The World's Thinnest Capacitor:

Anomalously Large capacitance in 2D electron gas and Ionic Liquid devices

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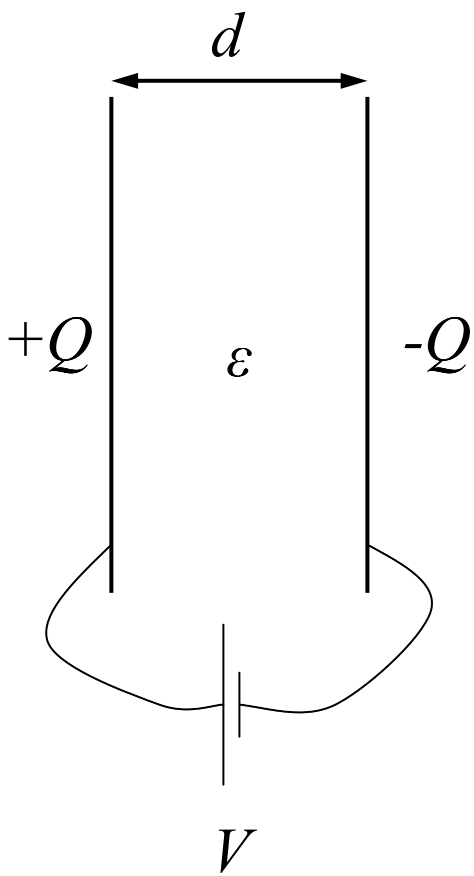
Part I:

A plane capacitor with a 2D electron gas: How large can its capacitance be?

Brian Skinner and B. I. Shklovskii

arXiv:1007.5308v2 (2010)

Normal "geometric" capacitance



Electrode area S

For two perfect metal electrodes:

$$C = C_g = \frac{\varepsilon S}{4\pi d}$$

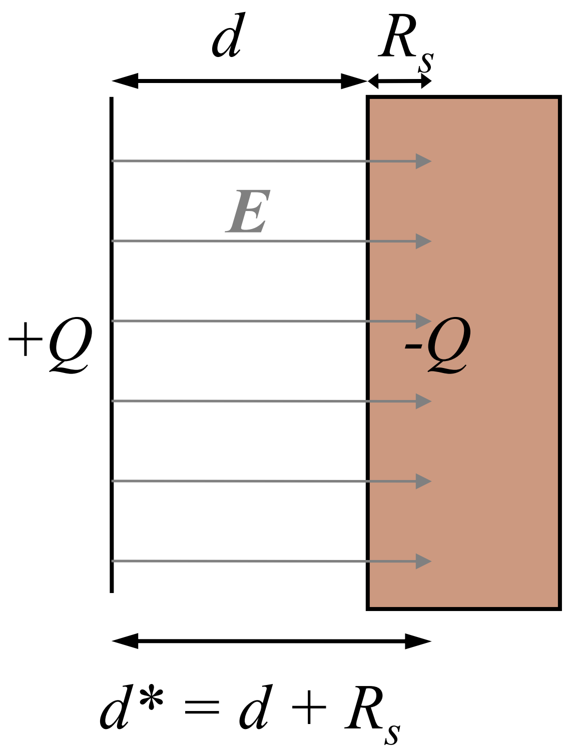
C is determined from total energy:

$$\frac{d}{dQ}(U - QV) = 0$$

$$\Rightarrow V = \frac{dU}{dQ}$$

$$\Rightarrow C = \frac{dQ}{dV} = \left(\frac{d^2U}{dQ^2}\right)^{-1}$$

Corrections to C_g from imperfect screening

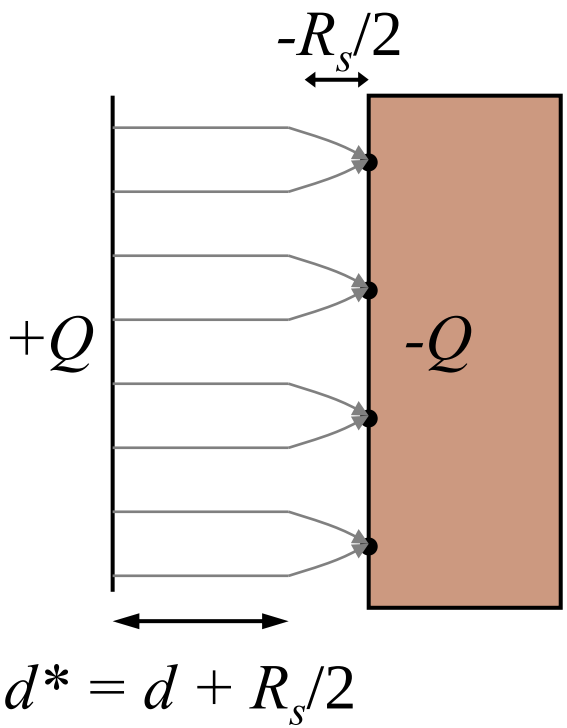


In 3D:

Electric field penetrates an imperfect electrode

$$C < C_g$$

The capacitance can be larger than C_g



At $n a_B^2 << 1$, a 2DEG is a classical system.

Strong electrostatic correlations lead to $\mu < 0$:

→ $\mu \sim -e^2 n^{1/2}/\varepsilon$

→ $R_s \sim d\mu/dn < 0$

→ $d^* = d - 0.12/n^{1/2} < d$

In 2D:

$$\frac{1}{C} = \frac{d^2U}{dQ^2} = \frac{1}{e^2S^2}\frac{d^2U}{dn^2}$$

$$d^* \equiv d \cdot \frac{C_g}{C} = d + \frac{1}{2} R_S^{(2D)}$$

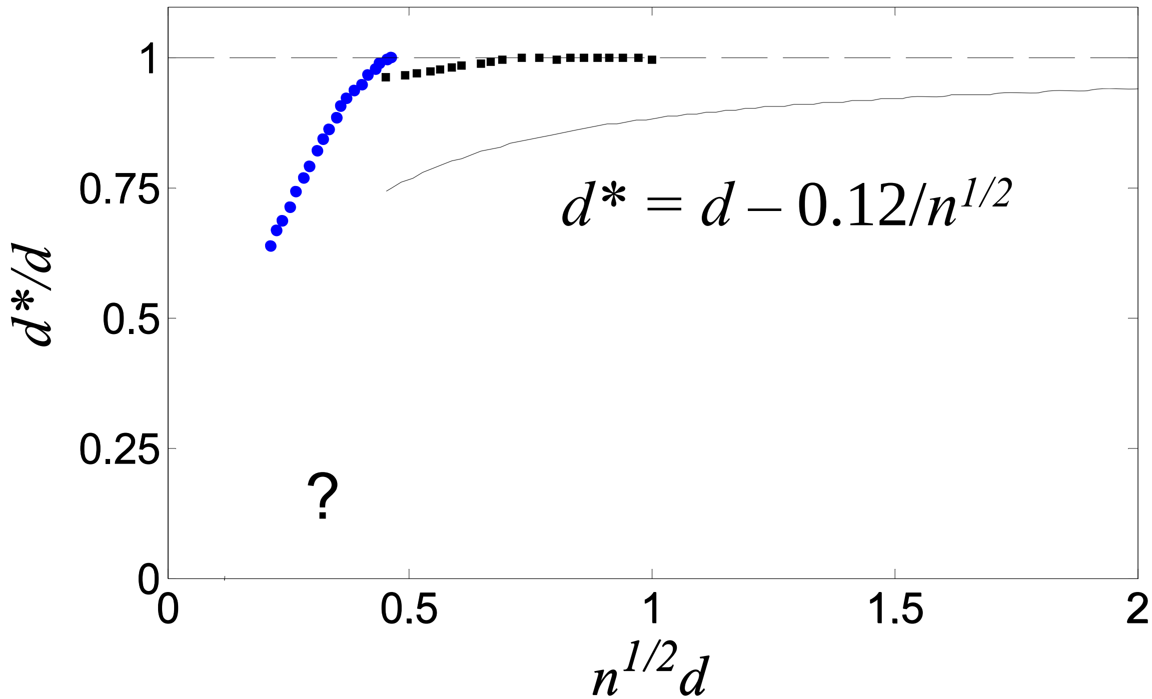
Theory: Bello, Levin, Shklovskii, and Efros, Sov. Phys.-JETP 53, 822 (1981).

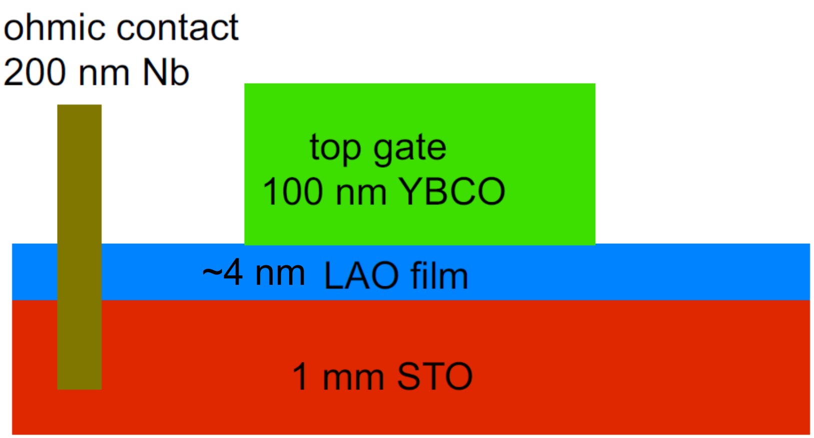
Experiment: Eisenstein, Pfeiffer, and West, PRL 68, 674 (1992).

How large can the capacitance be?

Most experiments operate at $n^{1/2}d > 1$

A recent experiment examined $n^{1/2}\, d < 1$

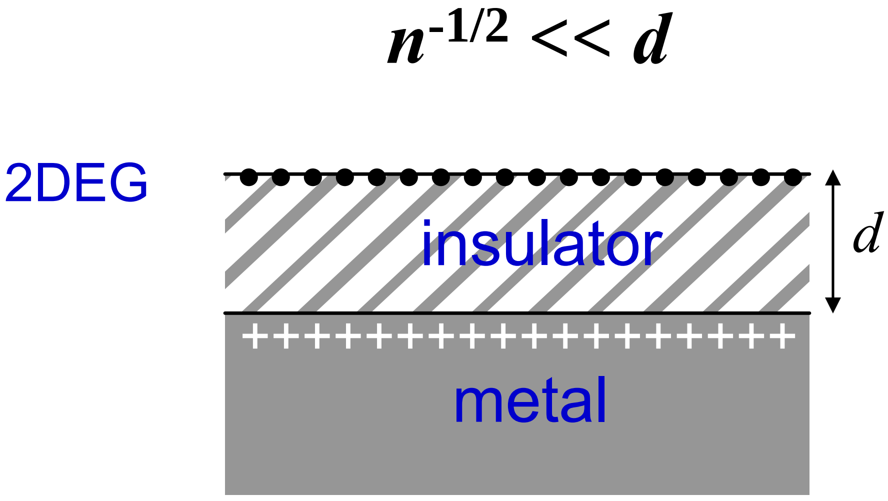


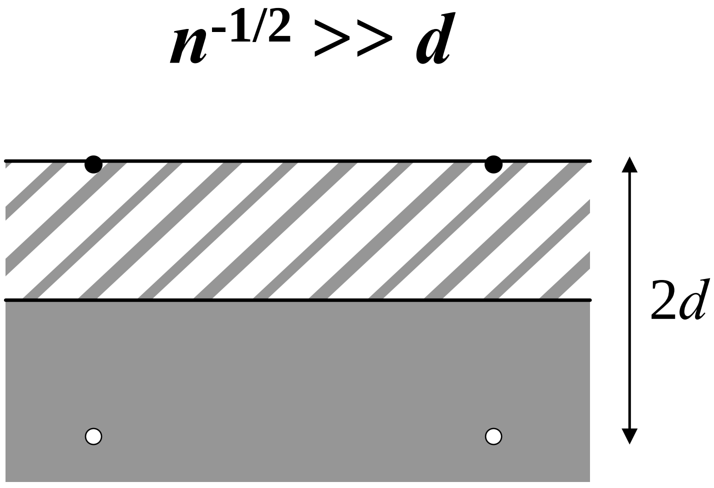


[Li, Richter, Paetel, Kopp, Mannhart, and Ashoori, arxiv:cond-mat/1006.2847 (2010)]

What happens in the limit $n^{1/2}\, d \rightarrow 0$? How large can C be?

Picture of the classical 2DEG





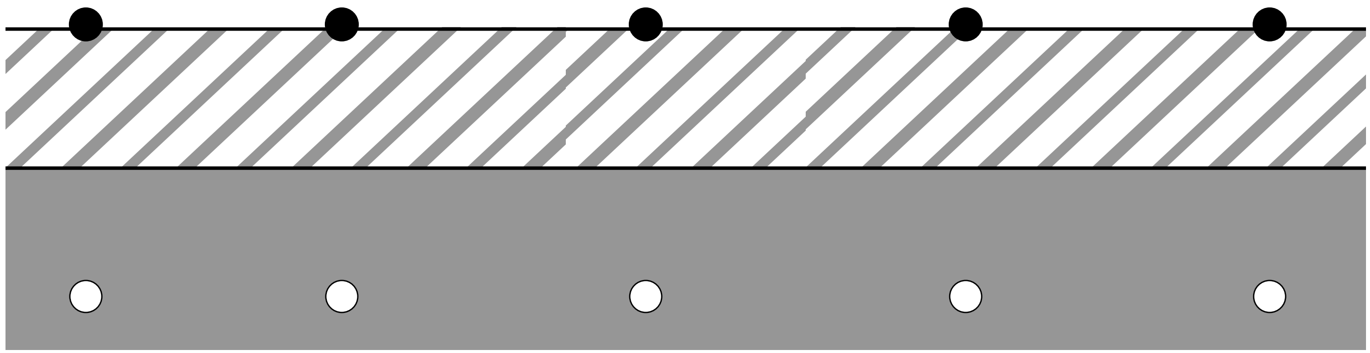
Metal charge is uniform.

Correlations produce a small correction to C.

Metal charge is discrete and correlated with the 2DEG.

Only a weak dipole-dipole repulsion resists capacitor charging.

Ground state of the classical 2DEG

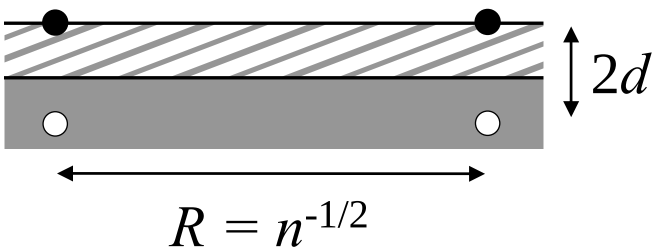


$$U = \frac{1}{2} e S n \phi_0$$

$$\phi_0 = \frac{e}{2\varepsilon d} - \sum_{\{i,j\} \neq \{0,0\}} \frac{e}{\varepsilon} \left(\frac{1}{r_{i,j}} - \frac{1}{\sqrt{r_{i,j}^2 + (2d)^2}} \right)$$

$$d^* = \frac{\varepsilon}{4\pi e^2 S} \frac{d^2 U}{dn^2}$$ can be computed for arbitrary nd^2.

Capacitance in the $nd^2 \rightarrow 0$ limit



one interaction:

$$u_{dd} = e^2(2d)^2/2\varepsilon R^3$$

all interactions:

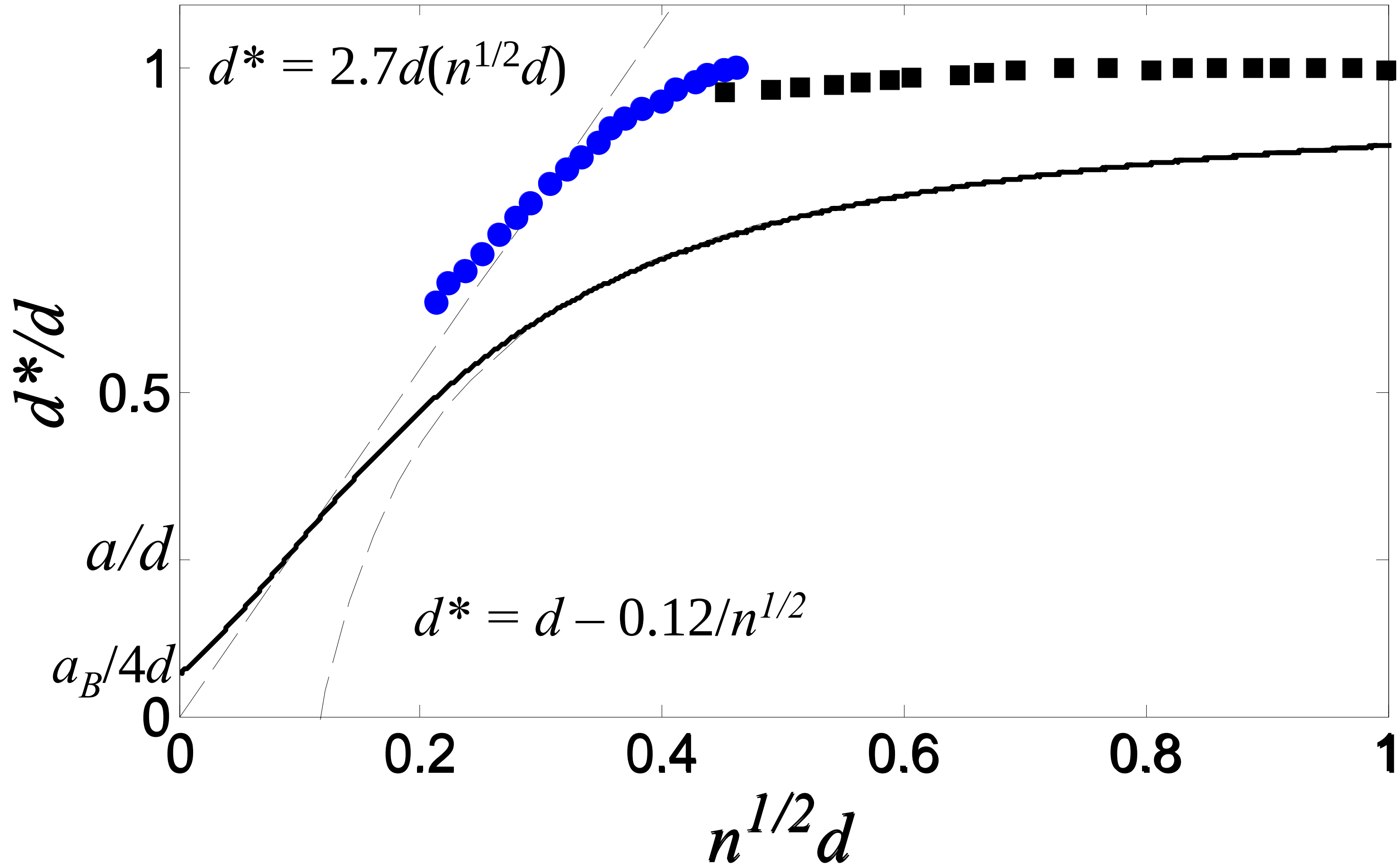
$$U = \frac{1}{2}\alpha n S u_{dd} = \alpha e^2 d^2 S n^{5/2}$$

$\alpha \approx 9.0$

$$d^* = \frac{\varepsilon}{4\pi e^2 S}\frac{d^2 U}{dn^2} = \frac{15\alpha}{16\pi} d \cdot n^{1/2} d$$

$$d^* \approx 2.7 d \cdot (n^{1/2} d)$$

Comparison with experiment



At $n^{1/2}d << a_B/d$, the quantum confinement energy ($\sim 1/R^2$) destroys the dipole correlations ($\sim 1/R^3$).

C is truncated at $d^* = a_B/4$.

Other experimental realizations of a capacitance substantially larger than geometrical

- GaAs-AlGaAs HIGFETs with n =10^9 cm^-2 holes and d=250nm, nd^2 =1.

- Two parallel quantum wells with separate contacts (a capacitor with *two* 2DEGs). At d =30 nm, n < 10^11 nd^2 <1. Jim Eisenstein says it can be measured. We generalized our theory for this case.

- Electrons floating on liquid helium surface with a close metal electrode under the surface. Presumably very low density n and low disorder.

Part II:

Anomalously large capacitance of an ionic liquid/metal capacitor

M. S. Loth, Brian Skinner and B. I. Shklovskii

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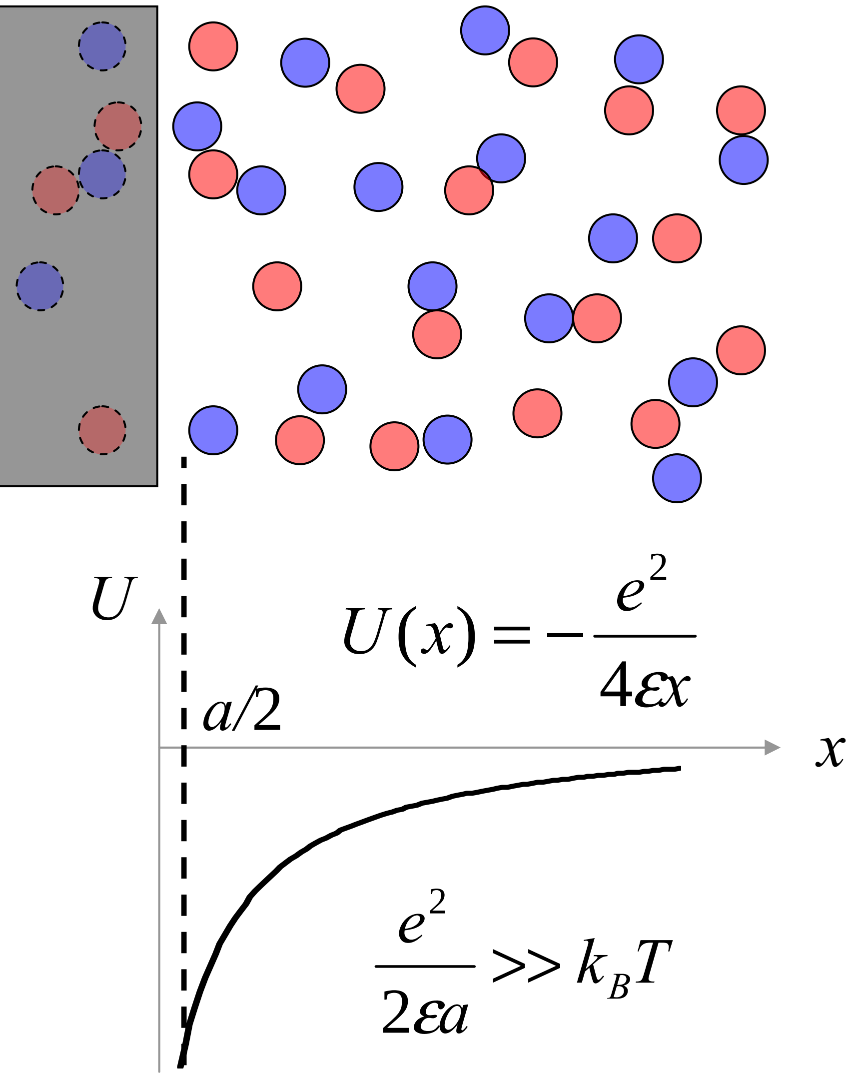
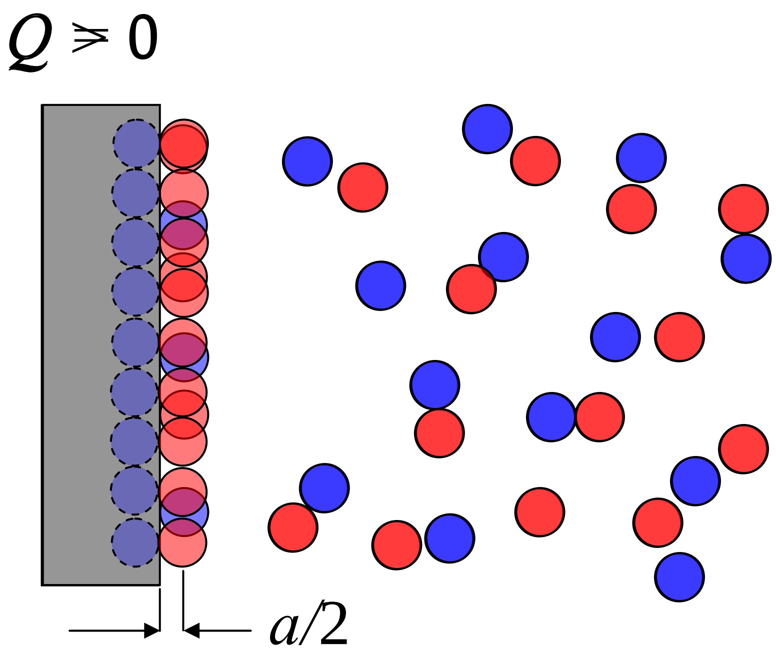
"Primitive model" of an ionic liquid



Ionic liquid is a molten salt -- liquid of classical charged hard spheres

Image charge attraction creates ion-image dipoles

The geometrical "limit" for *C*



At large Q, ions form a uniform layer

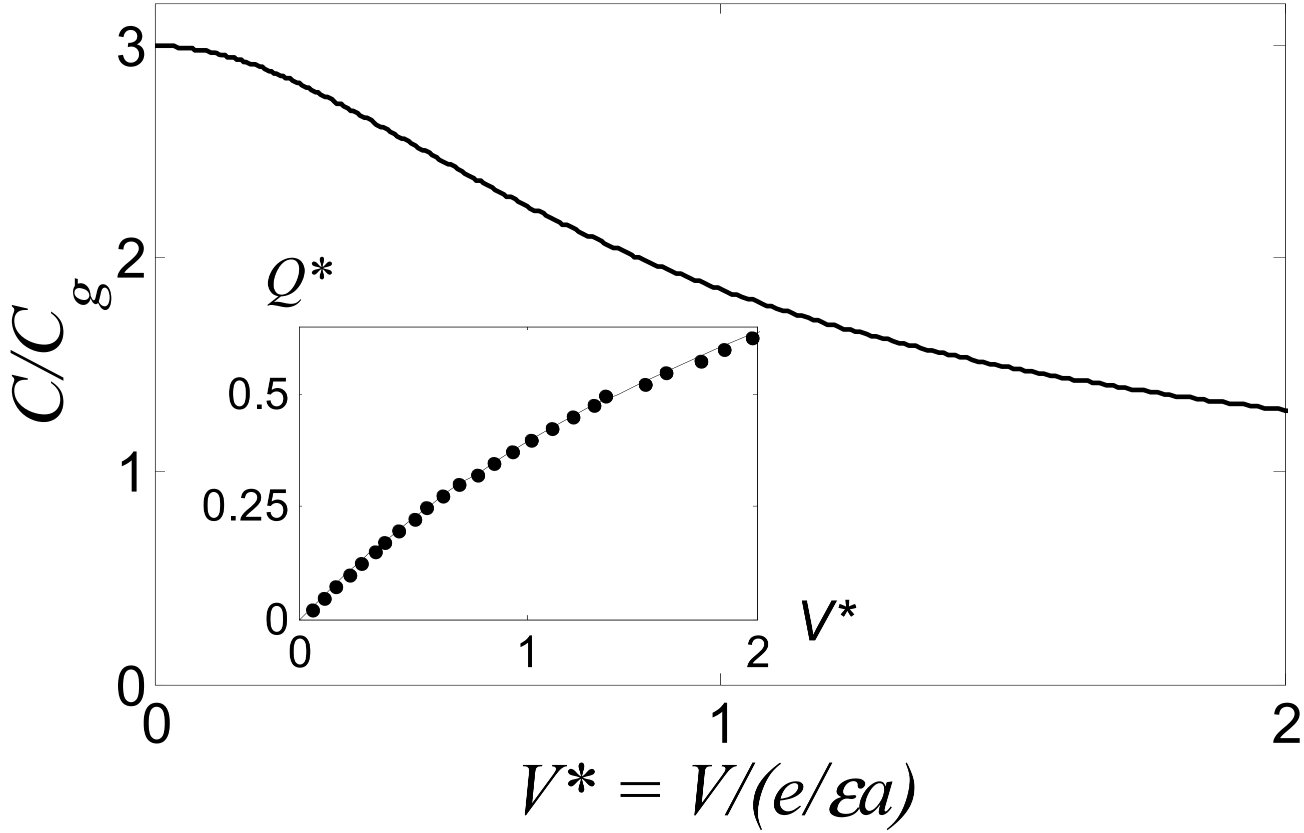
$$C_g = \frac{\varepsilon S}{4\pi(a/2)}$$

(Helmholtz, 1853)

$$d^* = a/2$$

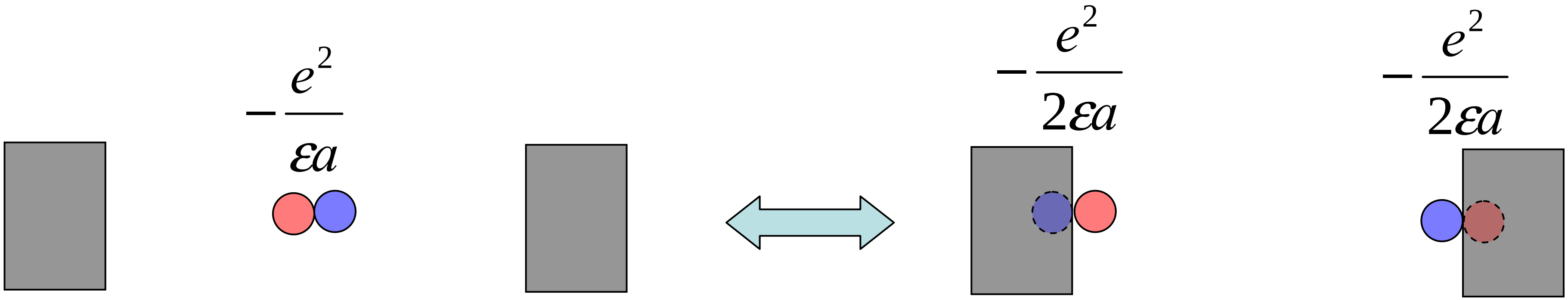
Monte Carlo results

MC simulations of nonlinear capacitance of the primitive model ($T^* = 0.05$):

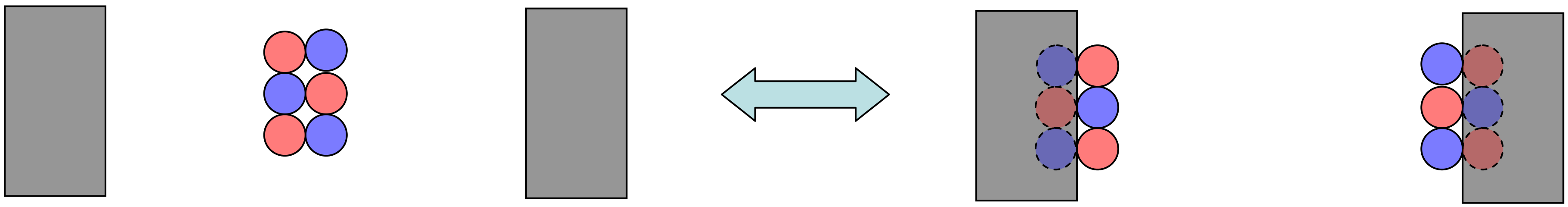


C **is as large as 3*C*$_g$!** **d*= a/6 !**

Binding to the metal surface



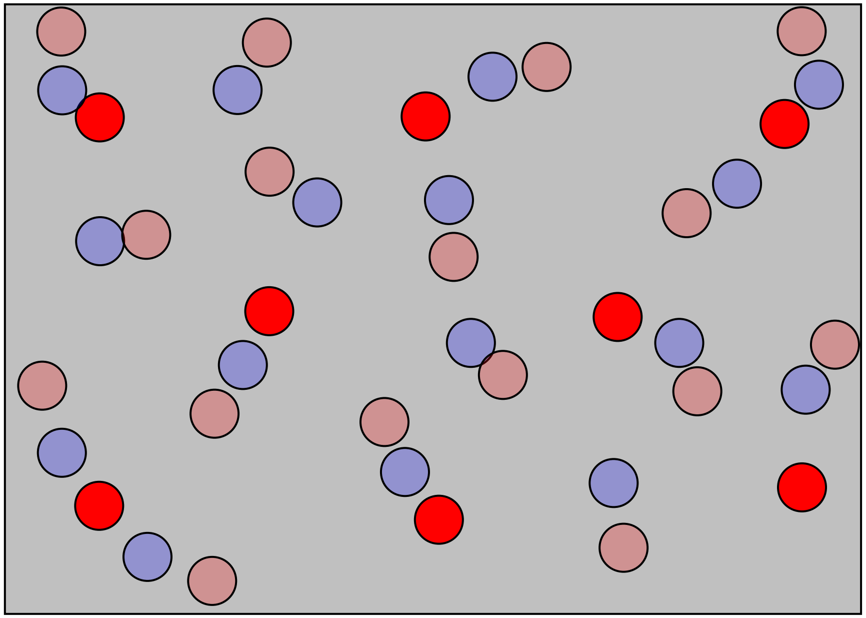
Perfect electrodes: pairs can be separated for zero energy



Likewise for any symmetric cluster

Charging through "excess dipoles"

$Q > 0$ comes as strongly-correlated "excess dipoles"



$u_{dd} = e^2 a^2 n^{3/2}/2\varepsilon$

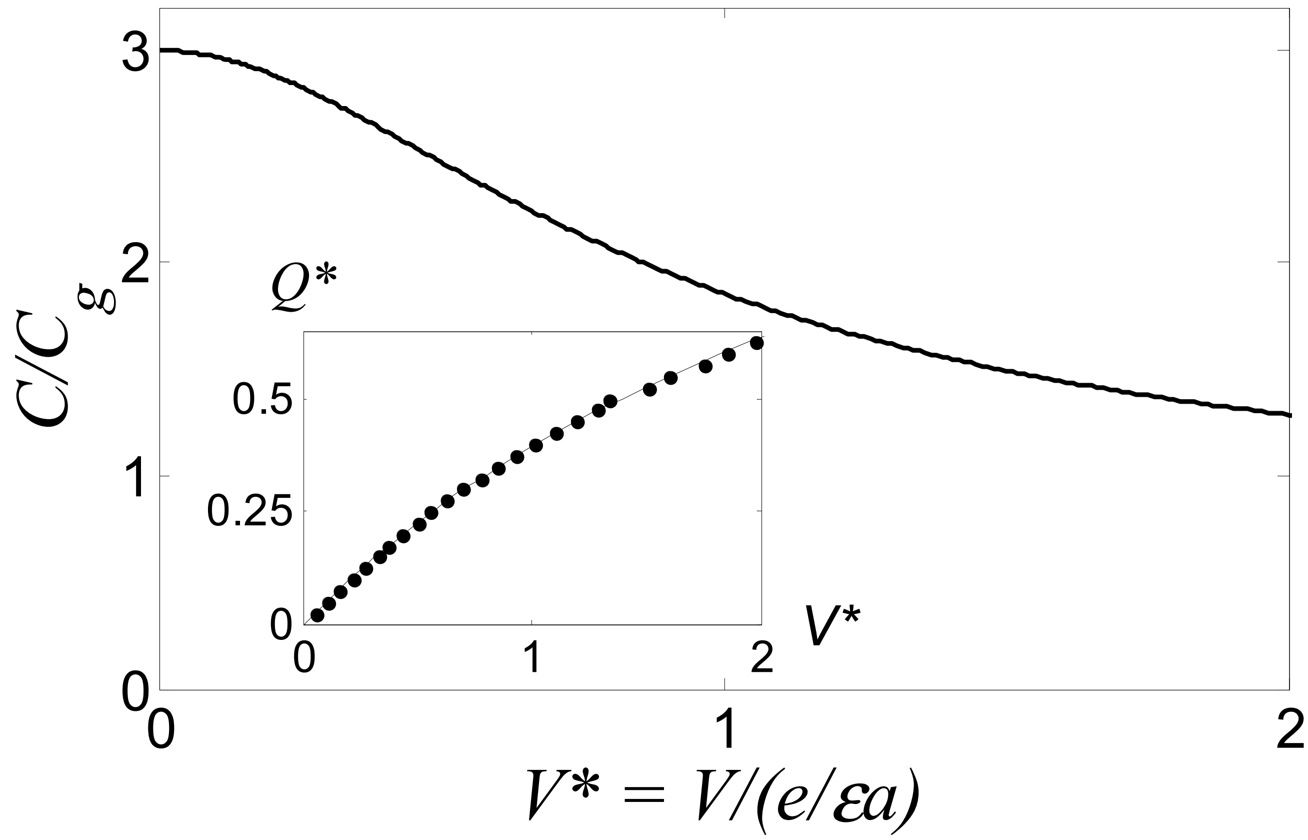
$$C = e^2 S \left(\frac{d^2(\alpha n u_{dd}/2)}{dn^2} \right)^{-1}$$

$$= 0.75 C_g / (n^{1/2} a)$$

$$V = \frac{dU}{d(enS)} = 5.6 \frac{e}{\varepsilon a} (n^{1/2} a)^3 \quad \Rightarrow C = 1.4 C_g \left(\frac{V}{e/\varepsilon a} \right)^{-1/3}$$

Monte Carlo results

MC simulations of nonlinear capacitance of the primitive model (T* = 0.05):



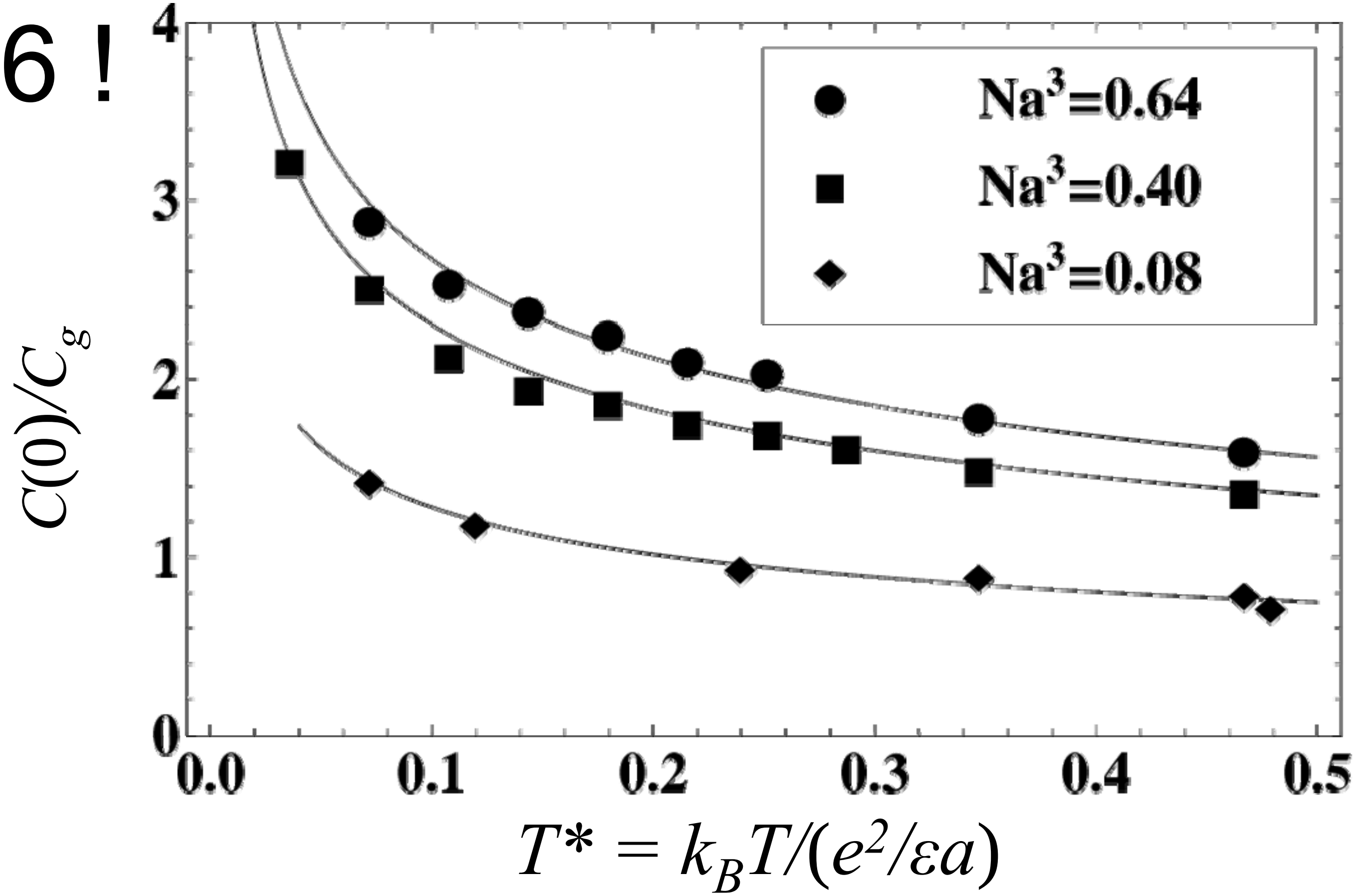
C is as large as $3C_g$! **d*= a/6 !**

Linear capacitance

Finite T truncates capacitance divergence:

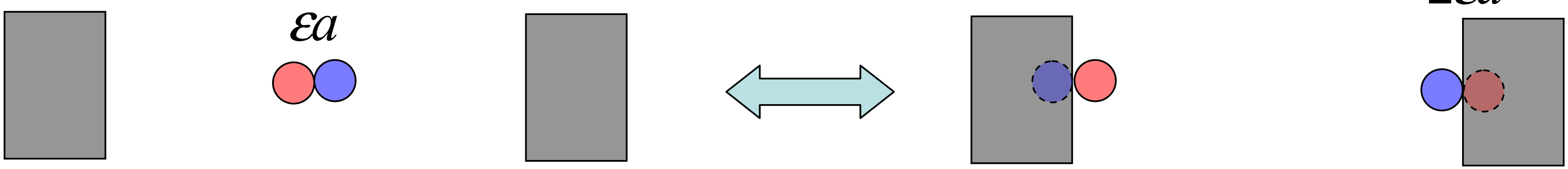
$$C(V \rightarrow 0) \sim 1/T^{1/3}.$$

d*=a/6 !



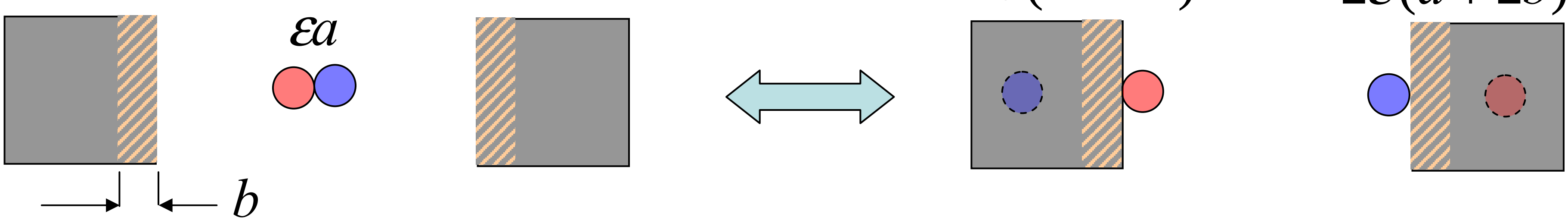
Effect of an "imperfect" metal surface

$$-\frac{e^2}{\varepsilon a} \qquad -\frac{e^2}{2\varepsilon a} \qquad -\frac{e^2}{2\varepsilon a}$$



Perfect electrodes: pairs can be separated for zero energy

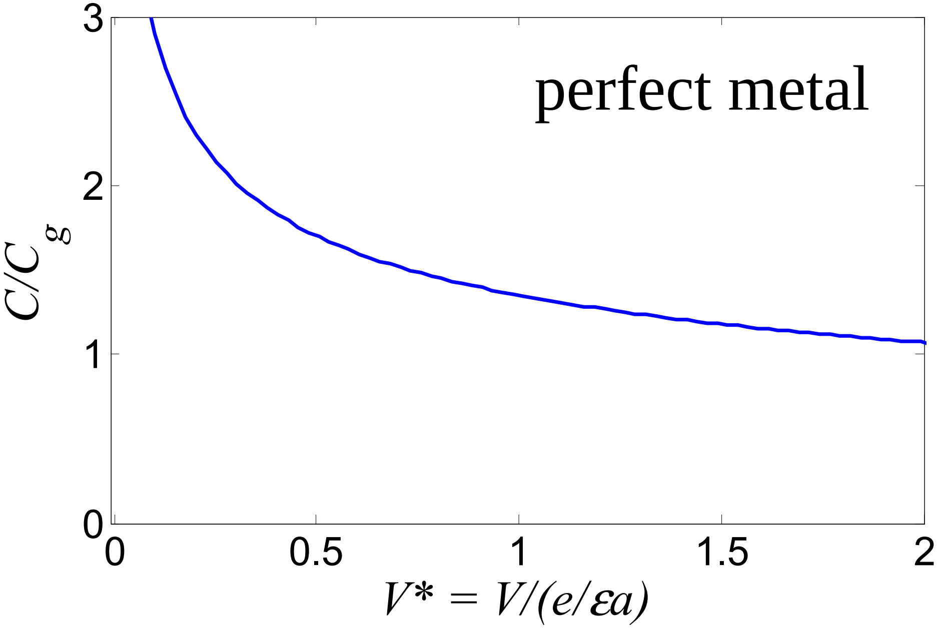
$$-\frac{e^2}{\varepsilon a} \qquad -\frac{e^2}{2\varepsilon(a+2b)} \qquad -\frac{e^2}{2\varepsilon(a+2b)}$$

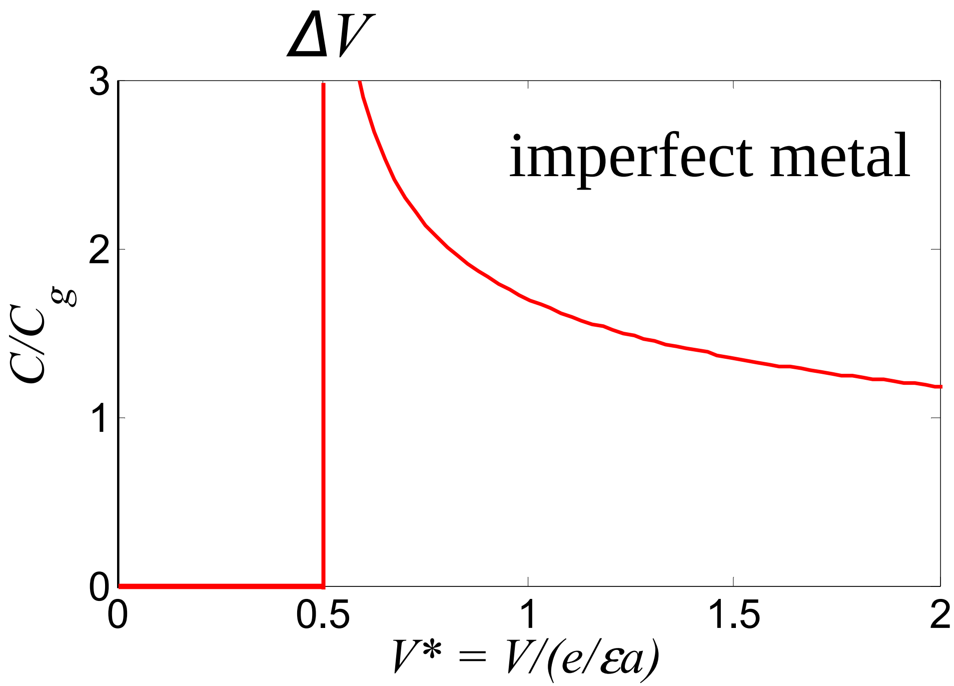


Imperfect electrodes: a finite voltage is required to ionize pairs

$$\Delta V = \frac{e}{\varepsilon a} - \frac{e}{\varepsilon(a+2b)}$$

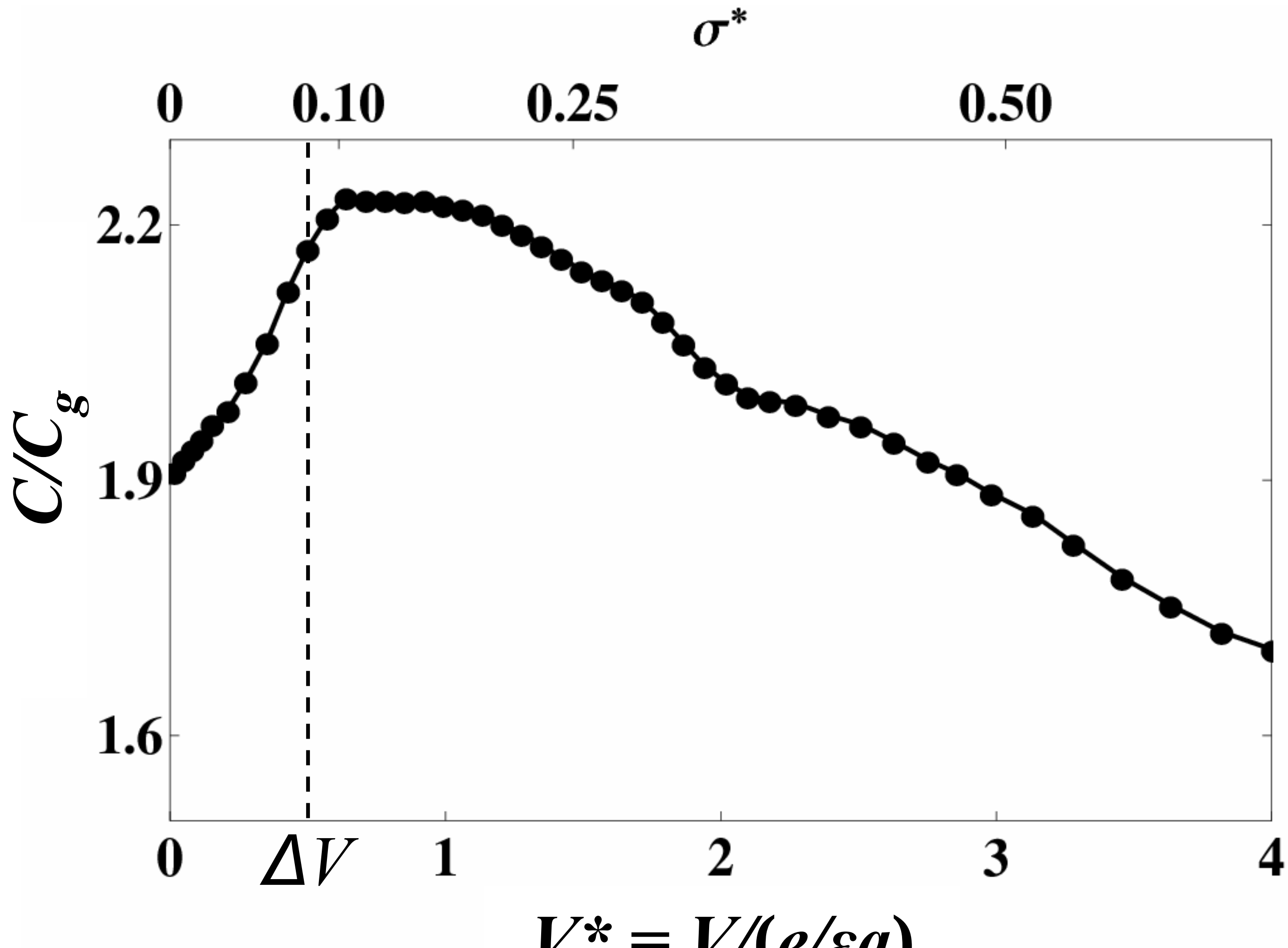
Effect of an "imperfect" metal surface





Finite V is required to bring free ions to the surface

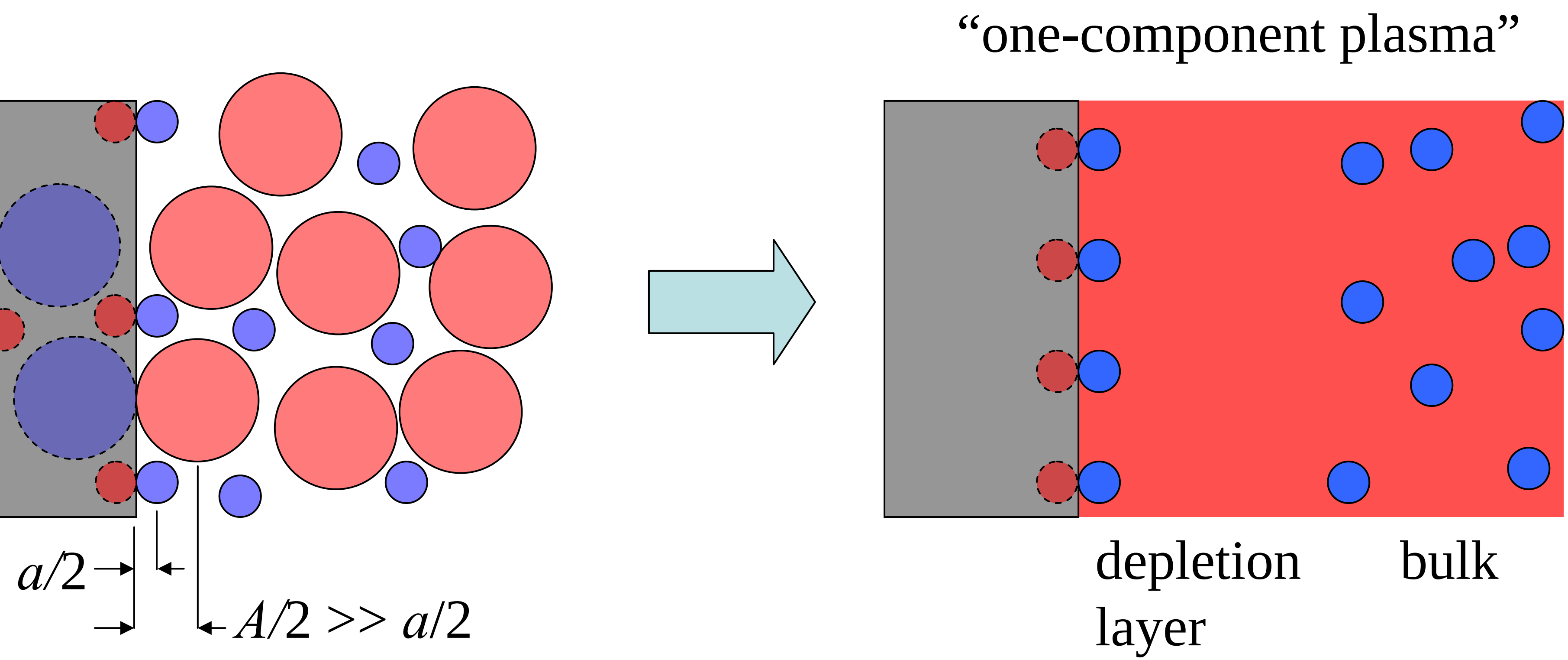
Imperfect electrode: MC results



b=a/2

At $V < \Delta V$, free pairs are sparse and C is reduced.

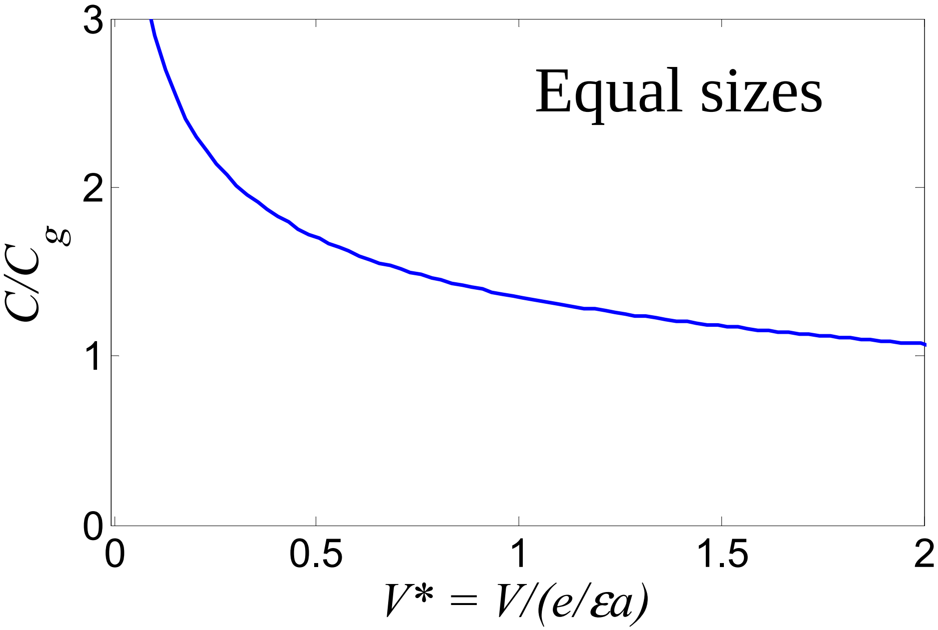
Effect of asymmetric ion size



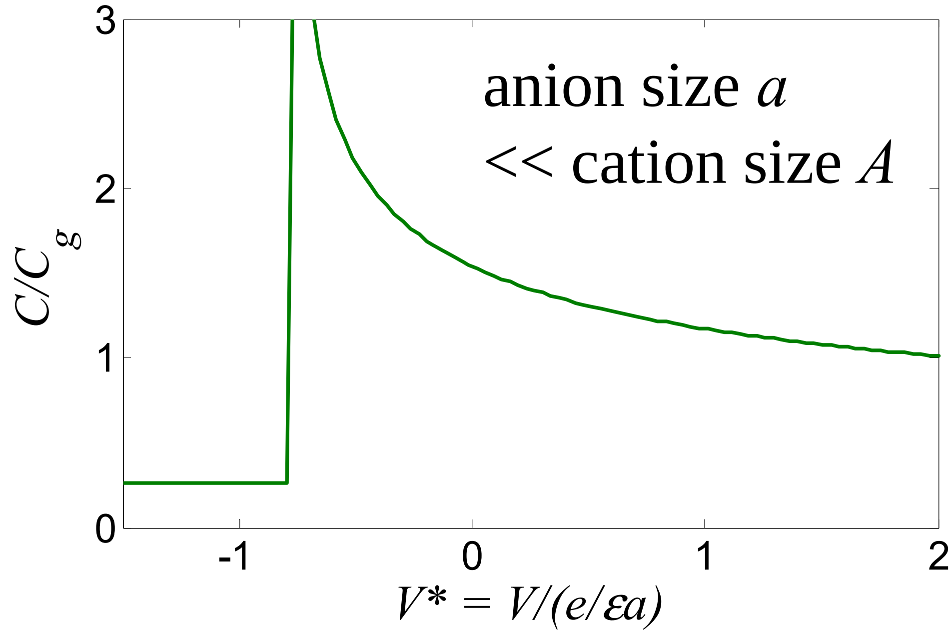
There is spontaneous polarization of the metal.

Finite voltage is required to *deplete* free ions.

Effect of asymmetric ion size



$V_c \sim -(a/A)^{9/4}\, e/\varepsilon a$



C diverges at $V \rightarrow V_c$,

where cations become depleted

Conclusions

Discrete charges form charge-image dipoles with the metal, producing $C > C_g$.

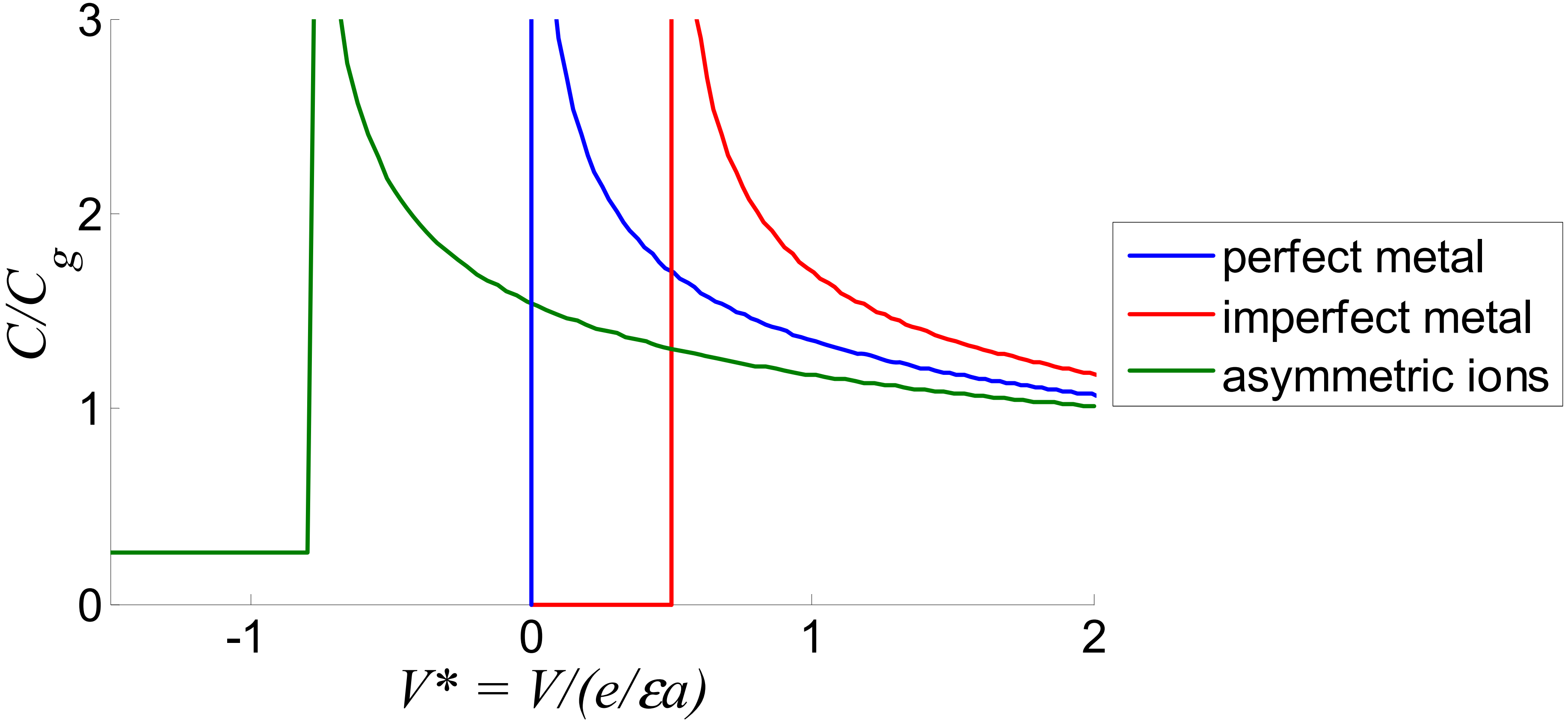
C grows sharply as $n \to 0$.

For a primitive model ionic liquid,

$C_{\mathrm{max}}/C_g \sim 1/T^{*1/3}$ and reaches 3 at small T.

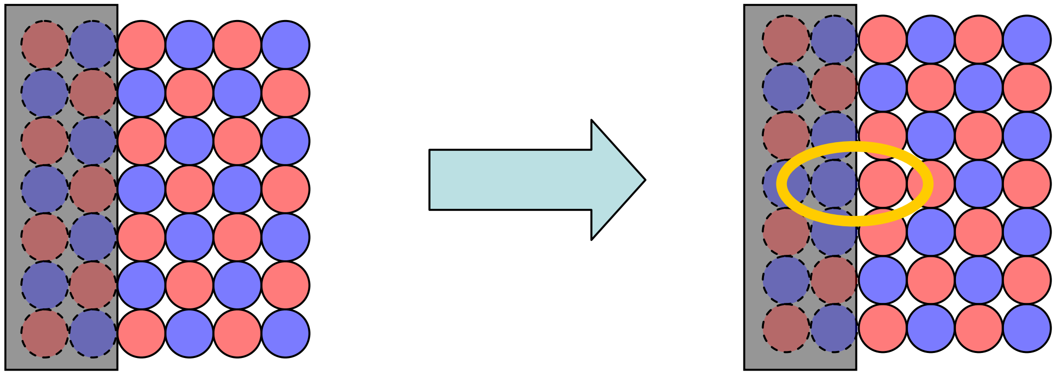
Questions?

Capacitance divergence

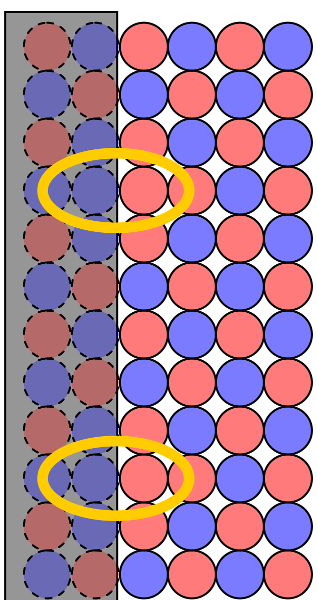


C diverges at the point where excess ions are depleted

Capacitance in the crystalline phase



Charging by $+2e$ defects

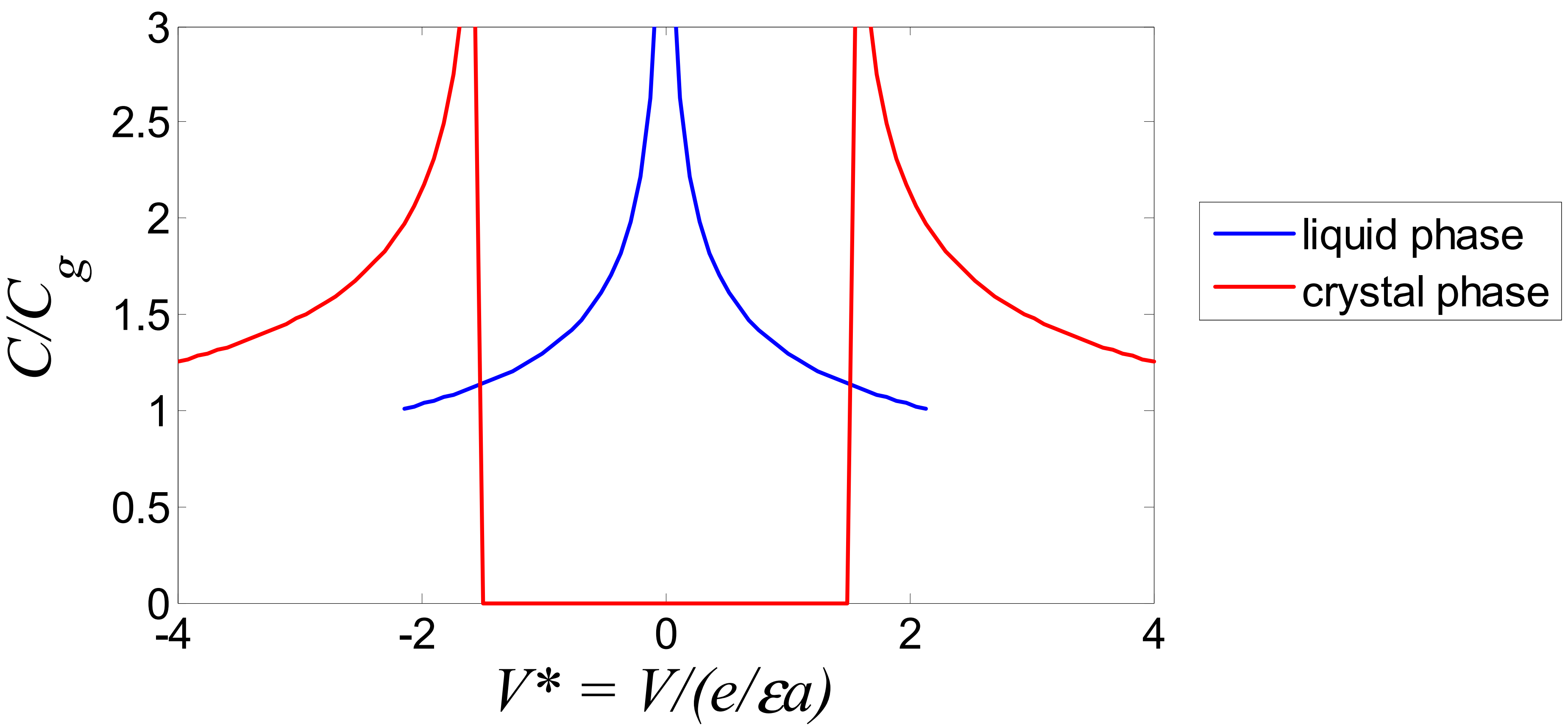


Energy cost: $2(U_{\text{Madelung}} - e^2/a)$

C is determined by interaction between $+2e$ dipoles:

$$u_{dd} = (2\text{e})^2 \text{a}^2 n^{3/2} / 2\varepsilon$$

Capacitance in the crystalline phase



A gap appears in voltage.

Capacitance at a given Q is $\sqrt{2}$ times larger.