Violation of the Fluctuation-Dissipation Theorem in Time-Dependent Mesoscopic Heat Transport

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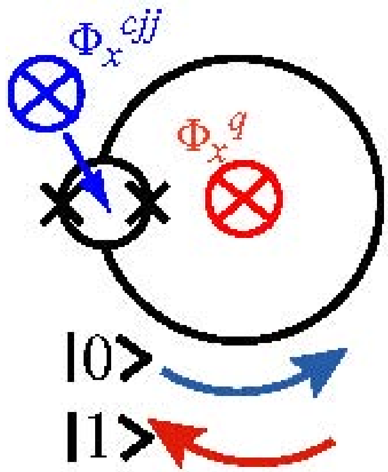
Outline

1. FDT, applications to qubits and status for thermal conductance.
2. Energy flux fluctuations in mesoscopic heat transport.
3. Finite-frequency heat conductance, no thermal FDT.

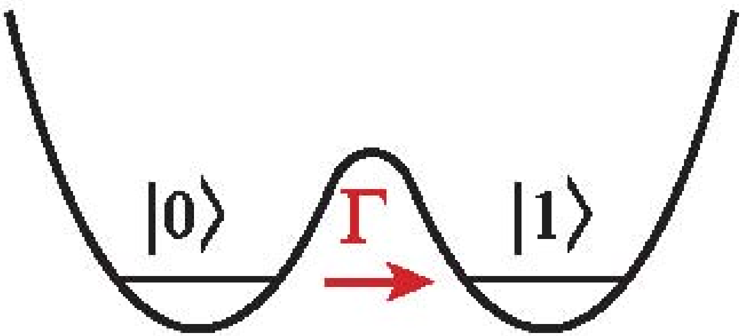
PRL **104**, 220601 (2010).

Applications of the FDT in studies of qubit decoherence



1. Microscopic origin of the low-frequency flux noise remains one of the main not fully resolved issues in superconducting qubits.
2. It can be addressed through macroscopic resonant tunneling as a convenient tool for measuring properties of the flux noise.

$$\Gamma(\epsilon) = \frac{\Delta^2}{2} \mathrm{Re} \int_0^\infty dt e^{i\epsilon t} \exp\left\{ \int d\omega S(\omega) \frac{e^{-i\omega t} - 1}{\omega^2} \right\}$$



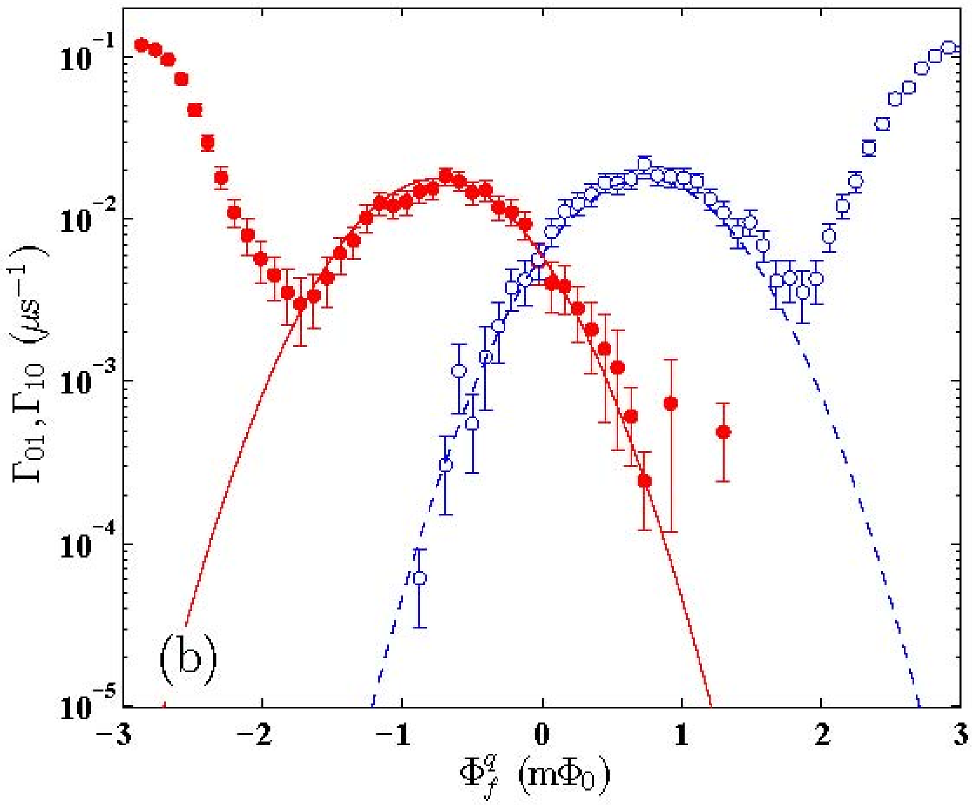
$$S(\omega) = \frac{1}{2\pi} \int dt e^{i\omega t} \langle E(t) E \rangle.$$

For low-frequency noise:

$$\Gamma(\epsilon) = \sqrt{\frac{\pi}{8}}\frac{\Delta^2}{W}\exp\left\{-\frac{(\epsilon-\epsilon_p)^2}{2W^2}\right\},$$

$$W^2 = \int d\omega S(\omega), \qquad \epsilon_p = \mathcal{P}\int d\omega \frac{S(\omega)}{\omega}.$$

Qualitative effect of the level renormalization by noise: splitting of the lowest tunneling peak.



R. Harris et. al., PRL 2008

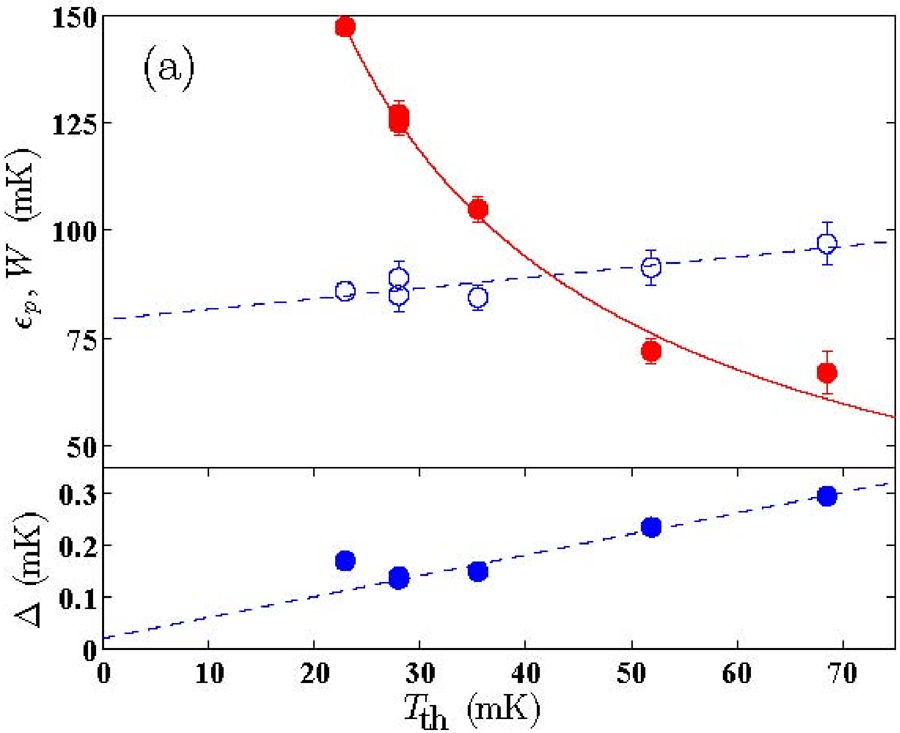
If the noise-generating environment is in equilibrium, FDT applies to the noise correlators and gives:

$$S_+(\omega) = S_-(\omega)\coth(\omega/2T), \qquad S_\pm(\omega) = [S(\omega) \pm S(-\omega)]/2$$

$$W^2 = \int d\omega S_+(\omega), \qquad \varepsilon_p = \int d\omega S_-(\omega)/\omega$$

$$T >> \omega \qquad \boxed{W^2 = 2T\varepsilon_p}$$

M.H.S. Amin and D.V.A., PRL 2008



R. Harris et. al., PRL 2008

Conclusion: equilibrium noise source with paramagnetic susceptibility.

Status of the thermal FDT

1. Microscopically, temperature is not a mechanical quantity.
2. There is still a common view that FDT should work for thermal conductance as well, e.g.

STATISTICAL PHYSICS

Part 2

Theory of the Condensed State

by

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Volume 9 of *Course of Theoretical Physics*

Thermodynamic motivation:

$$\left\langle \tilde{E}^2 \right\rangle = CT^2, \quad \dot{\tilde{E}} = -\tilde{E}/\tau + J, \quad \tau = C/G_{th}$$

$$S(0) = 2\left\langle \tilde{E}^2 \right\rangle / \tau = 2T^2 G_{th}.$$

This coincides with the ω=0 limit of the thermal FDT:

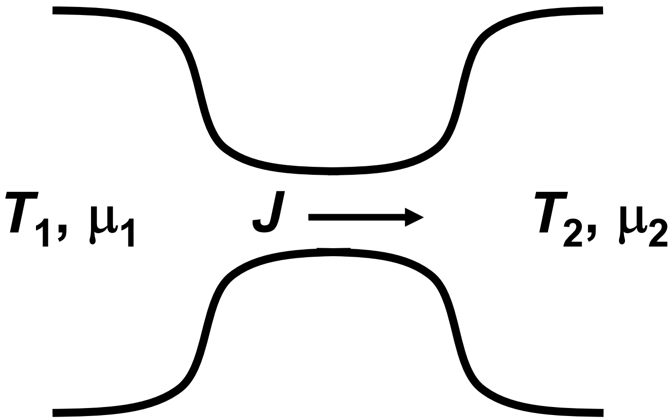
$$S(\omega) = \hbar\omega T \,\mathrm{Re}\, G_{th}(\omega) \coth\left(\hbar\omega / 2T\right).$$

the case of quantum fluctuations. According to the general rules of the fluctuation–dissipation theorem, such a generalization is obtained by including an extra factor $(\hbar\omega/2T)\coth(\hbar\omega/2T)$ (which is unity in the classical limit $\hbar\omega \ll T$).

$$(g_i^{(1)} g_k^{(2)})_\omega = \delta_{ik}\delta(\mathbf{r}_1 - \mathbf{r}_2)\,\hbar\omega T \coth(\hbar\omega/2T)\,\mathrm{re}\,\varkappa(\omega), \qquad (88.20)$$

Fluctuations of the energy flux in mesoscopic heat transport

Transport through a shot constriction:



$$S(\omega)=\int dt e^{-i\omega t}\left\langle \tilde{J}\tilde{J}(t)+\tilde{J}(t)\tilde{J}\right\rangle/2,$$

$$\tilde{J}\equiv J-\langle J\rangle.$$

<u>Phonons</u>

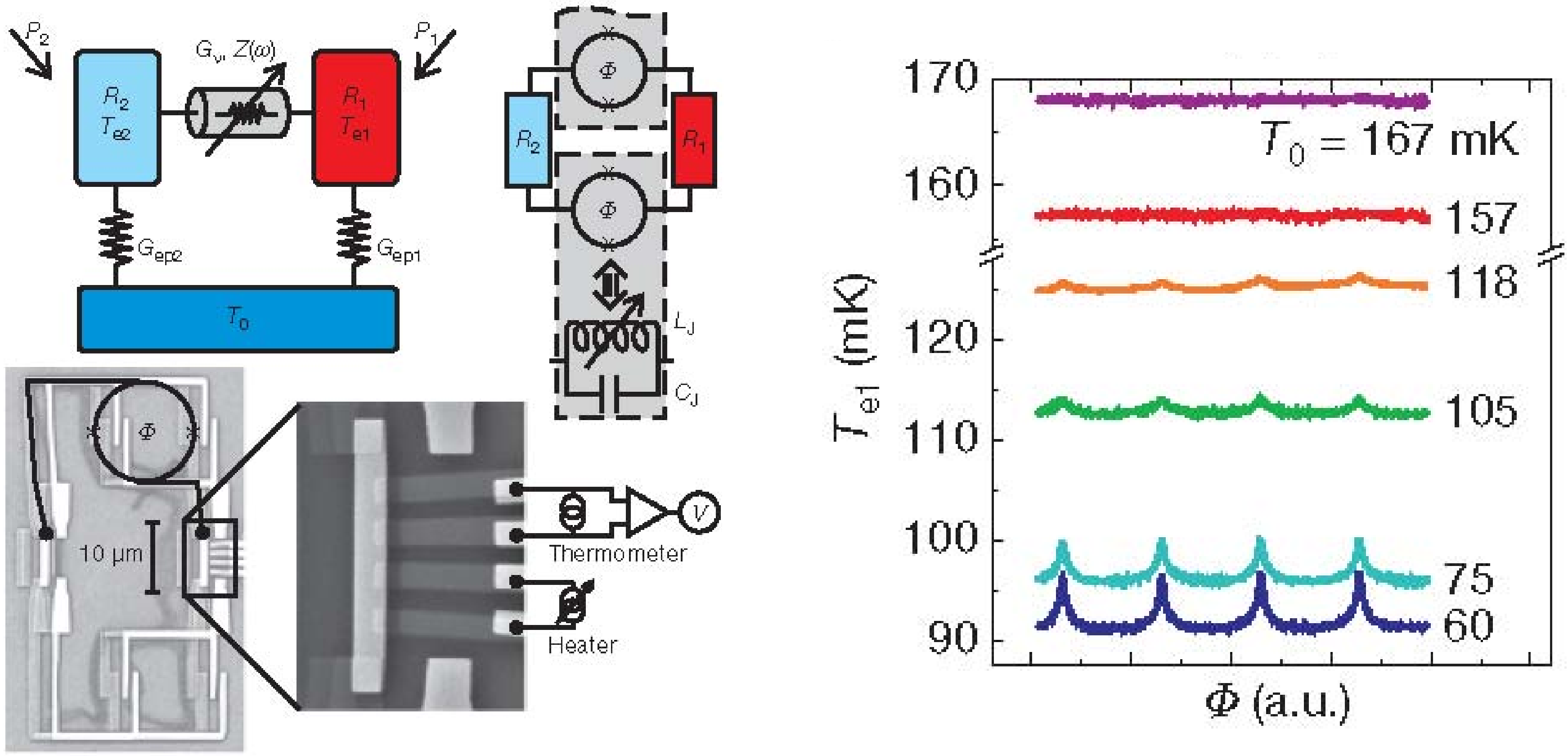
$$J=\frac{\hbar v}{2L}\sum_{k,p}\left(\omega_k\omega_p\right)^{1/2}\operatorname{sgn}(k)(a_k-a_k^+)(a_p^+-a_p).$$

$$S(\omega)=\frac{1}{48\pi\hbar}\sum_{j=1,2}\left[(2\pi T_j)^2+(\hbar\omega)^2\right]\hbar\omega\coth\left(\hbar\omega/2T_j\right).$$

In equilibrium, only the first part corresponds to the thermal FDT with

$$G_{th}=\pi T/6\hbar.$$

Mesoscopic heat conduction by photons



M. Meschke *et al.*, Nature (2006).

Electrons

Heisenberg equation of motion for the energy density

$$h(x) = \left[\psi^+ \hat{h}\psi + (\hat{h}\psi^+)\psi\right]/2, \quad \hat{h} = -(\hbar^2/2m)\partial^2/\partial x^2 + V(x)$$

gives the energy flux operator as

$$J = \frac{v_F}{L}\sum_{k,p}\frac{\varepsilon_k + \varepsilon_p}{2}\left[D(a_k^+ a_p - b_k^+ b_p) + \sqrt{DR}(a_k^+ b_p + b_k^+ a_p)\right],$$

where the energies are measured relative to the chemical potentials μ_j. With this operator one obtains the spectrum of the energy flux noise which in equilibrium reduces to

$$S(\omega) = \left(G/12e^2\right)\left[(2\pi T)^2 + (\hbar\omega)^2\right]\hbar\omega\coth\left(\hbar\omega/2T\right).$$

Both quantitatively and qualitatively, this result is similar to that for phonons.

Definition of the finite-frequency thermal conductance

Use the simplest electron tunnel model as a model of heat transport

$$H = H_1 + H_2 + H_T, \quad H_T = \sum_{k,p} (T_{kp} a_k^\dagger b_p + \mathrm{H.c.})$$

Conductance is defined as the linear response of the energy current

$$J = \dot{Q}/2 = (i/2\hbar) \sum_{k,p} (\epsilon_k + \epsilon_p)(T_{kp} a_k^\dagger b_p - \mathrm{H.c.})$$

to the time-dependent difference δT between the electrode temperatures

$$\delta\rho(t) = -\rho_0 Q(\delta T/2T^2) e^{-i\omega t}, \quad Q \equiv H_2 - H_1, \quad \rho_0 = (1/Z) e^{-(H_1+H_2)/T}.$$

This gives

$$G_{\mathrm{th}}(\omega) = \frac{i}{2\hbar T^2} \int_0^\infty dt e^{i\omega t} \langle J(t) H_T Q - Q H_T J(t) \rangle$$

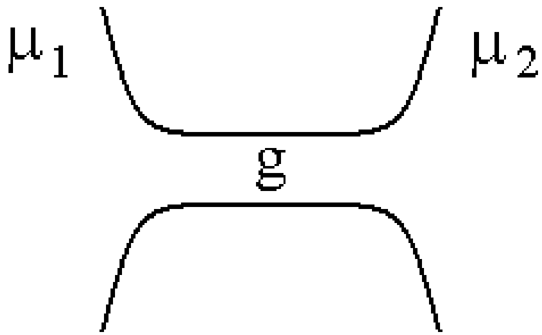
$$\mathrm{Re}\, G_{th}(\omega) = \pi^2 G T / 3e^2$$

Discussion

✓ Violation of the thermal FDT in "mesoscopic" context is a direct consequence of the finite relaxation energy.

✓ Lack of explicit treatment of this energy in the "thermal Kubo-Green formula" approache can invalidate its conclusions.

✓ Discrepancy between Kubo formula for conductivity and mesoscopic calculations of conductance is characteristic not only for the thermal, but also electric transport:



$$\sigma = g\, e^2/h$$

Conclusion

In mesoscopic heat transport, the fluctuations of the energy flux are not related to the thermal conductance by the fluctuation-dissipation theorem, but contains additional noise. The main physical consequence of this extra noise is that the fluctuations do not vanish at zero temperature together with the vanishing thermal conductance.