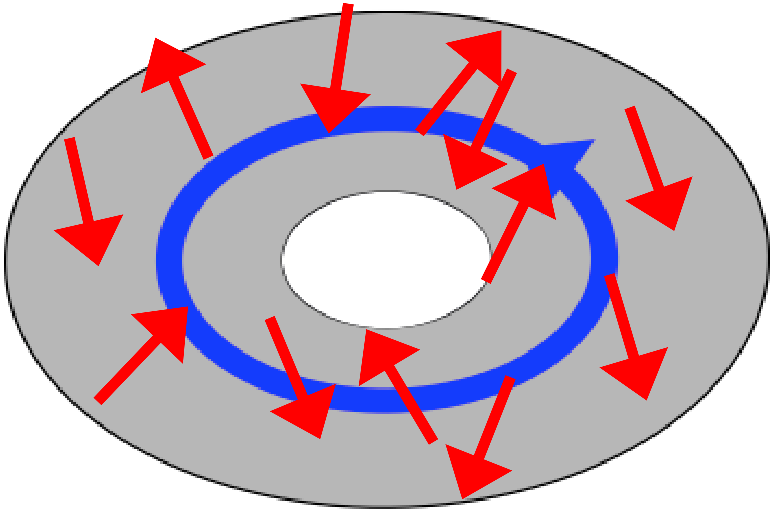
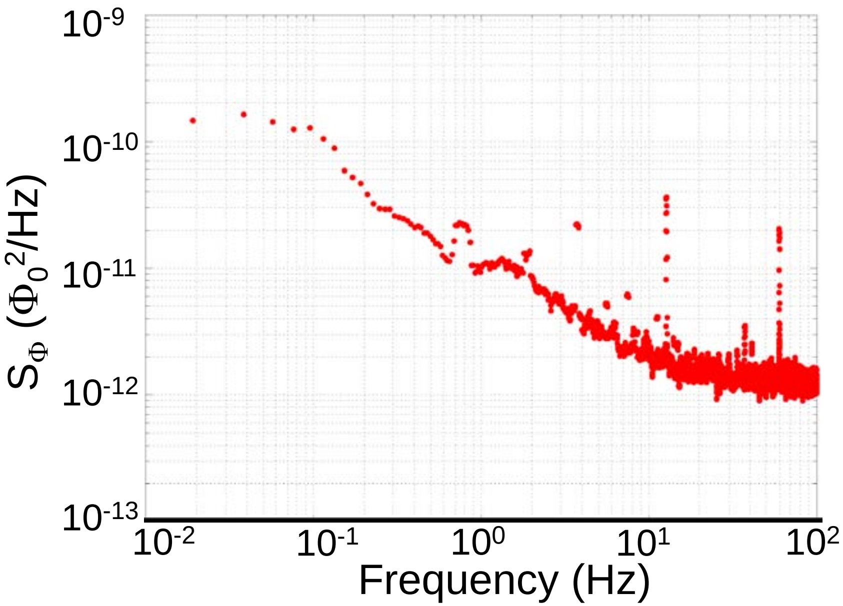
Spins on Metals: Noise in SQUIDs and Spin Glasses





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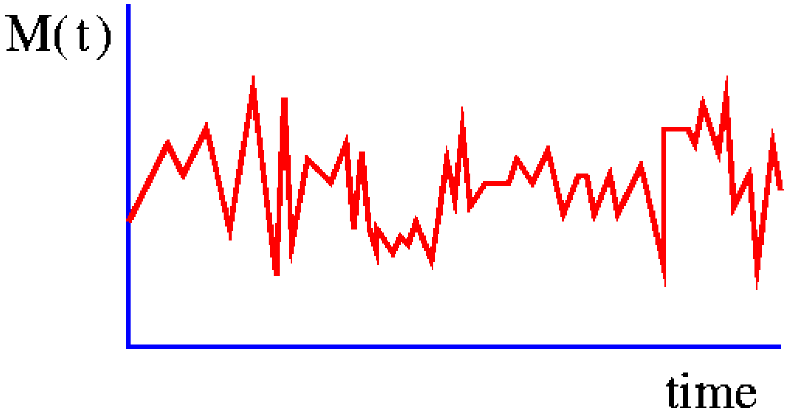
Noise Spectrum



Noise comes from fluctuations of some type. For example, let $\delta M(t)$ be a fluctuation of time t. The autocorrelation function is

$$\psi_M(t) = \langle \delta M(t) \delta M(0) \rangle$$

The noise spectral density is proportional to the Fourier transform:

$$S_M(\omega) = 2\psi_M(\omega) = 2\int dt e^{i\omega t} \psi_M(t)$$

1/f noise dominates at low frequencies, and corresponds to

$$S_M(\omega) \propto \frac{1}{\omega}$$

(Actually “1/f noise” refers to $S(f) \sim 1/f^a$ where a is approximately 1.)

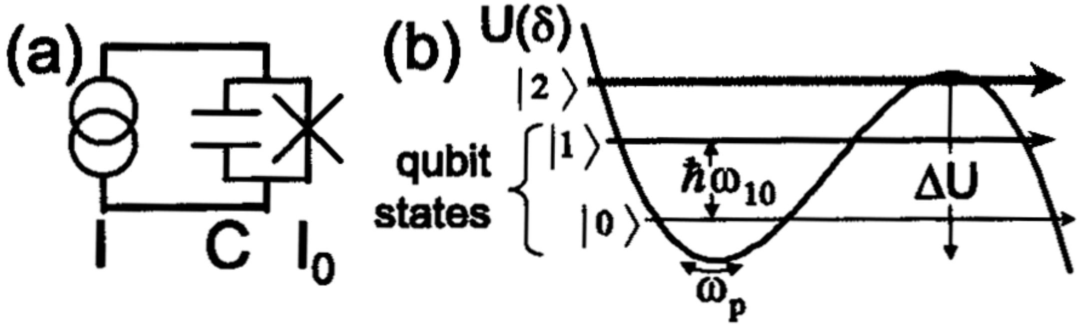
Quantum Computing and Qubits

Josephson junctions can be used to construct qubits.

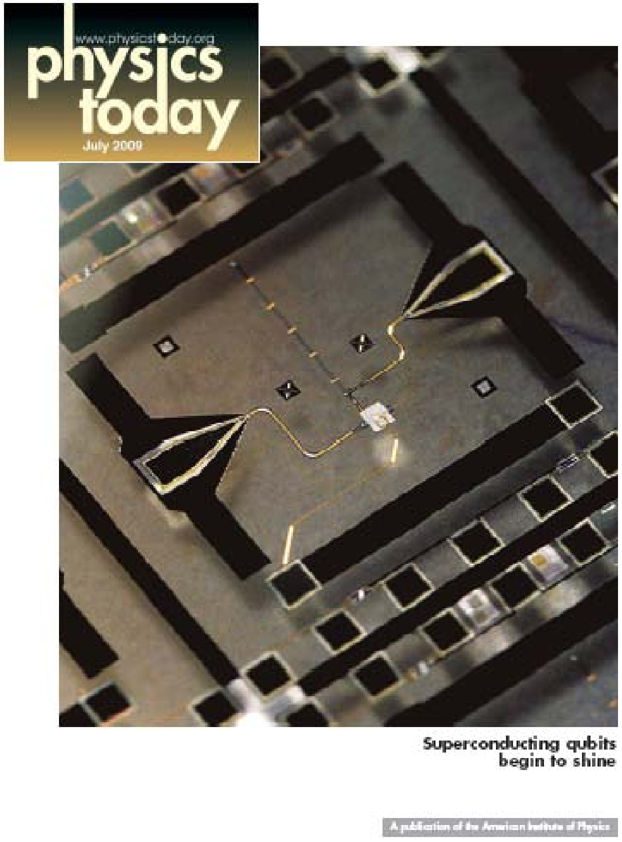
- Major Advantage: scalability using integrated circuit (IC) fabrication technology.
- Major Obstacle: Noise and Decoherence

$$\Psi = \cos\frac{\theta}{2}|0\rangle + \sin\frac{\theta}{2}e^{i\phi}|1\rangle$$

Qubit wavefunction



J. M. Martinis *et al.*, PRL 89, 117901 (2002).



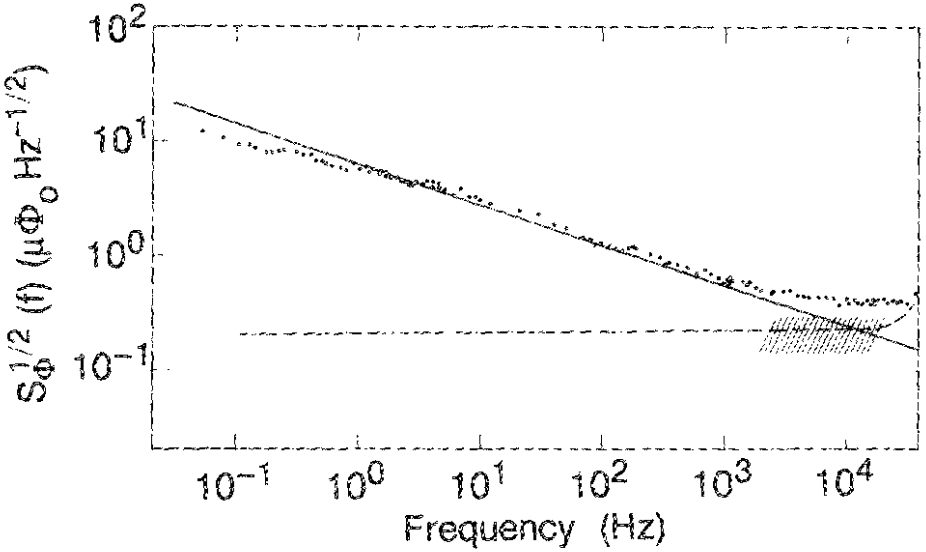
Flux Noise
Is a Major Source of Noise and Decoherence in SQUIDs

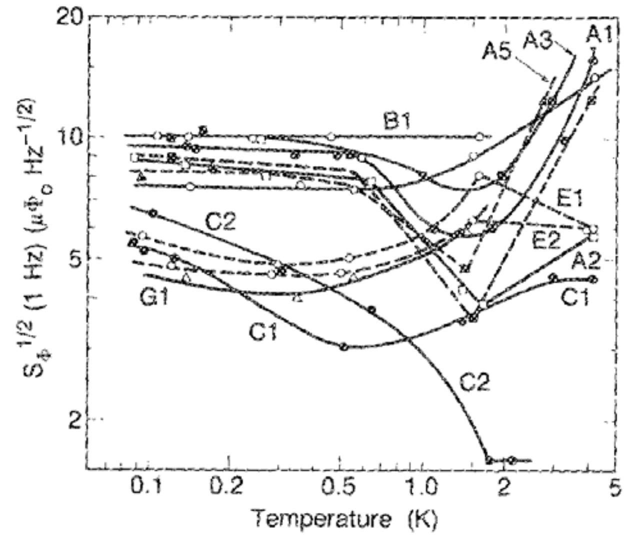
Flux noise looks like fluctuating vortices or fluxoids in the SQUID, but that is not the source of flux noise.

1/f Flux Noise in SQUIDs

[Wellstood *et al.*, APL **50** 772 ('87)]

$1/f^{\alpha}$ with $0.58 < \alpha < 0.80$





Hypothetical noise source	Properties of source
Noise from SQUID(2) or I_{b1}	Noise would not appear as flux noise
Noise from $I_{\Phi l}$	Noise would depend on M_i
Symmetric fluctuations in I_{01} & I_{02}, R_1 & R_2, or L_1 & L_2	Noise would not appear as flux noise
Antisymmetric fluctuations in I_{01} and I_{02}	a
Antisymmetric fluctuations in L_1 and L_2	S_Φ would scale as I^2
Antisymmetric fluctuations in R_1 and R_2	S_Φ would scale as V^2
Fluctuations in external magnetic field	$S_\Phi^{1/2}$ would scale as SQUID area
Noise from substrate	Should depend on material
Noise from SQUID support	Should depend on material
Liquid helium in cell	Should change in absence of helium
Heating effects	Should depend on power dissipated
Motion of flux lines trapped in SQUID	Should depend on material

"Universal" *1/f* flux noise

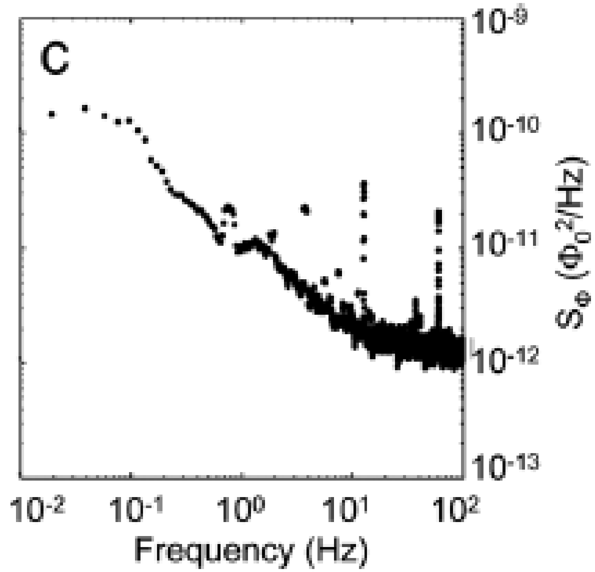
Independent of : inductance, materials, geometry

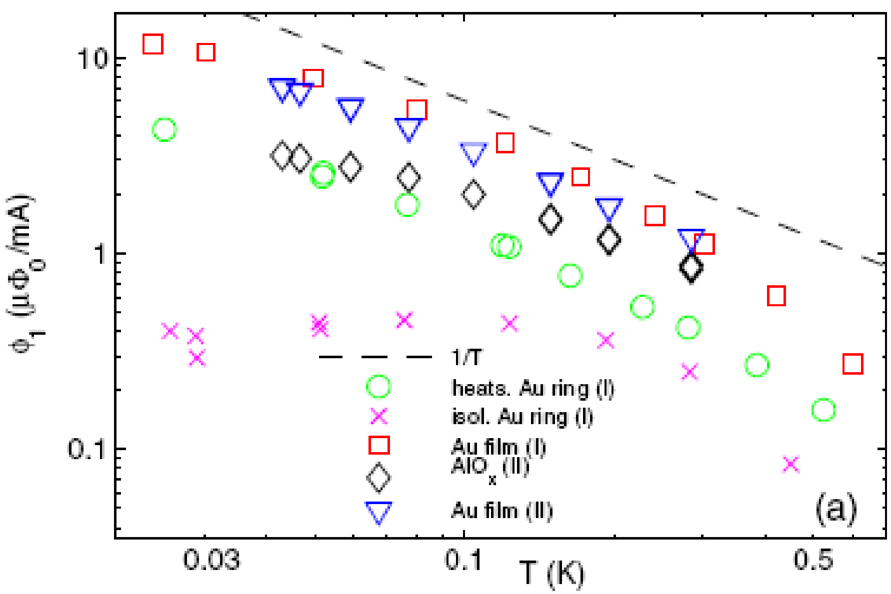
Not due to fluctuating vortices (seen in wires too thin to have a vortex)

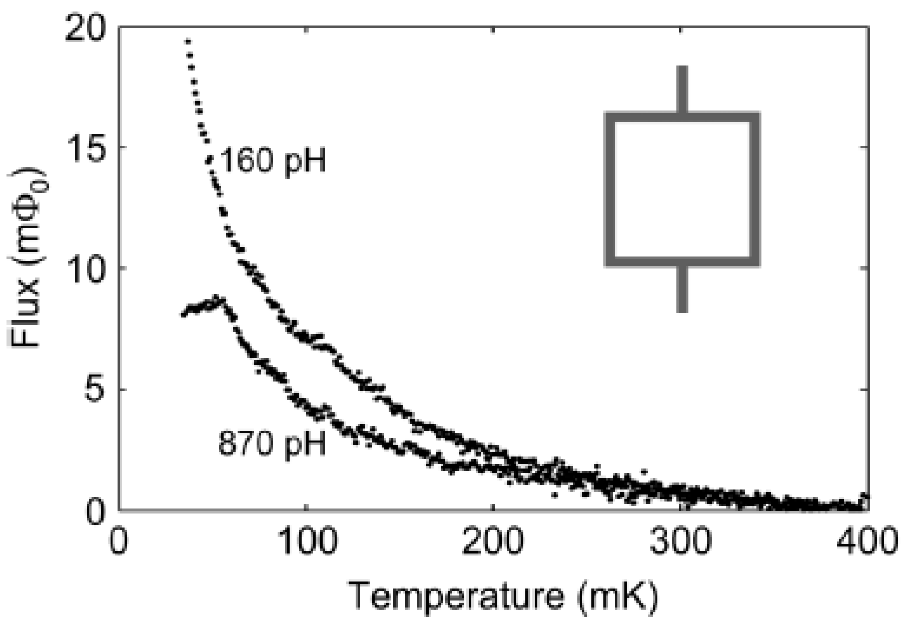
Mechanism was unknown

Flux Noise in SQUIDs

- Noise ~ $(1/f)^{\alpha}$ where $0.5 < \alpha < 1$.
- 1/f flux noise in SQUIDs is produced by fluctuating magnetic impurities.
- Paramagnetic impurities produce flux ~ 1/T on Al, Nb, Au, Re, Ag, etc.



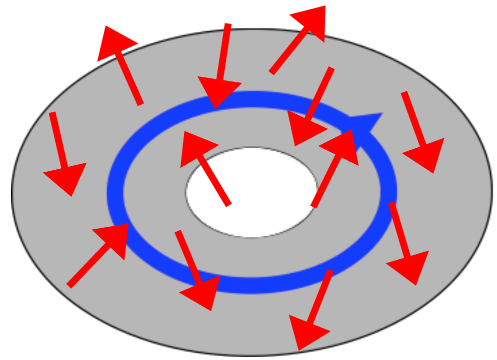




Bluhm *et al. PRL* (2009)

Sendelbach *et al. PRL (*2008)

Evidence Indicates Spins Reside on Metal Surface

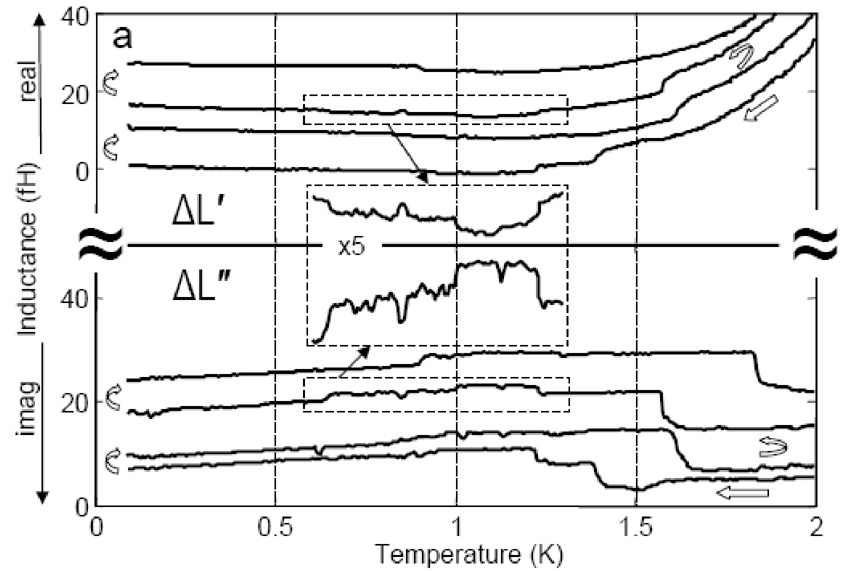


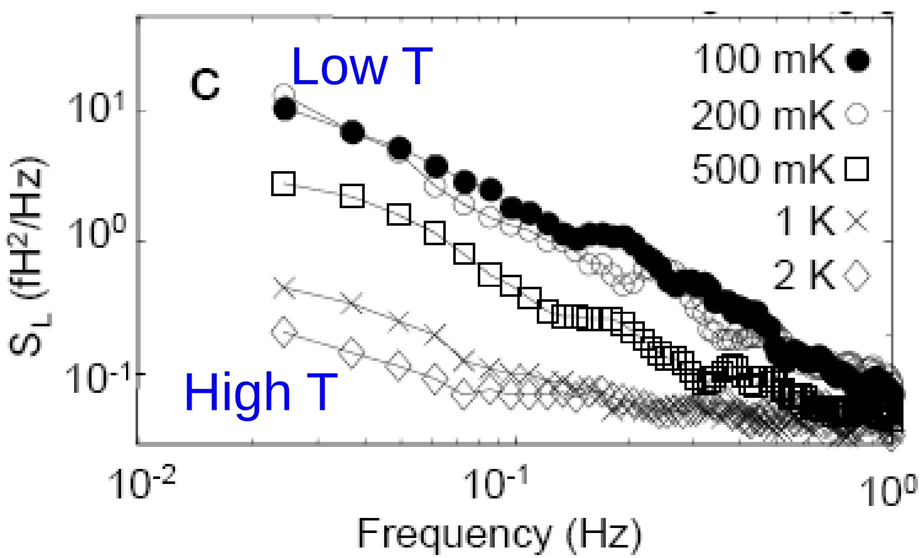
- Flux noise scales with surface area of the metal in the SQUID.
- Magnetic impurities in the bulk superconductor would be screened.
- Weak localization dephasing time τ_φ grows as T decreases (Bluhm *et al*.). If spin impurities in the bulk limited τ_φ, τ_φ would saturate at low T (Webb).
- Concentration ~ $5 \times 10^{17}/m^2$ implies a spacing of ~1 nm between impurities.
- May be due to states localized at the metal-insulator interface with magnetic moments (Choi *et al*.).

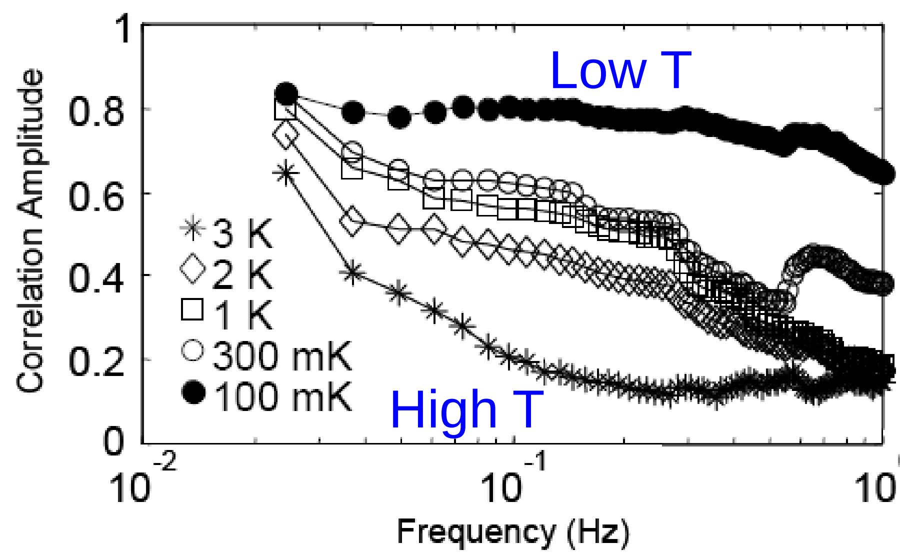
Inductance Noise

(Sendelbach *et al.*, PRL 2009)

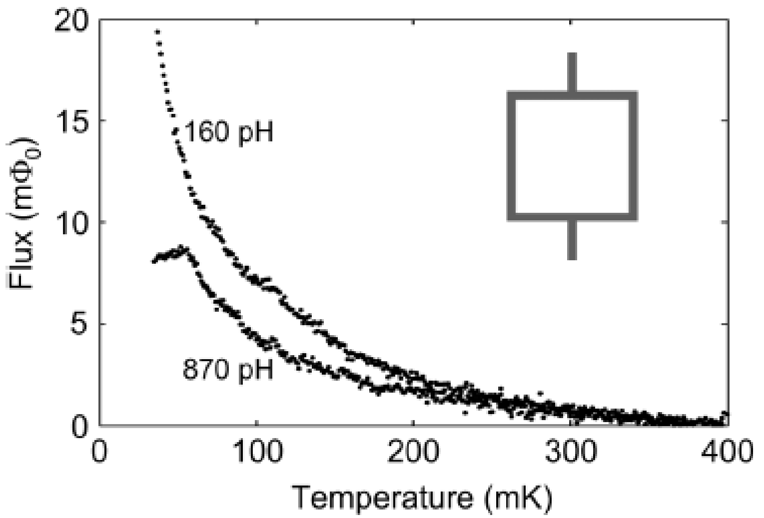
- 1/f inductance noise in SQUIDs driven by ac excitation current.
- Inductance is proportional to magnetic susceptibility.
- Inductance noise is correlated with flux noise.
- Implies magnetic impurities produce inductance and flux noise.

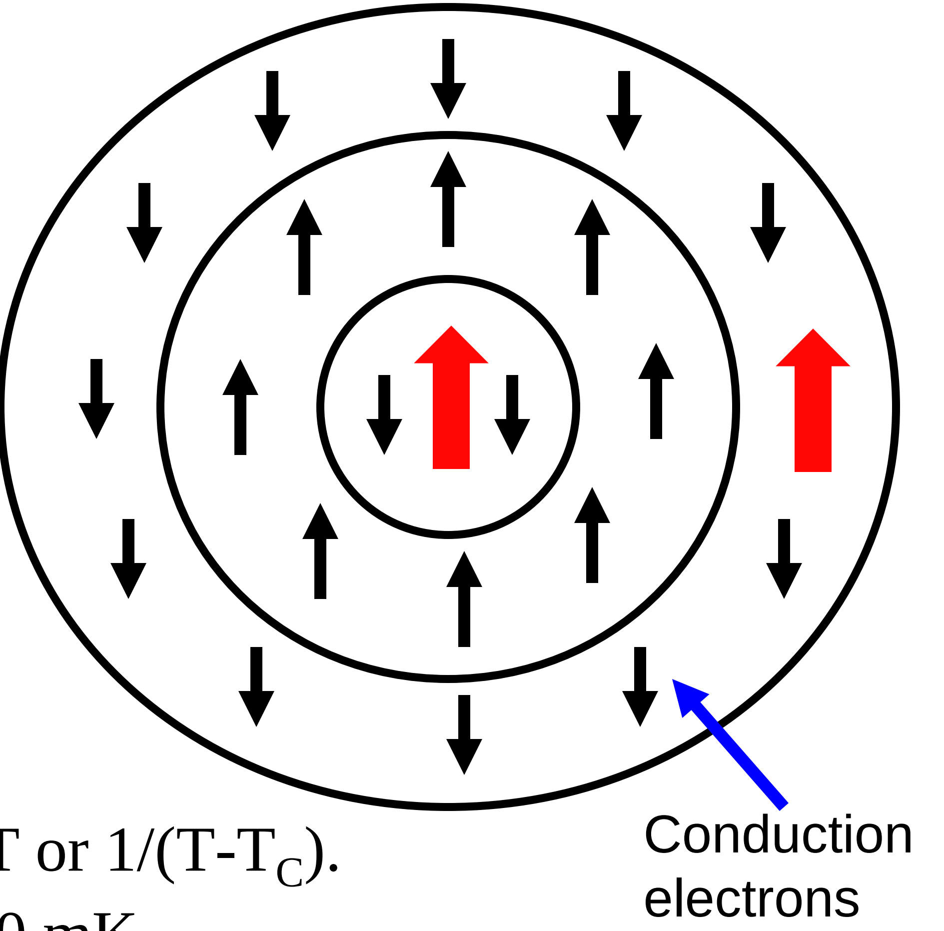






Spins May Interact Weakly via RKKY





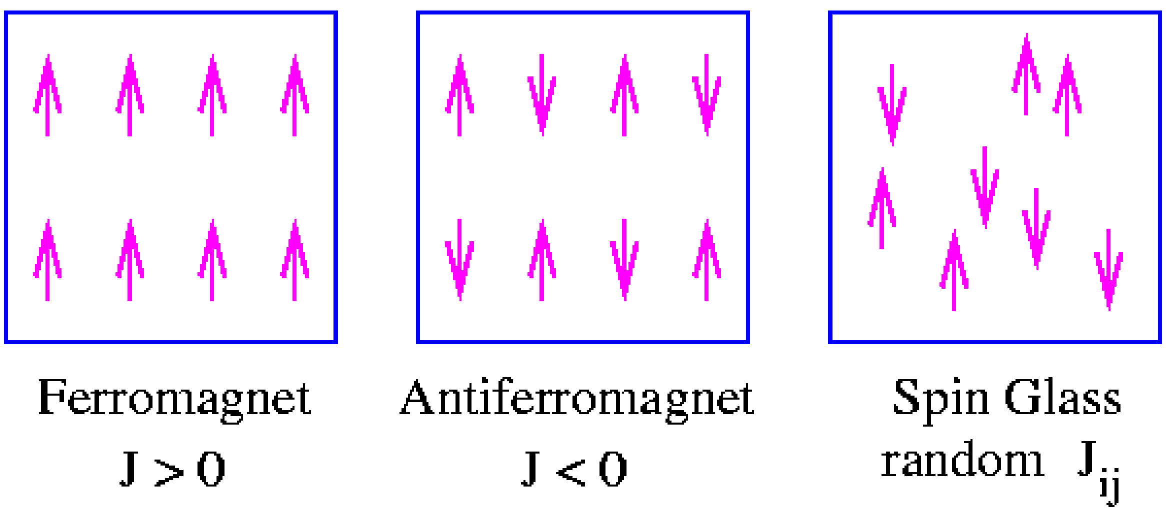
- Flux (susceptibility) goes as 1/T or $1/(T-T_C)$.
- If there is a T_C, estimate $T_C \sim 50$ mK.
- Implies there may be weak interaction between spins.
- Faoro and Ioffe proposed that the spins interact via RKKY which is oscillating spin polarization of the conduction electrons ($J_{RKKY} \sim \cos(2k_F r)/(2k_F r)^3$)
- RKKY leads to spin glass behavior.

Interacting Spin Systems

We can model interactions between spins with the Hamiltonian *H*:

$$H = -\sum_{i>j} J_{ij} \vec{S}_i \cdot \vec{S}_j$$

As the system is cooled, there is a phase transition at T_C from a high temperature paramagnetic phase to a low temperature phase. At low temperatures ($T \ll T_C$) the spins are frozen in one of the following configurations:

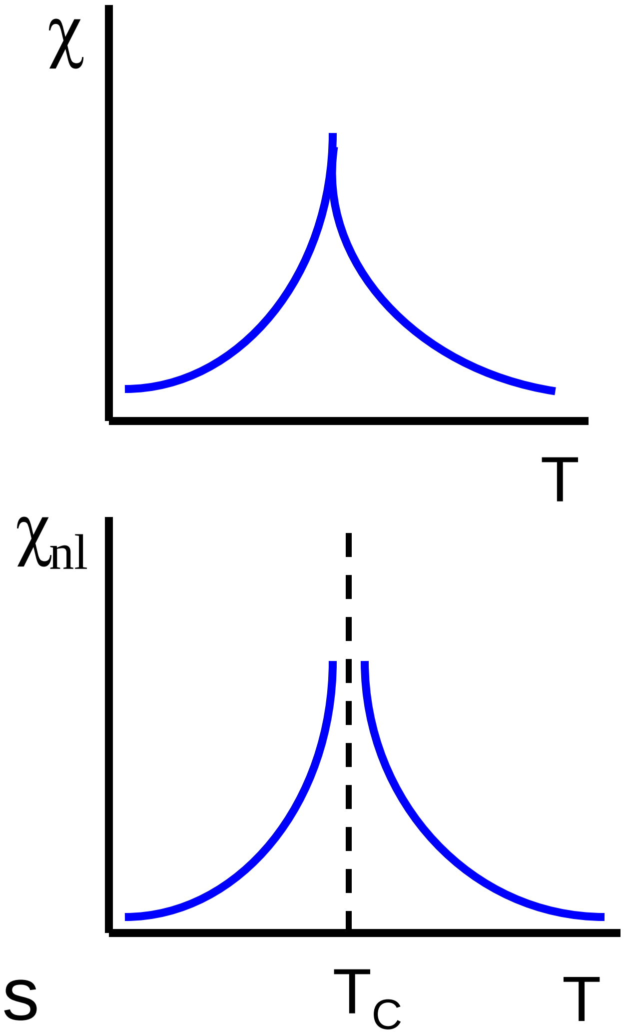


A spin glass is a collection of spins with random interactions between them.

Spin Glass Transition

$$H = -\sum_{i>j} J_{ij} \vec{S}_i \cdot \vec{S}_j$$

- Spin coupling J_{ij} random
- 2D Ising spin glass has $T_C=0$
- 3D Ising spin glass has $T_C > 0$
- 2nd order phase transition
- Specific heat and linear susceptibility do *not* diverge
- Nonlinear susceptibility χ_{nl} diverges



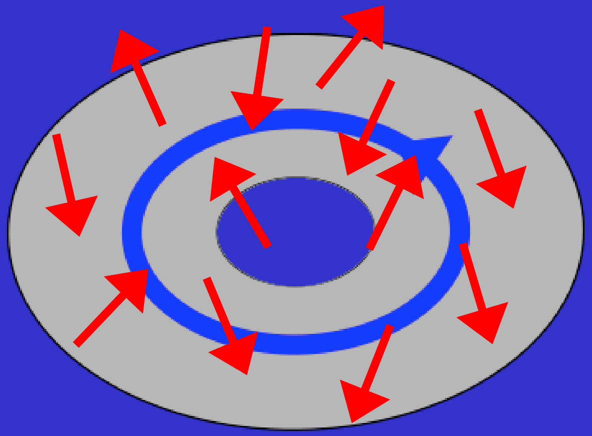
$$M(H) = \chi H - \chi_{nl} H^3 + \ldots$$

RKKY Spin Glass

- Surface spins are probably in 2D
- Lower critical dimension of RKKY spin glass is 3 (Bray *et al*. 1986)
- No RKKY spin glass transition in 2D
- But a spin glass transition is possible for other types of interactions
- For example: random power law interactions (~ $1/r_{ij}^{\sigma}$, $d/2 < \sigma < d$) can produce $T_C > 0$ in d dimensions (Katzgraber and Young, 2003)

Do spins on metals act like a spin glass?

- Does $\chi \sim 1/T$?
- Is flux noise in SQUIDs consistent with magnetization noise in a spin glass model?
- Is inductance noise in SQUIDs consistent with susceptibility noise in a spin glass model?
- (Inductance $L \sim \chi$)



Previous Experiments: Noise in Spin Glasses

- Spin glasses have low frequency magnetization noise $S_M(f) \sim 1/f$ (Ocio *et al.* 1986, Reim *et al.* 1986, Refregier *et al.* 1987).
- $S_M(f) \sim 1/f$ consistent with SQUID flux noise $\sim 1/f$
- Maximum 1/f noise near T_g (Refregier *et al.* 1987).

Reim *et al.* 1986

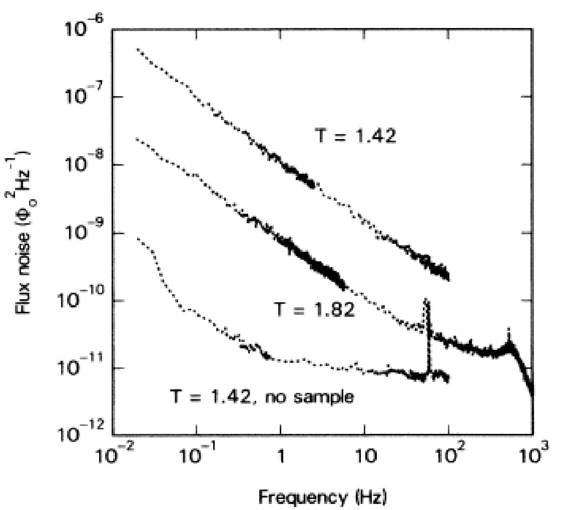
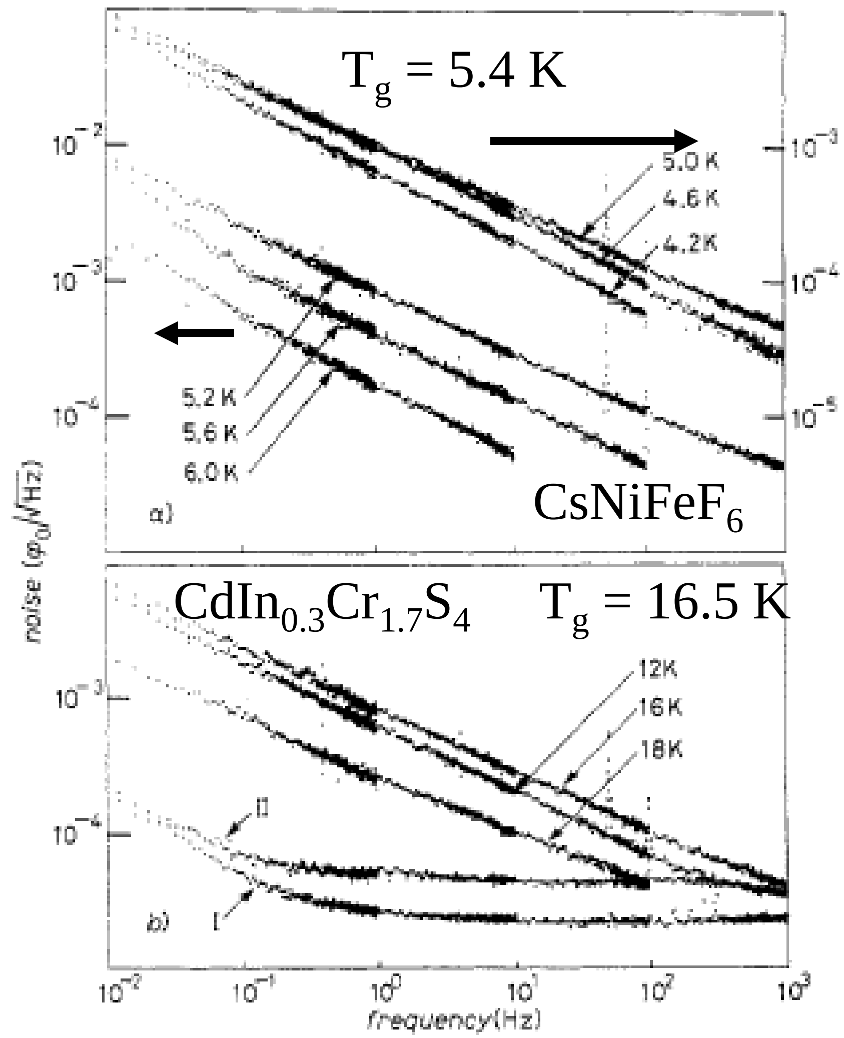


FIG. 1. Flux noise vs frequency for the detection system alone and for the spin-glass $Eu_{0.4}Sr_{0.6}S$ at two different temperatures above and below $T_f = 1.53$ K.



Refregier *et al.* 1987

Previous Theory: Noise in Spin Glasses

- $S_M(f)$ is magnetization noise.
- Infinite range (mean field) spin glass models: $S_M(f) \sim (1/f\,)^{-\alpha}$ with $\alpha \leq ½$ in the spin glass phase ($T \leq T_C$) (Kirkpatrick & Sherrington 1978, Ma & Rudnick 1978, Hertz & Klemm 1979, Sompolinsky & Zippelius 1982, Fischer & Kinzel 1984).
- Droplet model: $S_M(f) \sim (\ln f)/f$ (Fisher & Huse 1988).
- Hierarchical Model: $S_M(f) \sim 1/f$ (Weissman 1993).
- $S_M(f) \sim 1/f$ is consistent with 1/f flux noise

Previous Monte Carlo Simulations: Noise in Spin Glasses

- 2D and 3D Monte Carlo simulations of ± J Ising spin glass model (McMillan 1983, Marinari *et al.* 1984, Sourlas 1986)
- All simulations were at $T > T_C$
- High temperature magnetization noise is white
- As system is cooled, $S_M(f) \sim 1/f$
- No calculations to compare with SQUID inductance noise

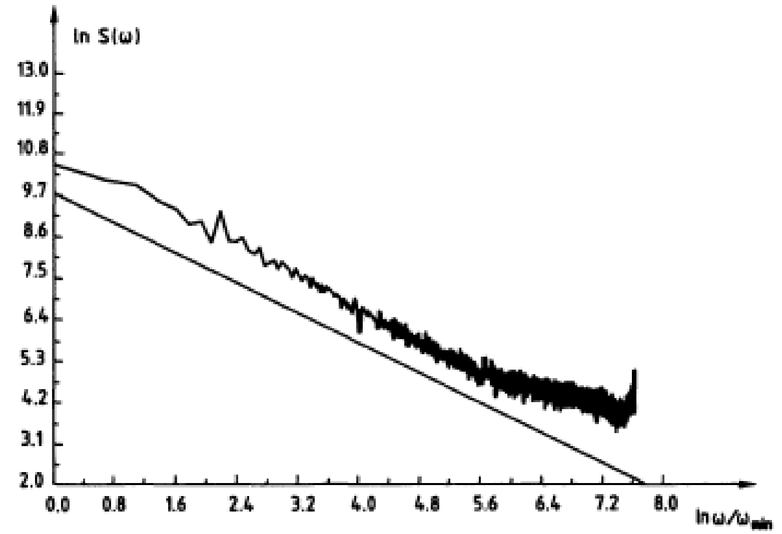
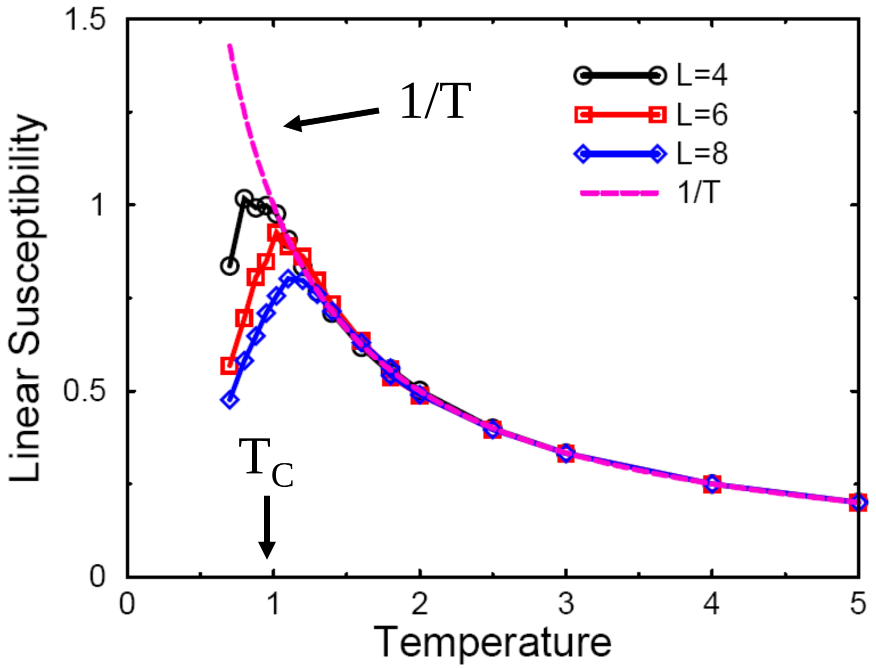


Fig. 2. — ln $S(\omega)$ (in arbitrary scale) *versus* ln $(\omega/\omega_{\text{min}})$, at $T = 1.0$. Same features as in figure 1, but MC dynamics. The straight line is the slope of $S(\omega) \sim 1/\omega$.

2D Ising Spin Glass
Marinari *et al.* (1984)

2D and 3D Ising Spin Glass Simulations

$$H = -\sum_{\{i>j\}} J_{ij} S_i S_j$$



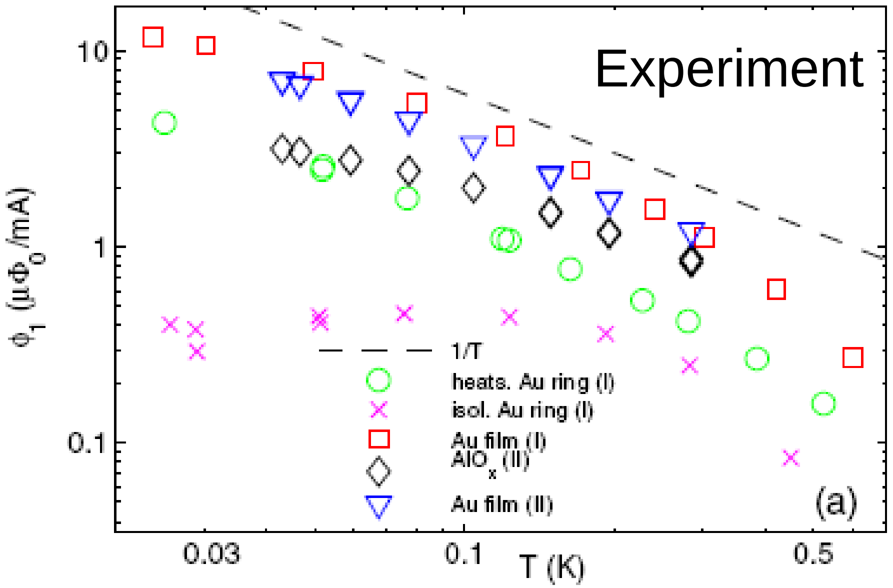
- ith spin S_i = -1, +1
- Nearest neighbor interactions
- 2^{nd} Order Phase Transition in 3D
- $k_B T_C$ = 0.95 J (3D); T_C= 0 (2D)
- Periodic boundary conditions
- $P(J_{ij})$ is a Gaussian distribution
- Parallel tempering Monte Carlo simulations to reach equilibrium
- 3D: $N = L^3$, L =4, 6, 8; 2D: $N = L^2$, L=8, 16
- After equilibrating, time series 1.5×10^6 Monte Carlo Steps per spin
- 200 samples for disorder average
- Obtain time series and noise spectra of magnetization.

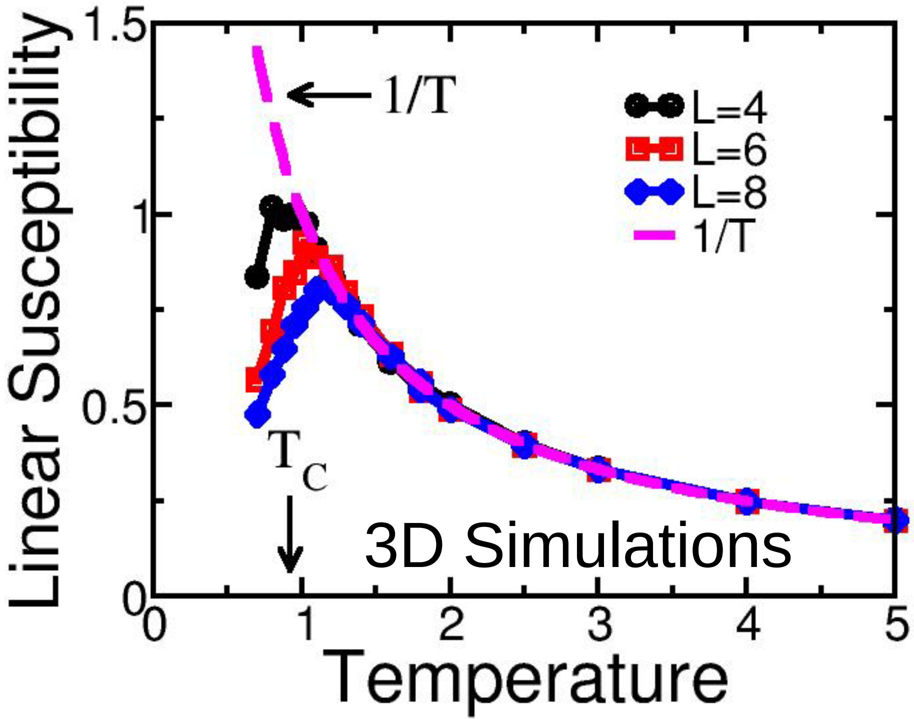
χ(T) and Φ(T) Consistent

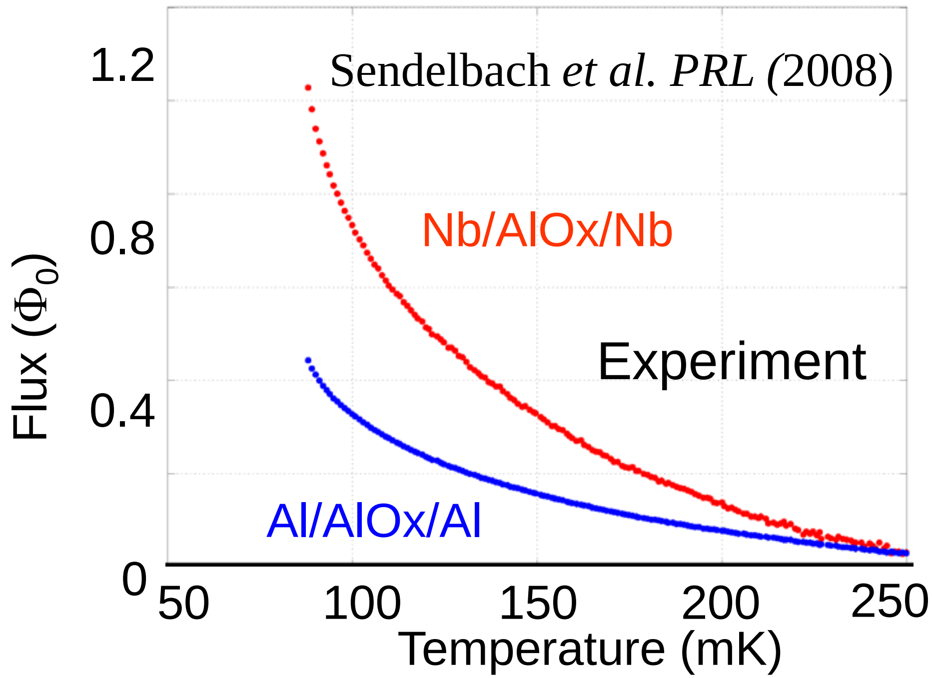
Susceptibility: $\chi = N\sigma_M^2/kT$

Flux Φ(T) ~ Magnetization M(T) ~ Susceptibility χ(T) ~ 1/T

Bluhm *et al.* *PRL* (2009)





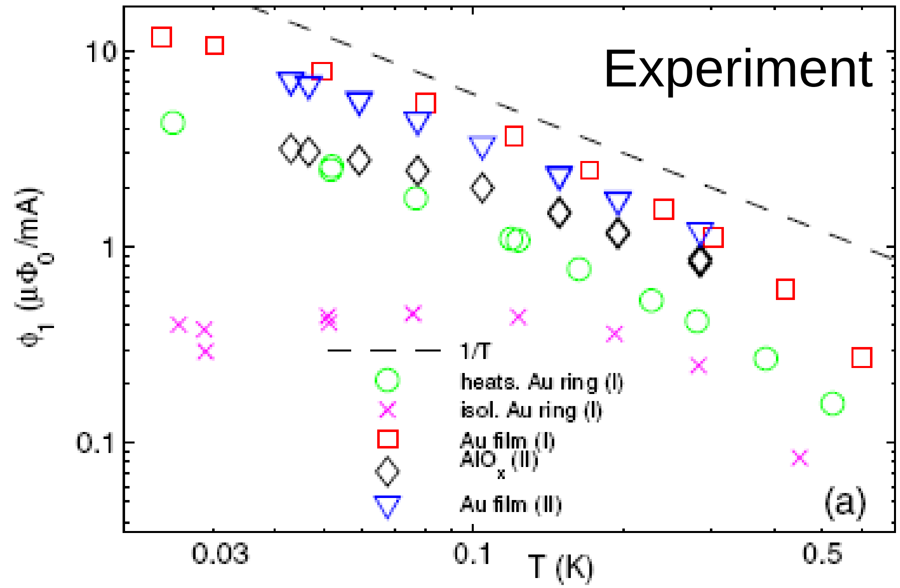


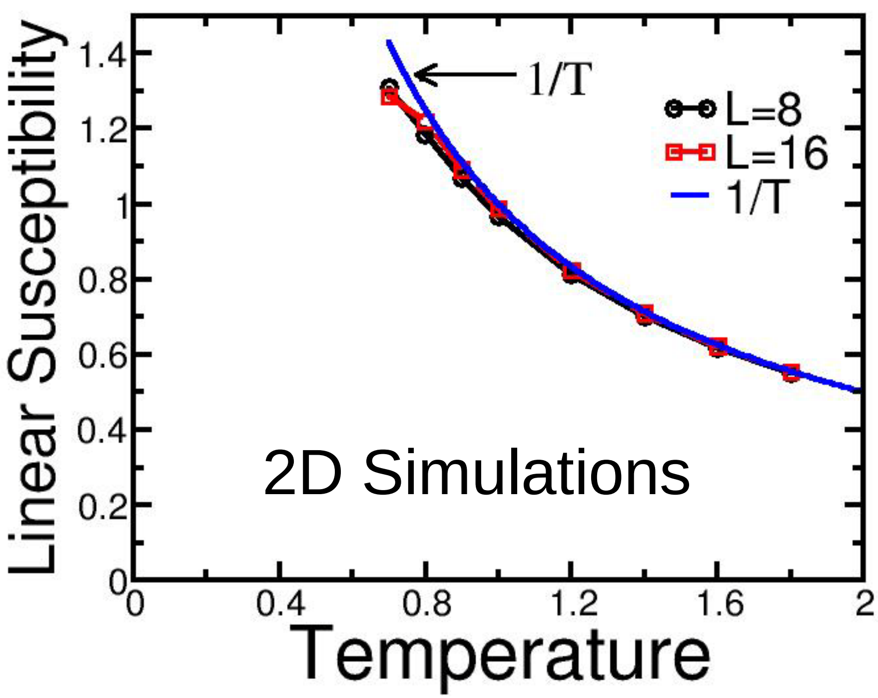
$\chi(T)$ and $\Phi(T)$ Consistent

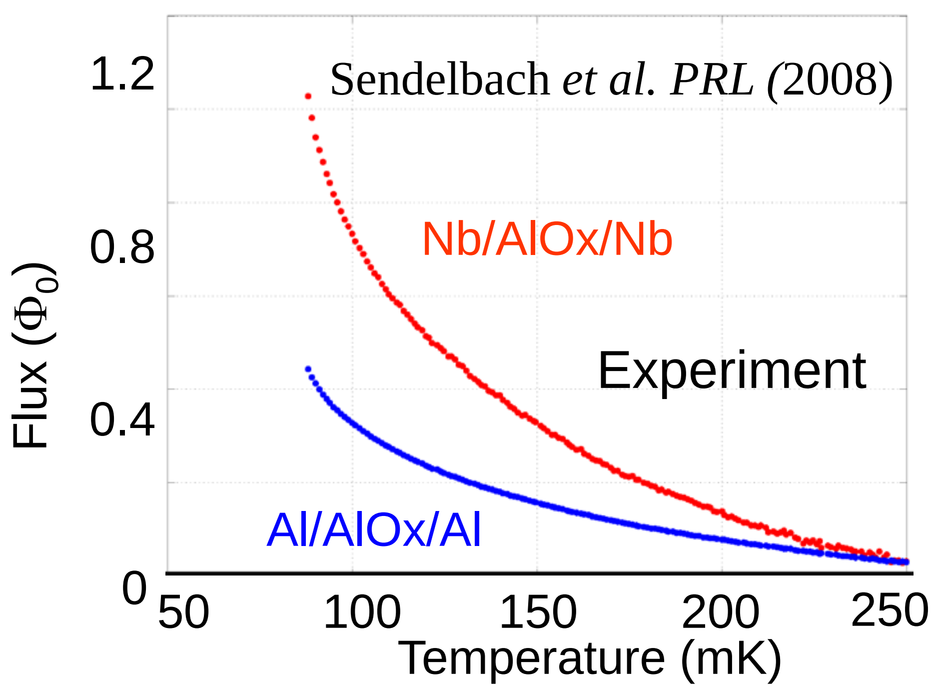
Susceptibility: $\chi = N\sigma_M^2/kT$

Flux $\Phi(T)$ ~ Magnetization $M(T)$ ~ Susceptibility $\chi(T)$ ~ $1/T$

Bluhm *et al. PRL* (2009)

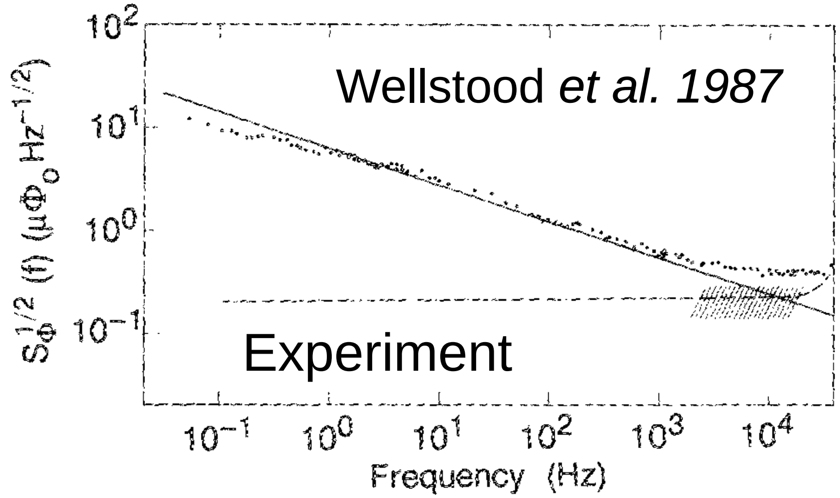


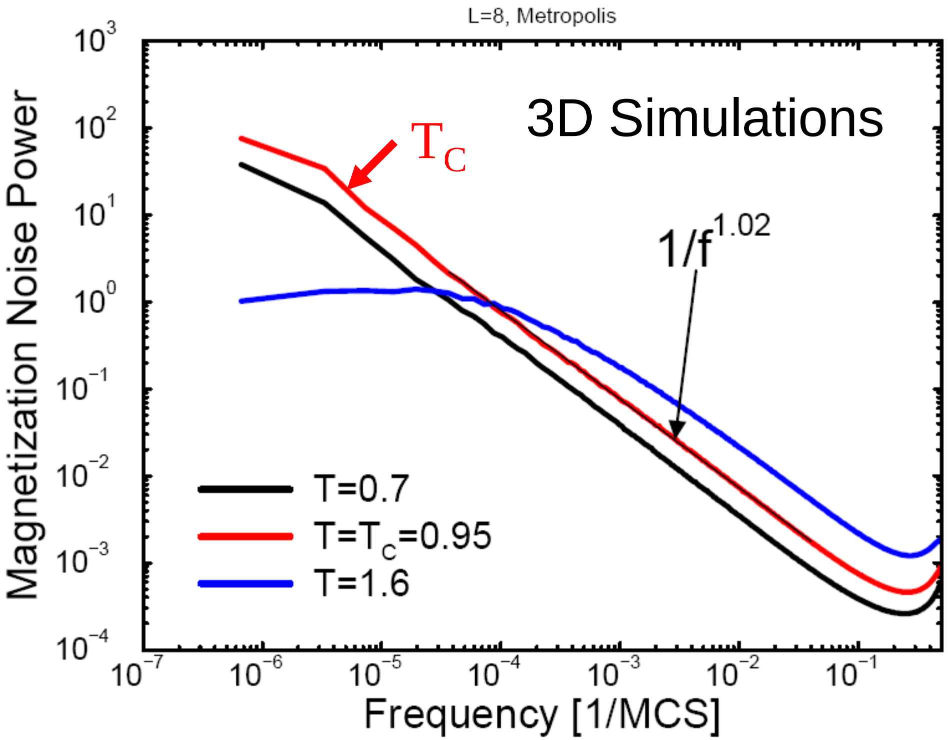


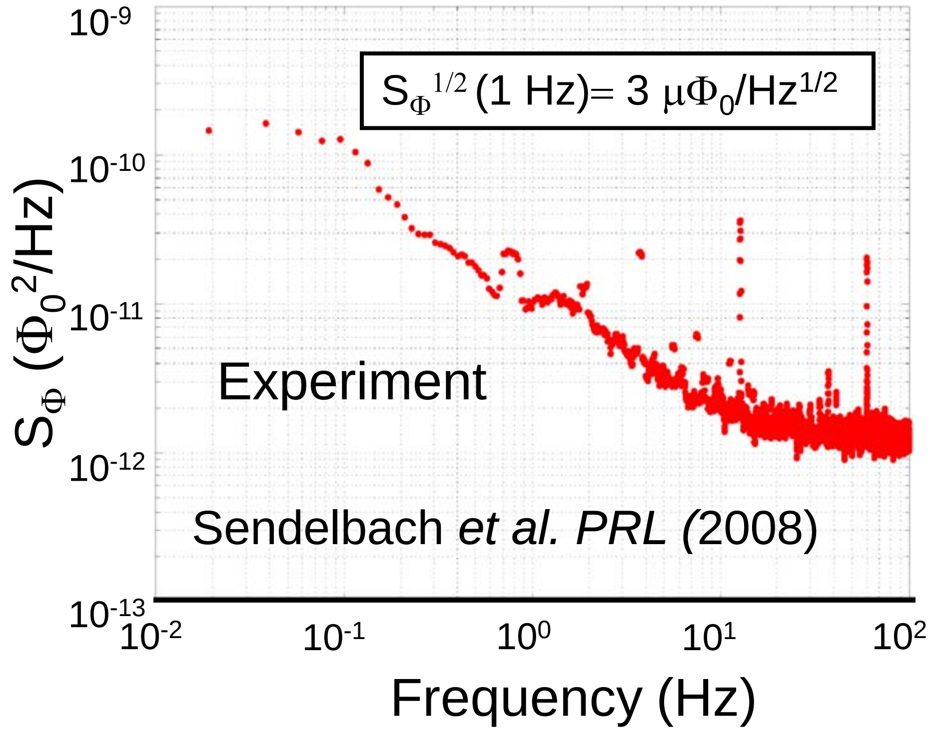


3D Magnetization and Flux Noise Consistent

- **Low frequency M noise max at T_C due to critical fluctuations**
- **Implies flux noise max identifies T_C**
- 1/f noise power spectrum
- $1/f^{\alpha}$ with $\alpha \sim 1.02$ (simulations)
- $\alpha \sim 0.95$ (UCSB expt); $0.58 < \alpha < 0.80$ (Wellstood *et al.*)

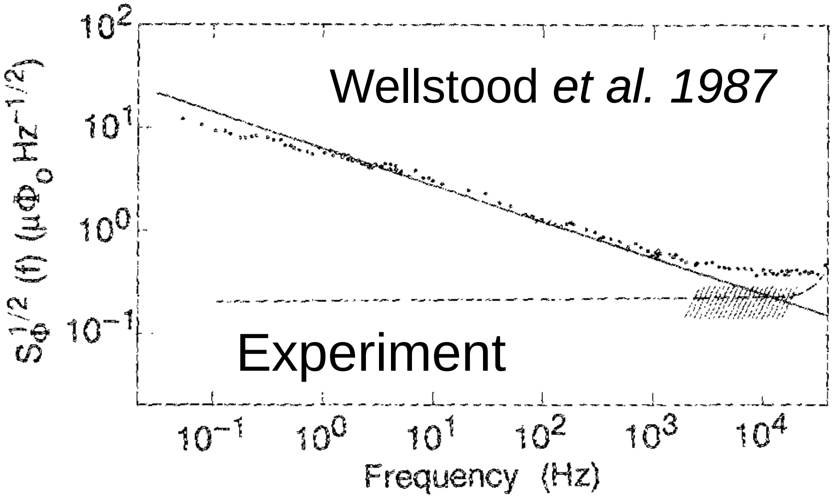


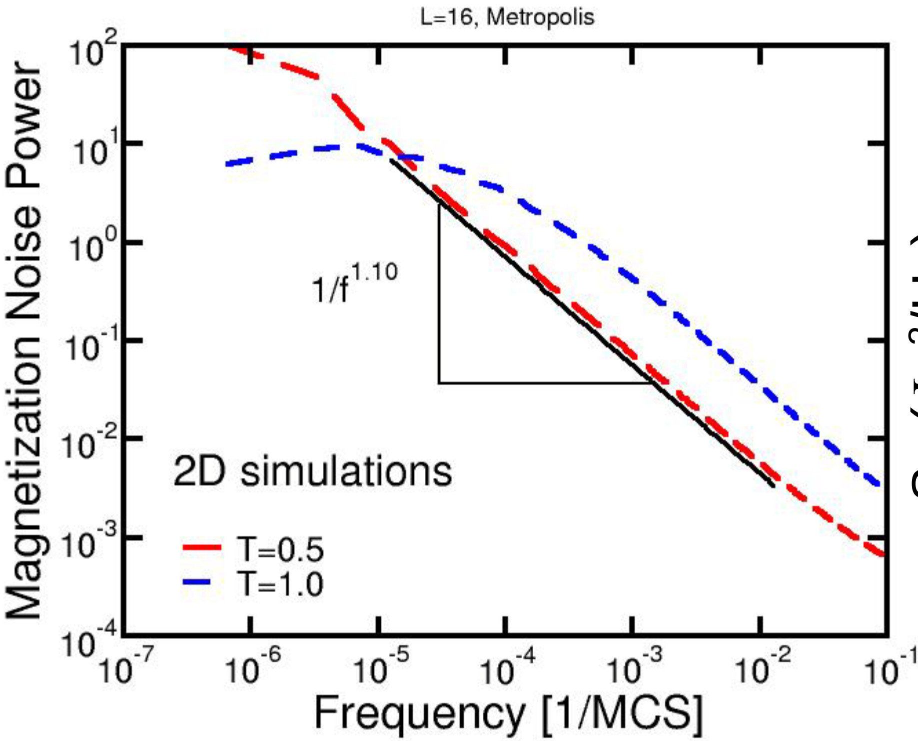


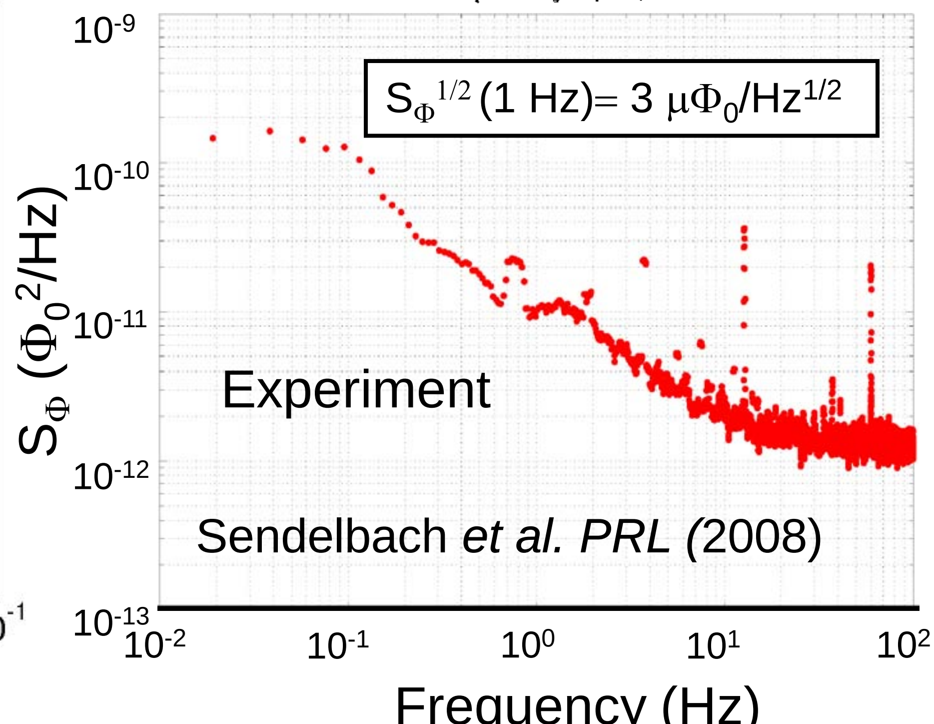


2D Magnetization and Flux Noise Consistent

- 1/f noise power spectrum
- $1/f^{\alpha}$ with $\alpha \sim 1$. (simulations)
- $\alpha \sim 0.95$ (UCSB expt); $0.58 < \alpha < 0.80$ (Wellstood *et al.*)







Inductance Noise

(Sendelbach *et al.*, PRL 2009)

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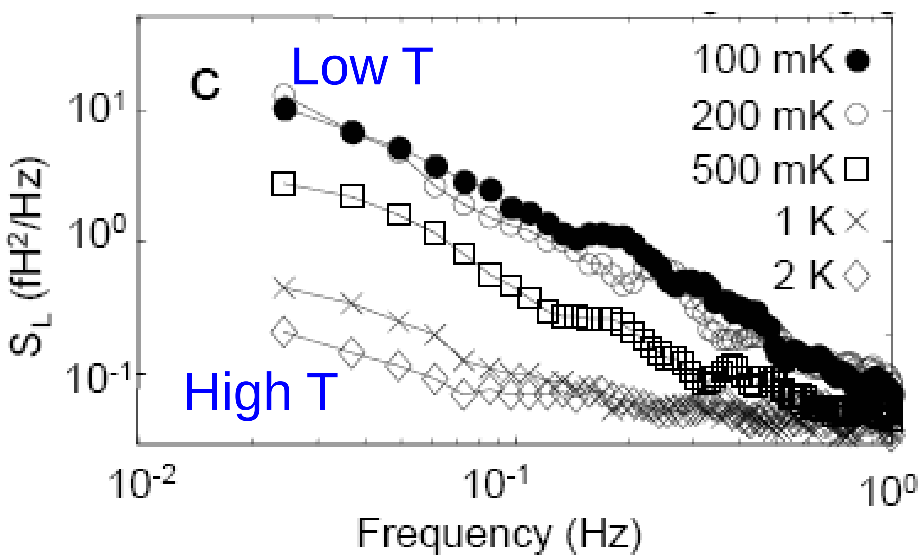
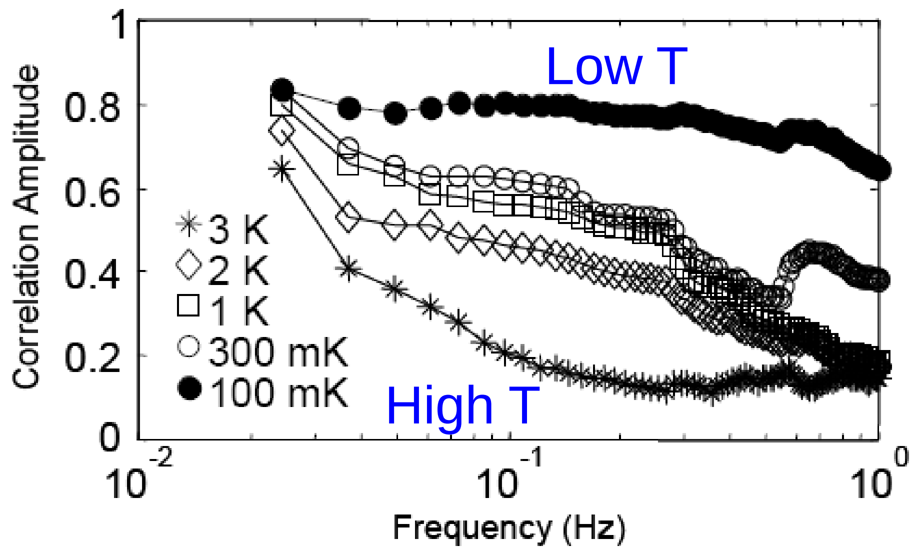
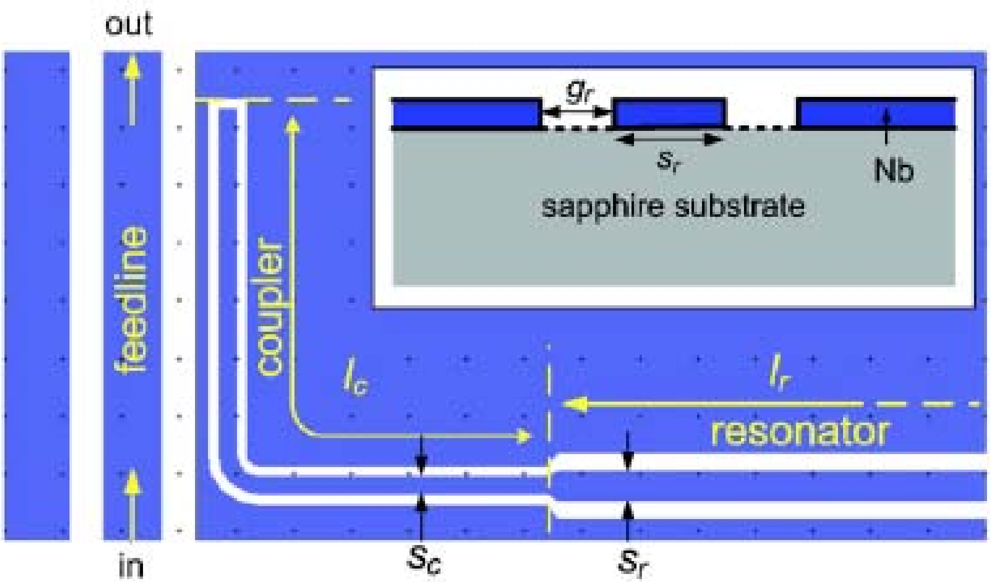




Phase Noise in Superconducting Resonators

(Gao *et al.*, Caltech, *Appl. Phys. Lett.* 2008)

- Inductance noise may explain resonant frequency (phase) noise in superconducting resonators.
- Resonant frequency $f_r = 1/\sqrt{LC}$.
- Noise in inductance L produces noise in f_r.



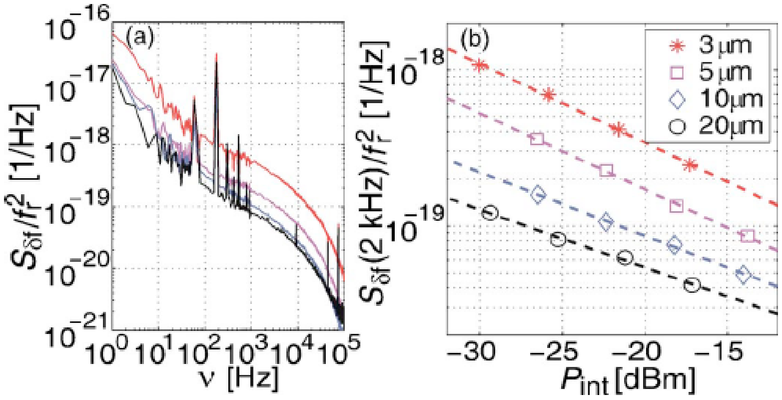
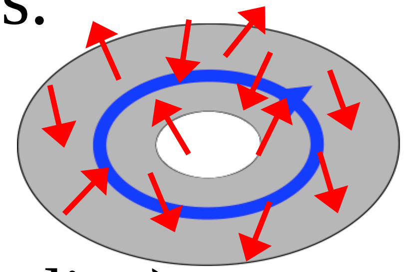


FIG. 1. (Color online) Fractional frequency noise spectra of the four CPW resonators measured at T=55 mK. (a) Noise spectra at $P_{\mu w}$=−65 dBm. From top to bottom, the four curves correspond to CPW center strip widths of s_r=3, 5, 10, and 20 μm. The various spikes seen in the spectra are due to pickup of stray signals by the electronics and cabling. (b) Fractional frequency noise at ν=2 kHz as a function of P_{int}. The markers represent different resonator geometries, as indicated by the values of s_r in the legend. The dashed lines indicate power law fits to the data of each geometry.

Inductance L ∝ Susceptibility χ

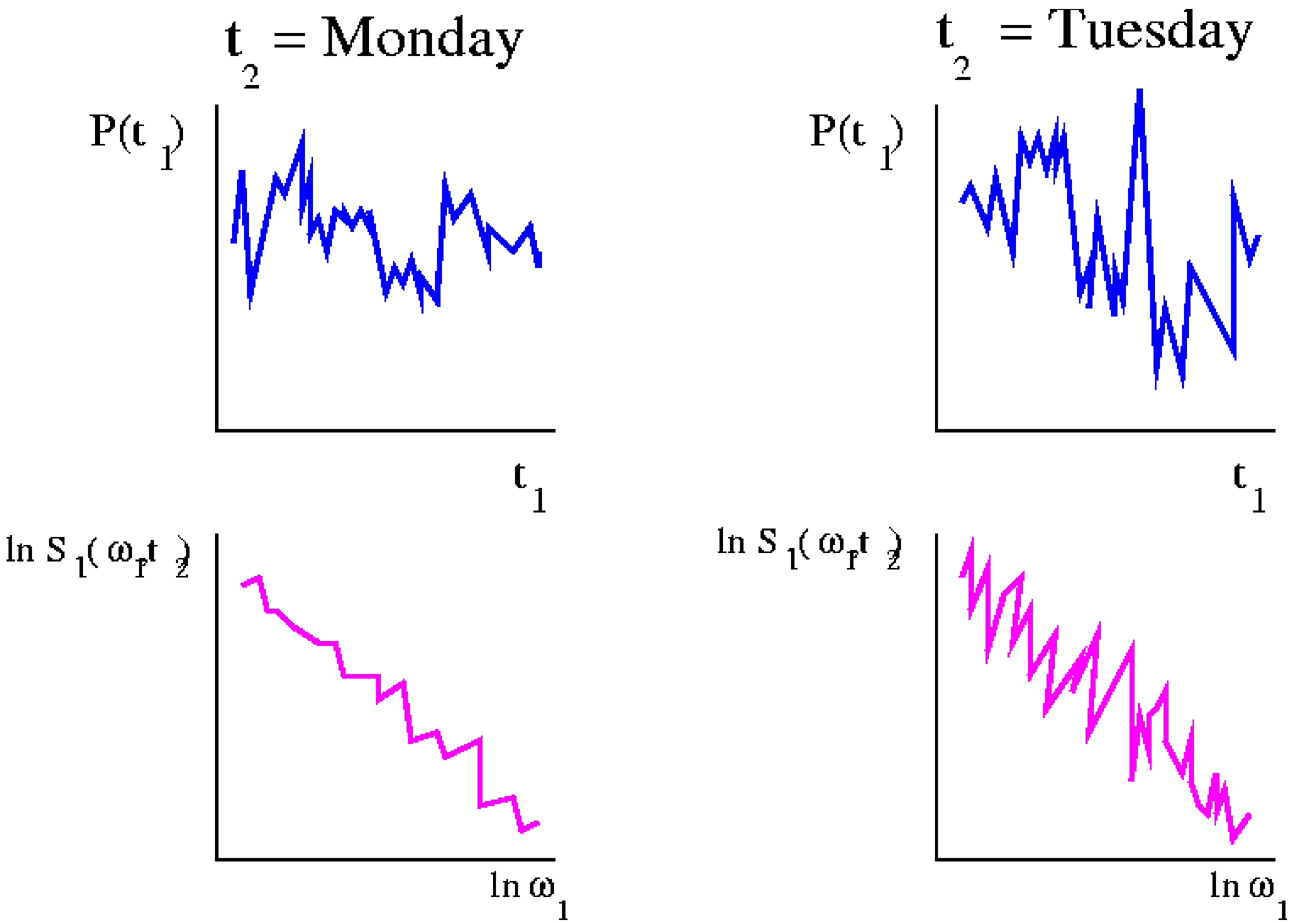


- Consider a toroidal current loop (SQUID) with spins on the surface.
- Current produces B field that polarizes spins.
- Polarized spins contribute to M and flux Φ.
- Flux $\Phi = LI \leftrightarrow$ Magnetization $M = \chi H$.
- $L = \mu_o \chi \times$ thickness $\times$ (loop radius/wire radius)
- Fluctuation-Dissipation theorem relates $S_M(\omega)$ to $\chi''(\omega)$:

$$S_M(\omega) = \frac{4kT}{\omega} \chi''(\omega)$$

- Noise in L” corresponds to noise in $\chi''(\omega)$ and $S_M(\omega)$.
- Noise in L” corresponds to the second spectrum of the noise.

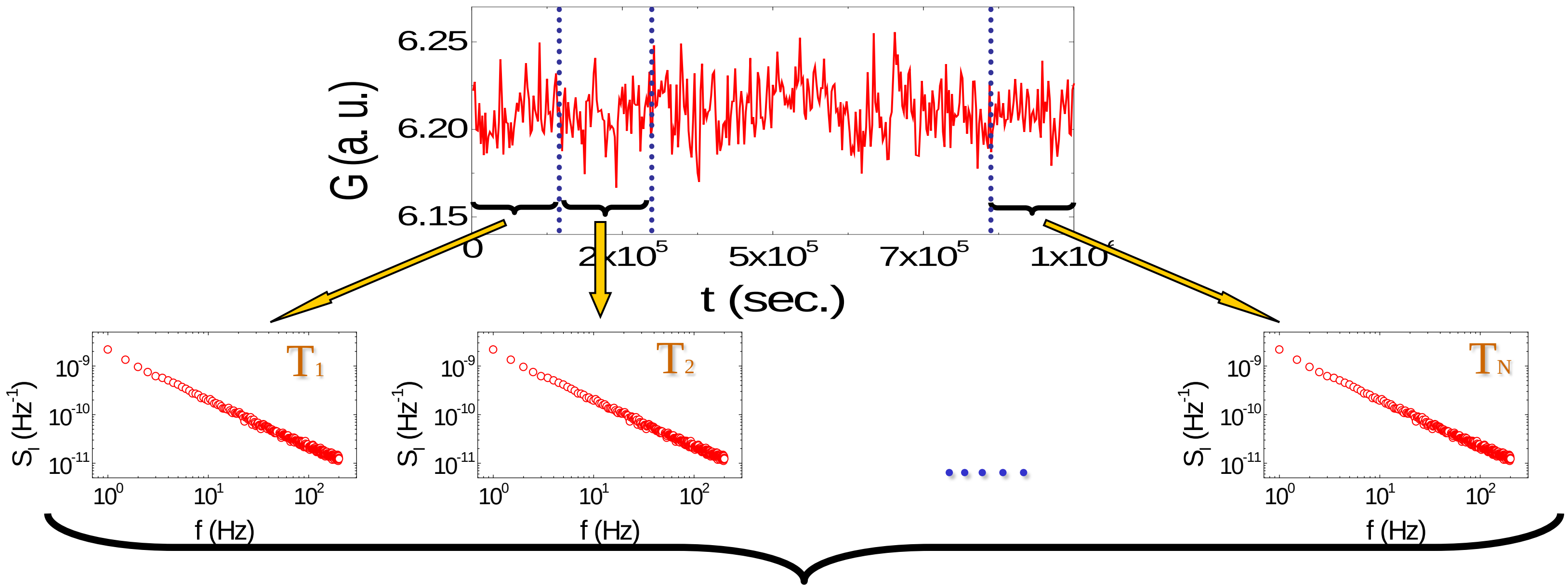
Second Spectrum of the Noise



The second spectrum is the power spectrum of the first spectrum

$$S_2(\omega_1, \omega_2) = 2\left\langle S_1(\omega_1, t_2 = t + \tau) S_1(\omega_1, t_2 = t) \right\rangle_{\omega_2}$$

Second Spectrum – "NOISE of the NOISE"



$S_1(T_1,f)$, $S_1(T_2,f)$, . . ., $S_1(T_N,f)$

Octave summing $f_H=2f_L$, and FFT with respect to T

$$S_2(f, f_2) = 2\left\langle S_1(T+\tau, f)\, S_1(T, f)\right\rangle_{f_2}$$

Uncorrelated fluctuators – **GAUSSIAN noise**

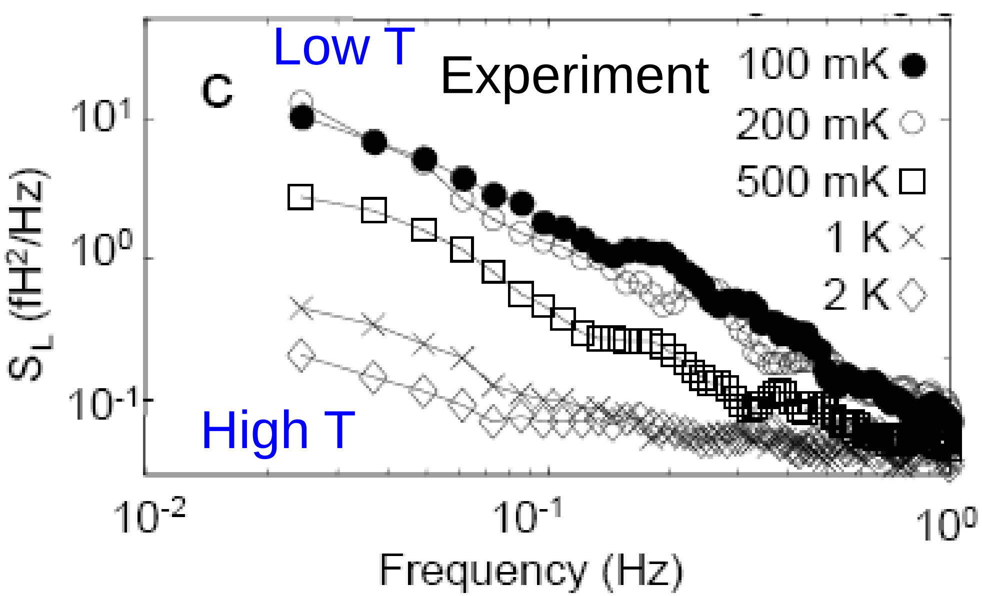
Correlated fluctuators – **NON-GAUSSIAN noise**

Inductance Noise Consistent with Noise in Imaginary Part of the Susceptibility

- Fluctuation-Dissipation Theorem:

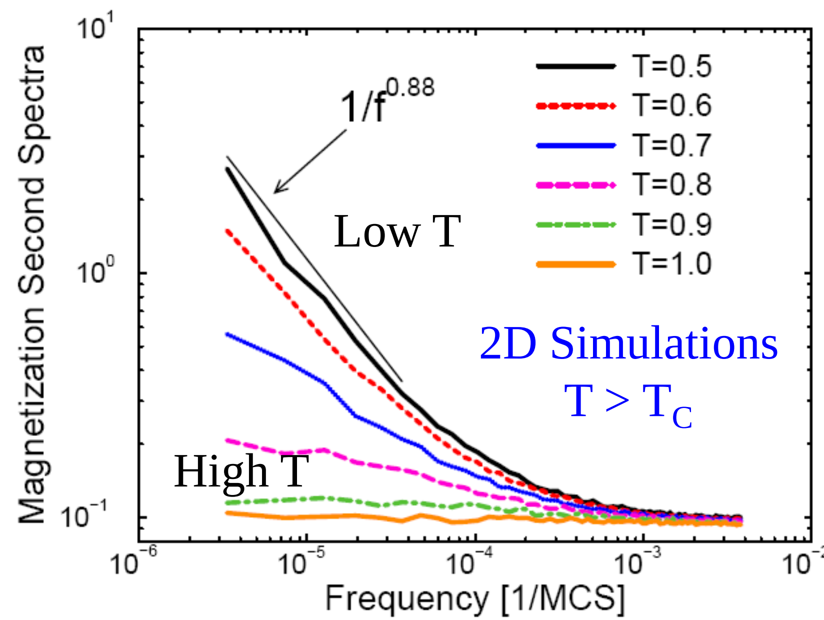
$$S_M(\omega_1) = \frac{4kT}{\omega_1}\chi''(\omega_1) \text{ implies } S_2(\omega_1,\omega_2) \propto S_{\chi''(\omega_1)}(\omega_2) \propto S_{L''(\omega_1)}(\omega_2)$$

- Inductance L ~ Susceptibility χ
- Biggest slope at low temperatures
- Slowly exploring metastable states in energy landscape at low T



(Sendelbach *et al.* 2009)

L = 16, f_1 frequency range 0.25-0.5

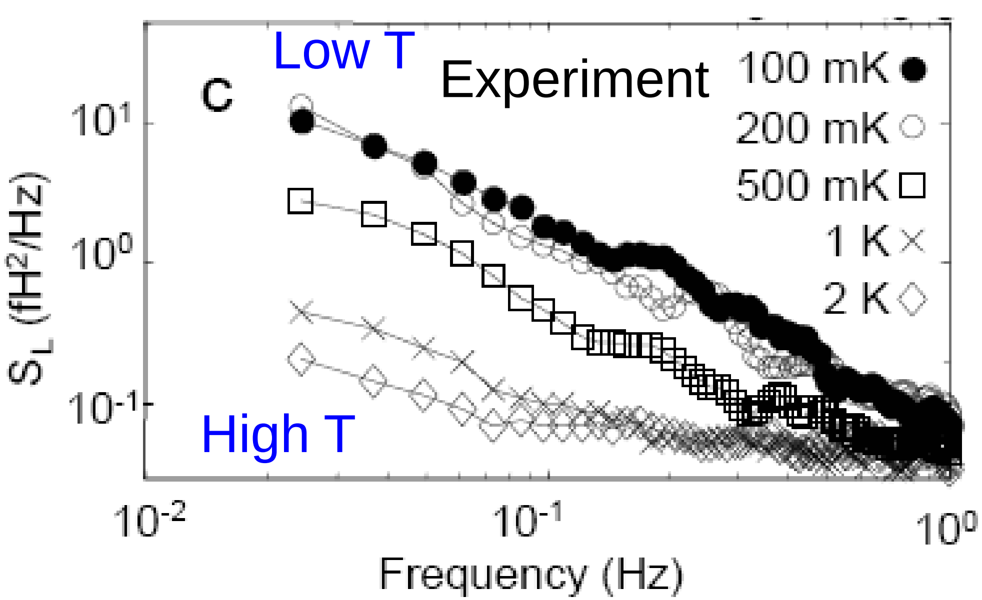


Inductance Noise Consistent with Noise in Imaginary Part of the Susceptibility

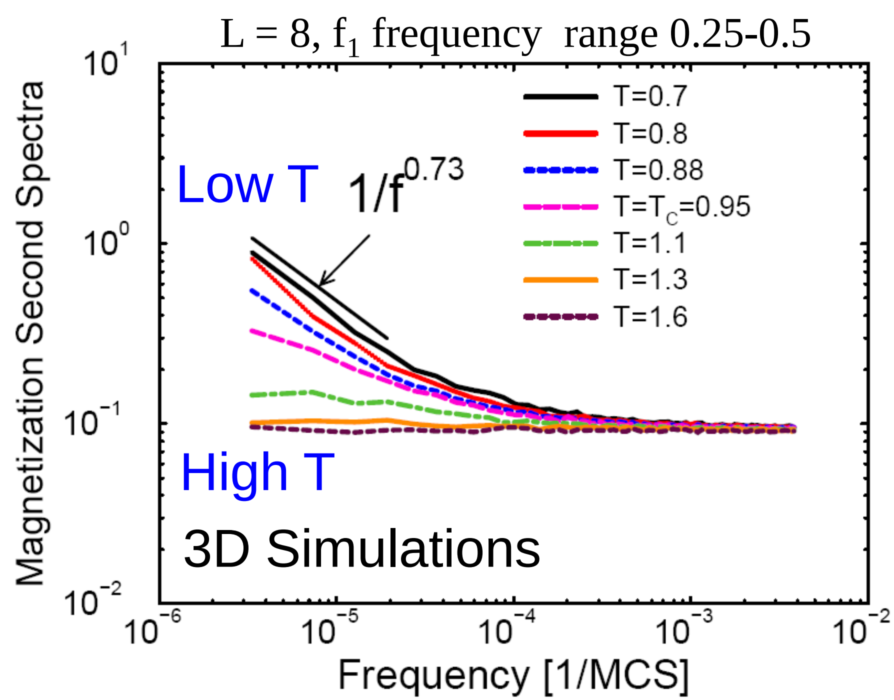
- Fluctuation-Dissipation Theorem:

$$S_M(\omega_1) = \frac{4kT}{\omega_1}\chi''(\omega_1) \text{ implies } S_2(\omega_1,\omega_2) \propto S_{\chi''(\omega_1)}(\omega_2) \propto S_{L''(\omega_1)}(\omega_2)$$

- Inductance L ~ Susceptibility χ
- Biggest slope at low temperatures
- Slowly exploring metastable states in energy landscape at low T

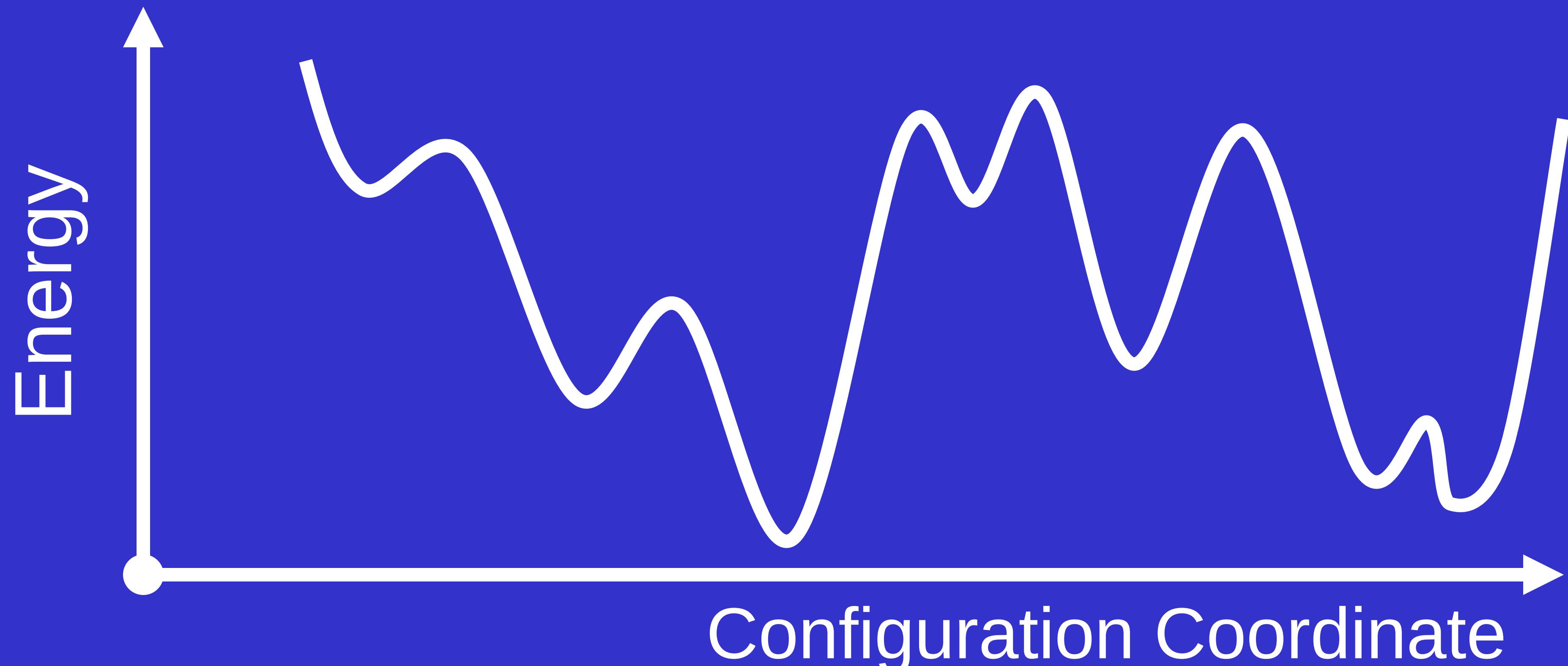


(Sendelbach *et al.* 2009)



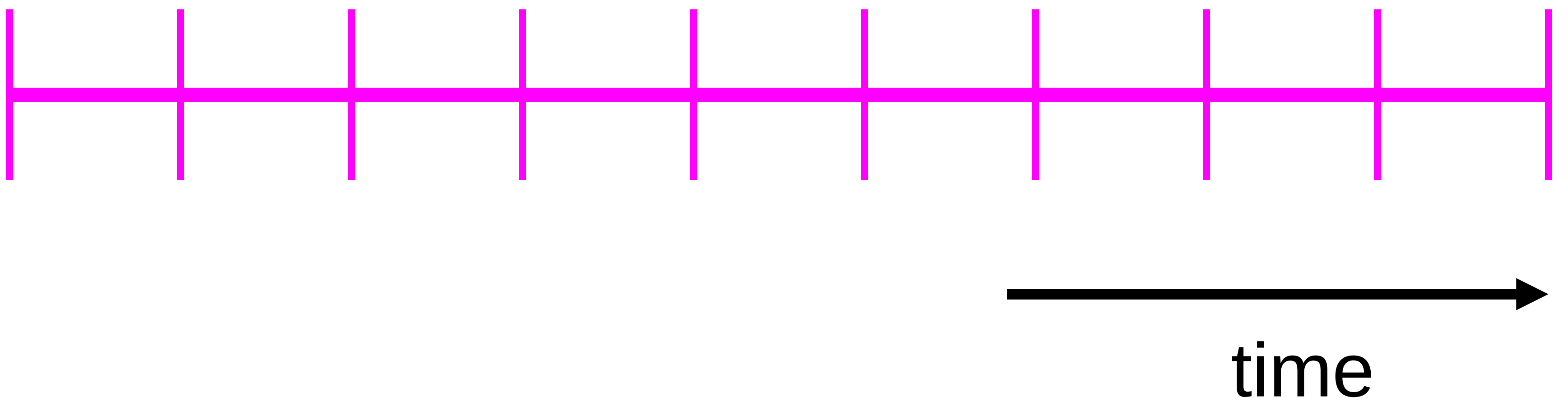
Energy Landscape

- System explores energy landscape.
- System spends a long time in metastable states at low temperatures.



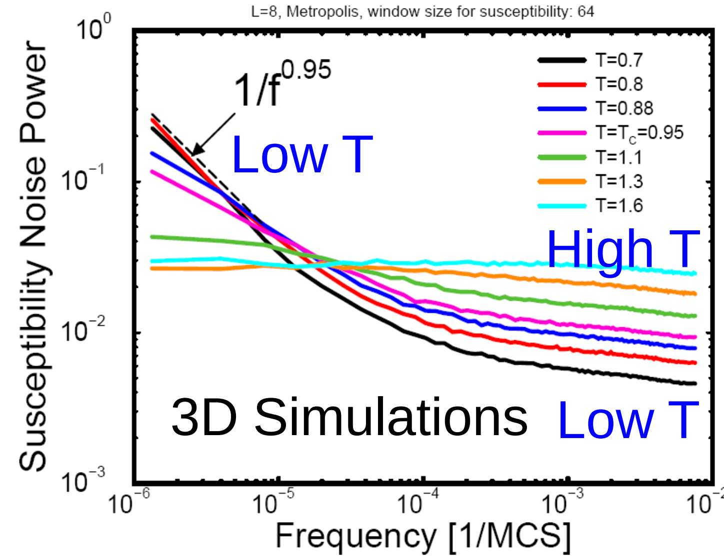
Noise in Real Part of Susceptibility χ'

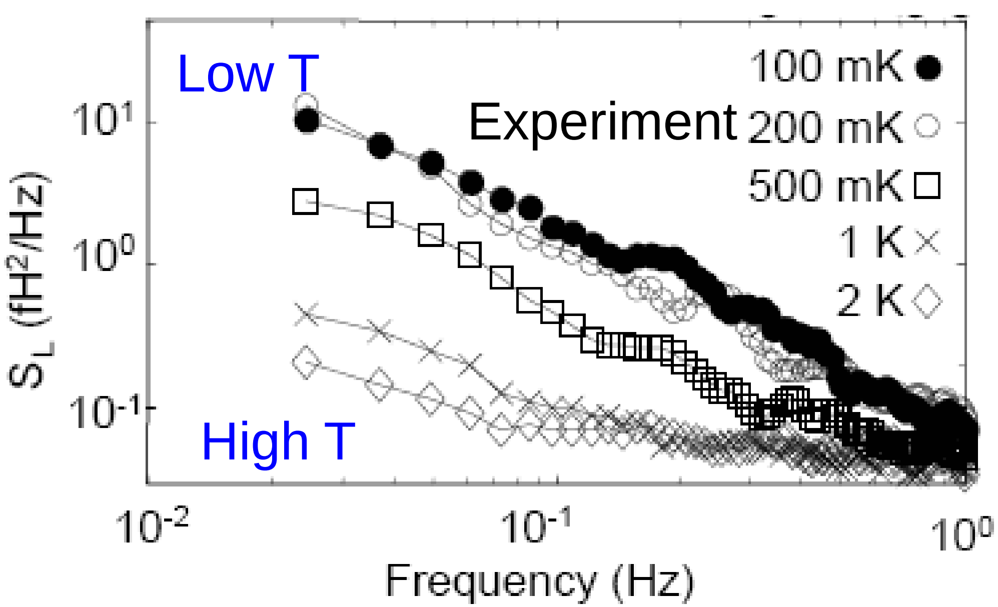
- To make time series of χ' (t)
  - Segment magnetization time series
  - Calculate $\chi = N\sigma_M^2/kT$ for each segment
- Calculate noise spectrum for χ'



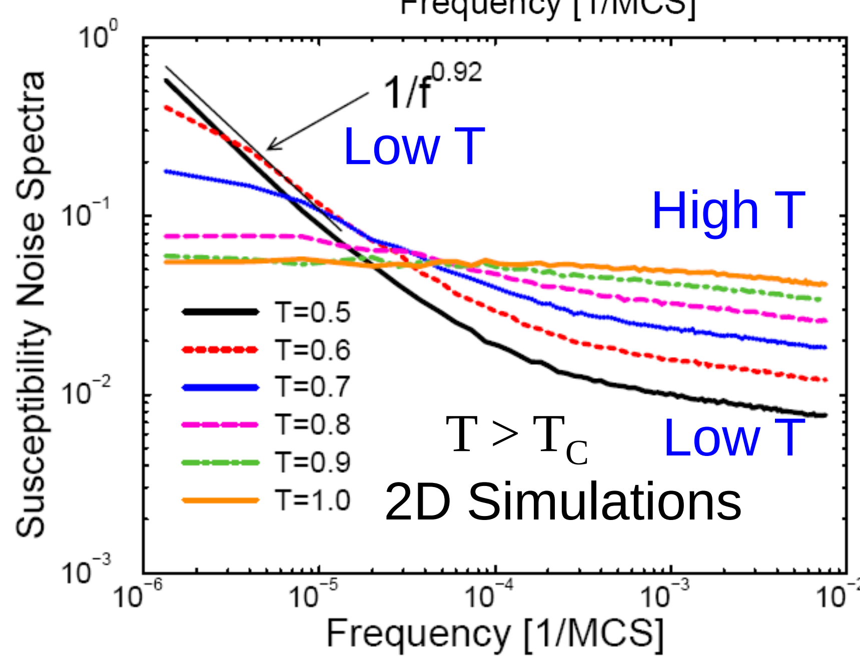
SQUID Inductance Noise Consistent with Noise in Real Part of Susceptibility χ'

Steepest slope at low temperatures: Slowly exploring metastable states in energy landscape at low T



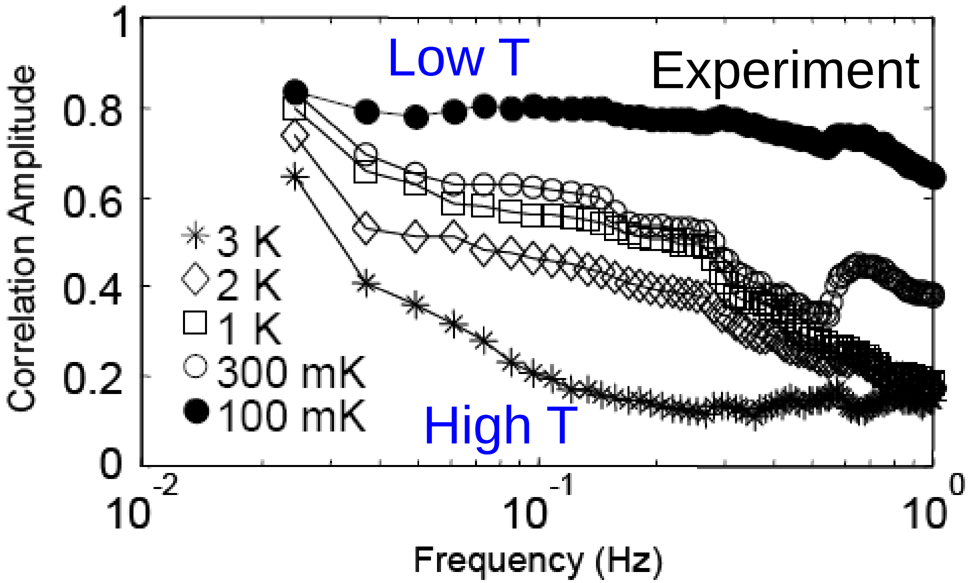


(Sendelbach *et al.* 2009)



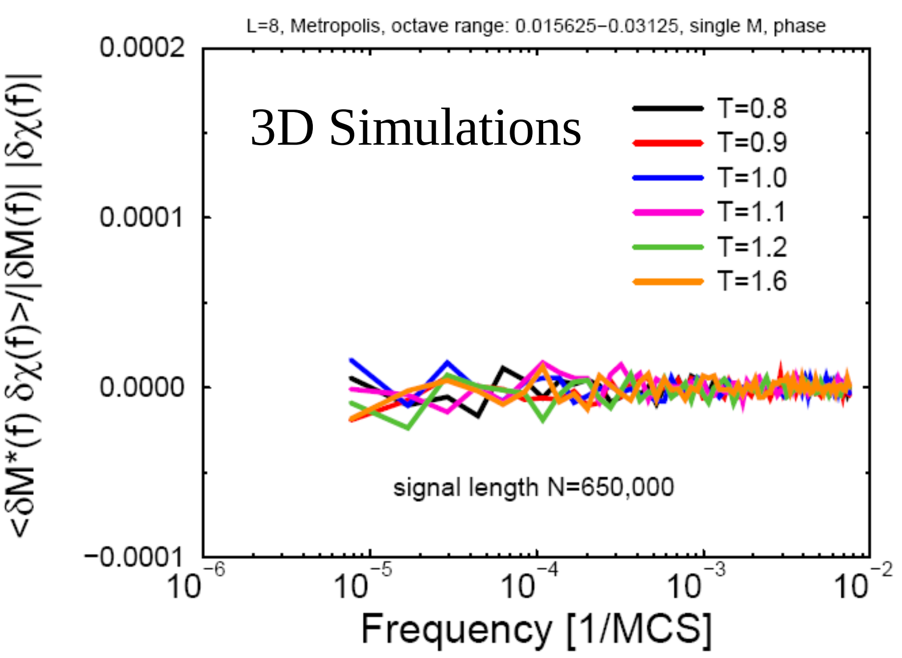
Cross Correlation of Inductance and Flux Fluctuations

- Cross correlation $\langle\delta\Phi\delta L\rangle \sim \langle\delta M\delta\chi\rangle \sim \langle(\delta M)^3\rangle$ is odd under time reversal.
- Large cross correlation and anti-correlation seen experimentally implies very slow fluctuators (Weissman).
- Correlation would average to zero over very long times.
- Cross correlation between magnetization and susceptibility is zero in spin glass simulations.



(Sendelbach *et al.* 2009)

Cross Spectra of M and χ



Summary of SQUID Noise Compared to Spin Glass Noise

- Flux noise in SQUIDs produced by mysterious magnetic impurities on metal surfaces.
- We used 2D and 3D Ising spin glass simulations to generate noise.
- $\chi \sim 1/T$ consistent with measured $\Phi \sim 1/T$.
- Magnetization noise consistent with measured flux noise.
- Low frequency noise in magnetization is a maximum at spin glass transition temperature.
- Susceptibility noise and 2nd spectrum of magnetization noise consistent with measured inductance noise.
- Magnetic impurities on metal surfaces act like interacting spins.

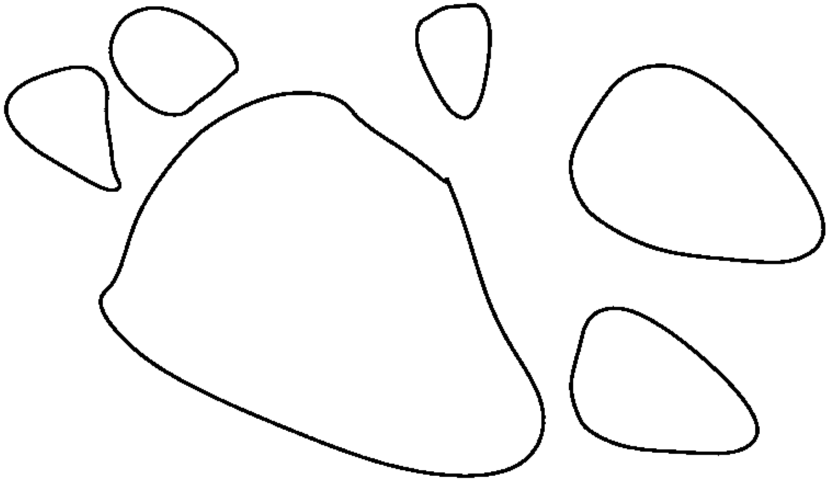
Z. Chen and C. Yu, *PRL* **104**, 247204 (2010).

Noise as a probe of microscopic fluctations:

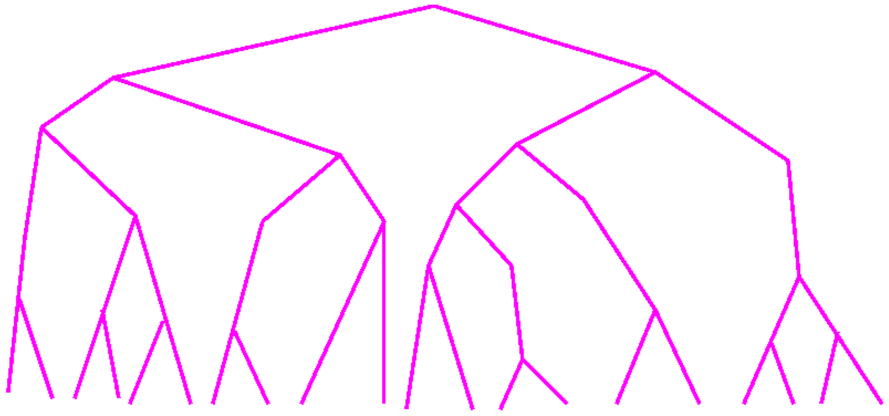
Using the noise second spectrum to differentiate between the droplet and hierarchical model of spin glasses

Droplet vs. Hierarchical Model of Spin Glasses

In the spin glass phase, the second spectrum can differentiate between the interacting droplet model and the hierarchical model (Weissman, Rev. Mod. Phys. **65**, 829 (1993)).



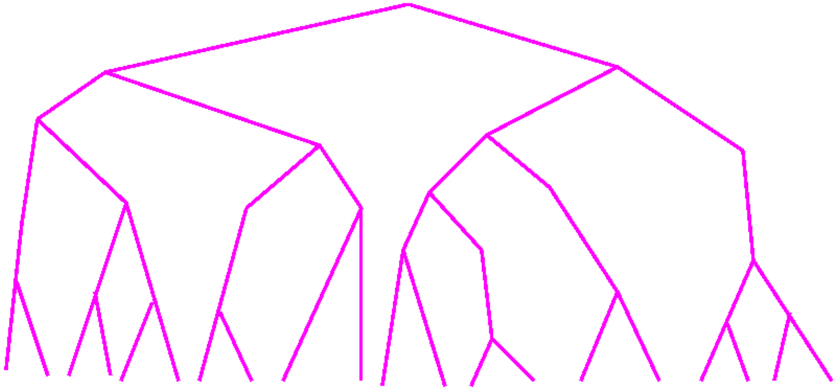
Droplet Model



Hierarchical Model

Hierarchical Model

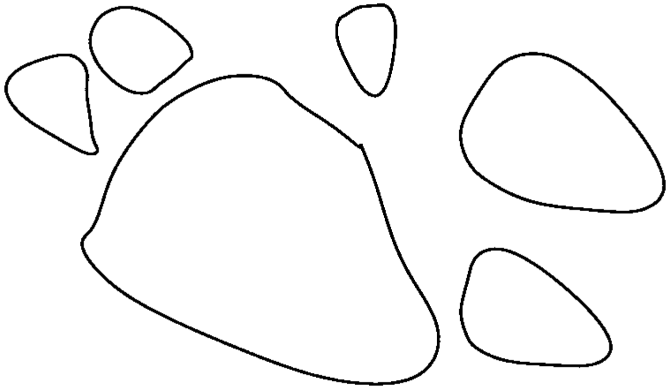
(Parisi and others)



- The states (configurations) of the system are represented by the end points of the lowest branches.
- The Hamming distance between 2 states is given by the highest vertex on the tree along the shortest path connecting the states.
- The farther 2 states are, the longer the time to go between them.
- The tree structure is self similar.
- The second spectrum should be scale invariant and only depend on f_2/f_1, not on f_1.

Droplet Model

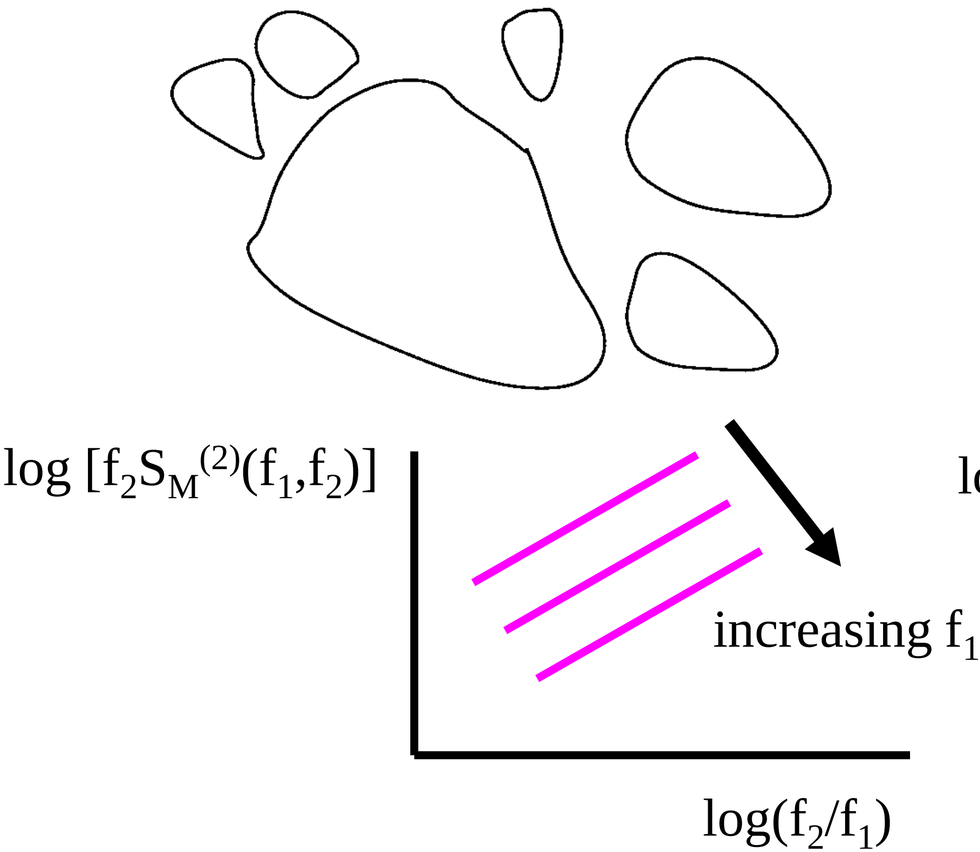
(Fisher and Huse)



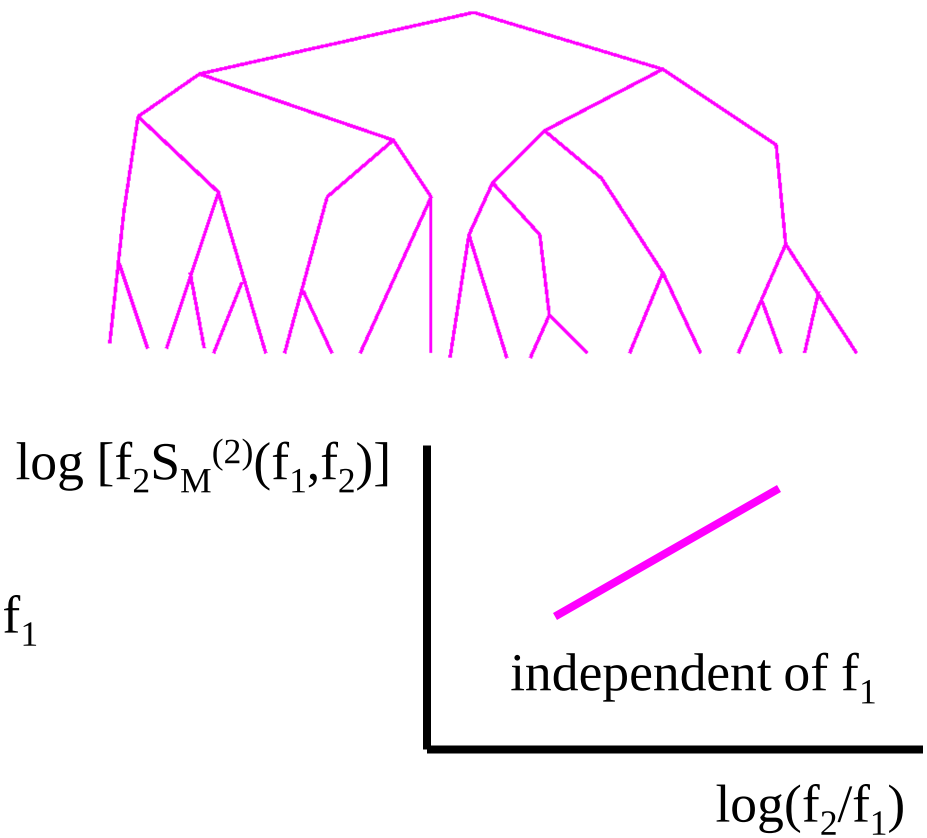
- In the droplet model there are droplets or clusters of coherently flipping spins. The energy for a cluster to flip scales as $L^{\rightarrow}$ where the power θ is small.
- Large clusters flip more slowly than small clusters. So the large clusters contribute to the low frequency noise and the small fast clusters to the high frequency noise.
- In the simplest version the droplets are noninteracting. If this is the case, the second spectrum would be white noise.
- A more sophisticated version has interacting droplets. Large droplets are more likely to interact than small droplets so the second spectrum will be larger at lower frequencies f_1.

Droplet vs. Hierarchical Model

In the spin glass phase, the second spectrum $S_M^{(2)}(f_1,f_2)$ can differentiate between the interacting droplet model and the hierarchical model (Weissman *et al.*).

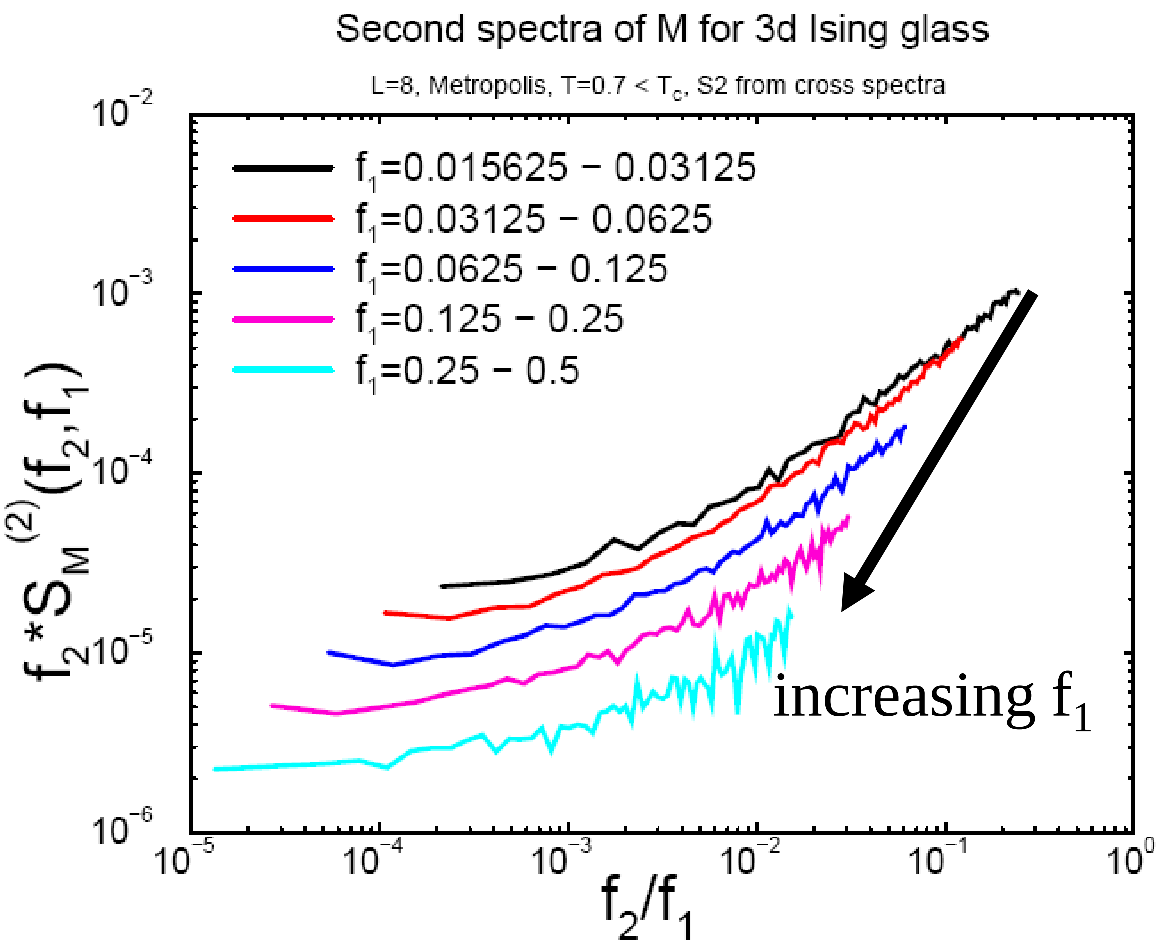


Droplet Model

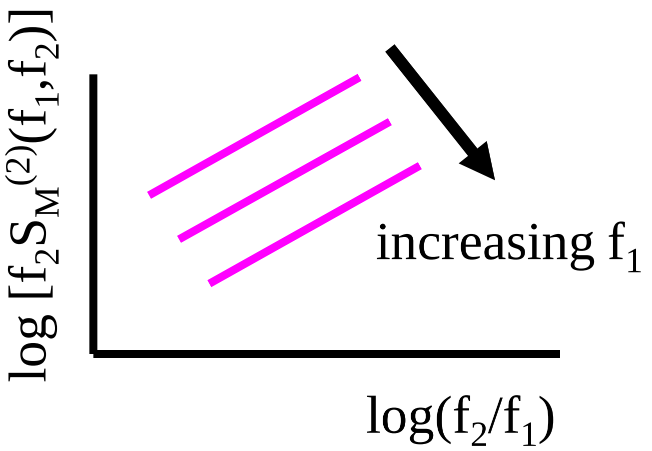


Hierarchical Model

3D Ising spin glass noise consistent with droplet model



Droplet Model



Evidence for Droplet vs. Hierarchical Model for 3D Ising Spin Glass

- Simulations in favor of droplet model:
  - Moore, Bokil, Drossel, *PRL* (1998)
  - Palassini and Young, *PRL* (1999)
  - Houdayer and Martin, *PRL (*1999)
- Simulations in favor of hierarchical model:
  - Marinari, Parisi, Ruiz-Lorenzo, Ritort, *PRL* (1996)
  - Contucci, Giardina, Giberti, Parisi, Vernia, *PRL* (2007)

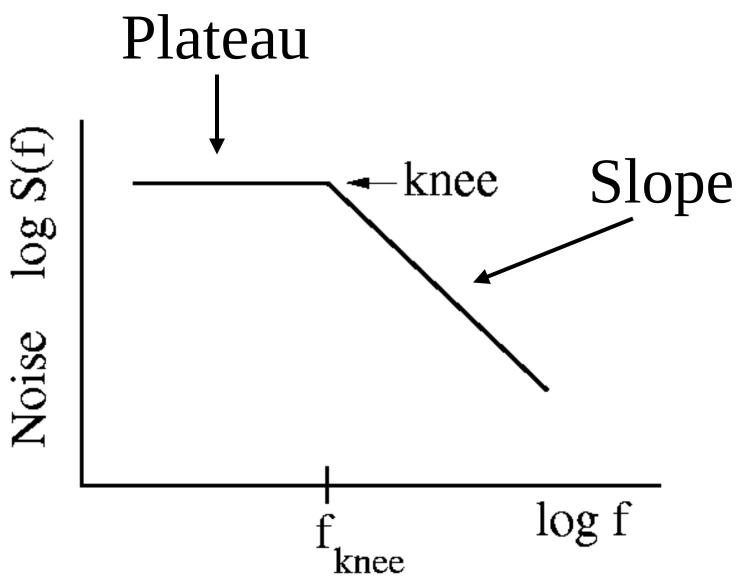
Still controversial whether the 3D Ising spin glass obeys the hierarchical or droplet model.

Summary

- Spins on metals produce flux noise.
- We used Monte Carlo simulations of Ising spin glasses to produce noise spectra.
- Flux and inductance noise consistent with noise produced by interacting spins.
- Noise in magnetization and order parameter q from spin glass simulations is maximum at $T_{C.}$
- Second spectrum of the magnetization noise consistent with droplet model.

THE END

Noise Spectrum Has 3 Parts

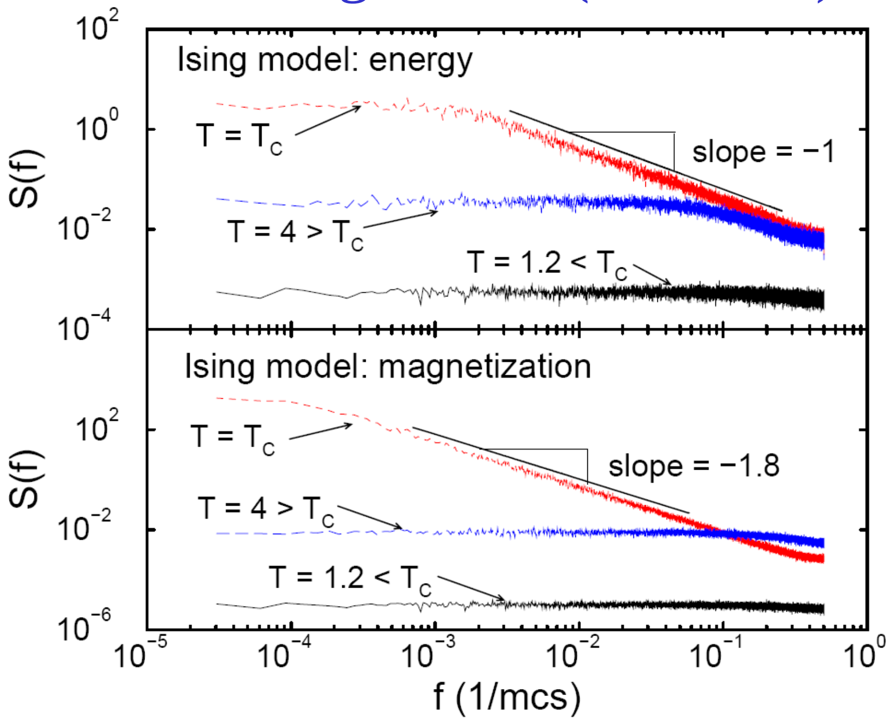


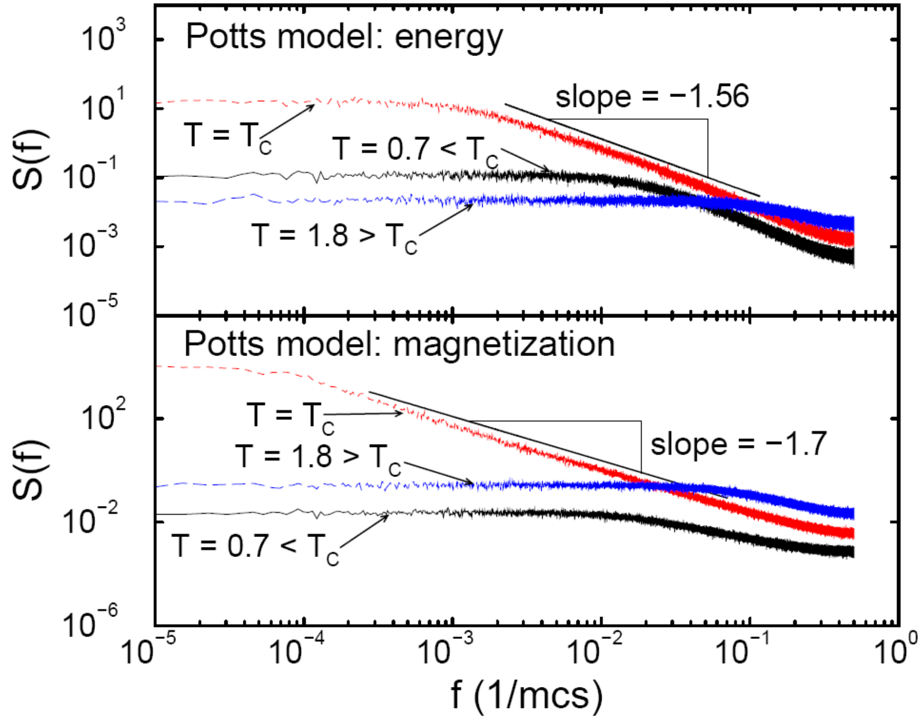
- High frequency: $S(f > f_{knee}) \sim 1/f^{\mu}$ (exponent μ determined by critical exponents for 2nd order transition)
- Crossover or knee frequency f_{knee} in S(f) (f_{knee} ~ inverse equilibration time)
- Low frequency: Plateau for $S(f < f_{knee})$ (maximum at T_C)

Noise Power Spectrum

(Chen and Yu, PRL 2007)

Ising Model (2nd order) — Potts Model (1st order)





- **Low frequency noise reaches maximum at T_C.**
- **Total noise power (σ^2) reaches maximum at T_C.**
- Away from T_C noise is low and white.
- Near T_C high frequency noise: $S(f) \sim 1/f^\mu$.
- μ given in terms of critical exponents for 2nd order transition.

Size Dependence of Noise

- Near T_C
- As $N \to \infty$
- $f_{knee} \sim 1/N^b \to 0 \quad (b \geq 1)$
- $S(f < f_{knee}) \sim 1/(N f_{knee}^{\mu}) \to \infty$
- $S(f > f_{knee}) \sim 1/(N f^{\mu}) \to 0$
- $S_{tot} = \sigma^2 \to 0$

Contradiction as $N \to \infty$ near T_C?
Noise increases
Noise $\to$ 0 from self-averaging
$\leftarrow$ Resolution

- Far away from T_C
- As $N \to \infty$
- $S(f) \sim 1/N \to 0$
- f_{knee} independent of N
- $S_{tot} = \sigma^2 \sim 1/N \to 0$