Glass phases of bosons in disordered and quasi-periodic potentials

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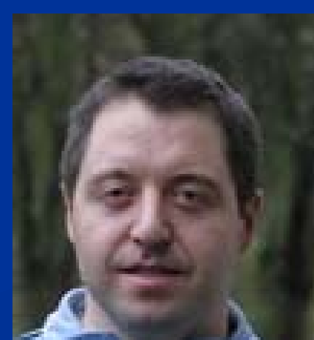
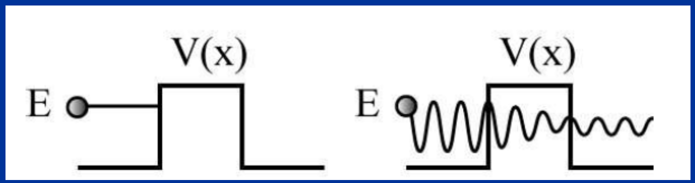
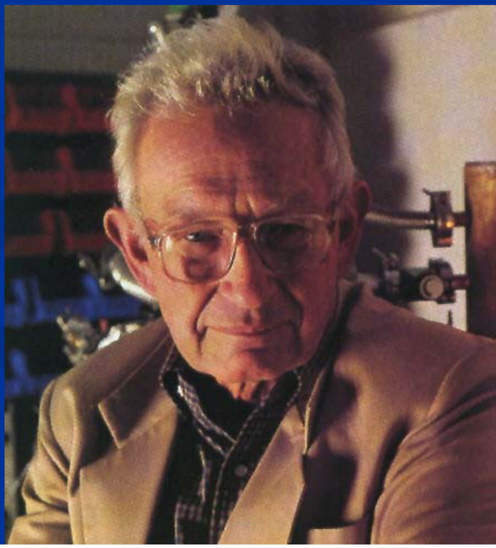


Disorder and quantum systems

Tunnelling: disorder less important ??



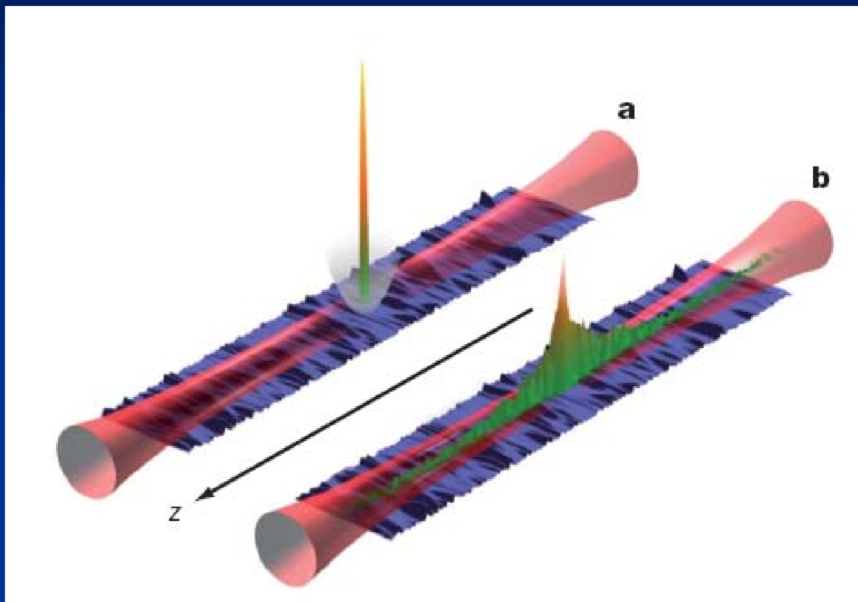


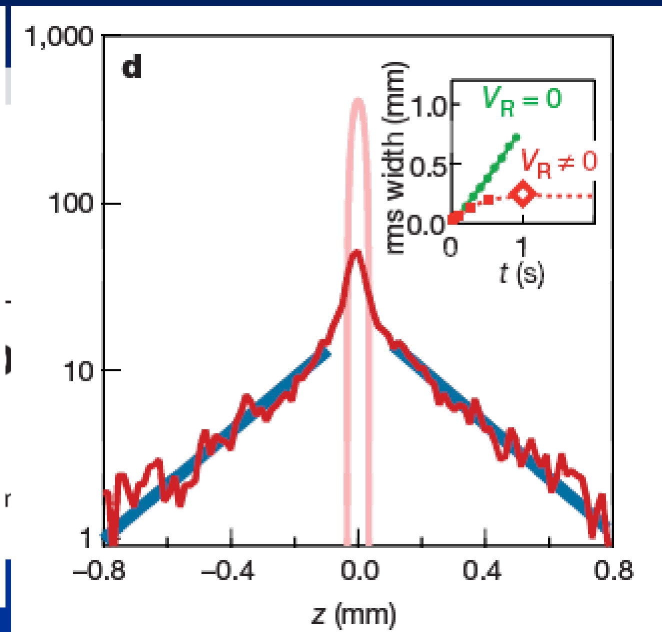
No !! (Anderson localization):

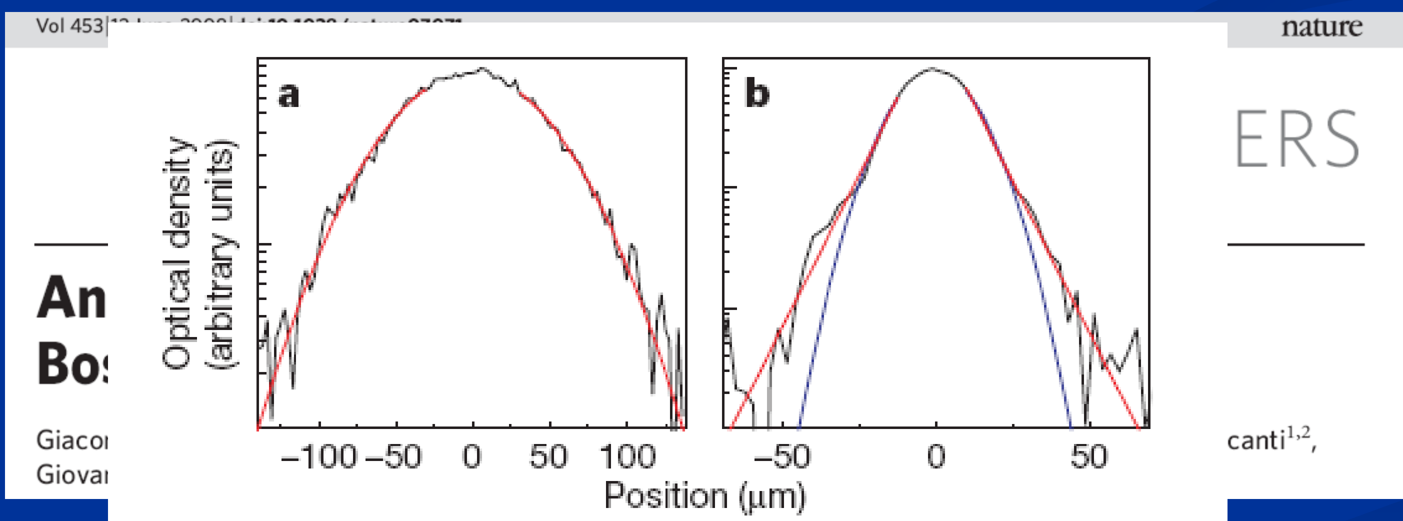
[P.W Anderson Phys Rev. 109 1492 (1958)]

interferences

Cold atomic gases

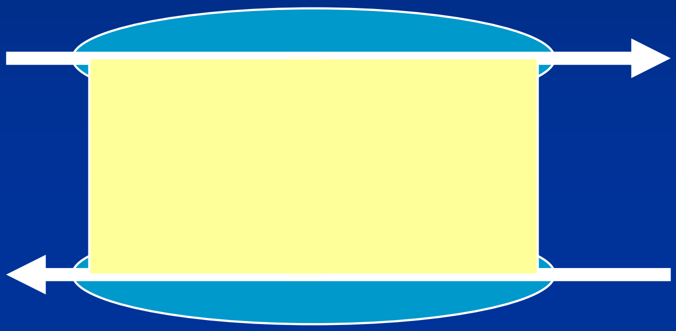






Aubry-Andre Model (Ann. Isr. Phys. Soc. **3, 133 1980**)

Challenge: disorder and interactions

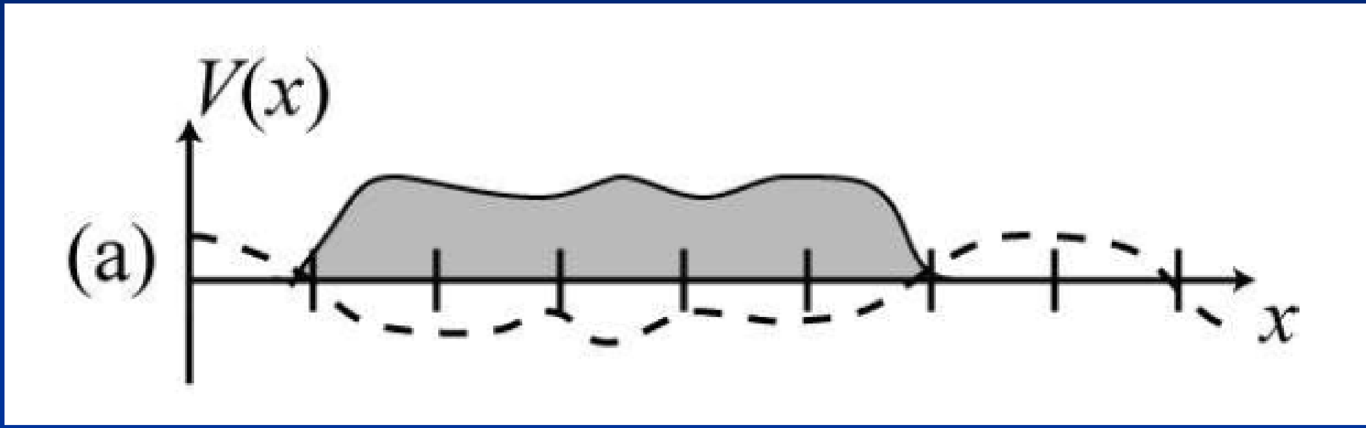


Interactions reinforced by disorder

- Localization ?
- Phases ?

One of the most important, difficult and controversial question in CM !!

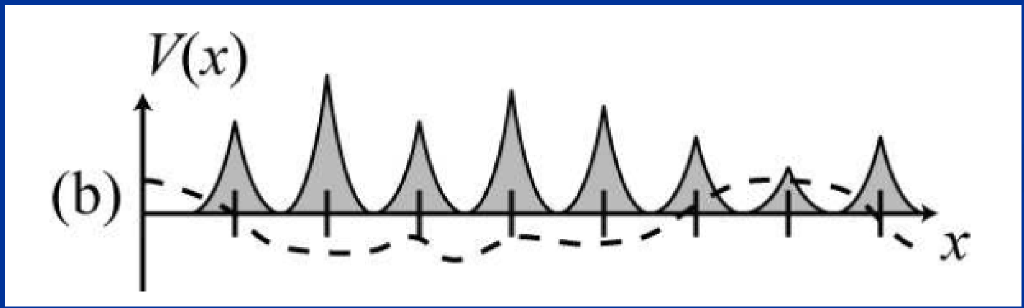
Why are bosons special ?



Free Bosons: pathological

$$H = \frac{1}{2m}\left(\frac{1}{L}\right)^2 - V_0$$

Rare events: Infinite density



Interactions needed from the start

Bose glass phase

1D : TG + H. J. Schulz PRB 37 325 (1988)

Microscopic derivation (bosonization)

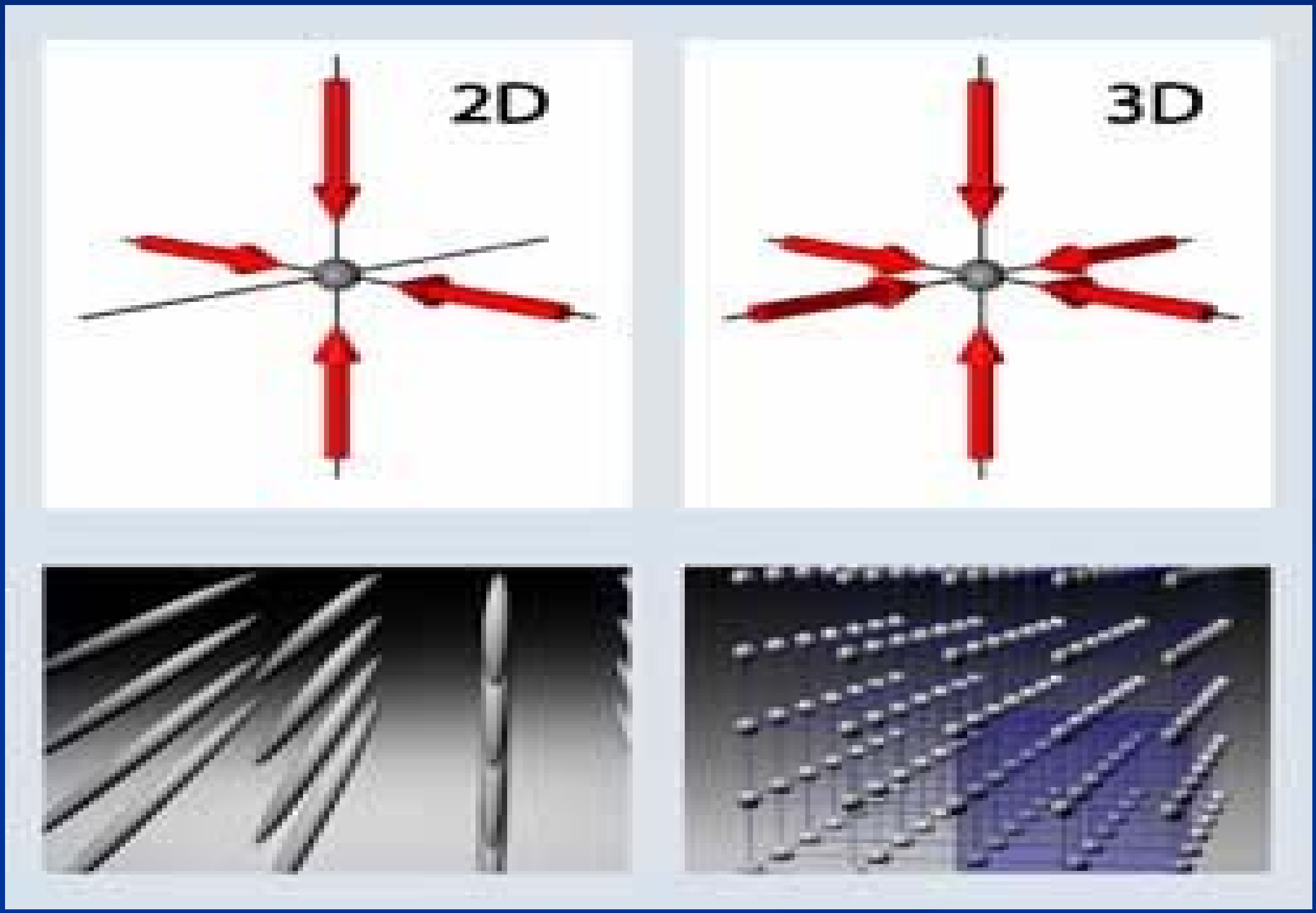
Higher dimensions: M.P.A. Fisher et al. PRB 40 546 (1989)

Scaling theory

Bose glass:

- Compressible
- Localized

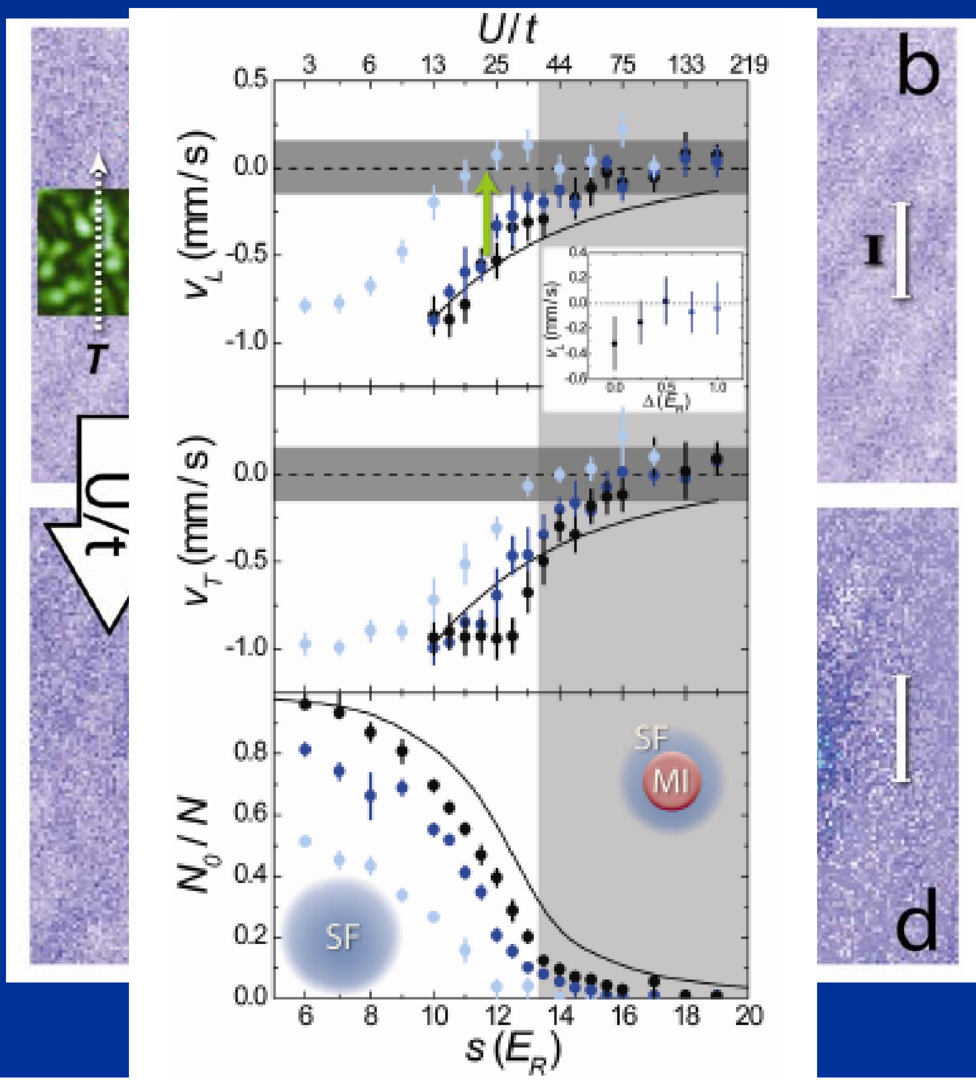
Cold atoms in optical lattices

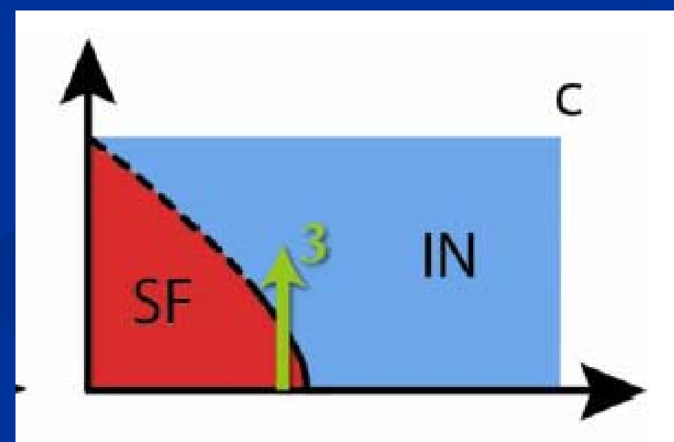


I. Bloch, Nat. Phys 1, 23 (2005)

Speckle

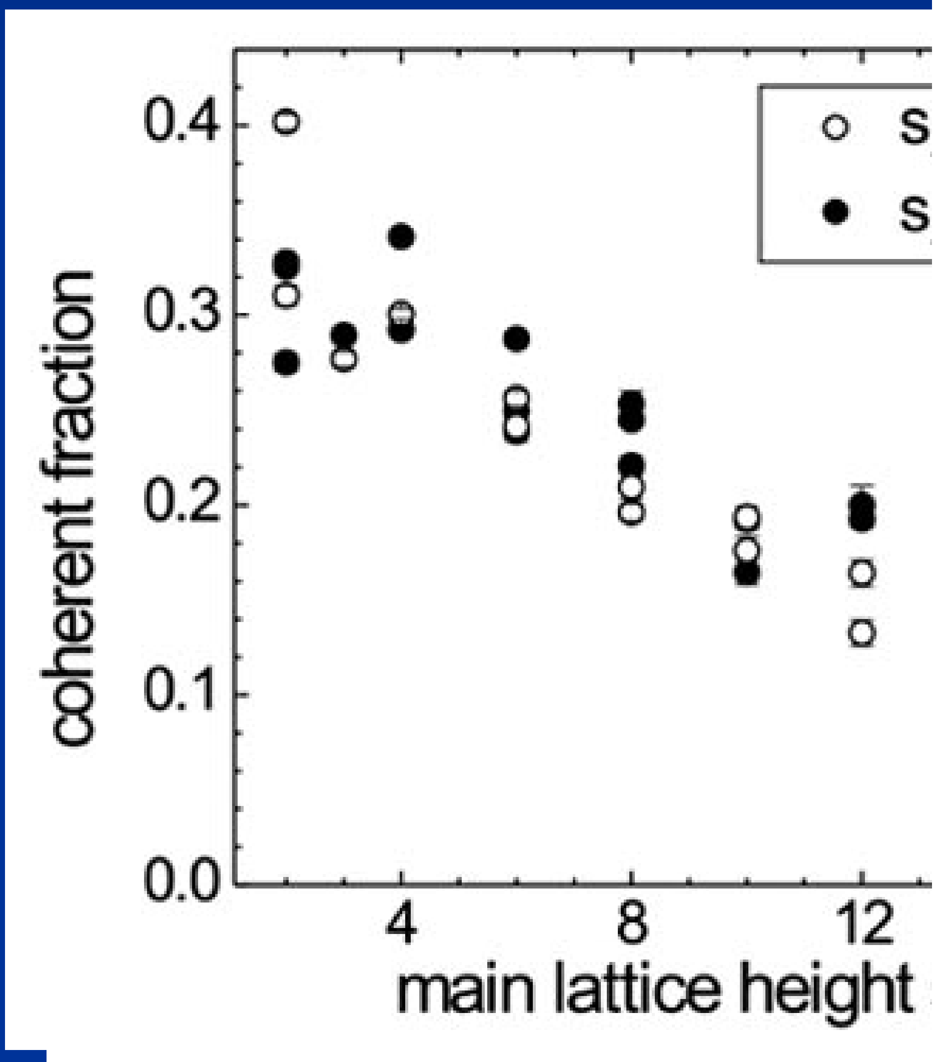
Pasienski et al. Nat. Phys 6 677 (2010)

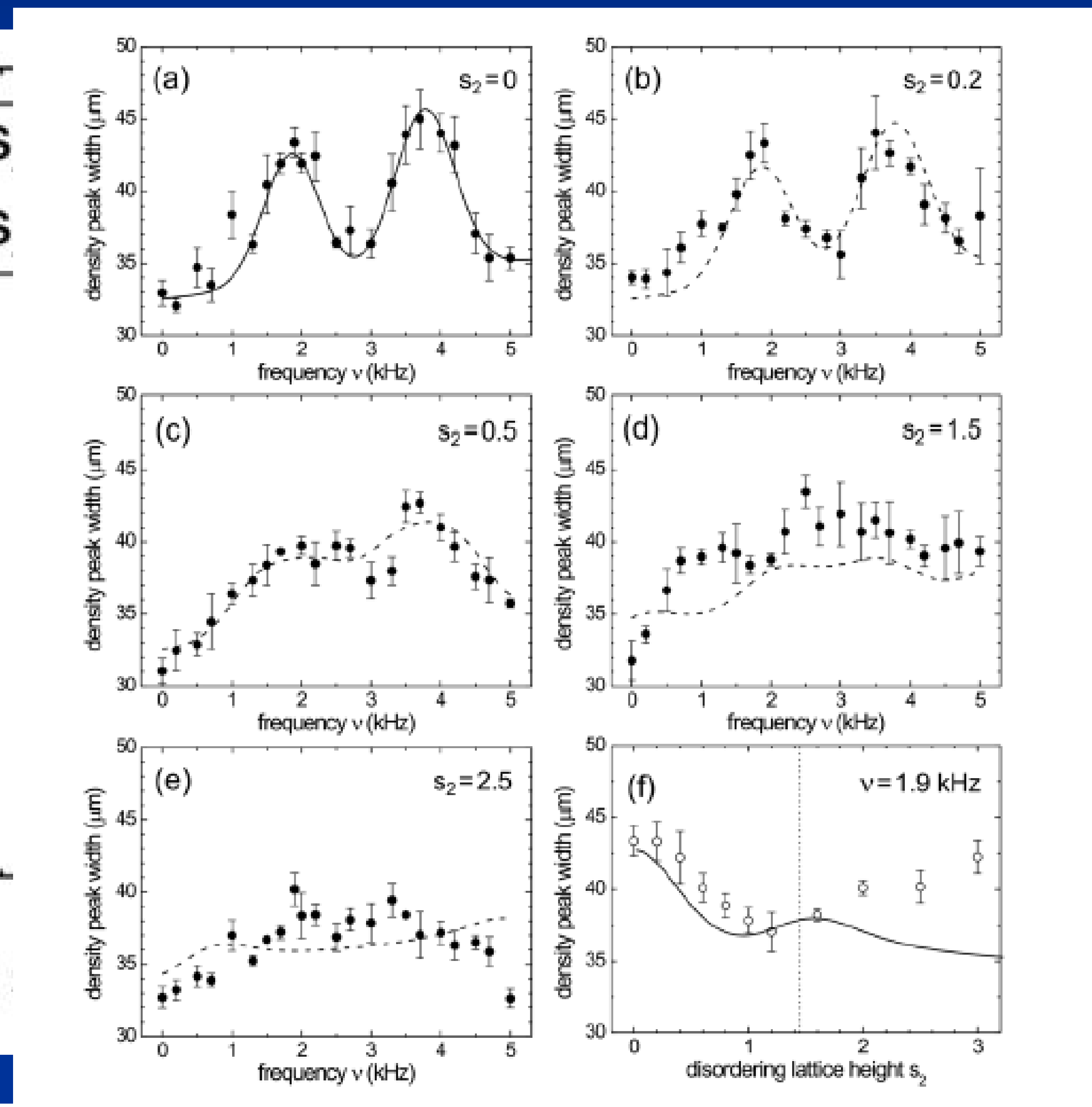




Quasi-periodic

Quasiperiodic (1D):





Questions

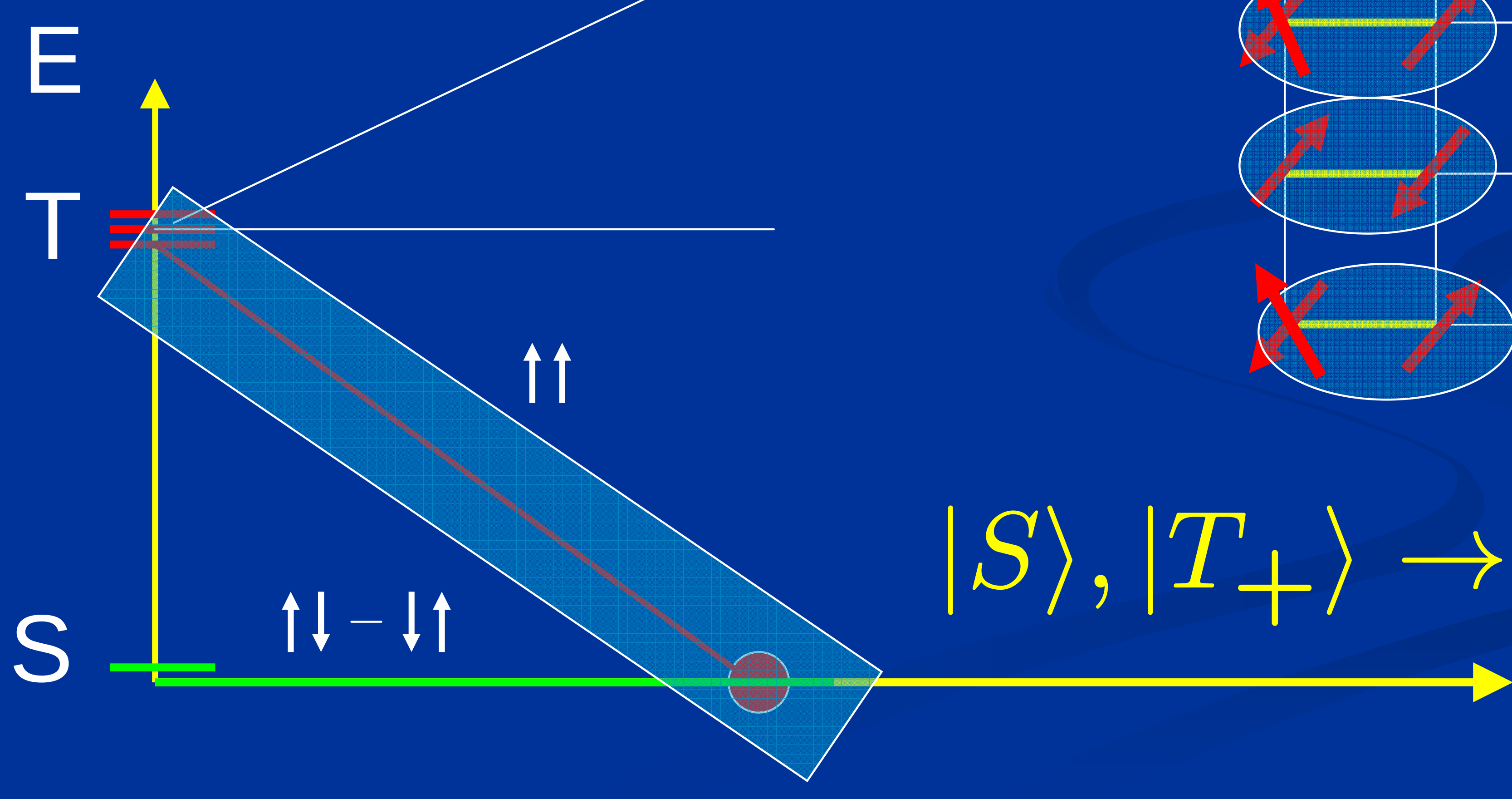
- Periodic, quasiperiodic, disordered ?
- Quasiperiodic: Bose glass ?
- Properties : expansion, shaking

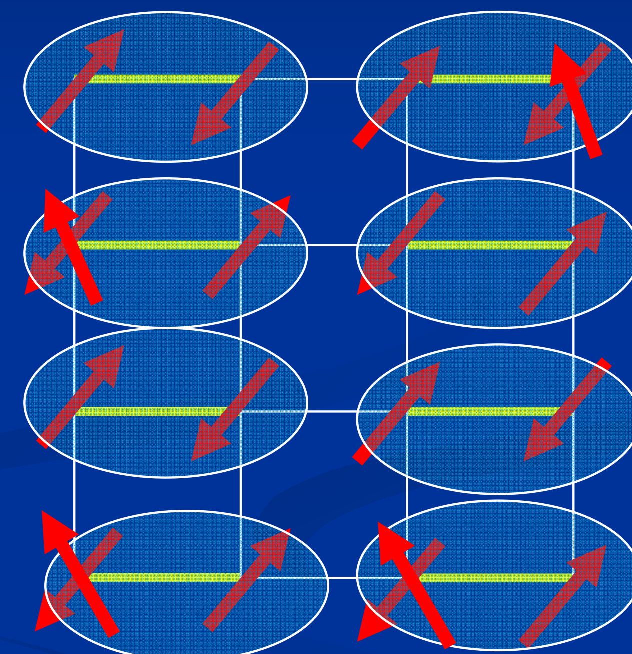
Localized spin systems

Dimer systems

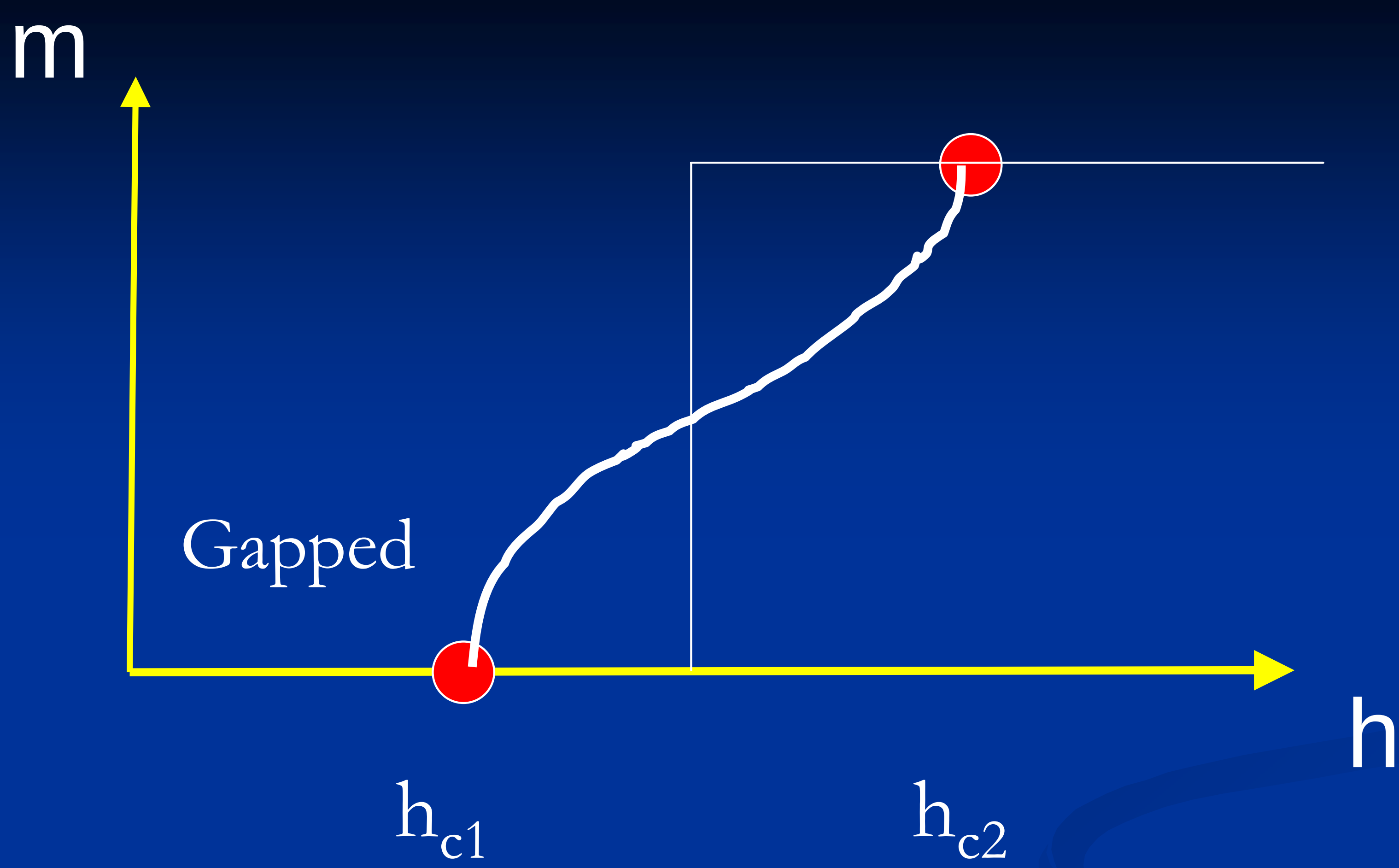
TG, Ch. Rüegg, O. Tchernyshyov, Nat. Phys. 4 198 (08)

$J_r \gg J$





$$|S\rangle, |T_+\rangle \to |0\rangle, |1\rangle$$

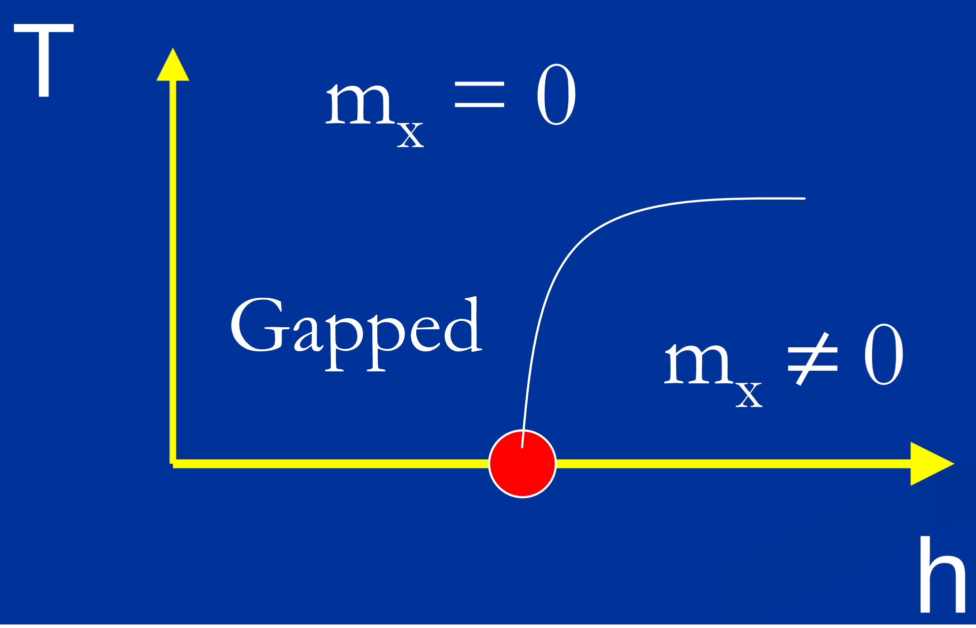


- Quantum phase transition
- Magnetic field: ``chemical potential” for the triplon band (interacting intinerant ``particles”)

Bose Einstein condensation

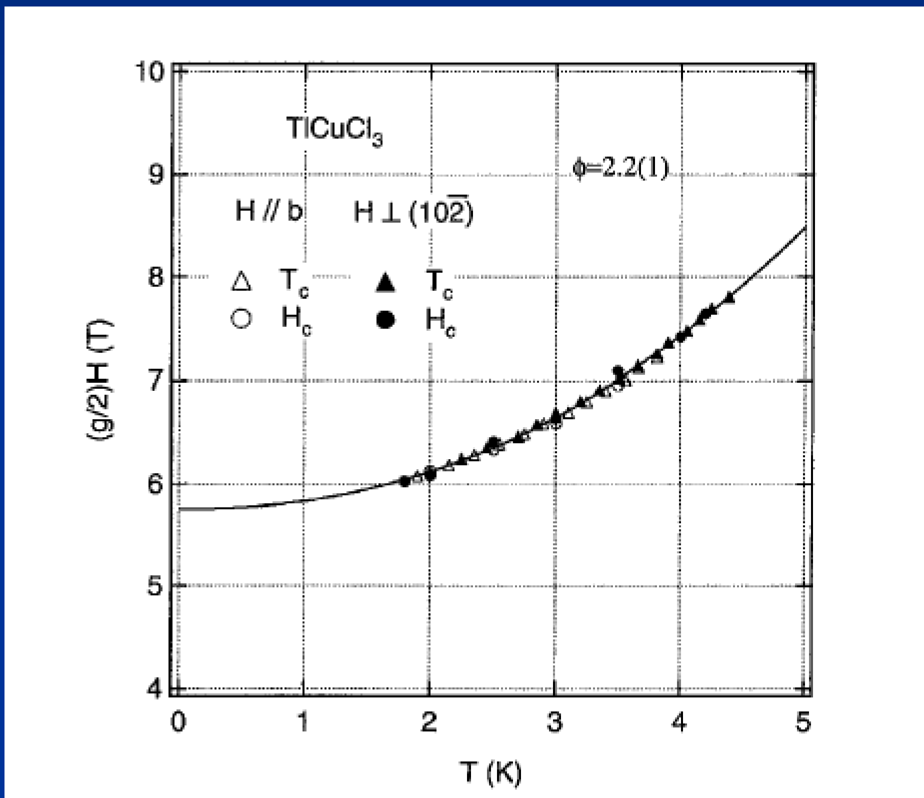
(TG and A. M. Tsvelik PRB 59 11398 (1999))

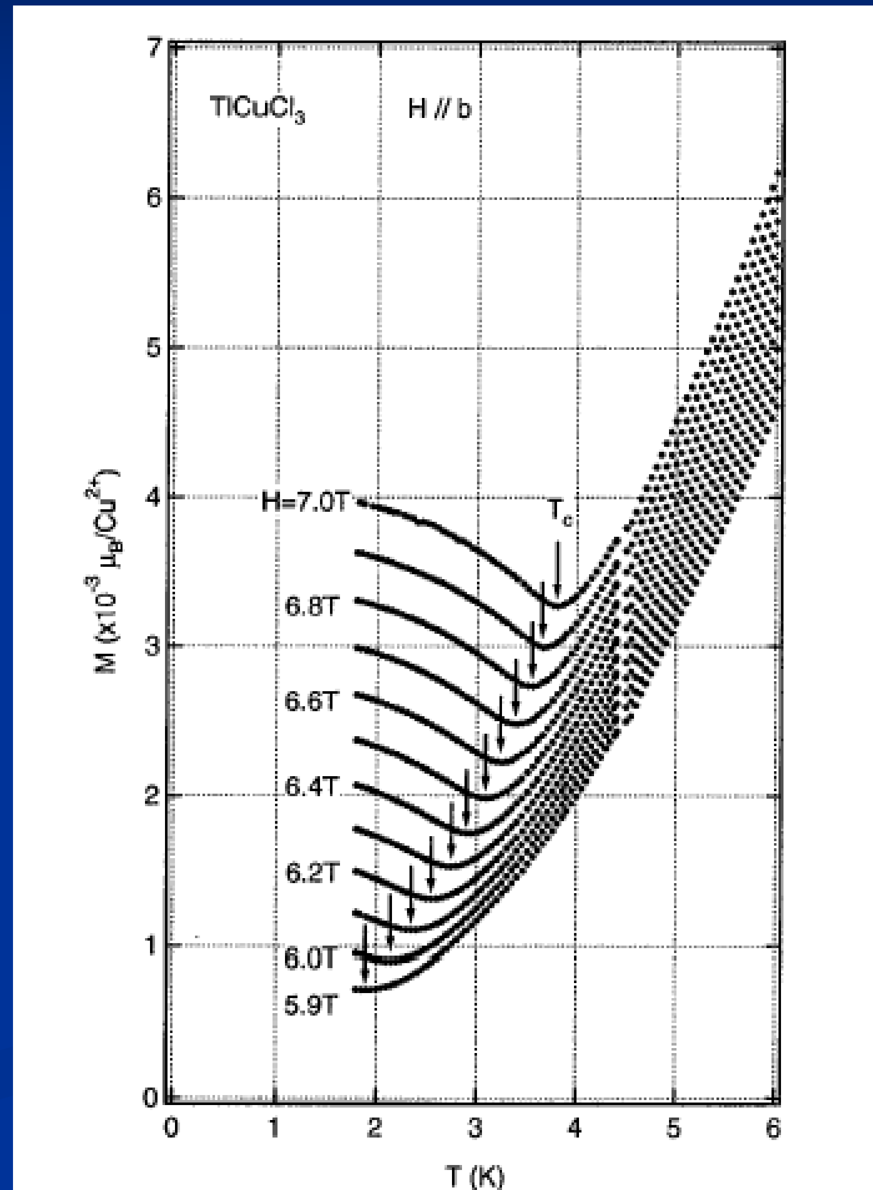
- Phase of the boson : order in the X-Y plane
- Why useful: close to hc1,h_{c2} : nearly free bosons



$$T_c \sim (h - h_c)^{2/d}$$

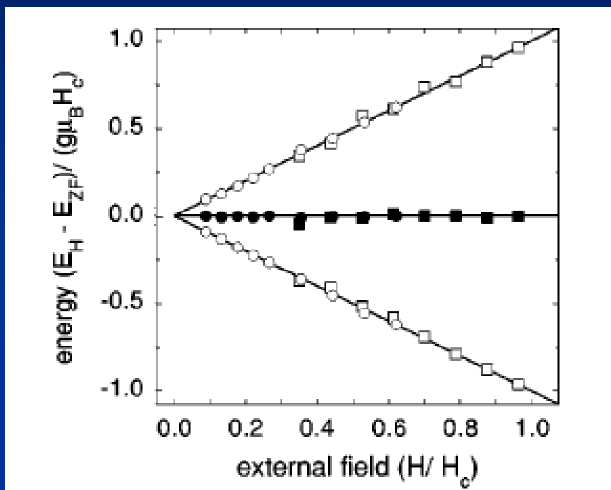
Experimental realization: $TlCuCl_3$



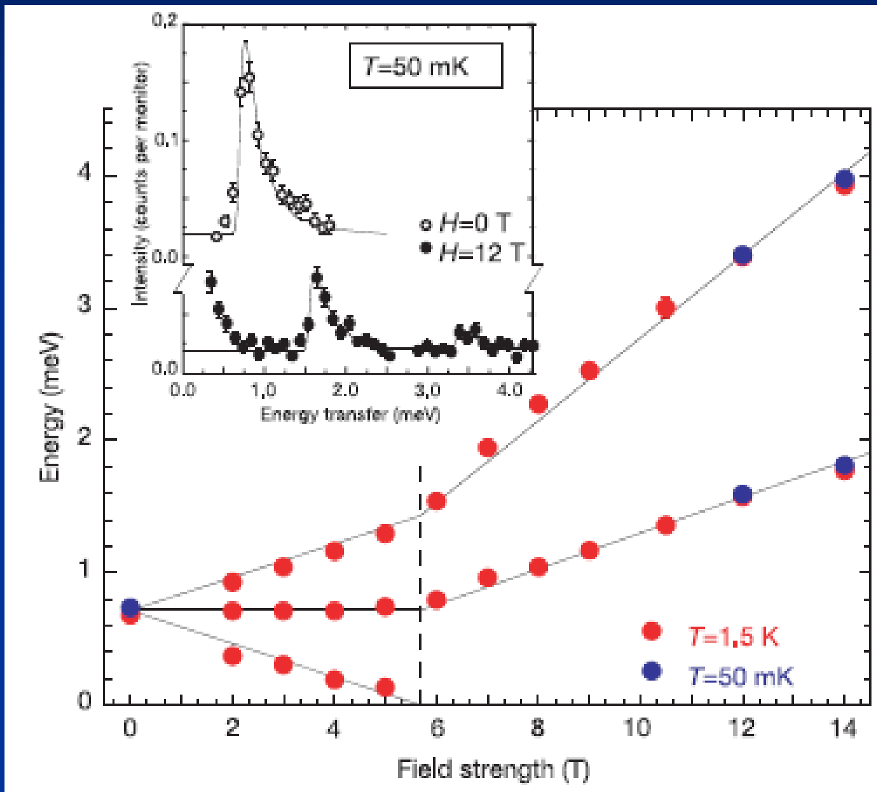


T. Nikuni et al., PRL **84** 5868 (2000)

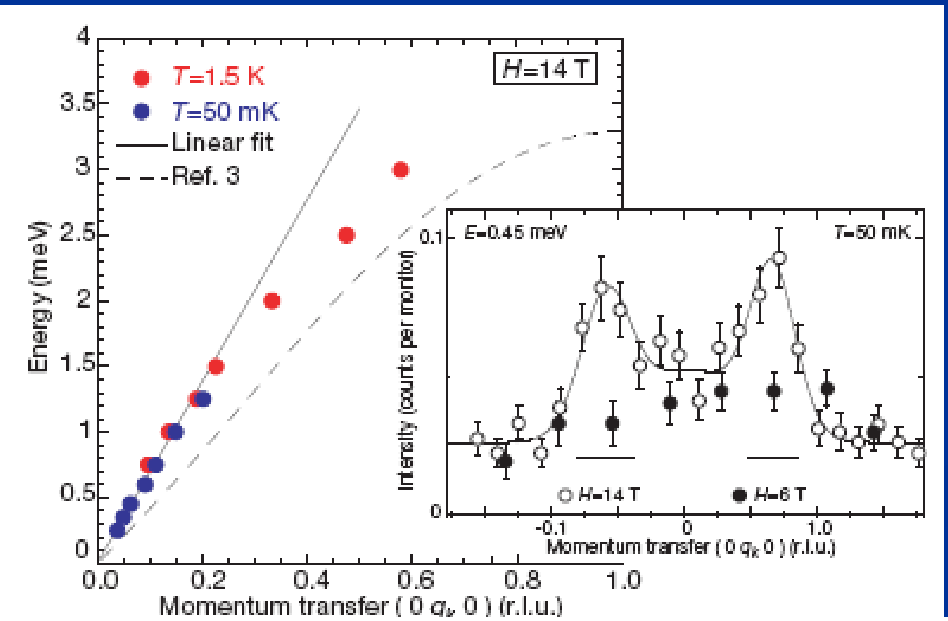
Neutron evidence



C. Ruegg et al., PRB **65** 132415 (2002)

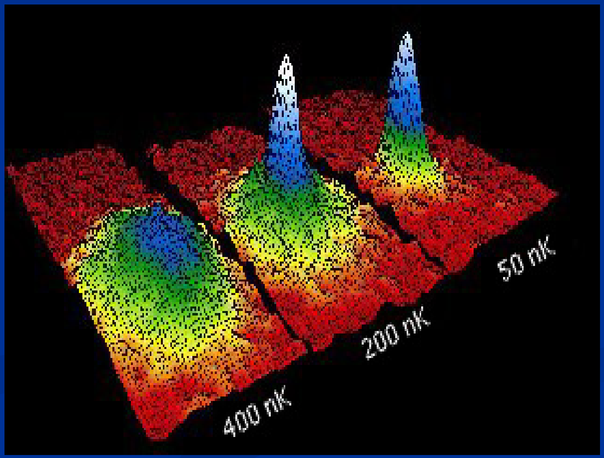


C. Ruegg et al., Nature 423 62 (2003)

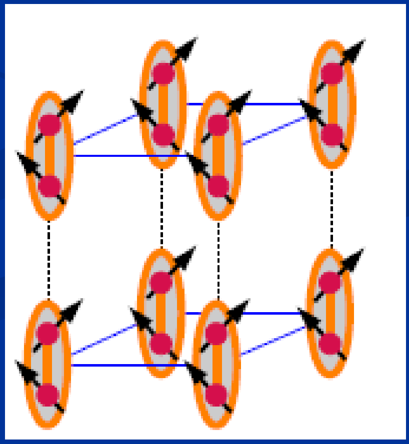


Spins – Cold atoms complementary

- Cold atoms:
  - control of lattice and parameters
  - short range interactions
  - inhomogeneous systems
  - probes

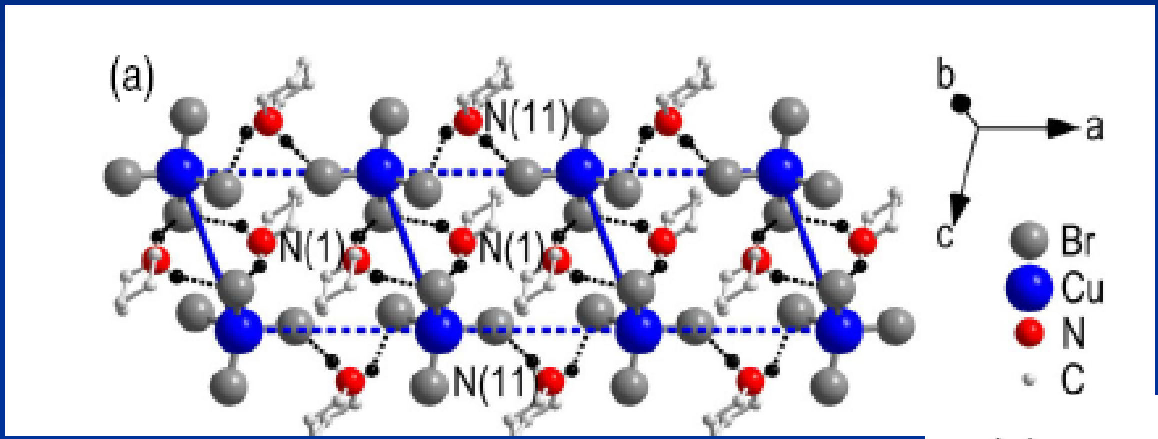


- BEC dimers/spins:
  - homogeneous, density control
  - probes
  - lattice fixed by chemistry



BPCB-HPIP

B. C. Watson et al., PRL 86 5168 (2001)

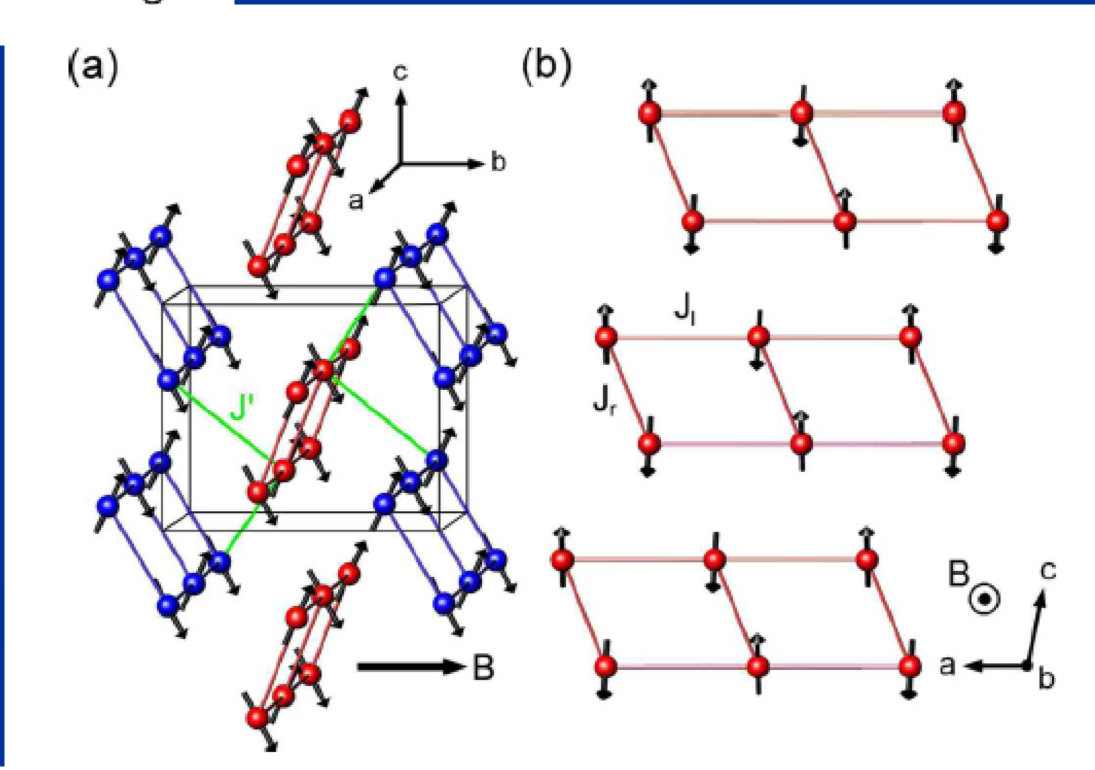


M. Klanjsek et al.,

PRL 101 137207 (2008)

B. Thielemann et al.,

PRB 79, 020408(R) (2009)



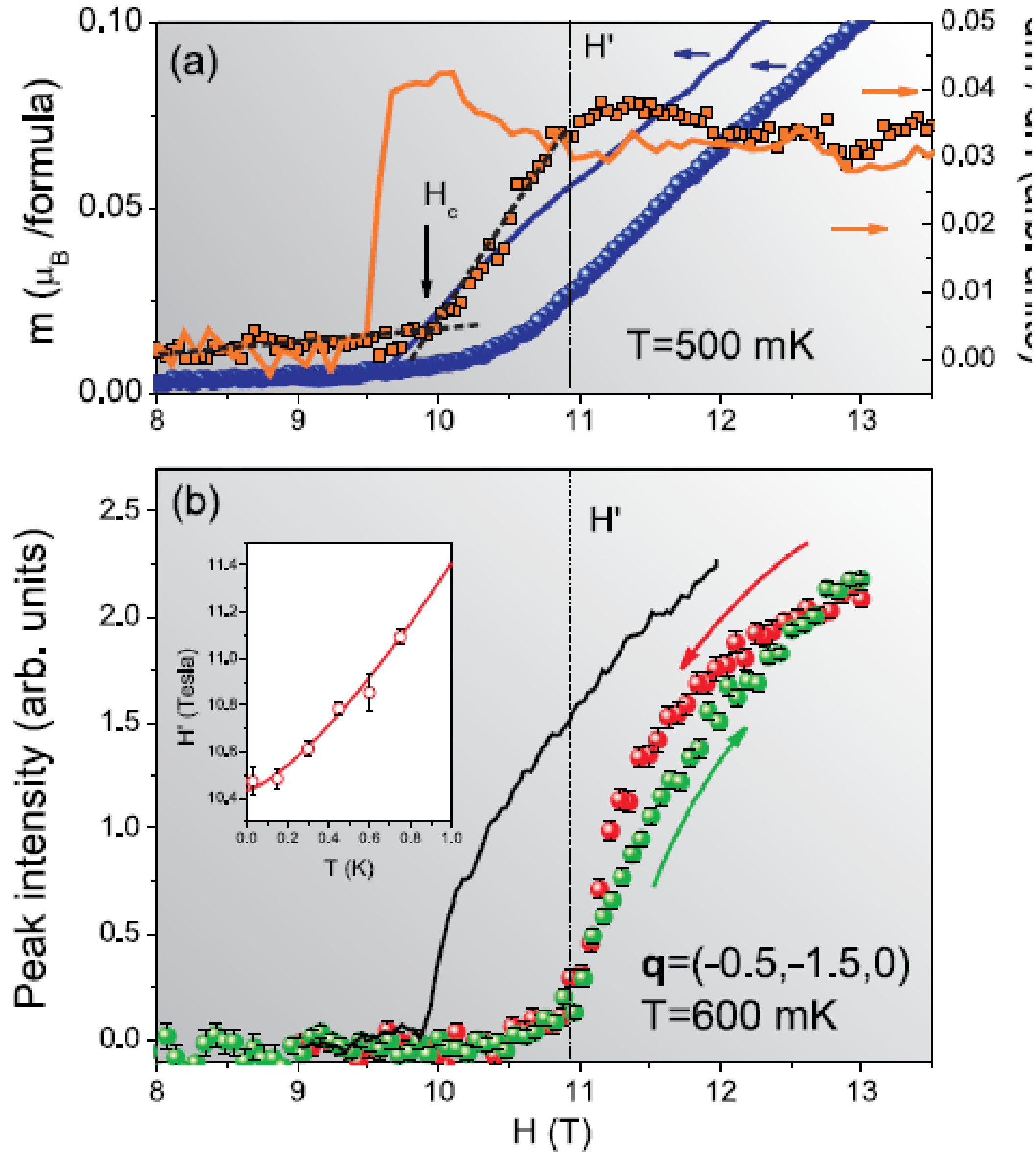
Bose glass phase

T. Hong, A. Zheludev
81 060410 (2010)

IPA-Cu(Cl0.95Br0.05)

d m/d h = compressi

$\langle S_x \rangle = \langle \psi \rangle$ superflu



Various ``disorders”

How to treat ?

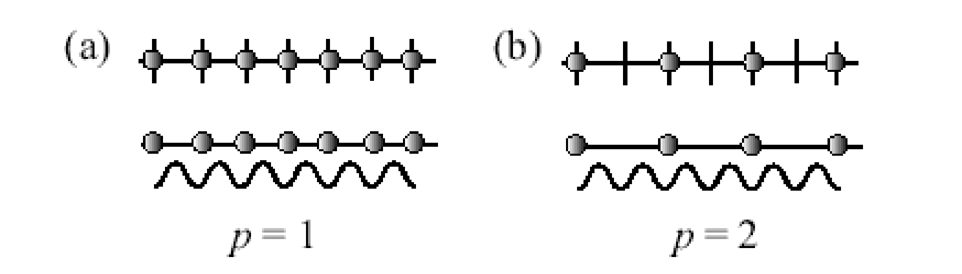
TG, Quantum physics in one dimension, Oxford (2004)

TG, cond-mat/0605472 (Salerno lectures)

- Continuum:

$$H = \int dx \frac{(\nabla\psi)^\dagger(\nabla\psi)}{2M} + \frac{1}{2}\int dx\, dx'\, V(x-x')\rho(x)\rho(x') - \mu \int dx\, \rho(x)$$

- Lattice:

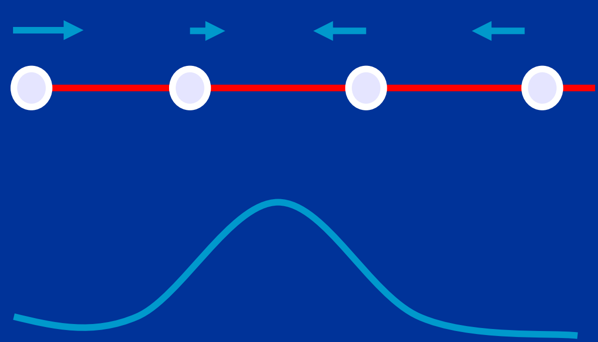


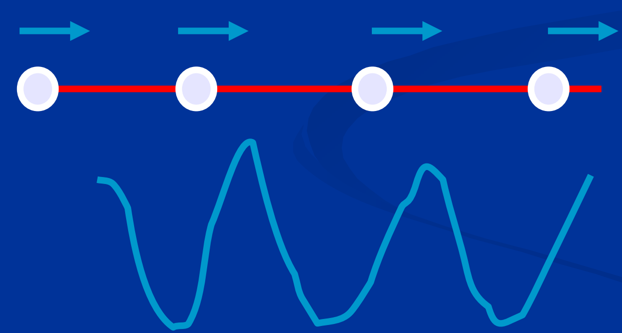
$$H = -J \sum_{\langle i,j \rangle} b_i^\dagger b_j + U \sum_i n_i(n_i - 1) - \mu \sum_i n_i$$

Bosonization

$$\rho(x) = \left[\rho_0 - \frac{1}{\pi}\nabla\phi(x)\right]\sum_p e^{i2p(\pi\rho_0 x - \phi(x))}$$

$\phi(x)$ varies slowly





CDW

$q \sim 0$

$q \sim 2\pi\rho_0$

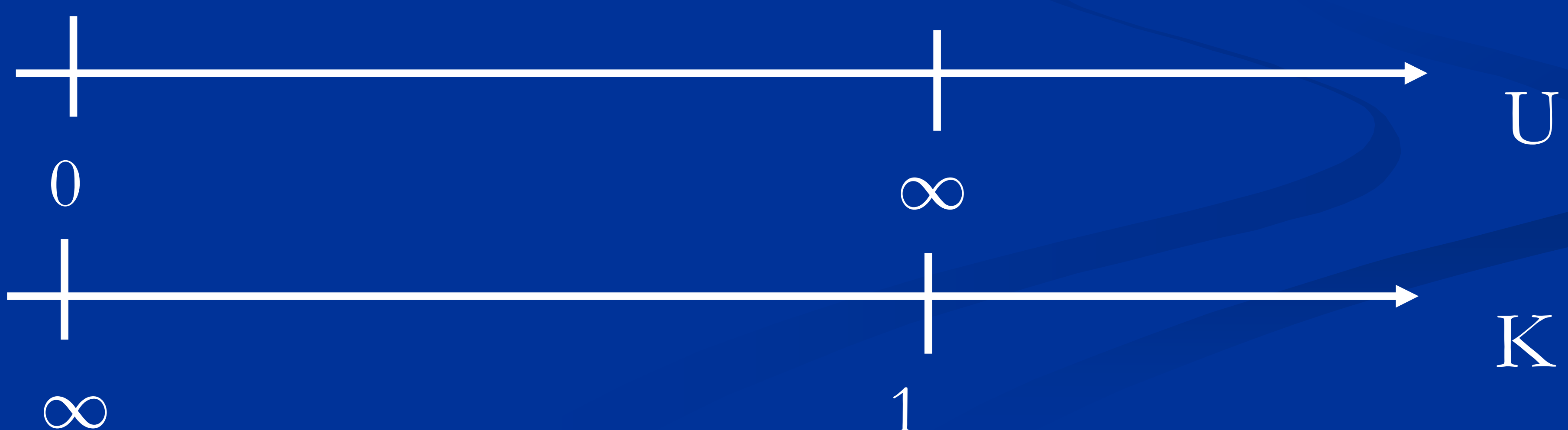
$$\psi^\dagger(x) = [\rho(x)]^{1/2} e^{-i\theta(x)}$$

θ: superfluid phase

$$[\frac{1}{\pi}\nabla\phi(x), \theta(x')] = -i\delta(x - x')$$

Quantum fluctuations

$$H = \frac{\hbar}{2\pi}\int dx[\frac{uK}{\hbar^2}(\pi\Pi(x))^2 + \frac{u}{K}(\nabla\phi(x))^2]$$



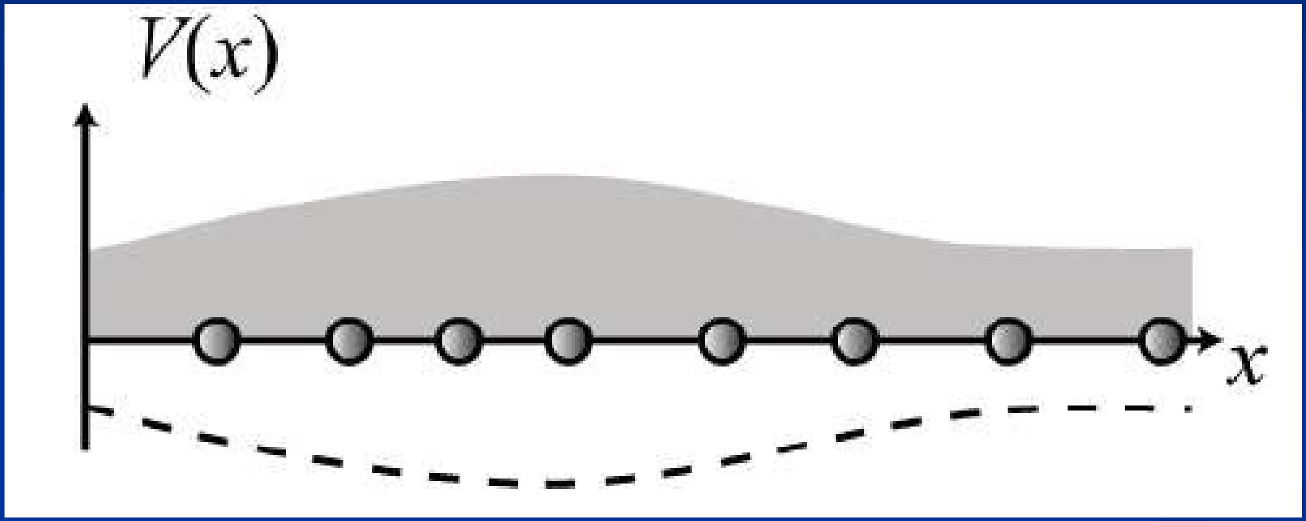
Coupling to an external potential

$$H_{\text{dis}} = \int dx\, V(x)\rho(x)$$

$$H_{\text{dis}} = \int dx\, V(x)[-\frac{1}{\pi}\nabla\phi(x) + \rho_0(e^{i(2\pi\rho_0 x - 2\phi(x))} + \text{h.c.})]$$

``Two" fourier components of disorder

Forward scattering (q ~ 0)

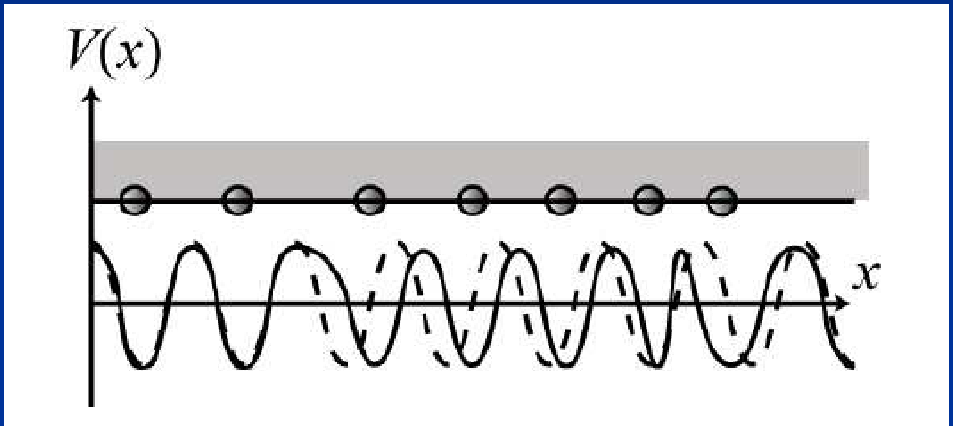


Random (smooth) chemical potential

No localization

Can break commensurate phases

Backward scattering ($q \sim 2\pi\rho_0$)



Responsible for localization (MI, Bose Glass)

Pinning of a CDW of bosons

Tonks limit: Free fermions in a potential

Different potentials

Commensurate (Mott transition): K=2

Disorder (Bose Glass): K=3/2

Quasiperiodic ??

Renormalization treatment

J. Vidal, D. Mouhanna, TG PRL 83 3908 (1999); PRB 65 014201 (2001)

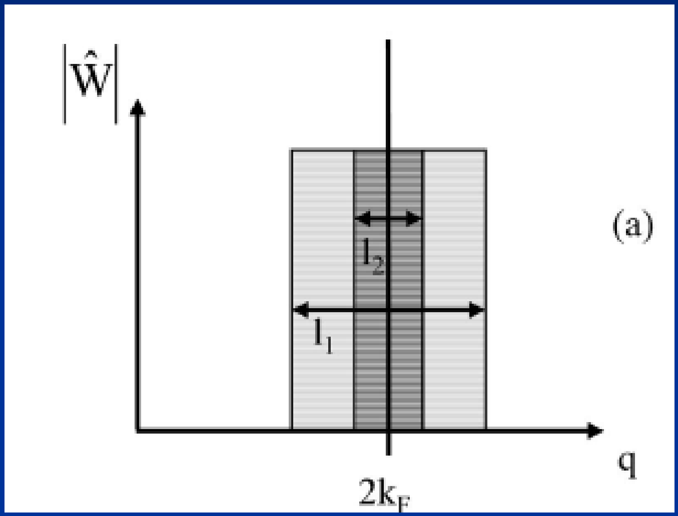
$$\frac{dK}{dl} = -K^2 \Xi(l),$$

$$\frac{dy_Q}{dl} = (2-K) y_Q,$$

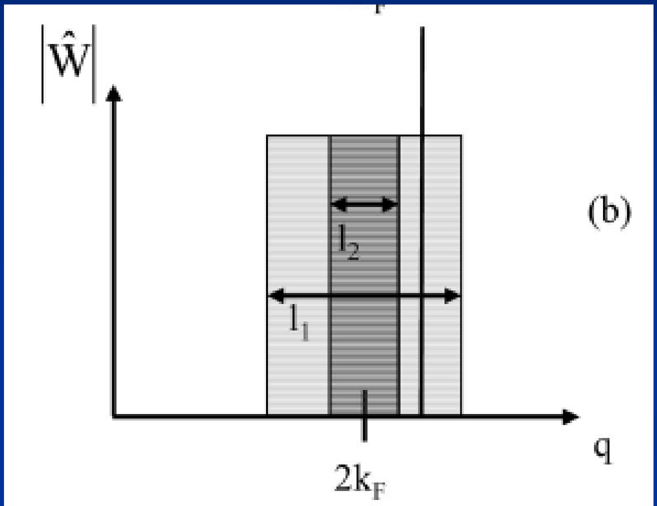
y_Q : Fourier components of potential

$$\Xi(l) = \frac{1}{2} \sum_Q y_Q^2 [\mathrm{J}(Q^+ \alpha(l)) + \mathrm{J}(Q^- \alpha(l))],$$

Periodic



commensurate



Incommensurate

$$\frac{dK}{dl} = -K^2 \Xi(l),$$

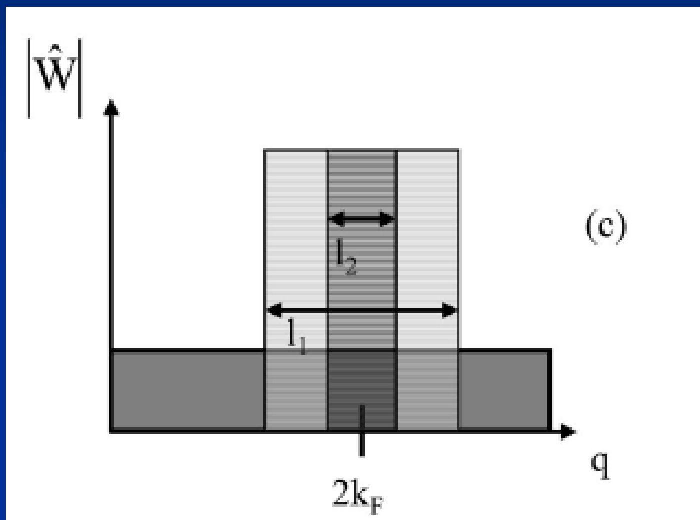
$$\frac{dy_Q}{dl} = (2-K) y_Q,$$

Comm: Ξ = Cste

Incomm: $\Xi = 0$

$K_c = 2$

Disordered



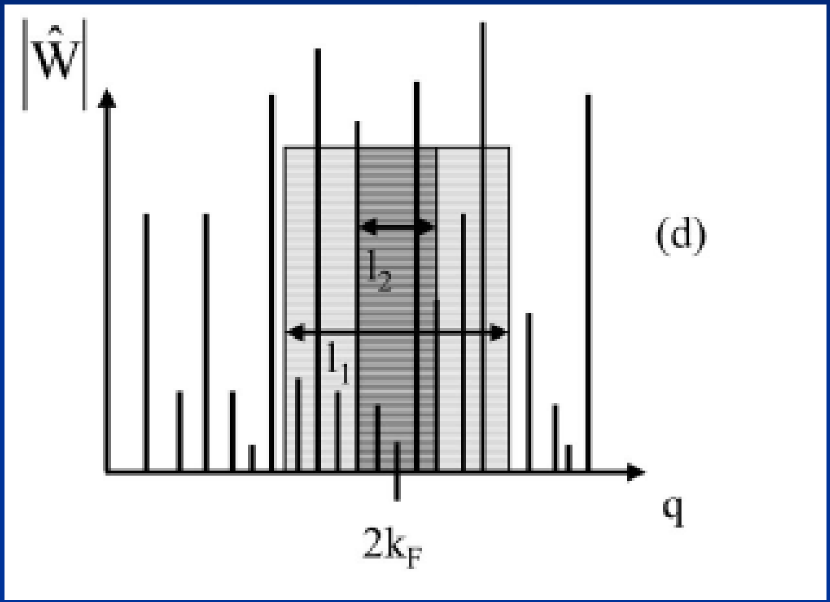
$$\frac{dK}{dl} = -K^2 \Xi(l),$$

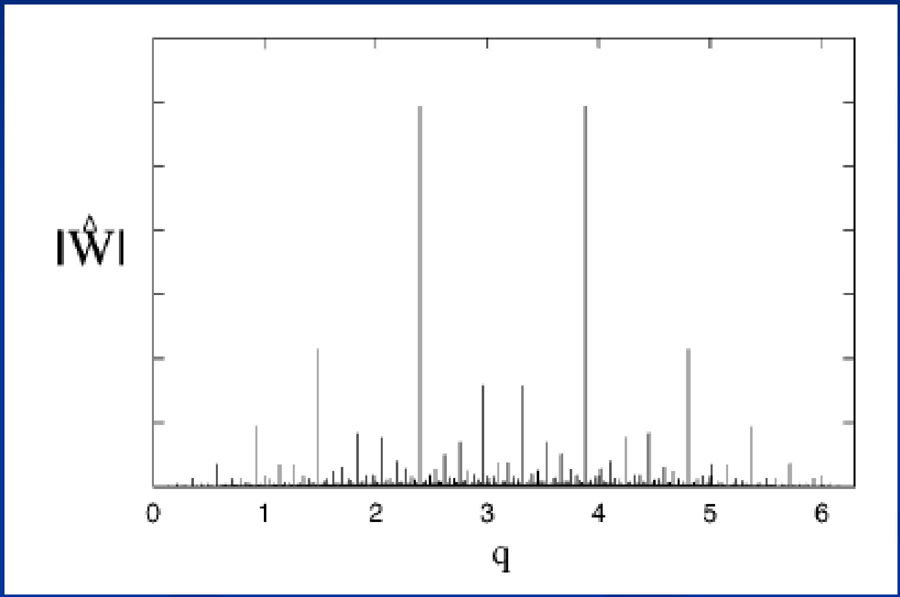
$$\frac{dy_Q}{dl} = (2-K)y_Q,$$

$$\Xi \propto y_Q^{\;2}\, e^{-l}$$

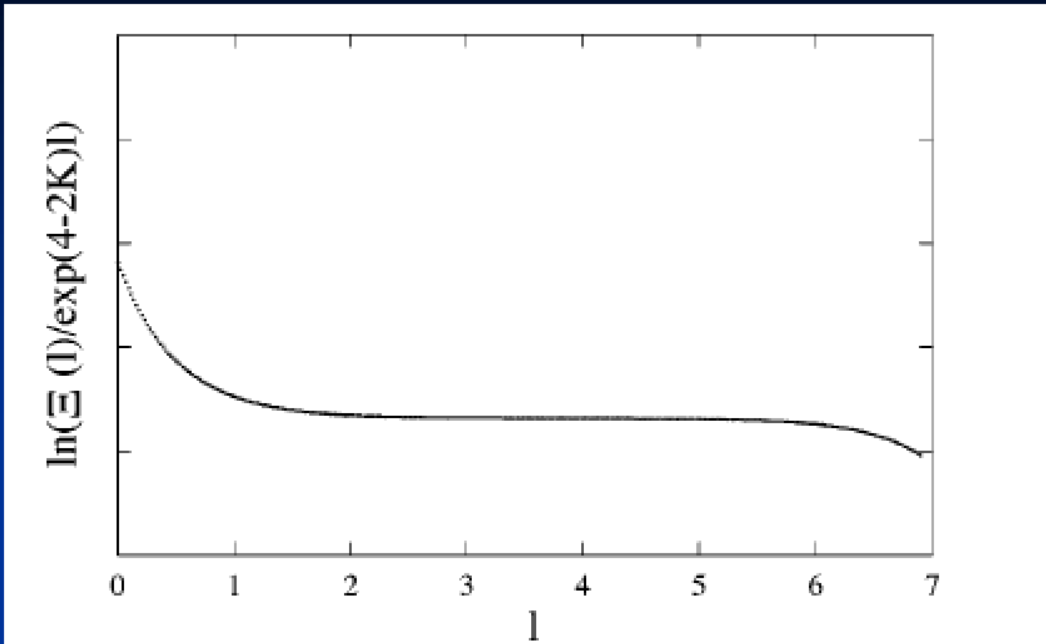
$$K_c = 3/2$$

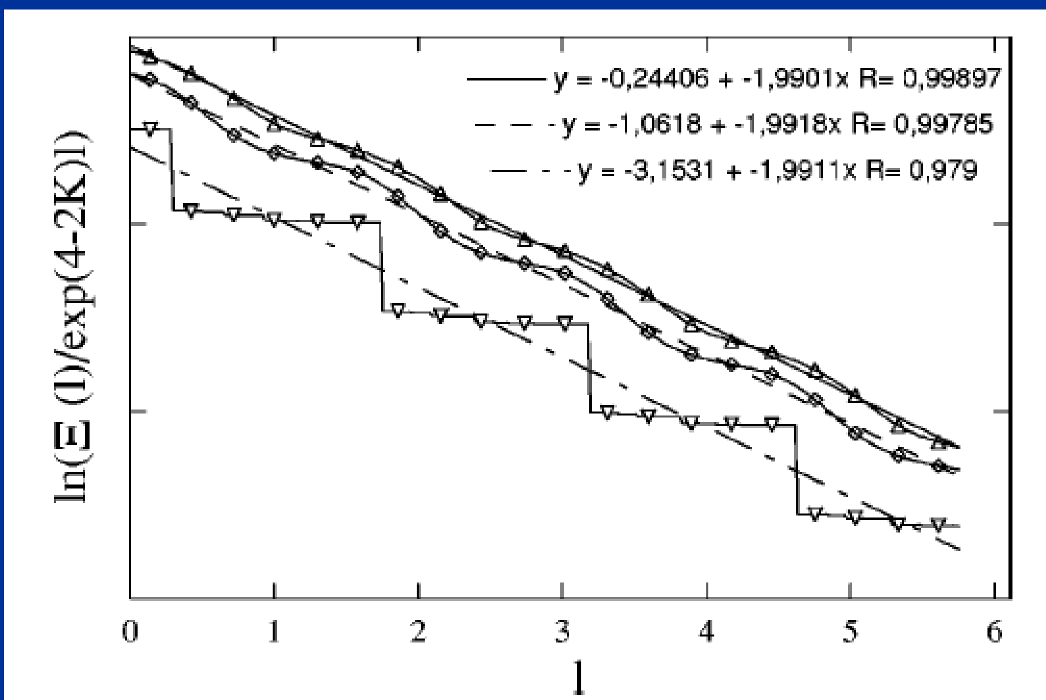
Quasiperiodic





Fibonacci potential





dominant peak

``commensurate”

$$\Xi(l) \sim e^{(4-2K-2)l}$$

$K_c = 1$

Harper potential

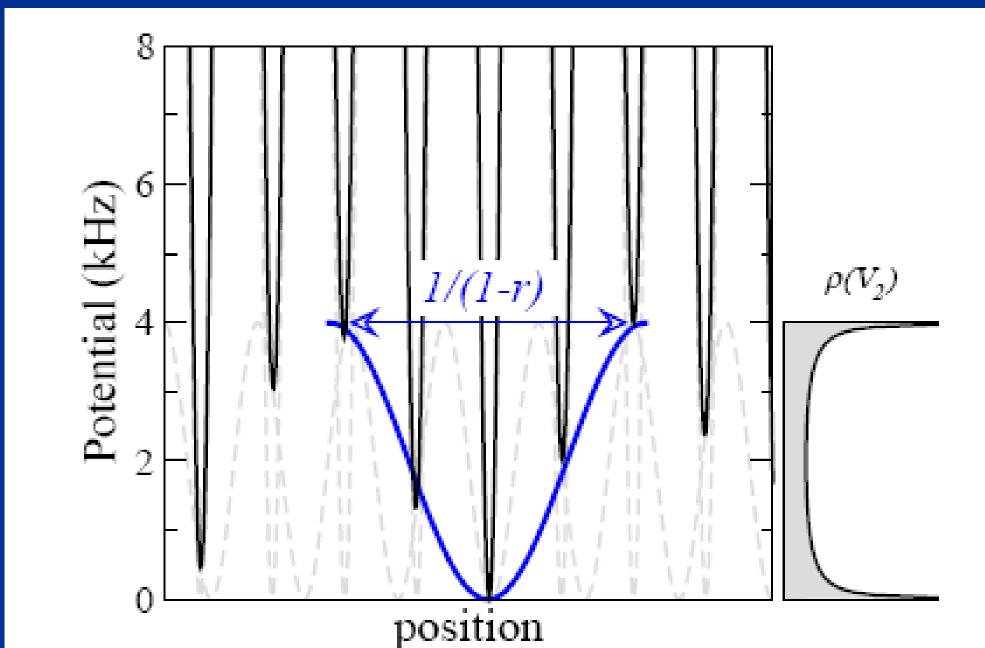
$V(x) = V_1 \, Cos(Q_1 x) + V_2 \, Cos(Q_2 x)$
[Harper potential]

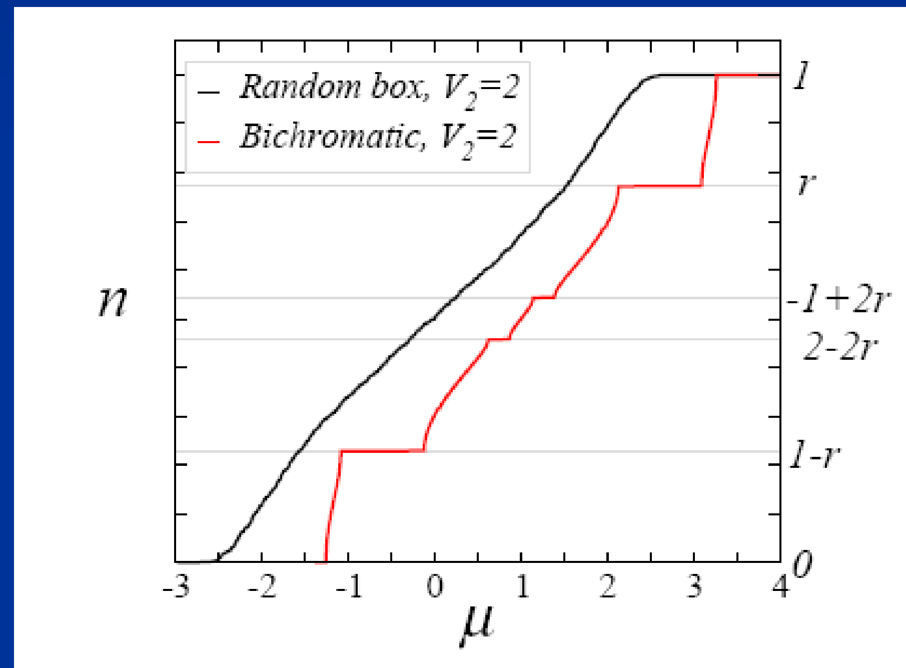
- Non perturbative: $p \, Q_1 + n \, Q_2$

- Noninteracting (Aubry-Andre): transition at $V_2 = 4$

Numerics: DMRG

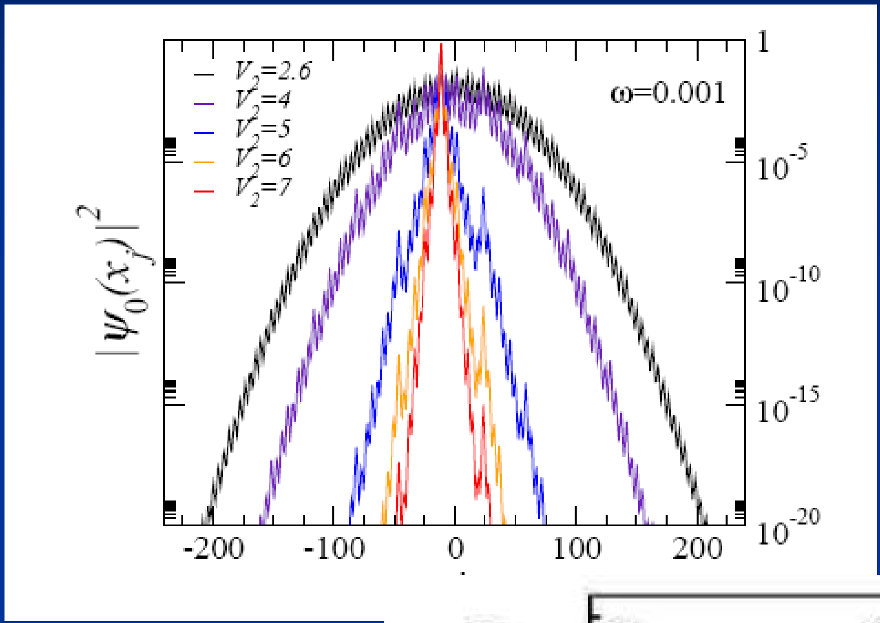
G. Roux et al. PRA 78 023628 (2008)

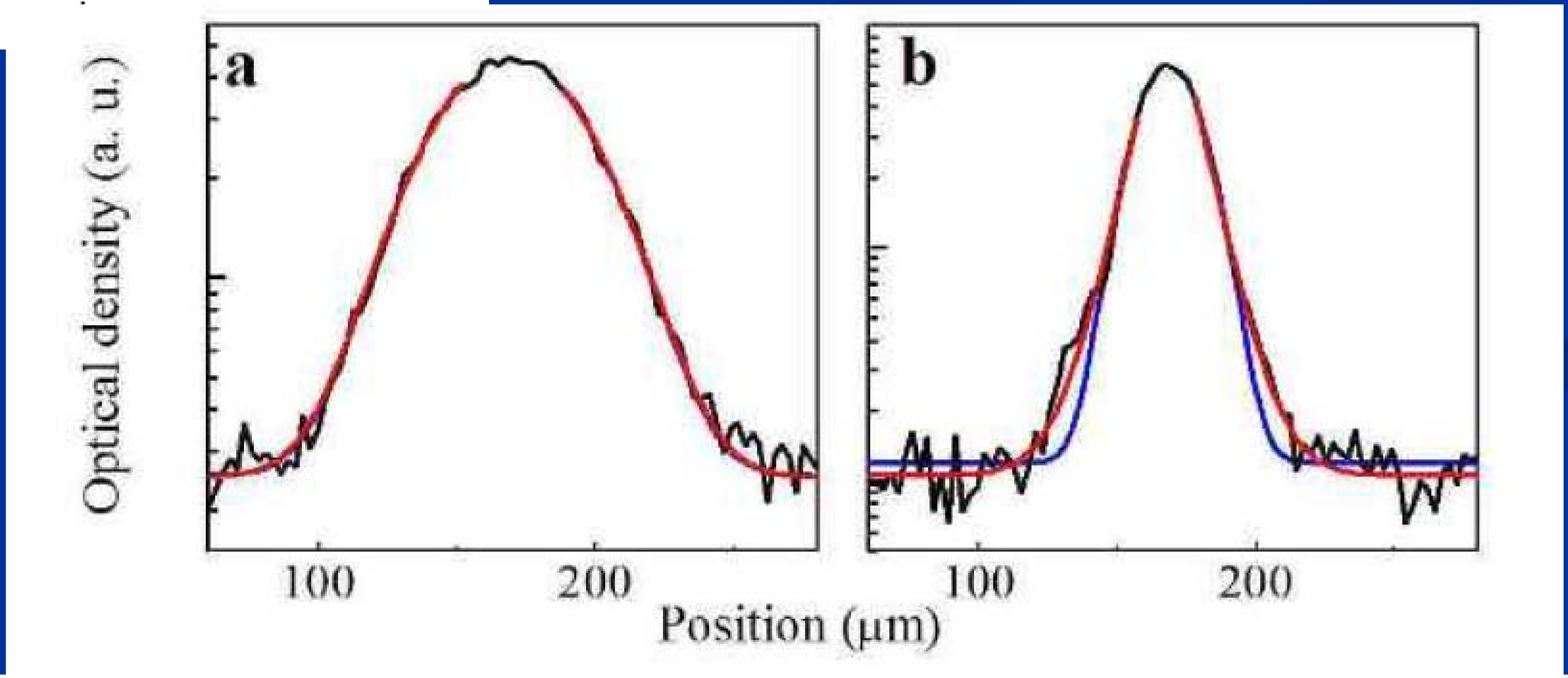




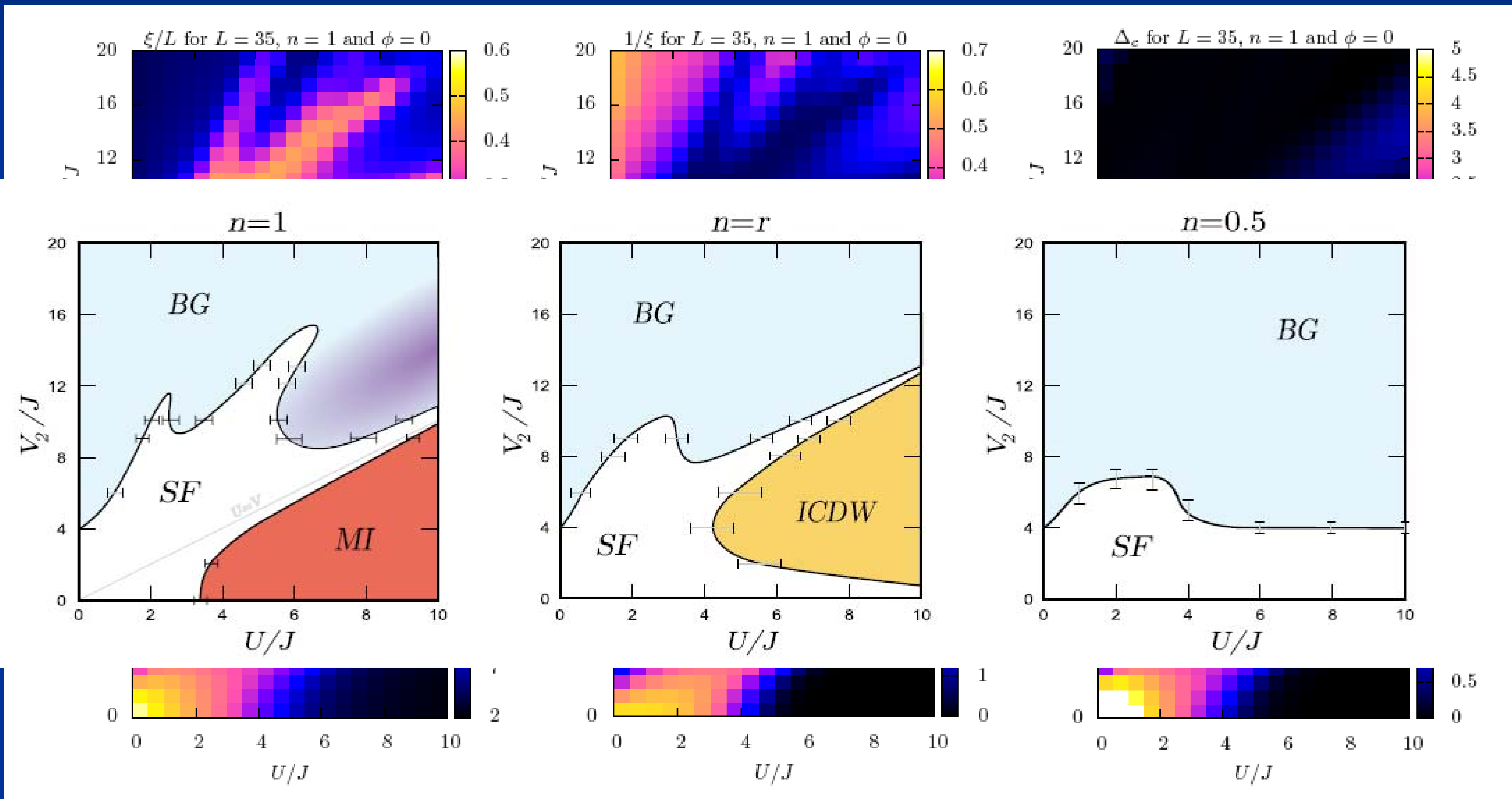
Related works: T. Roscilde, Phys. Rev. A 77, 063605 2008; X. Deng et al PRA 78, 013625 (2008); arXiv:0812.3479v1

Noninteracting

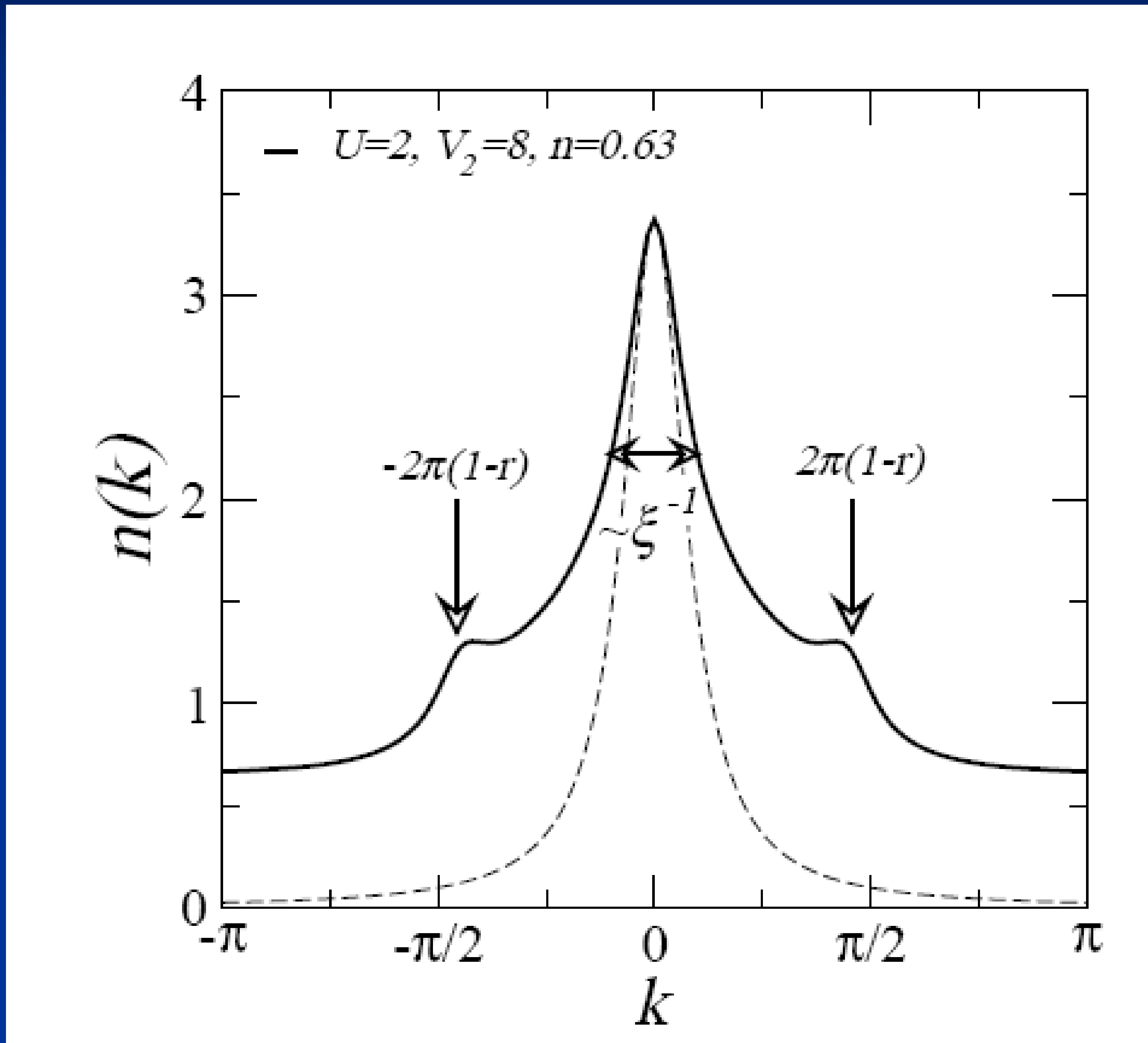




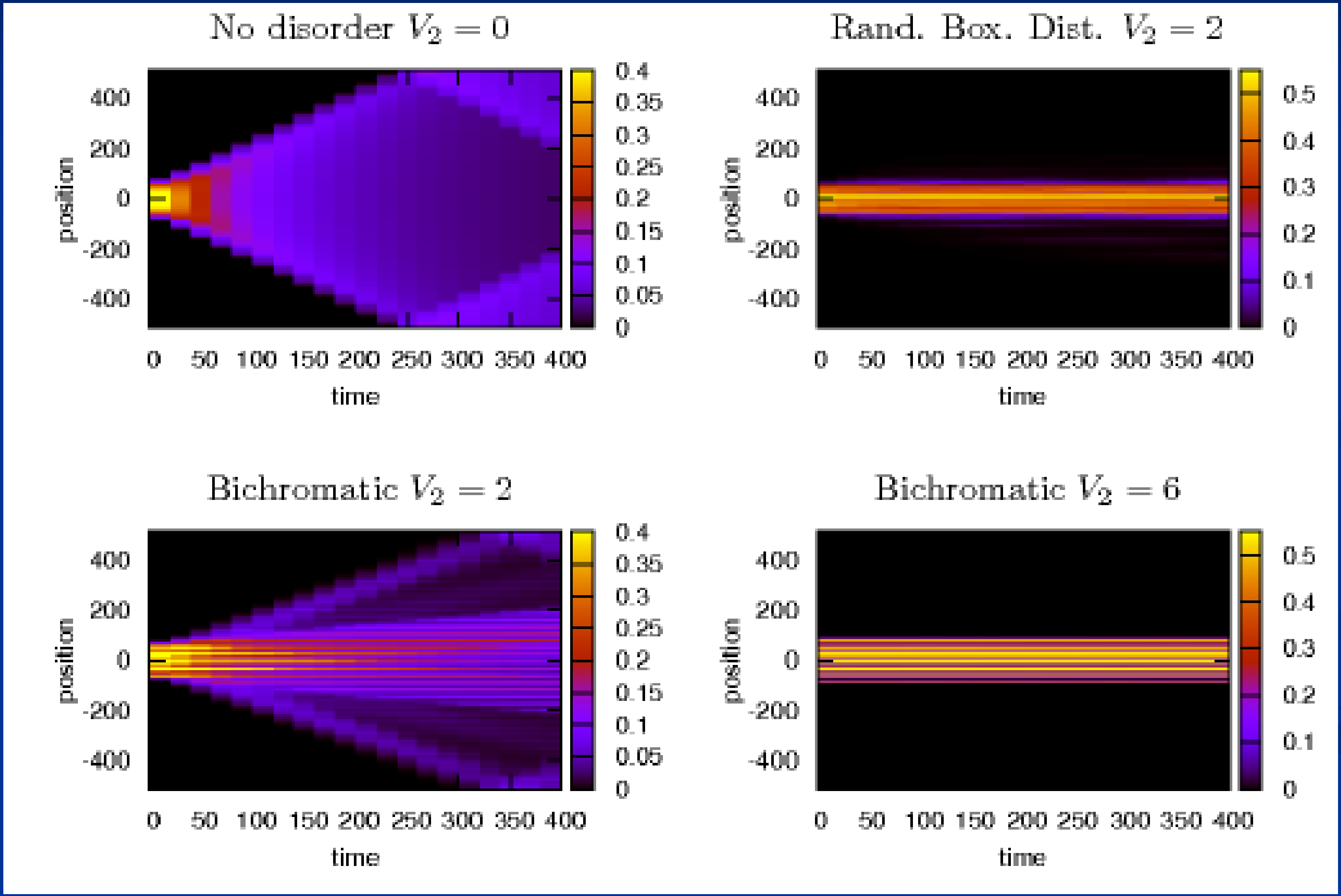
Phase diagram



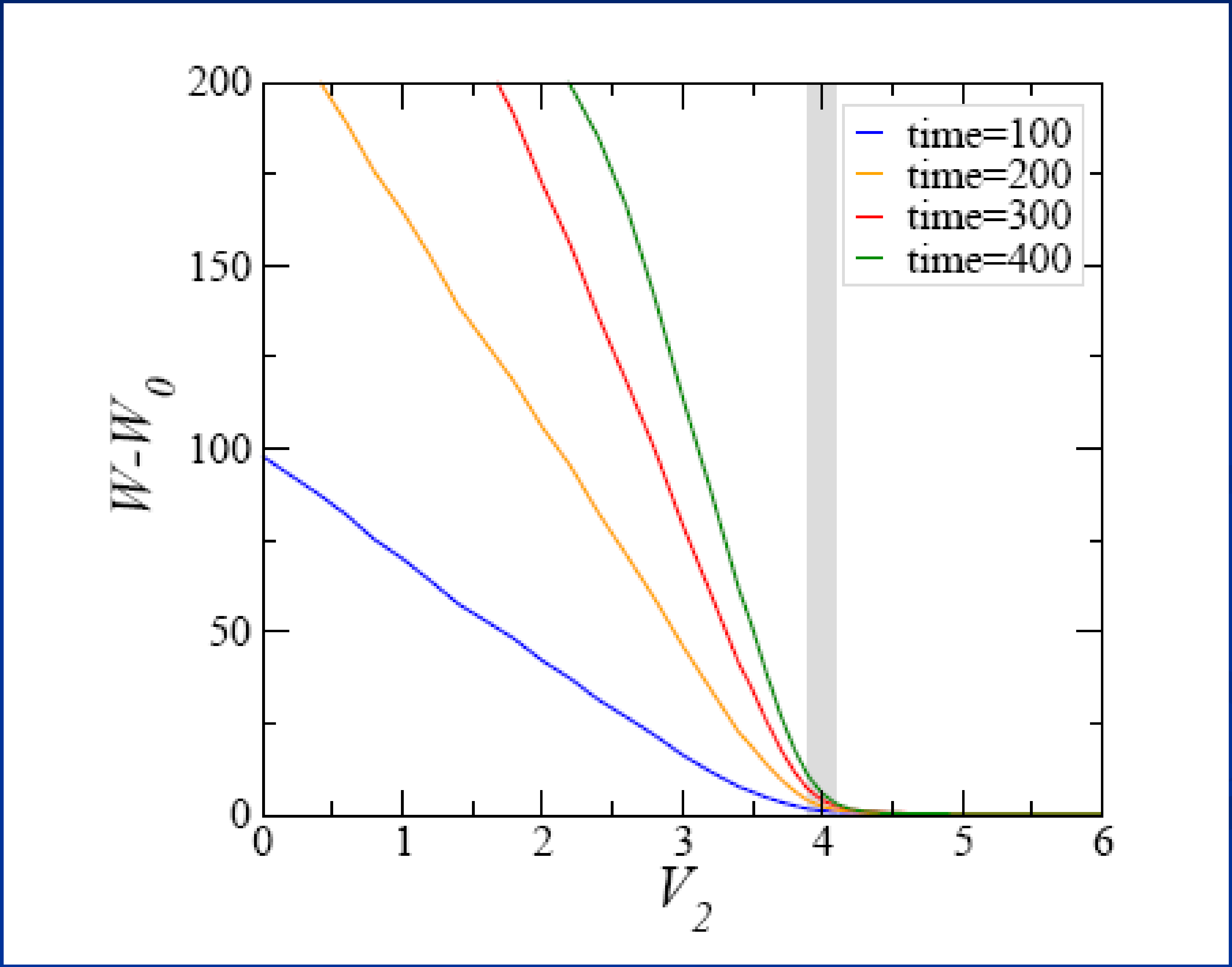
Momentum distribution



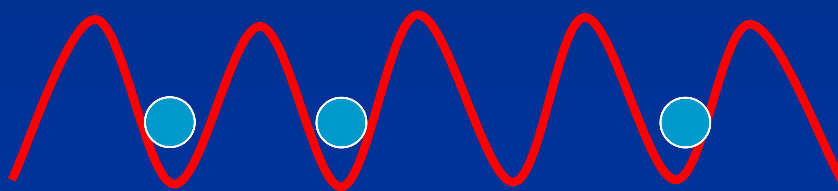
Expansion


No disorder $V_2 = 0$
Rand. Box. Dist. $V_2 = 2$
Bichromatic $V_2 = 2$
Bichromatic $V_2 = 6$
position
time
400
200
0
-200
-400
0 50 100 150 200 250 300 350 400
0.4
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
0.5
0.4
0.3
0.2
0.1
0

Dynamical criterion



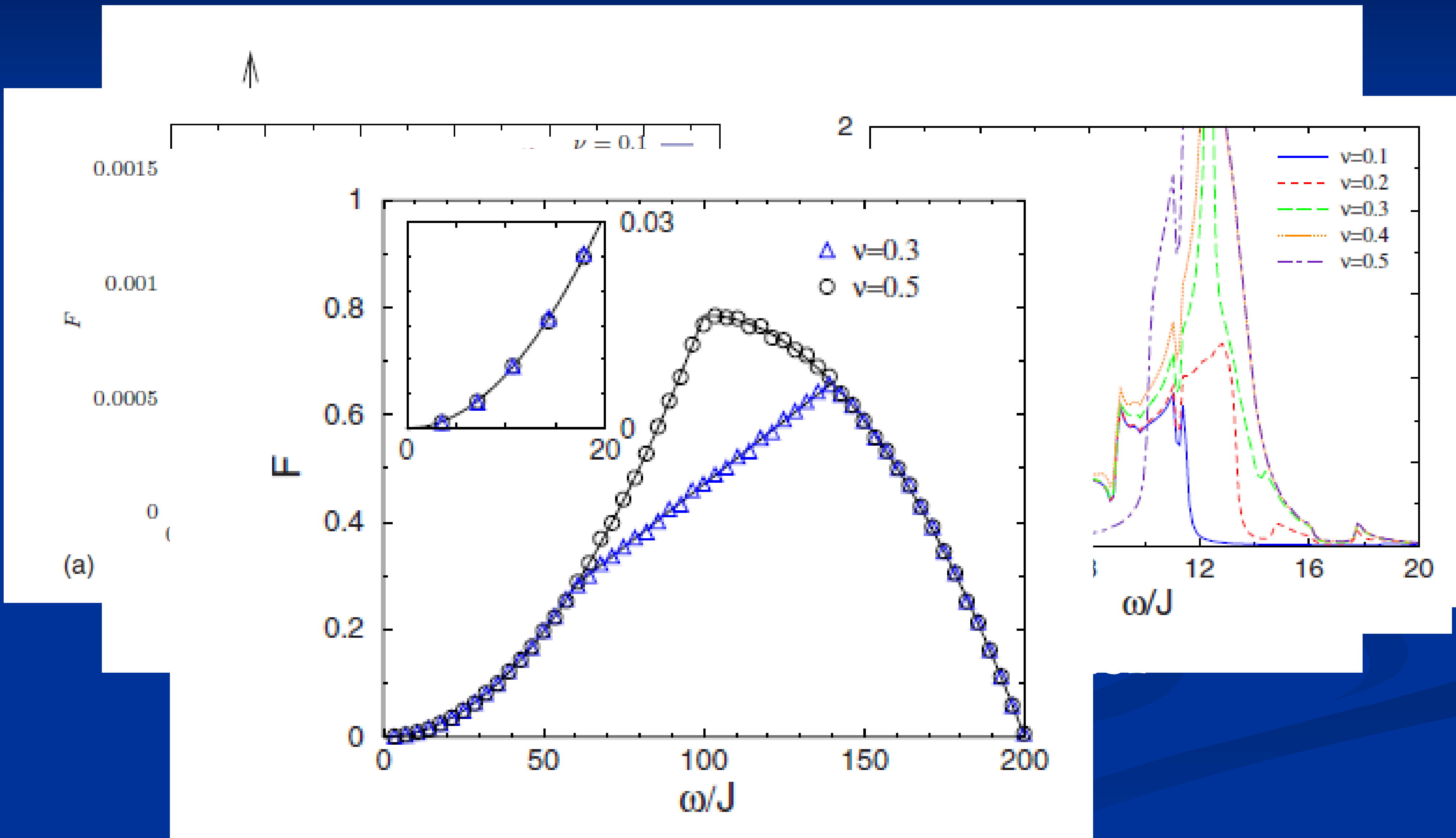
Shaking of the lattice



- Modulate amplitude of optical lattice at frequency ω
- Mesure the absorbed energy

$$\langle \frac{dE}{dt} \rangle \propto \omega \mathrm{Im} \langle [H_K, H_K] \rangle$$

G. Orso, A. Iucci, M. Cazalilla, TG PRA 80 033625 (2009)



Conclusions/Perspectives

- Quasiperiodic: many similarities with disorder
- ``Bose glass" phase (localized, compressible)
- Dynamical and thermodynamic criterion
- Shaking : marked differences
- Experiments (transport, compressibility)