



# Out-of-equilibrium dynamics in a two-dimensional Coulomb glass

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# Outline

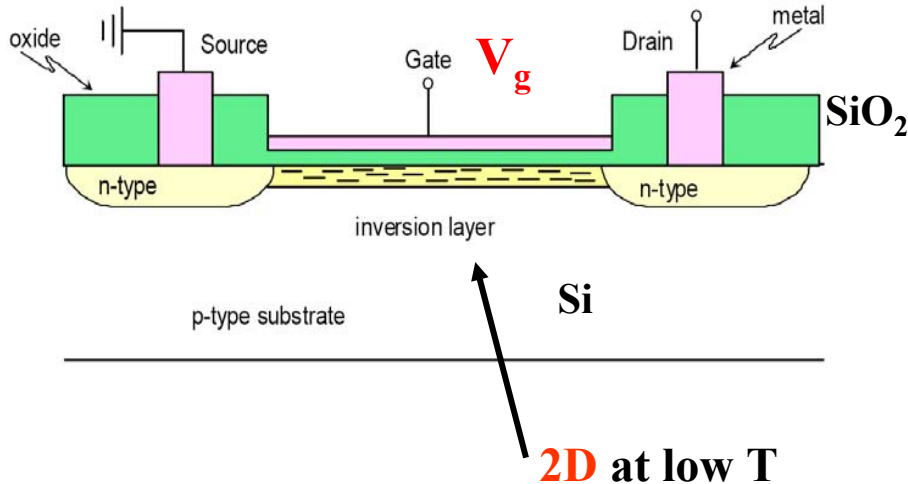


- 2D system of **strongly interacting** electrons in a **random** potential
- Electron density  $n_s$  varied from the **insulating** to the **metallic** regime, *i.e.* through the **metal-insulator transition (MIT)**
- Probing the glassy **dynamics**:
  - 1) measure **fluctuations** of conductivity – information on **correlations**  
⇒ **slowing down and correlated statistics** for  $n_s < n_g$  as  $T \rightarrow 0$
  - 2) measure **response** to a perturbation  
⇒ **nonexponential relaxations**  
⇒ **diverging equilibration times** for  $n_s < n_g$  as  $T \rightarrow 0$   
(**glass transition  $T_g=0$** )  
⇒ **aging and memory**  
⇒ **abrupt change in aging properties at the 2D MIT ( $n_c$ )**

# Samples: 2D electron system in Si MOSFETs



(metal-oxide-semiconductor field-effect transistor)



**Disorder due to:**

- 1) **Na<sup>+</sup> ions randomly distributed throughout SiO<sub>2</sub> (frozen out below ~100 K)**
- 2) **interface roughness**

**2D electrons move in a smooth random potential**

- **low densities ( $n_s \sim 10^{11} \text{ cm}^{-2}$ )**

**Fermi energy:**  $E_F = \pi \hbar^2 n_s / 2m^* \approx 0.6 \text{ meV}$

**Electron-electron interaction energy:**

$$E_{e-e} \sim (e^2/\epsilon)(\pi n_s)^{1/2} \approx 10 \text{ meV}$$

→  $r_s \equiv E_{e-e}/E_F \propto n_s^{-1/2} \sim 10!$

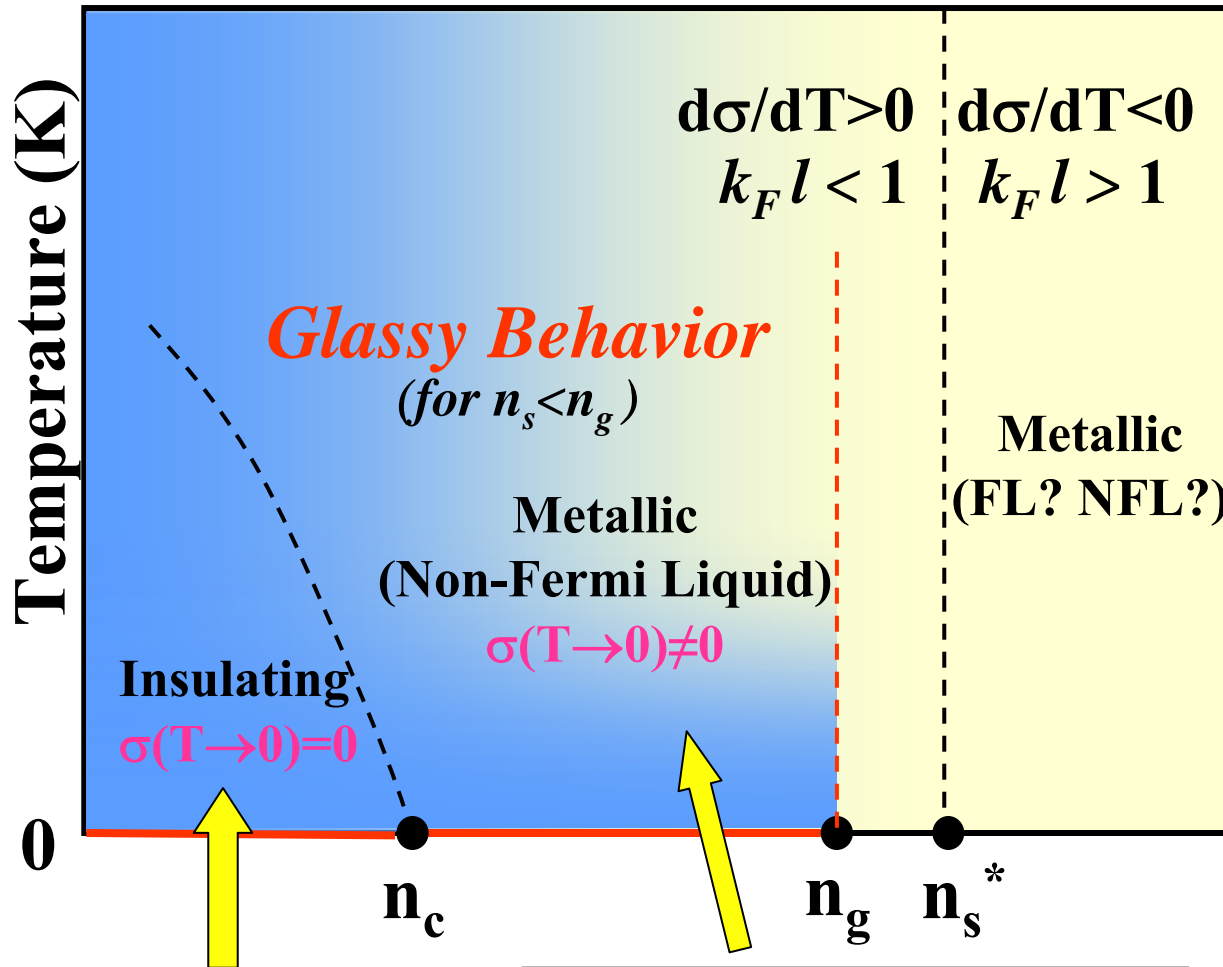
- **critical conductivity  $\sim e^2/h$**

$$\sigma \sim (e^2/h)(k_F l) \Rightarrow k_F l \sim 1$$

( $l$  – mean free path;  $k_F$  – Fermi wave vector)

**⇒ strong Coulomb interactions, strong disorder**

# Phase diagram of a 2DES in Si



Focus on the  $k_F l < 1$  region

(glassiness not observed for  $k_F l > 1$ )

$n_c$  – critical density for the MIT  
 $n_g$  – glass transition density

Exponential localization

“Bad” metal:  
 $\sigma(n_s, T) = \sigma(n_s, T=0) + b(n_s) T^{3/2}$

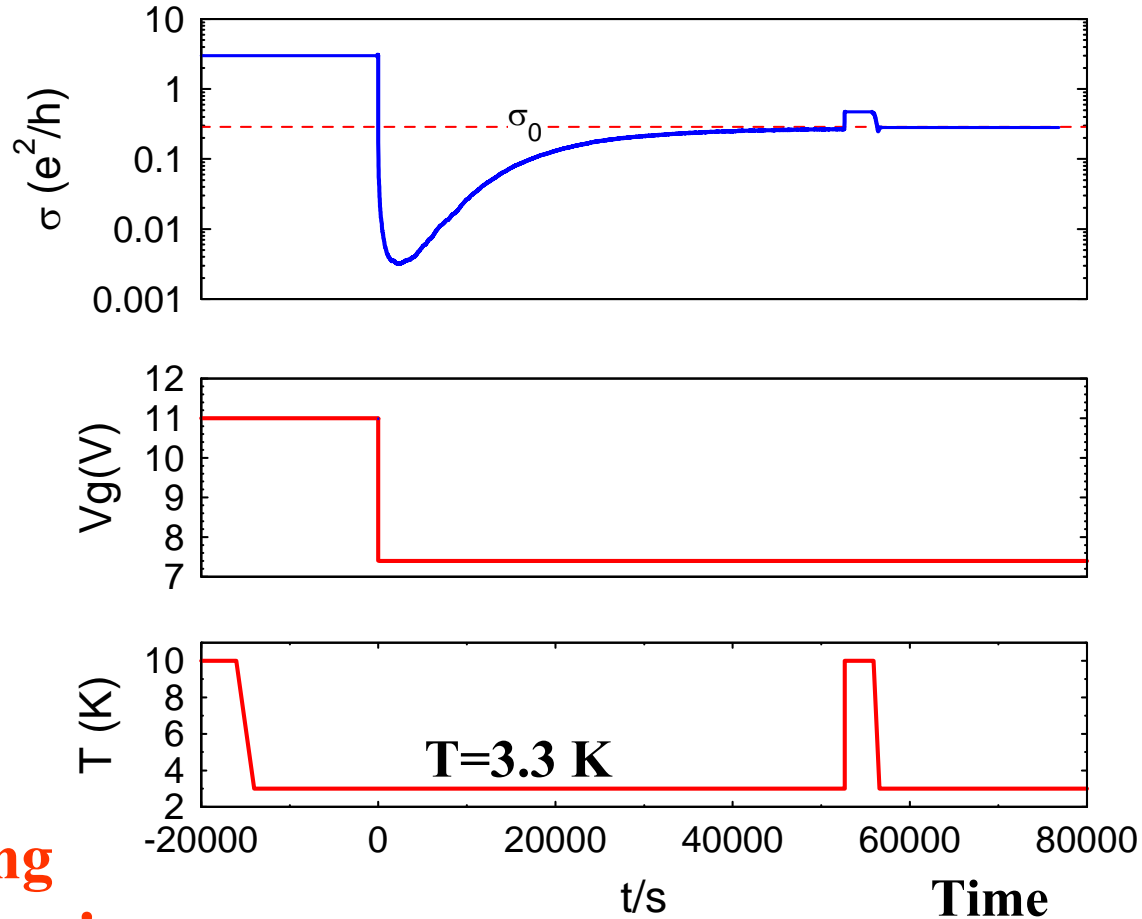
# Relaxations of conductivity after a rapid change of $n_s$



$$n_g \approx 7.5 \times 10^{11} \text{ cm}^{-2}, n_c \approx 4.5 \times 10^{11} \text{ cm}^{-2}$$

Low-mobility samples

Initial  
 $n_s (10^{11} \text{ cm}^{-2})$   
 $= 20.26 > n_g$   
 $k_F l \leq 1$



$\sigma_0$  – equilibrium conductivity at T and final  $n_s$

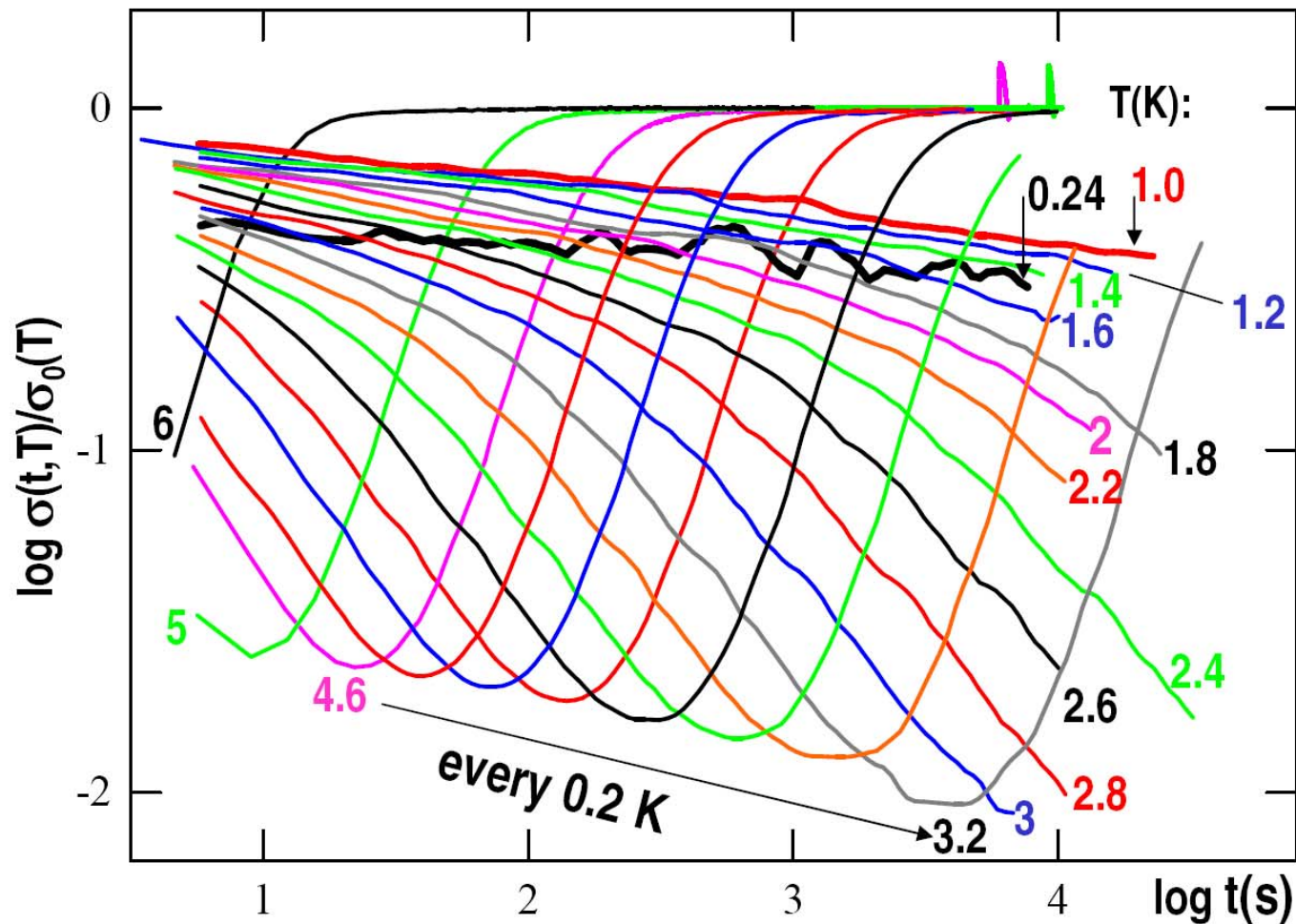
Final  
 $n_s (10^{11} \text{ cm}^{-2}) =$   
 $= 4.74 \geq n_c$

$$\Delta E_F \gg k_B T$$

Overshooting of equilibrium!

[J. Jaroszyński and D. Popović, PRL 96, 037403 (2006)]

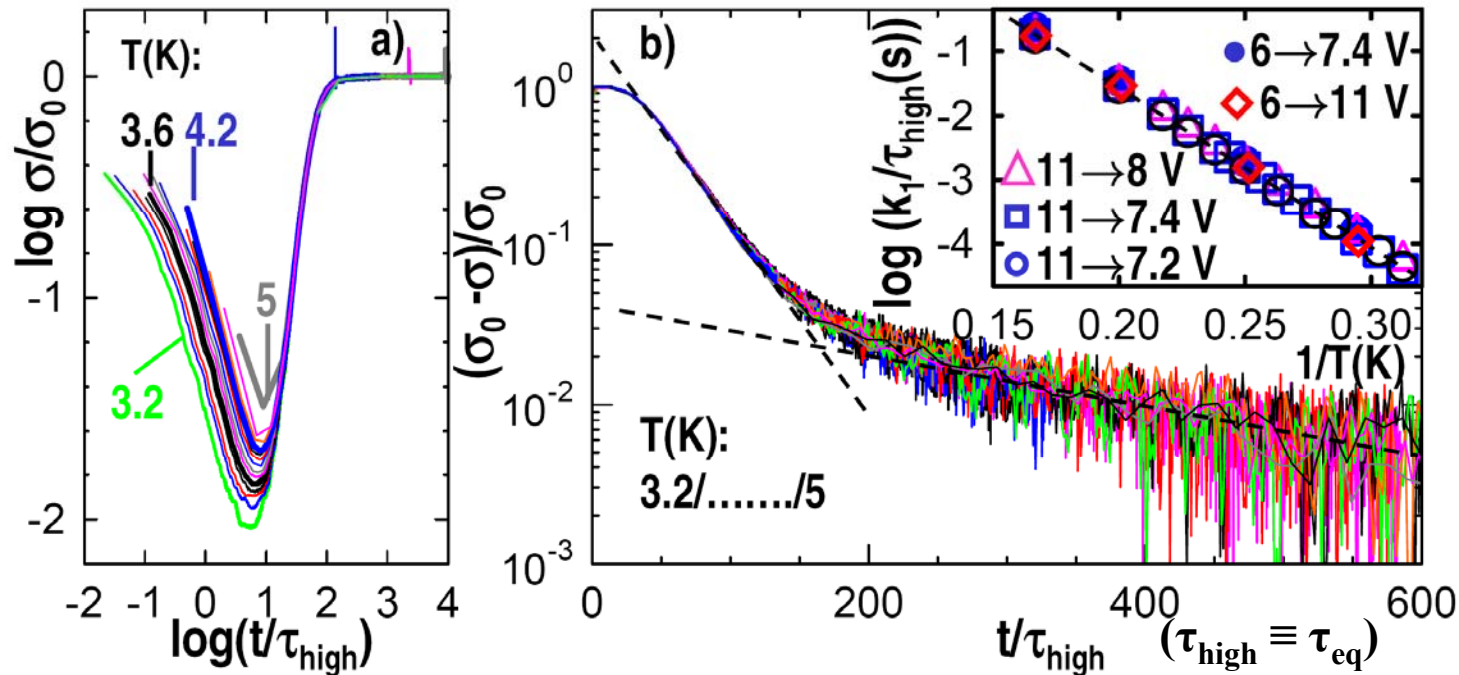
Repeat measurement at (many) different T (after warm-up to 10 K):



- minimum moves to longer times as T decreases – slower relaxations

Approach to equilibrium:

data (for different T) collapse for times after the minimum



• Relaxations exponential

- The system reaches equilibrium after a long enough  $t$
- Characteristic (equilibration) time  $\tau_{\text{eq}} \propto \exp(E_A/T)$ ,  $E_A \approx 57$  K

$\tau_{\text{eq}} \rightarrow \infty$  as  $T \rightarrow 0$ , i.e. glass transition  $T_g = 0$

[see Gempel, Europhys. Lett. 66, 854 (2004) for a 2D Coulomb glass; also showed aging]

**Initial relaxation:**

data (for different T) collapse for times before the minimum:



- for short enough  $t < \tau_{eq}$ ,

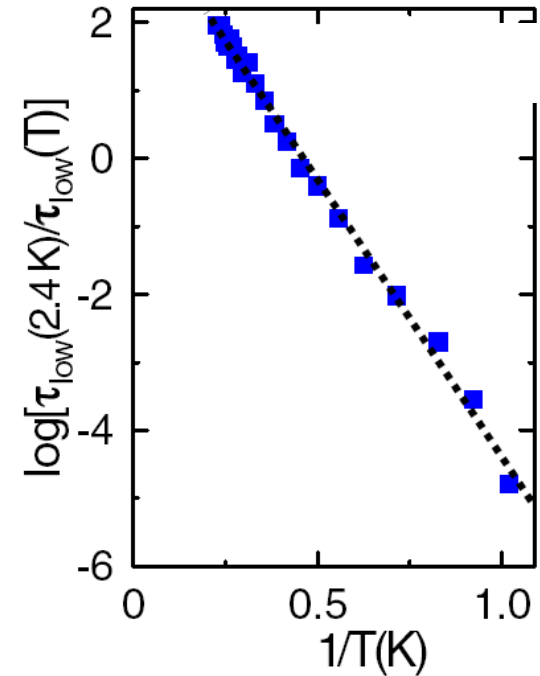
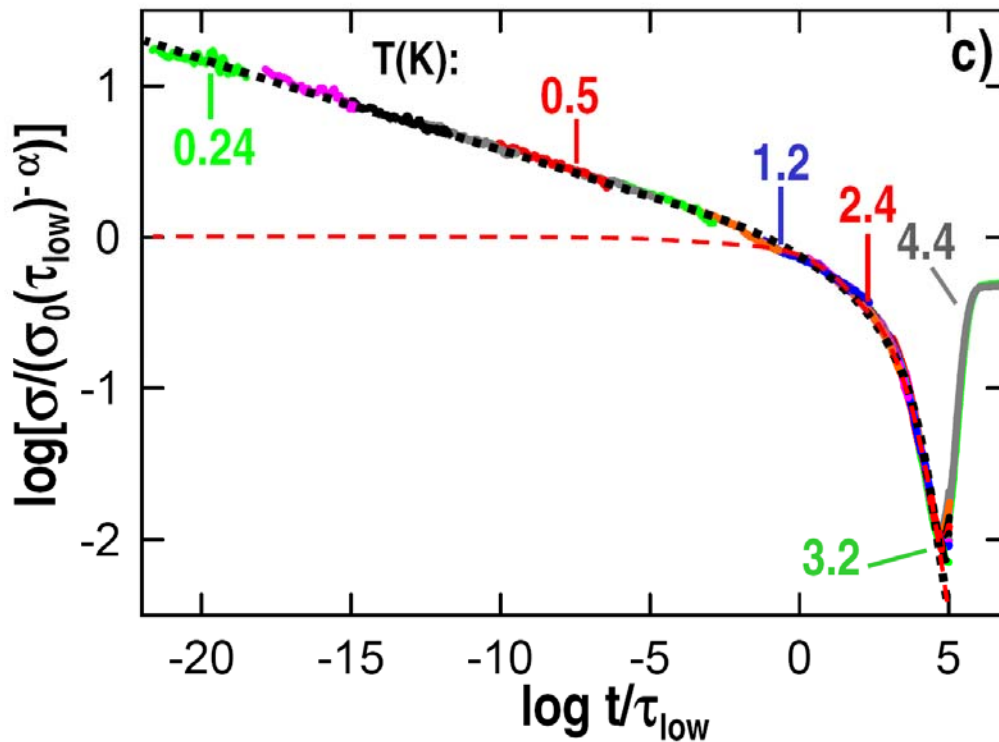
$$\sigma(t, T)/\sigma_0 \propto t^{-\alpha(n)} \exp\{-[t/\tau_{low}(n_s, T)]^{\beta(n)}\}$$

( $\alpha=0.07$ ,  $\beta < 0.3$  for this  $n_s$ )

( $n \equiv n_s$ )

glassy relaxation

$$\tau_{low} \propto f(n_s) \exp(E_a/T), E_a \approx 20 \text{ K}$$







Repeat everything for many different  $n_s$



$$\tau_{\text{low}} \propto \exp(an_s^{1/2}) \exp(E_a/T), E_a \approx 20 \text{ K}$$

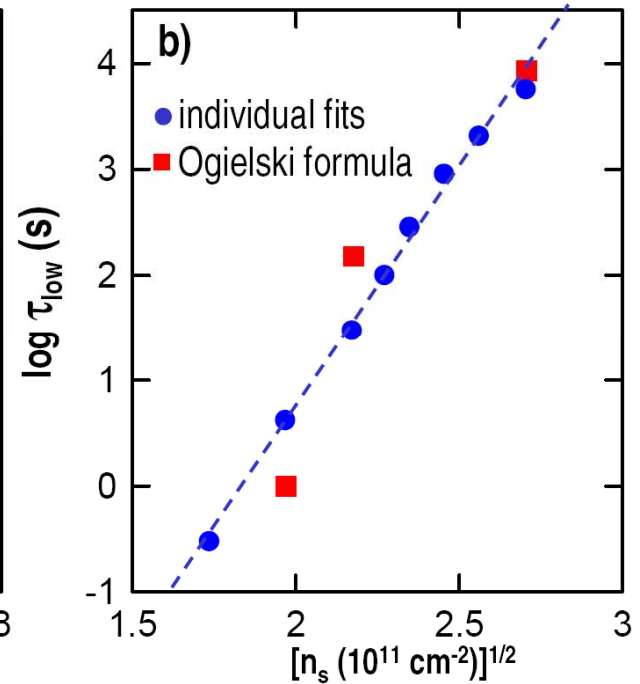
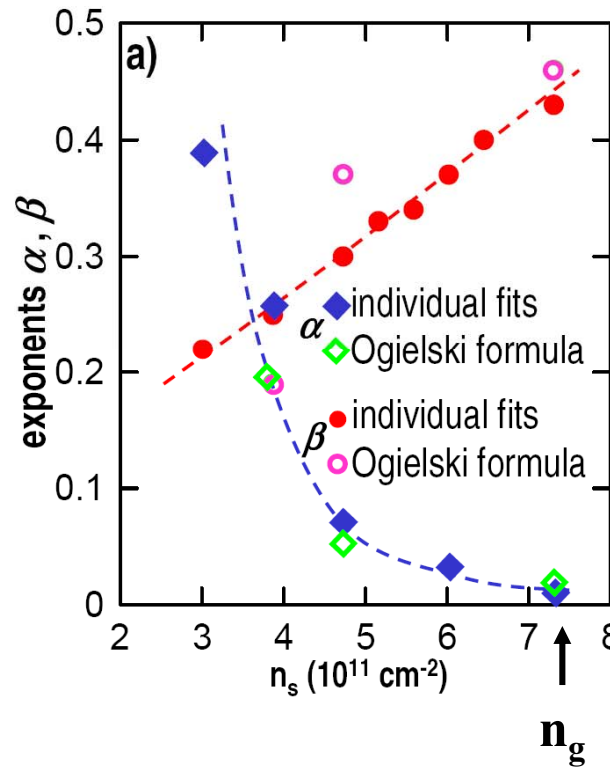
•  $T \rightarrow 0$ :

$$\sigma/\sigma_0 \propto t^{-\alpha}$$

as expected for a phase transition at  $T=0$

(previous slide: scaling as  $T \rightarrow 0$ )

• Coulomb interactions in 2D:  $E_F/U \sim n_s^{1/2}$



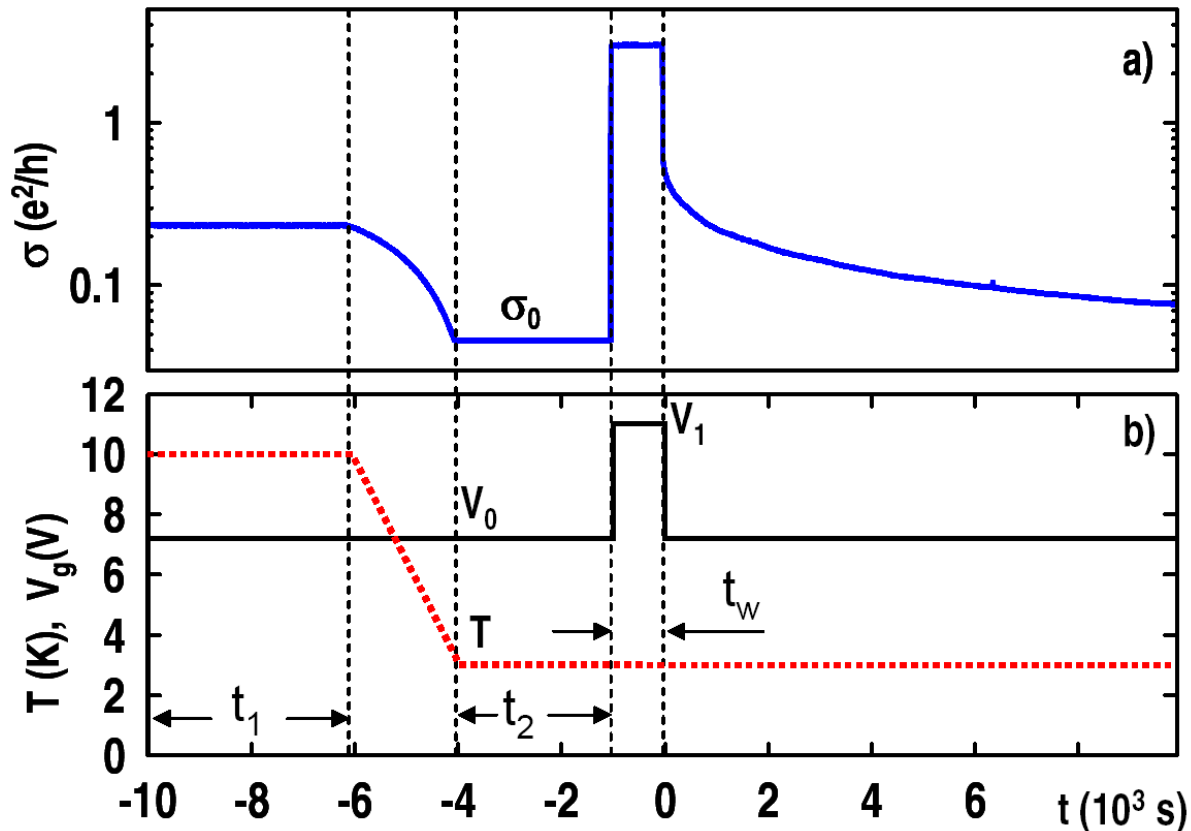
$\beta$  grows with  $n_s$ :  
relaxations faster with increasing  $n_s$

## What have we learned from relaxations?



- data strongly suggest  $T_g=0$  for  $n_s \leq n_g$  in a 2DES in Si  
(diverging equilibration time, scaling of nonexponential relaxations, power law as  $T \rightarrow 0 \Rightarrow T_g = 0$ ; similar behavior in spin glasses, where  $T_g \neq 0$ )
  - at finite  $T$ , the system appears glassy for short enough  $t$   
(e.g. at  $T=1$  K, equilibration time  $\sim 10^{13}$  years!  
age of the Universe  $\sim 10^{10}$  years)
  - Coulomb interactions between 2D electrons – a dominant role in the out-of-equilibrium dynamics
  - as  $T \rightarrow 0$ , no relaxations for  $n_s > n_g$ ; no relaxations for  $k_F l > 1$
- Note: system equilibrates only after it first goes farther away from equilibrium!

# Relaxations of conductivity after a waiting time protocol: aging and memory



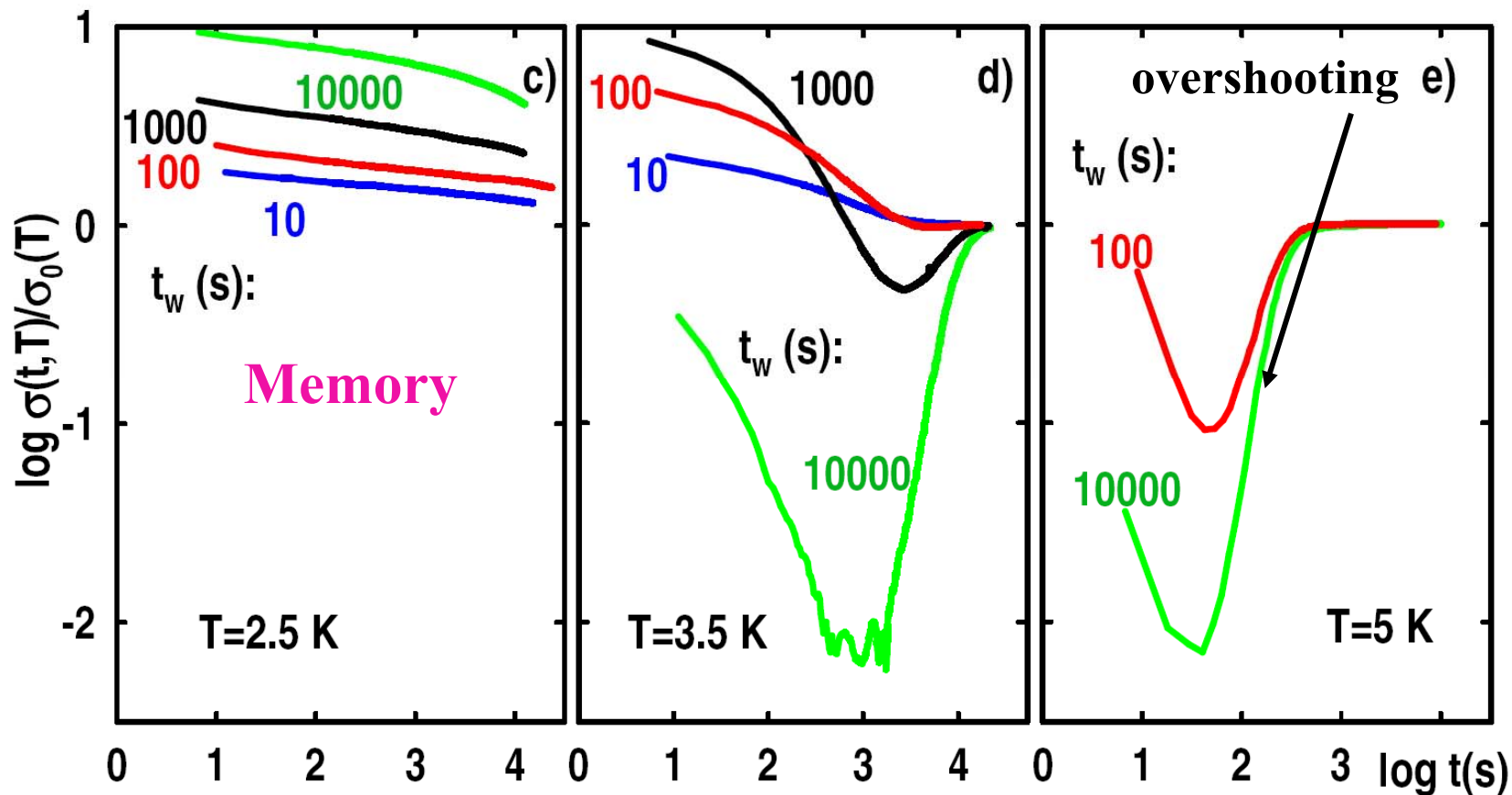
Initial and final  
 $n_s(10^{11}\text{cm}^{-2})=3.88 < n_c$ ;  
 density during  $t_w=1000$  s:  
 $n_s(10^{11}\text{cm}^{-2})=20.26 > n_g$

- change history by varying  $T$  and  $t_w$

[J. Jaroszyński and D. Popović, Phys. Rev. Lett. 99, 046405 (2007)]

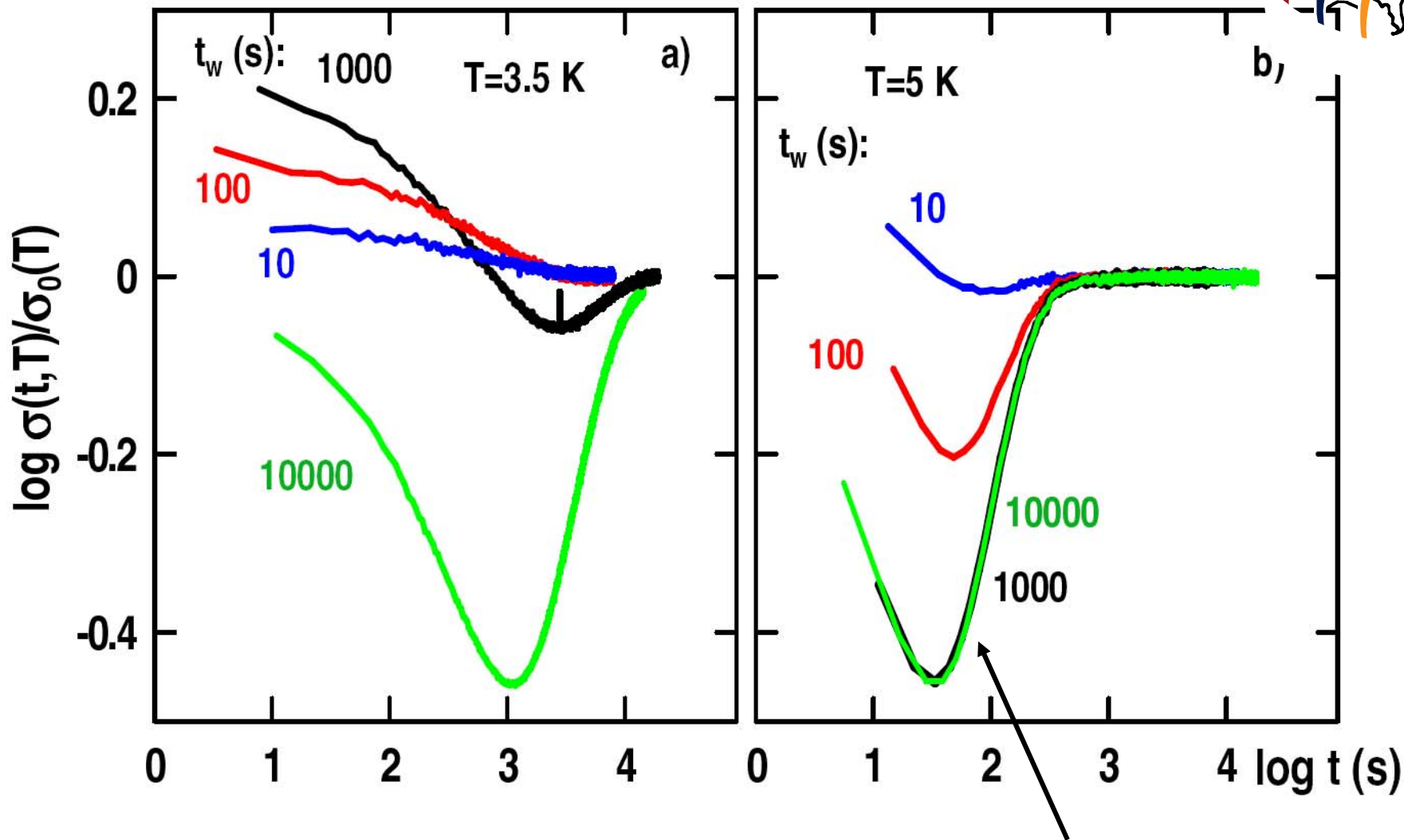


## Relaxations for a few different T and $t_w$ :



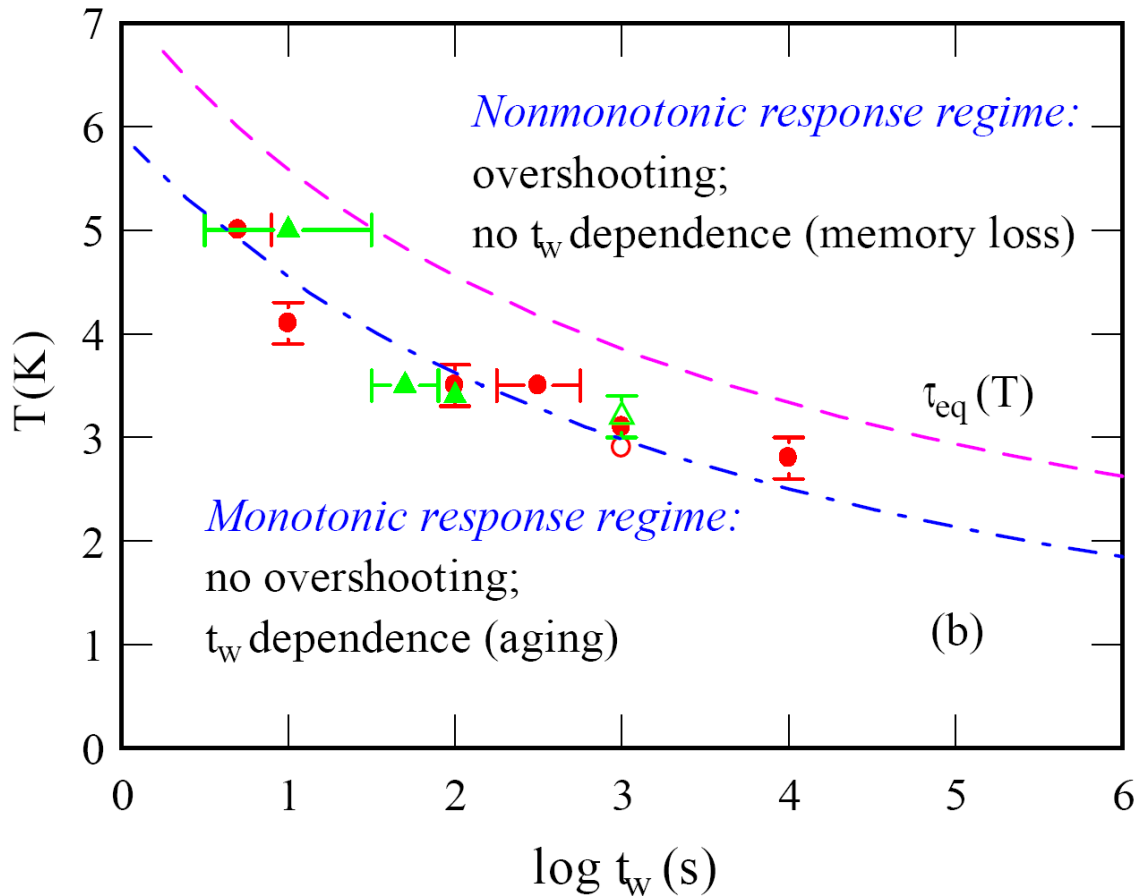
**Response (conductivity) depends on the system history ( $t_w$  and T) in addition to the time  $t$  – *aging* – a key characteristic of relaxing glassy systems.**

And a few more... [for  $n_s(10^{11}\text{cm}^{-2}) = 7.33 \leq n_g$ ]:



Memory loss

# When is the overshooting observed?



- **overshooting** only when the system is excited out of a thermal equilibrium ( $t_w \gg \tau_{eq}$ ); no memory
- no OS when excited out of a relaxing (nonequil.) state ( $t_w \ll \tau_{eq}$ ): **aging and memory**

# What is the origin of overshooting???

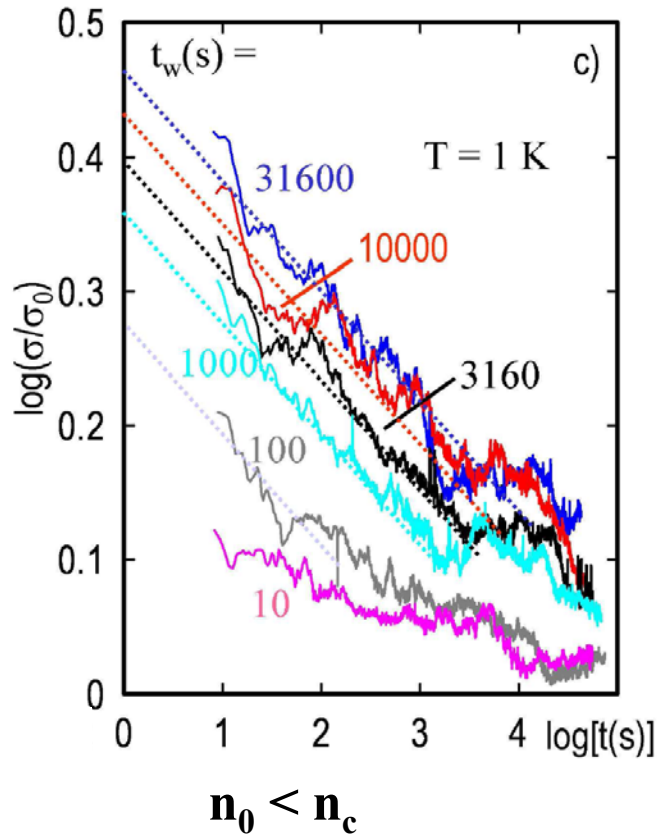


- **observed** in a variety of systems (*e.g.* insulating granular metals, manganites, biological systems)
- some theoretical **models**  
[Morita *et al.*, PRL 94, 087203 (2005); Mauro *et al.*, PRL 102, 155506 (2009)]
- **large** perturbations out of equilibrium?
- here  $\Delta E_F \gg T$  should trigger **major charge rearrangements**  
( $n_s$  changed up to a factor of 7; in  $\text{InO}_x$ , density change  $\sim 1\%$ )

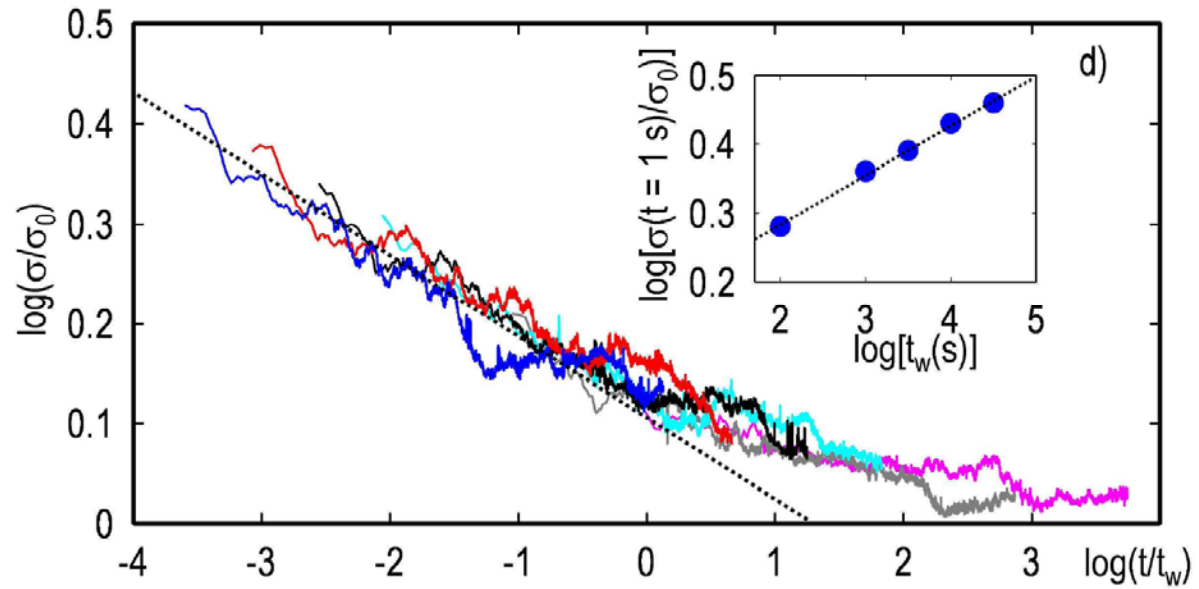
# Aging regime (no OS, T=1 K)

[J. Jaroszyński and D. Popović, Phys. Rev. Lett. 99, 216401 (2007)]

(T= 1 K:  $\tau_{eq} \sim 10^{13}$  years!  
Age of the Universe  $\sim 10^{10}$  years)



## Full (simple) aging: $\sigma(t/t_w)$



$$\sigma(t)/\sigma_0 \propto (t/t_w)^{-\alpha} \quad \text{for } t \leq t_w$$

$\Rightarrow$  a memory of  $t_w$  is imprinted on each  $\sigma(t)$

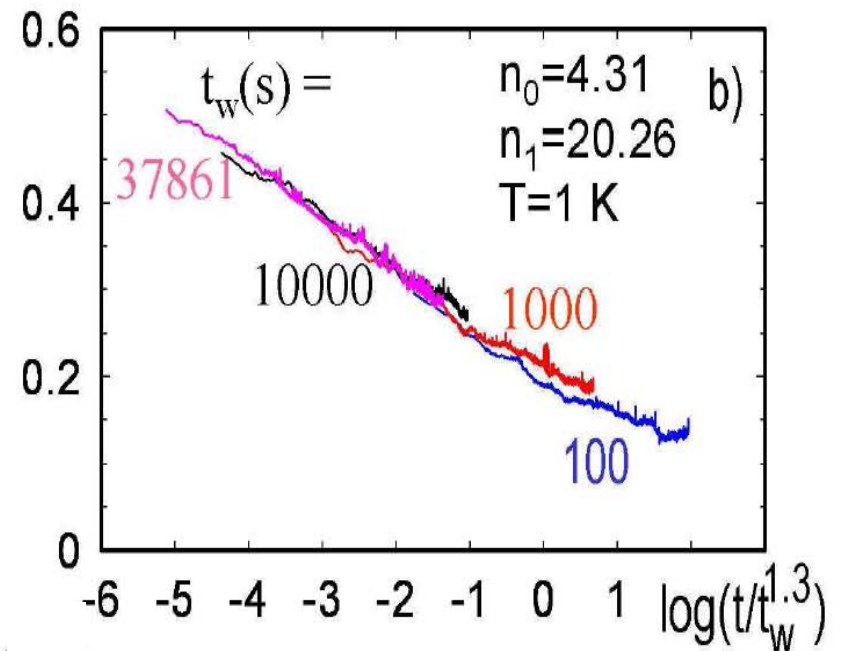
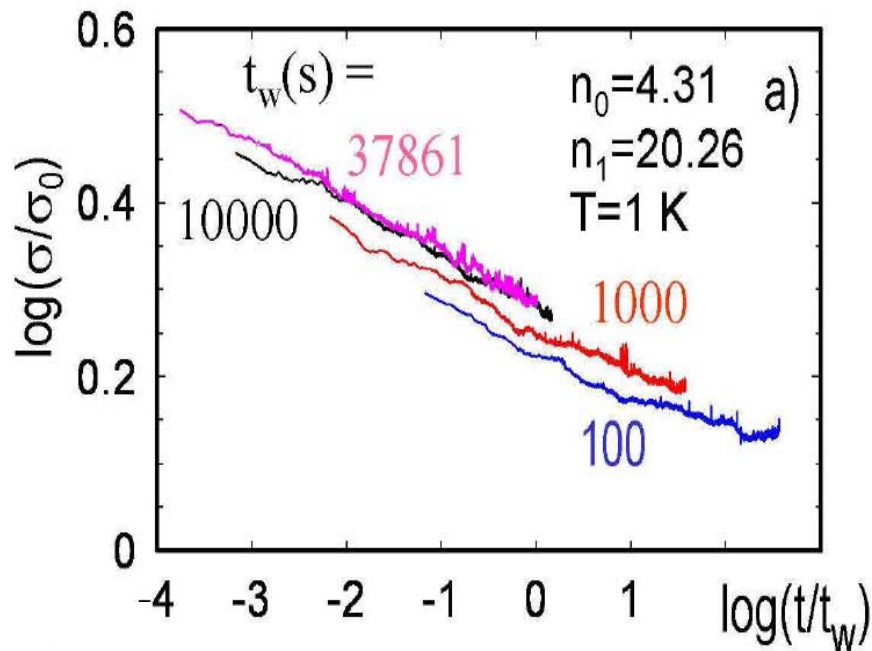




- $\sigma(t, t_w)$  exhibit **full aging** for  $n_s < n_c$
- for  $n_s > n_c$ , an increasingly strong **departure from full aging**

aging function:  $\sigma(t/t_w^\mu)$

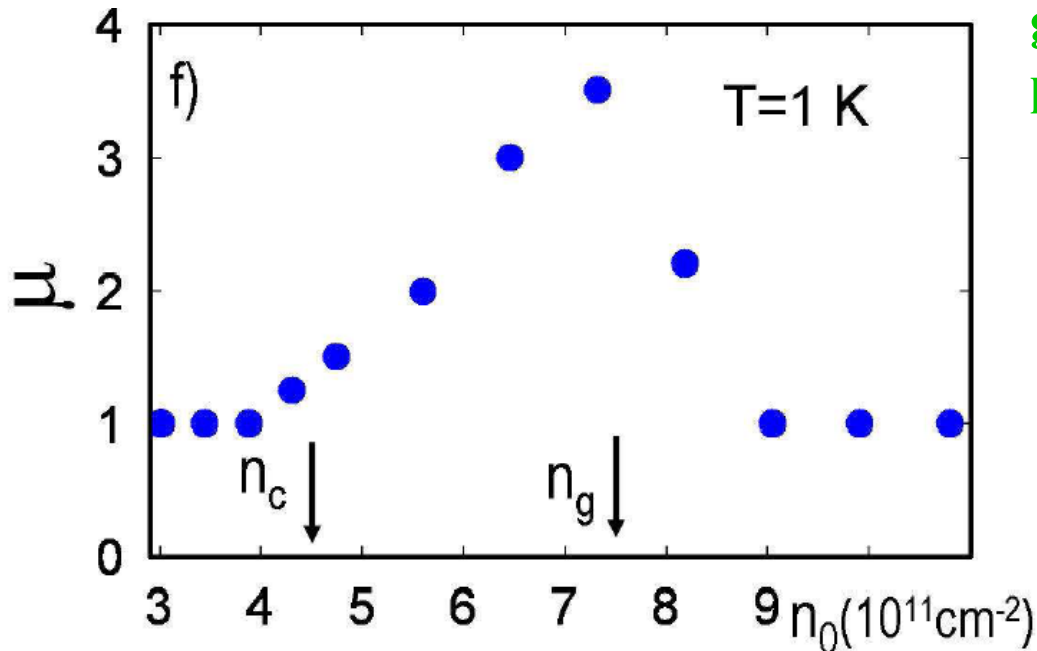
( $\mu$ -scaling useful in studies of other glasses; may not have a clear physical meaning)





- $\sigma(t, t_w)$  exhibit **full aging** for  $n_s < n_c$
- for  $n_s > n_c$ , an increasingly strong **departure from full aging** that reaches maximum at  $n_g$

aging function:  $\sigma(t/t_w^\mu)$



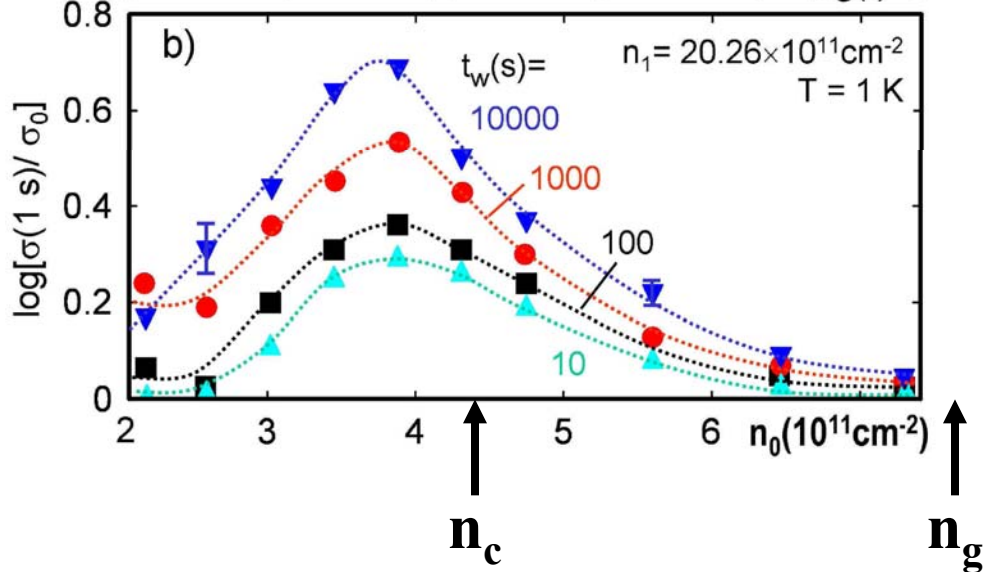
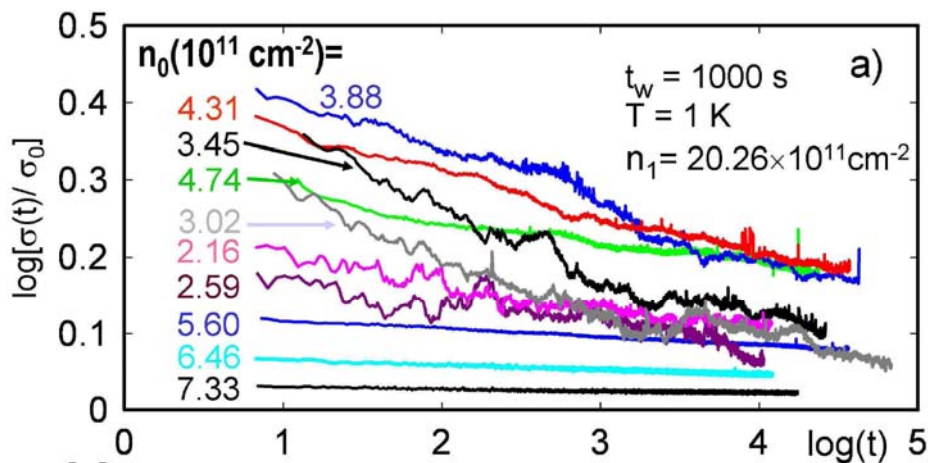
( $\mu$ -scaling useful in studies of other glasses; may not have a clear physical meaning)

full aging:  $\mu=1$

- an abrupt change in aging at the 2D MIT ( $n_c$ )
- insulating glassy phase and metallic glassy phase are different!

**NOTE:** mean-field models of glasses, for example, include both those that show full aging and those where no  $t/t_w$  scaling is expected.

Fixed  $t_w$  and  $n_1$ ; vary  $n_0$



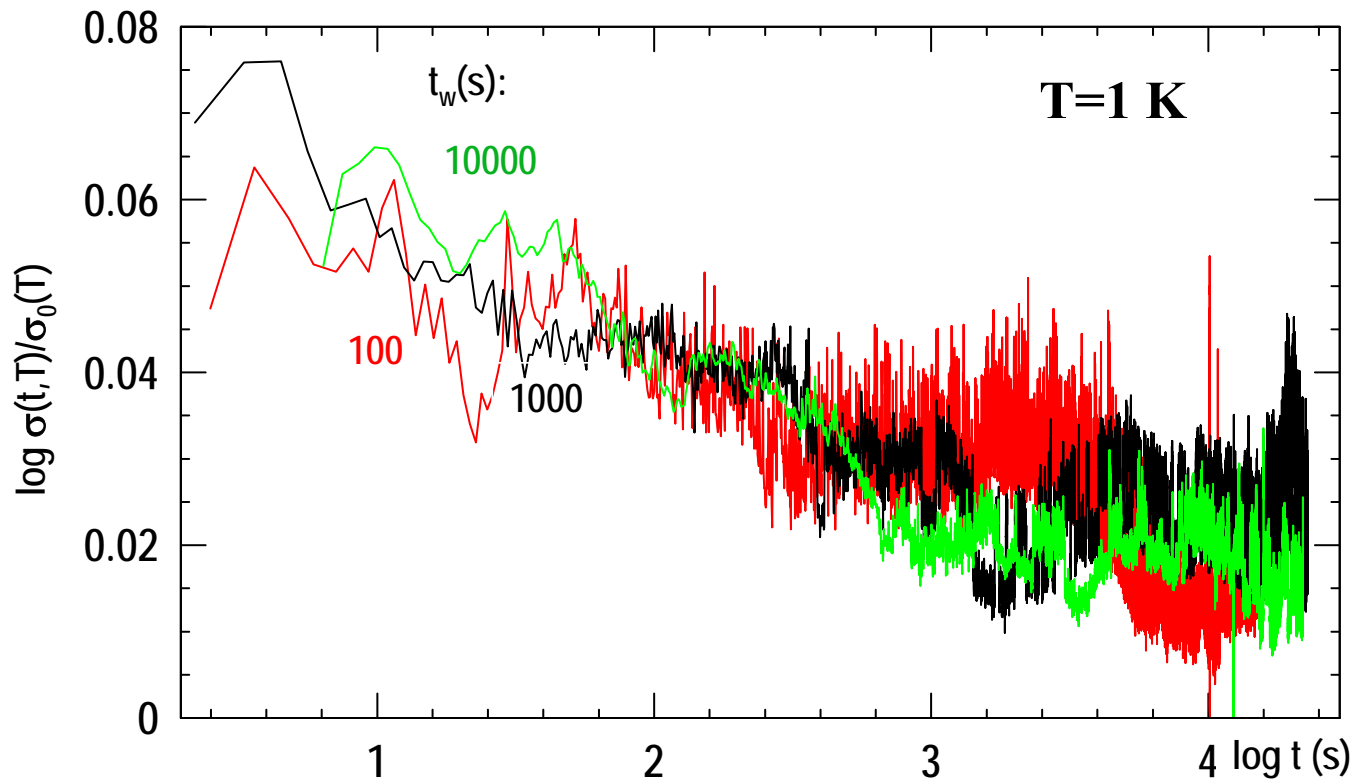
$$\sigma(t)/\sigma_0 = [\sigma(t=1s)/\sigma_0] t^{-\alpha}$$

- both **relaxation amplitudes**  $\sigma(t=1s)/\sigma_0$  and slopes  $\alpha$  depend **nonmonotonically** on  $n_0$
- **another change in aging properties** at  $n_s \approx n_c$

**Relaxation amplitudes peak just below  $n_c$ , and they are suppressed in the insulating regime!**



Remove all 2D electrons from the inversion layer during  $t_w$   
( $V_1 < V_T$ ):



**No  $t_w$  dependence, *i.e.* no memory!**

**$\Rightarrow$  Glassiness from 2DES, not from background charges**

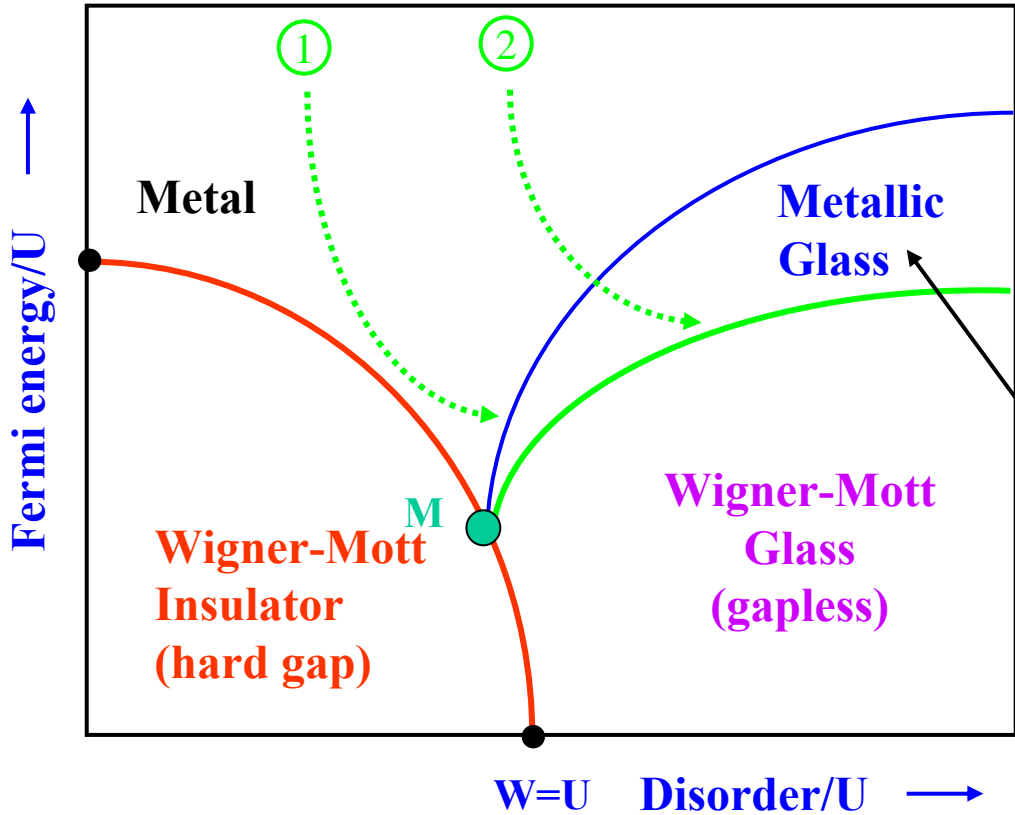
## Summary: 2D Coulomb glass



- Emergence of an **intermediate, (NFL) metallic phase** ( $n_c < n_g$ ) between the metal and the insulator
- **Glassy behavior for  $n_s < n_g$**  (in the insulator and in the intermediate phase) – glassy ordering as a **precursor of the MIT in a 2DES in Si**
- Manifestations of glassiness:  
**nonexponential relaxations, diverging equilibration times ( $T_g=0$ ), aging and memory** (abrupt changes in aging at the MIT)
- 2DES in Si:
  - similarities to other glassy systems (*e.g.* spin glasses)
  - a “simple”, **model system** for exploring the dynamics of strongly correlated systems (**free of complications associated with changes in magnetic or structural symmetry**)

[V. Dobrosavljevic *et al.*: PRL 83, 4642 (1999); PRB 66, 081107 (2002); PRL 90, 016402 (2003); PRL 91, 066603 (2003); EPL 67, 226 (2003); PRL 94, 046402 (2005)]

**Global phase diagram (theory)**



- MIT as a Mott transition with disorder (DMFT picture)
- glass as a precursor of MIT
- melting of glass even at T=0 (by quantum fluctuations)
- Metallic glass phase:  $\sigma(T) - \sigma(0) \sim T^{3/2}$
- hierarchical, correlated dynamics

Physical trajectory:  $E_F \sim n_s$ ;  $U \sim n_s^{1/2}$ ;  $W \sim \text{const.}$   $\Rightarrow (E_F/U) \sim (W/U)^{-1}$

① High-mobility samples, ② Low-mobility samples

# Simulations



- **Molecular Dynamics** [C. Reichhardt and C. J. Olson Reichhardt, *PRL*, **93**, 176405 (2004)]: a classical model of interacting electrons in 2D with disorder
- **increase of noise power and  $\alpha$**  with decreasing density and  $T$
- **non-Gaussianity** at low  $n_s$  and  $T$

Similar to experiments in 2DES in Si

Trajectories change with time:  
dynamical inhomogeneities

Noise power and  $\alpha$  maximum

- **Monte Carlo** [Kolton, Grempel, Dominguez, *PRB* **71**, 024206 (2005)]: 3D Coulomb glass – heterogeneous dynamics

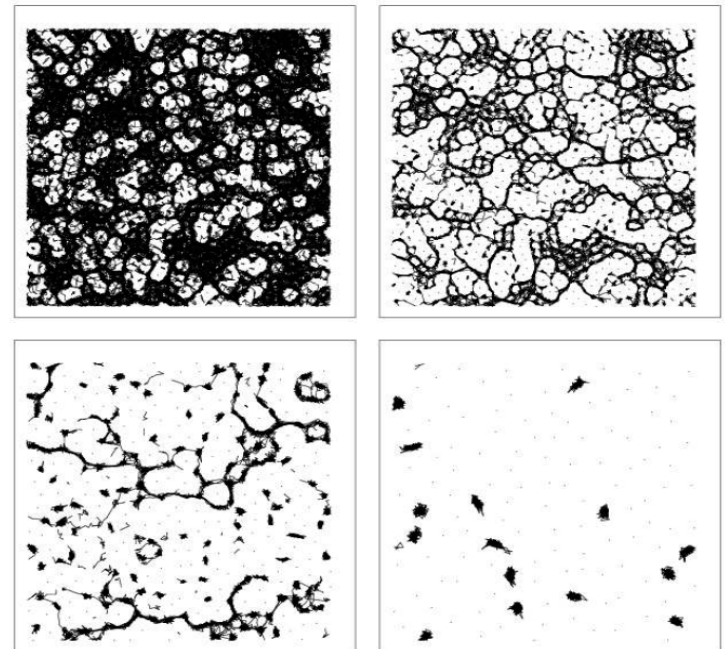


FIG. 5. Electron trajectories for a fixed period of time for fixed  $T = 0.09$  at (a)  $N_s/N_p = 1.67$ , (b) 1.37, (c) 0.5, and (d) 0.3.



## Monte Carlo – aging in a 2D Coulomb glass:

- Grepel, Europhys. Lett. 66, 854 (2004)
- Shimer, Täuber, Pleimling, arXiv: 1007.1929 (2010) –  
density autocorrelation function

The aging function obeys power-law scaling

$$\sim t_w^{-b} (t/t_w)^{-\alpha}$$

where the exponents depend on the density and T