Understanding the stability of topologically-protected quantum computing proposals using spin glasses

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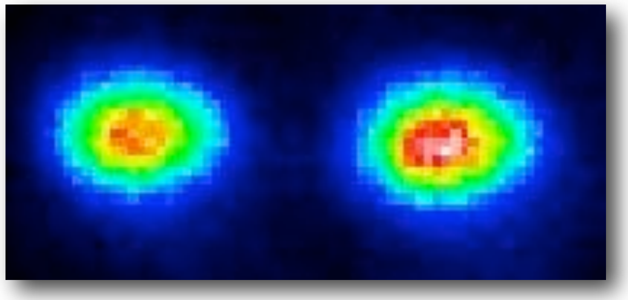
Disclaimer

- What is this talk about?
  - Understand the stability of topologically-protected quantum computing proposals using spin glasses.
  - New applications of the glass machinery.
- What is this talk not about?
  - A talk on quantum computing.
  - A talk on spin glasses.
- Brief outline:
  - Error correction using topology.
  - Topological color codes.
  - Stability against bit flip and measurement errors.

Motivation

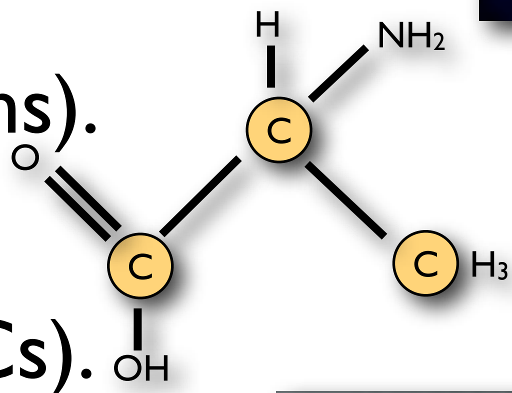
General motivation

- Why should we care about quantum computers?
  - Faster computations (prime decomposition, search algorithms…).
  - Quantum cryptography.
  - Quantum simulators (Fermionic models, …).



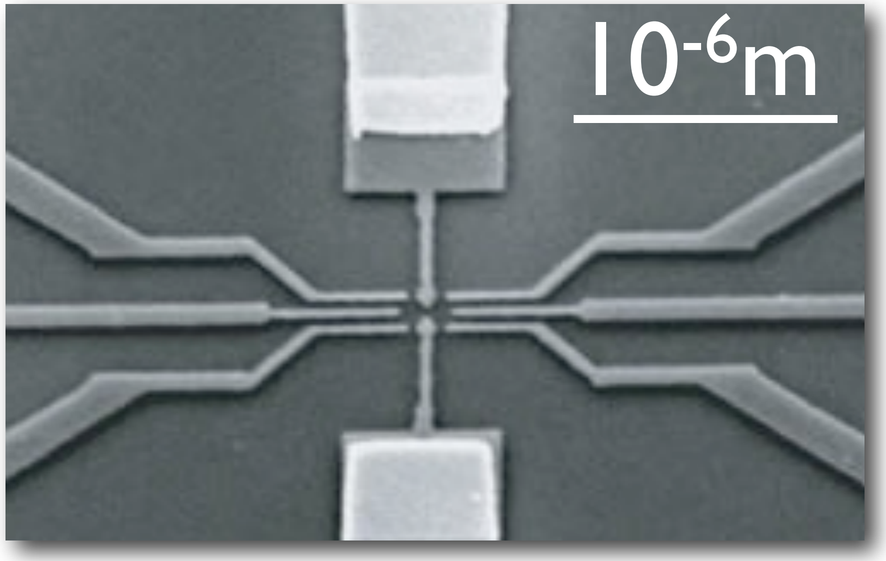
NIST (95)

- Current "working" implementations
  - Trapped ions (e.g., XOR via Be ions).
  - Nuclear Magnetic Resonance.
  - Solid state (quantum dots, JJAs, SCs).



IBM (01)

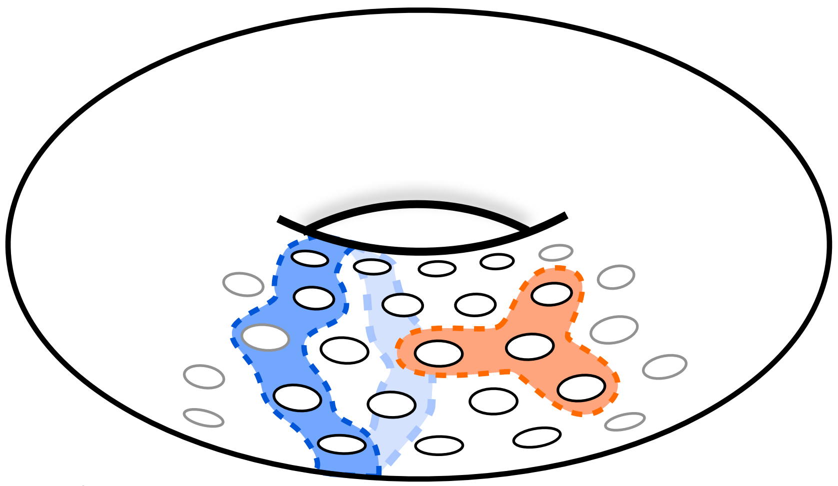
- Problems:
  - Scalability (~ 100 qubits).
  - Decoherence.



Decoherence

- Sources of decoherence:
  - Initial state preparation & faulty gate execution.
  - Local noise, interaction with a bath.

- How can we overcome decoherence?
  - Software: Better codes, smarter quantum error correction
  - Hardware: More qubits, error correction via redundancy
- Problem:
  - More qubits → more errors.
- Solution: Use topology!
  - Hardware encoding to protect states.
  - Software approach via active error correction.



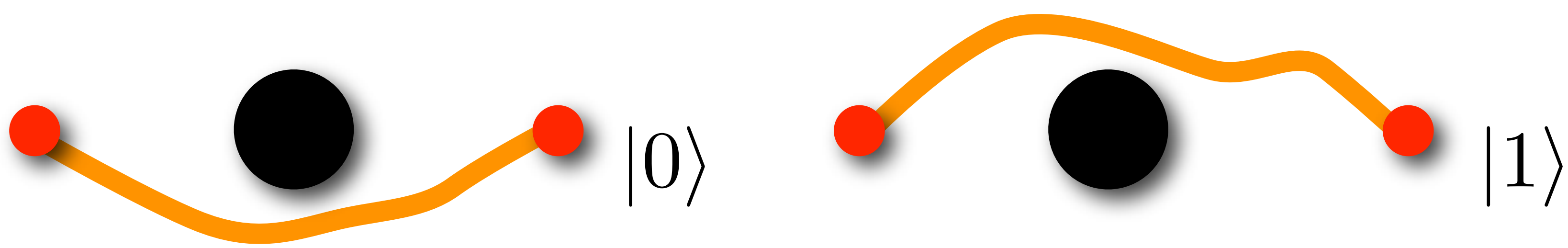
Using topology for quantum computation

- Fault-tolerant quantum computer:

  "A device that works efficiently even when its elementary components are imperfect."

  Preskill (97)

- Topologically-protected quantum computation:
  - Errors happen *locally* (e.g., bit flips).
  - Exploit the *global* (topological) properties of a system.
  - Introduce active error correction (here software level).



- First proposal: Toric code Kitaev Ann. Phys. (03)
  - Ground state is a (topological) loop gas.
  - CNOT, X and Z Pauli gates can be implemented, $p_c \sim 10.9\%$.

Dennis *et al.* (02)

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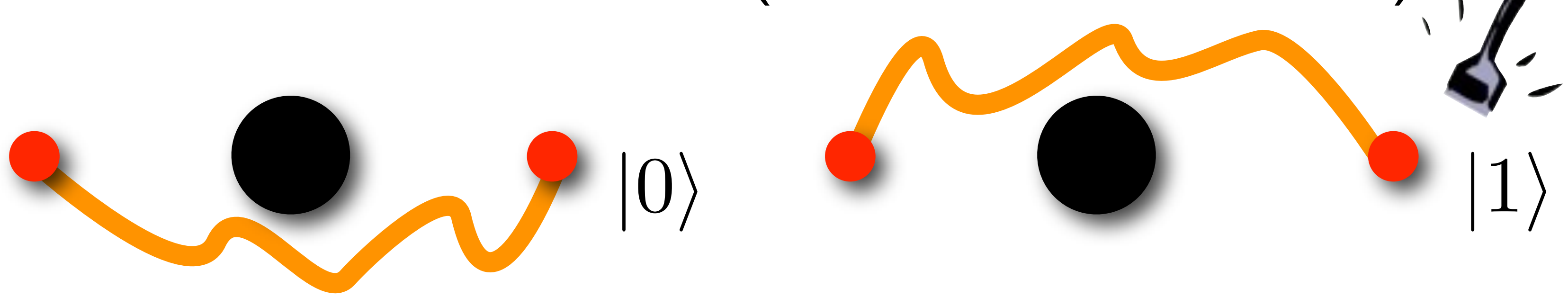

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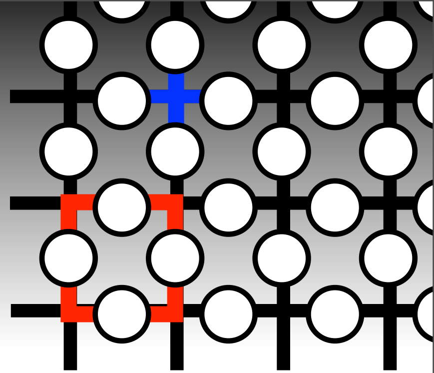
Topological color codes

Topological color codes

- Alternative: Topological color codes
  - Similar to the Kitaev proposal.
  - Encodes twice the number of qubits as a Toric (Kitaev) Code.
  - The whole Clifford group of gates can be implemented.
  - The phase gate *K* can be implemented transversally.

$$K = \begin{pmatrix} 1 & 0 \\ 0 & i \end{pmatrix}$$



Toric Code

Kitaev (97)

$$\mathcal{H} = -\sum_s A_s - \sum_p B_p$$

stabilizer

$$A_s = \prod_{j \in +_s} Z_j$$

$$B_p = \prod_{j \in \square_p} X_j$$

- How do color codes work?
  - Defined in terms of a (local) stabilizer group.
  - Measurement detects the errors.
  - Active error correction applies (up to a threshold).

Bombin & Martin-Delgado, PRL (06)

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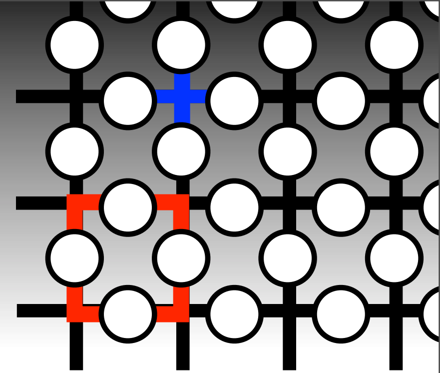

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Do wider computational capabilities imply a lower resistance to noise?

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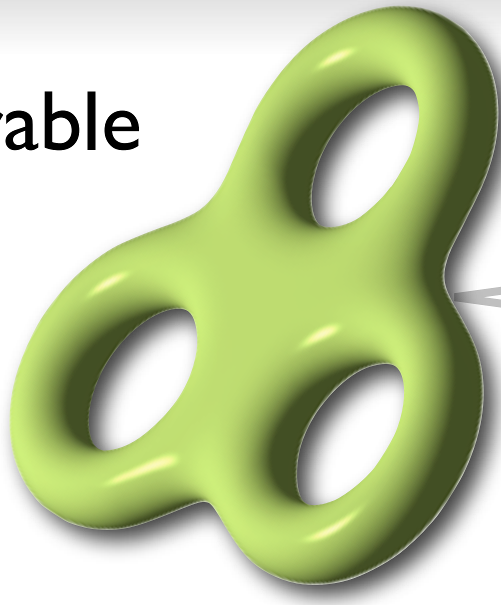
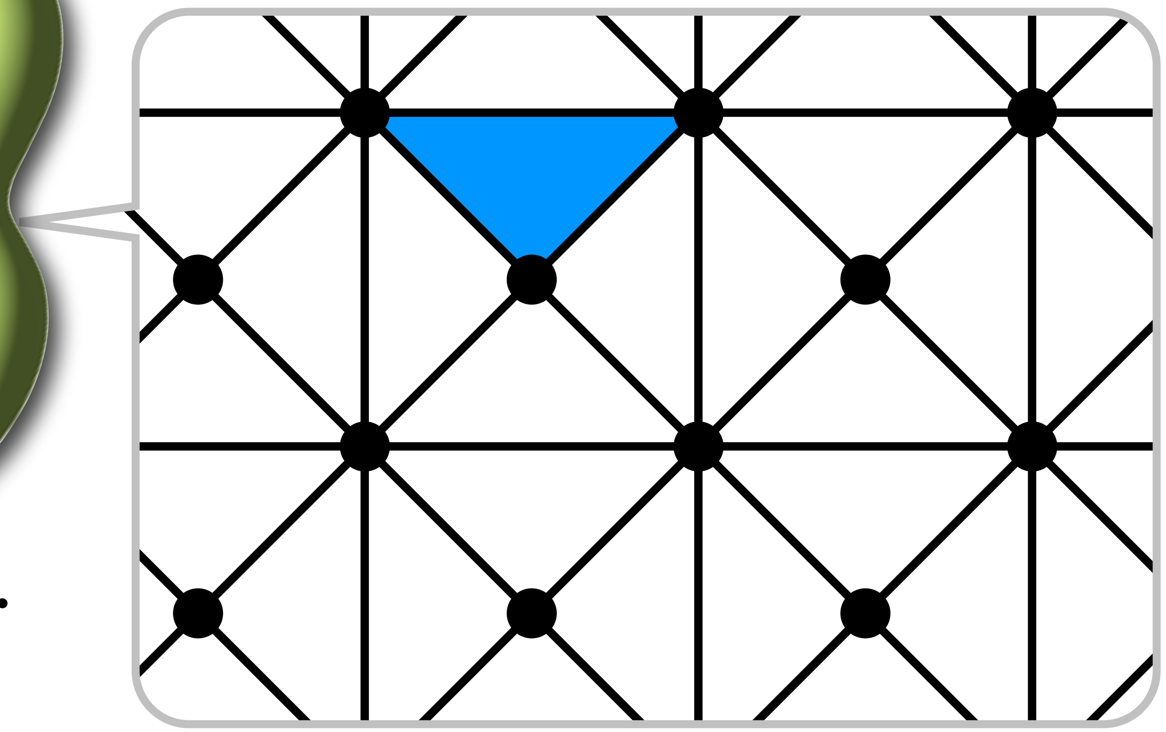
Bombin & Martin-Delgado, PRL (06)

Topological color codes (the details)

Bombin & Martin-Delgado, PRL (06)





- Start from a 2D 3-colorable triangular lattice.
- Embed the lattice in a nontrivial compact surface.
- 4/8 triangles per vertex (phase gate).
- A qubit is placed on each triangle.
- Stabilizer group:

$$X_v := \bigotimes_{\triangle : v \in \triangle} X_\triangle \qquad Z_v := \bigotimes_{\triangle : v \in \triangle} Z_\triangle$$

Note: vertex operators pairwise commute and square to unity.

- The code is defined on the subspace with $X_v = Z_v = 1 \quad \forall\, v$.
- Error syndrome: collection of ±1 eigenvalues.
- X (bit-flip) and Z (phase) operators do not mix: *study only bit flips*.

Bit-flip errors

Threshold: map to a statistical model

- Error correction is achievable if:

$$\sum_E P(E)P(\overline{E}|\partial E) \to 1 \qquad N \to \infty$$

Dennis *et al.*, J Math Phys (02)

  - $P(E) \propto [p/(1-p)]^{|E|}$ — E is a bit-flip error with probability p
  - $P(\overline{E}|\partial E)$ — probability that a syndrome ∂E was caused by an error in the homology class $\overline{E}$.
- Mapping:
  - Nishimori line: $\exp(-2J) = p/(1-p)$ $\longrightarrow$ $P(E) \propto \exp(\sum_\triangle \tau_\triangle)$
  - $\tau_\triangle = \pm 1$; negative when $\triangle \in E$.
  - Insert classical spin variables $S^i = \pm 1$ at the vertices to obtain:

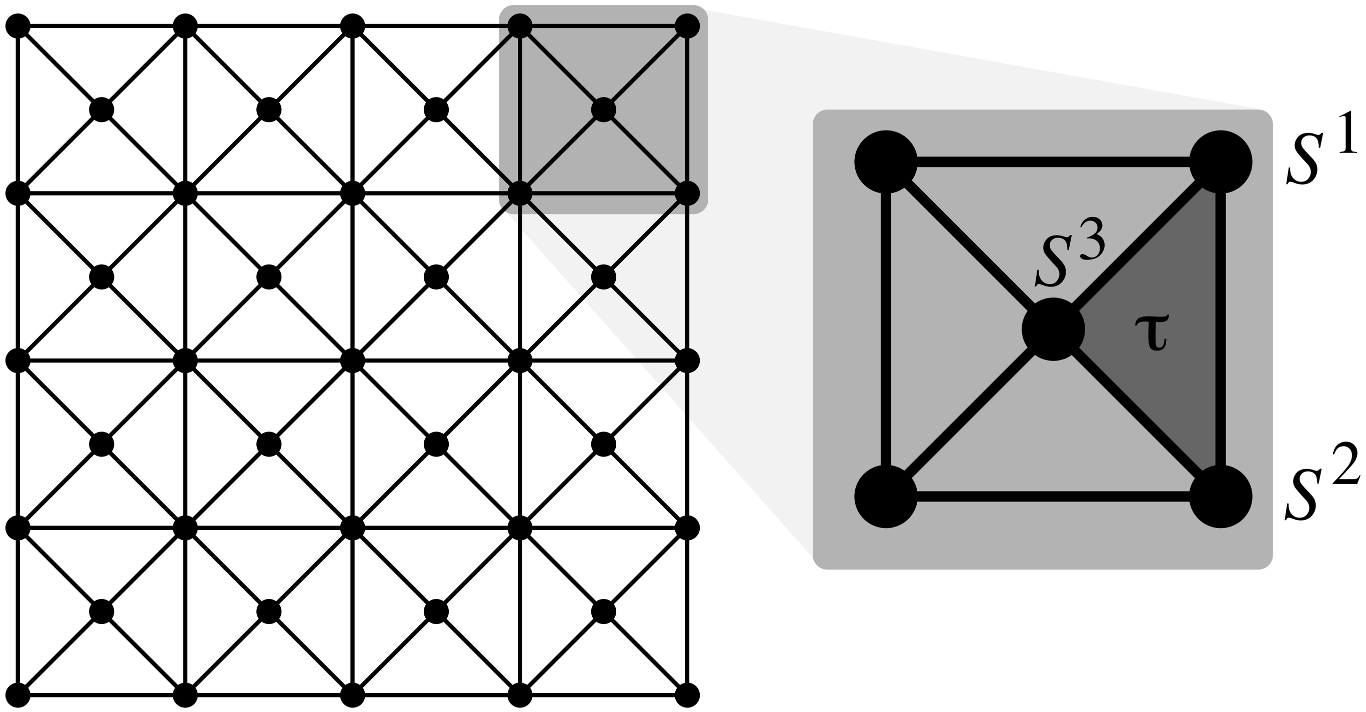
$$P(\bar{E}) \propto Z[J,\tau] := \sum_S e^{J\sum_{\langle ijk\rangle} \tau_{ijk} S^i S^j S^k}$$

Katzgraber *et al.*, , PRL (09), PRA (10)

Bit-flip errors: random 3-body Ising model

- Hamiltonian:

$$\mathcal{H} = J \sum_{\langle ijk \rangle} \tau_{ijk} S^i S^j S^k$$



- Details:
  - Ising spins *S* on the vertices of a 2D Union Jack lattice.
  - A bit-flip error corresponds to $\tau_{ijk} = -1$ with probability *p*.
  - *p* > 0: glassy Ising model (3-body interactions).
  - Note: the Toric Code maps onto a 2D random-bond Ising model.

- Error threshold:
  - Compute the p–T_c phase diagram of the model.
  - p_c corresponds to the critical *p* along the Nishimori line where *ferromagnetic* order is *lost*. Dennis *et al.*, J Math Phys (02)

Interlude: Algorithms

Monte Carlo & the Metropolis algorithm

THE JOURNAL OF CHEMICAL PHYSICS VOLUME 21, NUMBER 6 JUNE, 1953

Equation of State Calculations by Fast Computing Machines

Nicholas Metropolis, Arianna W. Rosenbluth, Marshall N. Rosenbluth, and Augusta H. Teller,
Los Alamos Scientific Laboratory, Los Alamos, New Mexico

and

Edward Teller,* *Department of Physics, University of Chicago, Chicago, Illinois*
(Received March 6, 1953)

A general method, suitable for fast computing machines, for investigating such properties as equations of state for substances consisting of interacting individual molecules is described. The method consists of a modified Monte Carlo integration over configuration space. Results for the two-dimensional rigid-sphere system have been obtained on the Los Alamos MANIAC and are presented here. These results are compared to the free volume equation of state and to a four-term virial coefficient expansion.

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II. THE GENERAL METHOD FOR AN ARBITRARY POTENTIAL BETWEEN THE PARTICLES

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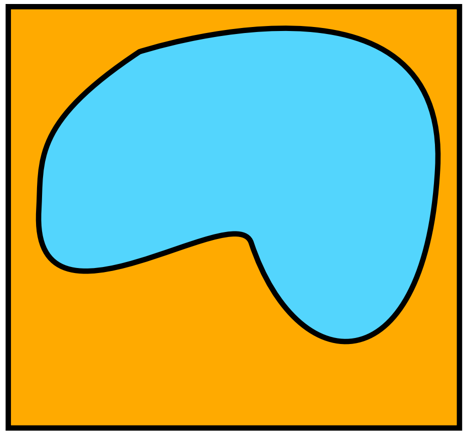
Standard Monte Carlo

- **Goal:** Compute a thermodynamic average of an observable *O*:

$$\langle O \rangle = \sum_n P_n^{\mathrm{eq}} O_n \qquad P_n^{\mathrm{eq}} = \frac{e^{\beta E_n}}{\sum_n e^{-\beta E_n}}$$



- **Problem:** The number of states is exponentially large (*N* Ising spins ⟶ 2^N states).
- **Solution:** Statistically sample a subset of smartly chosen states but with a statistical error.

  - Select the states according to P_n^{eq} to obtain a Markov chain for $\langle O \rangle_{\mathrm{est}}$

$$\langle O \rangle_{\mathrm{est}} = \frac{1}{M} \sum_i^M O_i$$

  M is the number of trials.

  - Metropolis algorithm: accept new configuration

$$\mathrm{if}(e^{-\Delta E/T} > \mathrm{rand}()) \qquad P_{\mathrm{accept}} = \min(1, e^{-\Delta E/T})$$

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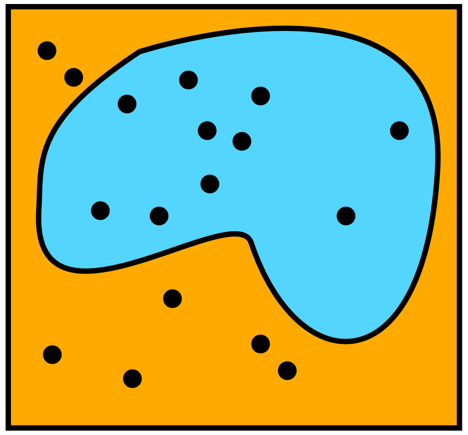

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Why does simple Monte Carlo fail here?

- The systems we are interested in have rugged energy landscapes.
- At low temperature, when ΔE is large

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is "never" accepted.

- How can we resolve the problem?
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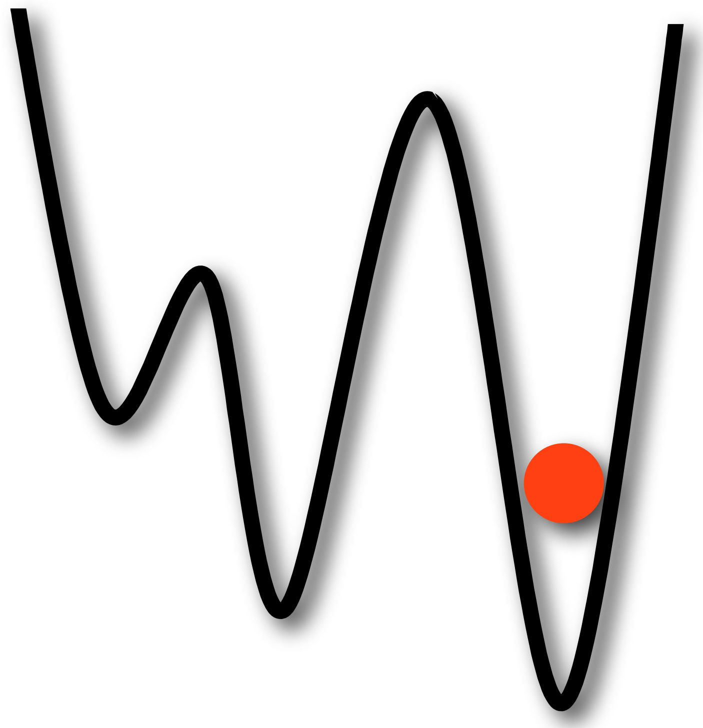

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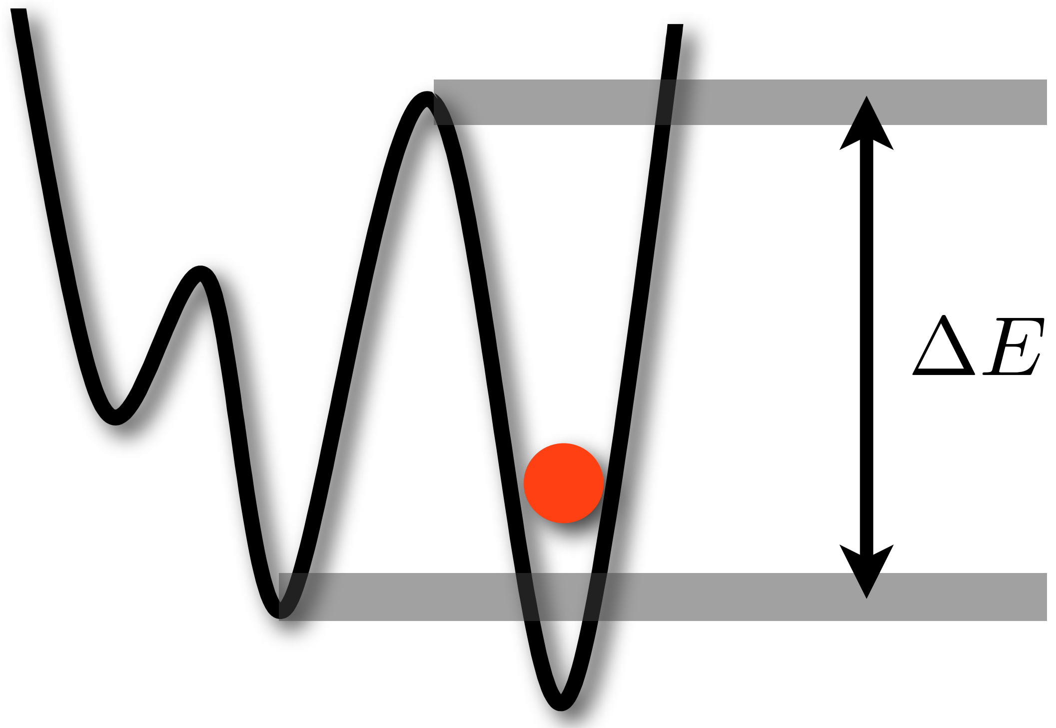

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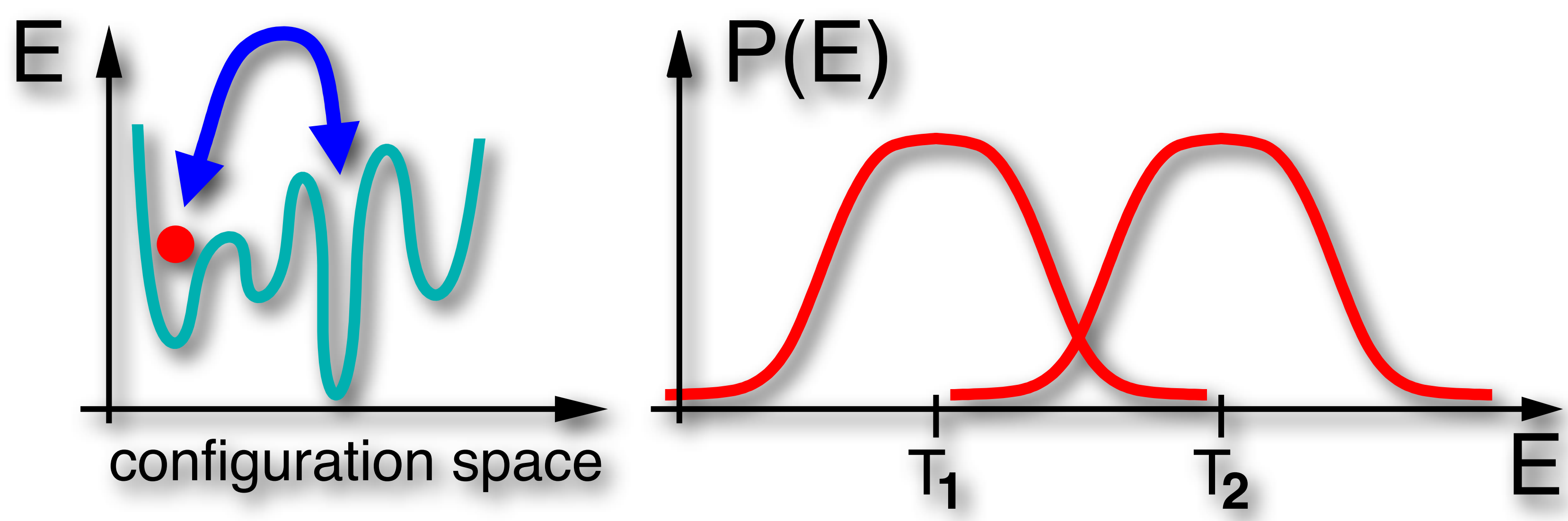
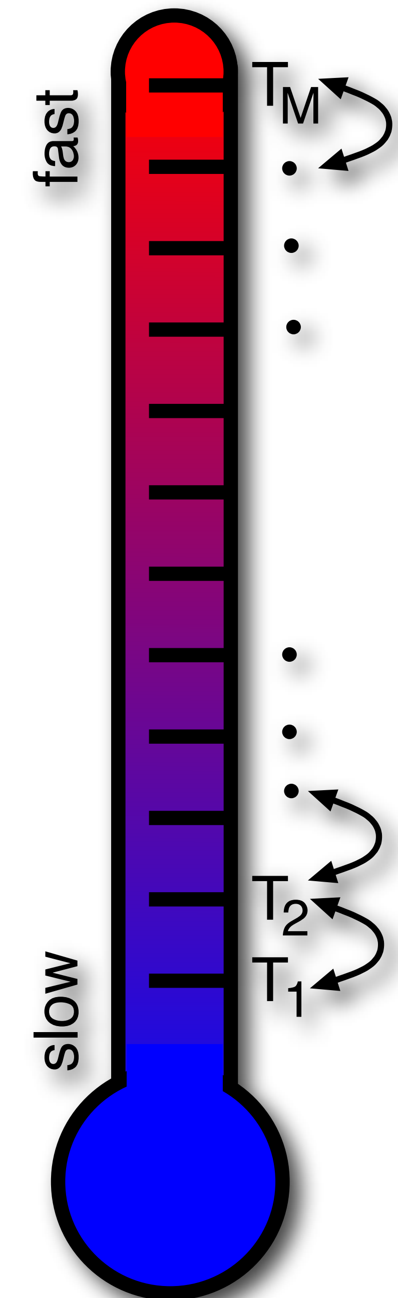
Exchange (parallel tempering) Monte Carlo

Hukushima & Nemoto (96)
Geyer (91)

- Efficient algorithm to treat spin glasses at finite T.
- Idea:
  - Simulate M copies of the system at different temperatures with $T_{\max} > T_c$ (typically $T_{\max} \sim 2T_c^{MF}$).
  - Allow swapping of neighboring temperatures: easy crossing of barriers. see, e.g., Katzgraber *et al.*, JSTAT (06)





- Extremely fast equilibration at low temperatures ($\sim 10^4$).
- Transition probabilities:

$$\mathcal{T}[(E_i, T_i) \to (E_{i+1}, T_{i+1})] = \min\{1, \exp[\Delta E_{i+1,i} \Delta\beta_{i+1,i}]\}$$

Back to bit-flip errors…

Probing criticality: correlation length

- Study the finite-size two-point correlation function.

Cooper (82)

- *k*-space susceptibility of the magnetization...

$$\chi(\mathbf{k}) = \frac{1}{N} \sum_{ij} \langle S^i S^j \rangle_T e^{i\mathbf{k}(\mathbf{R}_i - \mathbf{R}_j)}$$

- Perform an Ornstein-Zernicke approximation...

$$[\chi(k)/\chi(0)]^{-1} = 1 + \xi^2 k^2 + \mathcal{O}[(\xi k)^4]$$

- Compute the two-point correlation function:

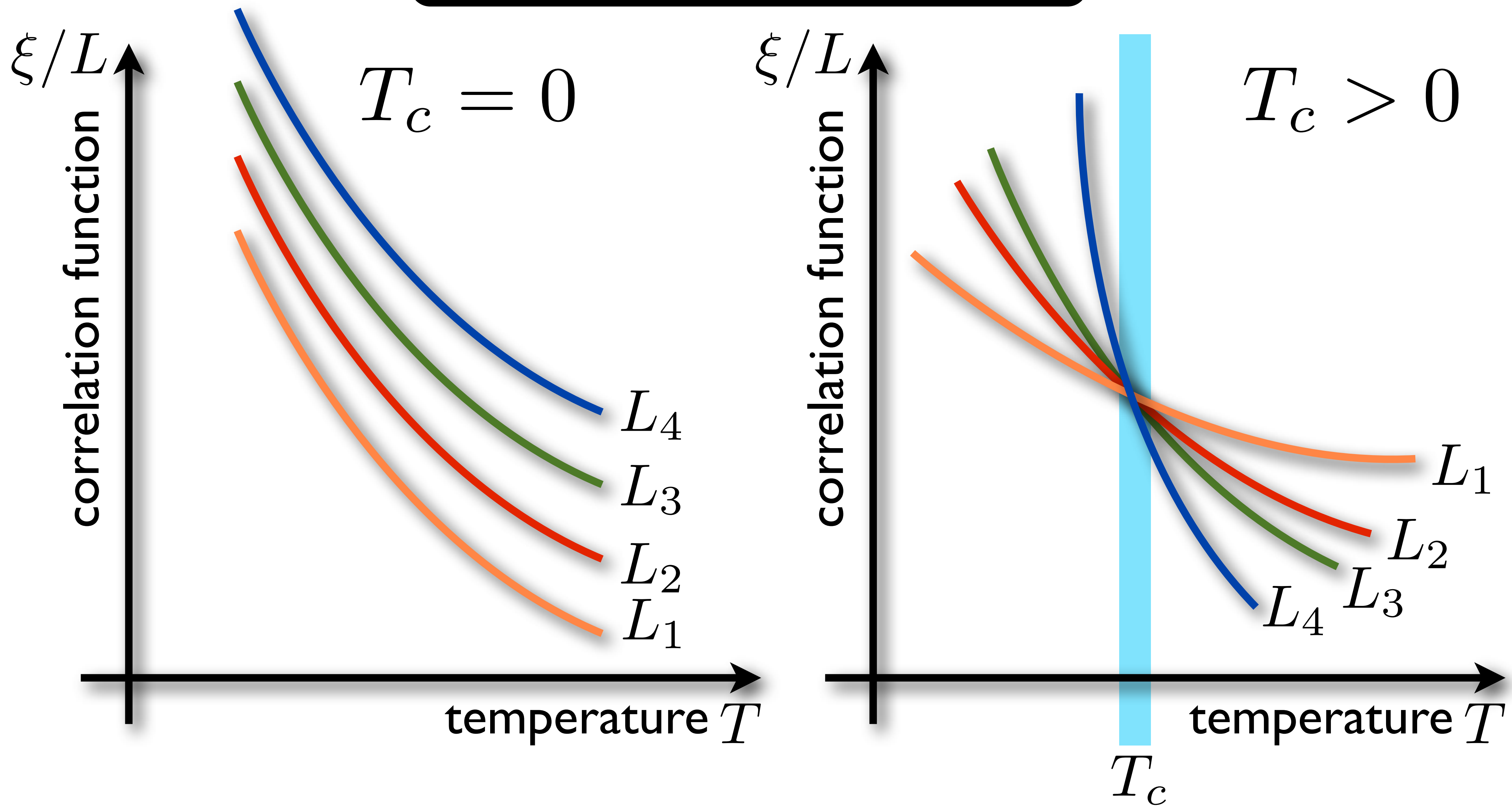
$$\xi = \frac{1}{2\sin(k_{\mathrm{min}}/2)} \sqrt{\frac{[\chi(0)]_{\mathrm{av}}}{[\chi(k_{\mathrm{min}})]_{\mathrm{av}}} - 1}$$

Probing criticality: correlation length

Cooper (82)

- Study the finite-size two-point correlation function.
- Scaling behavior:

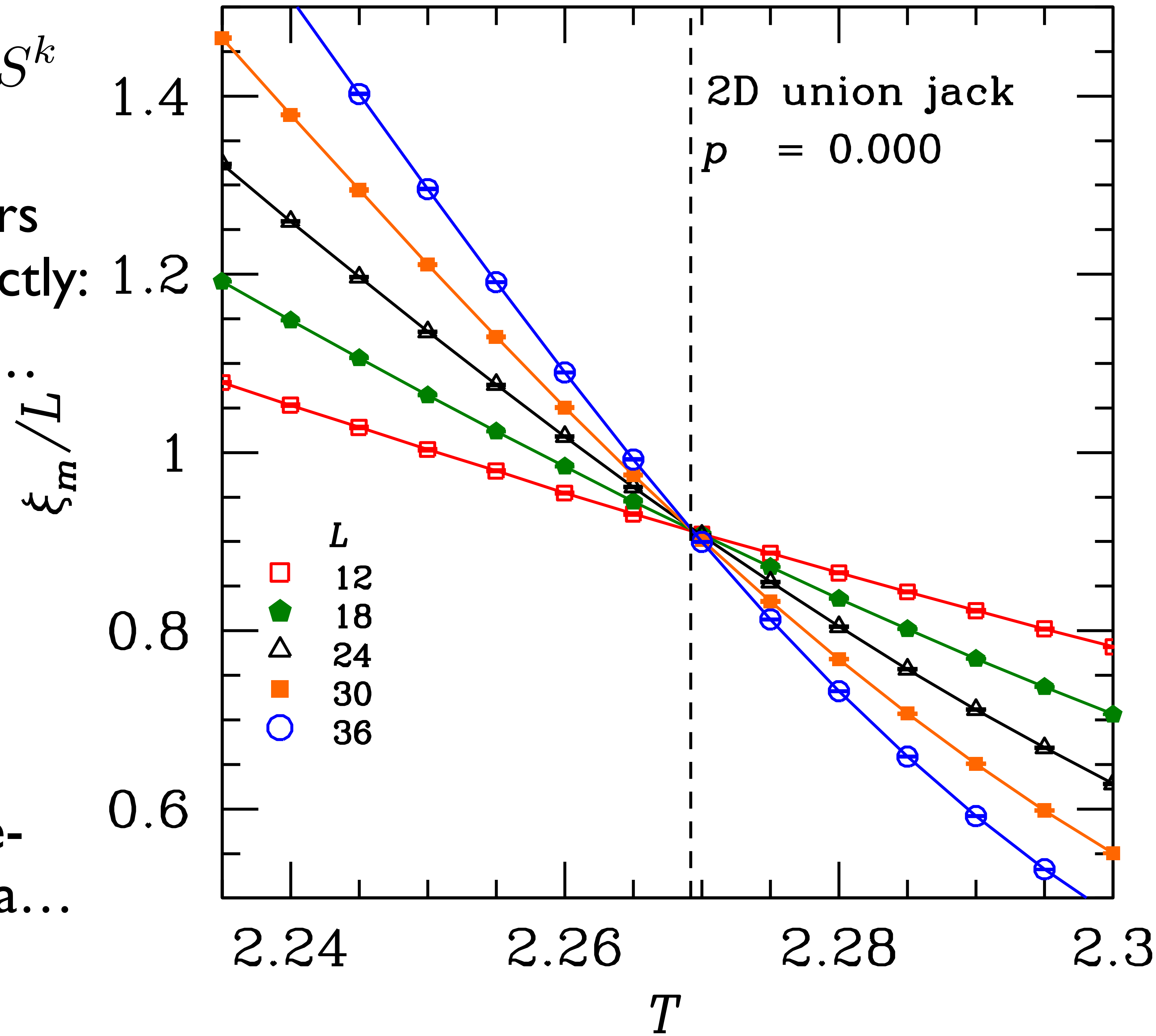
$$\frac{\xi}{L} = \tilde{X}\left(L^{1/\nu}[T - T_c]\right)$$



Benchmark case: $p = 0$

$$\mathcal{H} = J \sum_{\langle ijk \rangle} \tau_{ijk} S^i S^j S^k$$

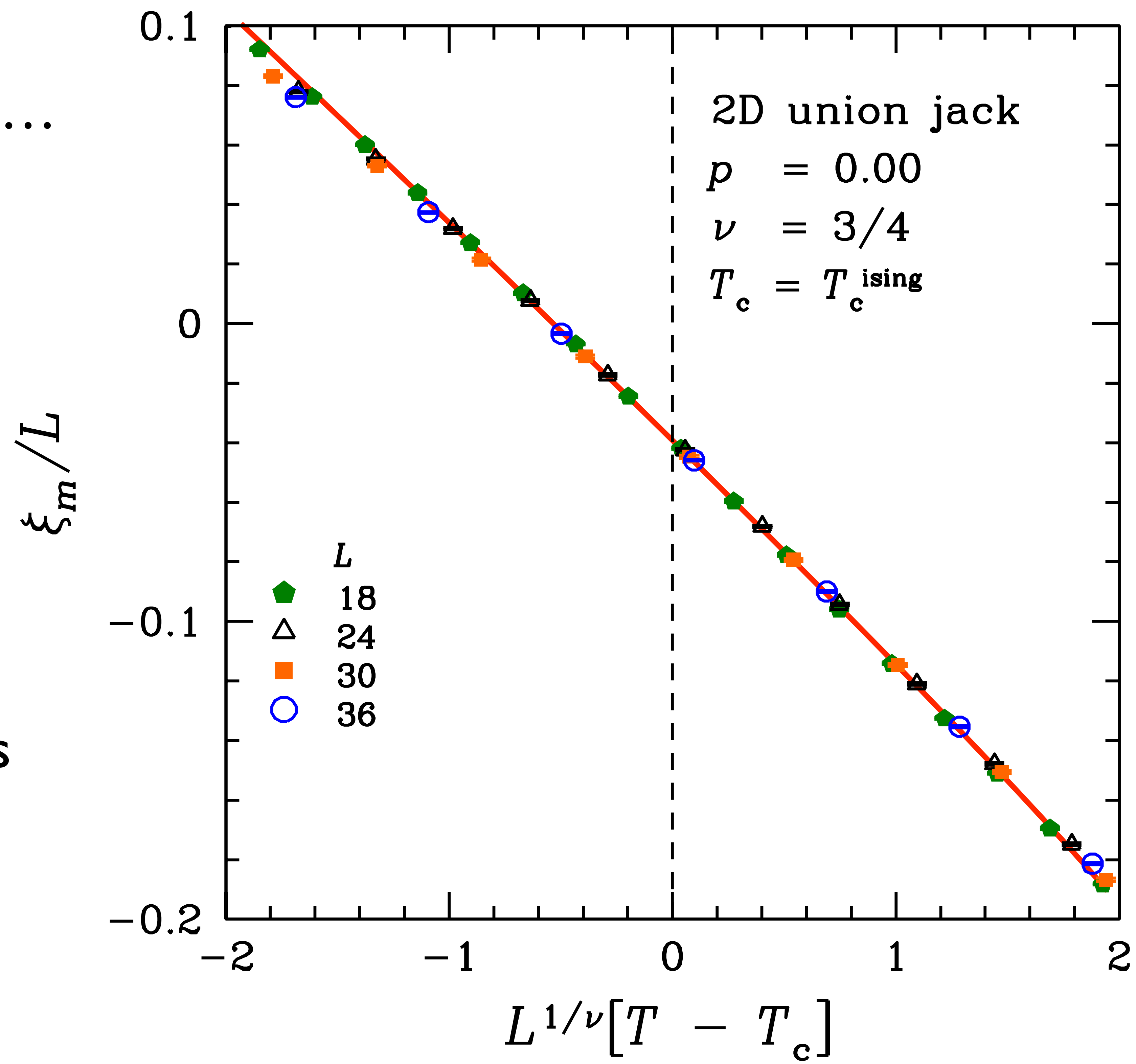
- The critical parameters can be computed exactly:
  - $T_c = T_c^{\text{ising}} = 2.269\ldots$
  - $\nu = 3/4$
  - $\alpha = 1/2$
- Agreement with exact results.
- Next: Perform a finite-size scaling of the data…



Scaling with known exponents ($p = 0$)

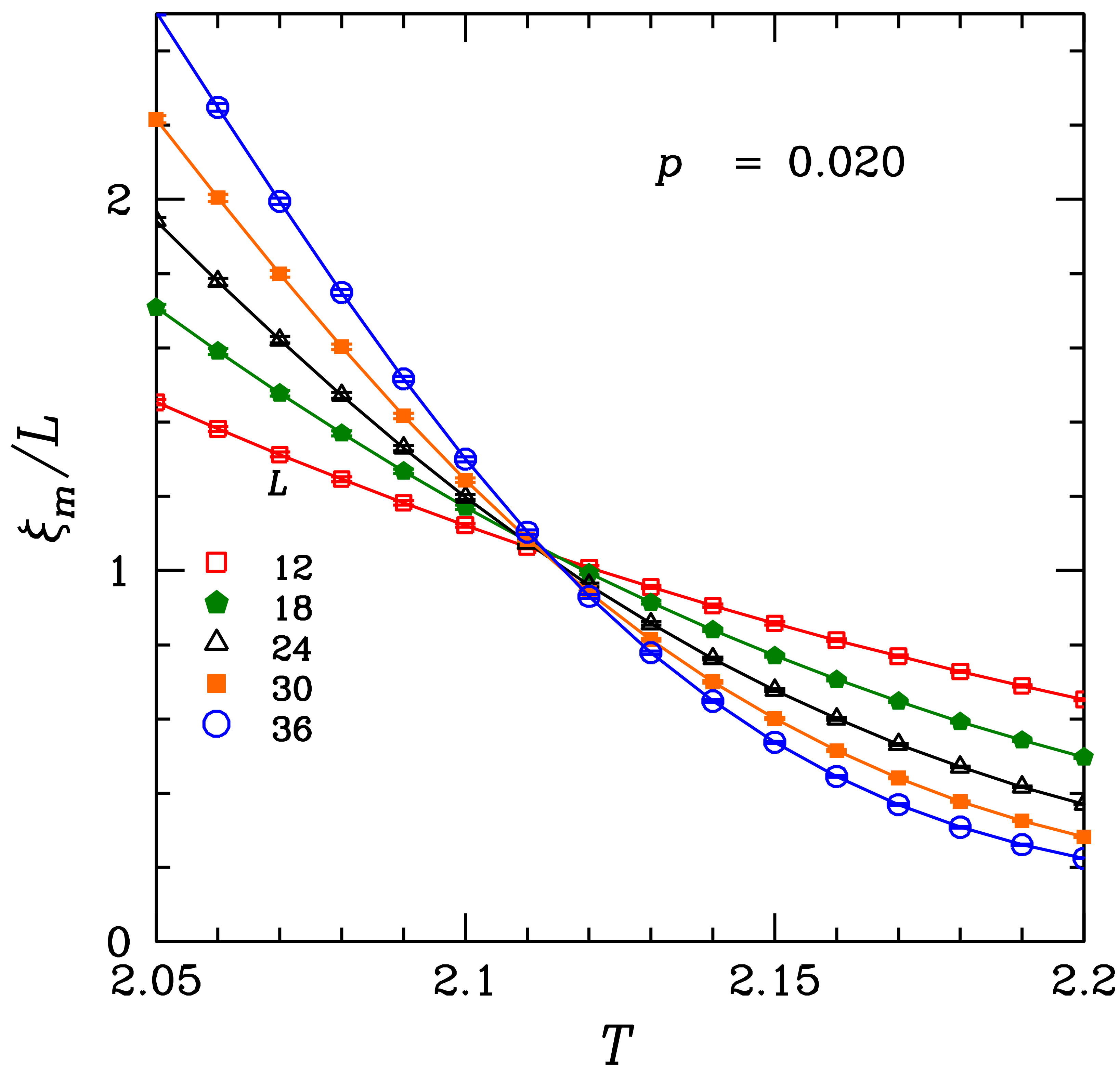
- Fixed parameters:
  - $T_c = T_c^{\text{ising}} = 2.269\ldots$
  - $\nu = 3/4$
- The finite-size scaling is perfect.
- Next: introduce errors by flipping bits

$$\tau_{ijk} \longrightarrow -\tau_{ijk}$$



Introduce qubit errors with $p > 0$

- For each value of p compute T_c.
- Corrections around $p \sim 0.108$.



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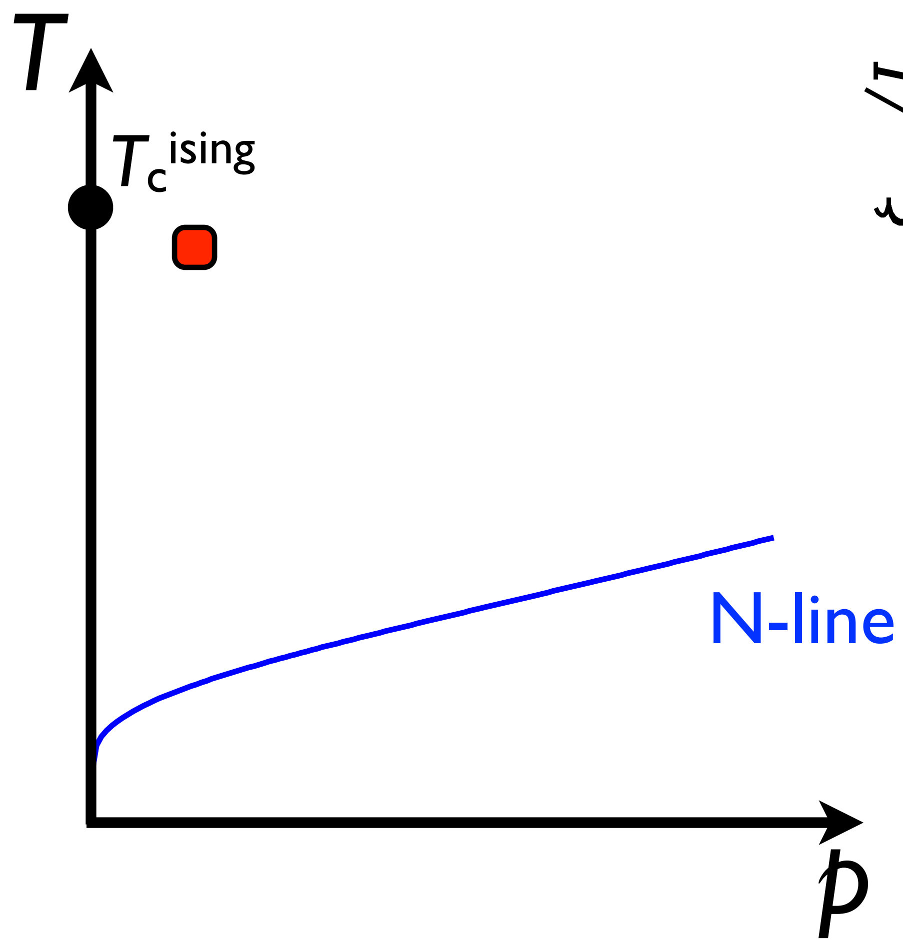

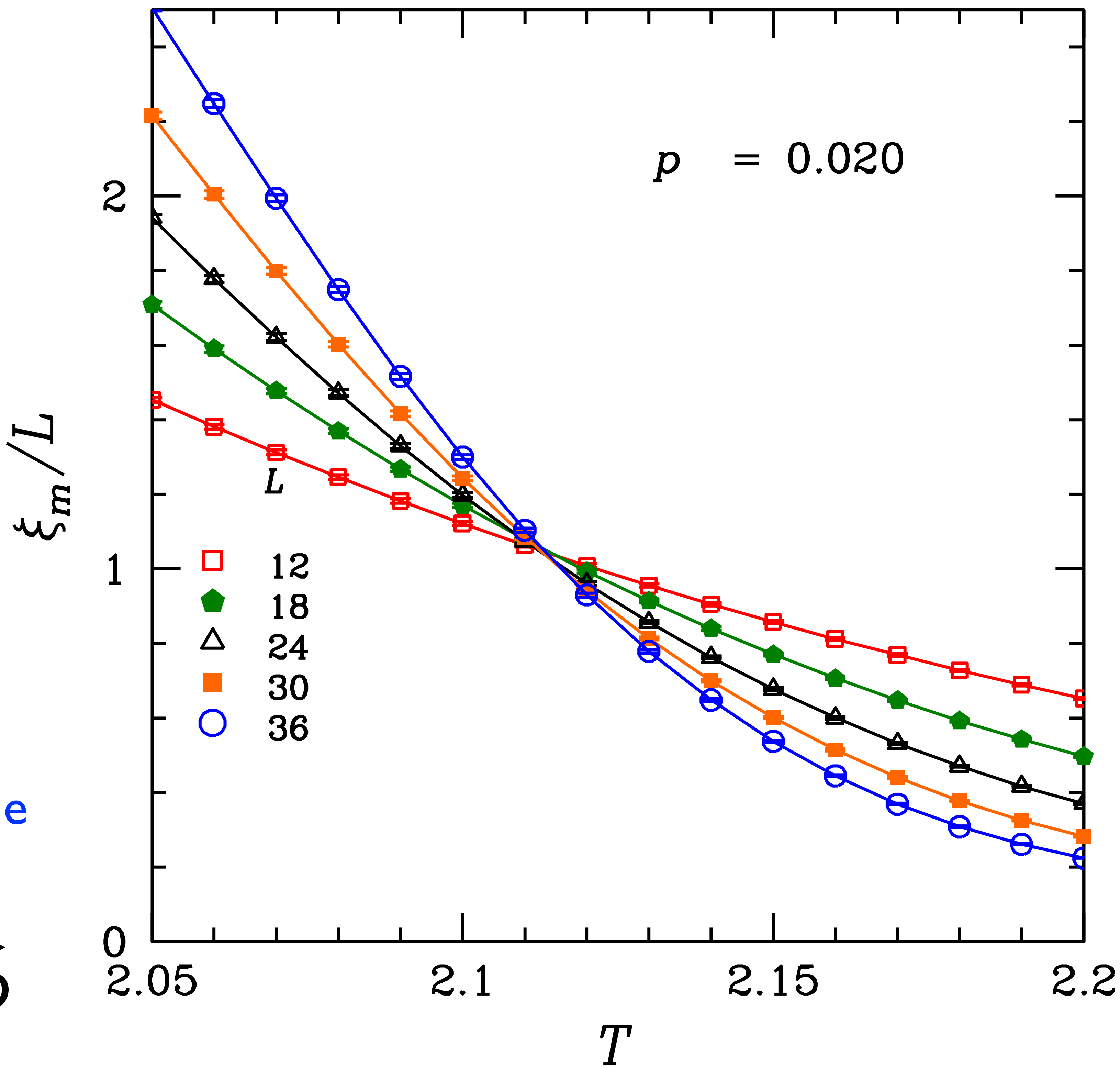

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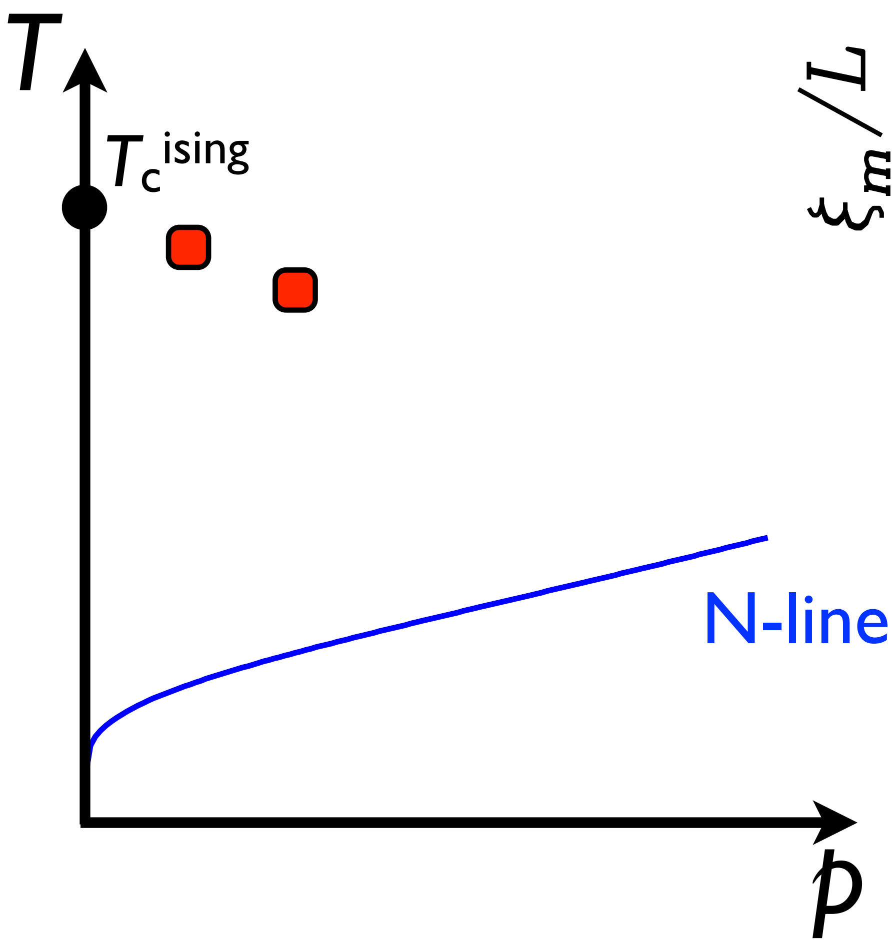

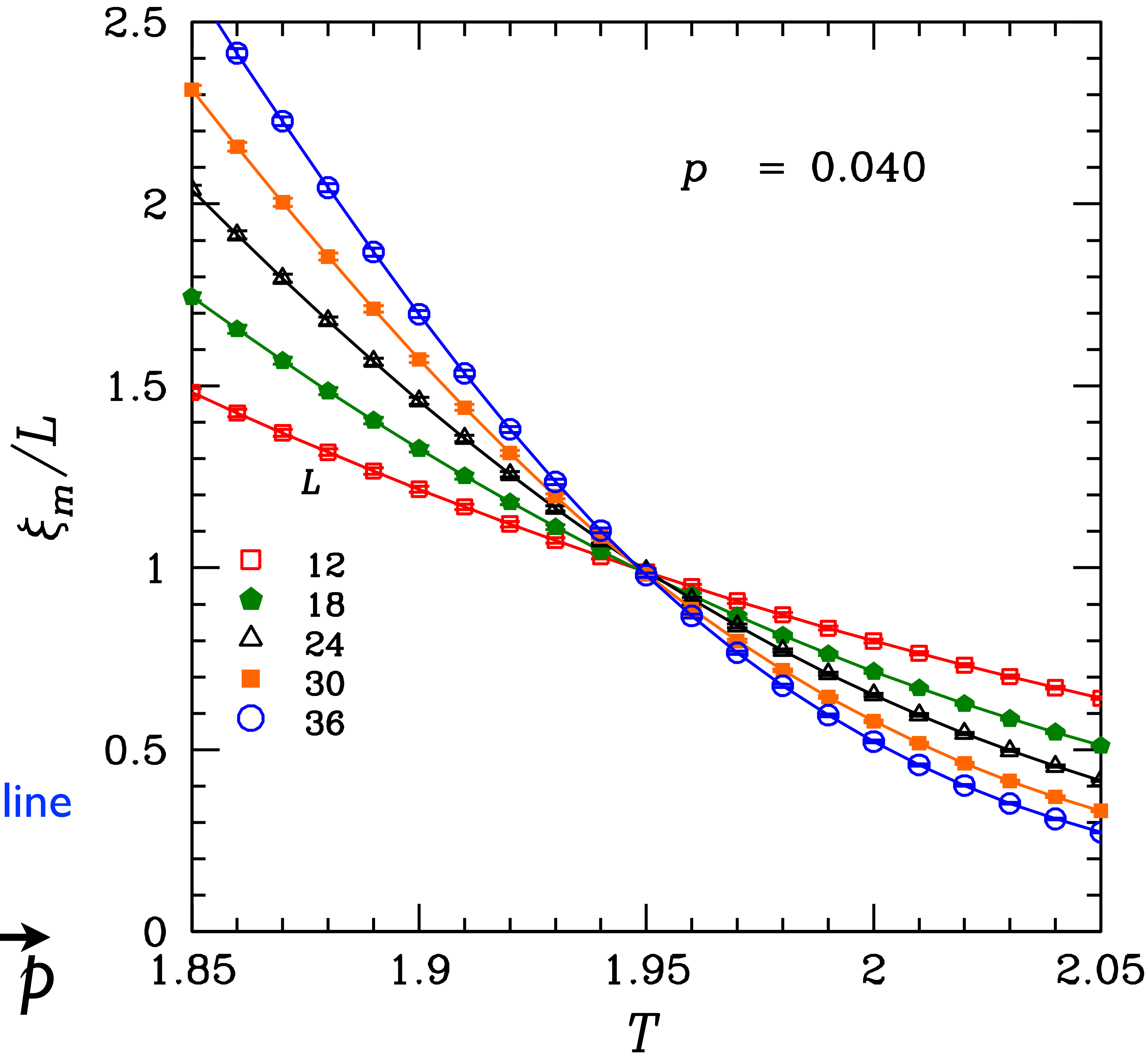

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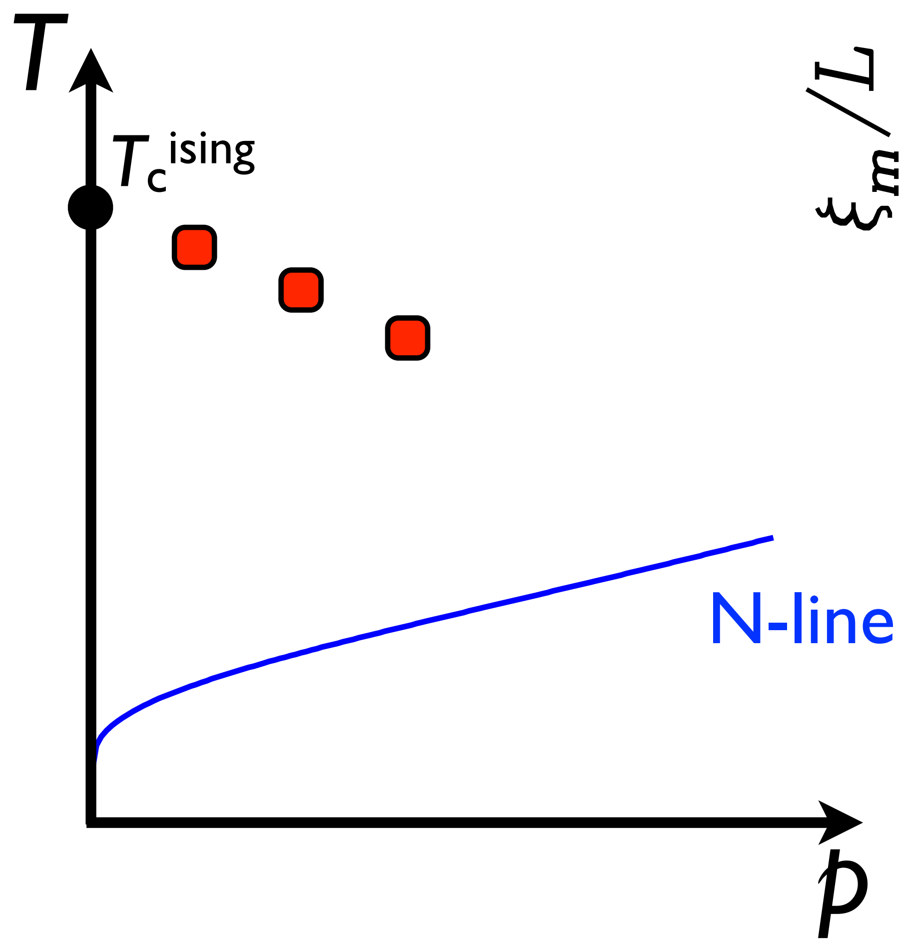

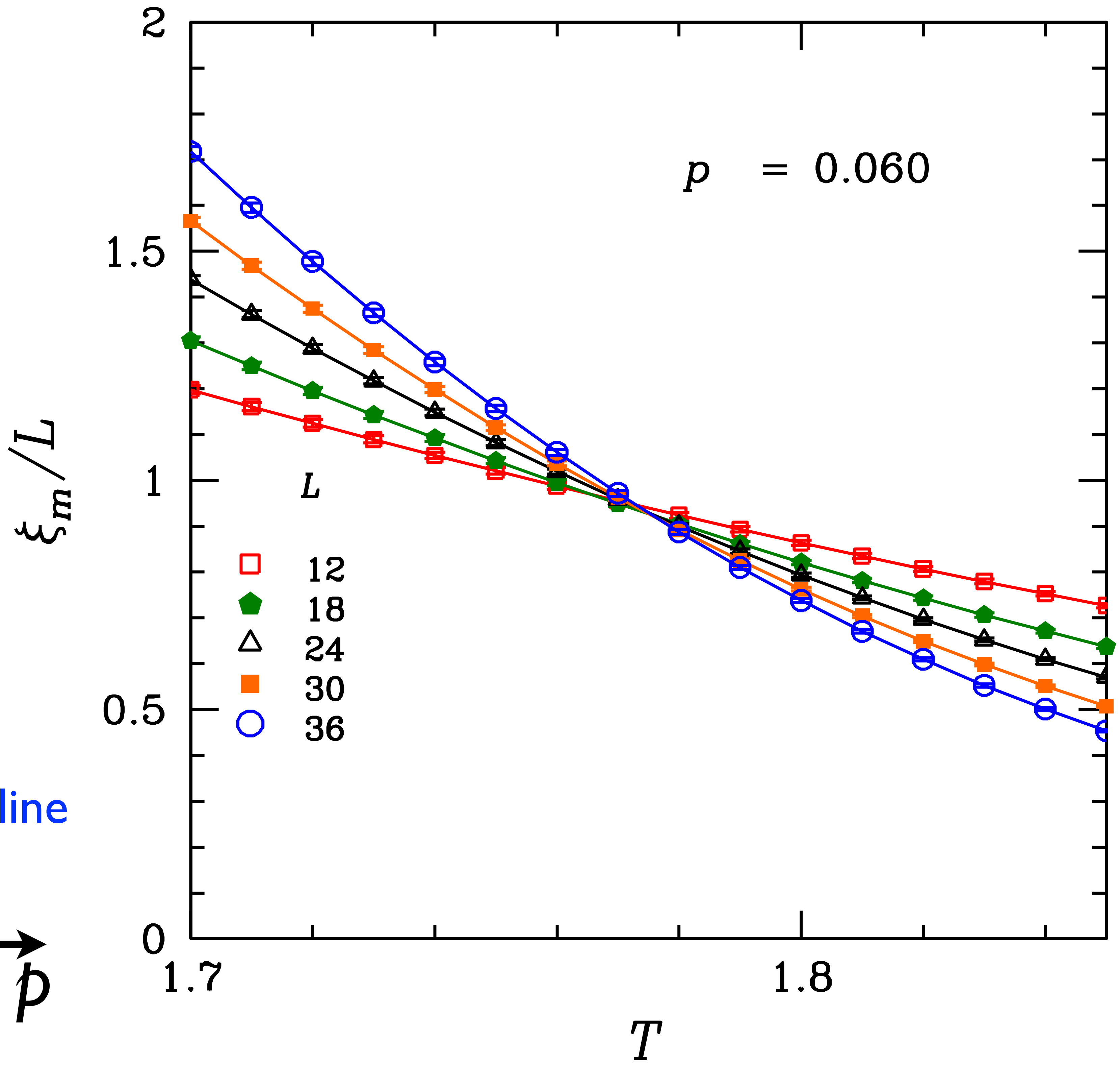

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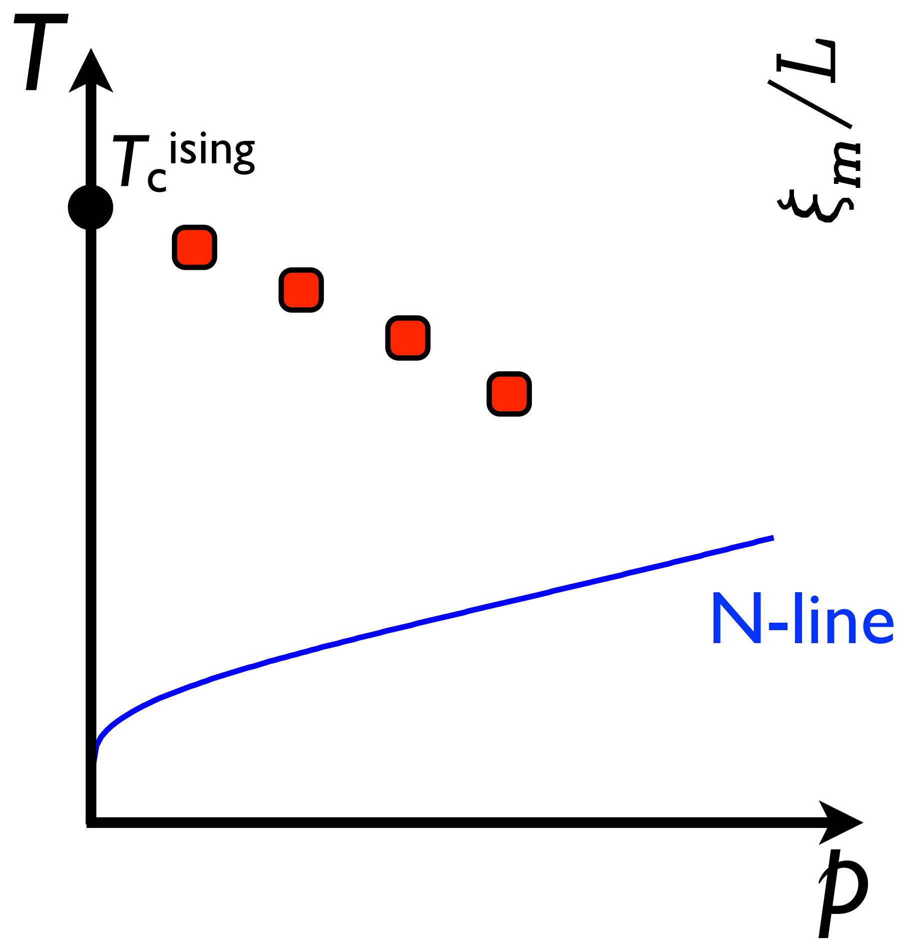

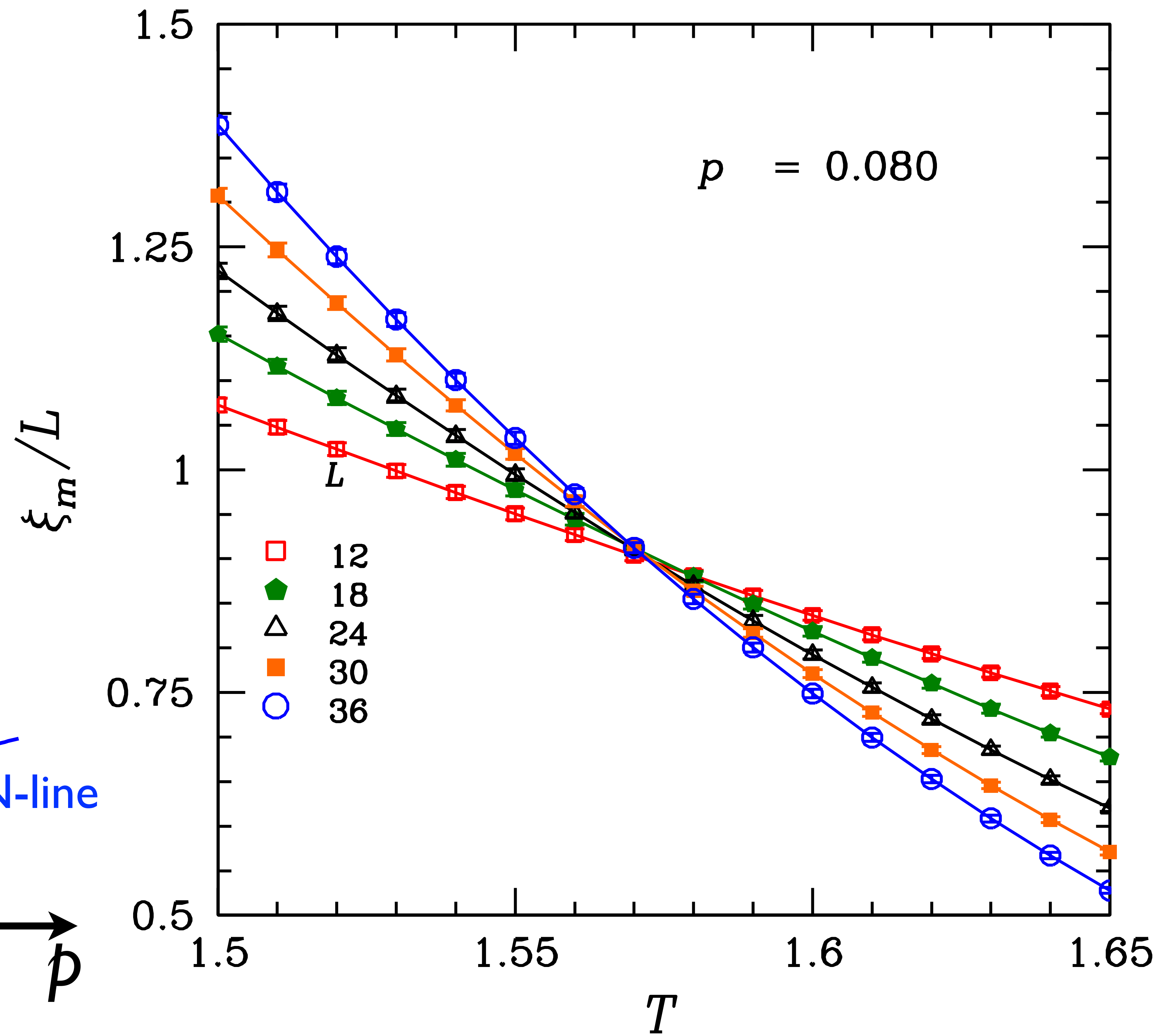

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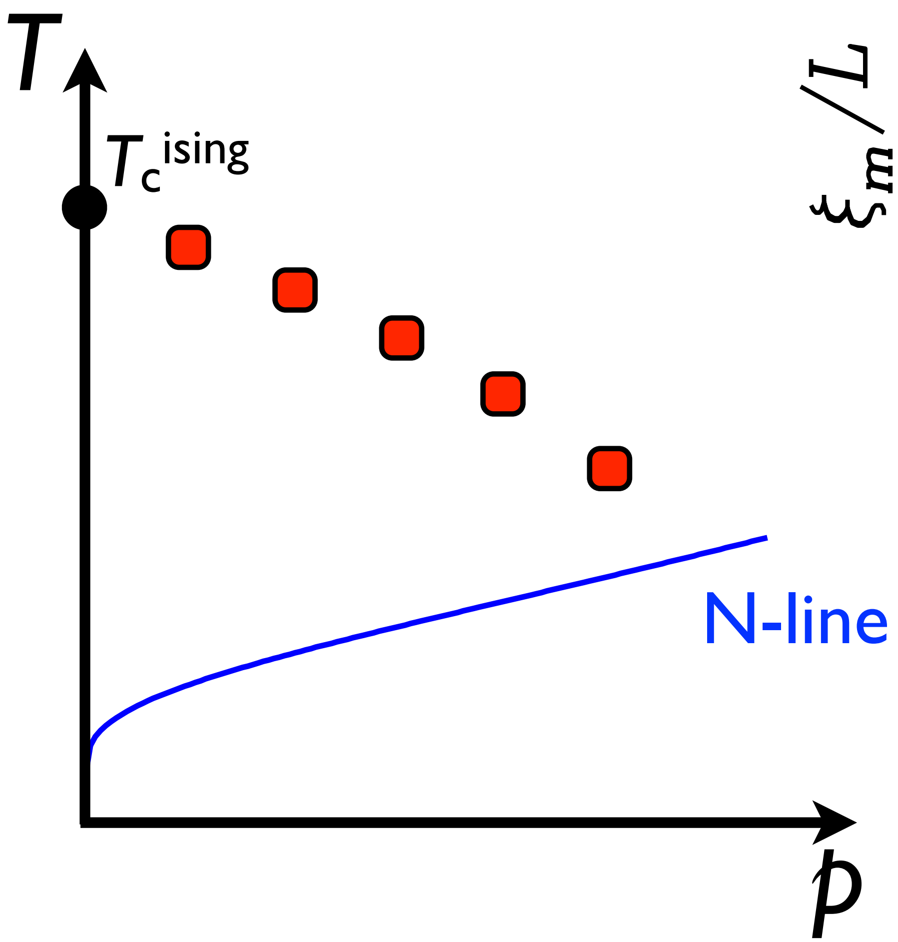

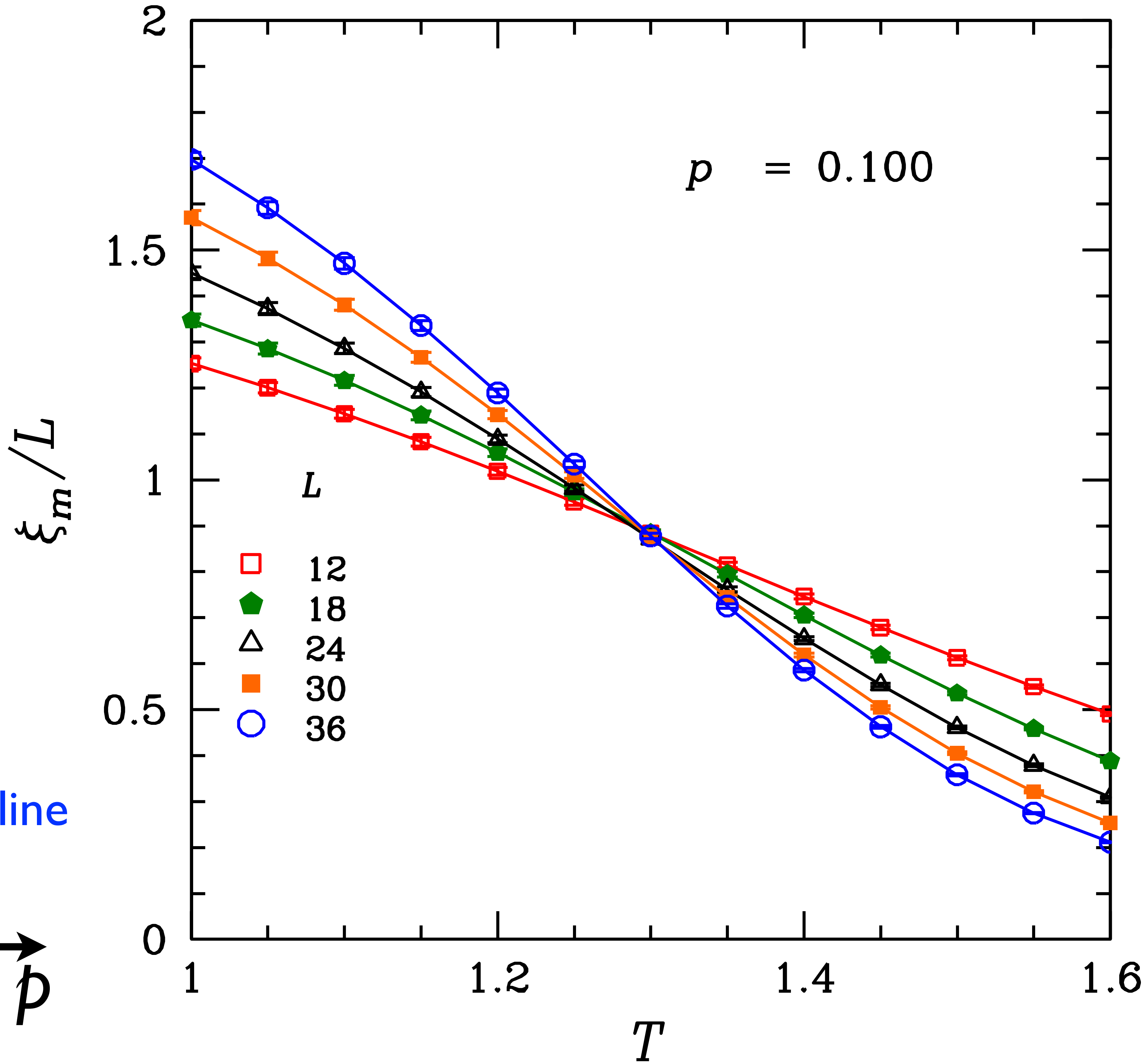

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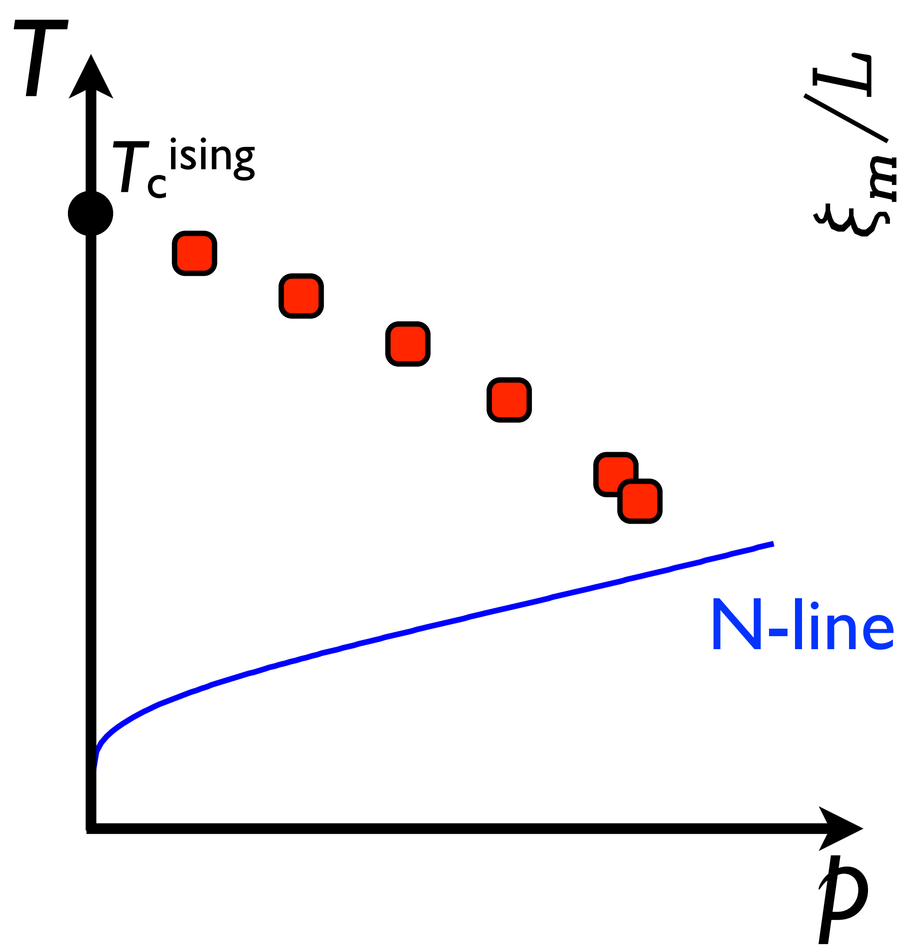

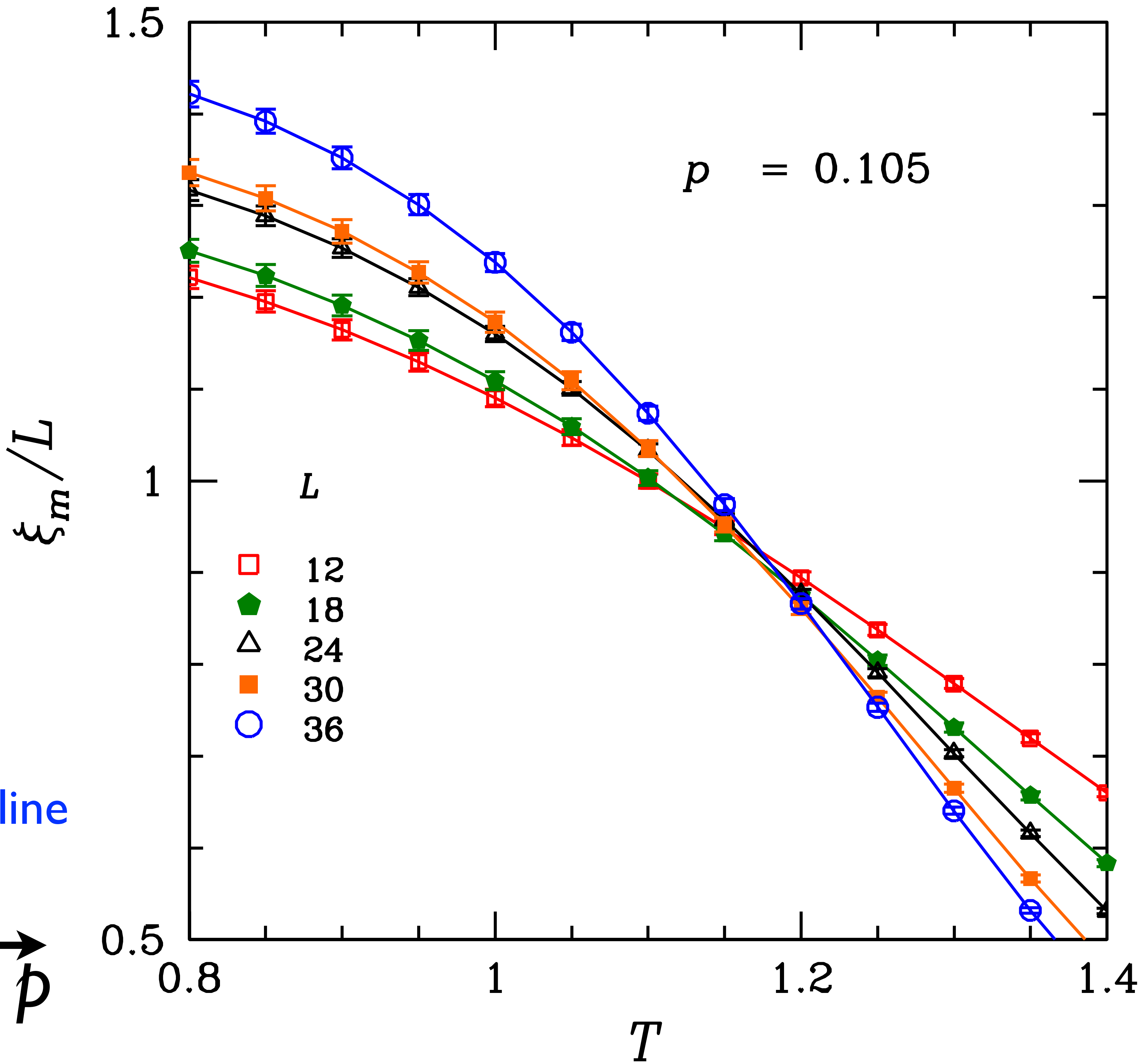

Introduce qubit errors with $p > 0$

- For each value of p compute T_c.
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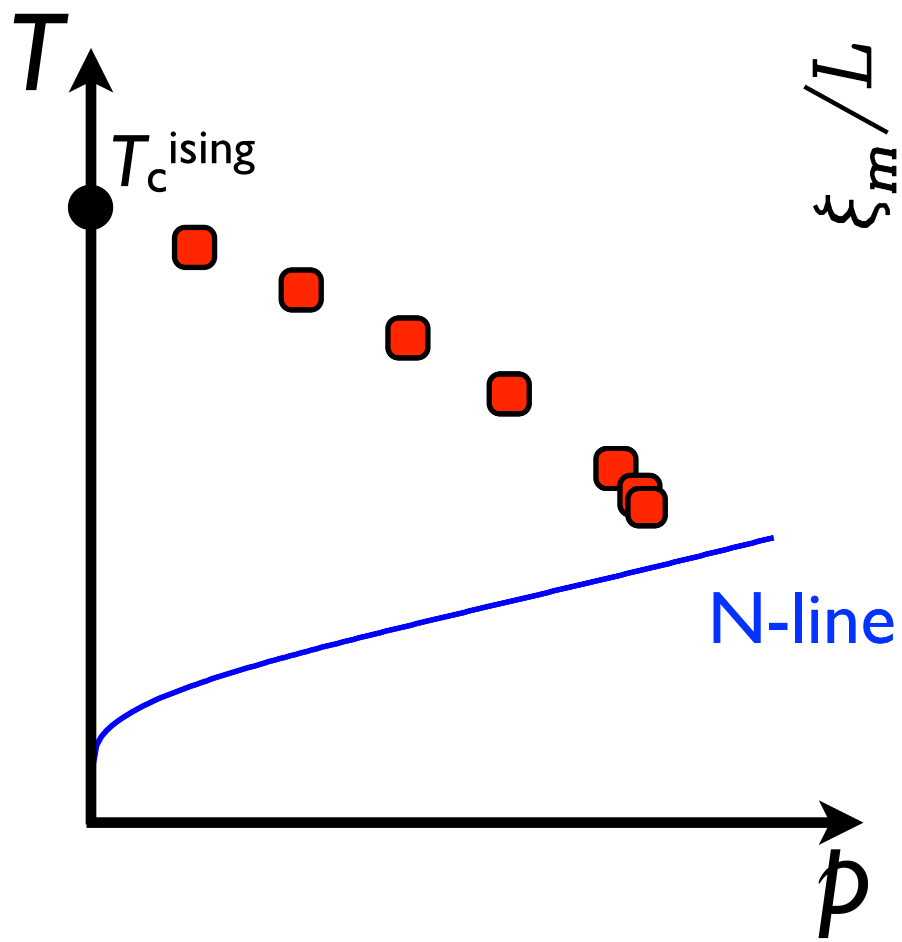

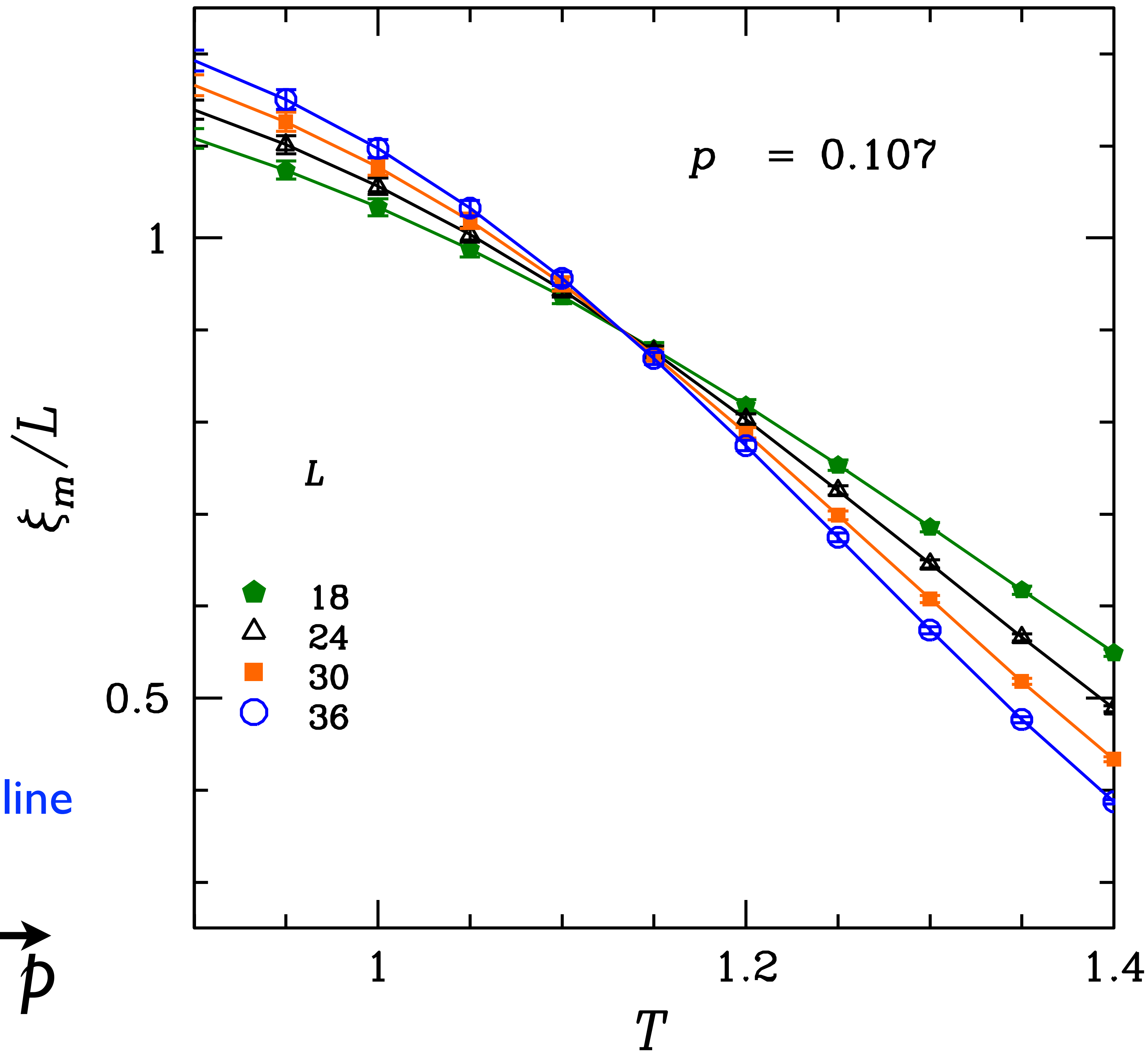

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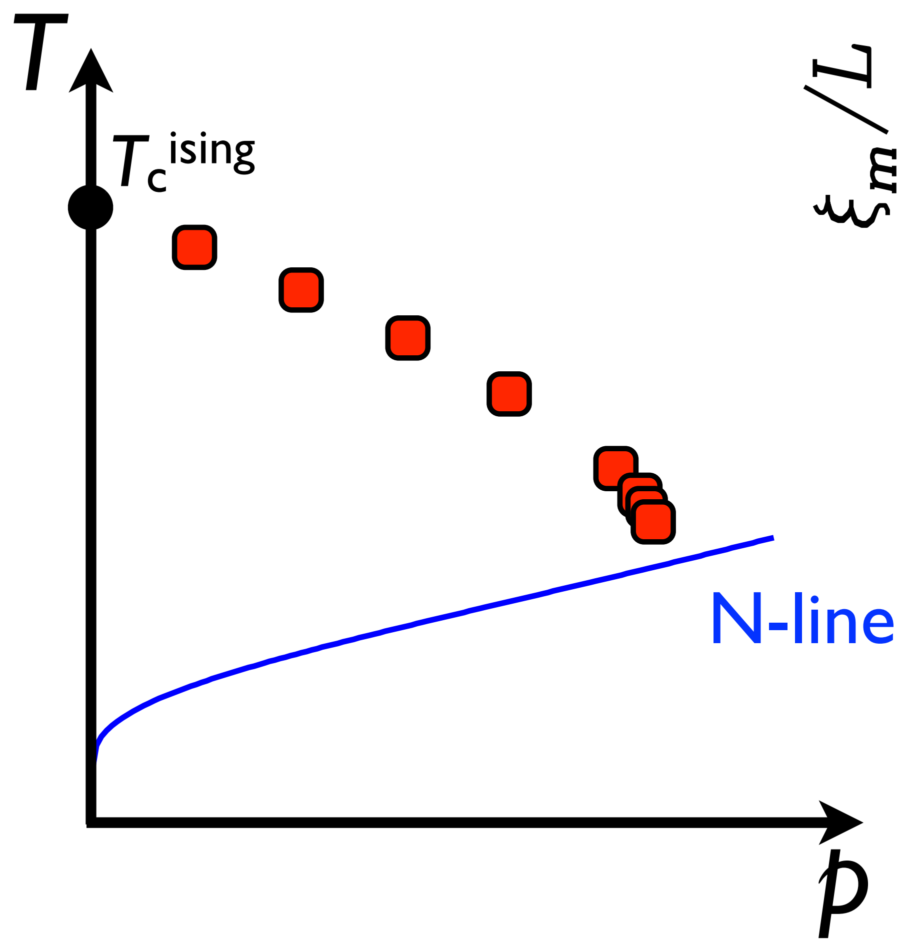

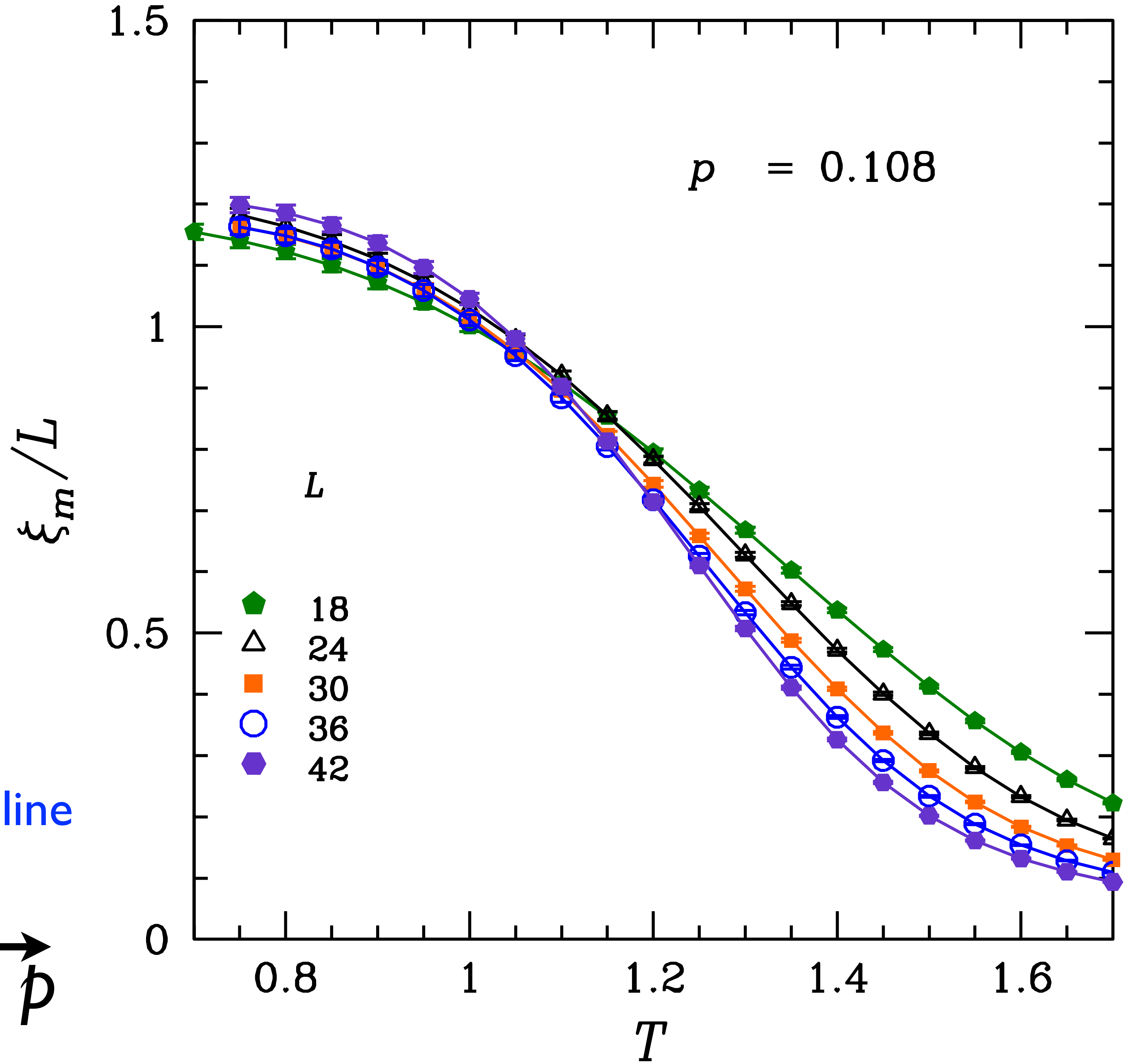

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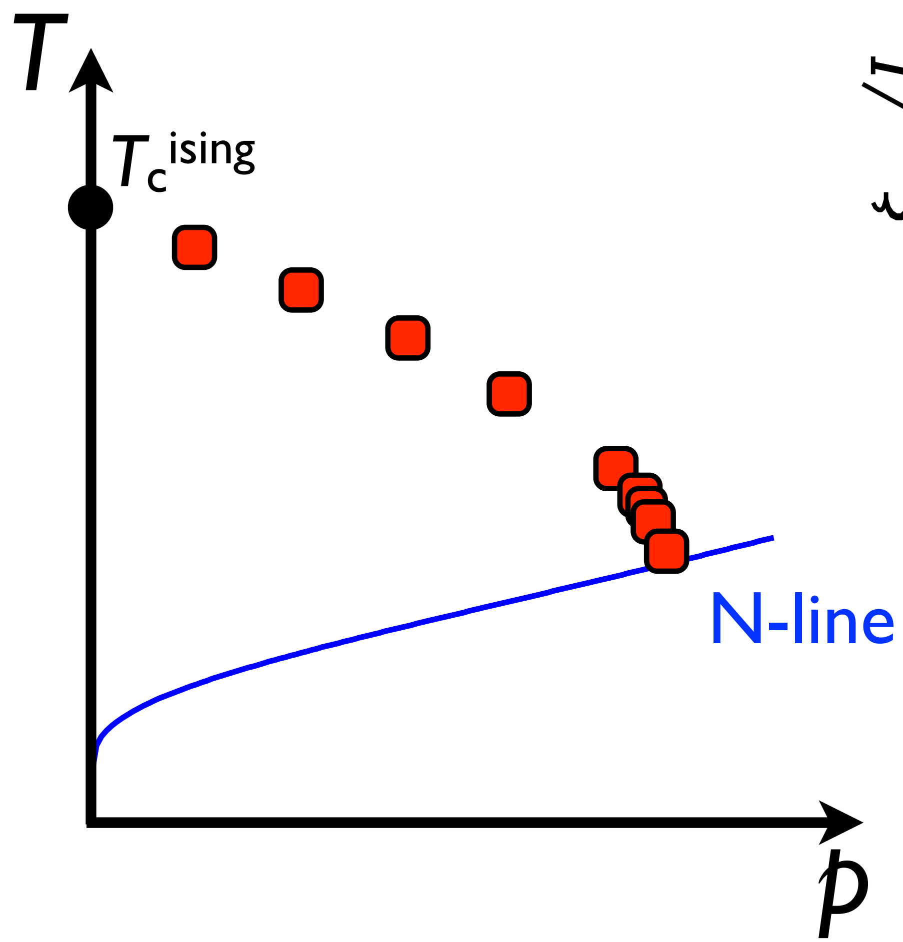

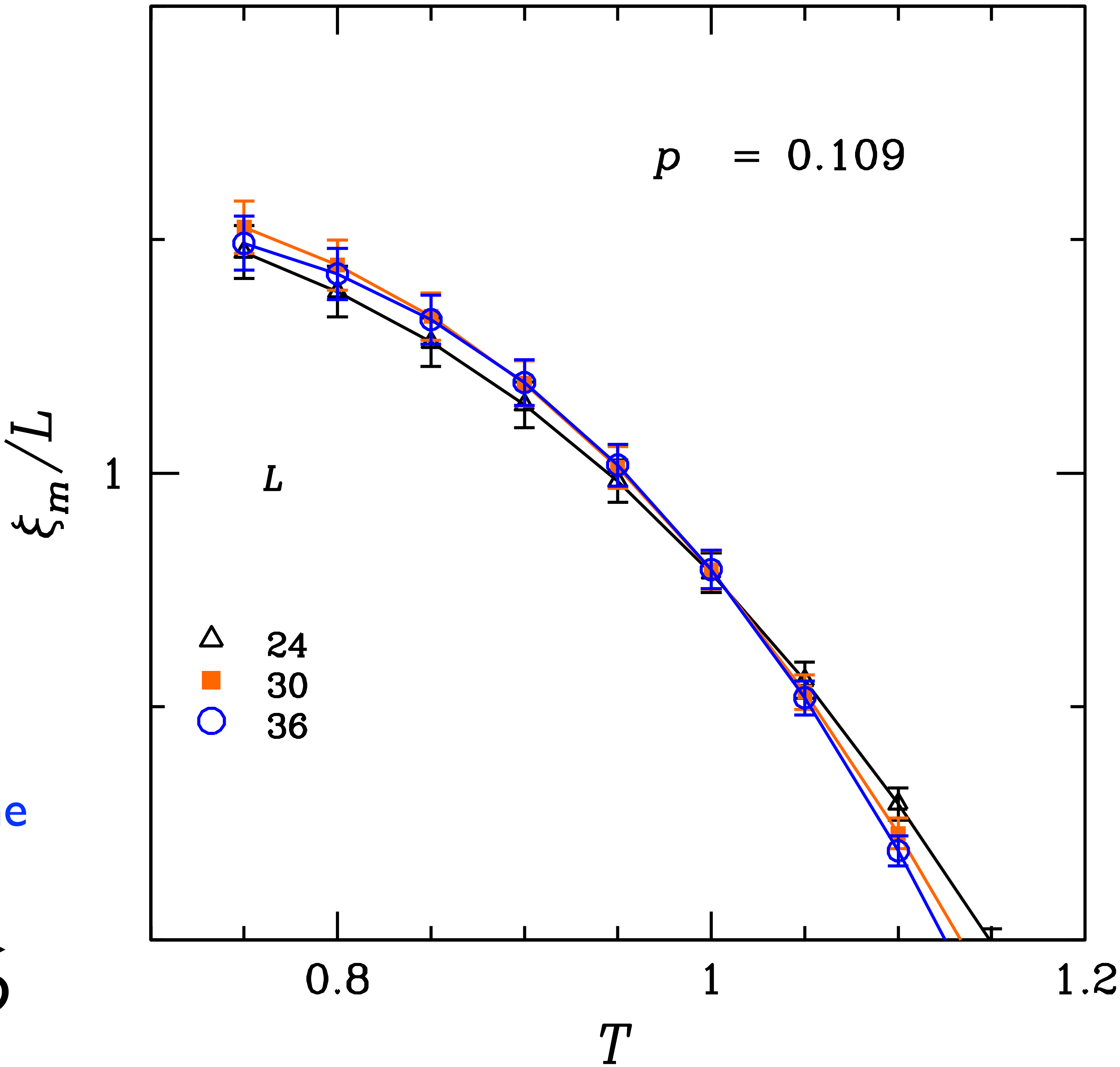

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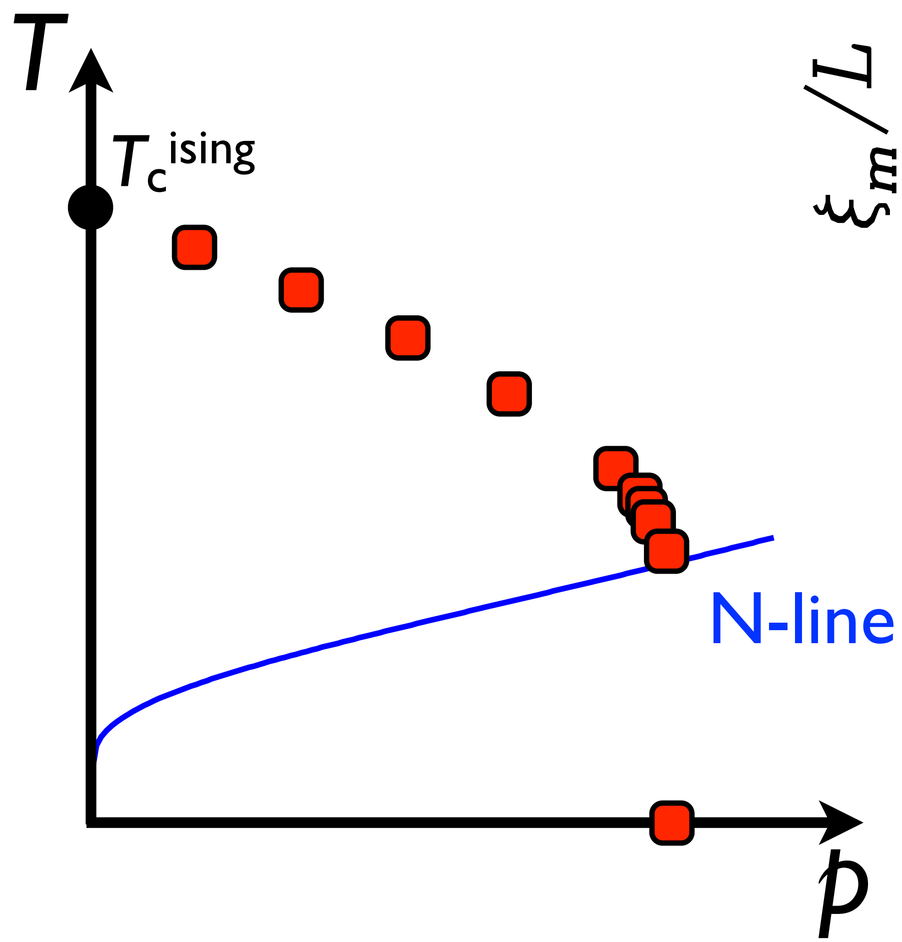

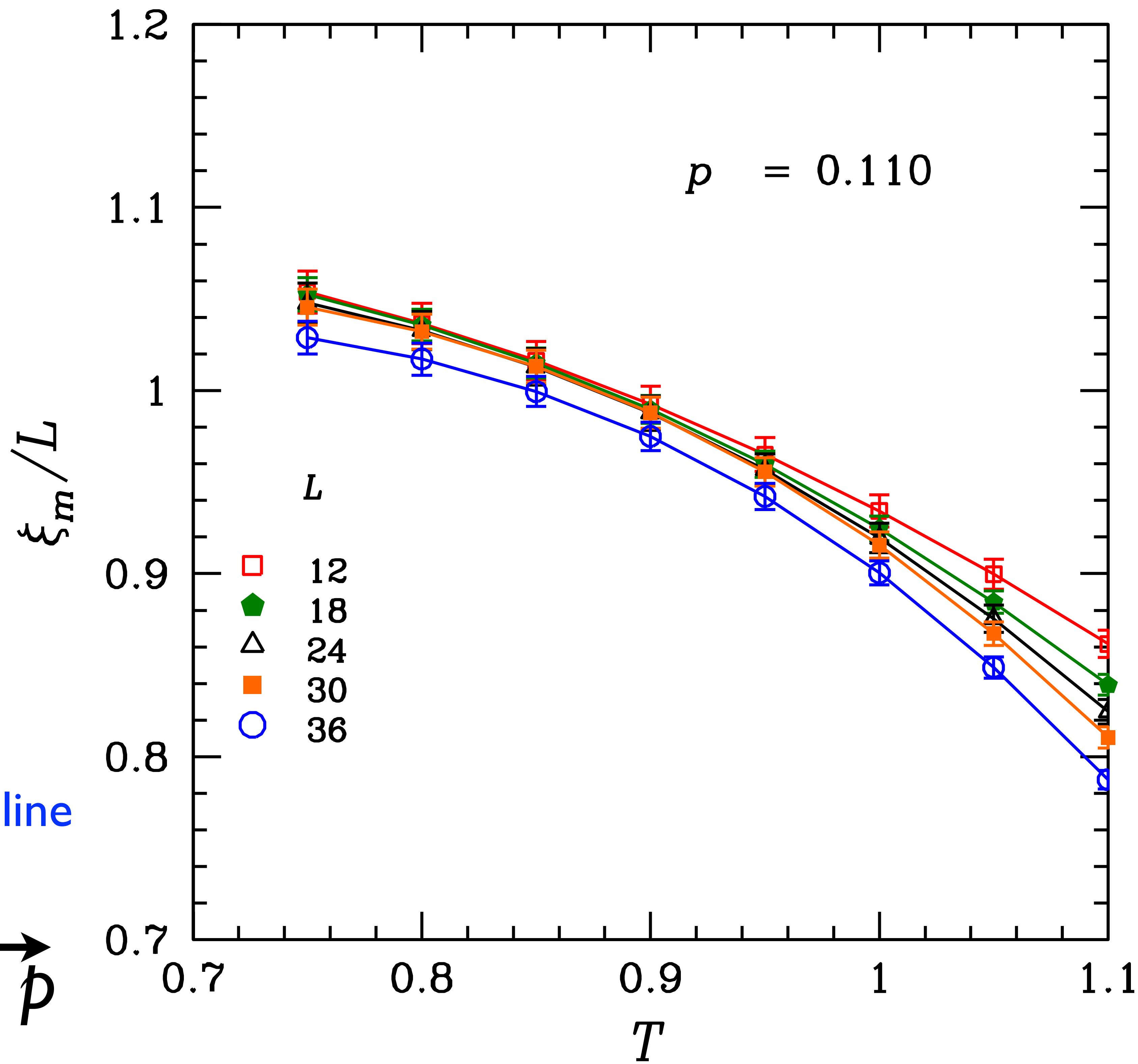

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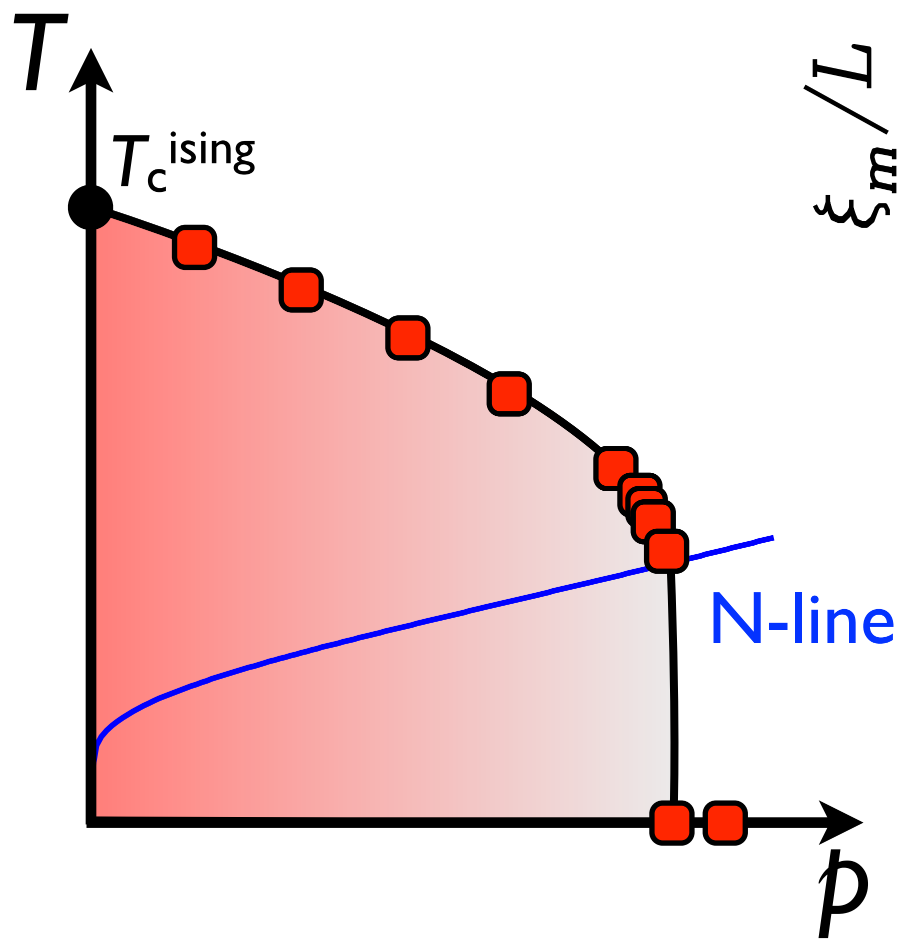
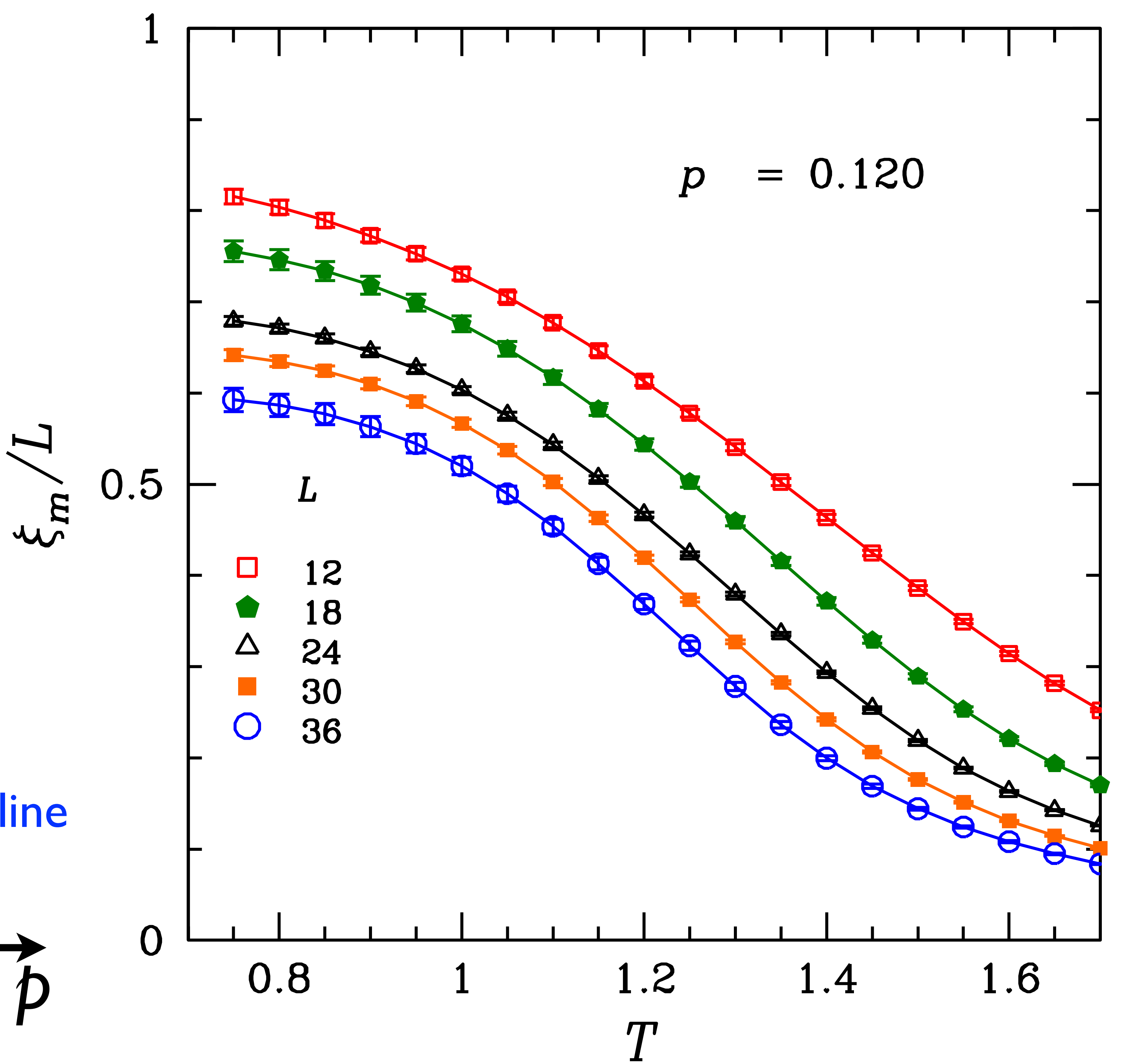


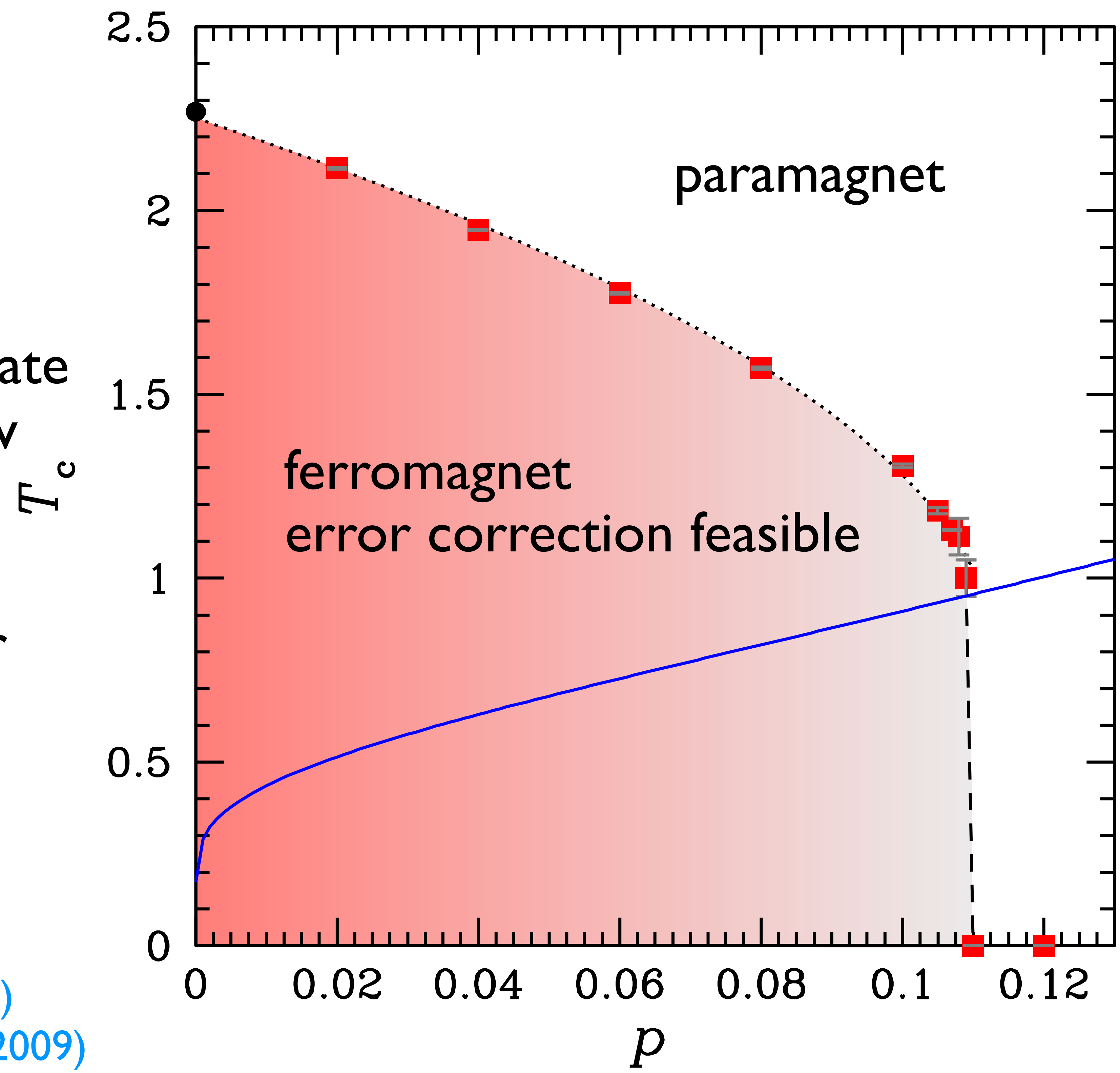


p–T phase diagram & error threshold

- 54 CPU years later…
- Error threshold:

$$p_c = 0.109(2)$$

- Note: p_c does not violate the Gilbert-Varshamov bound $p \sim 0.110027$.
- Same as Kitaev model and TCC on triangular lattices.



Phys. Rev. A 81, 012319 (2010)
Phys. Rev. Lett. 103, 090501 (2009)

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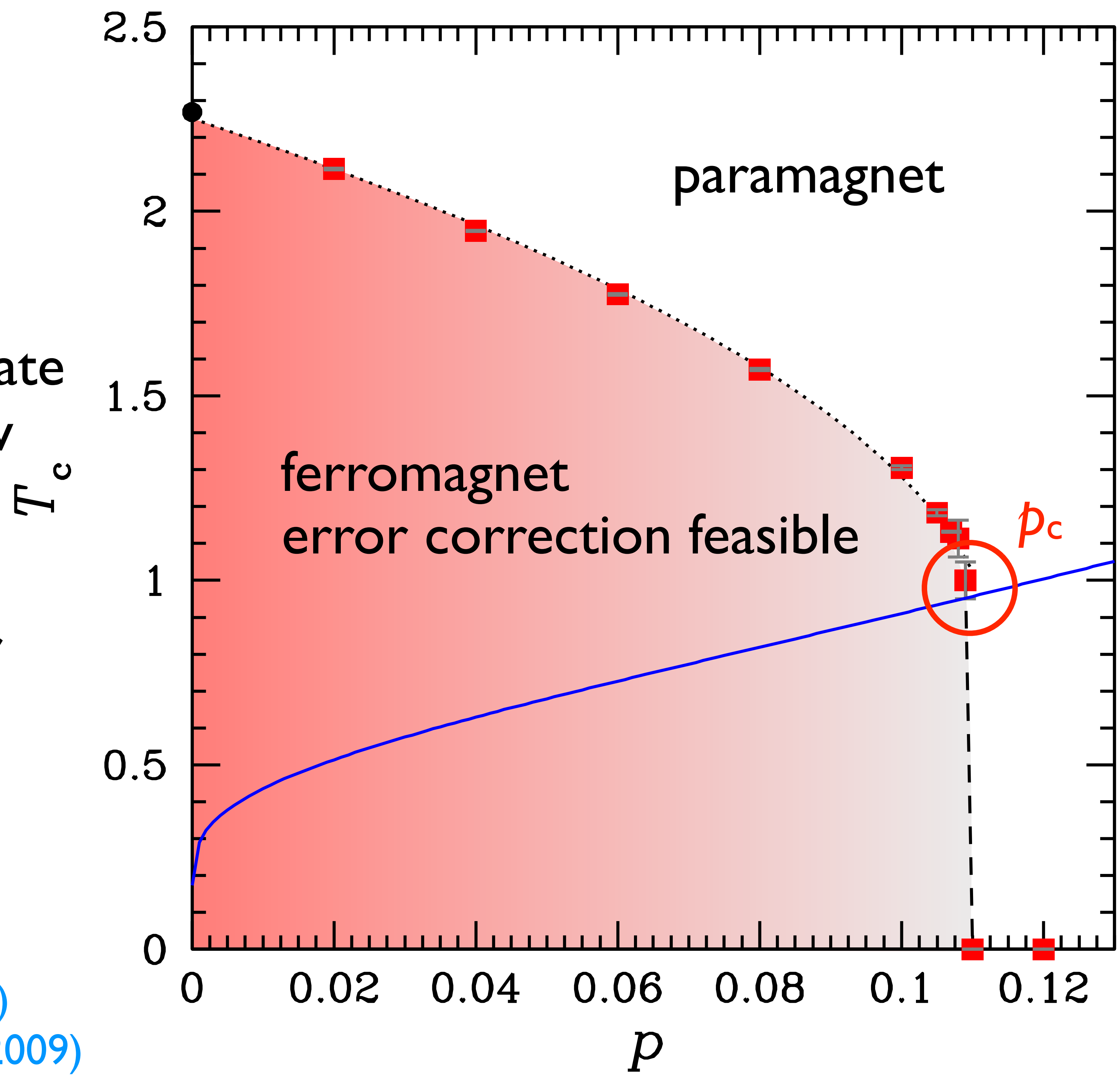

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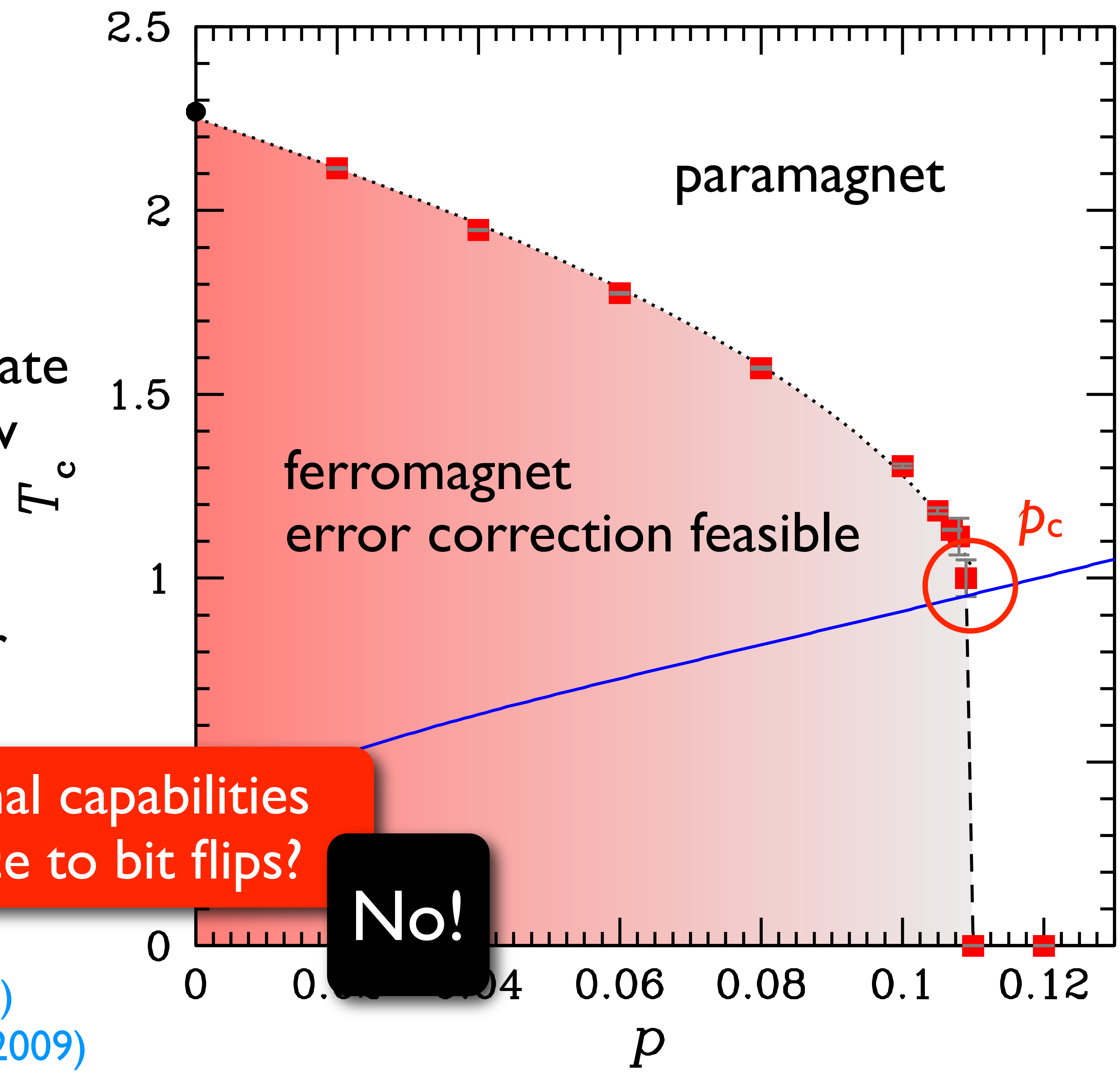

Do wider computational capabilities imply a lower resistance to bit flips?

No!

Phys. Rev. A 81, 012319 (2010)
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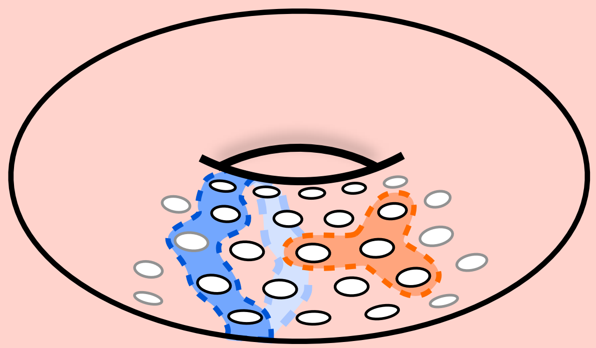
Measurement errors

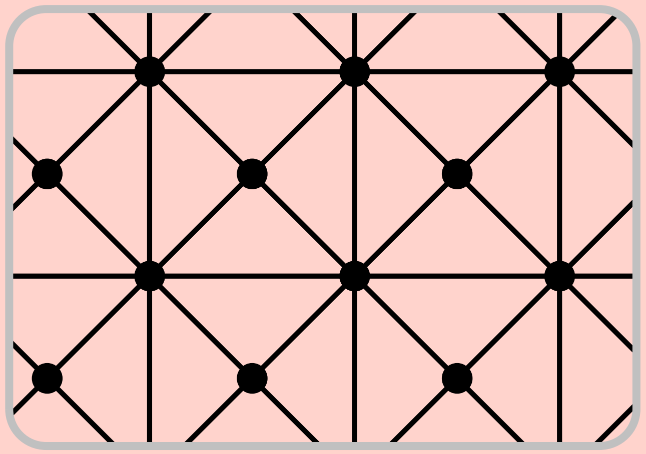
Add measurement errors…

bit-flip errors

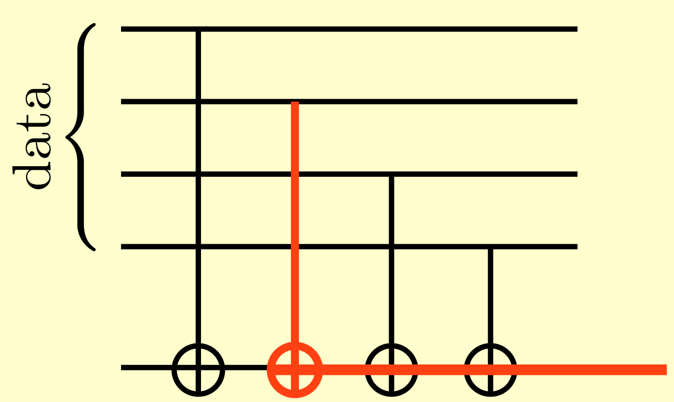


error rate p

2D random model

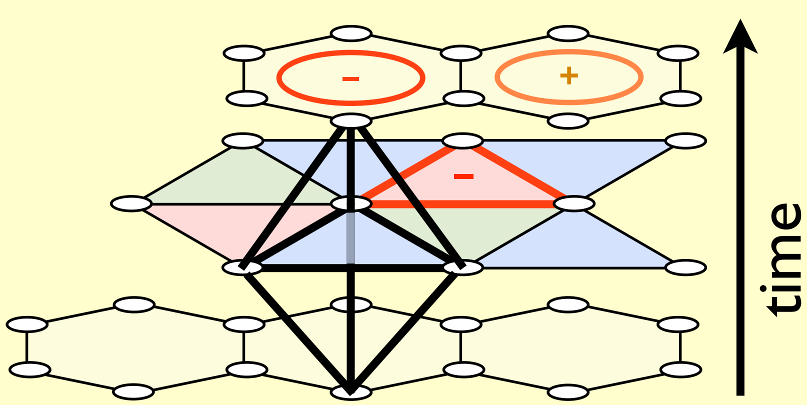


measurement errors



error rate q

3D lattice gauge theory



3D disordered lattice gauge theory

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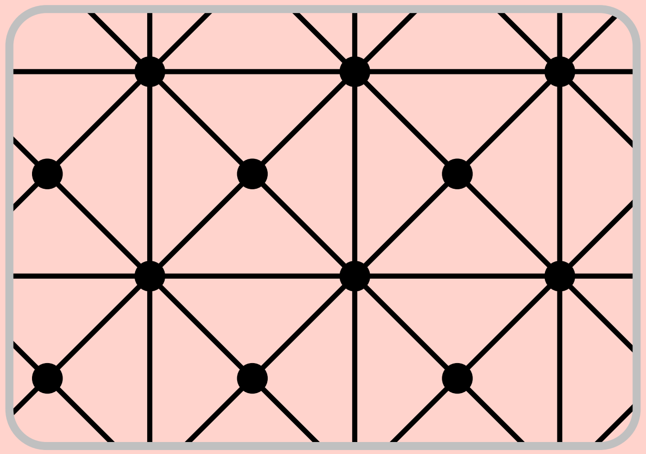
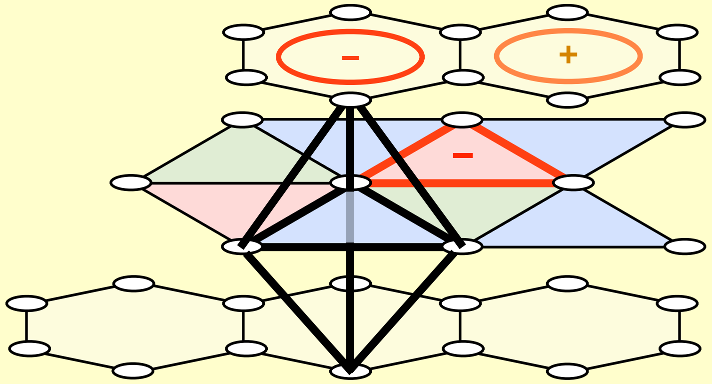
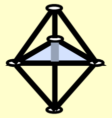
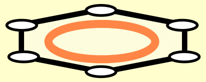


$$\mathcal{H} = J \sum_{\langle ijk \rangle} \tau_{ijk} S^i S^j S^k$$

3D lattice gauge theory



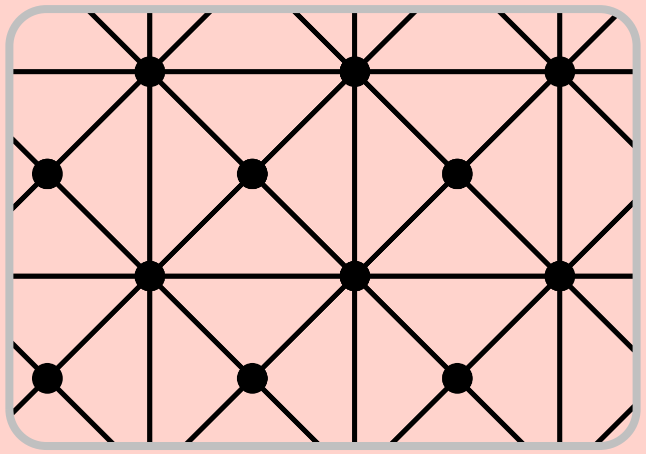
$$\mathcal{H} = -\sum J_j [S^j]_5 - \sum K_k [S^k]_6$$

$$J_j, K_k = \begin{cases} -1 & \text{probability } p, q \\ +1 & (1-p), (1-q) \end{cases}$$

3D disordered lattice gauge theory

2D random model



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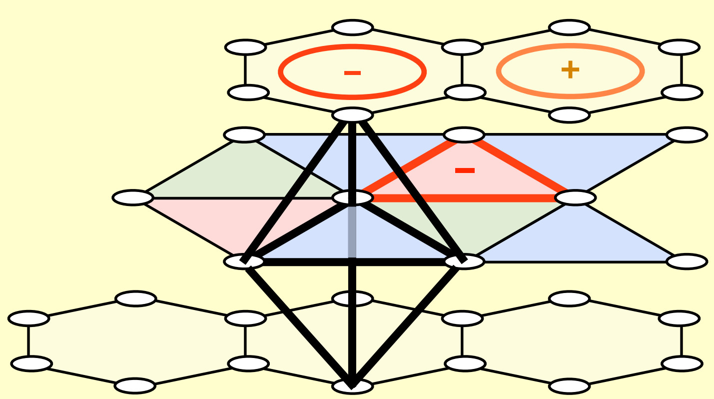
for simplicity $p = q$…

3D lattice gauge theory



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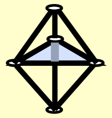 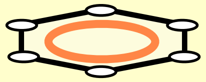

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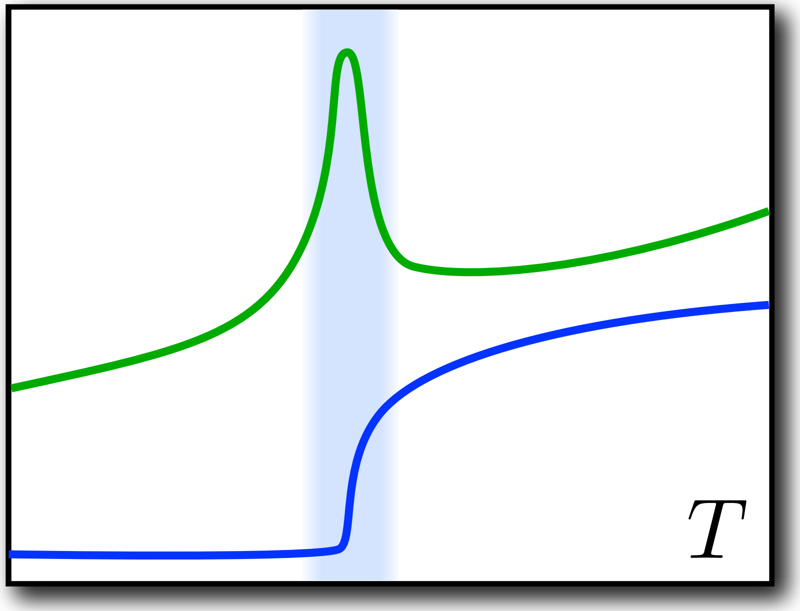
Order parameter

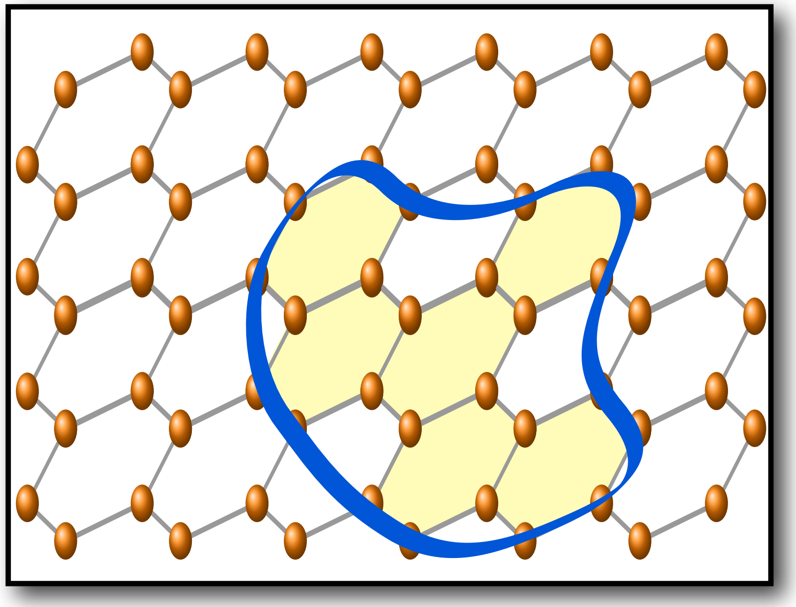
- Problems:
  - Local order parameters (magnetization) do not work for LGTs.
  - The transition is *first order*.
  - Both specific heat and energy imprecise.



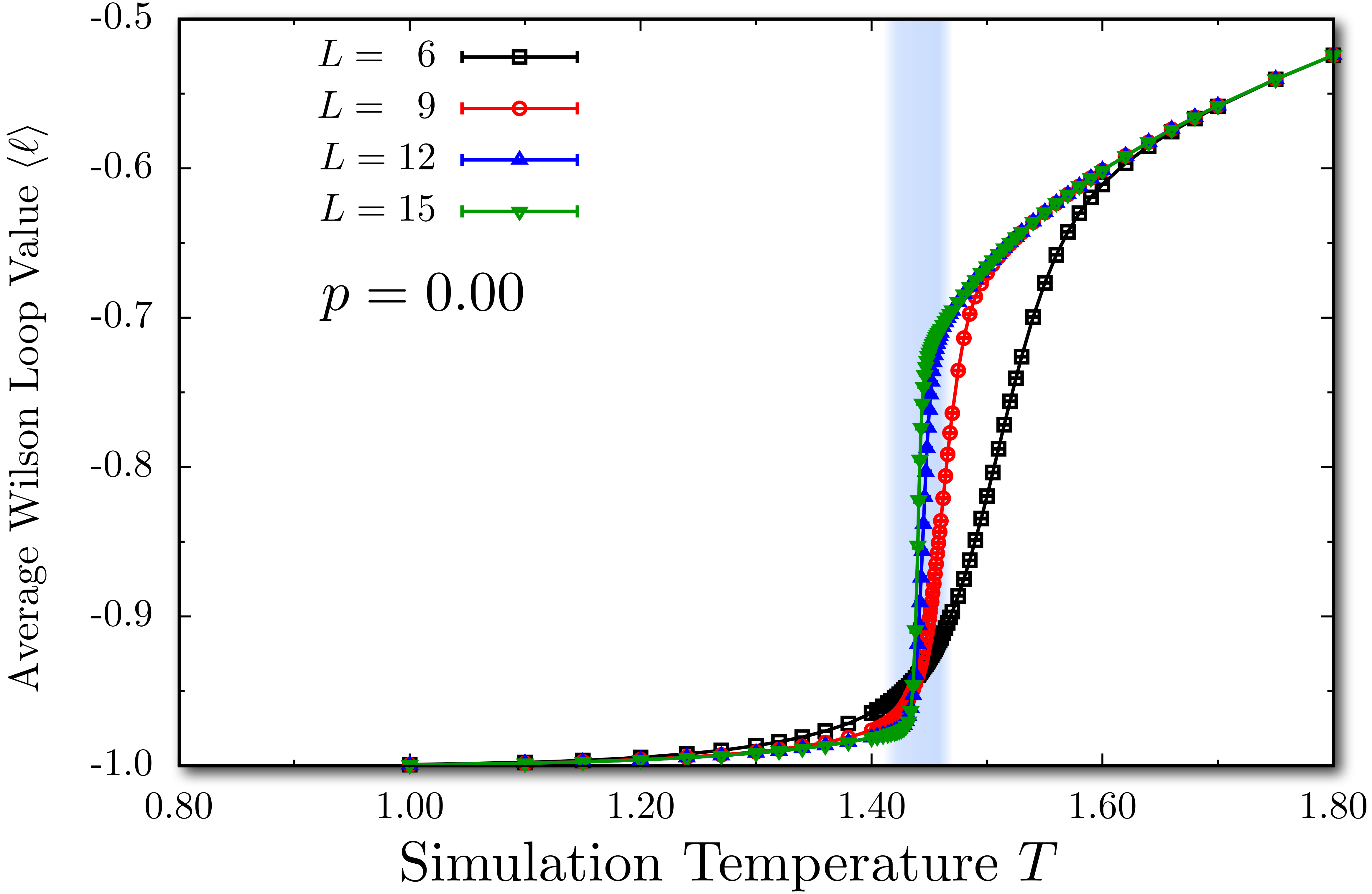
- Solution:
  - Wilson loops in the hexagon plane

$$\ell = \frac{1}{N_{\text{loops}}} \sum_{\text{loops}} \prod_{j \in \text{loop}} S_j$$

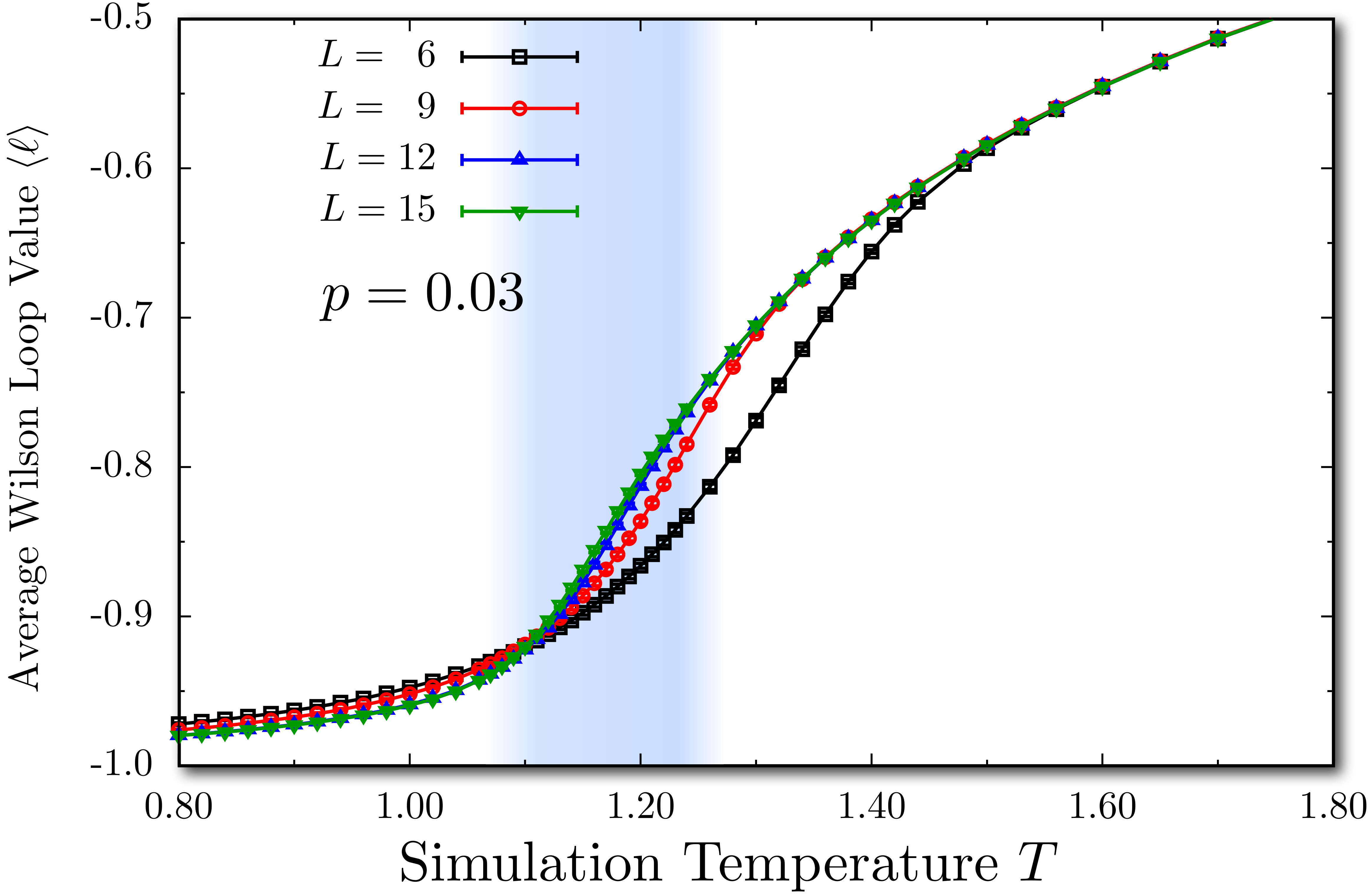
  - Note: here we use minimal loops over one plaquette to reduce corrections.



Average Wilson loop value (no errors)



Average Wilson loop value (p = 3%)



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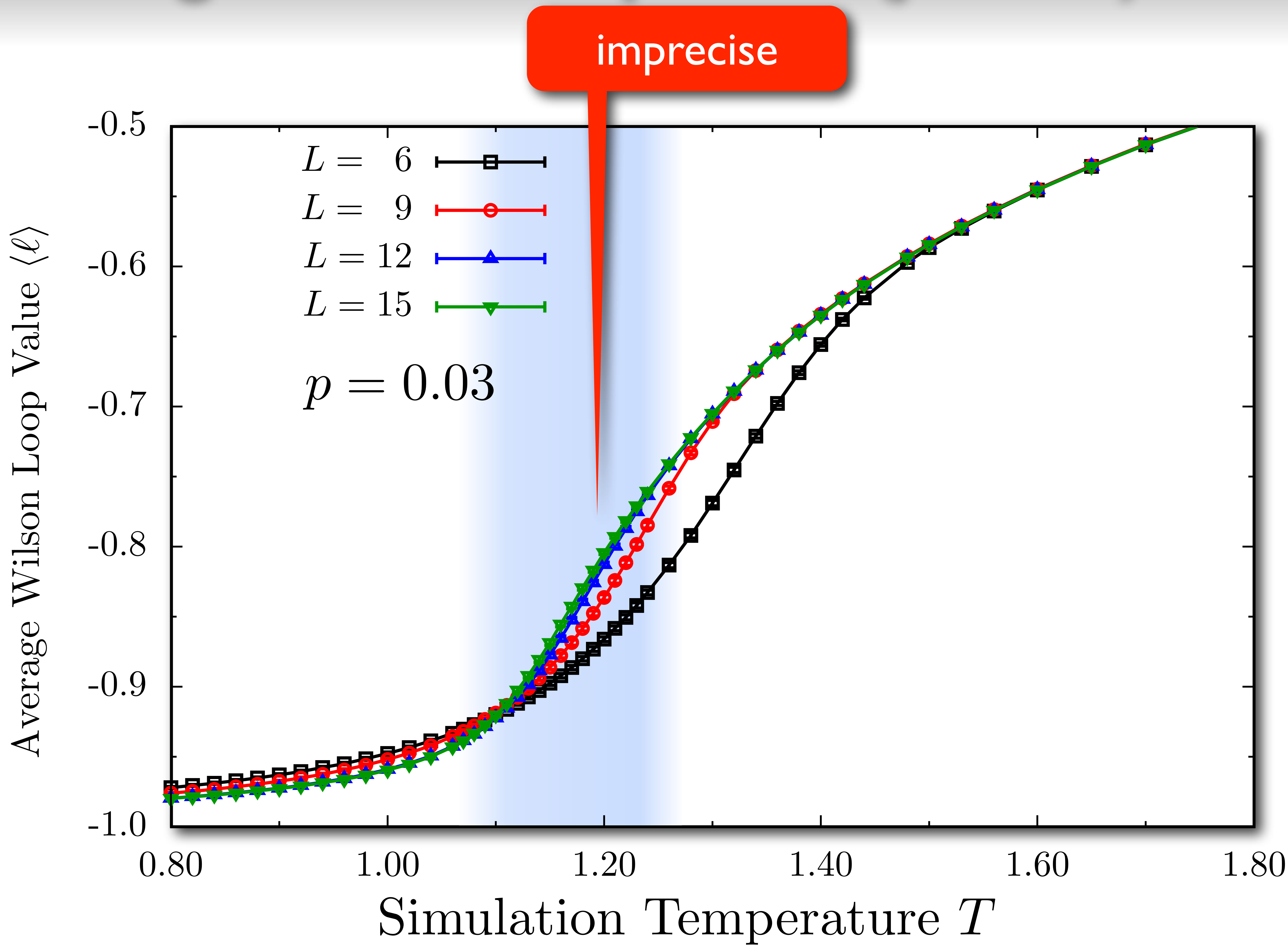
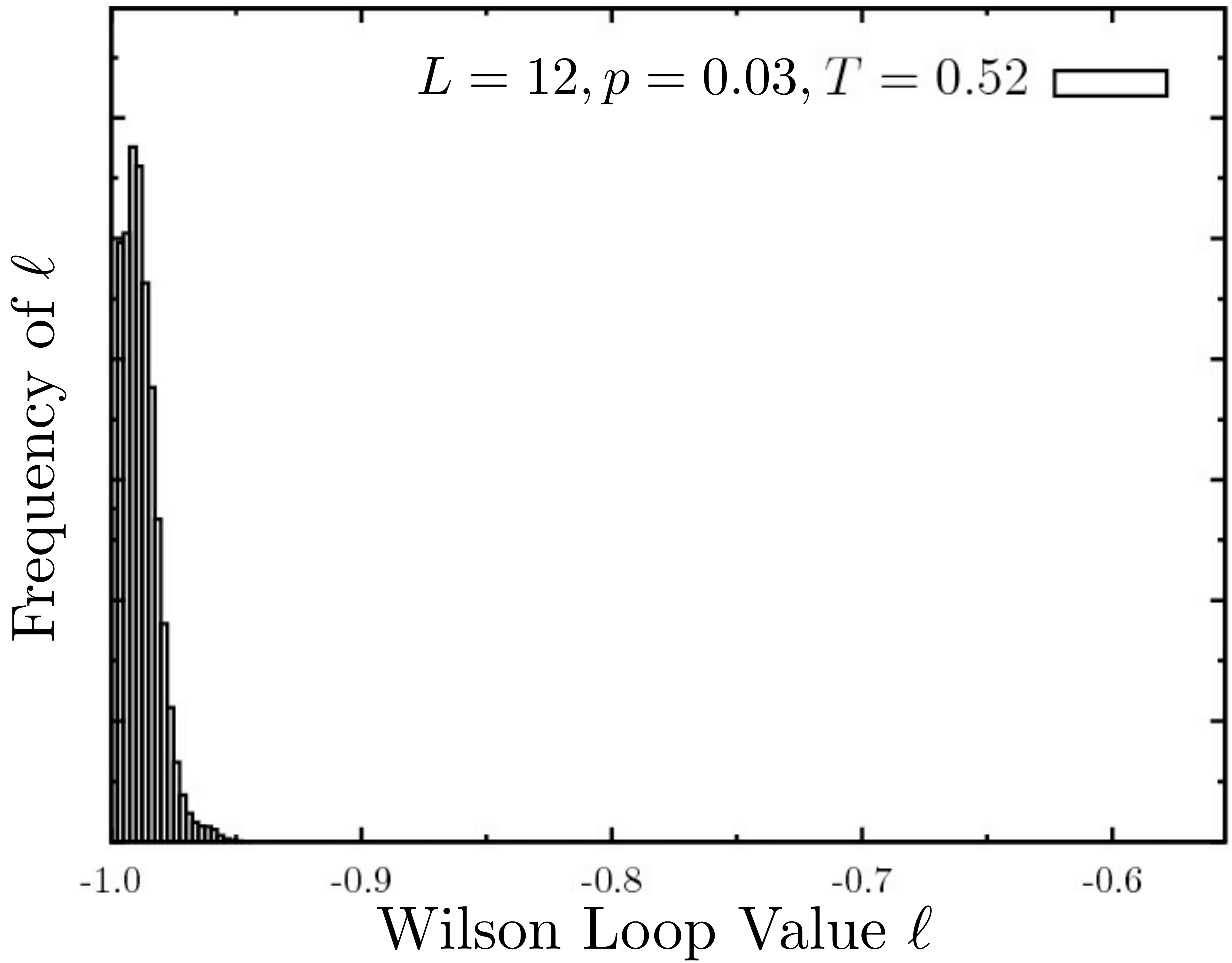


Wilson loop distribution (p = 3%)



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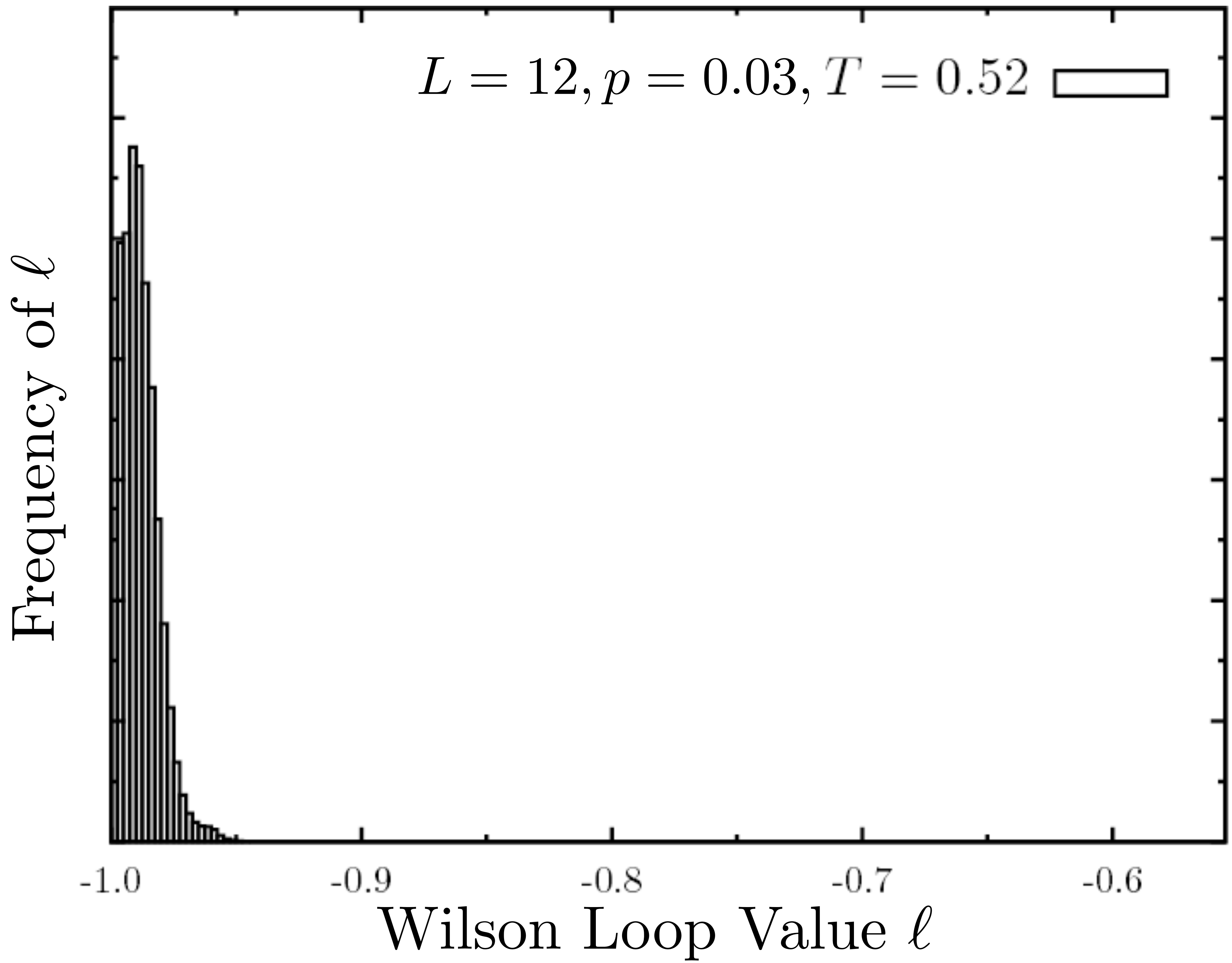

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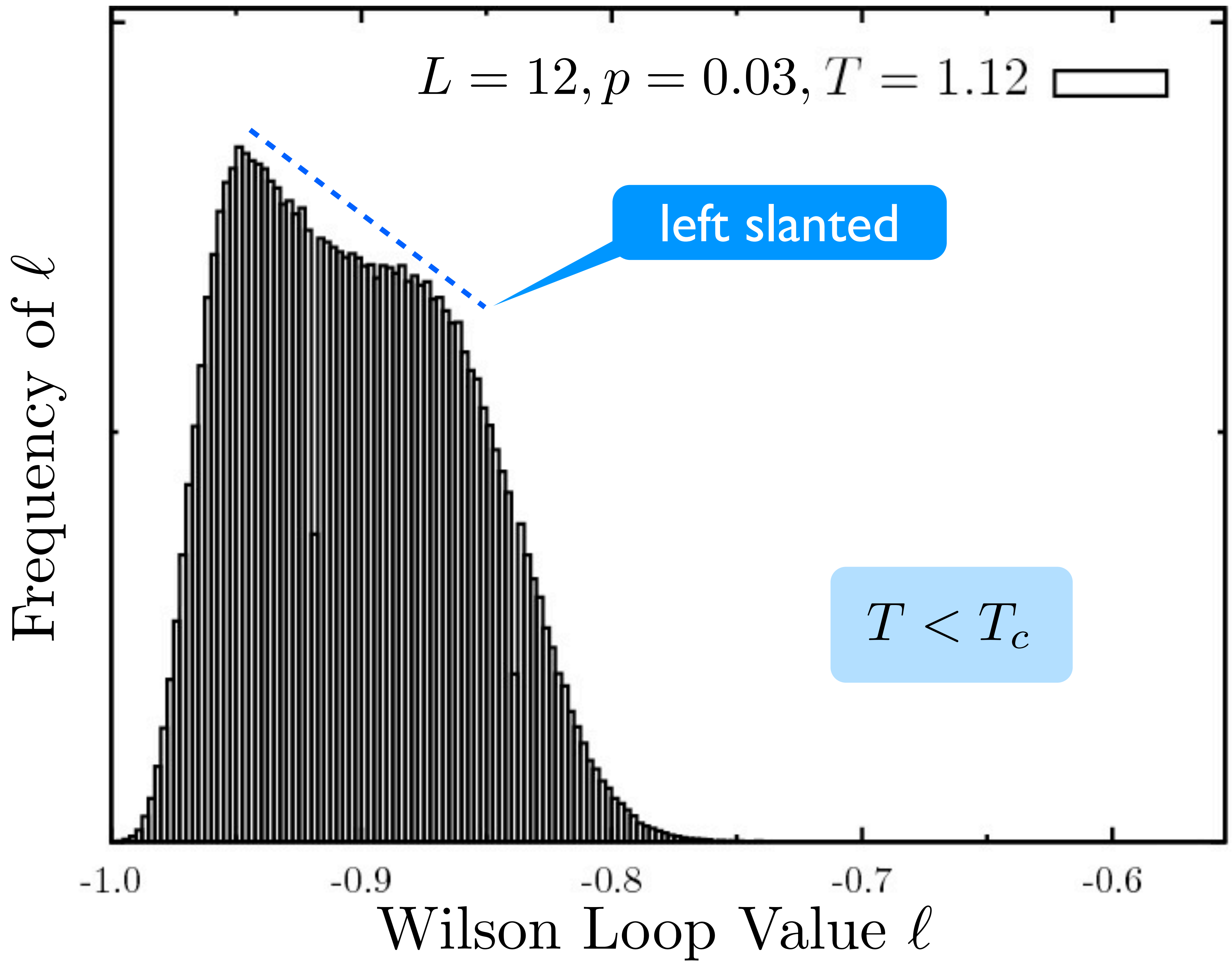

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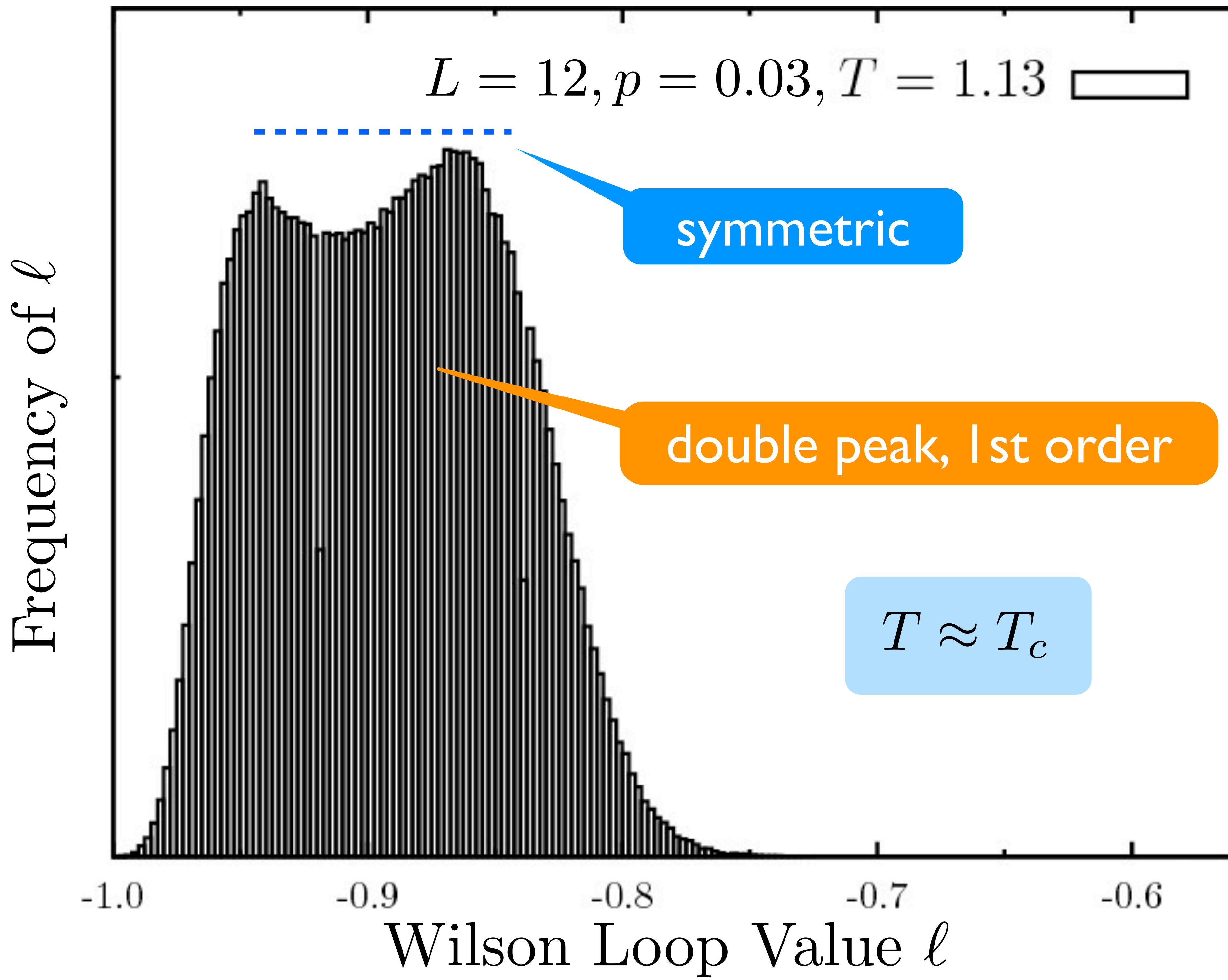

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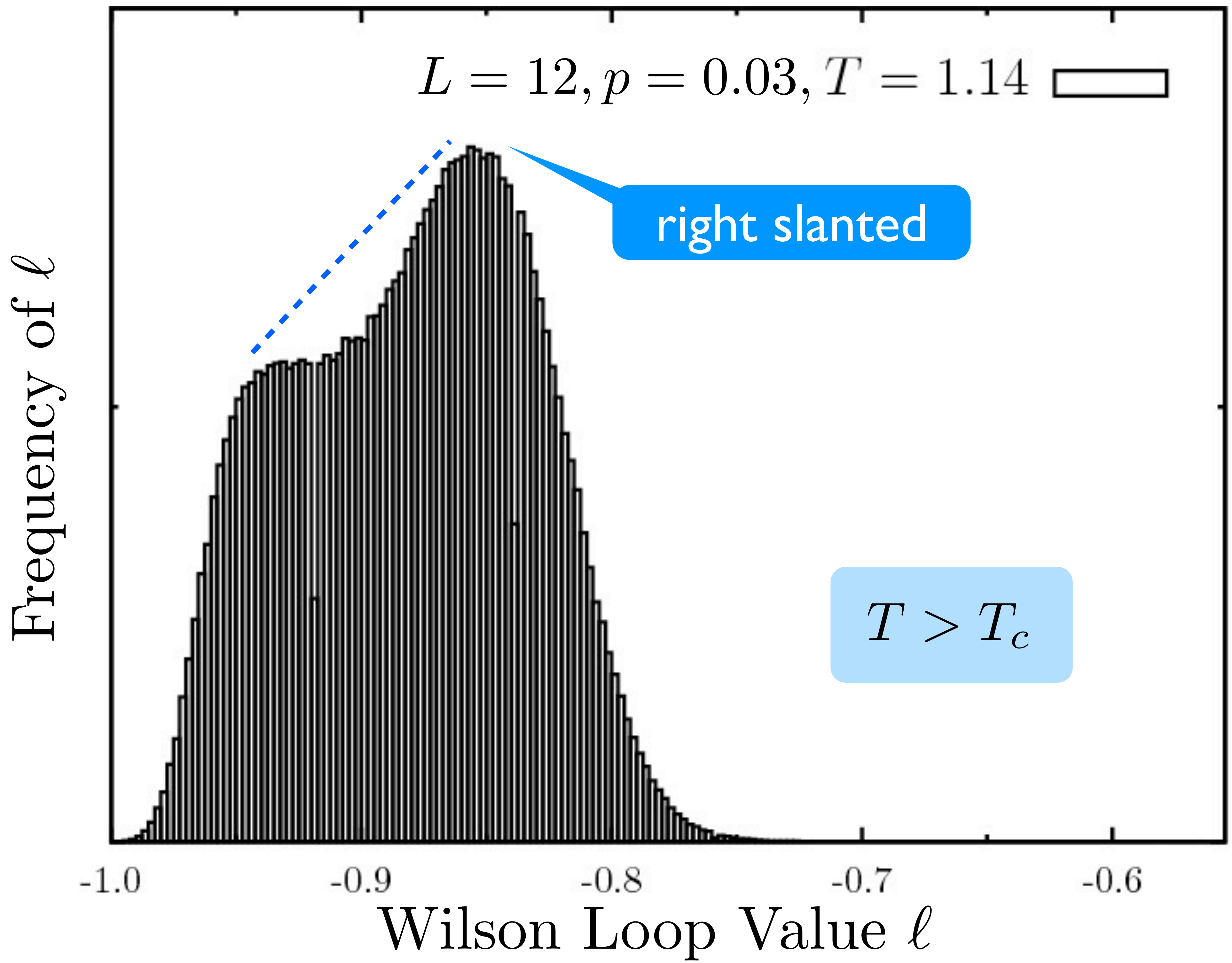

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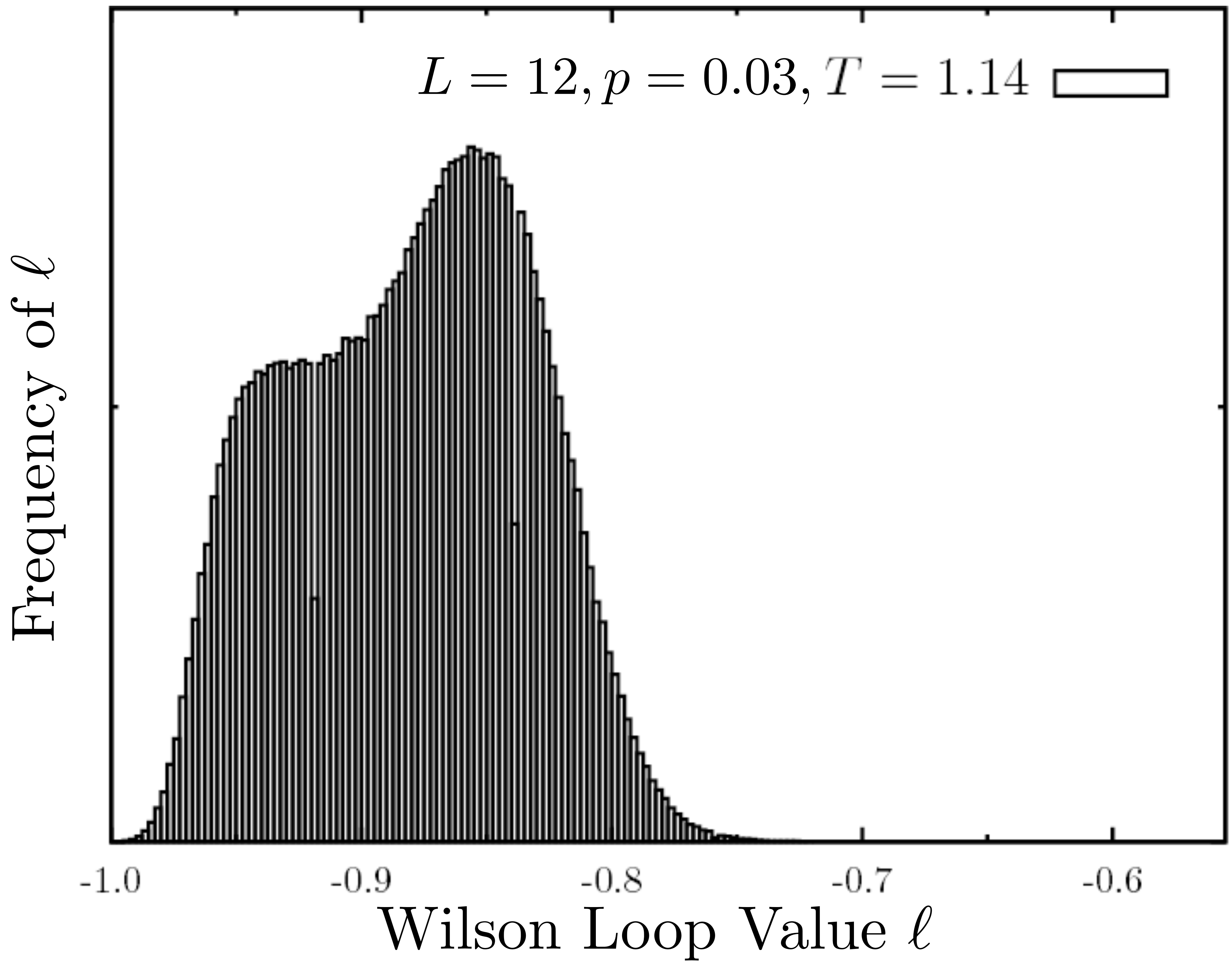
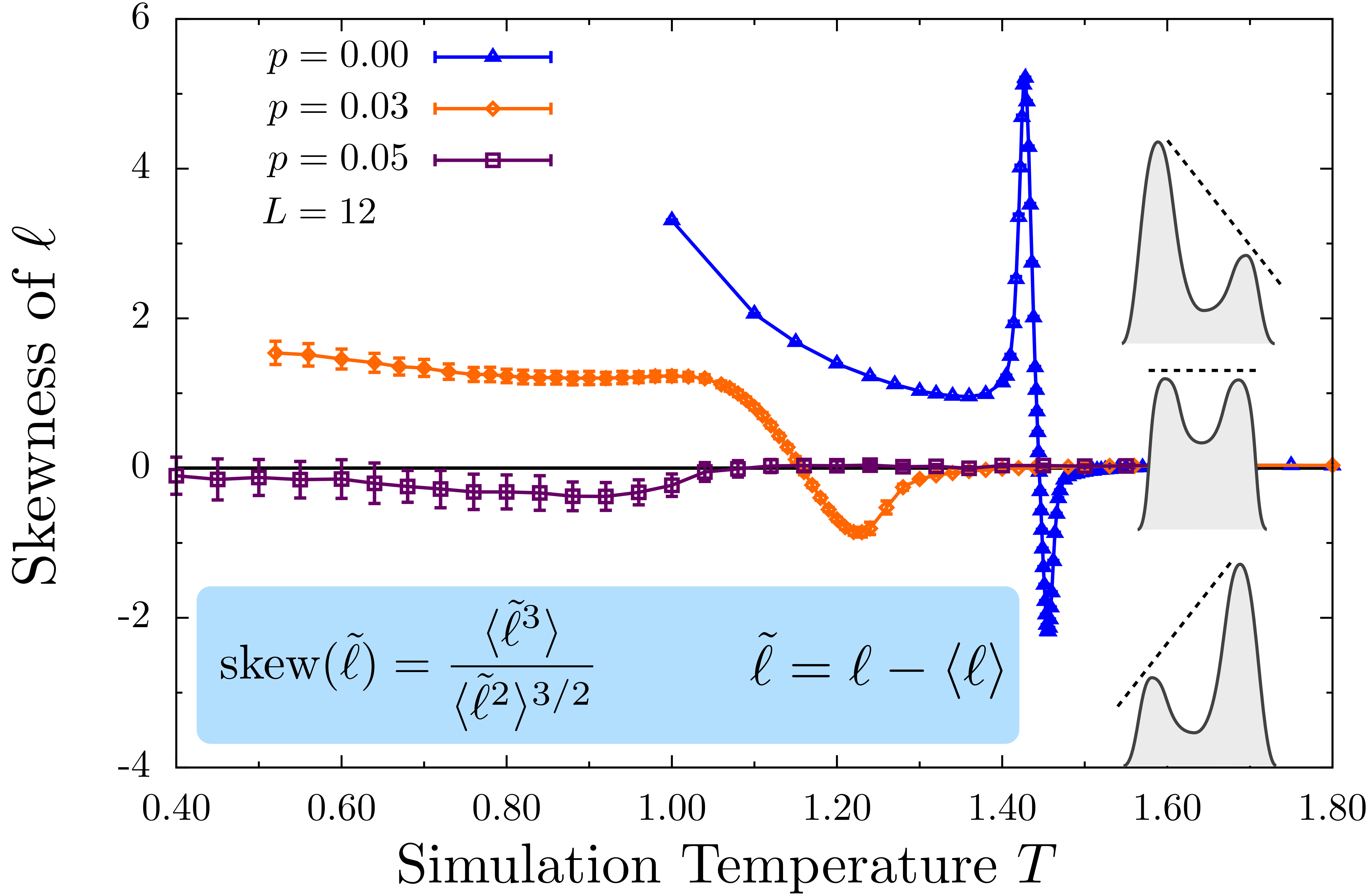
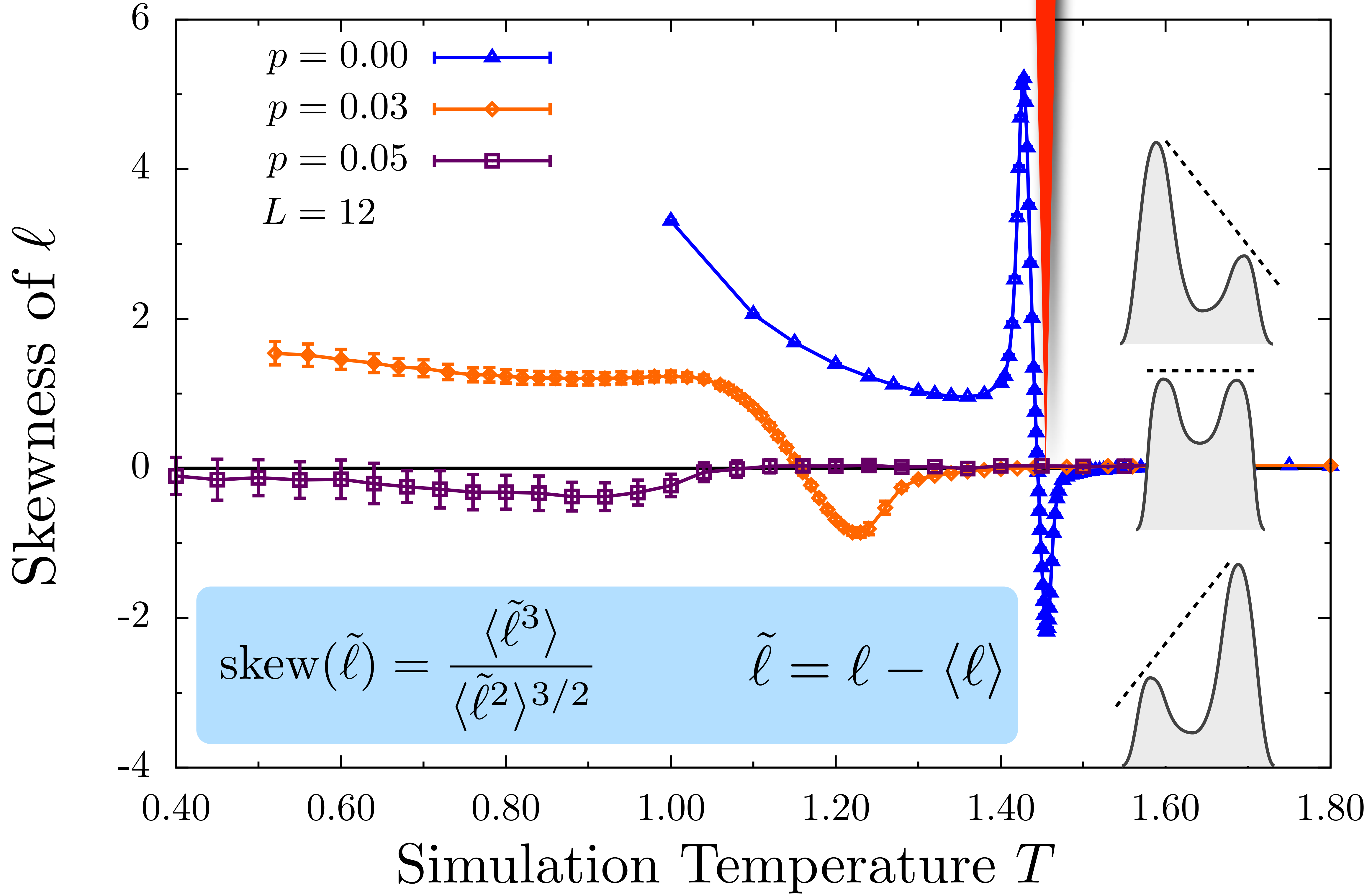


Skewness as a "Binder parameter"



Skewness as a “Binder parameter”

agrees with C_v and E, Maxwell



$$\mathrm{skew}(\tilde{\ell}) = \frac{\langle \tilde{\ell}^3 \rangle}{\langle \tilde{\ell}^2 \rangle^{3/2}} \qquad \tilde{\ell} = \ell - \langle \ell \rangle$$

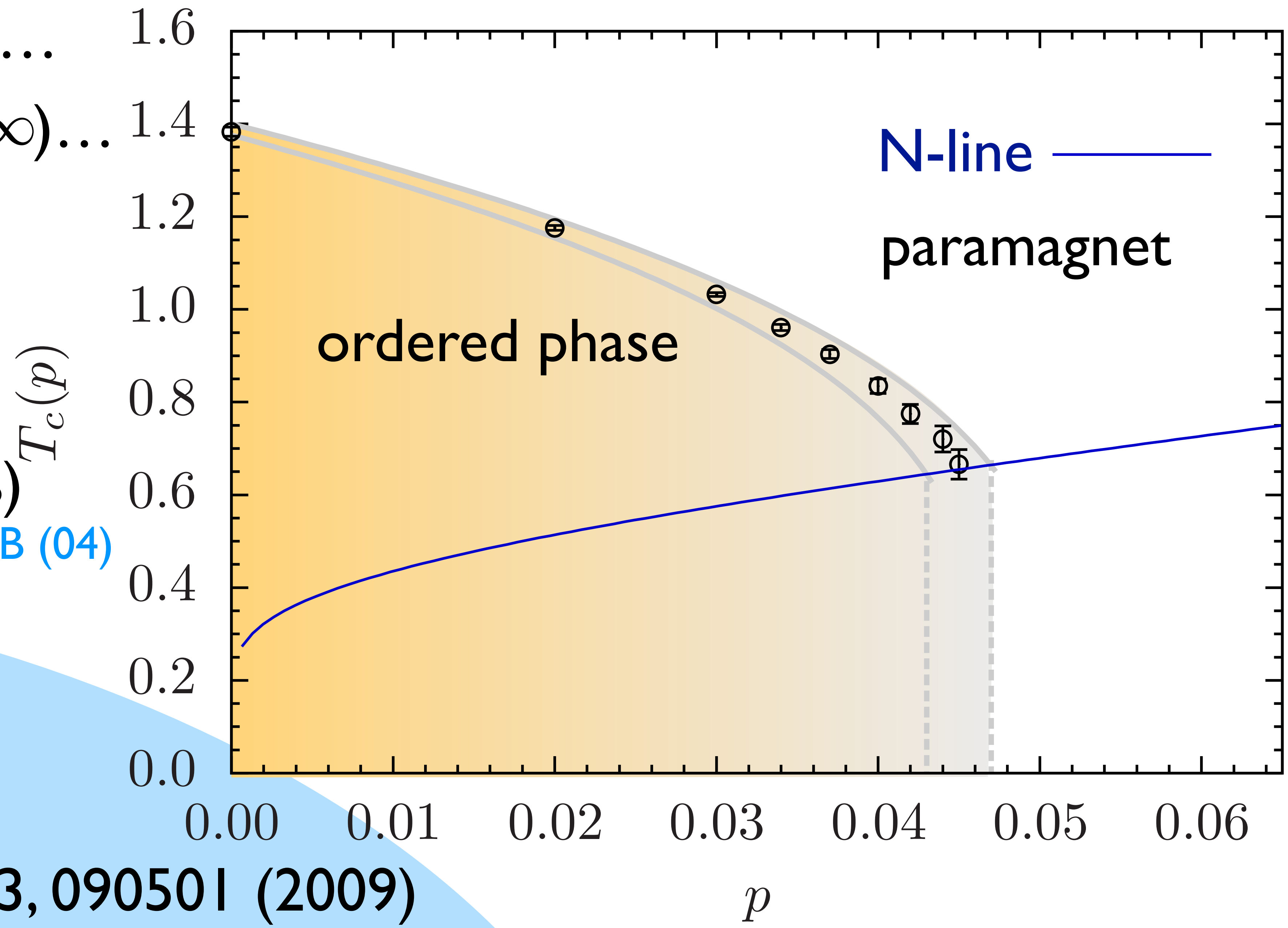
Error threshold with measurement errors

- 23 CPU years later…
- Extrapolate ($L \to \infty$)…
- Threshold:

$$p_c = 0.045(2)$$

- Revisit TC (p_c = 3%)

Ohno *et al.*, Nuc Phys B (04)



arXiv:quant-phys/1005.0777

- See also:
  - Phys. Rev. Lett. 103, 090501 (2009)
  - Phys. Rev. A 81, 012319 (2010)
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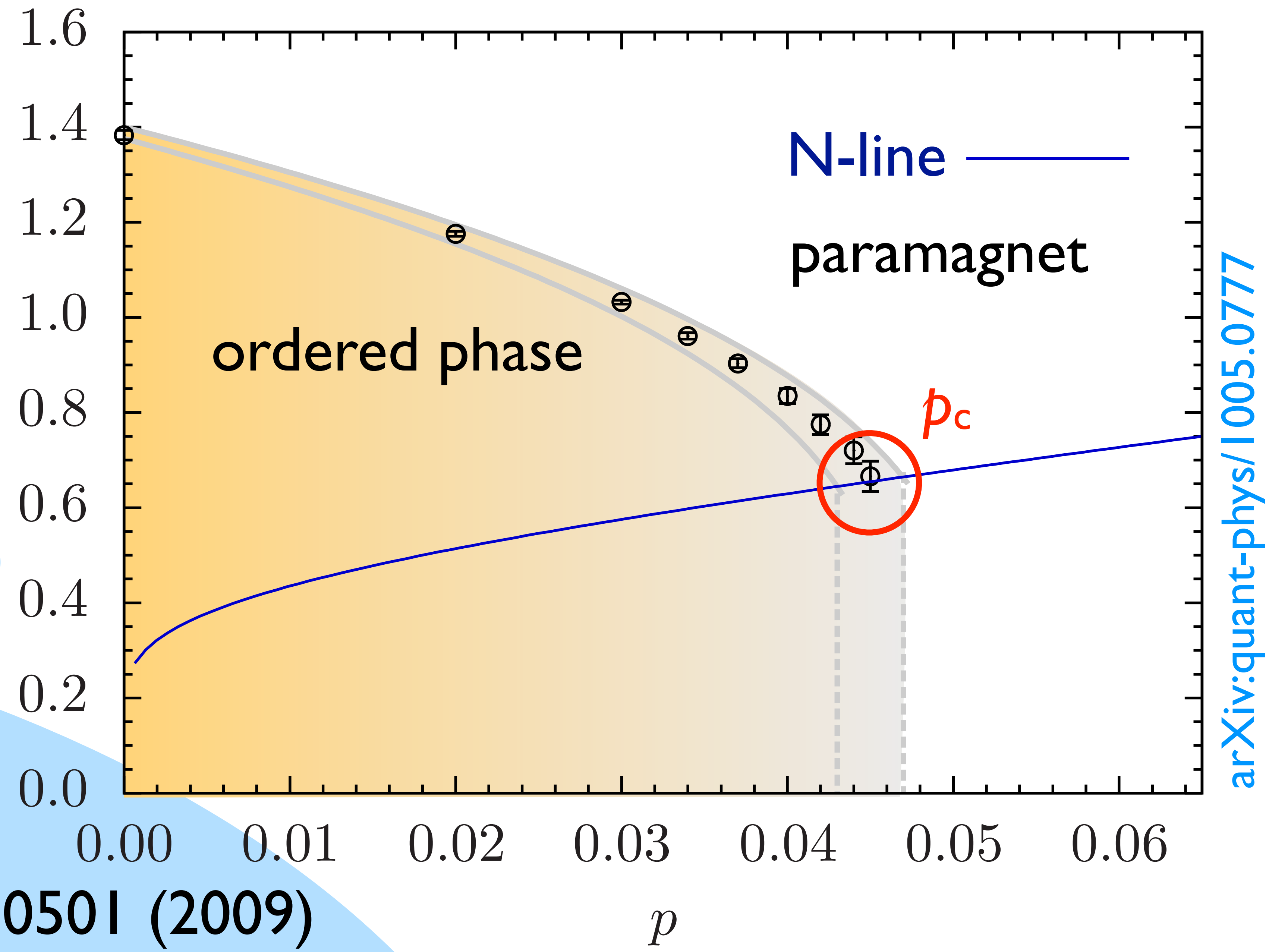

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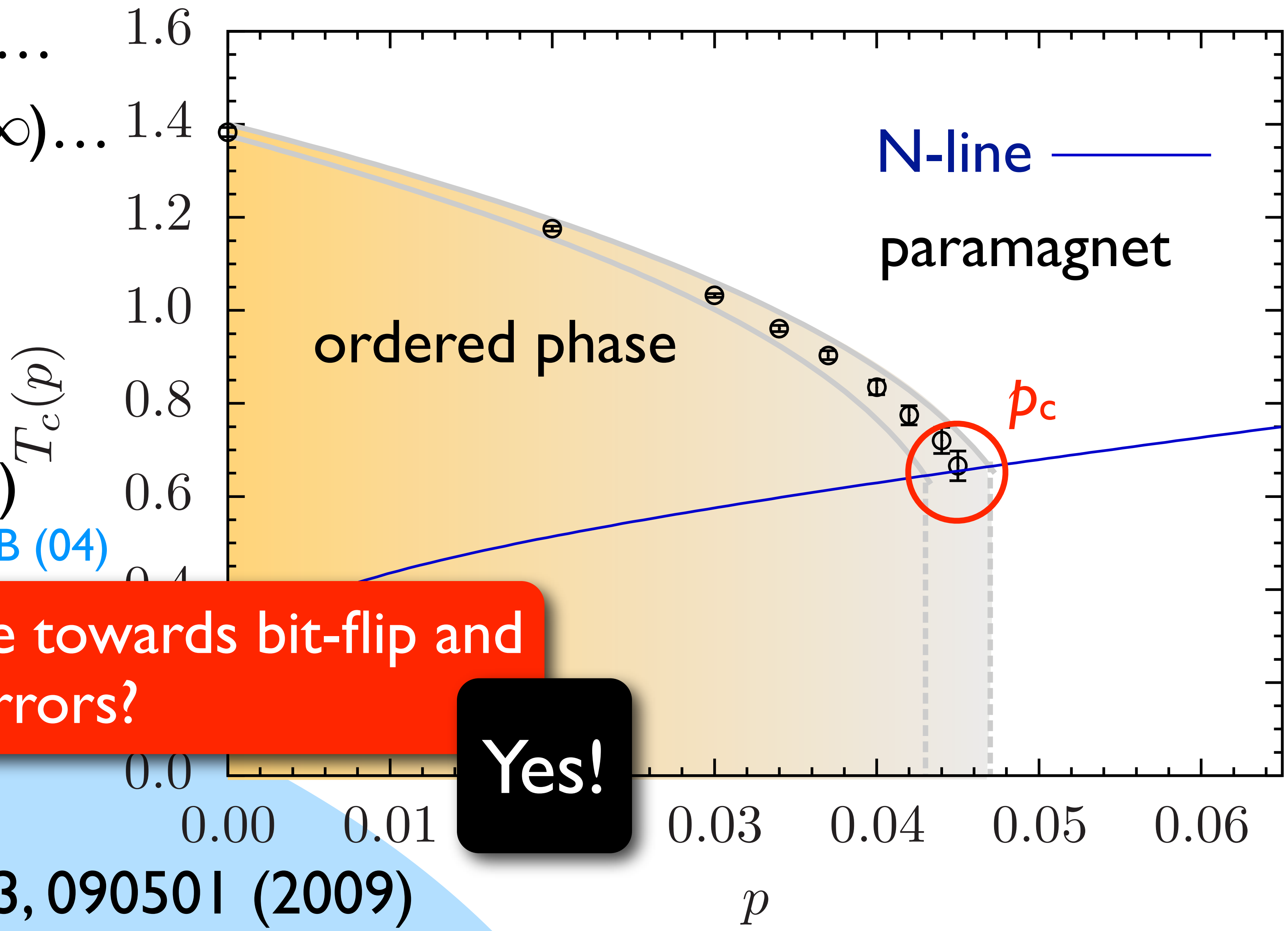
Are TCCs stable towards bit-flip and measurement errors?

Yes!



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