Many Body Anderson Localization

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Electron Glasses

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Outline:

1. Introduction to Anderson Localization
2. Phononless conductivity
3. Many – Body Localization
4. Disordered bosons in 1D
5. Metal -Perfect Insulator transition in electronic systems

Introduction

Philip W. Anderson
The Nobel Prize in Physics 1977

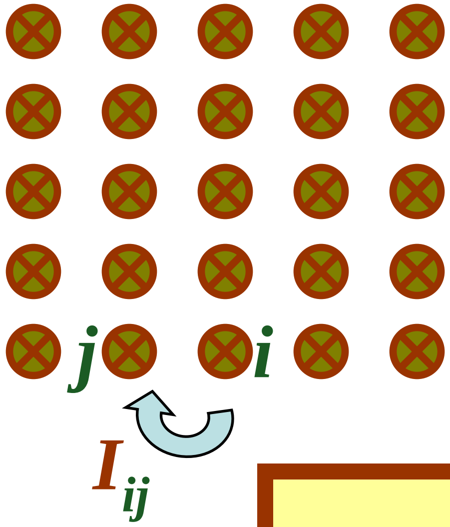
Nobel Lecture
Nobel Lecture, December 8, 1977

Local Moments and Localized States

I was cited for work both. in the field of magnetism and in that of disordered systems, and I would like to describe here one development in each held which was specifically mentioned in that citation. The two theories I will discuss differed sharply in some ways. The theory of local moments in metals was, in a sense, easy: it was the condensation into a simple mathematical model of ideas which. were very much in the air at the time, and it had rapid and permanent acceptance because of its timeliness and its relative simplicity. What mathematical difficulty it contained has been almost fully- cleared up within the past few years.

Localization was a different matter: very few believed it at the time, and even fewer saw its importance; among those who failed to fully understand it at first was certainly its author. It has yet to receive adequate mathematical treatment, and one has to resort to the indignity of numerical simulations to settle even the simplest questions about it .

Anderson Model



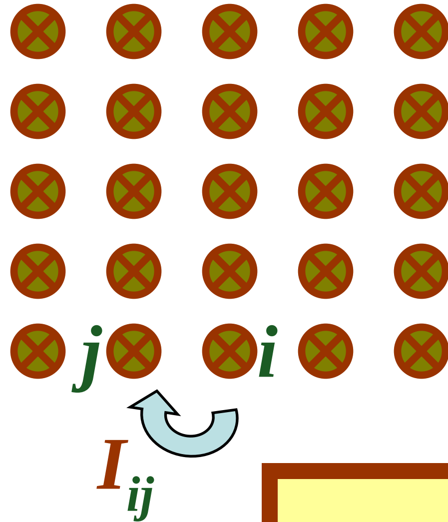
- *Lattice - tight binding model*
- *Onsite energies* ε_i *- random*
- *Hopping matrix elements* I_{ij}

$$-W < \varepsilon_i < W$$

uniformly distributed

$$I_{ij} = \begin{cases} I & i \text{ and } j \text{ are nearest neighbors} \\ 0 & \text{otherwise} \end{cases}$$

Anderson Model



- *Lattice - tight binding model*
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$-W < \varepsilon_i < W$

uniformly distributed

$$I_{ij} = \begin{cases} I & i \text{ and } j \text{ are nearest neighbors} \\ 0 & \text{otherwise} \end{cases}$$

Anderson Transition

$$I_c = f(d) * W$$

$I < I_c$

Insulator

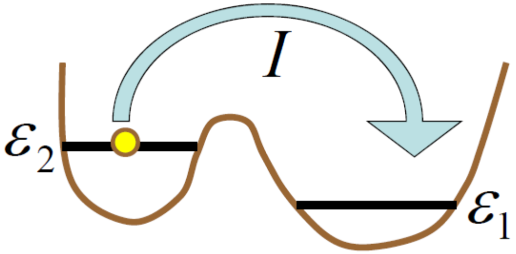
All eigenstates are localized

Localization length ξ

$I > I_c$

Metal

There appear states extended all over the whole system



$$\hat{H} = \begin{pmatrix} \varepsilon_1 & I \\ I & \varepsilon_2 \end{pmatrix}$$

$$\varphi_1 \varepsilon_1; \varphi_2 \varepsilon_2 \Leftarrow \psi_1, E_1; \psi_2, E_2$$

$$\varepsilon_2 - \varepsilon_1 >> I$$

$$\psi_{1,2} = \varphi_{1,2} + O\left(\frac{I}{\varepsilon_2 - \varepsilon_1}\right)\varphi_{2,1}$$

Off-resonance

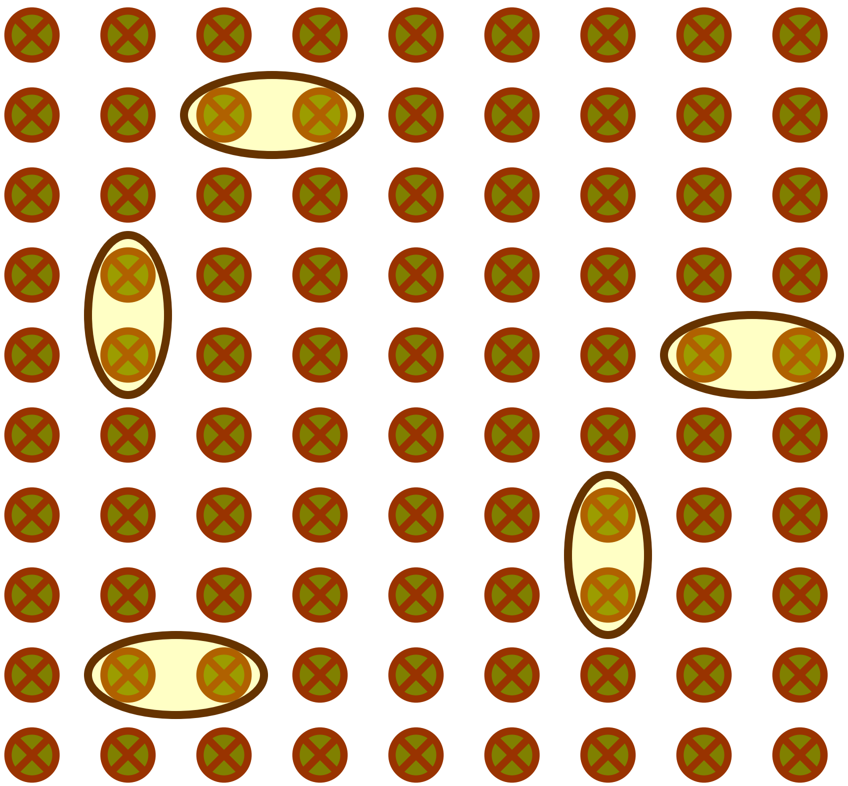
Eigenfunctions are close to the original on-site wave functions

$$\varepsilon_2 - \varepsilon_1 << I$$

$$\psi_{1,2} \approx \varphi_{1,2} \pm \varphi_{2,1}$$

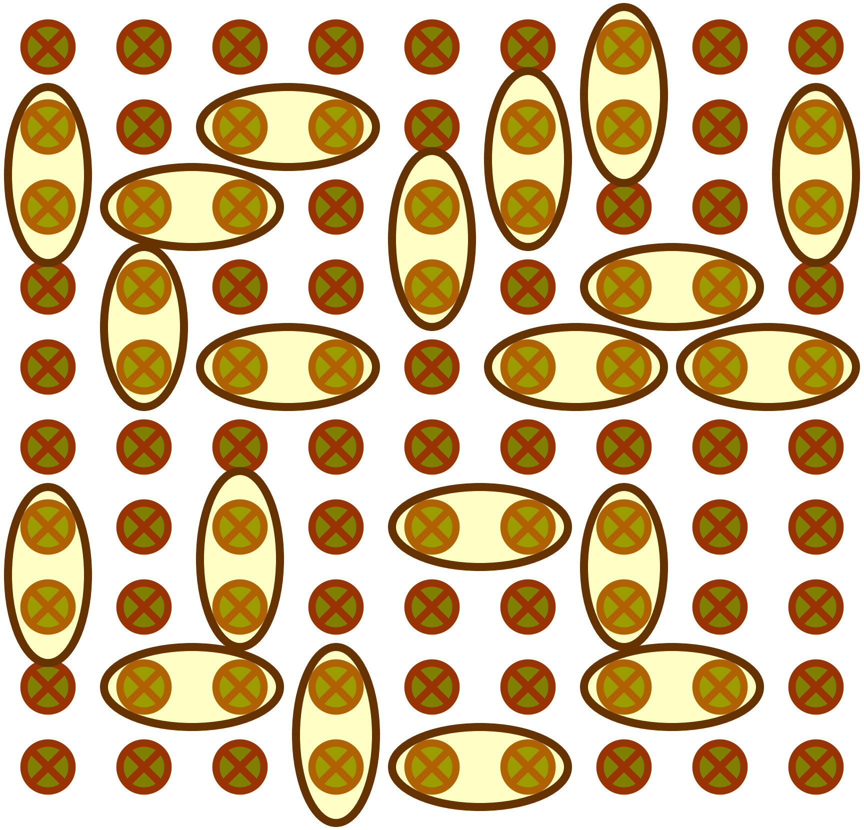
Resonance

The probability is equally shared between the sites



Anderson insulator

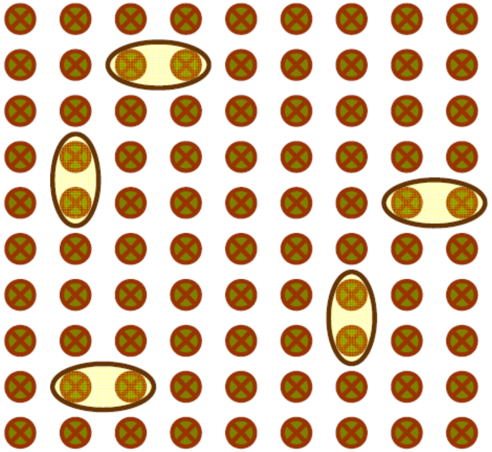
Few isolated resonances



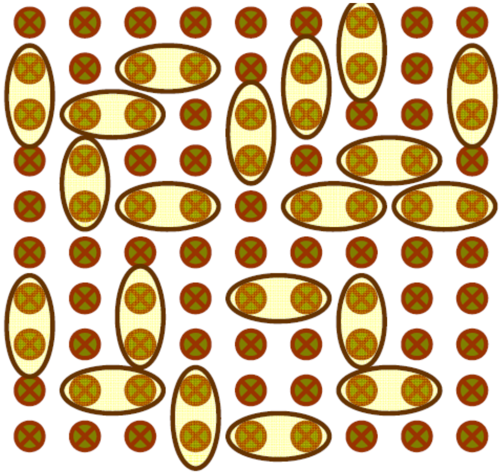
Anderson metal

There are many resonances and they overlap

Transition: Typically each site is in the resonance with some other one



Anderson insulator
Few isolated resonances



Anderson metal
There are many resonances and they overlap

Condition for Localization:

$$I < \frac{\text{energy mismatch}}{\text{\# of n.neighbors}}$$

energy mismatch $= \left|\varepsilon_i - \varepsilon_j\right|_{typ} = W$

\# of nearest neighbors $= 2d$

Transition: Typically each site is in the resonance with some other one

A bit more precise:

$$\frac{I_c}{W} \simeq \left(\frac{1}{2d}\right)\left(\frac{1}{\ln d}\right)$$

Logarithm is due to the resonances, which are not nearest neighbors

Condition for Localization:

$$\frac{I_c}{W} \simeq \left(\frac{1}{2d}\right)\left(\frac{1}{\ln d}\right)$$

Q: Is it correct?

A1: For low dimensions – **NO**. $I_c = \infty$ for $d = 1, 2$
All states are localized. Reason – loop trajectories

A2: Works better for larger dimensions $d > 2$

A3: Is exact on the Cayley tree (Bethe lattice)

$$I_c = \frac{W}{K \ln K},$$

K is the branching number

J. Phys. C: Solid State Phys., Vol. 6, 1973. Printed in Great Britain. © 1973

Anderson Model on a Cayley tree

A selfconsistent theory of localization

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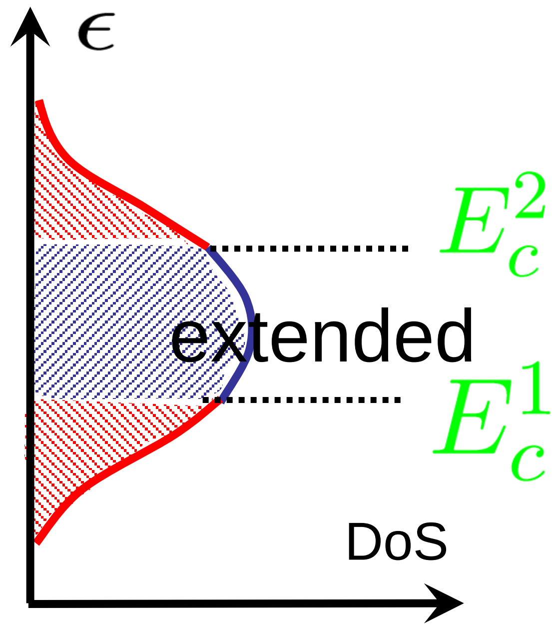
Received 12 January 1973

Abstract. A new basis has been found for the theory of localization of electrons in disordered systems. The method is based on a selfconsistent solution of the equation for the self energy in second order perturbation theory, whose solution may be purely real almost everywhere (localized states) or complex everywhere (nonlocalized states). The equations used are exact for a Bethe lattice. The selfconsistency condition gives a nonlinear integral equation in two variables for the probability distribution of the real and imaginary parts of the self energy. A simple approximation for the stability limit of localized states gives Anderson's 'upper limit approximation'. Exact solution of the stability problem in a special case gives results very close to Anderson's best estimate. A general and simple formula for the stability limit is derived; this formula should be valid for smooth distribution of site energies away from the band edge. Results of Monte Carlo calculations of the selfconsistency problem are described which confirm and go beyond the analytical results. The relation of this theory to the old Anderson theory is examined, and it is concluded that the present theory is similar but better.

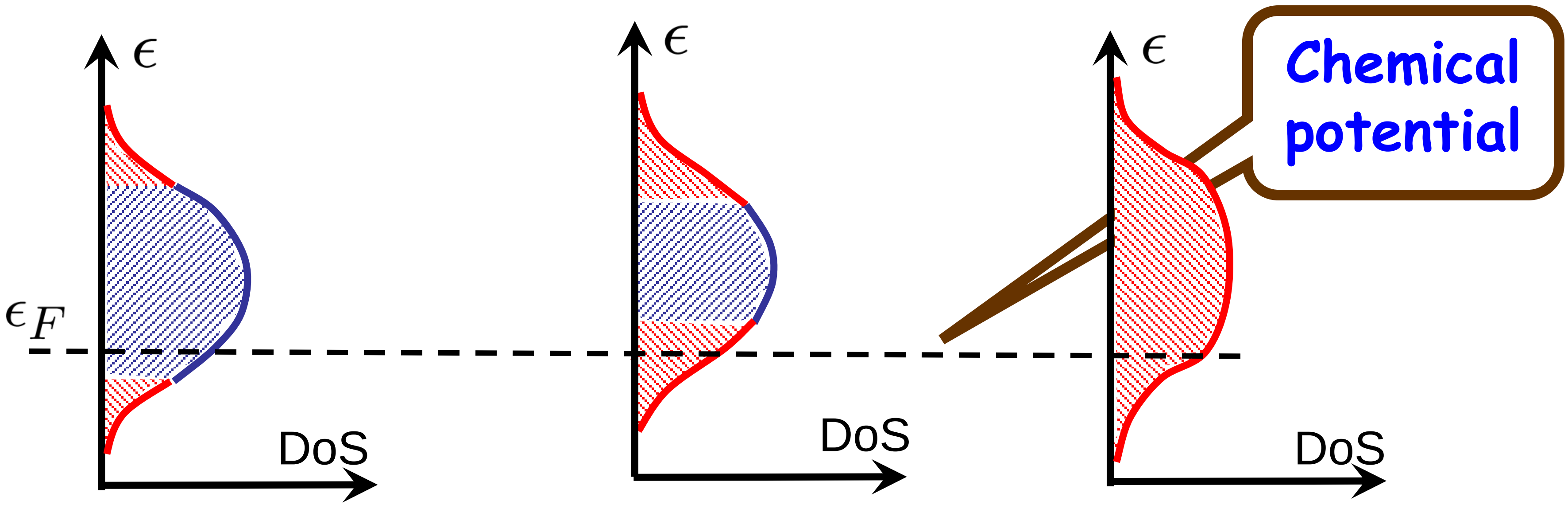
Anderson Transition

$I > I_c$ $\qquad$ $I < I_c$



E_c - mobility edges (one particle)

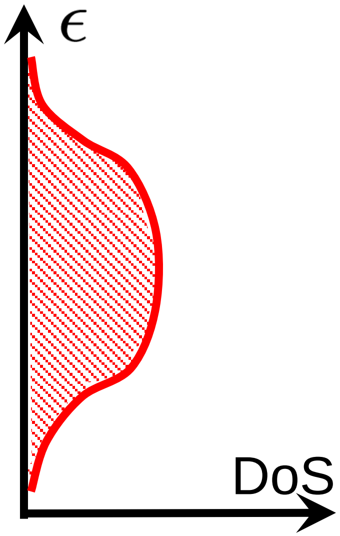
Temperature dependence of the conductivity
one-electron picture



$$\sigma(T \to 0) > 0 \qquad \sigma(T) \propto e^{-\frac{E_c - \varepsilon_F}{T}} \qquad \sigma(T) = 0 \ \ \forall T$$

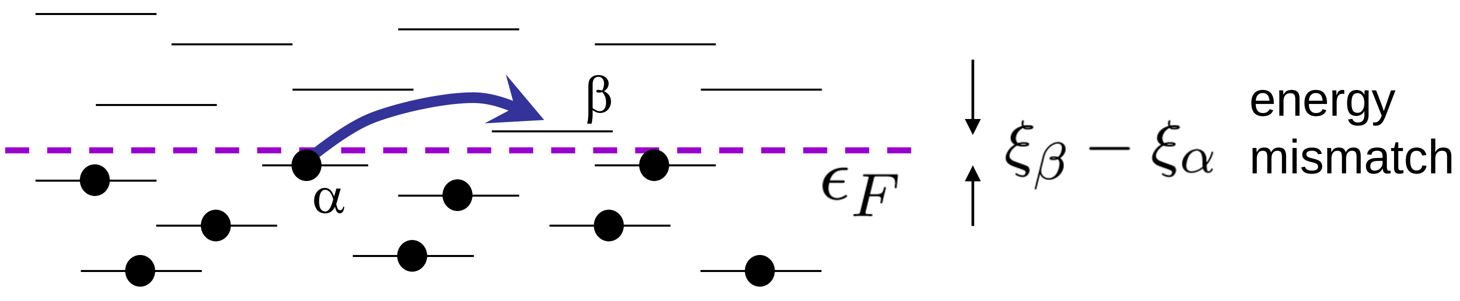
Temperature dependence of the conductivity
one-electron picture

Assume that all the states are localized; e.g. $d = 1,2$



$$\sigma(T) = 0 \quad \forall T$$

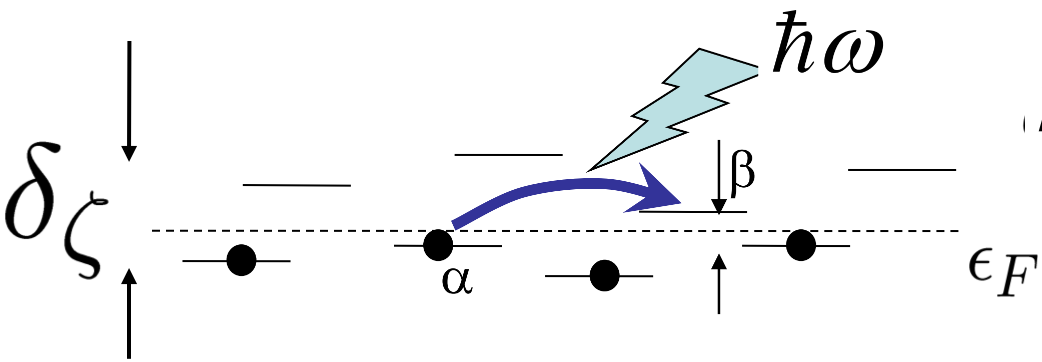
Inelastic processes
transitions between localized states



$$T = 0 \quad \Rightarrow \quad \sigma = 0$$

(any mechanism)

Phonon-assisted hopping



$$\hbar\omega = \varepsilon_\alpha - \varepsilon_\beta$$

$$\sigma(T=0)=0$$

Variable Range Hopping

N.F. Mott (1968)

$$\sigma(T) \propto T^{\gamma} \exp\left[-\left(\frac{\delta_\zeta}{T}\right)^{\frac{1}{d+1}}\right]$$

Mechanism-dependent prefactor

Optimized phase volume

Any bath with a continuous spectrum of delocalized excitations down to $\omega = 0$ will give the same exponential

Phononless conductance in Anderson insulators with e-e interaction

Common belief: Anderson Insulator weak e-e interactions ⇨ Phonon assisted hopping transport

Can hopping conductivity exist without phonons ?

Given:

1. All one-electron states are localized
2. Electrons interact with each other
3. The system is closed (no phonons)
4. Temperature is low but finite

Find: DC conductivity $\sigma(T,\omega=0)$ (zero or finite?)

Q: Can e-h pairs lead to phonon-less variable range hopping in the same way as phonons do ?

A#1: Sure

1. Recall phonon-less
   AC conductivity:
   Sir N.F. Mott (1970)

$$\sigma\left(\omega\right)=\frac{e^2\zeta_{loc}^{d-2}}{\hbar}\left(\frac{\hbar\omega}{\delta_\zeta}\right)^2\ln^{d+1}\left|\frac{\delta_\zeta}{\hbar\omega}\right|$$

2. Fluctuation Dissipation Theorem:
   there should be Johnson-Nyquist noise

3. Use this noise as a bath instead of phonons

4. Self-consistency (whatever it means)

Q: Can e-h pairs lead to phonon-less variable range hopping in the same way as phonons do ?

A#1: **Sure**

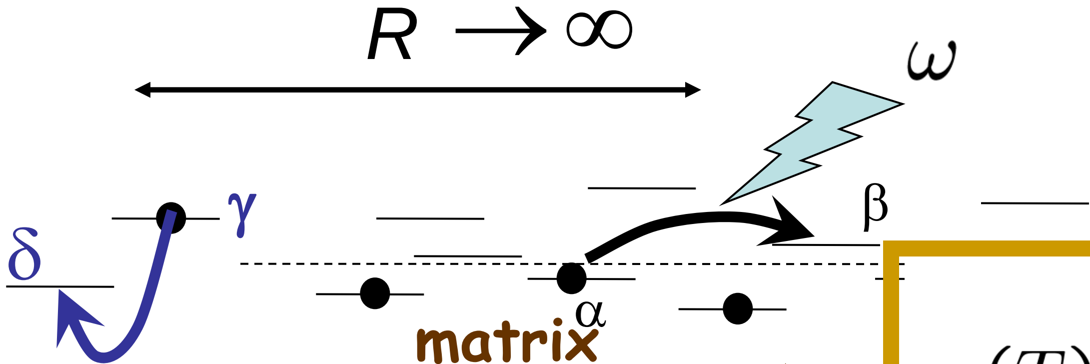
A#2: **No way** (L. Fleishman. P.W. Anderson (1980))

Except maybe Coulomb interaction in 3D

$$\sigma(\omega) \simeq \frac{e^2 \zeta_{loc}^{d-2}}{\hbar} \left(\frac{\hbar\omega}{\delta_\zeta}\right)^2 \ln^{d+1}\left|\frac{\delta_\zeta}{\hbar\omega}\right|$$

is contributed by rare resonances

$R \to \infty$



$$\omega = \xi_\beta - \xi_\alpha = \xi_\gamma - \xi_\delta$$

$R \to \infty \quad \Rightarrow$ matrix element vanishes

$$\sigma(T) \propto 0 \ \exp\left[-\left(\frac{\delta_\zeta}{T}\right)^{\frac{1}{d+1}}\right]$$

No phonons → ??? → No transport $\forall T$

Problem:

- If the localization length exceeds L_φ, then – metal.
- In a metal e-e interaction leads to a finite L_φ

⟹ At high enough temperatures conductivity should be **finite** even **without phonons**

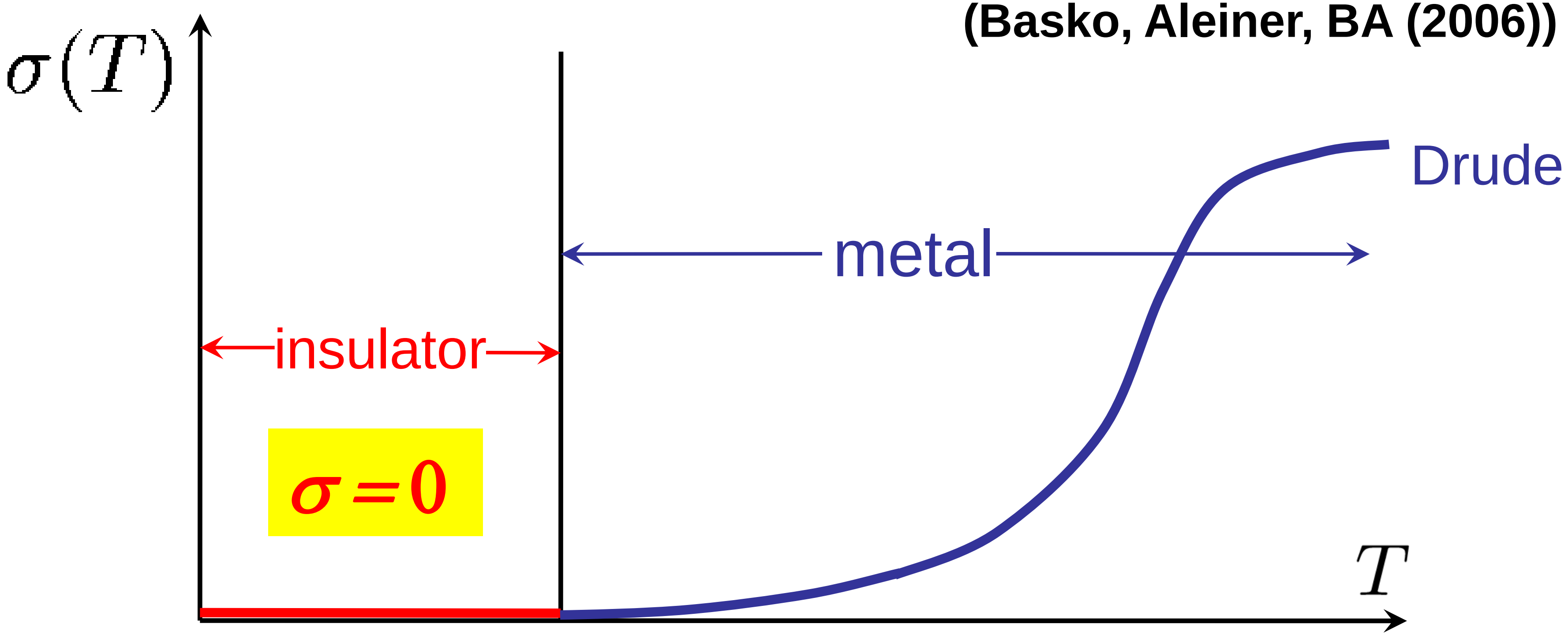
Q: Can e-h pairs lead to phonon-less variable range hopping in the same way as phonons do ?

A#1: Sure

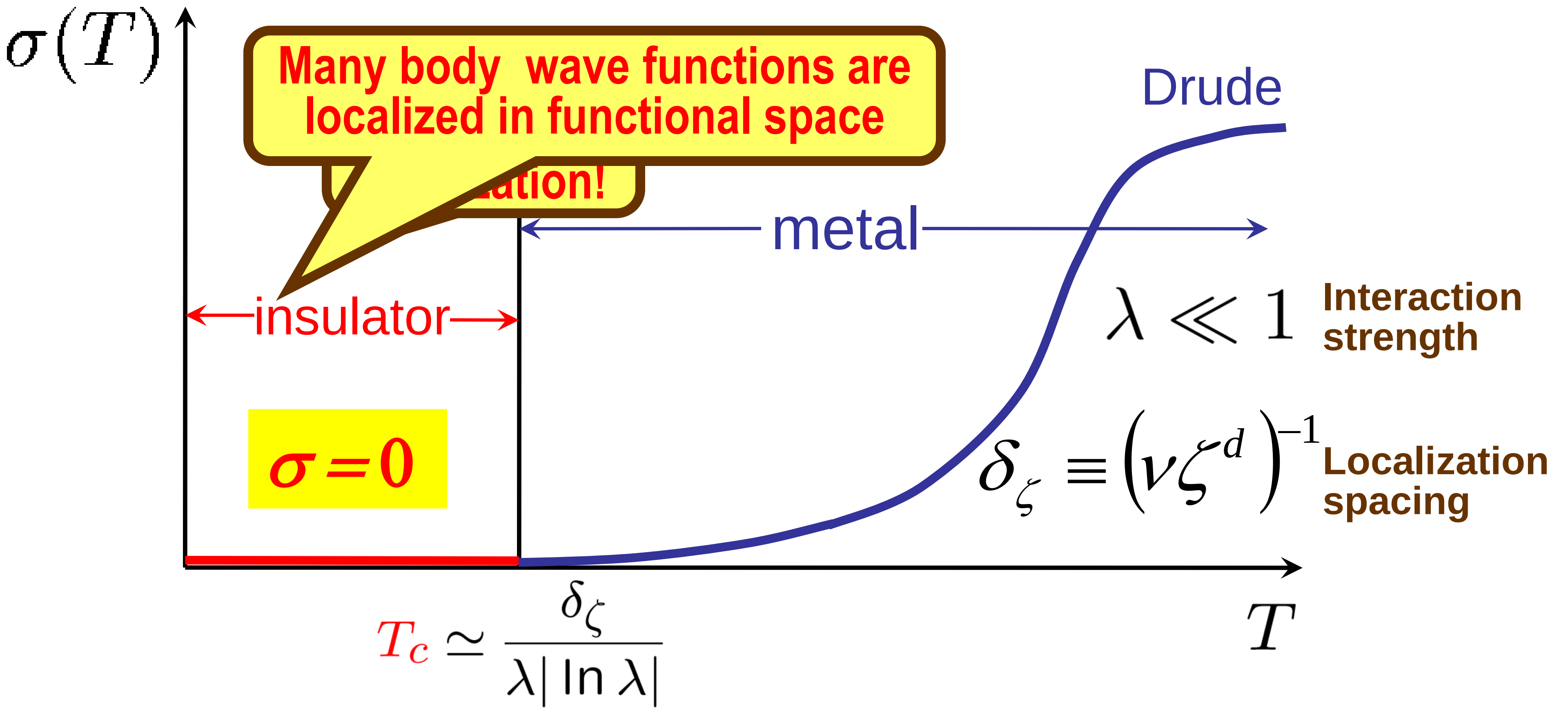
A#2: No way **(L. Fleishman. P.W. Anderson (1980))**

A#3: Finite temperature Metal-Insulator Transition

(Basko, Aleiner, BA (2006))



Finite temperature Metal-Insulator Transition



Definitions:

Insulator $\sigma = 0$ Metal $\sigma \neq 0$

not $d\sigma/dT < 0$ not $d\sigma/dT > 0$

Many-Body Localization

BA, Gefen, Kamenev & Levitov, 1997
Basko, Aleiner & BA, 2005. . .

Example: Random Ising model in the perpendicular field

Will not discuss today in detail

$$\hat{H} = \sum_{i=1}^{N} B_i \hat{\sigma}_i^z + \sum_{i \neq j} J_{ij} \hat{\sigma}_i^z \hat{\sigma}_j^z + I \sum_{i=1}^{N} \hat{\sigma}_i^x \equiv \hat{H}_0 + I \sum_{i=1}^{N} \hat{\sigma}_i^x$$

Random Ising model in a parallel field

Perpendicular field

$\vec{\sigma}_i$ - Pauli matrices, $\sigma_i^z = \pm \frac{1}{2}$

$i = 1, 2, \ldots, N; \quad N >> 1$

$$\hat{H} = \sum_{i=1}^{N} B_i \hat{\sigma}_i^z + \sum_{i \neq j} J_{ij} \hat{\sigma}_i^z \hat{\sigma}_j^z + I \sum_{i=1}^{N} \hat{\sigma}_i^x \equiv \hat{H}_0 + I \sum_{i=1}^{N} \hat{\sigma}_i^x$$

Random Ising model in a parallel field

Perpendicular field

$\vec{\sigma}_i$ - Pauli matrices

$i = 1, 2, \ldots, N; \quad N >> 1$

Anderson Model on *N*-dimensional cube

$\left\{ \sigma_i^z \right\}$ determines a site

$H_0\left(\left\{\sigma_i\right\}\right)$

onsite energy

$\hat{\sigma}^x = \hat{\sigma}^+ + \hat{\sigma}^-$

hoping between nearest neighbors

$$\hat{H} = \sum_{i=1}^{N} B_i \hat{\sigma}_i^z + \sum_{i \neq j} J_{ij} \hat{\sigma}_i^z \hat{\sigma}_j^z + I \sum_{i=1}^{N} \hat{\sigma}_i^x \equiv \hat{H}_0 + I \sum_{i=1}^{N} \hat{\sigma}_i^x$$

Anderson Model on *N*-dimensional cube

Usually:

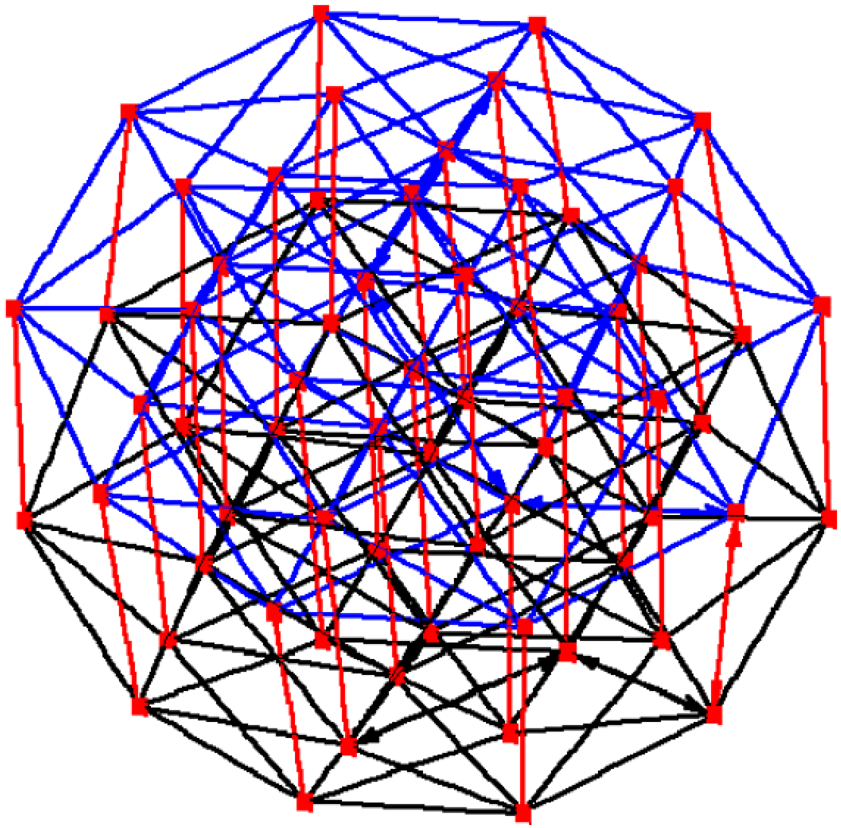
\# of dimensions $d \to const$

system linear size $L \to \infty$

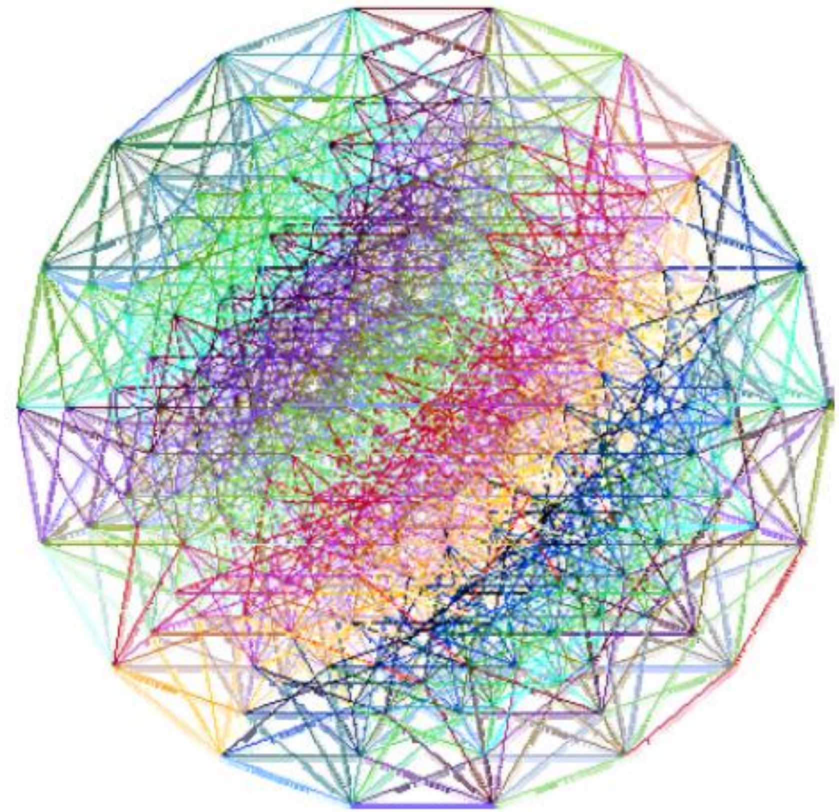
Here:

\# of dimensions $d = N \to \infty$

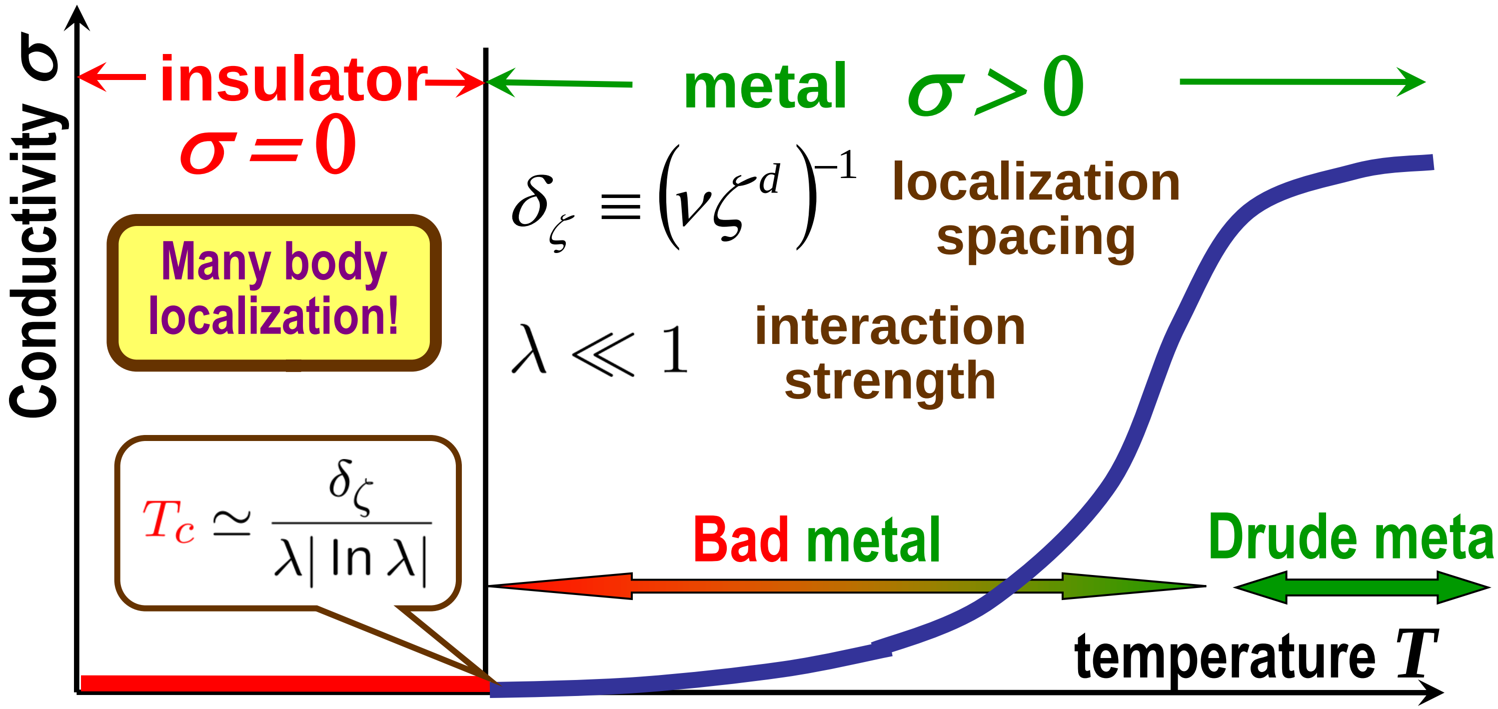
system linear size $L = 1$



6-dimensional cube



9-dimensional cube



Definitions:

Insulator $\sigma = 0$ **Metal** $\sigma \neq 0$

not $d\sigma/dT < 0$ **not** $d\sigma/dT > 0$

Many-Body Localization

1D bosons + disorder

1D Localization

Exactly solved:
all states are localized

Gertsenshtein & Vasil'ev, 1959

Conjectured:

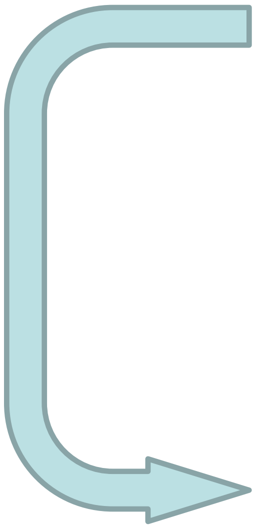
Mott & Twose, 1961

-
-
-

1-particle problem → correct for bosons as well as for fermions

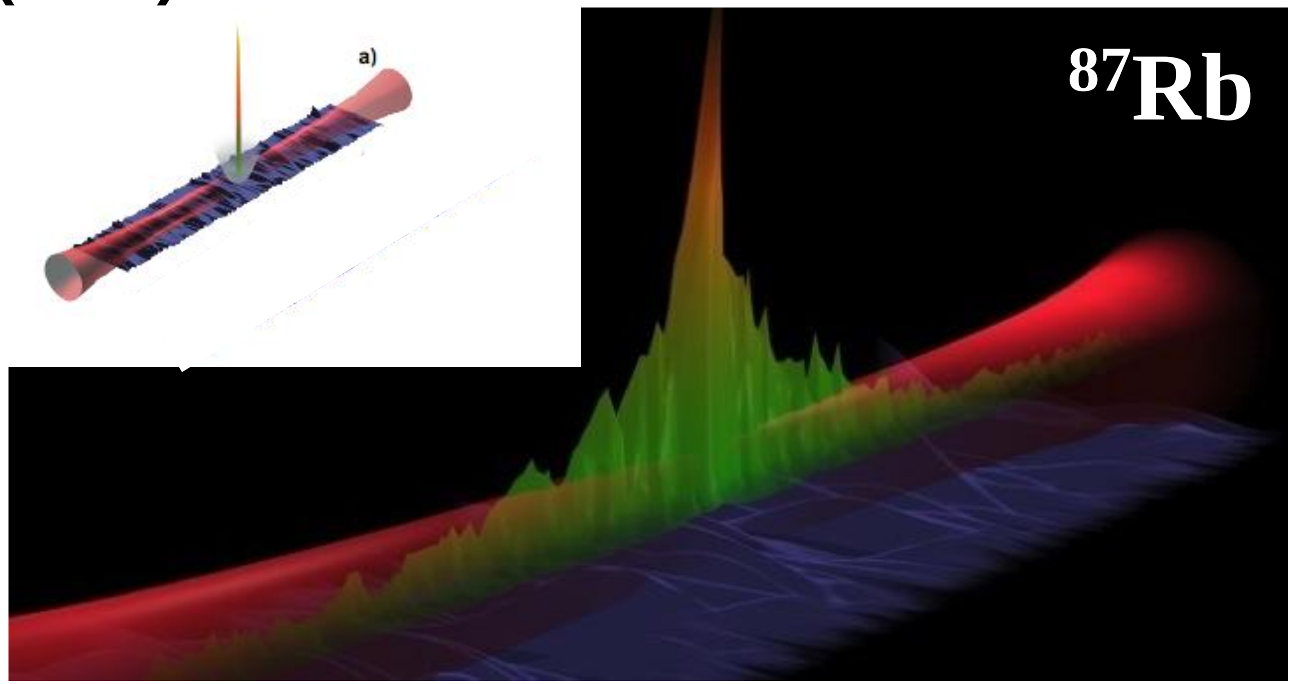
Bosons without disorder

- Bose - Einstein condensation
- Bose–condensate even at weak enough repulsion
- Even in 1*d* case at $T=0$ – "algebraic superfluid"
- Finite temperature – Normal fluid



Localization of cold atoms

Billy et al. "Direct observation of Anderson localization of matter waves in a controlled disorder". Nature <u>453</u>, 891- 894 (2008).



Roati et al. "Anderson localization of a non-interacting Bose-Einstein condensate". Nature <u>453</u>, 895-898 (2008).

No interaction !

Thermodynamics of ideal Bose-gas in the presence of disorder is a pathological problem: all particles will occupy the localized state with the lowest energy

→

Need repulsion

Weakly interacting bosons

- Bose - Einstein condensation
- Bose-condensate even at weak enough repulsion
- Even in 1D case at T=0 – "algebraic superfluid"

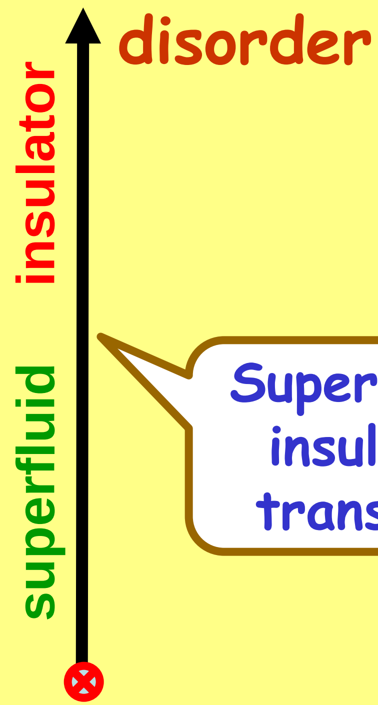
1. No interaction

For any energy at finite disorder 1D localization

2. No disorder

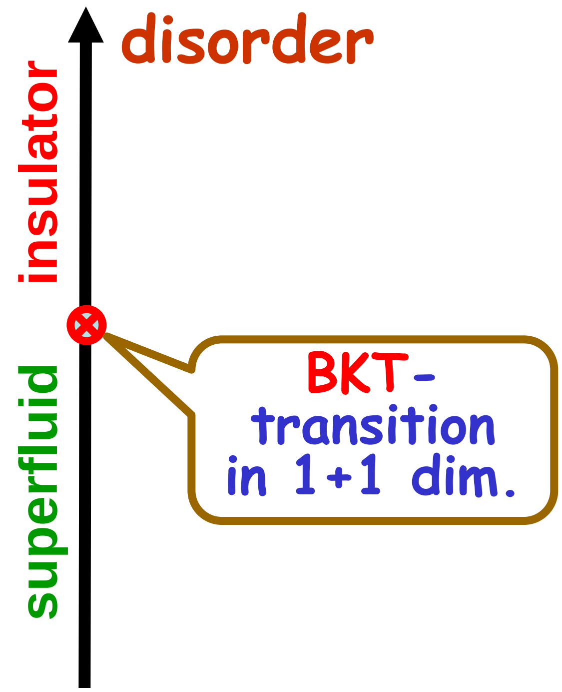
Superfluid-insulator transition

3. Weak repulsion



Superfluid-insulator transition

$T=0$ Superfluid - Insulator Quantum Phase Transition



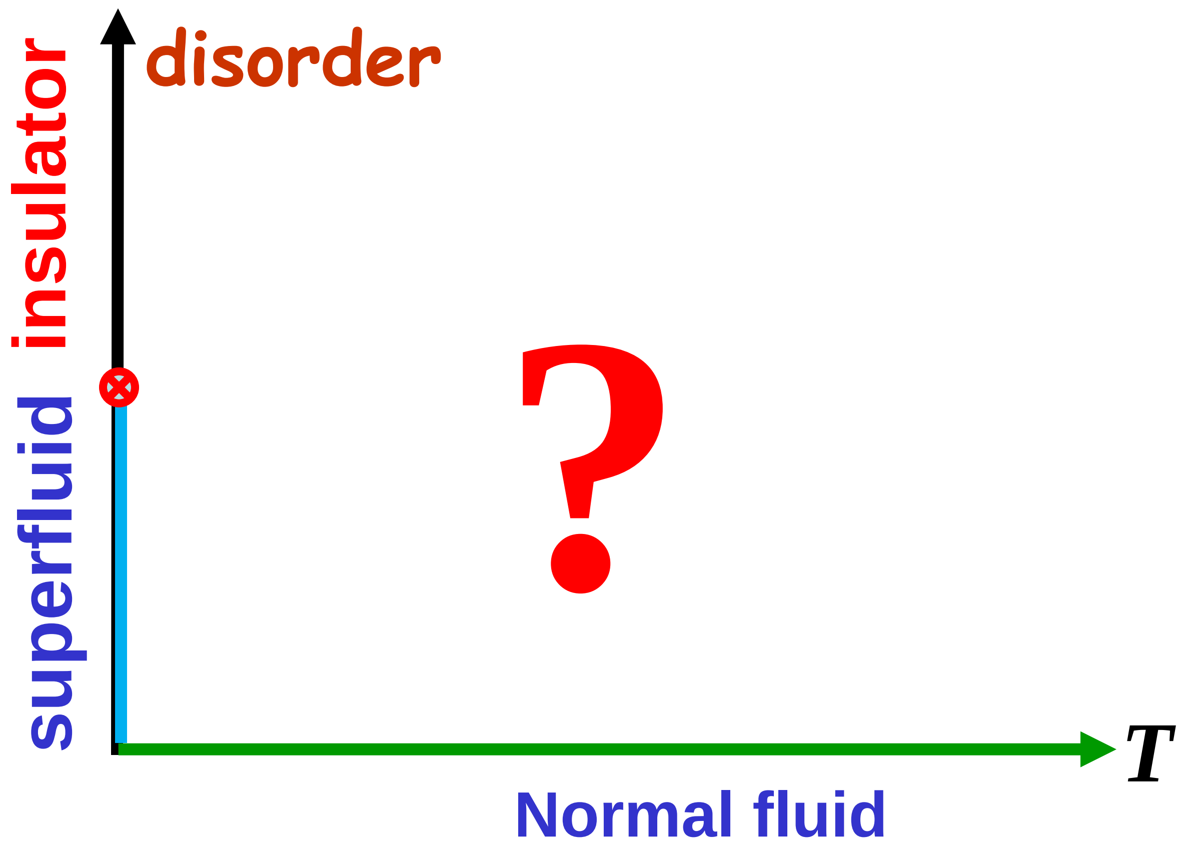
T. Giamarchi and H. J. Schulz, *Phys. Rev.*, **B37**, #1(1988).

relatively strong interaction

E. Altman, Y. Kafri, A. Polkovnikov & G. Refael, *Phys. Rev. Lett.*, **100**, 170402 (2008).

G.M. Falco, T. Nattermann, & V.L. Pokrovsky, *Phys. Rev.*, **B80**, 104515 (2009).

weak interaction



Is it a normal fluid at any temperature?

Dogma There can be no phase transitions at a finite temperature in 1D

Van Howe, Landau

Reason Thermal fluctuation destroy any long range correlations in 1D

$T \neq 0$ Normal fluid – Insulator Phase Transition:

Neither normal fluids nor glasses (insulators) exhibit long range correlations

still

True phase transition: singularities in transport (rather than thermodynamic) properties

What is insulator?

Perfect Insulator

Zero DC conductivity at finite temperatures

Possible if the system is decoupled from any outside bath

Normal metal (fluid)

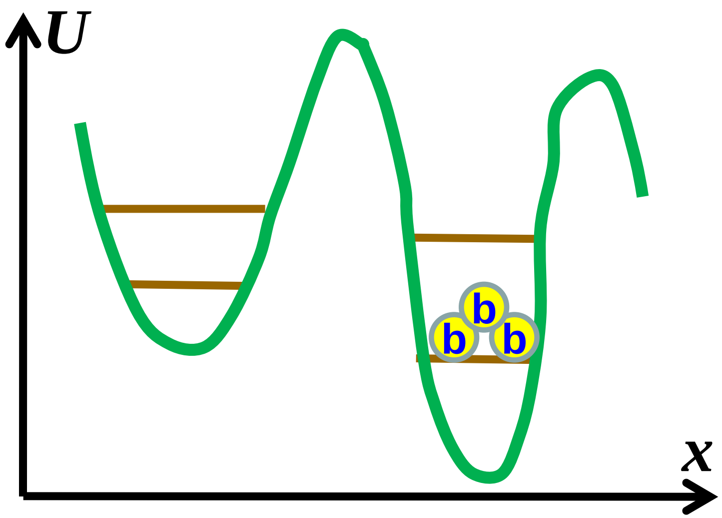
Finite (even if very small) DC conductivity at finite temperatures

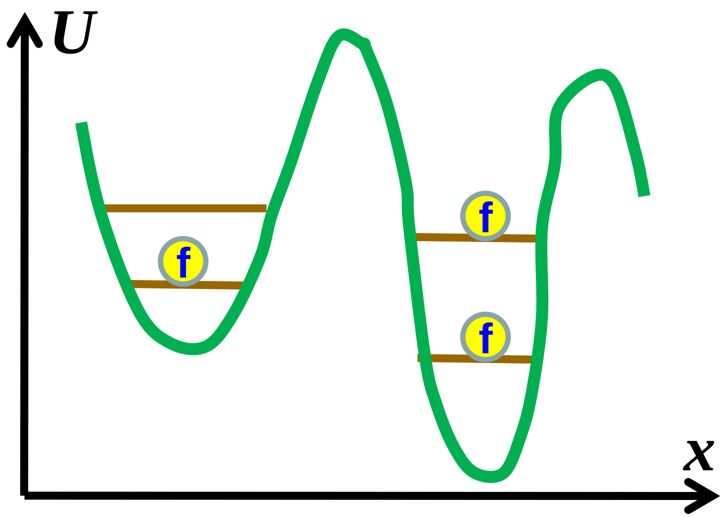
1D Luttinger liquid: bosons = fermions ?

Bosons with infinitely strong repulsion ≈ Free fermions

Free bosons ≈ Fermions with infinitely strong attraction

Weakly interacting bosons ≈ Fermions with strong attraction



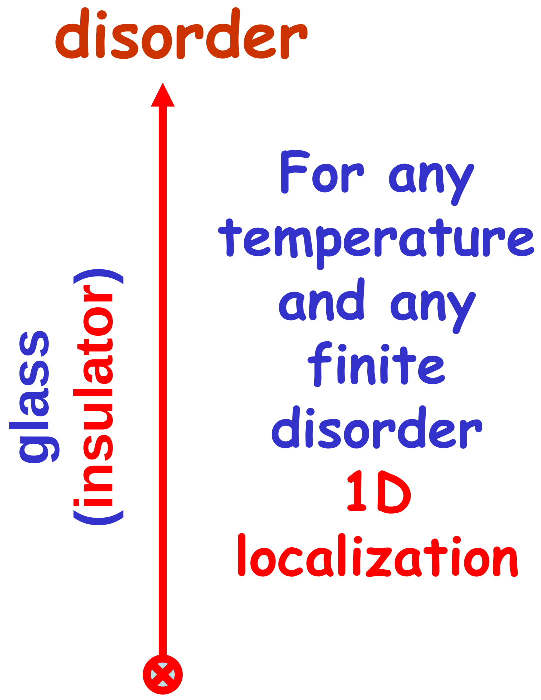


As soon as the occupation numbers become large the analogy with fermions is not too useful

1D Weakly Interacting Bosons + Disorder

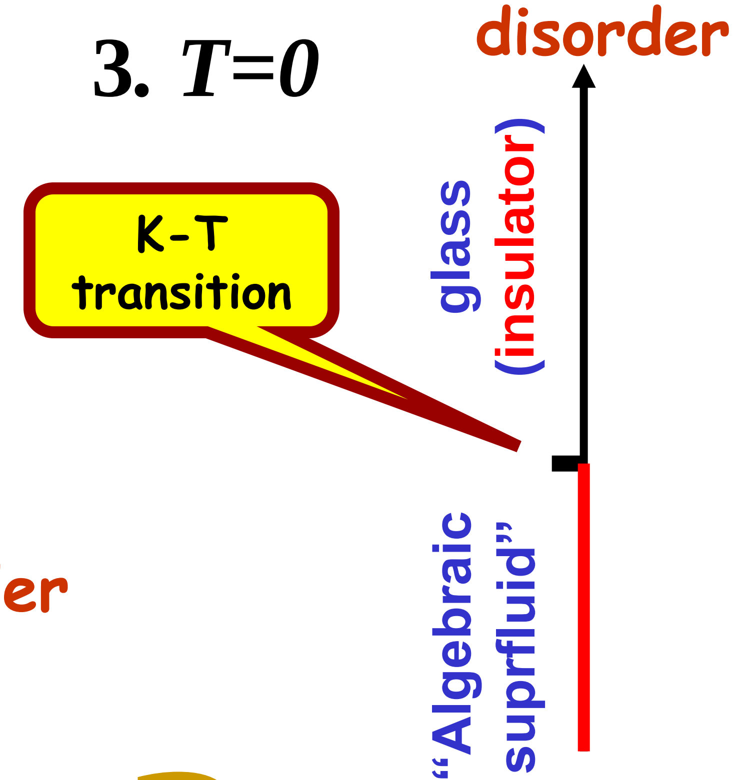
Aleiner, BA & Shlyapnikov, 2010, Nature Physics, to be published
cond-mat 0910.4534

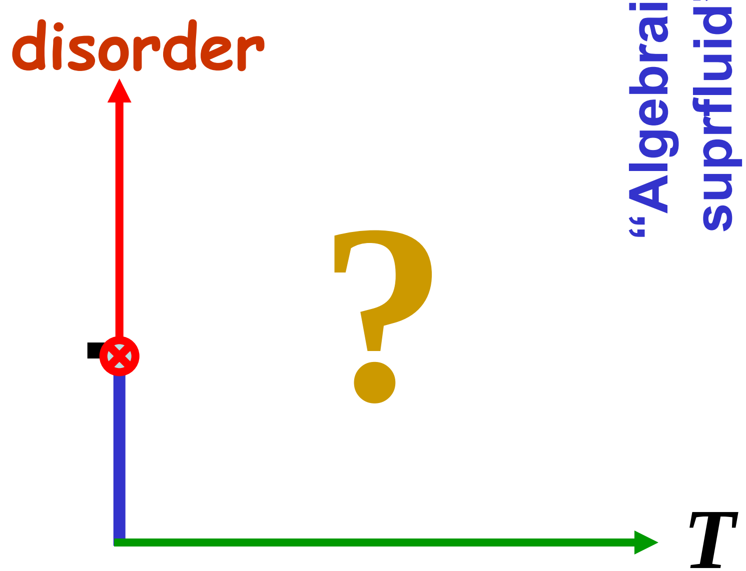
1. No interaction



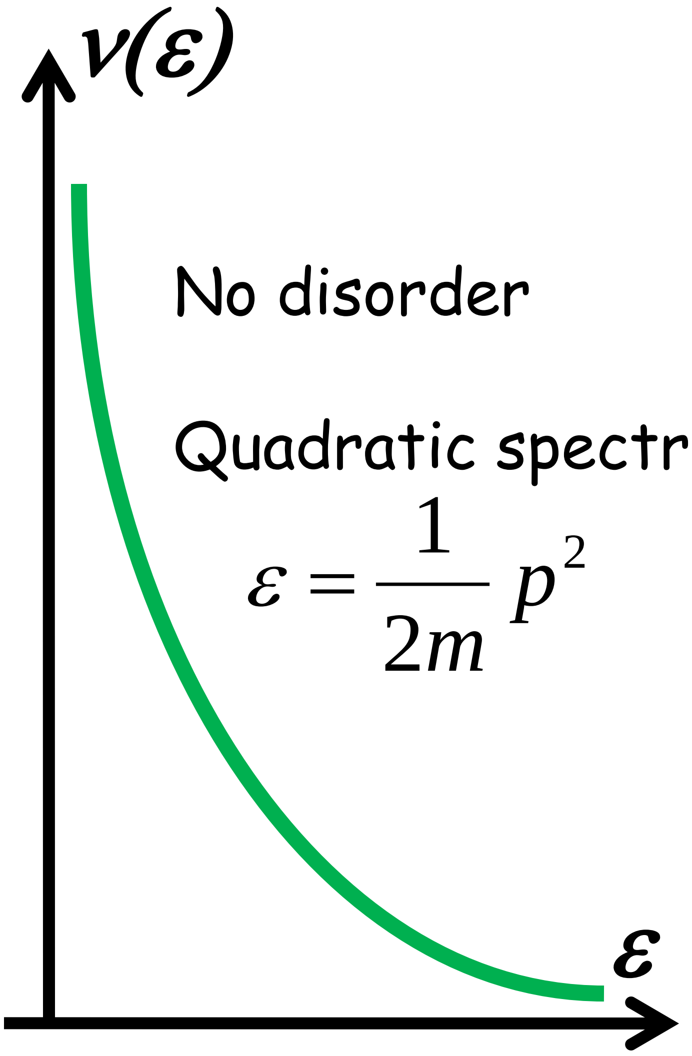
2. No disorder

3. *T=0*





Density of States $\nu(\varepsilon)$ in one dimension



No disorder

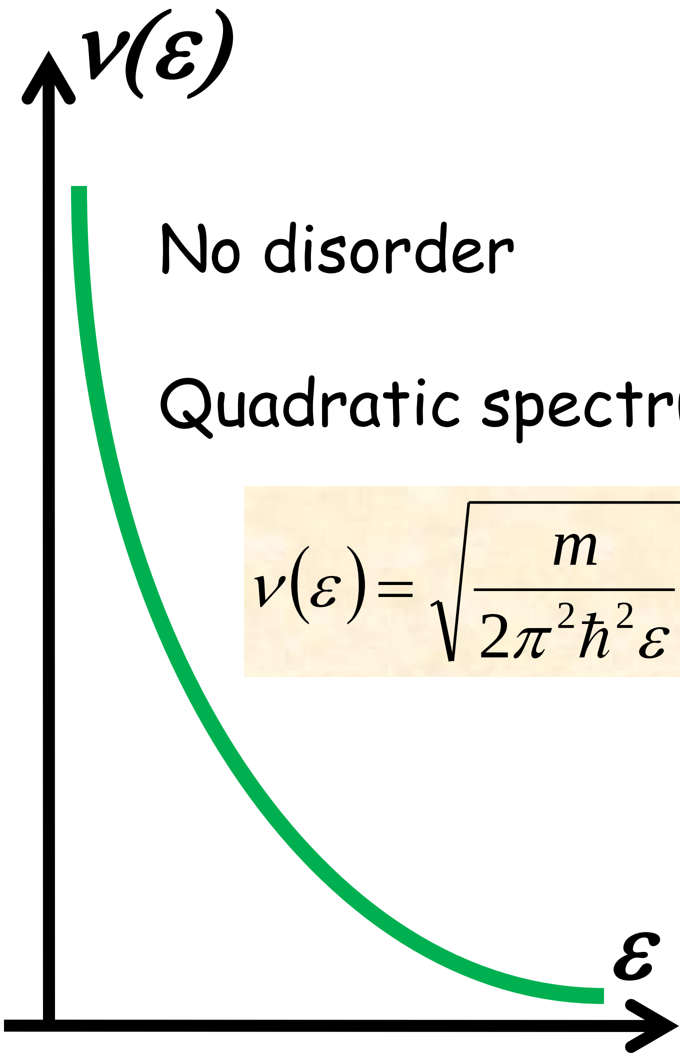
Quadratic spectrum

$$\varepsilon = \frac{1}{2m} p^2$$

$$\nu(\varepsilon) = \sqrt{\frac{m}{2\pi^2 \hbar^2 \varepsilon}}$$

$\sqrt{\ }$ - singularity

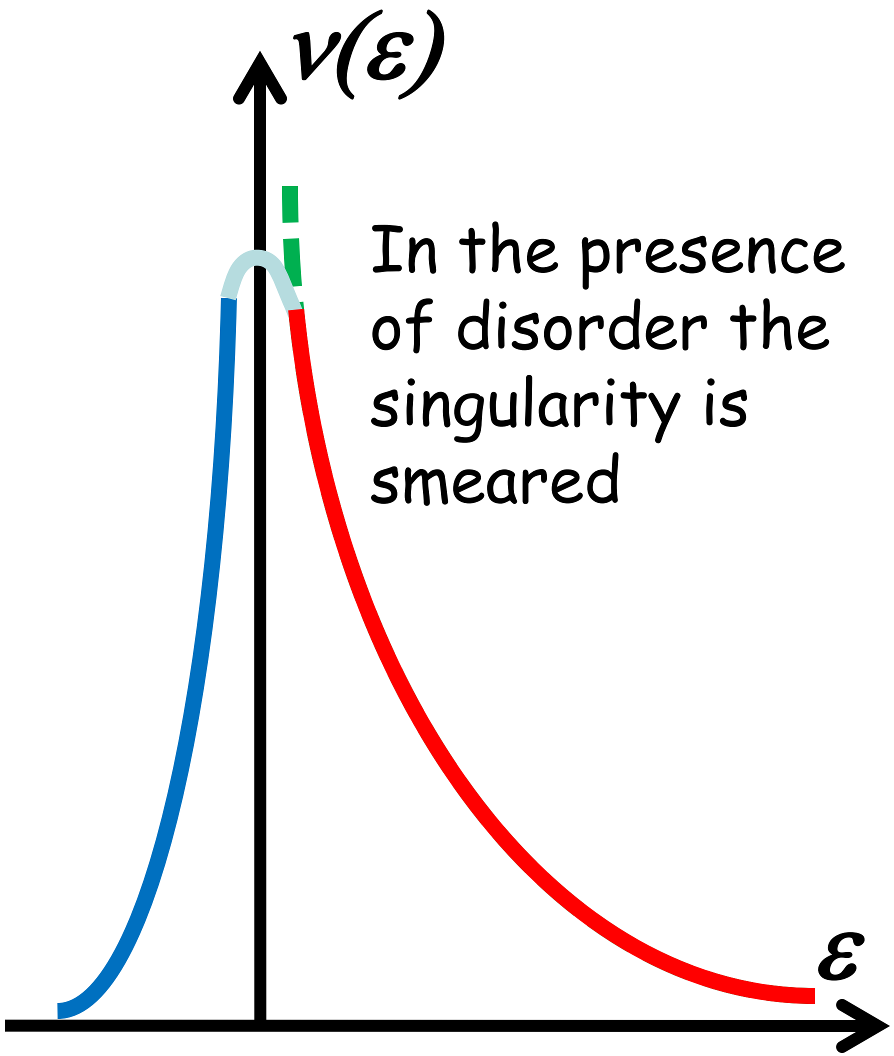
Density of States $\nu(\varepsilon)$ in one dimension



No disorder

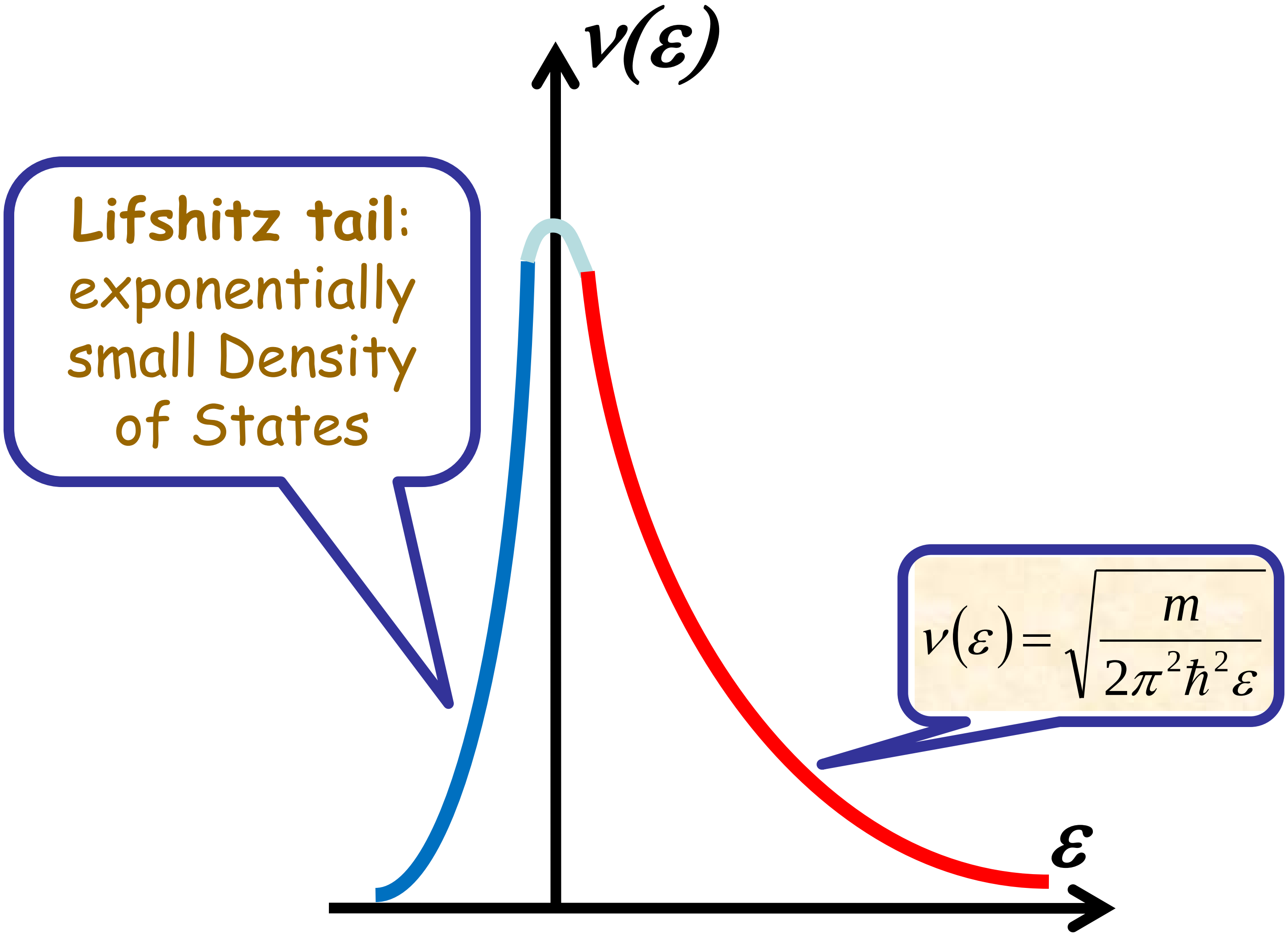
Quadratic spectrum

$$\nu(\varepsilon) = \sqrt{\frac{m}{2\pi^2\hbar^2\varepsilon}}$$

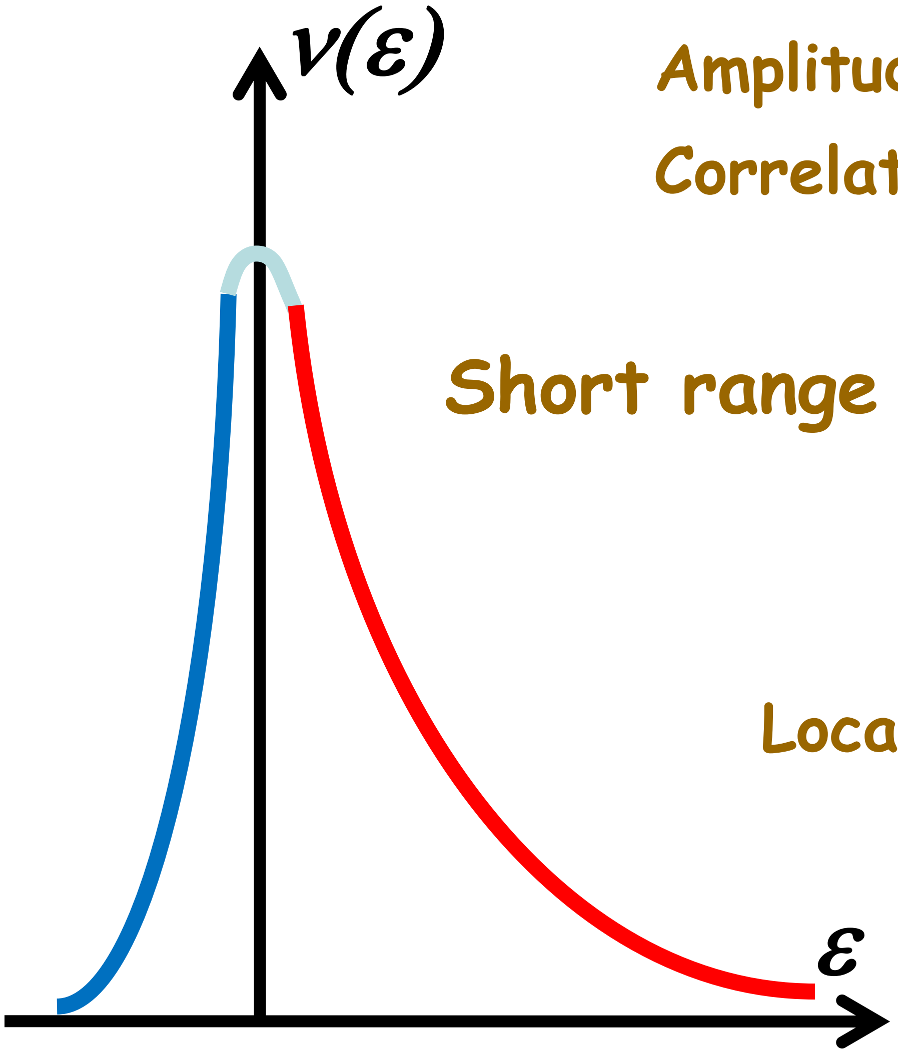


In the presence of disorder the singularity is smeared

Density of States $\nu(\varepsilon)$ in one dimension



Weak disorder – random potential $U(x)$



Random potential $U(x)$:

Amplitude U_0

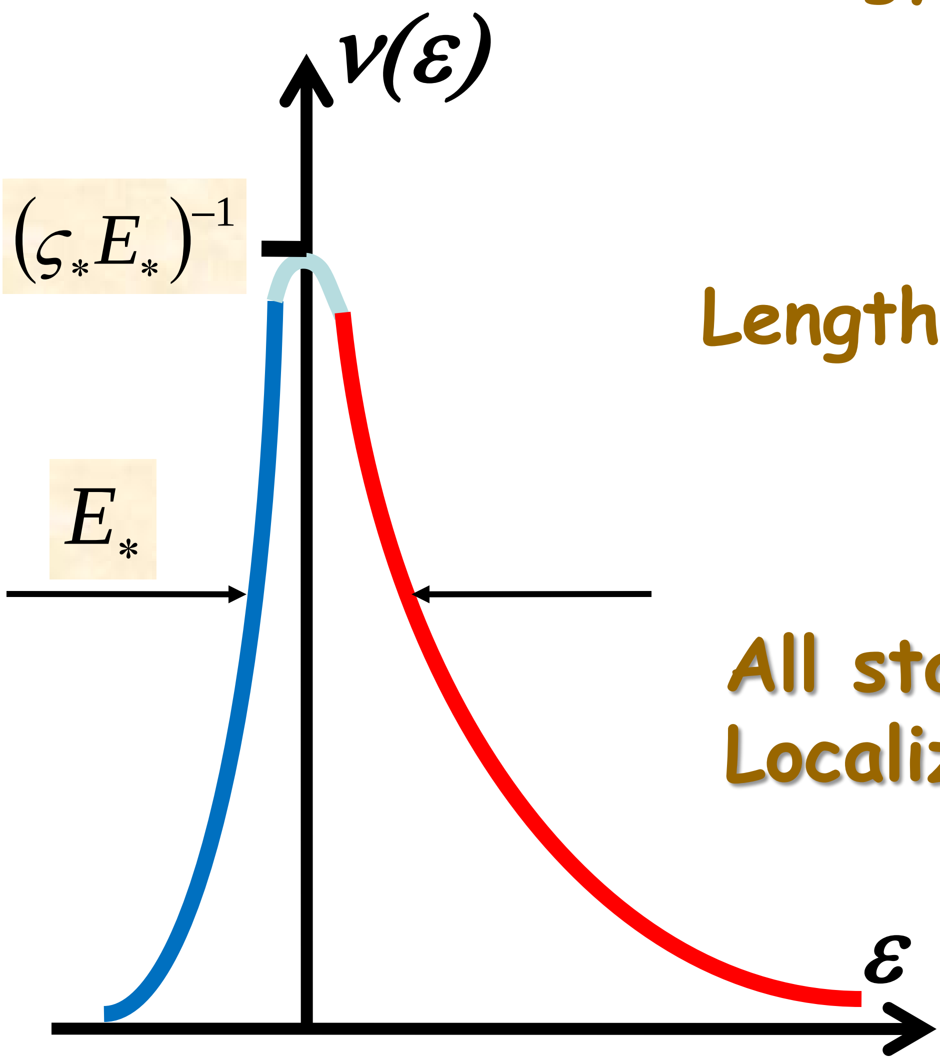
Correlation length σ

Short range disorder: $U_0 << \frac{\hbar^2}{m\sigma^2}$

⇓

Localization length $\zeta >> \sigma$

Characteristic scales:



Energy

$$E_* \equiv \left(\frac{U_0^4 \sigma^2 m}{\hbar^2} \right)^{1/3}$$

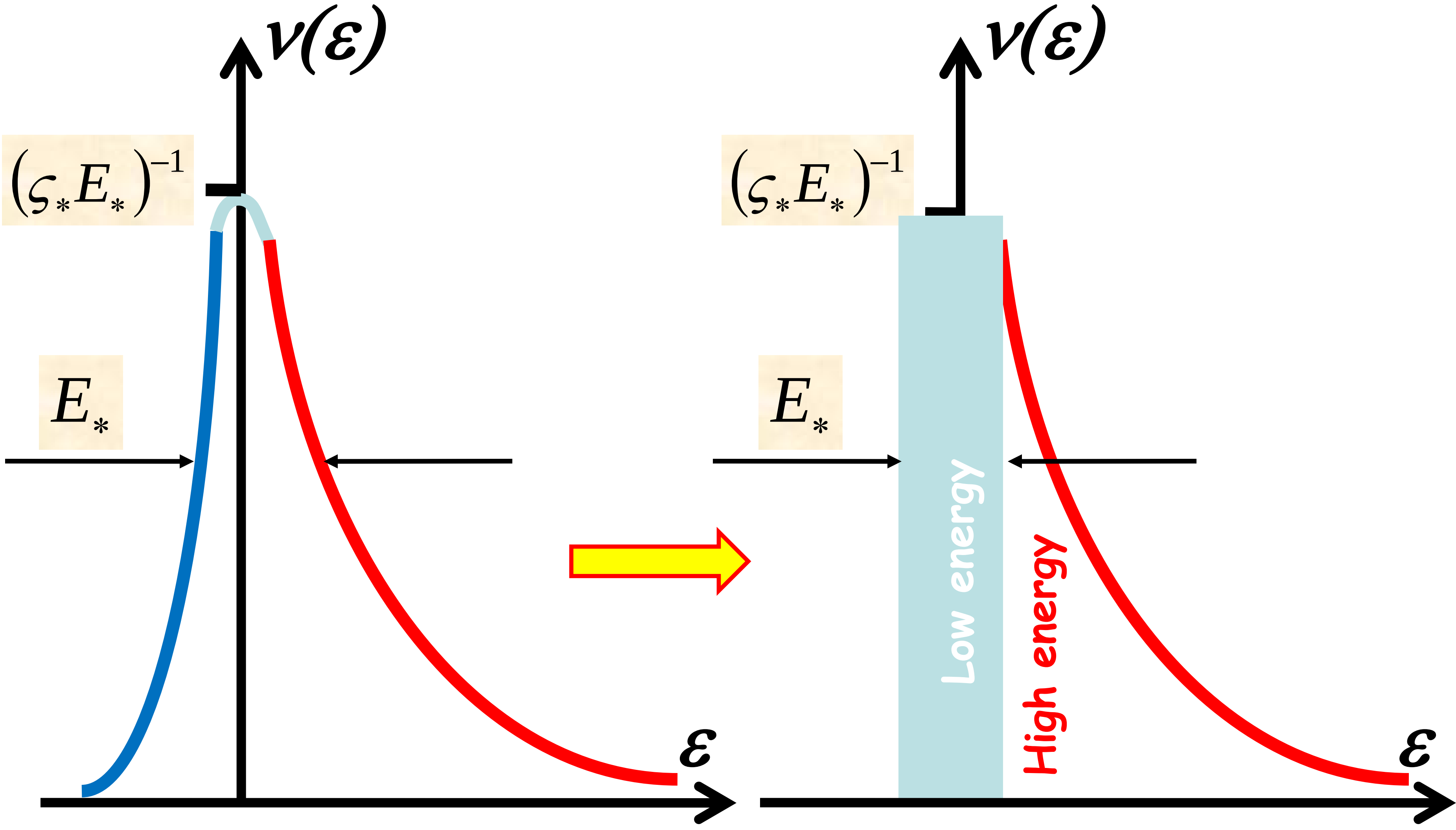
Length

$$\varsigma_* \equiv \left(\frac{\hbar^4}{U_0^2 \sigma m} \right)^{1/3} >> \sigma$$

All states are localized

Localization length:

$$\varsigma(\varepsilon) \sim \begin{cases} \varsigma_* & \varepsilon \sim E_* \\ \varsigma_* \dfrac{\varepsilon}{E_*} & \varepsilon >> E_* \end{cases}$$


$\nu(\varepsilon)$
$(\varsigma_* E_*)^{-1}$
E_*
ε
$\nu(\varepsilon)$
$(\varsigma_* E_*)^{-1}$
E_*
Low energy
High energy
ε

Finite density Bose-gas with repulsion

Density n

Two more energy scales

Temperature of quantum degeneracy $T_d \equiv \frac{\hbar^2 n^2}{m}$

Interaction energy per particle ng

Two dimensionless parameters

$\kappa \equiv E_* / ng$ Characterizes the strength of **disorder**

$\gamma \equiv ng / T_d$ Characterizes the **interaction** strength

Strong disorder $\kappa >> 1$

Weak interaction $\gamma << 1$

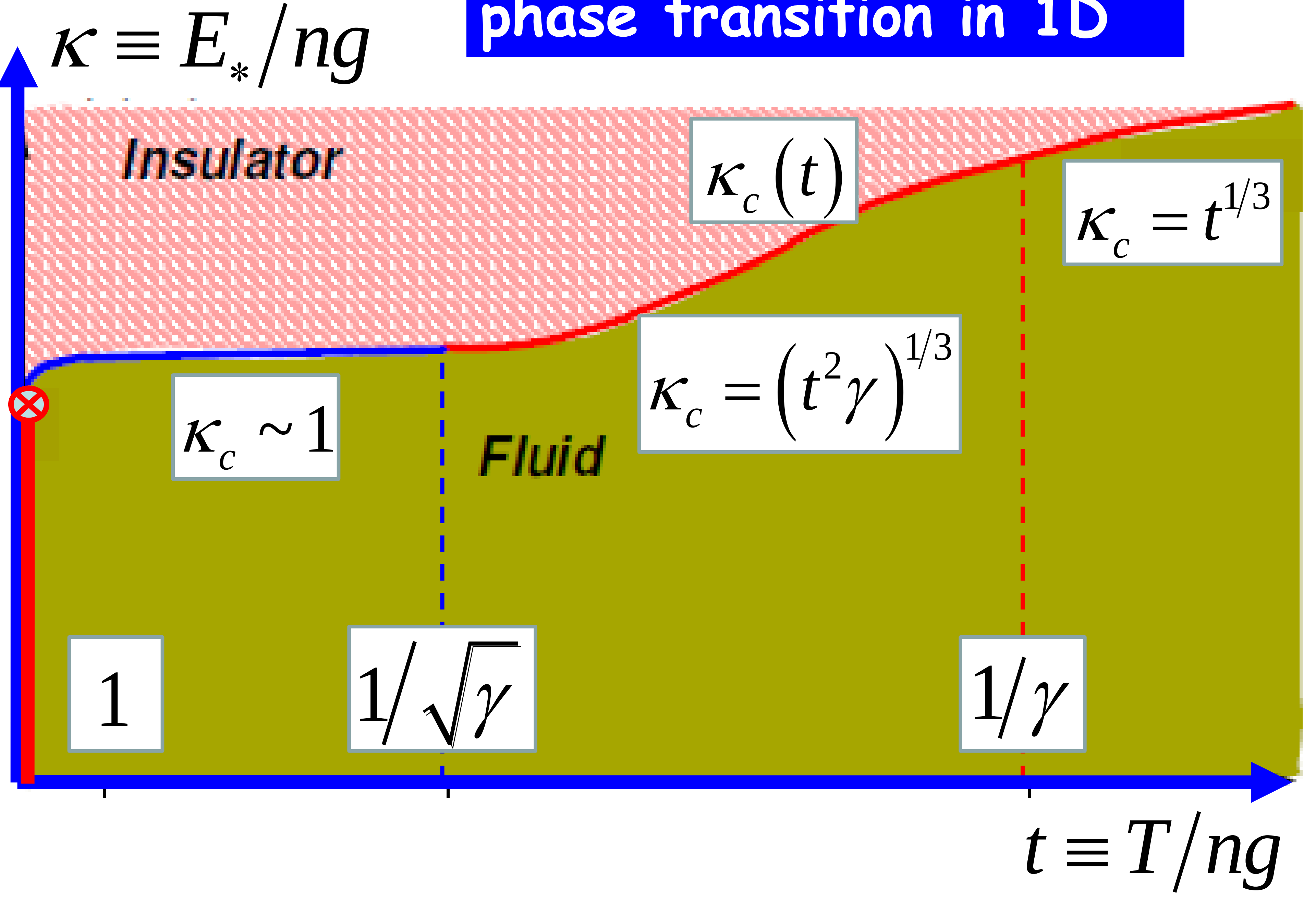
Dimensionless temperature $t = T/ng$

Critical temperature T_c $t_c = t_c(\kappa,\gamma)$

Critical disorder $\kappa_c = \kappa_c(t,\gamma)$

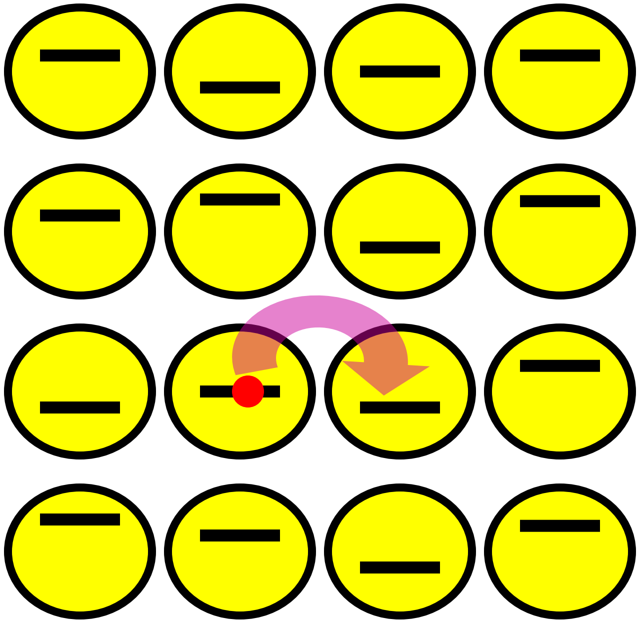
Phase transition line on the t,κ - plane

Finite temperature phase transition in 1D



Conventional Anderson Model

- one particle,
- one level per site,
- onsite disorder
- nearest neighbor hoping



Basis: $\left| i \right\rangle, \quad i$ labels sites

Hamiltonian: $\hat{H} = \hat{H}_0 + \hat{V}$

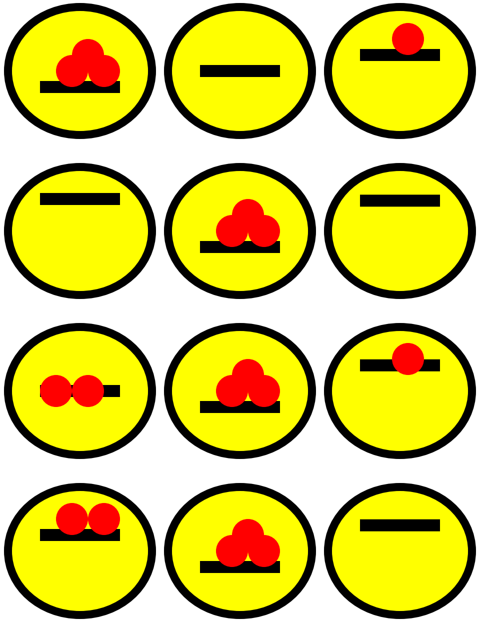
$$\hat{H}_0 = \sum_i \varepsilon_i \left| i \right\rangle \left\langle i \right| \qquad \hat{V} = \sum_{i,j=n.n.} I \left| i \right\rangle \left\langle j \right|$$

Transition: happens when the hoping matrix element exceeds the energy mismatch

The same for many-body localization

Many body Anderson-like Model

- many particles,
- several particles per site.
- interaction



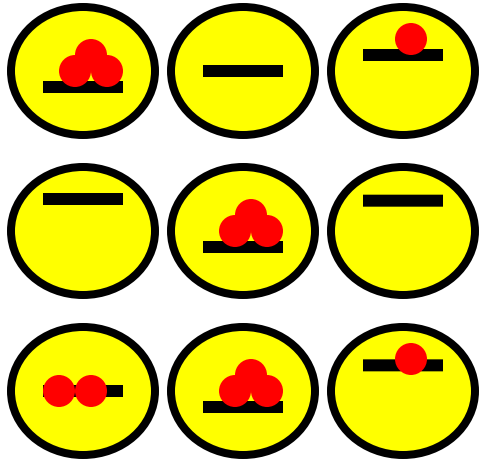
Basis: $|\mu\rangle \equiv \left|\{n_i\}\right\rangle$

i label sites

$n_i^\alpha = 0, 1, 2, 3, \ldots$
occupation numbers

Many body Anderson-like Model

- many particles,
- several particles per site.
- interaction



Basis: $\left|\mu\right\rangle$

$$\mu = \left\{ n_i \right\}$$

i labels sites

$n_i = 0, 1, 2, \ldots$ occupation numbers

Hamiltonian:

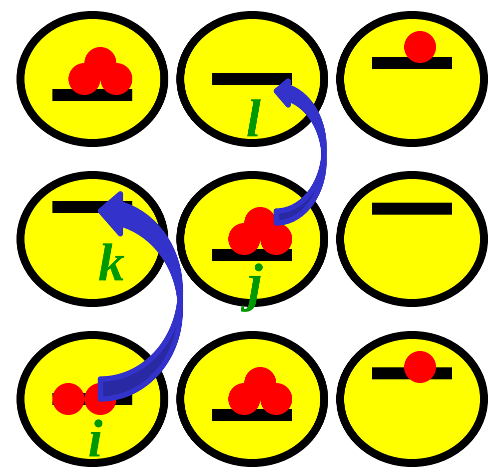
$$\hat{H} = \hat{H}_0 + \hat{V}$$

$$\hat{H}_0 = \sum_{\mu} E_{\mu} \left|\mu\right\rangle\left\langle\mu\right|$$

$$\hat{V} = \sum_{\mu,\eta(\mu)} I \left|\mu\right\rangle\left\langle\eta(\mu)\right|$$

$$\left|\nu(\mu)\right\rangle = \left|.., n_i - 1, .., n_j - 1, .., n_k + 1, .., n_l^{\delta} + 1, ..\right\rangle$$

$$i, j, k, l = n.n.$$



Conventional Anderson Model

Basis: $\left| i \right\rangle$

i labels sites

$$\hat{H} = \sum_i \varepsilon_i \left| i \right\rangle \left\langle i \right| + \sum_{i,j=n.n.} I \left| i \right\rangle \left\langle j \right|$$

Many body Anderson-like Model

Basis: $\left| \mu \right\rangle, \quad \mu = \left\{ n_i^\alpha \right\}$

i labels sites

$n_i = 0, 1, 2, \ldots$ occupation numbers

$$\hat{H} = \sum_\mu E_\mu \left| \mu \right\rangle \left\langle \mu \right| + \sum_{\mu, \nu(\mu)} I \left| \mu \right\rangle \left\langle \nu(\mu) \right|$$

"nearest neighbors":

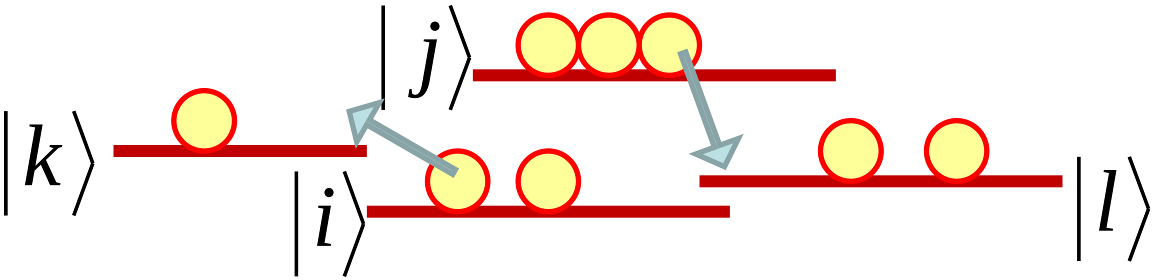
$$\left| \nu(\mu) \right\rangle = \left| .., n_i - 1, .., n_j - 1, .., n_k + 1, .., n_l^\delta + 1, .. \right\rangle$$

$$i, j, k, l = n.n.$$

Transition temperature: $T_c \equiv t_c(ng)$

$|i\rangle, |j\rangle \Rightarrow |k\rangle, |l\rangle$

transition



Transition temperature: $T_c \equiv t_c(ng)$

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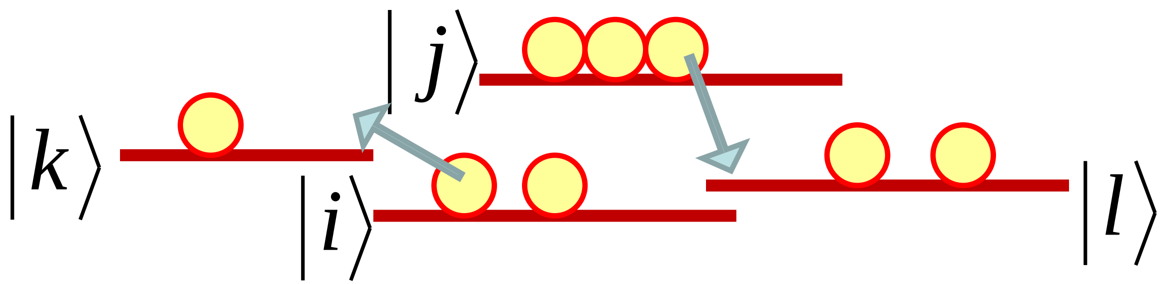
transition



$\Delta_{ij,kl} \equiv \varepsilon_i + \varepsilon_j - \varepsilon_k - \varepsilon_l$ energy mismatch

$I_{ij,kl}$ matrix element

Decay of a state $|i\rangle$

Δ typical mismatch

N_1 typical # of channels

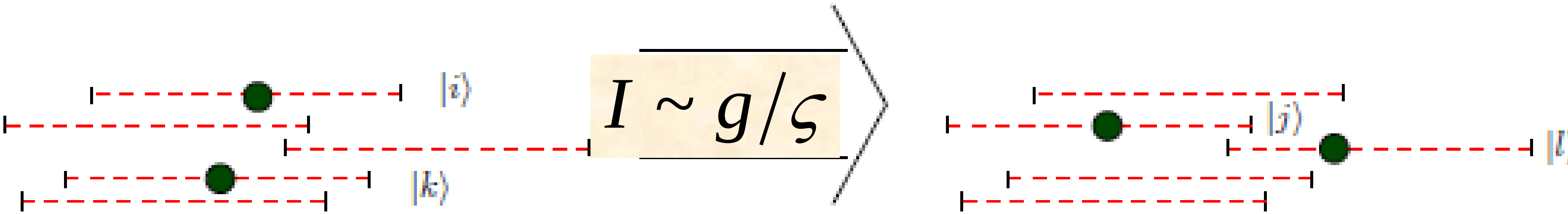
I typical matrix element

Anderson condition:

$$I(T) \begin{matrix} \gg \Delta(T)/N_1(T) \\ \ll \Delta(T)/N_1(T) \end{matrix} \quad \begin{matrix} \text{extended} \\ \text{localized} \end{matrix}$$

High temperatures: $T >> T_d \Leftrightarrow t >> \gamma^{-1}$

Bose-gas is not degenerated;
occupation numbers either $\mathbf{0}$ or $\mathbf{1}$



Matrix element of the transition

$$I \sim g/\varsigma(\varepsilon = T) \sim (gE_*)/(\varsigma_* T)$$

should be compared with the minimal energy mismatch

$$(v\varsigma)^{-1}/(n\varsigma) \sim \left(vn\varsigma_*{}^2T^2\right)^{-1} E_*^2$$

Localization spacing δ_ς

Number of channels

$$\kappa_c(t) \propto t^{1/3} \qquad t\gamma >> 1$$

Intermediate temperatures: $\gamma^{-1/2} << t << \gamma^{-1}$

1. $T << T_d$ ⟺ $t\gamma << 1$

2. Bose-gas is degenerated; occupation numbers either $>>1$.

3. Typical energies $|\mu| = T^2/T_d$, μ is the chemical potential. Correct as long as

$|\mu| >> ng, E_*$ ⟺ $t\sqrt{\gamma} >> 1$

multiple occupation $N(\varepsilon) \sim \frac{T}{\varepsilon}$

4. Characteristic energies $\varepsilon \sim |\mu|$ $<< T$, $>> ng, E_*$

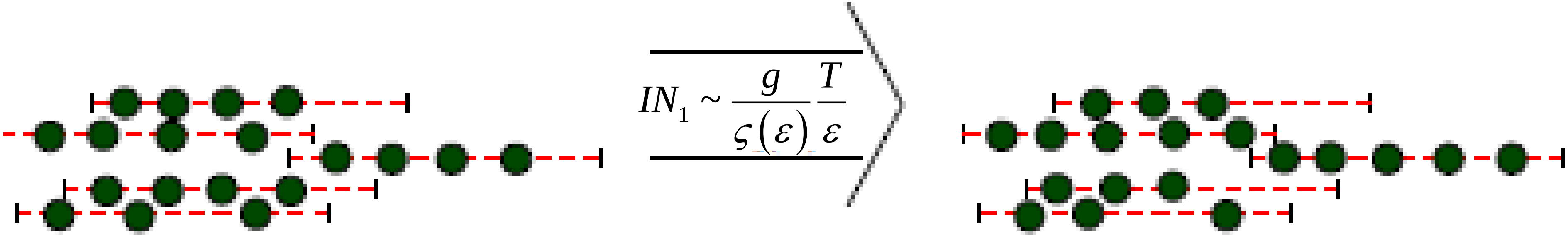
We are still dealing with the high energy states

Intermediate temperatures: $\gamma^{-1/2} << t << \gamma^{-1}$

$$|\mu| = T^2/T_d >> ng, E_*$$

$$T << T_d$$

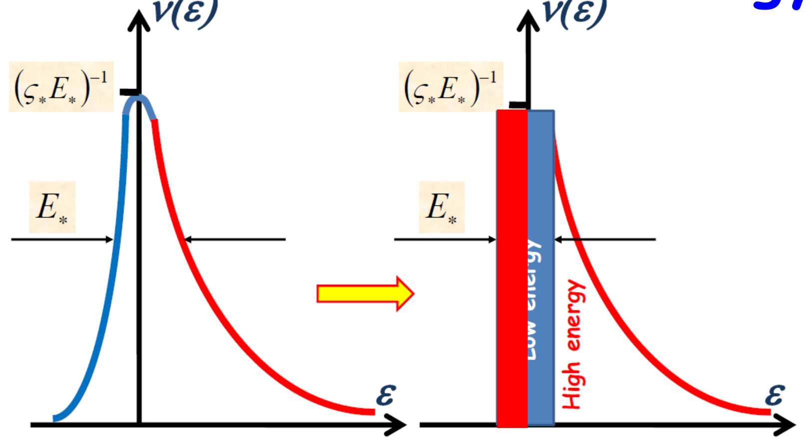
Bose-gas is degenerated; typical energies ~ $|\mu| >> T \Longrightarrow$ occupation numbers $>>1 \Longrightarrow$ matrix elements are enhanced



$$\kappa_c(t) \propto t^{2/3}\gamma^{1/3} \qquad \sqrt{\gamma} << t\gamma << 1$$

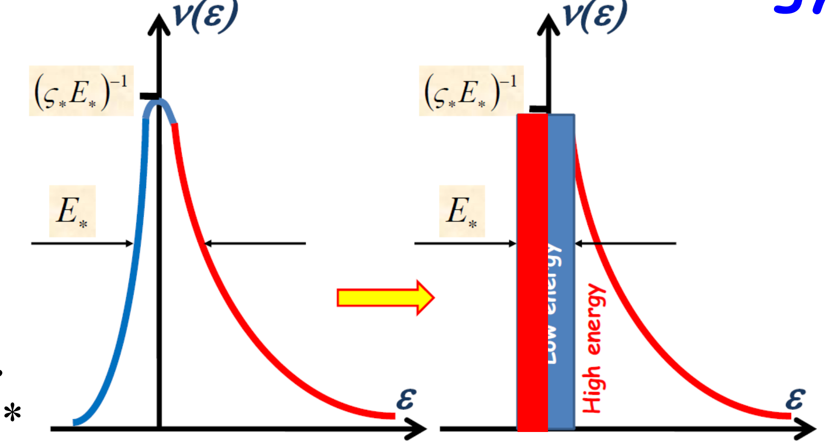
Low temperatures: $t << \gamma^{-1/2}$ Start with $T=0$

Suppose $\kappa \equiv E_*/ng >> 1 \Longrightarrow |\mu| << E_* \Longrightarrow$ **Bosons occupy only small fraction of low energy states** $\varepsilon_i < \mu$



Low temperatures: $t << \gamma^{-1/2}$ Start with $T=0$

Suppose $\kappa \equiv E_* / ng >> 1 \Rightarrow |\mu| << E_* \Rightarrow$ Bosons occupy only small fraction of low energy states $\varepsilon_i < \mu$

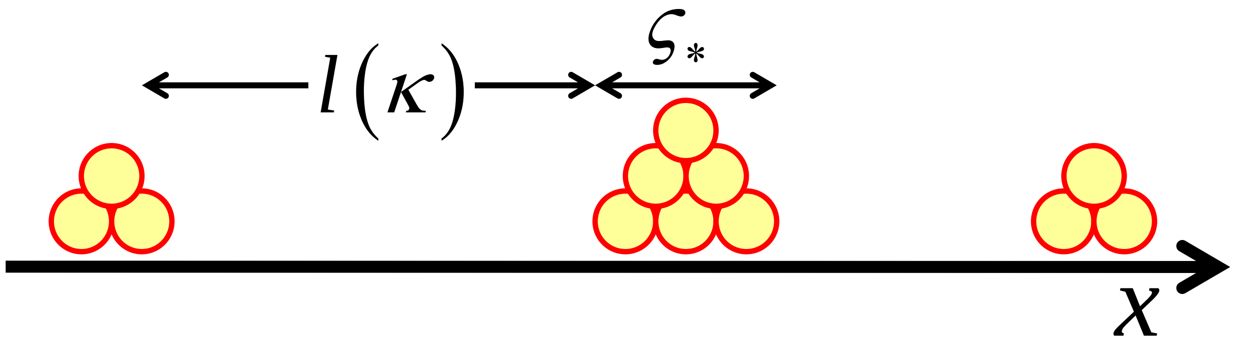


Localization length ς_*

Occupation #: $(\mu - \varepsilon_i)\varsigma_* / g$

DoS: $\nu(\varepsilon) = (E_* \varsigma_*)^{-1}$

$$\Rightarrow n = \frac{\mu^2}{2gE_*} \Rightarrow \mu = E_* \sqrt{\kappa}$$



$l(\kappa) = \varsigma_* \sqrt{\kappa} >> \varsigma_*$

Occupation

$nl(\kappa)/\varsigma_* = \gamma^{-1/2} >> 1$

Low temperatures: $t << \gamma^{-1/2}$

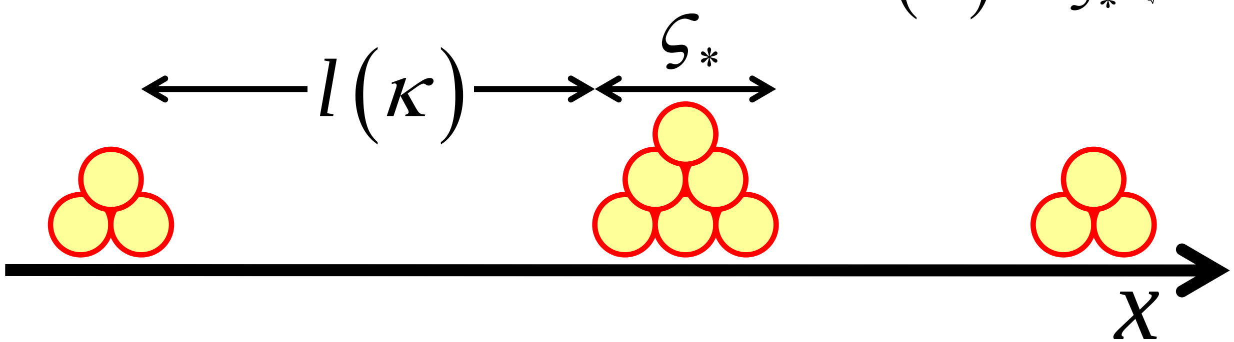
$\kappa \equiv E_* / ng >> 1$ ⟹ **"lakes"**

Occupation

$nl(\kappa)/\varsigma_* = \gamma^{-1/2} >> 1$

Distance

$l(\kappa) = \varsigma_* \sqrt{\kappa} >> \varsigma_*$



$l(\kappa) >> \varsigma_*$ ⟹ **Strong insulator**

$\kappa \to \kappa_c$

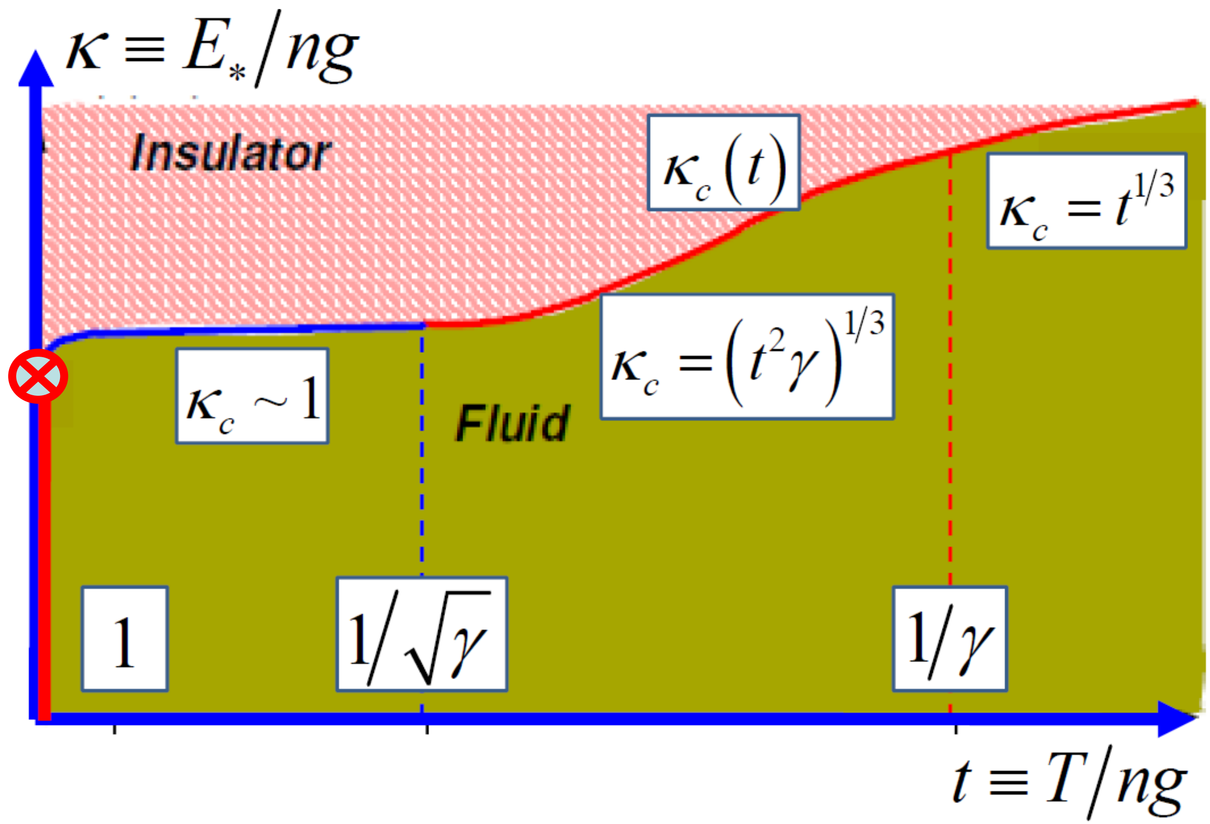
$l(\kappa) << \varsigma_*$ ⟹ **Insulator – Superfluid transition in a chain of "Josephson junctions"**

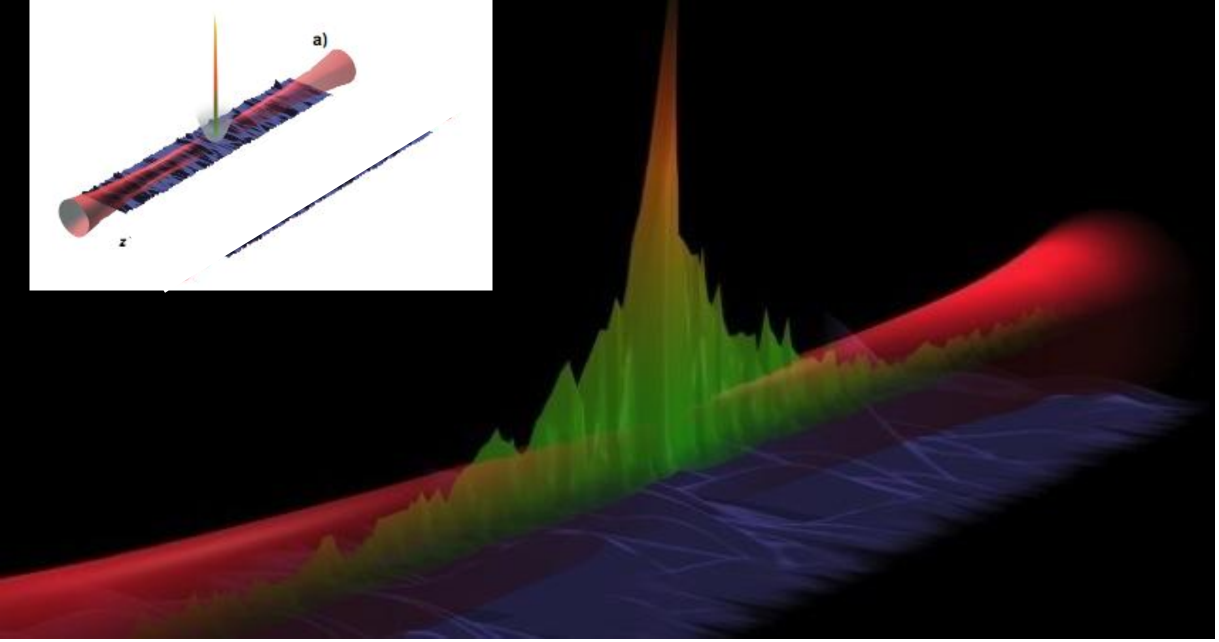
Low temperatures: $t << \gamma^{-1/2}$

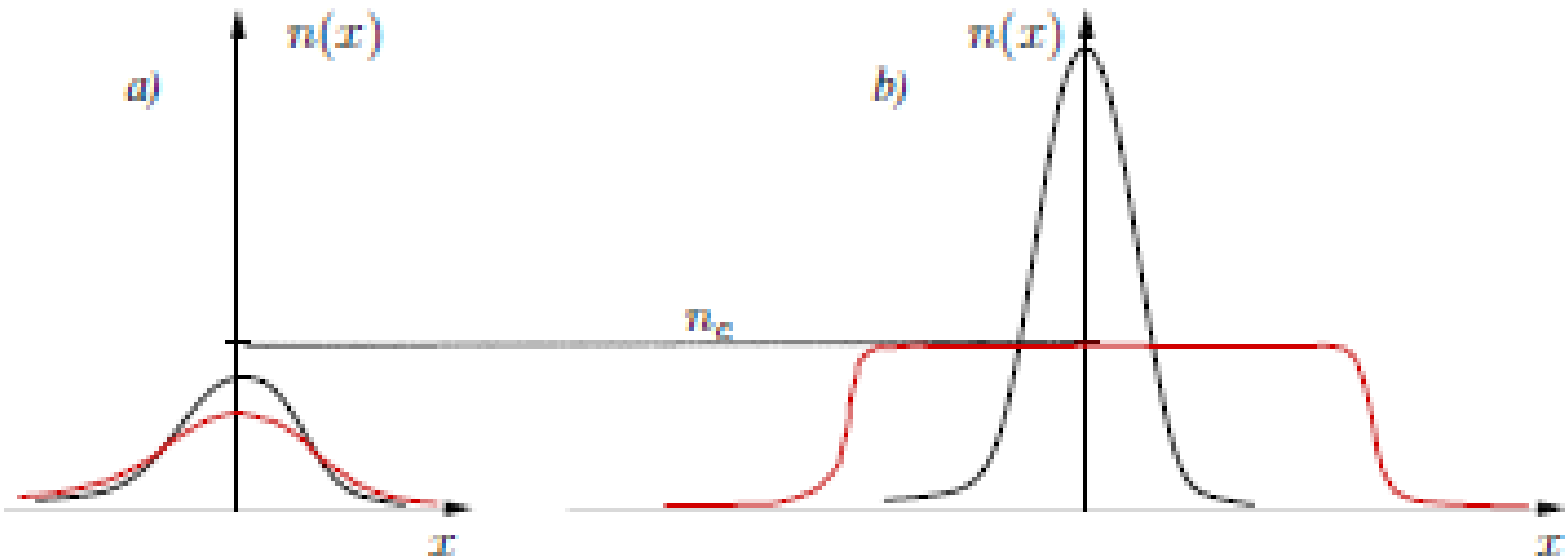
$\kappa \equiv E_*/ng >> 1$ ⟹ **Strong insulator**

$T = 0$ **transition** $\kappa_c \sim 1$

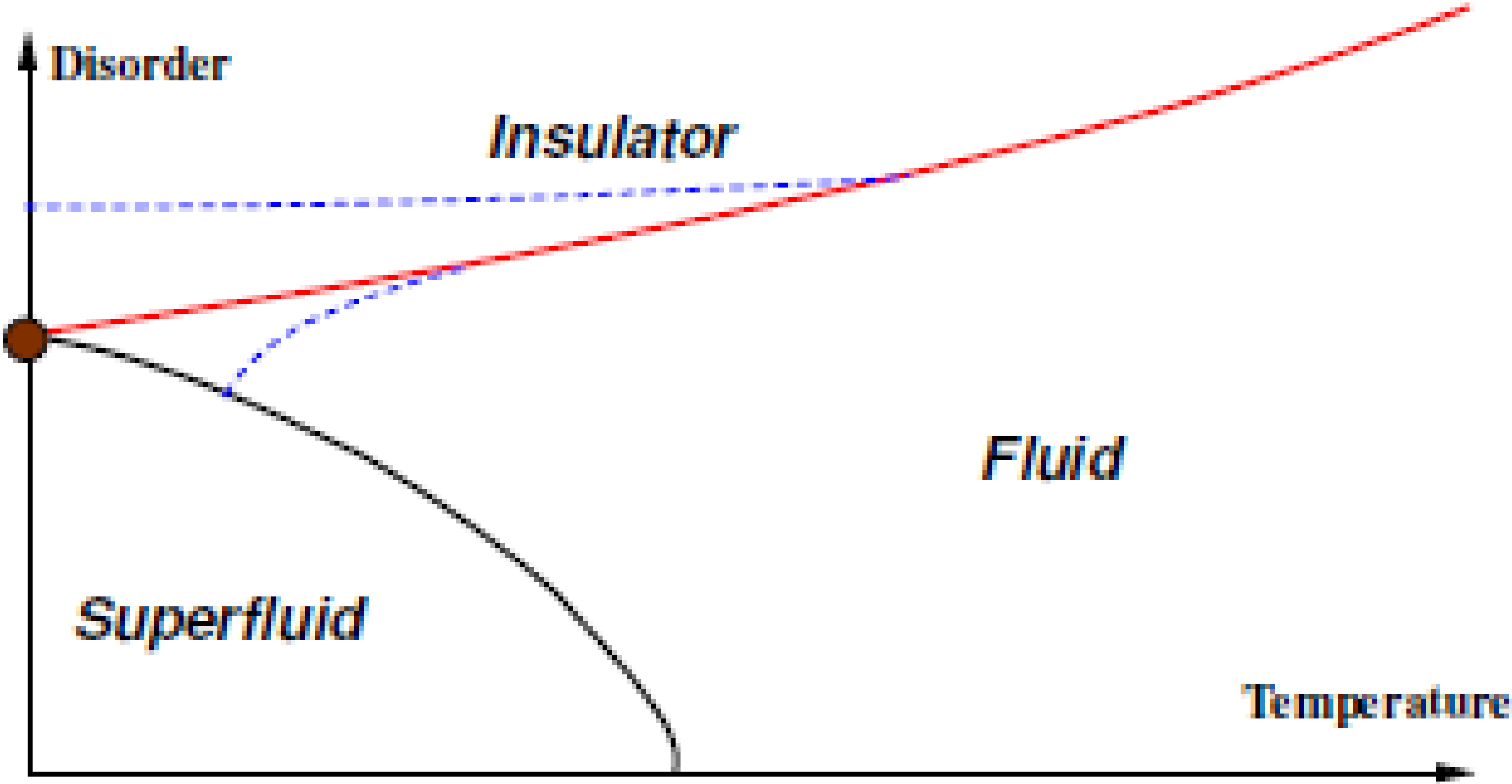
⟹ $\kappa_c \sim 1$ **for** $t << \gamma^{-1/2}$



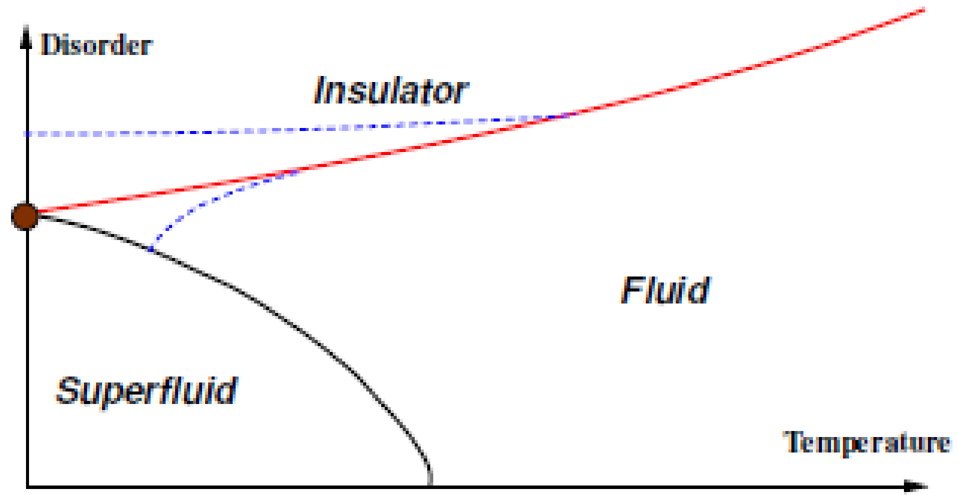

a)


$n(x)$
a)
$n(x)$
b)
n_c
x
x

Disordered interacting bosons in two dimensions



Disordered interacting bosons in two dimensions



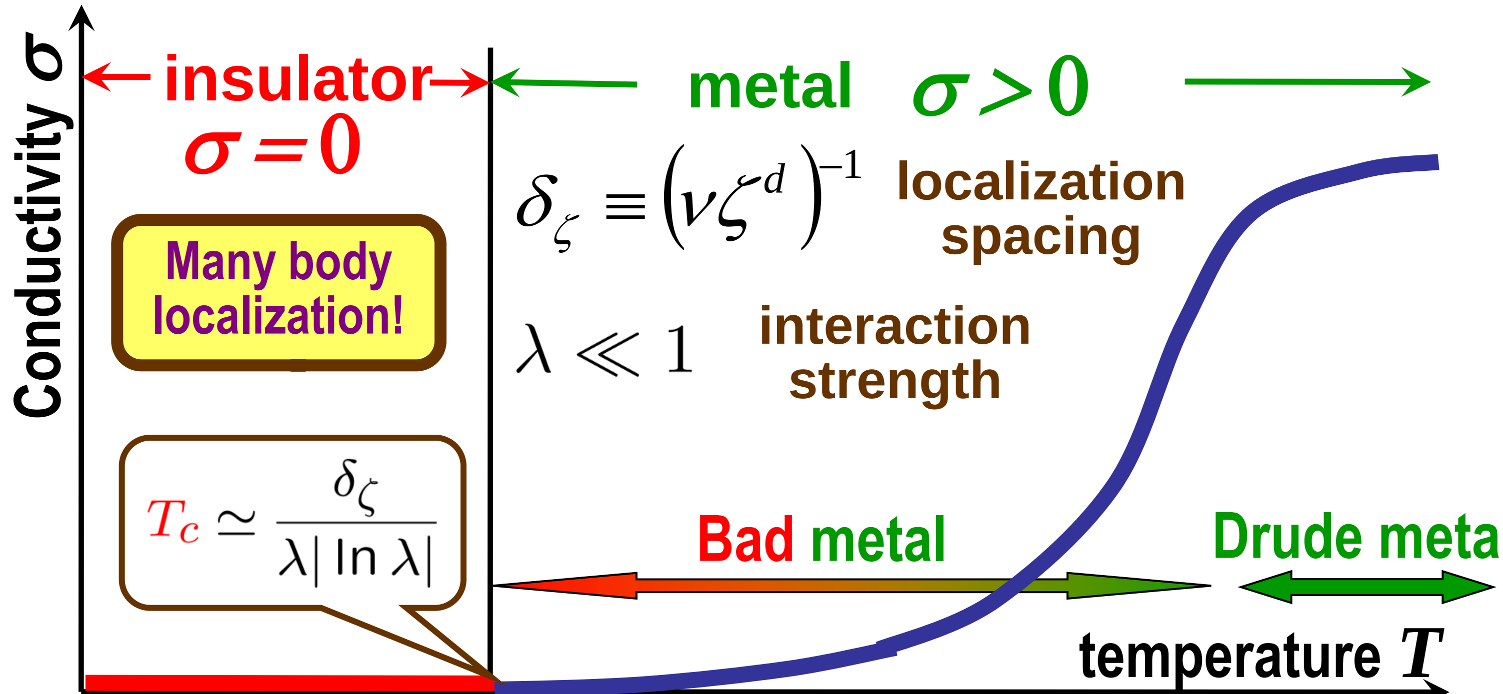
Justification:

1. At $T=0$ normal state is unstable with respect to either insulator or superfluid.

2. At finite temperature in the vicinity of the critical disorder the insulator can be thought of as a collection of "lakes", which are disconnected from each other. The typical size of such a "lake" diverges. This means that the excitations in the insulator state are localized but the localization length can be arbitrary large. Accordingly the many –body delocalization is unavoidable at an arbitrary low but finite T.

Phononless conductance

Many-body Localization of fermions



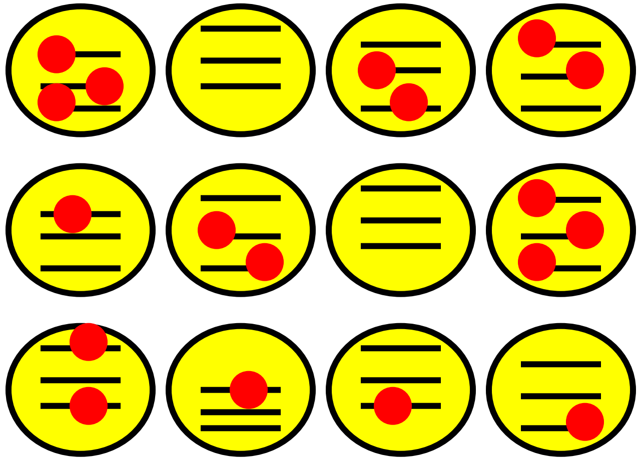
Definitions:

Insulator $\sigma = 0$ **Metal** $\sigma \neq 0$

not $d\sigma/dT < 0$ **not** $d\sigma/dT > 0$

Many body Anderson-like Model

- many particles,
- several levels per site,
- onsite disorder
- local interaction



Basis: $\left|\mu\right\rangle$

$$\mu = \left\{ n_i^{\alpha} \right\}$$

i labels sites

α labels levels

$n_i^{\alpha} = 0,1$ occupation numbers

Hamiltonian:

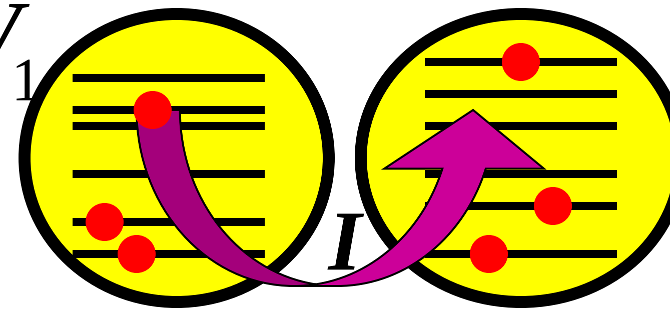
$$\hat{H} = \hat{H}_0 + \hat{V}_1 + \hat{V}_2$$

$$\hat{H}_0 = \sum_{\mu} E_{\mu} \left|\mu\right\rangle\left\langle\mu\right|$$

$$\hat{V}_1 = \sum_{\mu,\nu(\mu)} I \left|\mu\right\rangle\left\langle\nu(\mu)\right|$$

$$\left|\nu(\mu)\right\rangle = \left|.., n_i^{\alpha} - 1, .., n_j^{\beta} + 1, ..\right\rangle, \quad i, j = n.n.$$

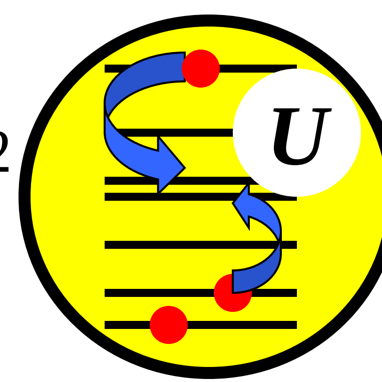
$\hat{V}_1$



$$\hat{V}_2 = \sum_{\mu,\eta(\mu)} U \left|\mu\right\rangle\left\langle\eta(\mu)\right|$$

$$\left|\nu(\mu)\right\rangle = \left|.., n_i^{\alpha} - 1, .., n_i^{\beta} - 1, .., n_i^{\gamma} + 1, .., n_i^{\delta} + 1, ..\right\rangle$$

$\hat{V}_2$



Conventional Anderson Model

Basis: $|i\rangle$

i **labels sites**

$$\hat{H} = \sum_i \varepsilon_i |i\rangle\langle i| + \sum_{i,j=n.n.} I|i\rangle\langle j|$$

Many body Anderson-like Model

Basis: $|\mu\rangle, \quad \mu = \{n_i^\alpha\}$

i **labels sites** α **labels levels** $n_i^\alpha = 0,1$ **occupation numbers**

$$\hat{H} = \sum_\mu E_\mu |\mu\rangle\langle\mu| + \sum_{\mu,\nu(\mu)} I|\mu\rangle\langle\nu(\mu)| + \sum_{\mu,\eta(\mu)} U|\mu\rangle\langle\eta(\mu)|$$

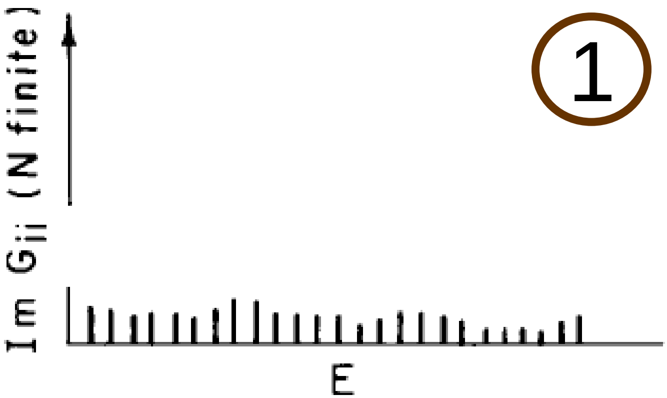
Two types of "nearest neighbors":

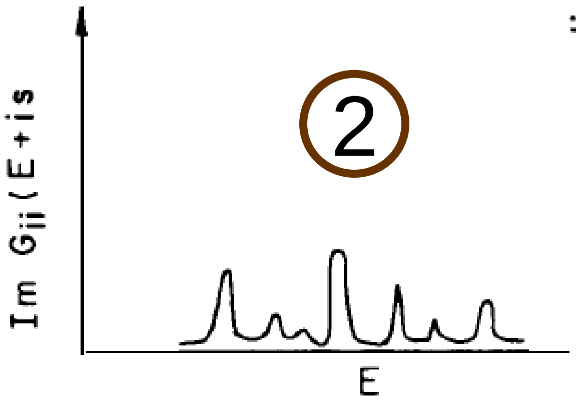
$$|\nu(\mu)\rangle = |.., n_i^\alpha - 1, .., n_j^\beta + 1, ..\rangle, \quad i, j = n.n.$$

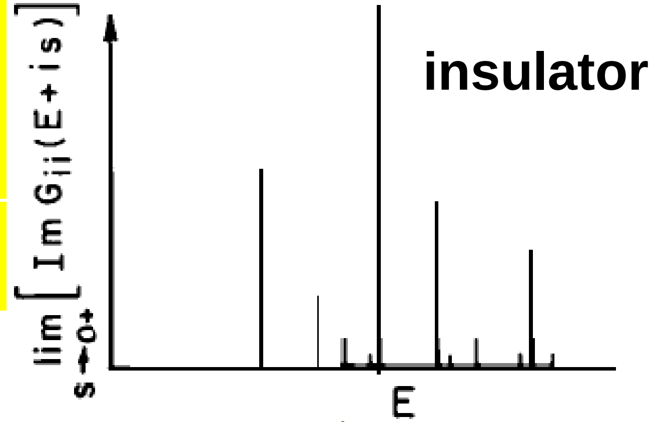
$$|\eta(\mu)\rangle = |.., n_i^\alpha - 1, .., n_i^\beta - 1, .., n_i^\gamma + 1, .., n_i^\delta + 1, ..\rangle$$

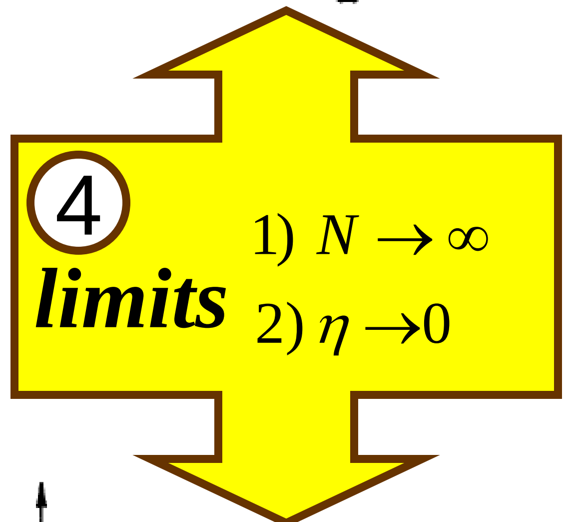
Anderson's recipe:

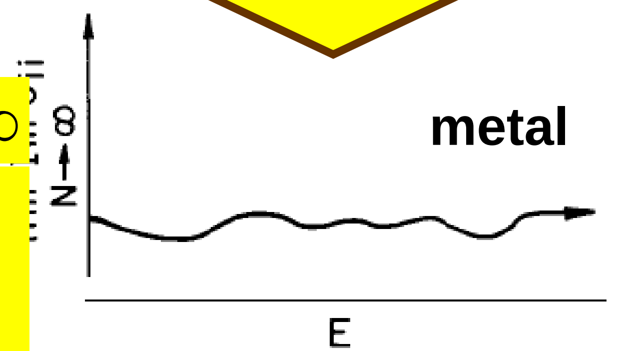
1. take descrete spectrum E_μ of H_0
2. Add an infinitesimal Im part $i\eta$ to E_μ
3. Evaluate $Im\Sigma_\mu$





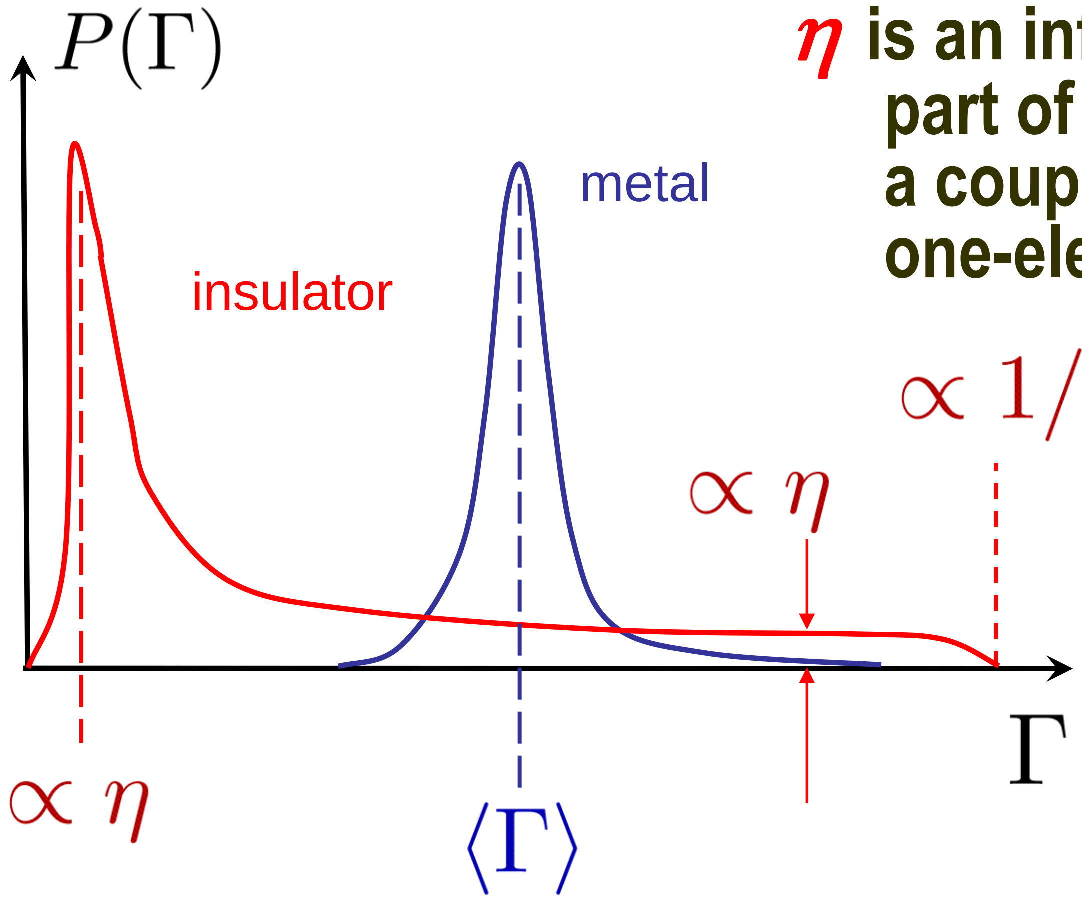






4. take limit $\eta \to 0$ but only after $N \to \infty$
5. "What we really need to know is the *probability distribution* of $Im\Sigma$, *not* its average…" !

Probability Distribution of Γ=Im Σ



η is an infinitesimal width (***Im*** part of the self-energy due to a coupling with a bath) of one-electron eigenstates

Look for:

$$\lim_{\eta \to +0} \lim_{V \to \infty} P(\Gamma > 0) = \begin{cases} > 0; & metal \\ 0; & insulator \end{cases}$$

Stability of the insulating phase: NO spontaneous generation of broadening

$$\Gamma_\alpha(\varepsilon) = 0$$

is always a solution

$$\varepsilon \to \varepsilon + i\eta$$

linear stability analysis

$$\frac{\Gamma}{(\varepsilon-\xi_\alpha)^2+\Gamma^2} \to \pi\delta(\varepsilon-\xi_\alpha) + \frac{\Gamma}{(\varepsilon-\xi_\alpha)^2}$$

After n iterations of the equations of the **S**elf **C**onsistent **B**orn **A**pproximation

$$P_n(\Gamma) \propto \frac{\eta}{\Gamma^{3/2}}\left(const\,\frac{\lambda T}{\delta_\zeta}\ln\frac{1}{\lambda}\right)^n$$

first $n \to \infty$

then $\eta \to 0$

$(\ldots) < 1$ – insulator is stable !

Physics of the transition: cascades

Conventional wisdom:

For phonon assisted hopping one phonon – one electron hop

Baron Münchhausen regime

Cascade regime

Physics of the transition: cascades

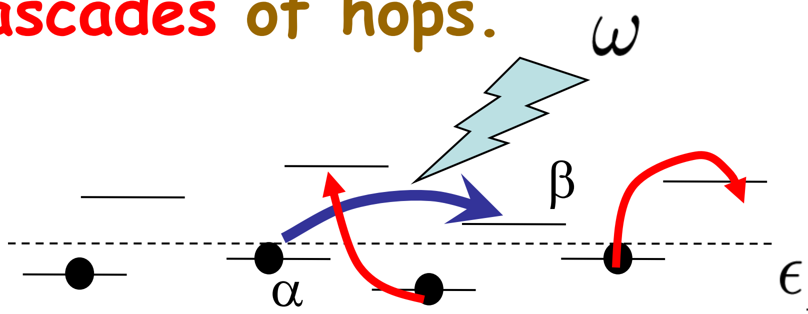
Conventional wisdom:

For phonon assisted hopping one phonon – one electron hop

It is maybe correct at low temperatures, but the higher the temperature the easier it becomes to create e-h pairs.

Therefore with increasing the temperature the typical number of pairs created n_c (i.e. the number of hops) increases. Thus phonons create cascades of hops.

Typical size of the cascade $\approx$ Localization length



Physics of the transition: cascades

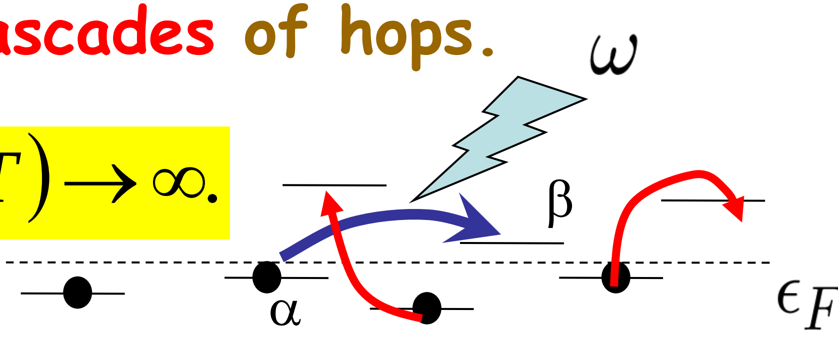
Conventional wisdom:

For phonon assisted hopping one phonon – one electron hop

It is maybe correct at low temperatures, but the higher the temperature the easier it becomes to create e-h pairs.

Therefore with increasing the temperature the typical number of pairs created n_c (i.e. the number of hops) increases. Thus phonons create cascades of hops.

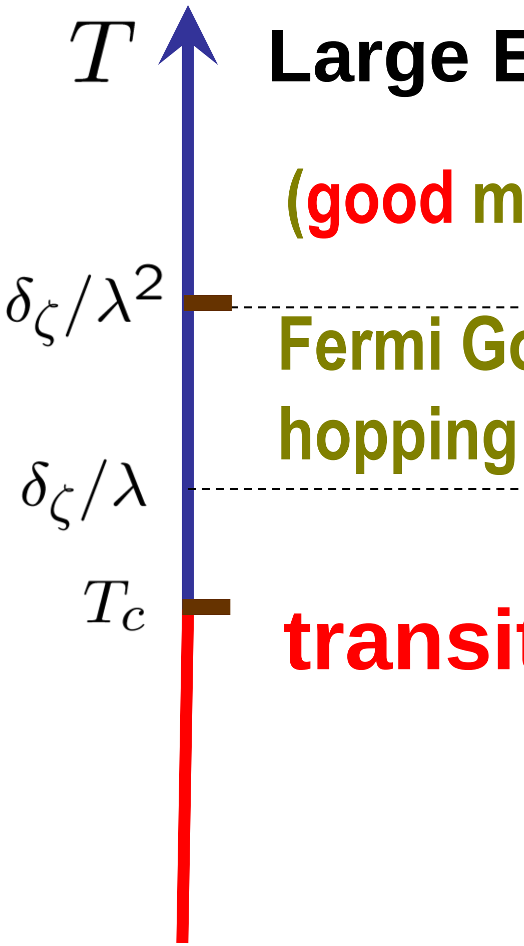
At some temperature $T = T_c \quad n_c(T) \to \infty.$
This is the critical temperature.
Above T_c one phonon creates infinitely many pairs, i.e., phonons are not needed for charge transport.



Many-body mobility edge

transition ! mobility edge

Many-body mobility edge



Large E (high T): extended states

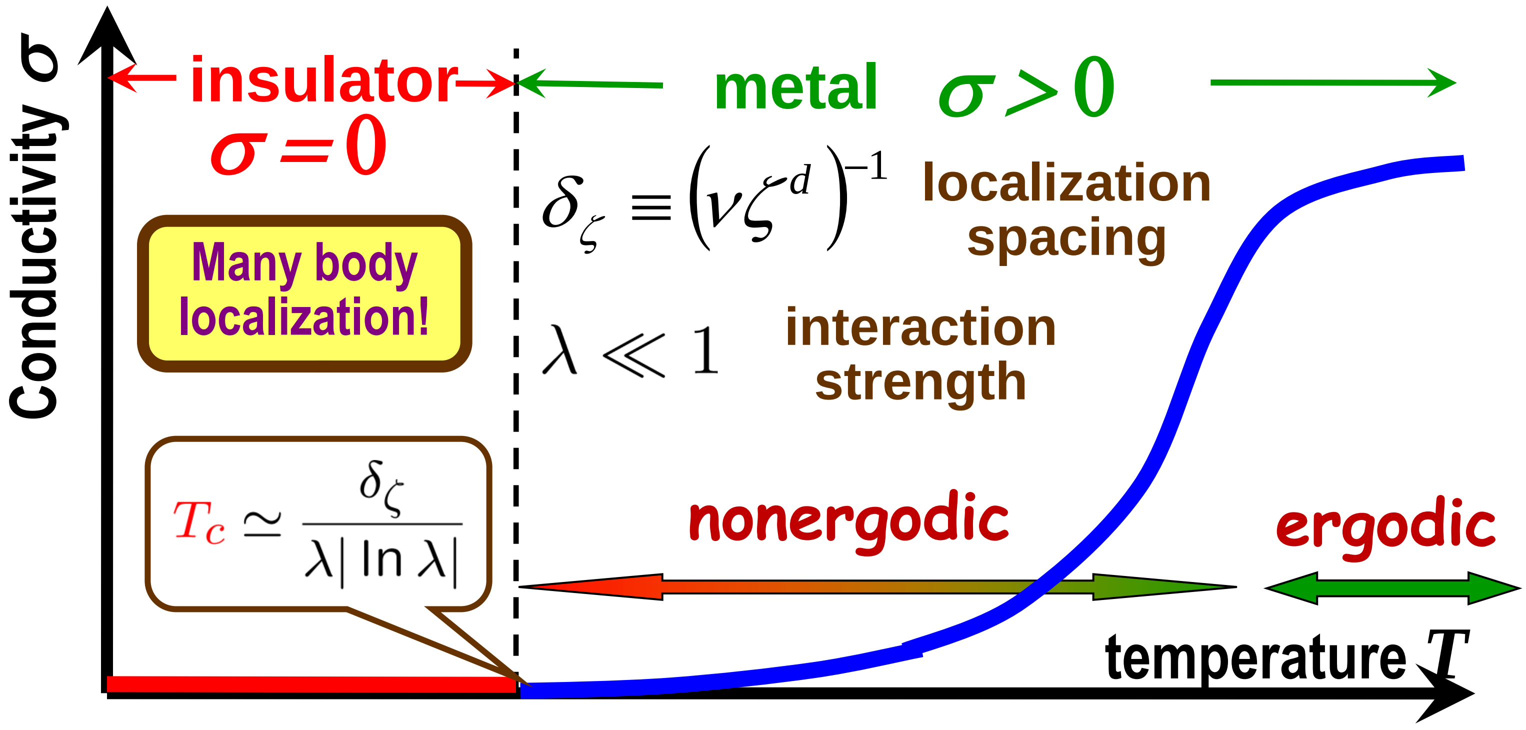
(good metal)

Fermi Golden Rule
hopping (bad metal)

mobility
edge

transition !

Finite T normal metal – insulator transition is another example of the many-body localization



Definition: We will call a quantum state $|\mu\rangle$ **ergodic** if it occupies the number N_μ of sites N_μ on the Anderson lattice, which is proportional to the total number of sites N:

$$\frac{N_\mu}{N} \xrightarrow[N\to\infty]{} 0$$

nonergodic

$$\frac{N_\mu}{N} \xrightarrow[N\to\infty]{} const > 0$$

ergodic

Localized states are obviously not ergodic: $N_\mu \xrightarrow[N\to\infty]{} const$

Q: Is each of the extended state ergodic **?**

A: In **3D** probably **YES**, for $d>4$ - probably **NO**

Nonergodic states

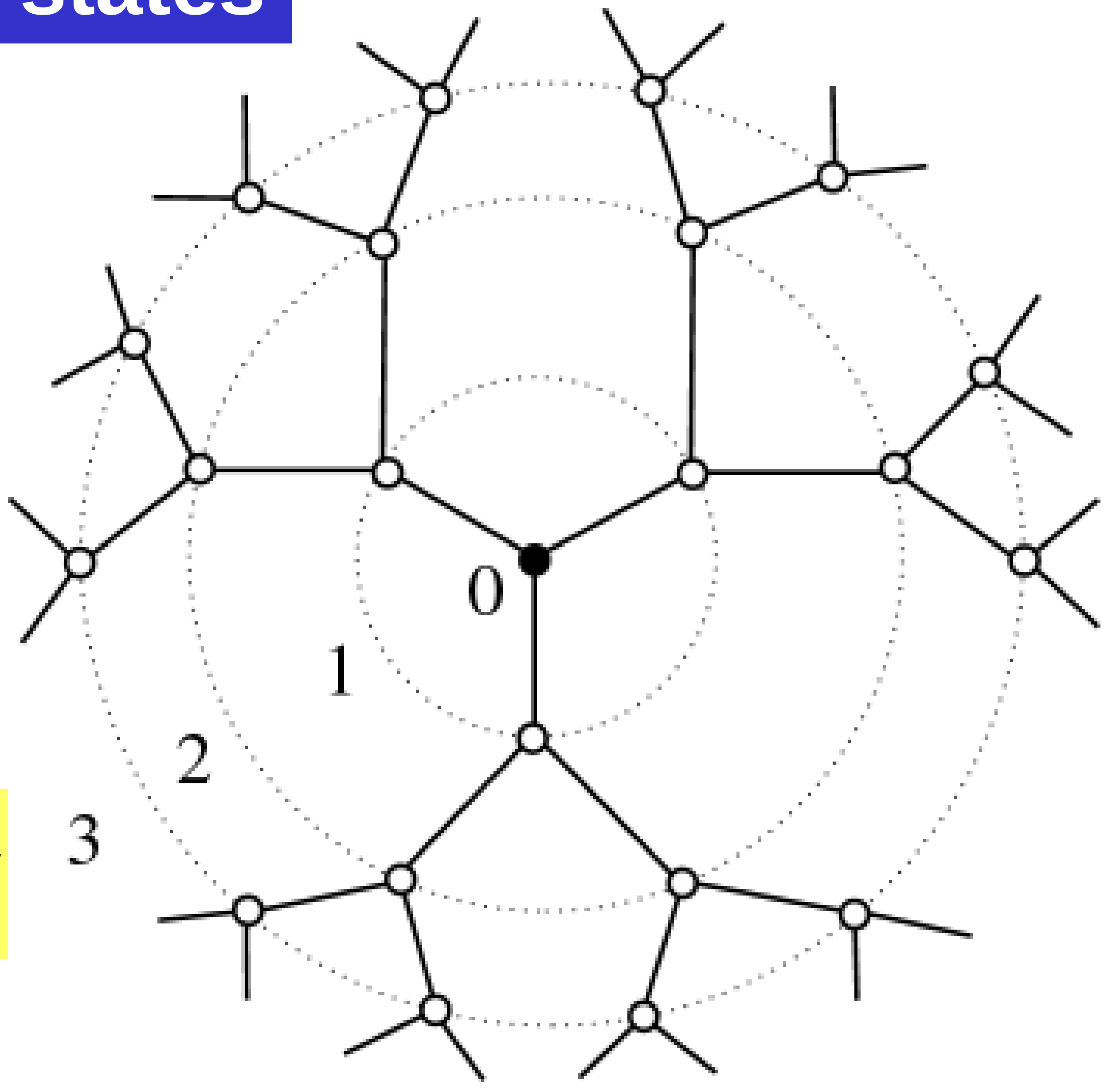
Cayley tree (Bethe lattice)

$$I_c = \frac{W}{K \ln K}$$

K is the branching number

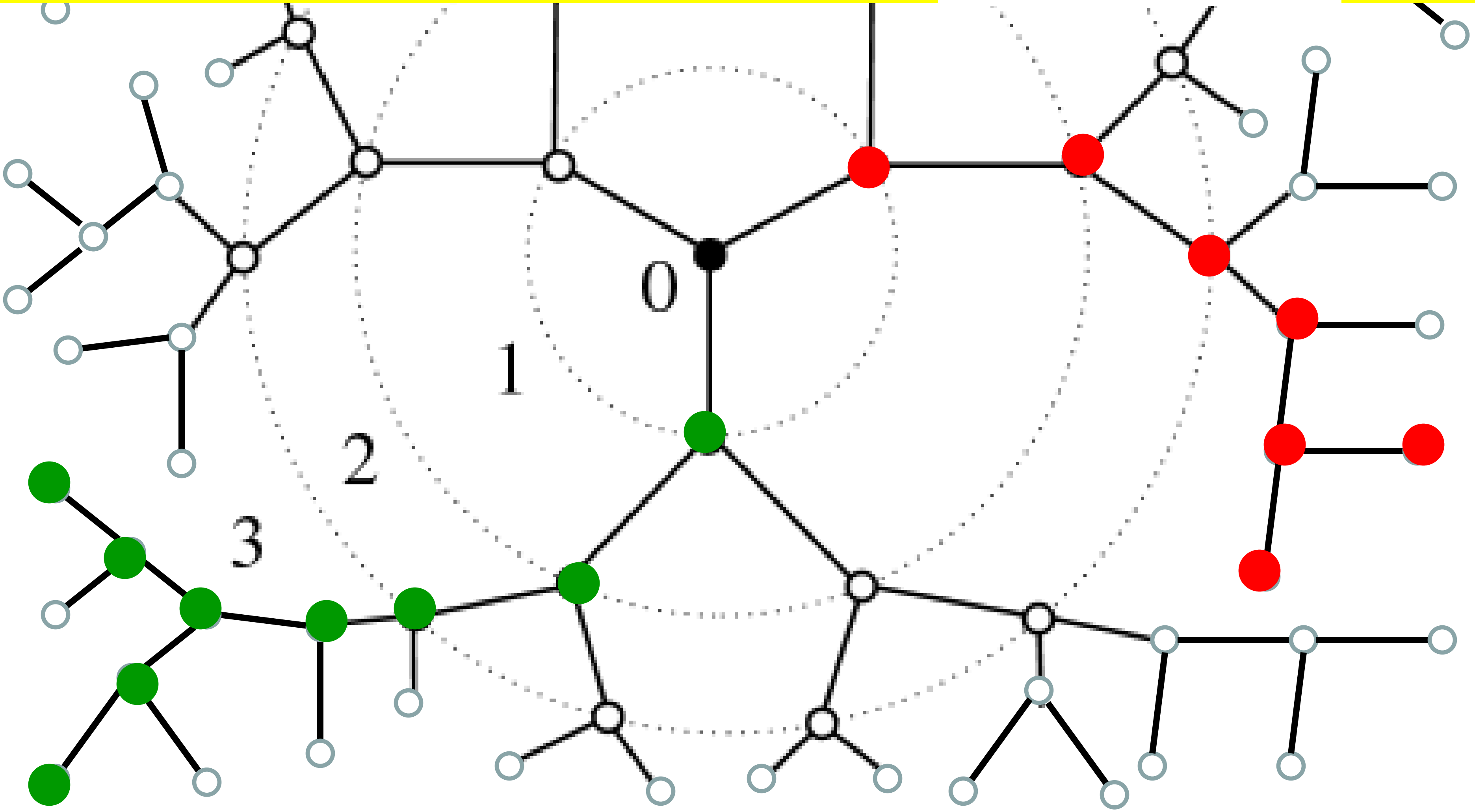
$$I_c < I < W$$

Extended but not ergodic



$$I \approx \frac{W}{K} \quad \Rightarrow \quad N_\mu \approx \ln N << N$$

nonergodic



nonergodic ⟹ ??? ⟹ glassy