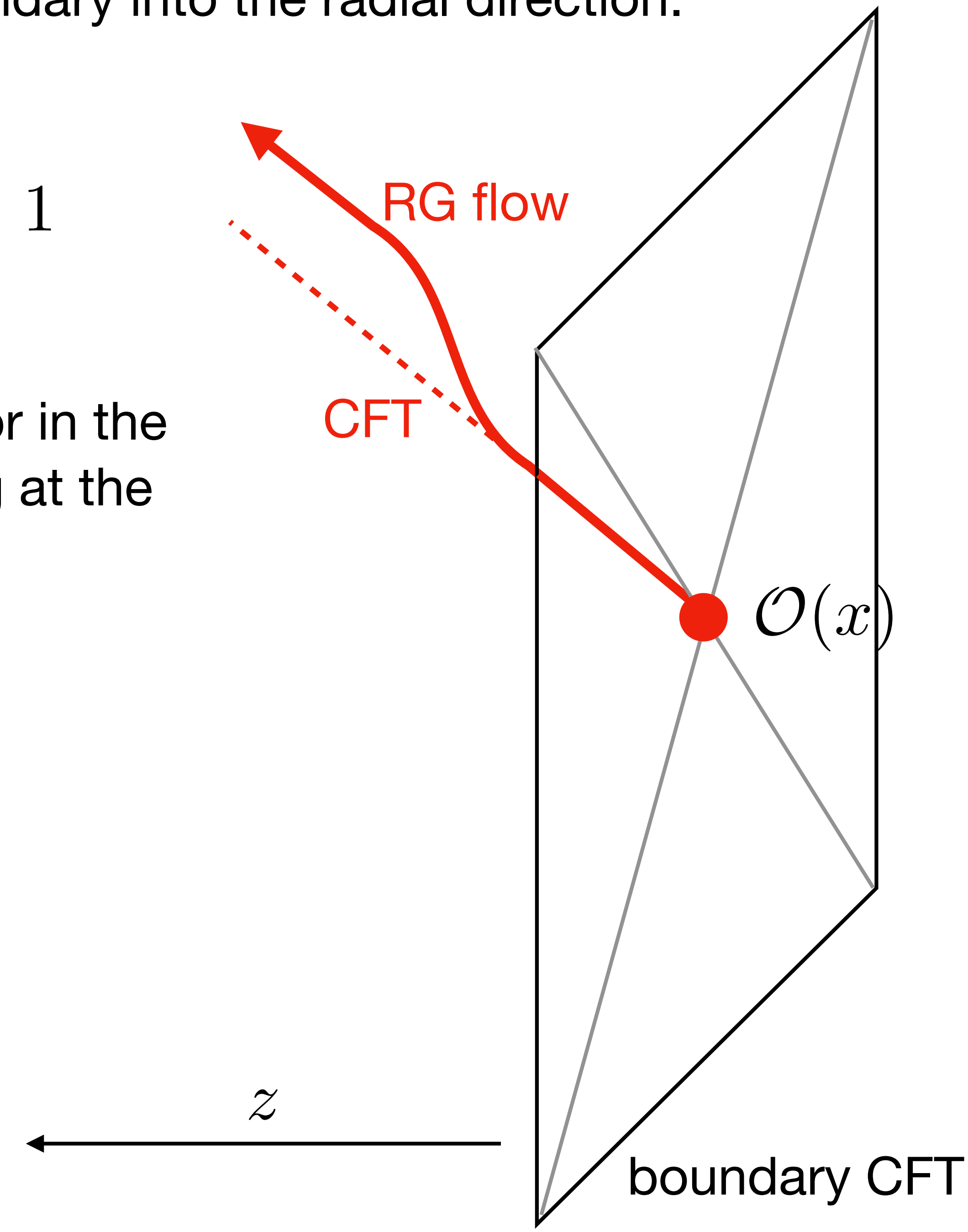
Drop an object from the boundary into the radial direction.

In the CFT vacuum, $\dfrac{dz}{dt} = 1$

If we reconstruct this operator in the boundary CFT, it is spreading at the speed of light.

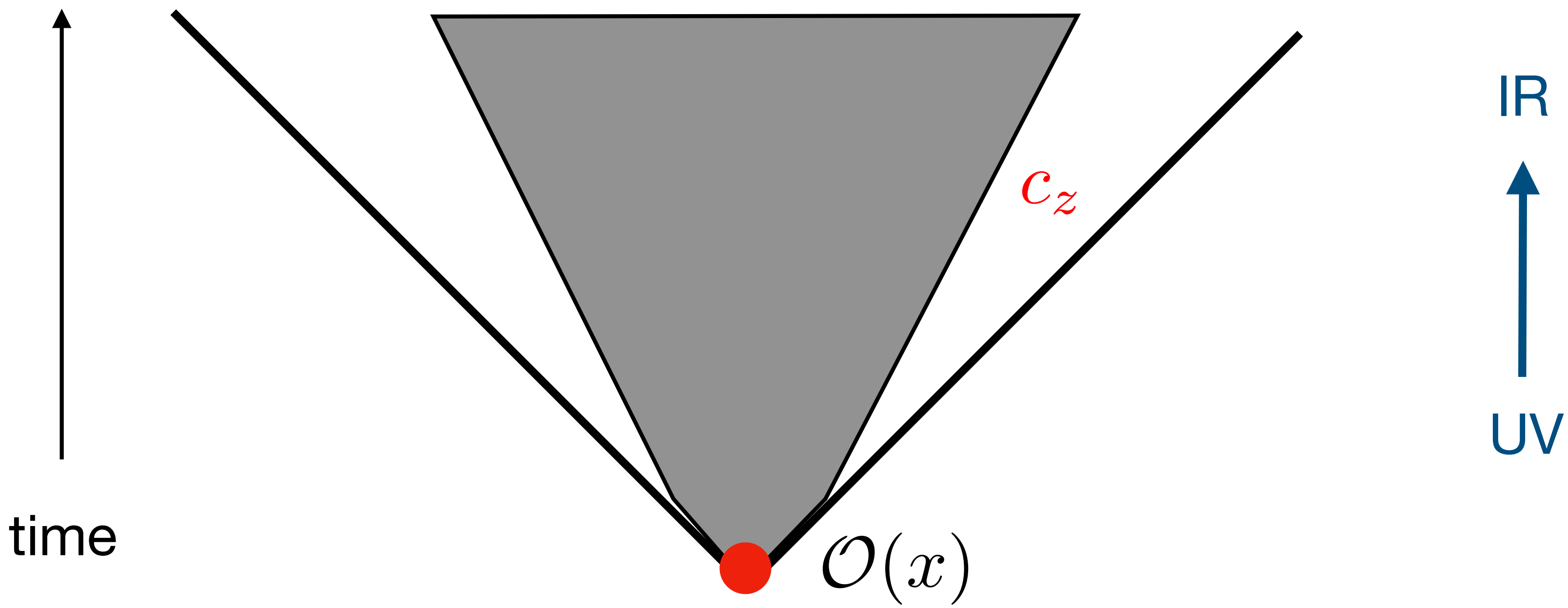
In an RG flow,

$$c_z = \frac{dz}{dt} < 1$$



Boundary QFT interpretation

The infall experiment is dual to a local operator that spreads out in time.



"dynamical probe of the RG flow"

cf. "butterfly velocity"

There is an EFT of fluctuations in holographic RG flows, like the EFT of inflation: [Kaplan, Wang '15]

$$S_\pi \sim \int d^{d+1}x a^4 \dot{H} \Big[c_z^2 \dot{\pi}^2 + a^{-2}(\partial\pi)^2$$

speed of infall

$$+(c_z^2 - 1)\dot{\pi}(\partial\pi)^2 + \#\dot{\pi}^3$$

$$+(c_z^2 - 1)(\partial\pi)^4 + \#\dot{\pi}^2(\partial\pi)^2$$

suggests a sum rule

$$+\cdots \Big]$$

The goldstone π is dual to the QFT operator T^μ_μ

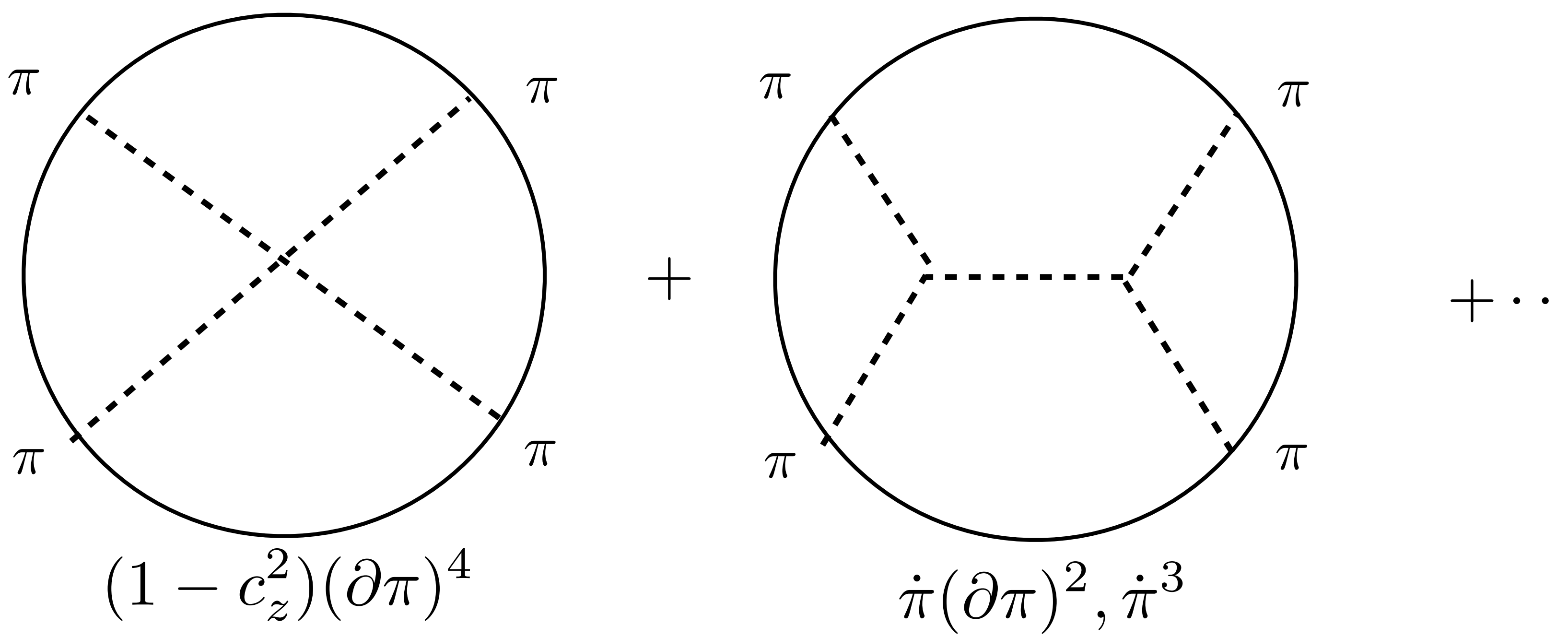
Strategy

Conformal invariance is broken, so the bulk theory is complicated and non-relativistic.

But the boundary theory is just a CFT in the UV, so all the leading-Regge technology still holds.

We will apply it to the trace of the stress tensor and derive a spin-2 sum rule:

$$\mathcal{A}_{DIS} = \langle T^{\mu}_{\mu}(P) T^{\mu}_{\mu}(q) T^{\mu}_{\mu}(-q) T^{\mu}_{\mu}(-P)\rangle'$$



$$1 - c_z^2 + \frac{(9 + 24\lambda_3 - 20c_z^2 + 11c_z^4)^2}{64c_z^6} = \int_0^1 dx\, x \mathrm{Im}\ \mathcal{A}_{DIS}^{T_\mu^\mu}$$
$$\geq 0$$

This is a dispersion relation for the infall speed / spreading speed (plus exchange terms).

$0 \leq c_z \leq 1$ inequality is obeyed

Freedom at light-speed

If $c_z = 1$ then an infinite class of operators vanish from the EFT.

Conjecture: Therefore it is free.

Inflation?

RG sum rule

$$1 - c_z^2 + \cdots = \int \mathrm{Im}\, \mathcal{A}_{DIS}$$

EFT of inflation

$$1 - \frac{1}{c_s^2} + \cdots = -\int \mathrm{Im}\, \mathcal{A}_{DIS}$$

$c_s =$ Speed of sound during inflation

constraint from Planck:
$c_s \geq 0.024$

sum rule: ✔

positivity: **?** *UV properties in dS are unknown*

Thus a measurement of $c_s \approx 1$ would tell us something interesting about the UV.

But we cannot conclude that it's free.

Summary

Bootstrap techniques constrain EFT in anti-de Sitter. (ANEC, etc.)

Some of these techniques apply even when conformal invariance is broken —> RG sum rules

Inflationary sum rules, but no definitive low-energy constraints.

The future

Continue to develop the CFT technology; apply to

- Gravity Bootstrap
- non-conformal QFTs

Thank you.