Creep dynamics via exact transition pathways in disordered media

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REFS:
PRL 97, 057001 (2006); **PRB 79, 184207 (2009).**

Outline

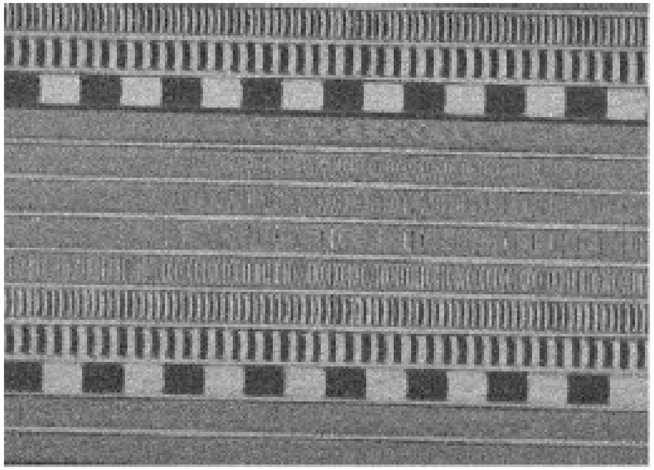
- Motivation
- Models & Universality
- Exact algorithm for ultra-slow creep motion
- Steady-State Geometry
- Barriers & steady-state transport
- Conclusions

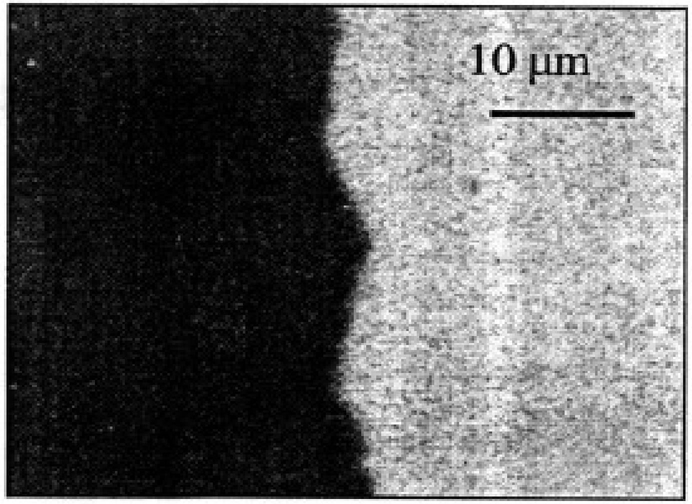
Motivation

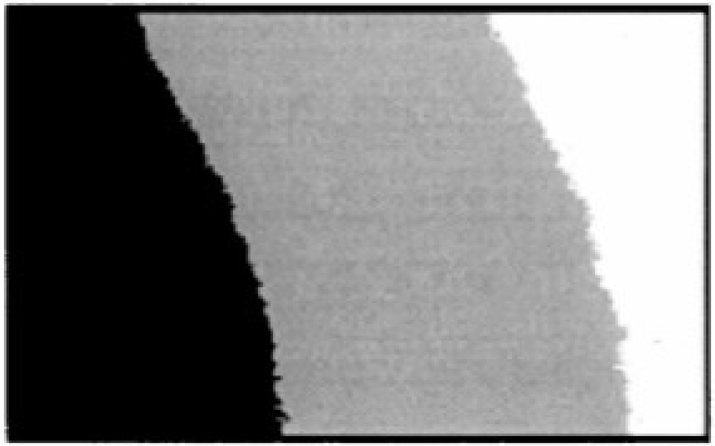
Motivation

Lemerle, Jamet, Ferre, et al (LPS Orsay)

Magnetic Domain Walls

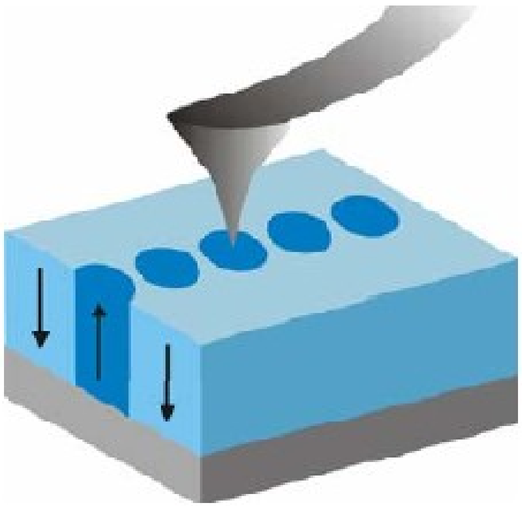


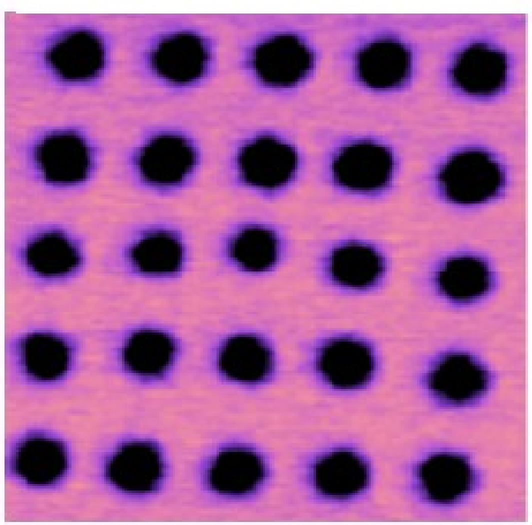


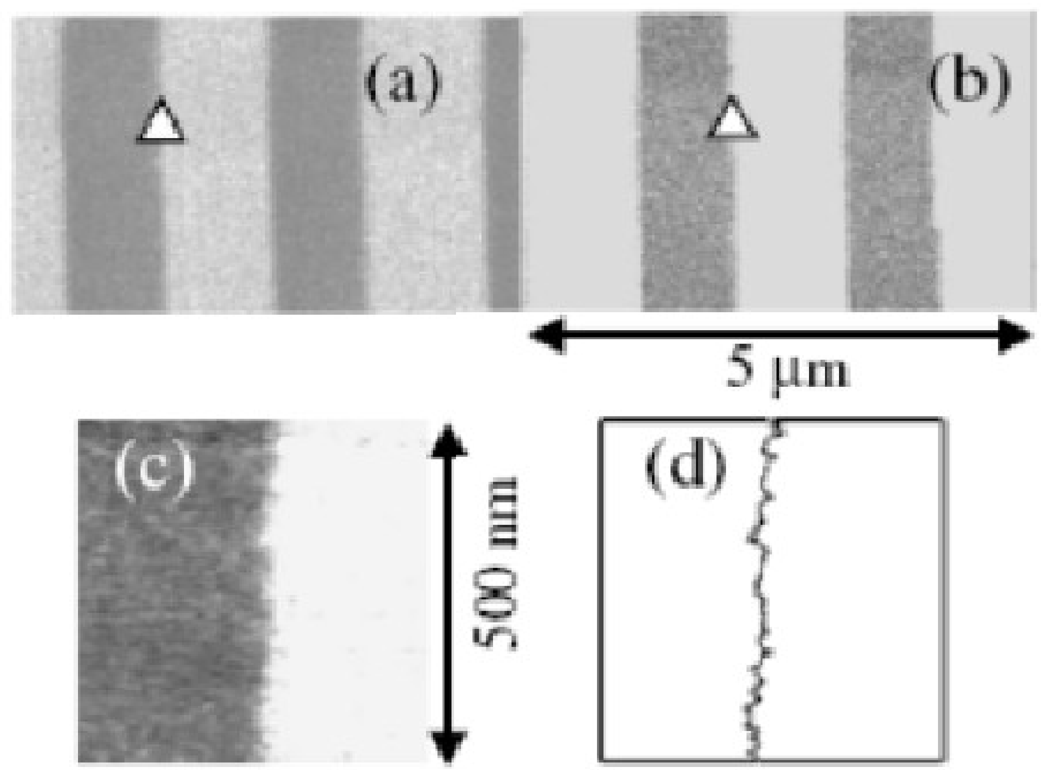


Lemerle, Jamet, Ferre, et al (LPS Orsay)

Electric domain walls

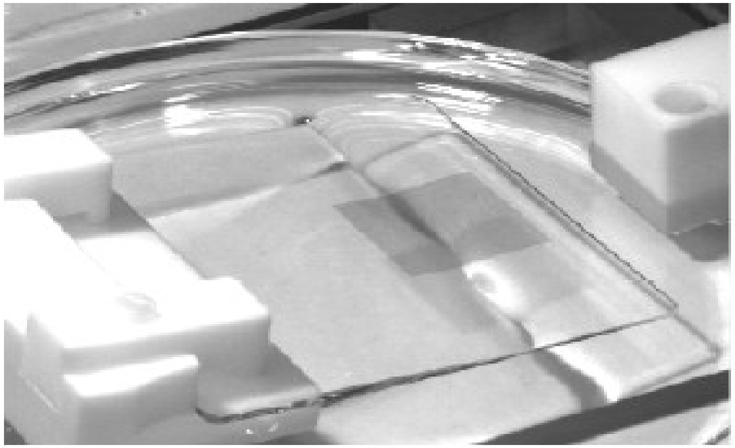


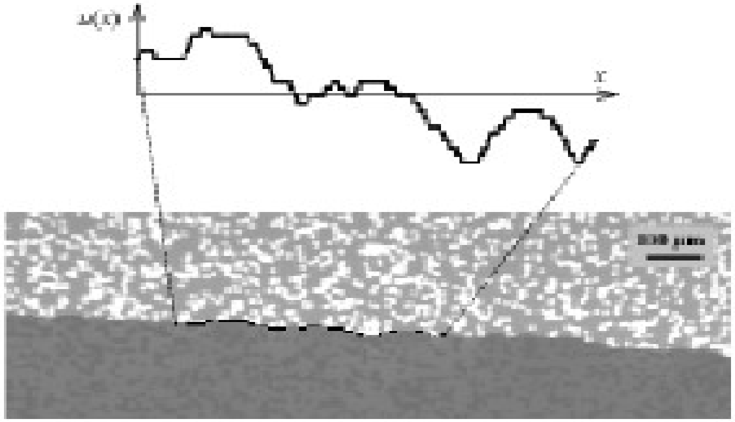




Paruch, Tybell, Triscone, University of Geneva. (2002,2005)

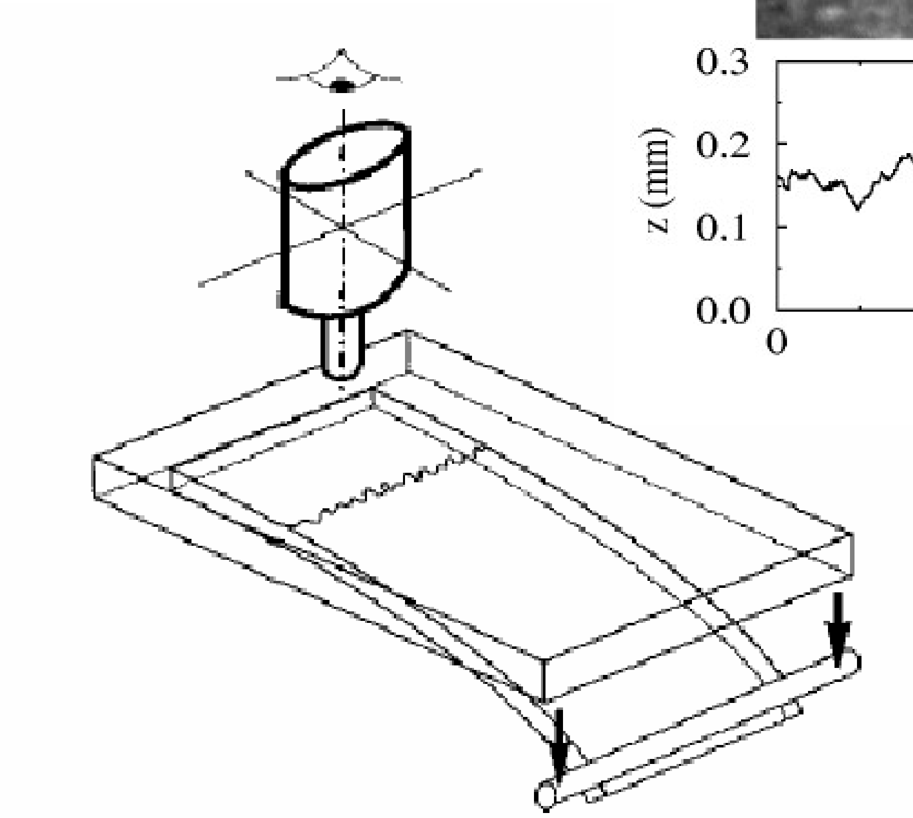
Contact lines in partial wetting

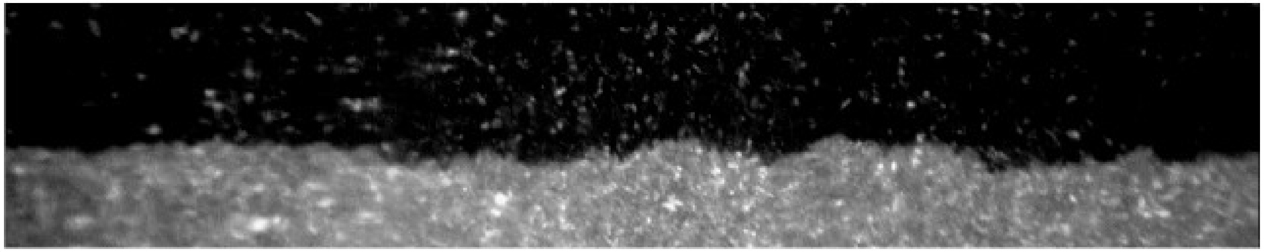


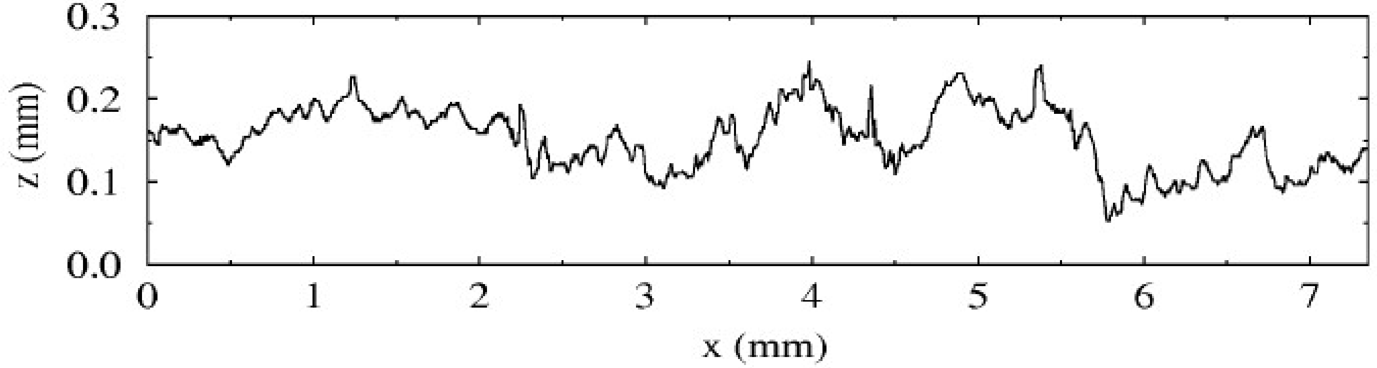


Moulinet, Rolley *et al*, LPS-ENS Paris. (2004)

Crack propagation







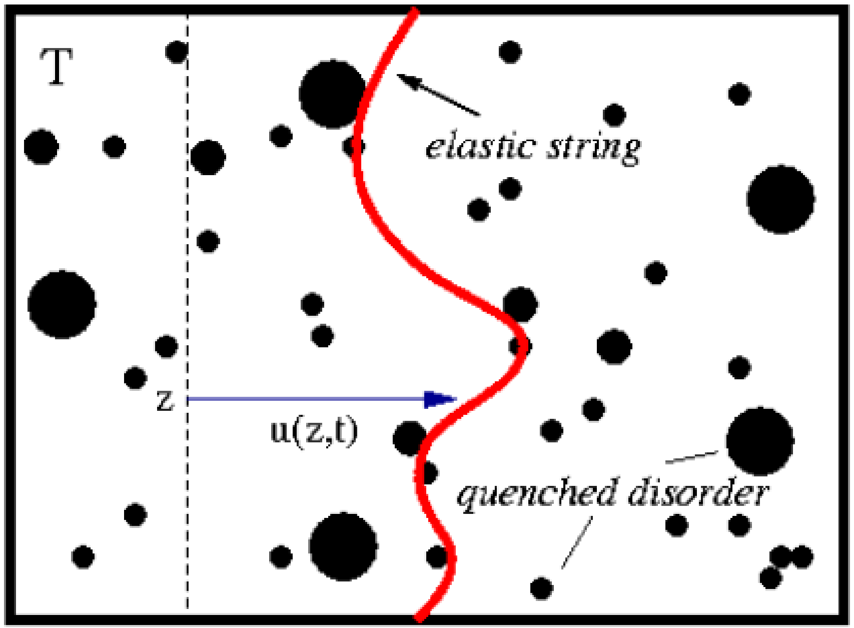
J. Schmittbuhl and K. J. Måløy (1997)

“Universal” dynamics of disordered elastic systems

- Many experimental situations share the same effective physics ...
- The effective physics can be described by minimal models...
- Quantitative predictions can (still) be made ...

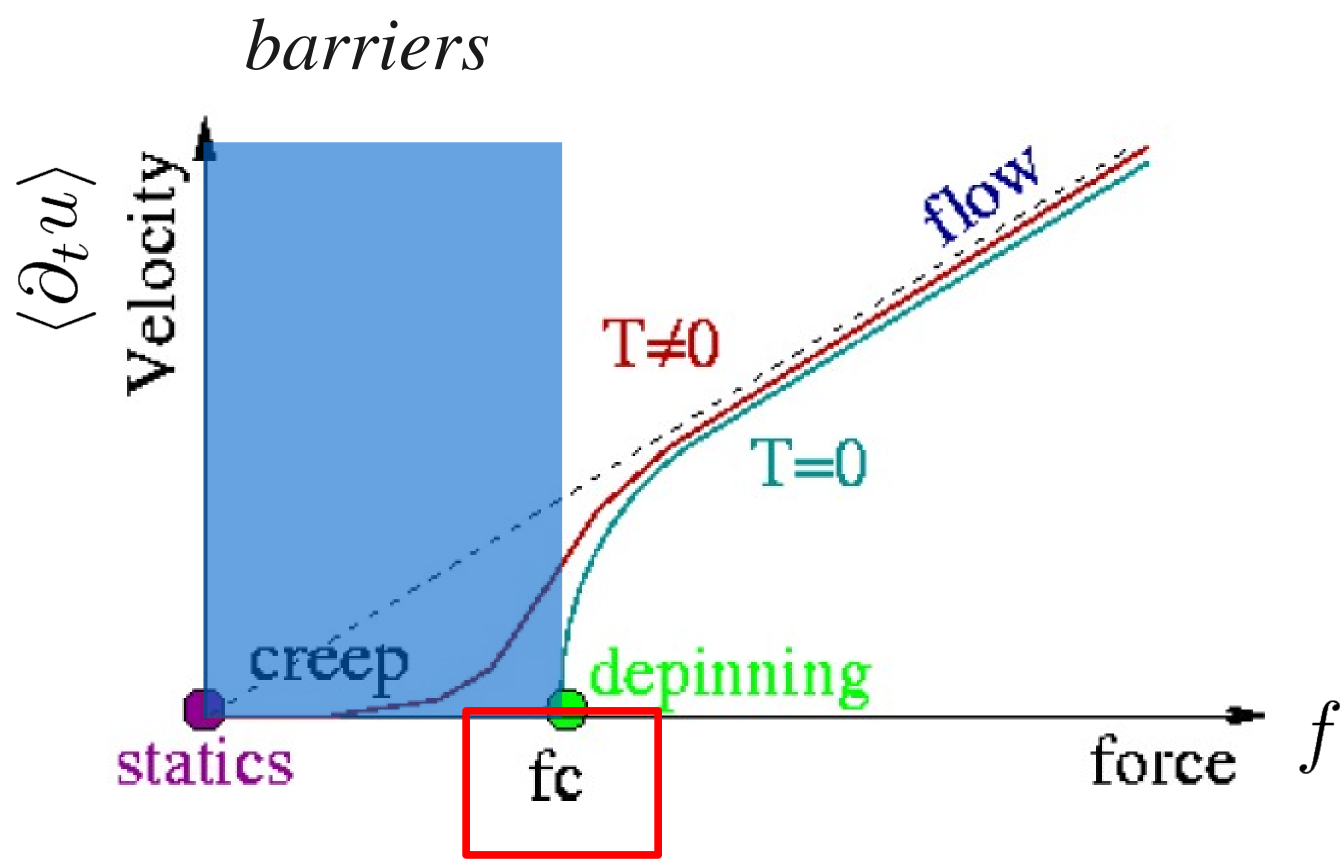
Minimal models, Universality

$$\gamma \partial_t u(z,t) = -\frac{\delta H_{el}}{\delta u} + F_p(u,z) + f + \eta_T(z,t)$$



- Short range:
  $H_{el} = \int dz [\partial_z u(z,t)]^2 + \ldots$
- Long range:
  $H_{el} = \int dz_1 dz_2 \frac{[u(z_1,t)-u(z_2,t)]^2}{(z_1-z_2)^2} + \ldots$
- Disorder:
  $\overline{F_p(u,z)F_p(u',z')} = \Delta[u-u']\delta(z-z')$

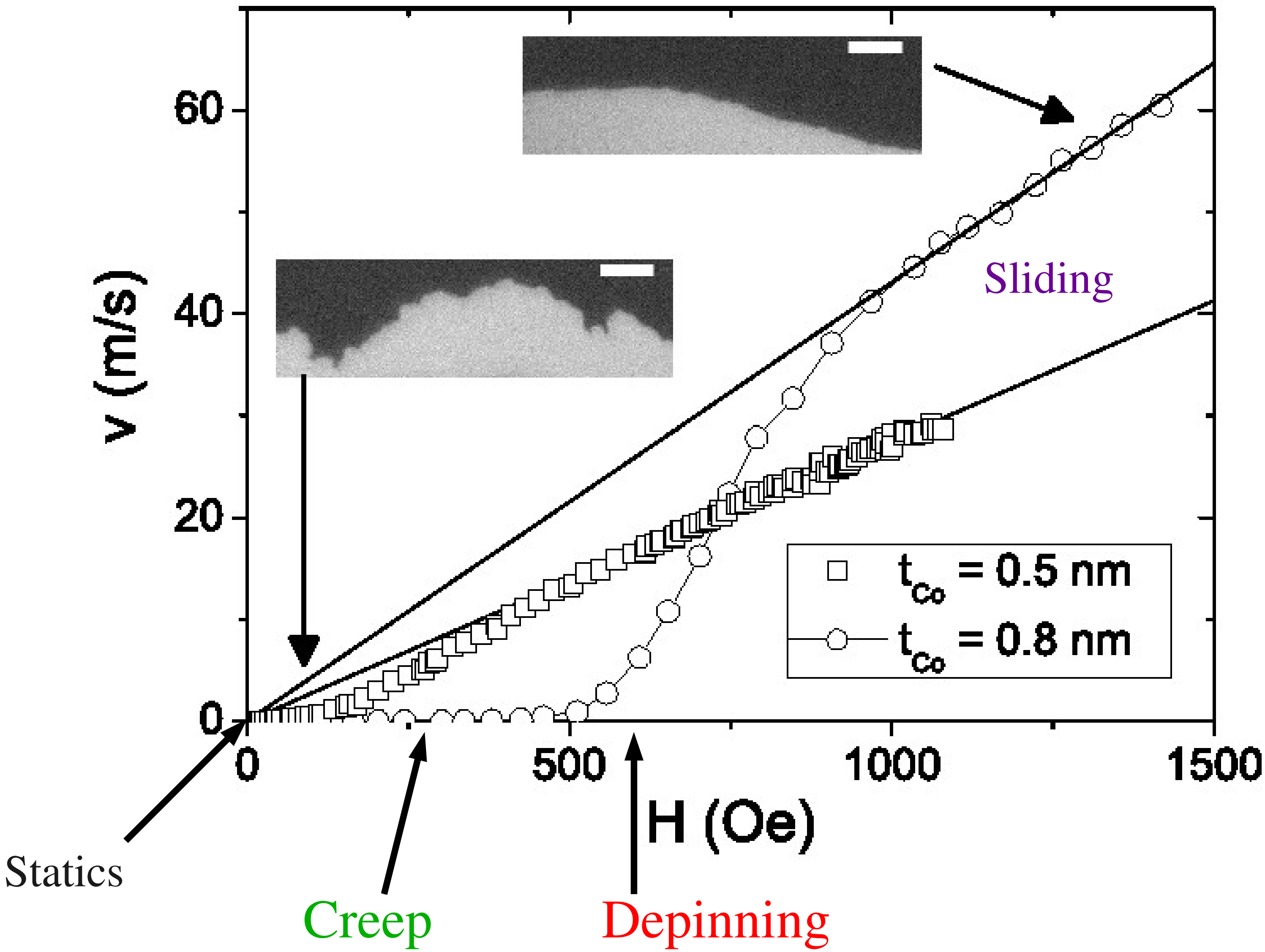
Collective transport in a disordered medium



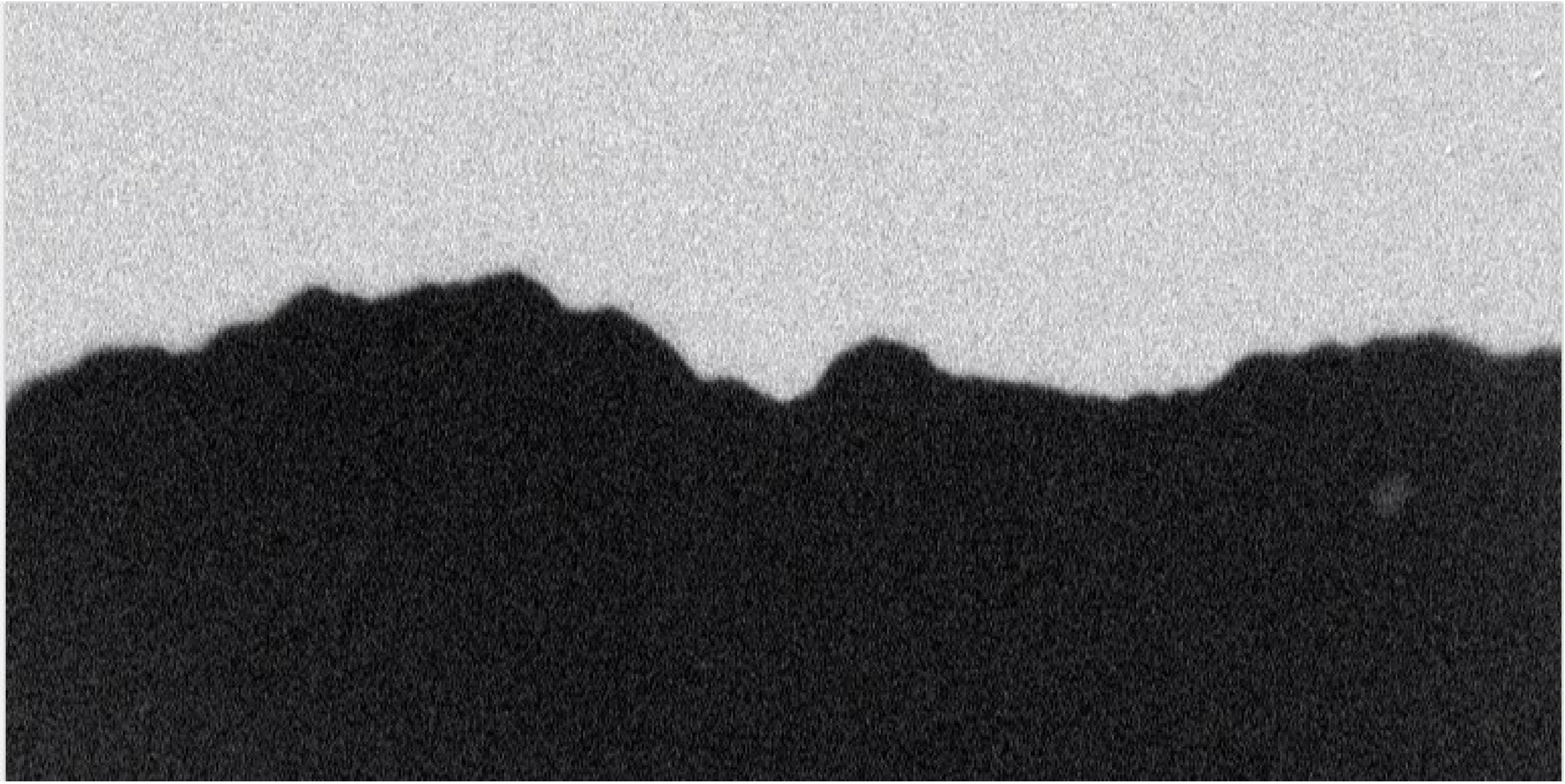
$$\gamma \partial_t u(z,t) = c \partial_z^2 u(z,t) + F_p(u,z) + f \ \ + \eta_T(z,t)$$

Geometry and transport

P.J. Metaxas et al, LPS Orsay/CEA Grenoble (2007)

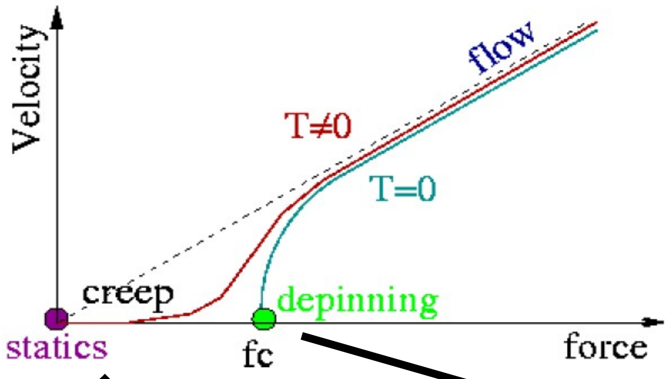


Steady-State dynamical structure

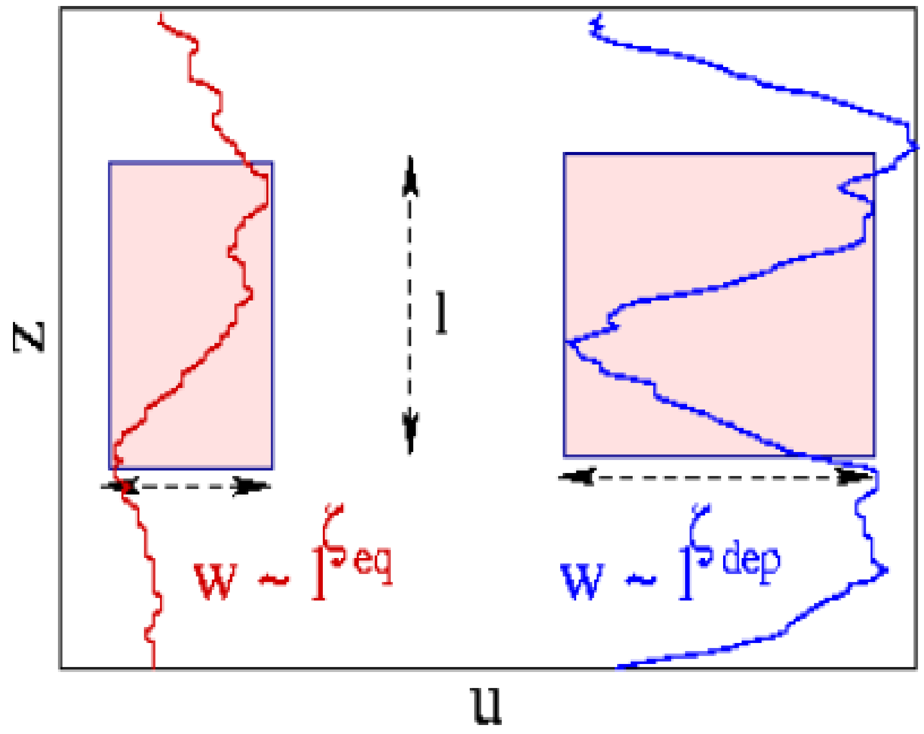


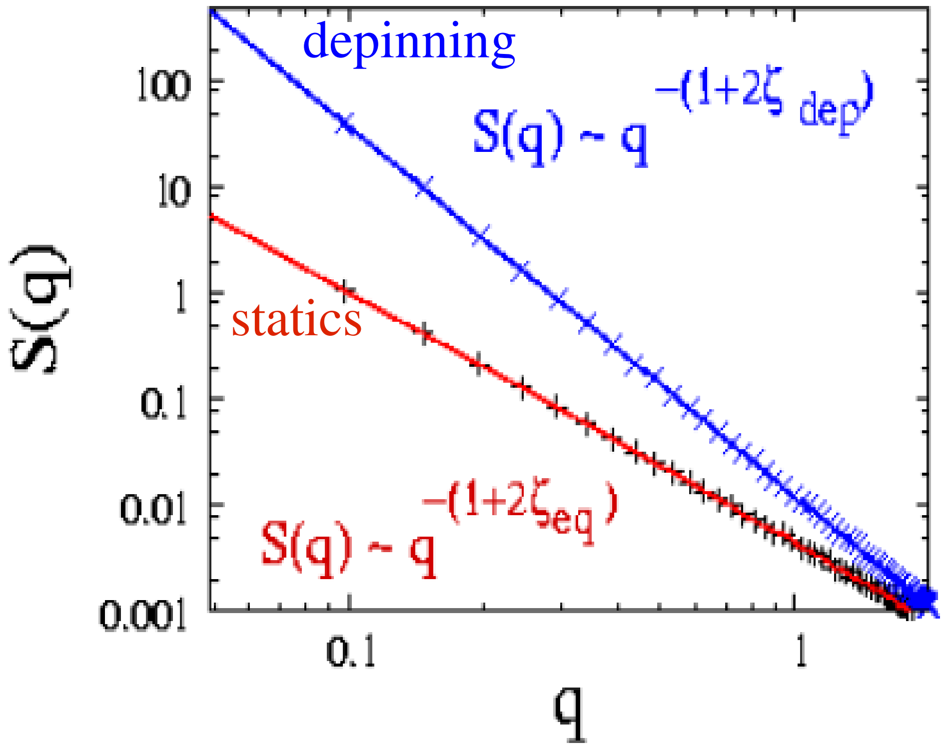
- How does the steady-state geometry depend on applied field, temperature, and disorder strength?.
- How can we describe the rough geometry quantitatively?.

“Rough Geometry”



Equilibrium (f=0) Depinning (f=fc)

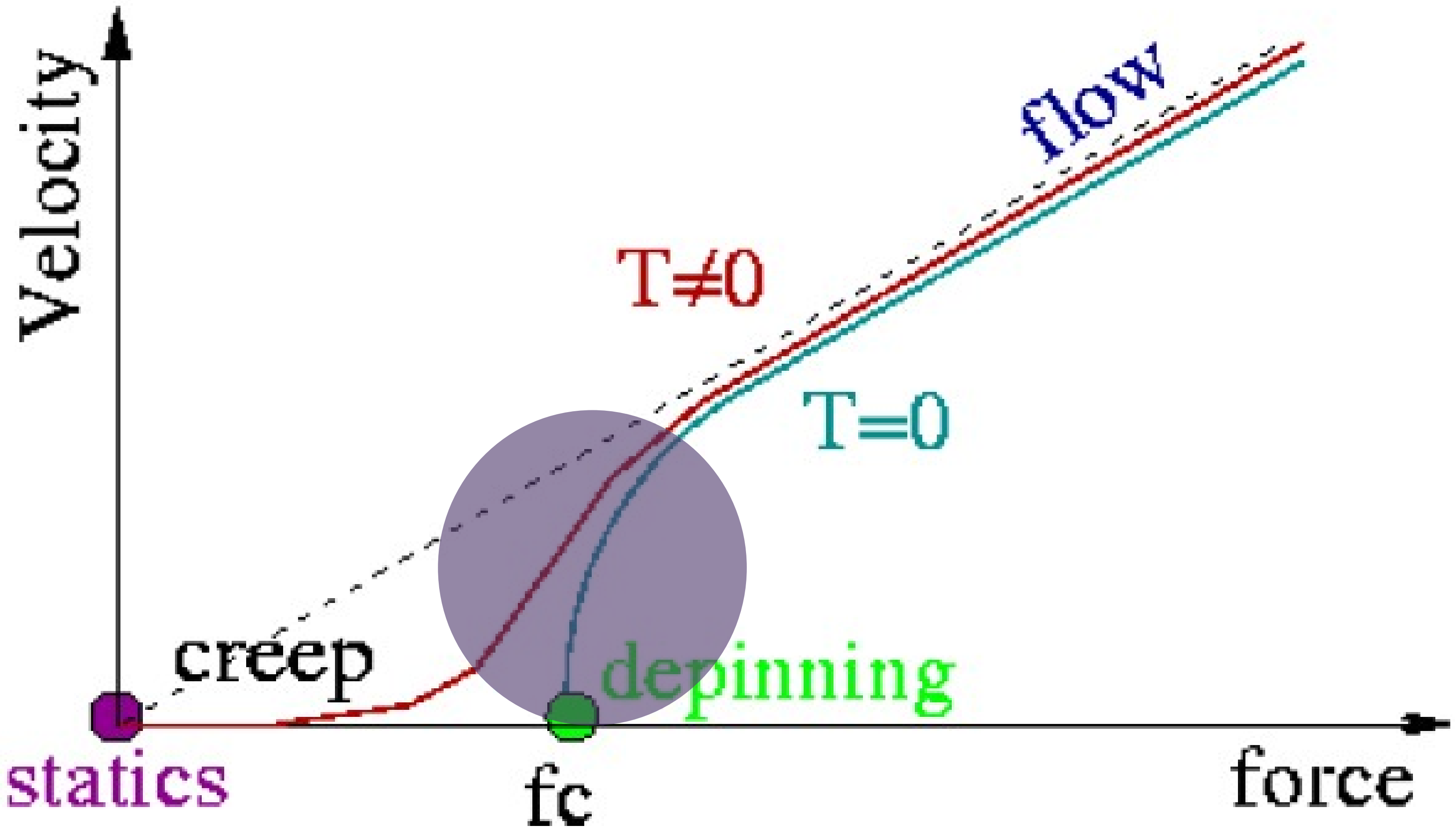




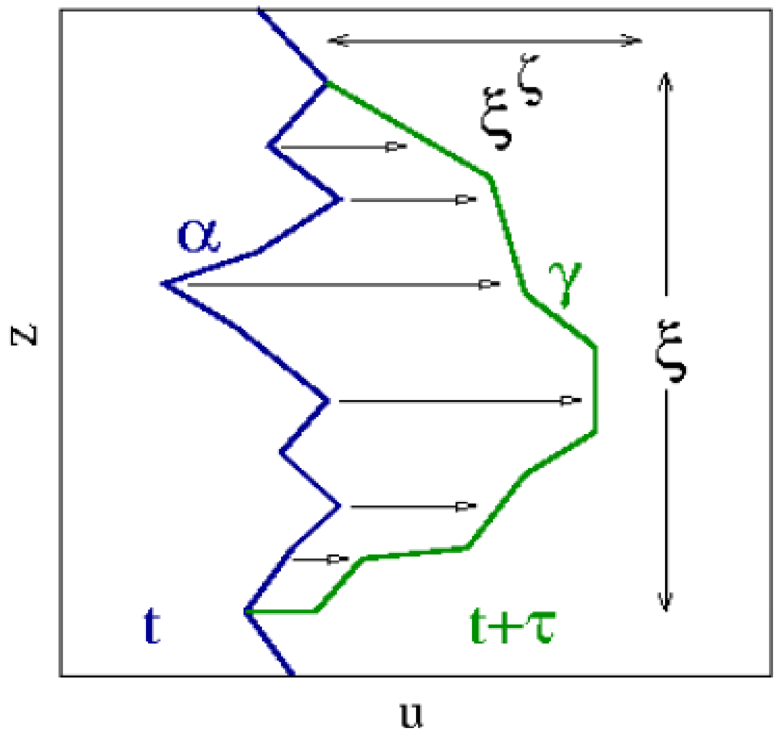
Statistical self-affinity

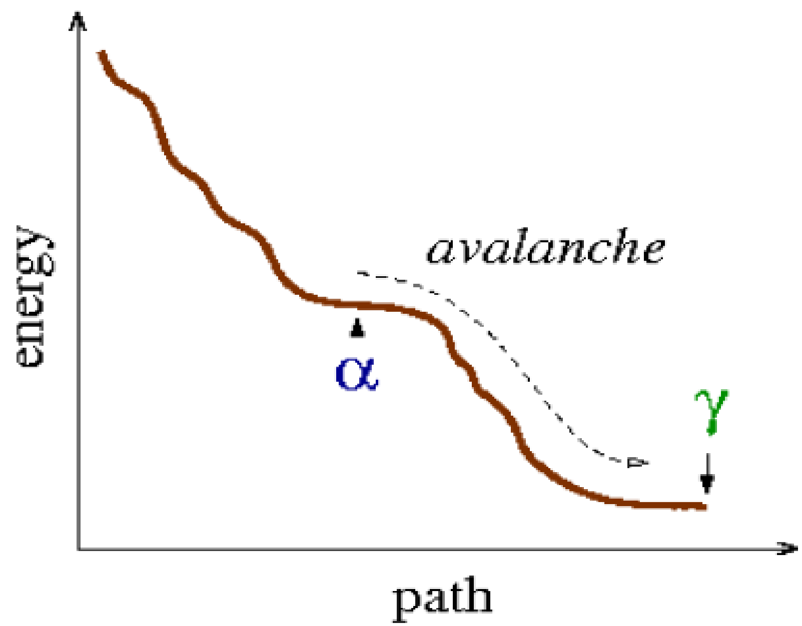
$$z \longrightarrow bz\,,\; u \longrightarrow b^{\zeta} u$$

Depinning



Depinning





Critical Phenomenon [D. Fisher 1985]

Order parameter

$$v \sim (f - f_c)^\beta$$

Divergent length

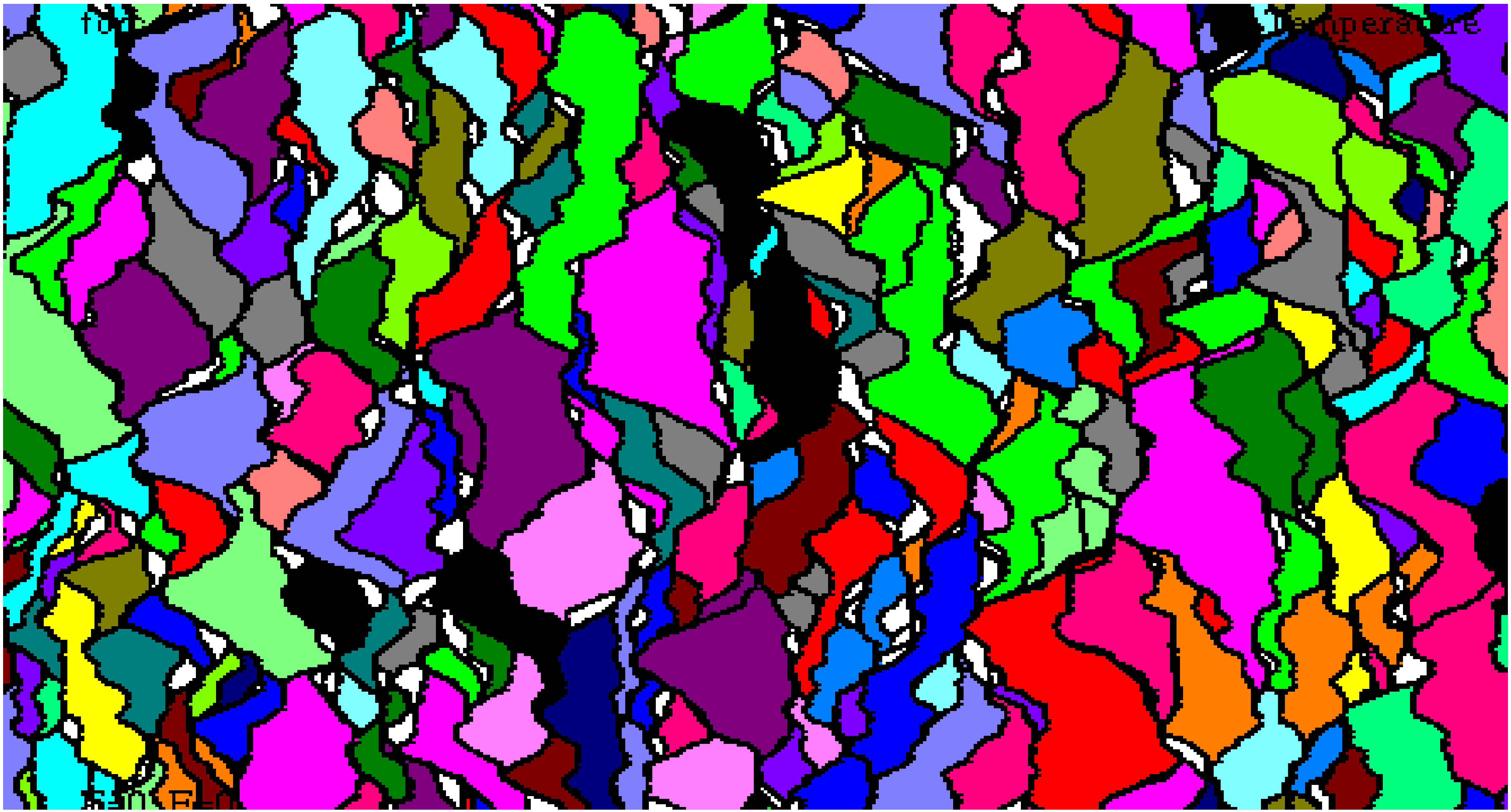
$$\xi \sim (f - f_c)^{-\nu}$$

Divergent time

$$\tau \sim \xi^z$$

Avalanches at depinning f=fc

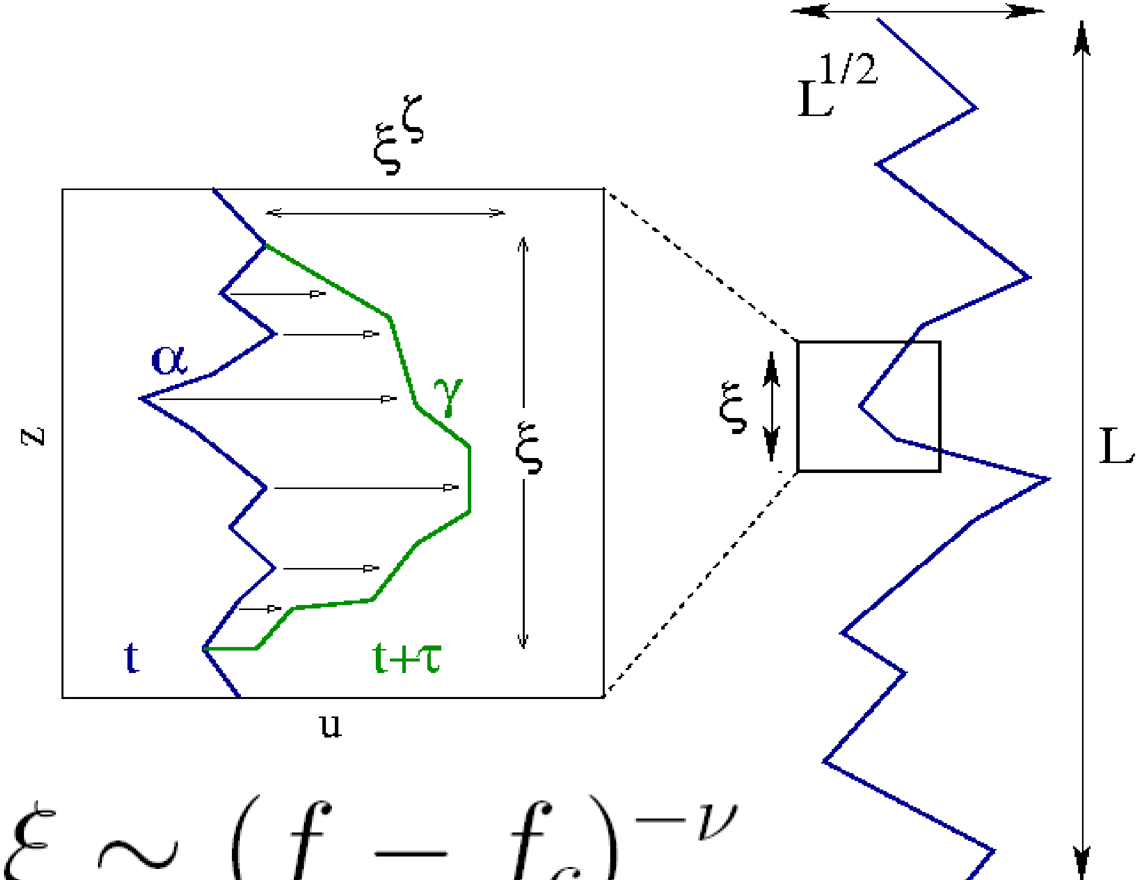
- Le Doussal, Wiese, Rosso.



Above Depinning

Chauve, Giamarchi, Le Doussal PRB (2000)

"typical" Avalanche size => roughness change

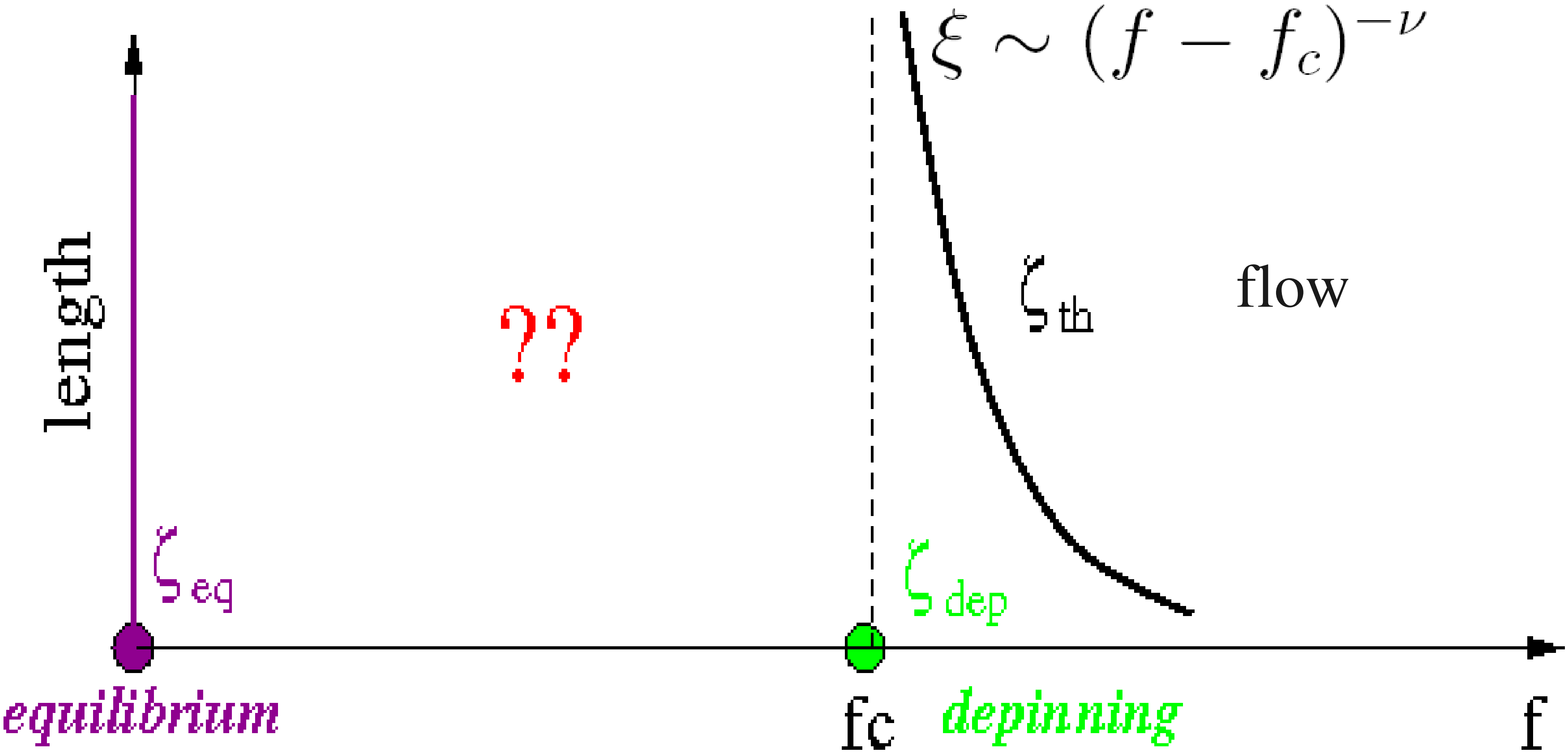


$$\xi \sim (f - f_c)^{-\nu}$$

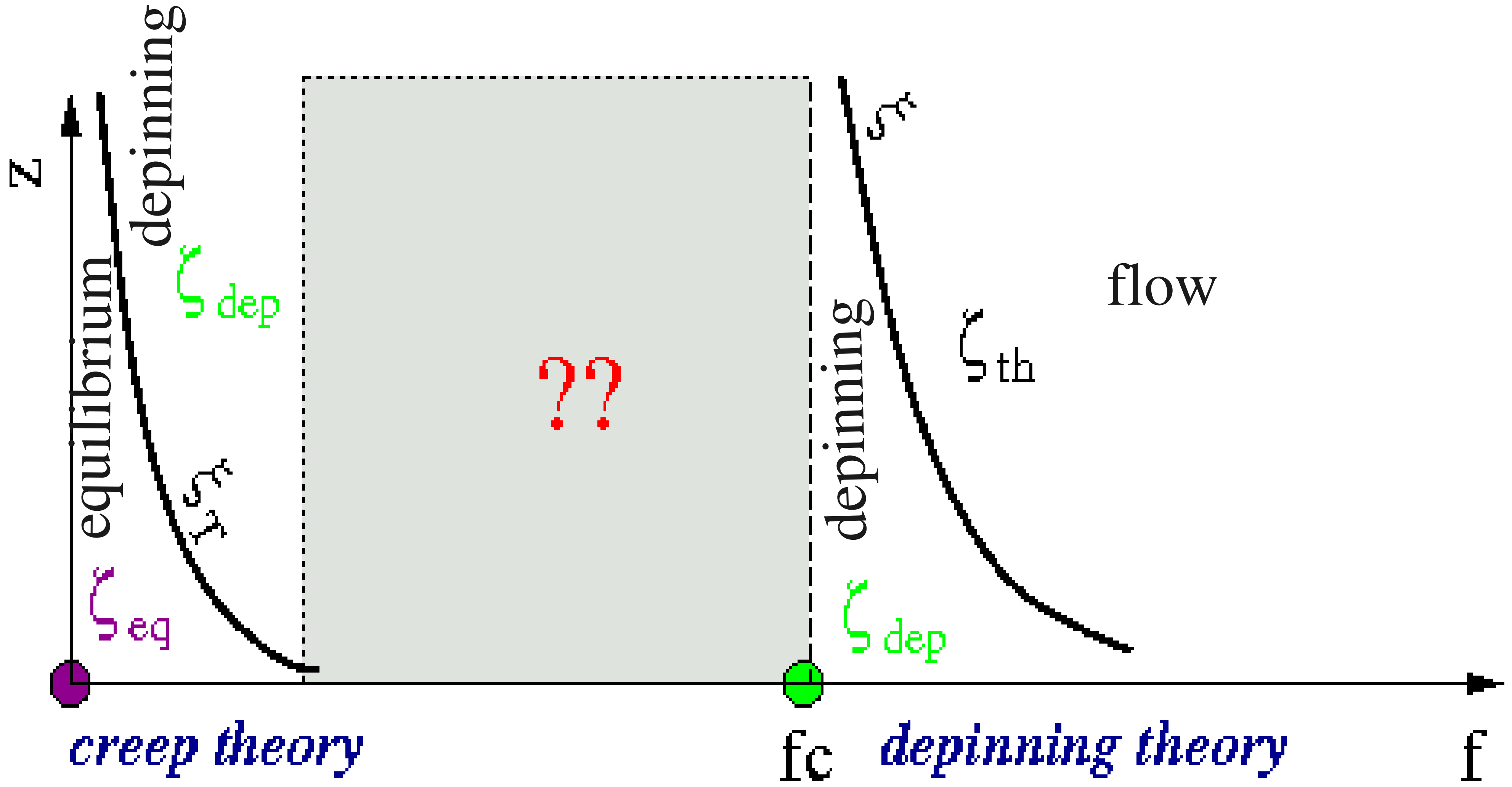
Large scale :
Normal random walk = high speed geometry = "thermally" excited line

Short scales :
Anomalous random walk = critical geometry (f=fc)

“Dynamical Phase diagram”

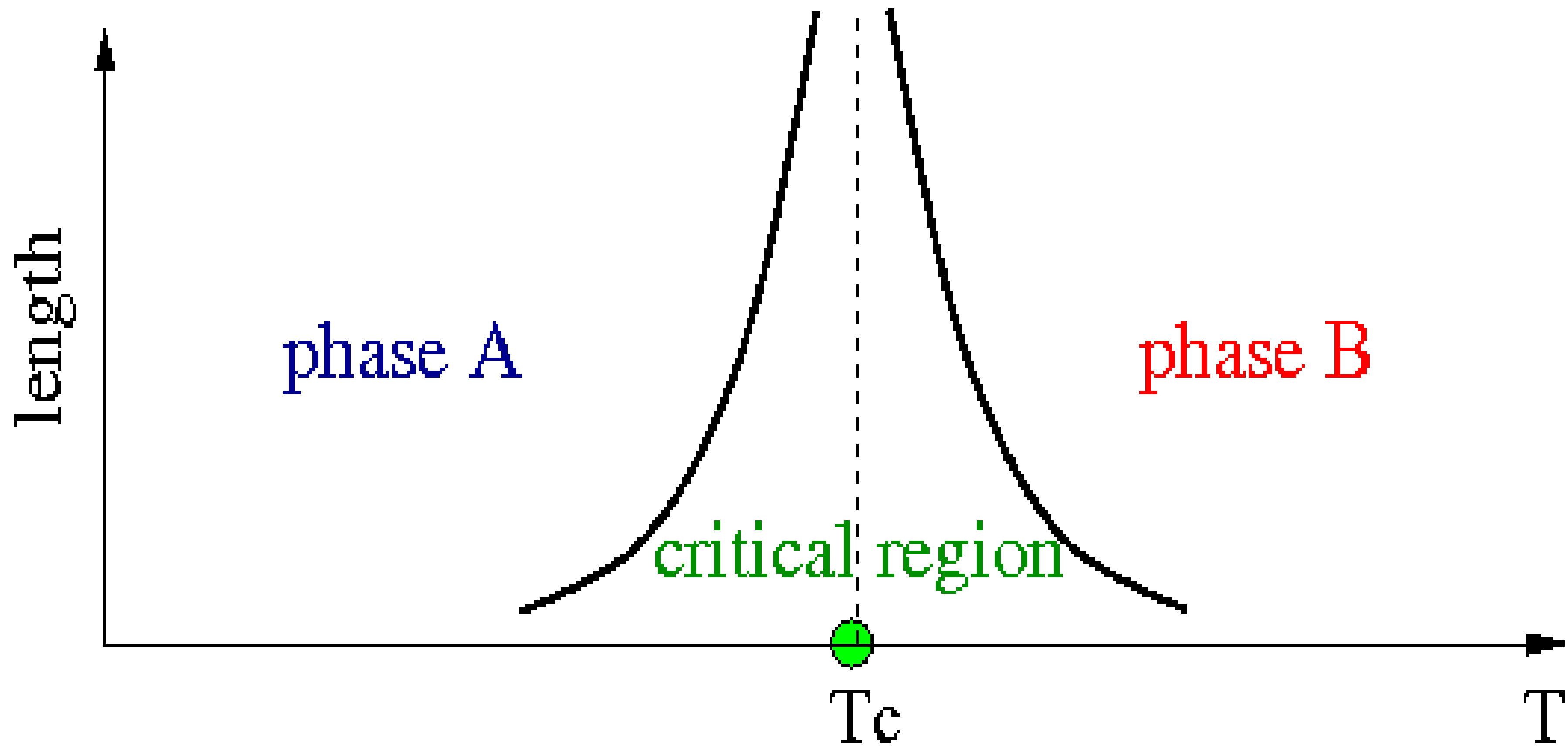


Phase diagram?

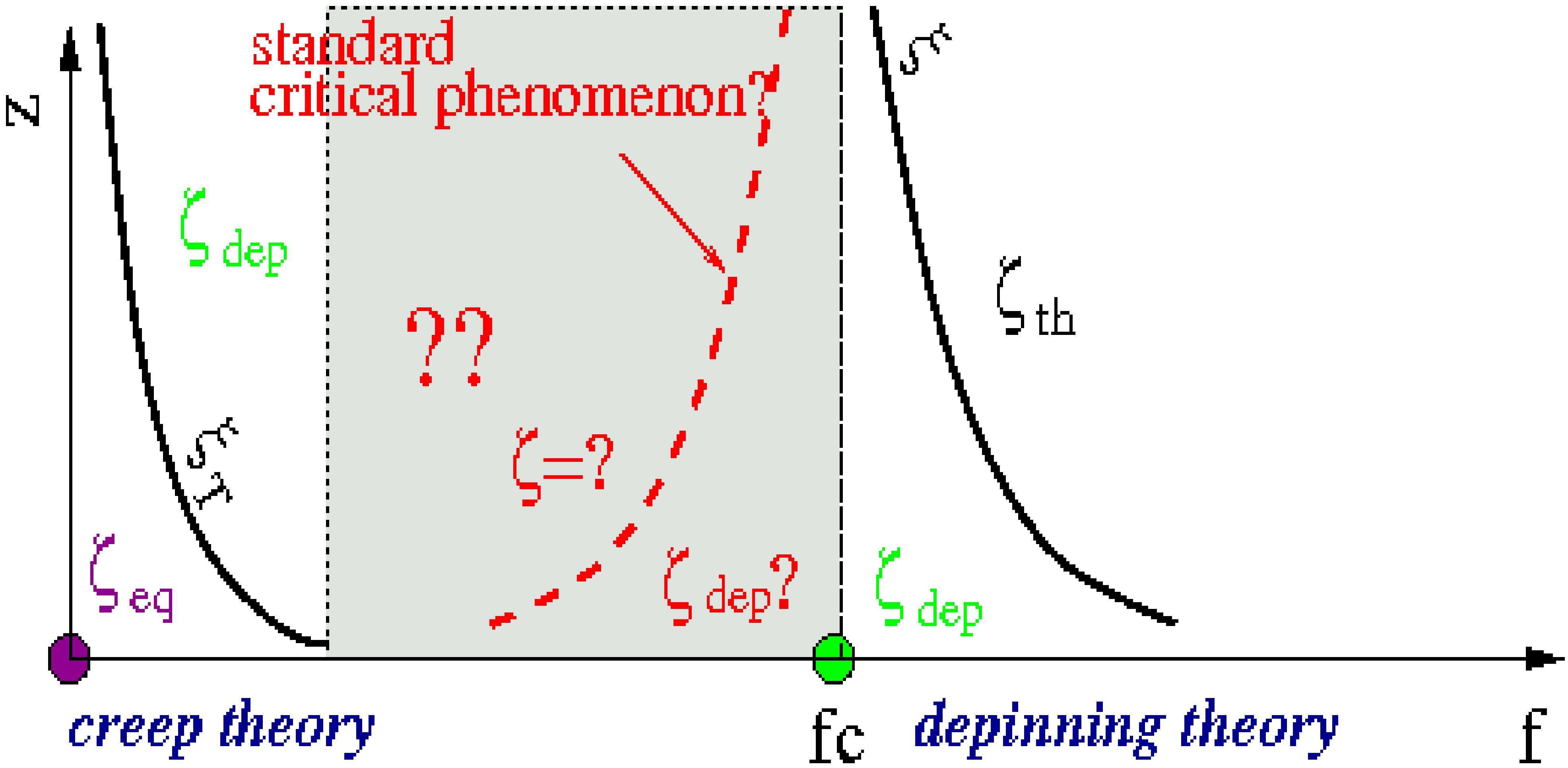


FRG predictions: P. Chauve, T. Giamarchi, P. Le-Doussal (2000)

“standard” continuous phase transitions

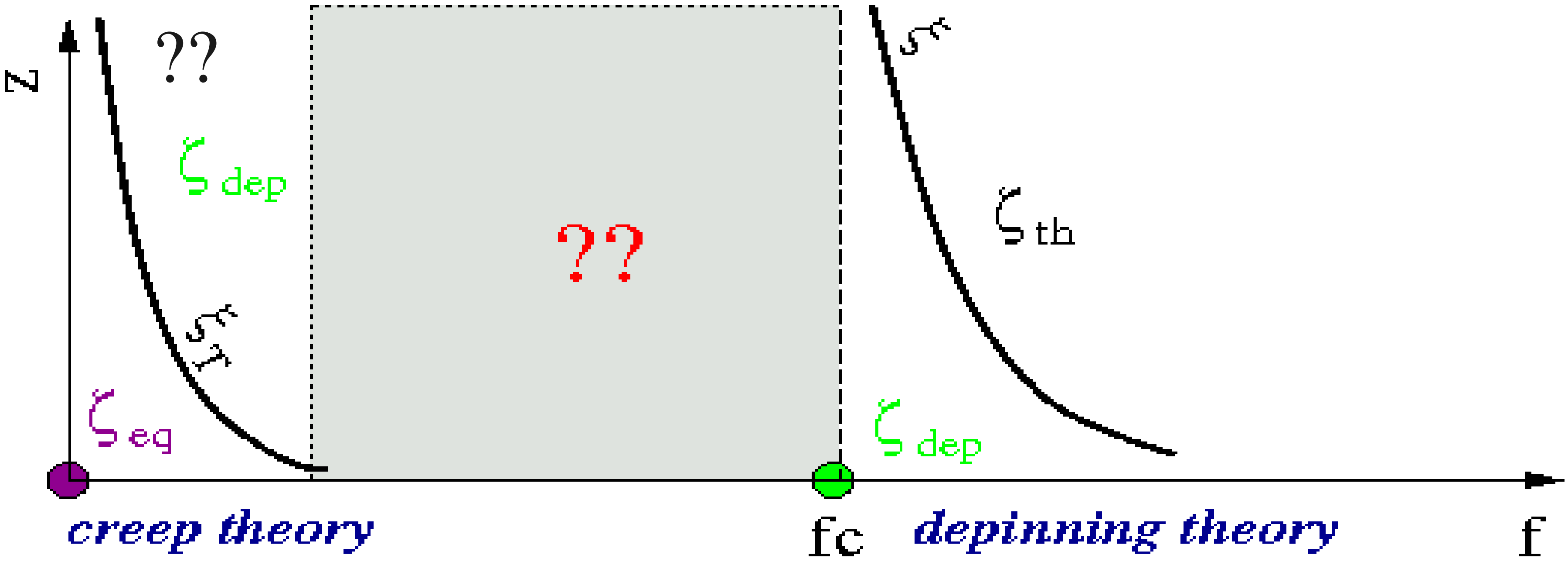


Phase diagram?



- How to study the steady state at $f < f_c$ for very low temperatures?.

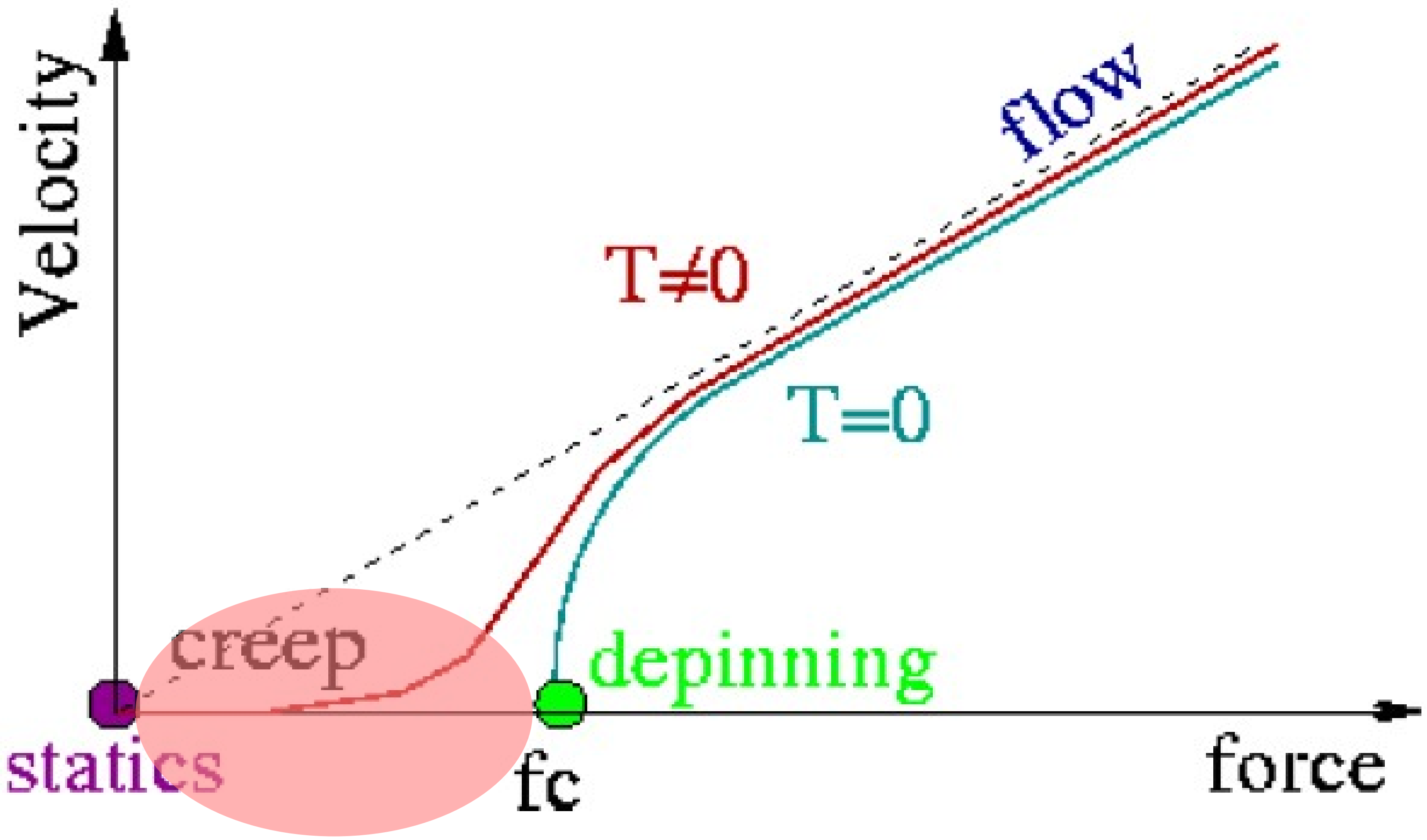
Experimentally relevant, low T, d=1 phase diagram?



Challenge:

- ***Ultra slow non-equilibrium dynamics***
- ***Divergent barriers with decreasing f***
- ***Traditional numerical methods fail***
- ***Analytical predictions only for:***
  $f << fc$ or $f-fc << fc$, around $d=4-\varepsilon$

T→0 creep: ultra slow



How to analyze numerically the steady-state creep?

Langevin dynamics

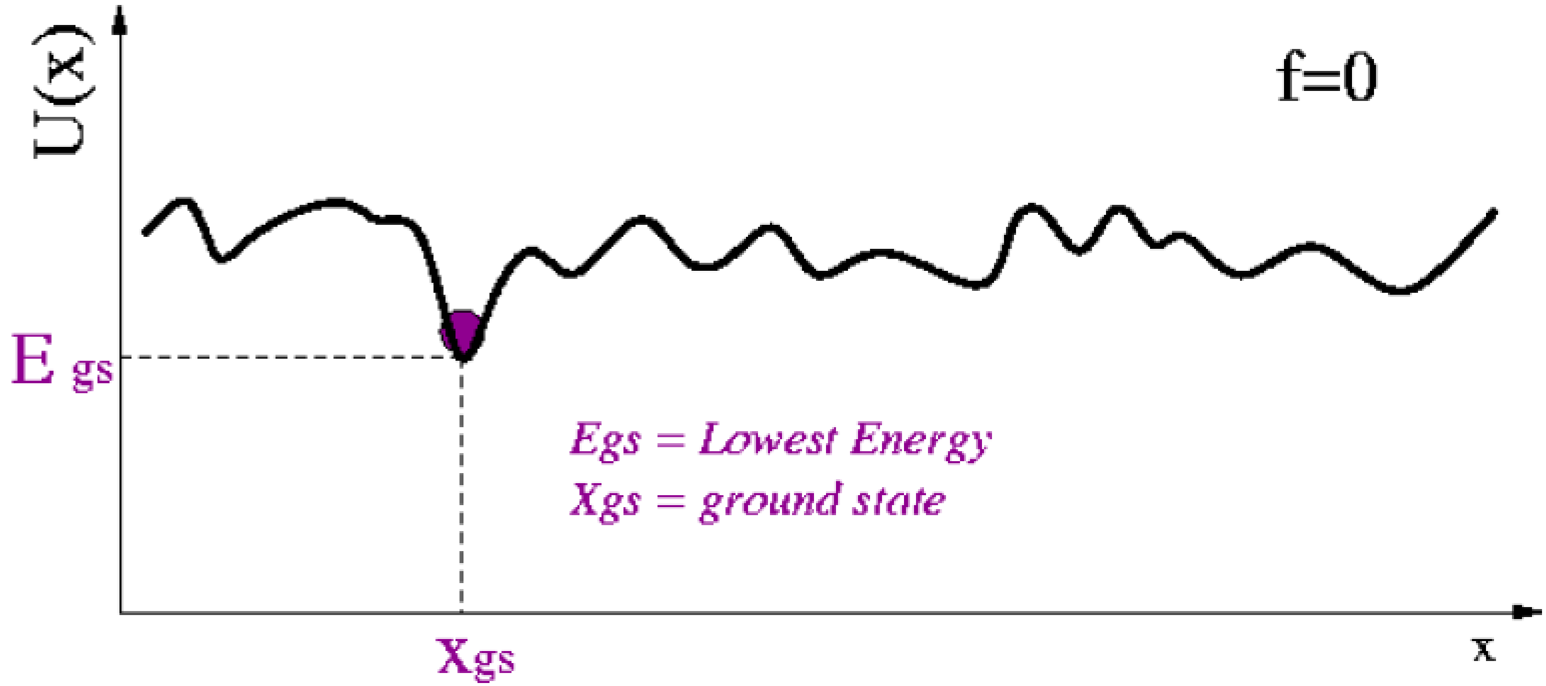
$$\gamma \partial_t u(z,t) = c\partial_z^2 u(z,t) + F_p(u,z) + f \quad +\eta_T(z,t)$$

Too slow when T ->0

New Exact Algorithm for ultra-slow motion

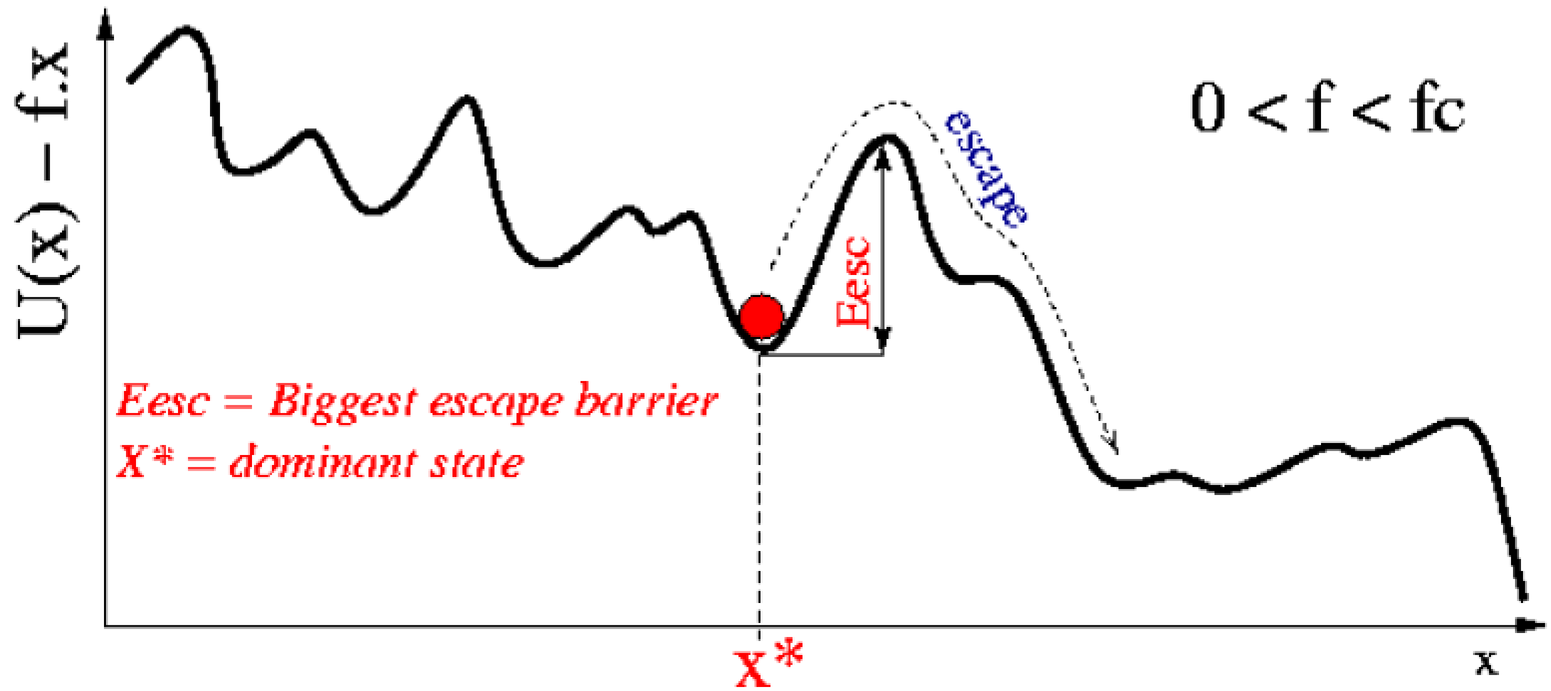
T→ 0 Steady State: one particle

- At Equilibrium, $f = 0$, Boltzmann impose $P(GS) \to 1$ for $T \to 0$:



- Occupation probabilities also exist for the $0 < f < f_c$ steady-state dynamics in a finite system. The $T \to 0$ limit imposes a dominant configuration.

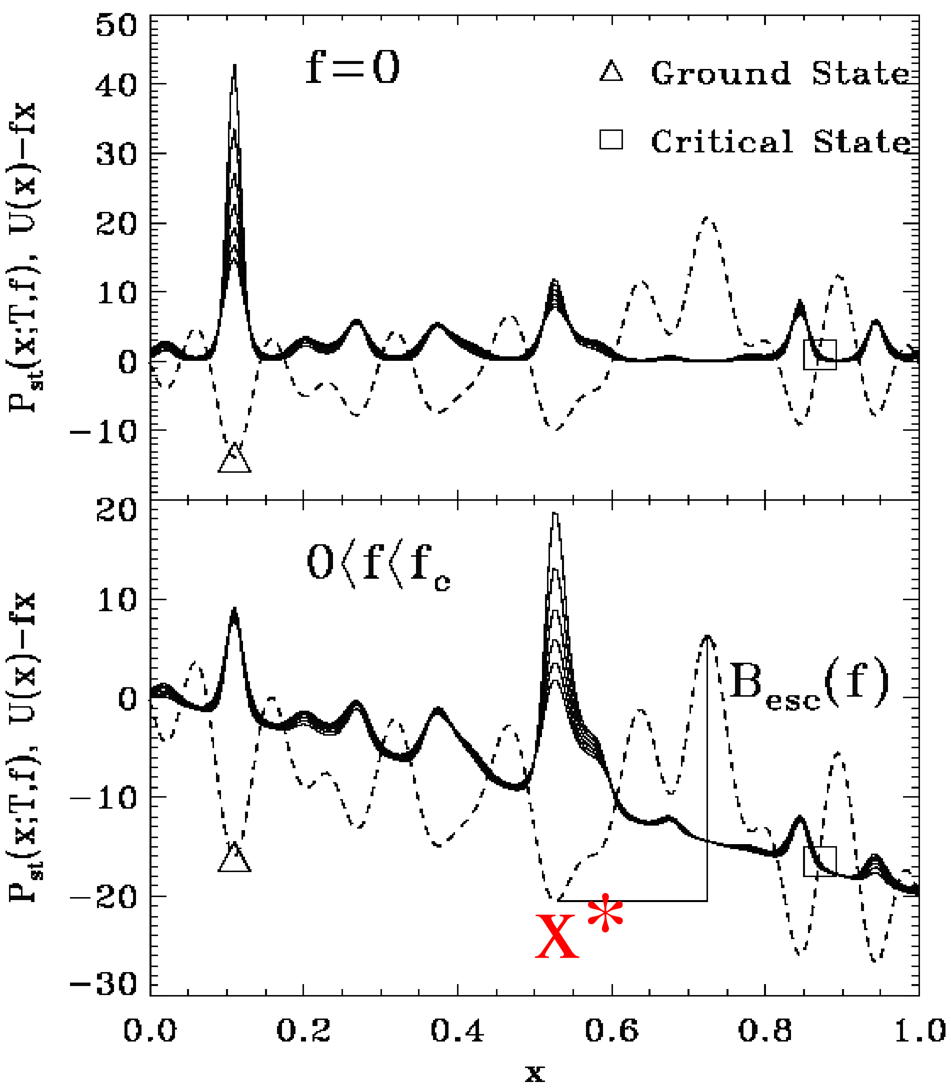
T→ 0 Steady State: one particle

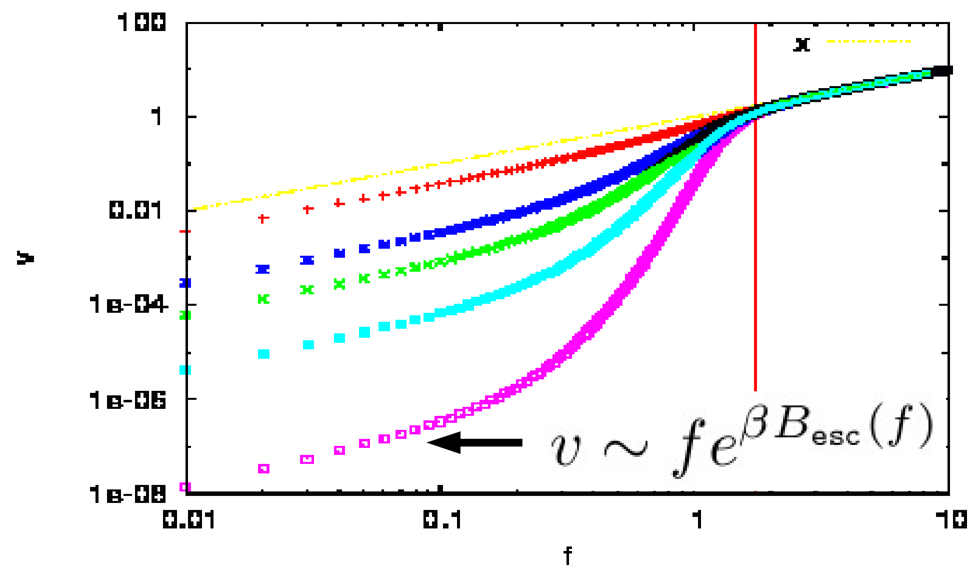


- $P(X^*) \to 1$, when $T \to 0$

transparent for a particle on a 1D ring [Derrida (83); Le Doussal, Vinokur (95)]

Low Temperature Steady State: one particle



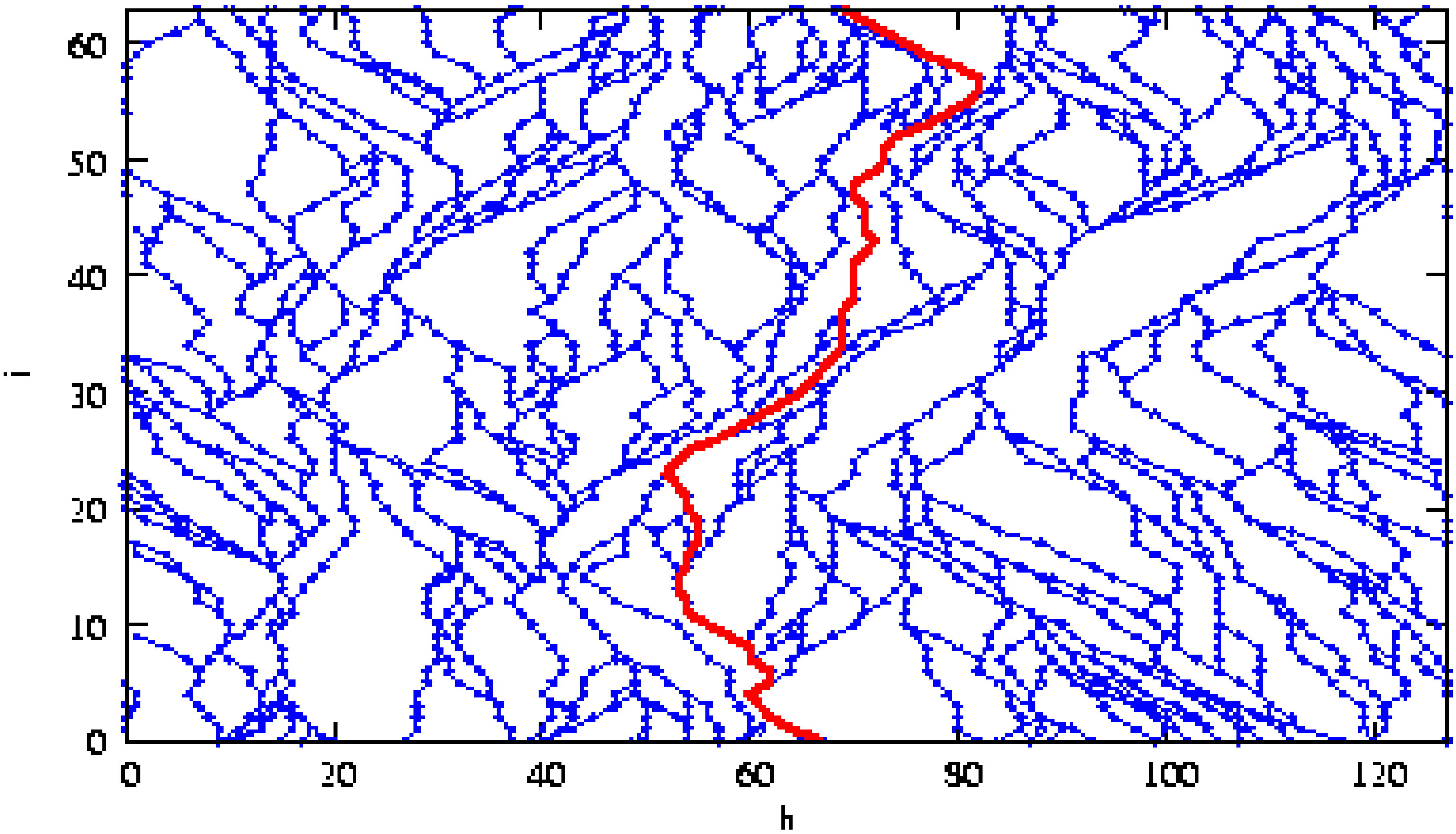


x^*

Determina todas las propiedades del estado estacionario a baja T en un sistema periódico finito

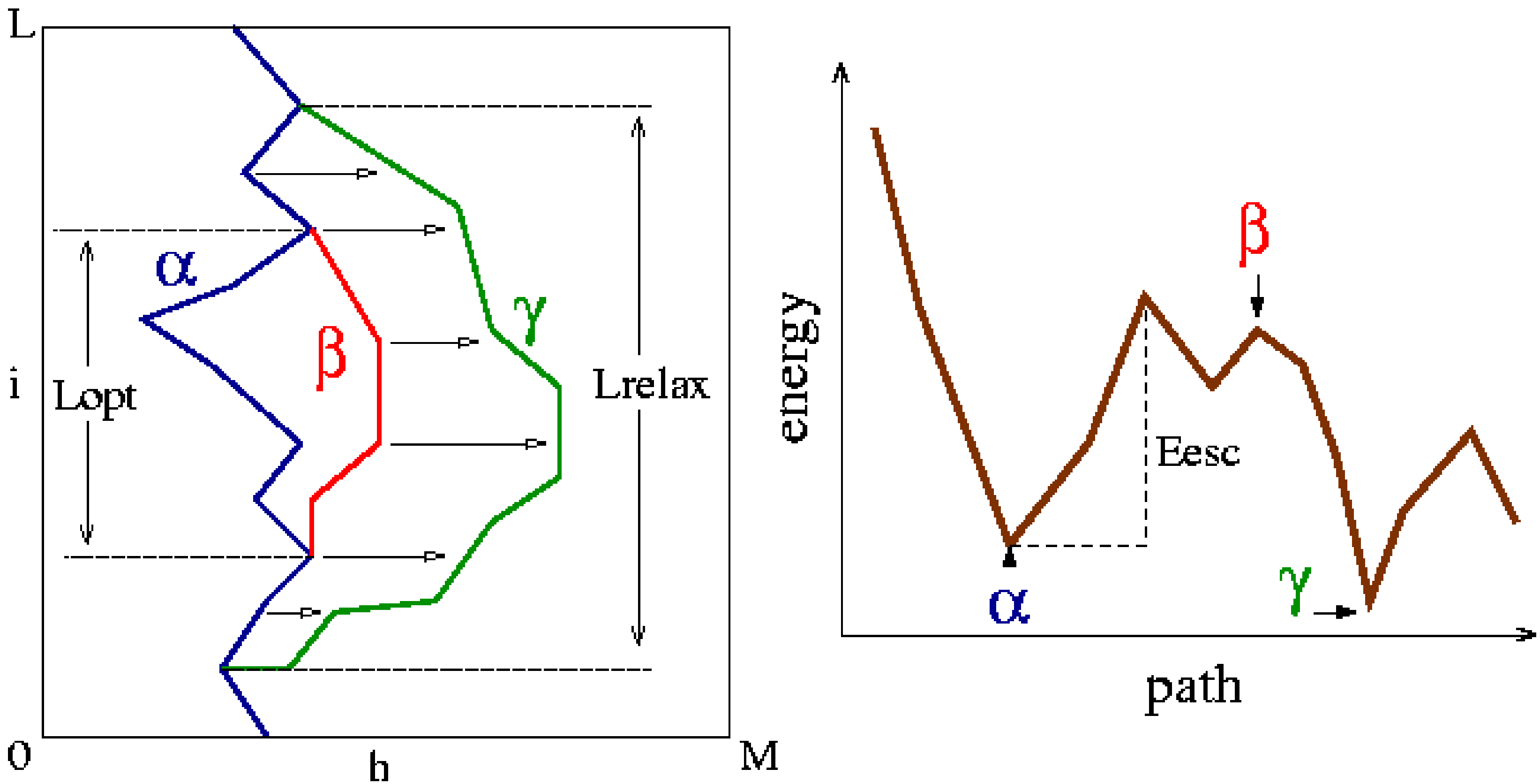
Can we do the same for the elastic interface?

"Dominant Configuration" f<fc



The *dominant configuration* *is the only statistically relevant configuration of the* $T \to 0$ *steady state dynamics: Under the conditions of Arrhenius activation the system will spend much more time on it than in any other configuration.*

“Optimal” T→ 0 Path

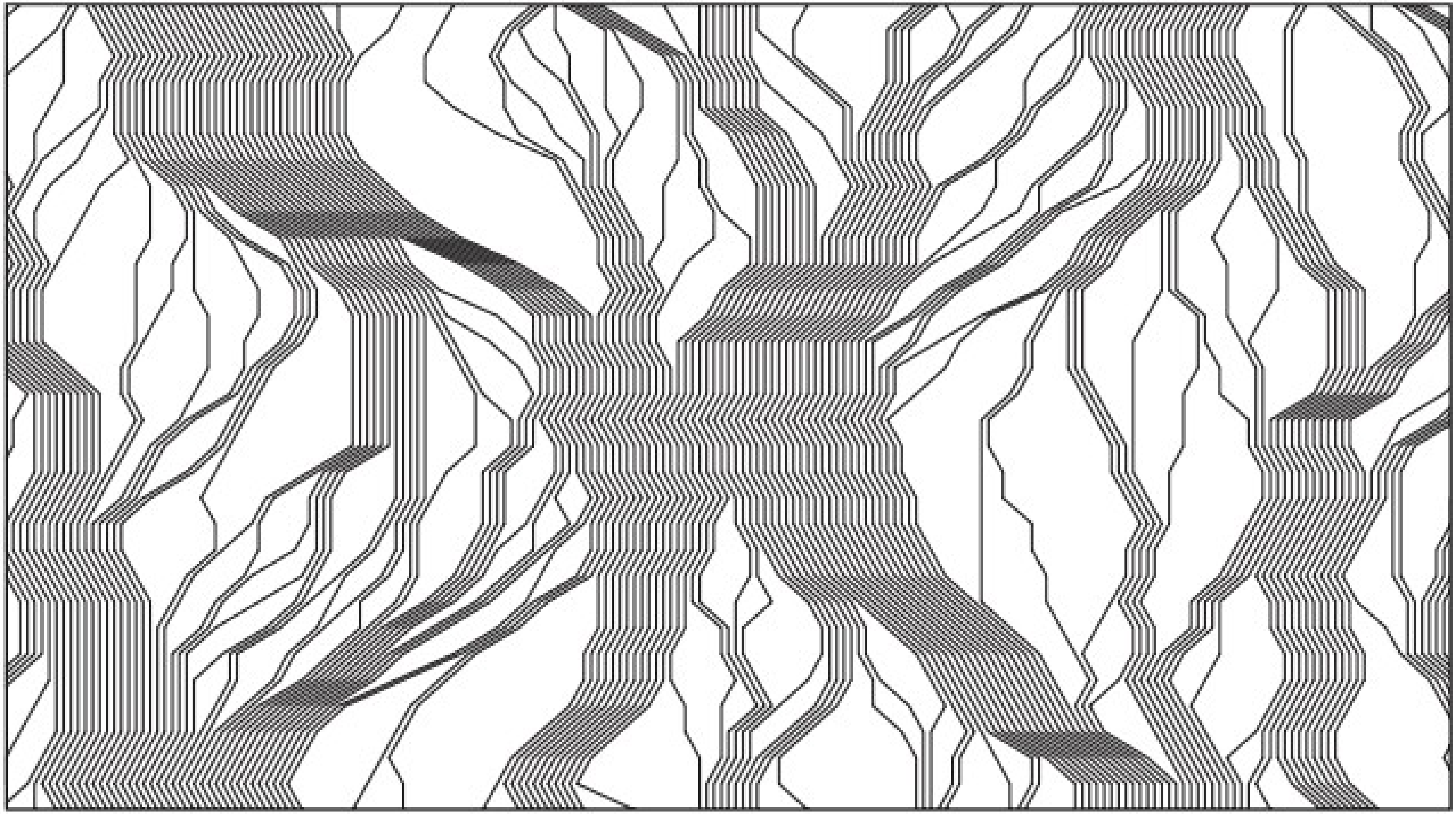


Optimal path: minimal barriers, relaxing in valleys and connecting two metastable states α *and* γ*, such that* $E_\alpha > E_\gamma$.

Creep as a nucleation process: Ioffe & Vinokur, Nattermann (1987)

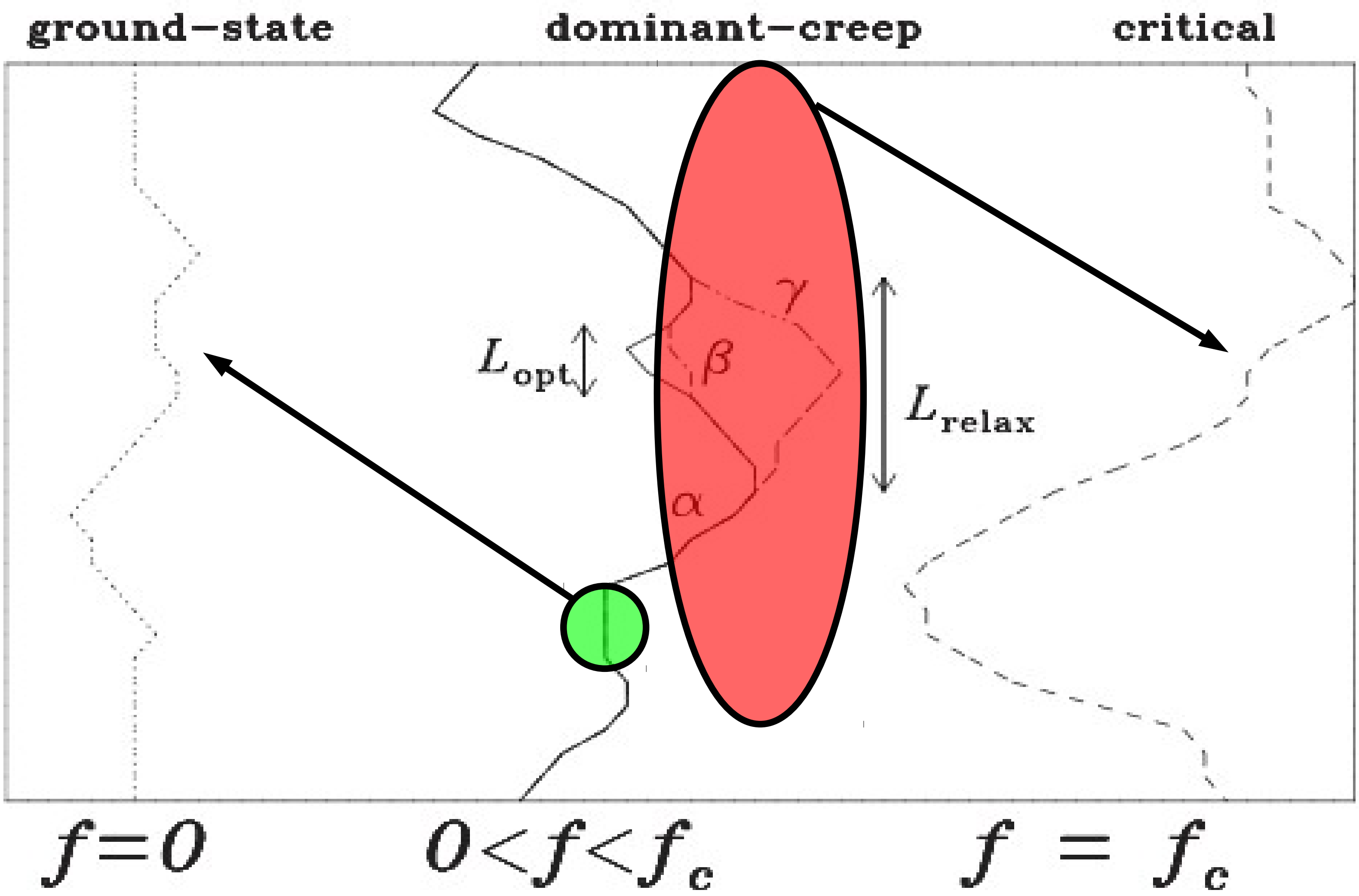
Thermal nucleus triggers an avalanche: Chauve, Le Doussal, Giamarchi (2000)

Unique sequence of metastable states

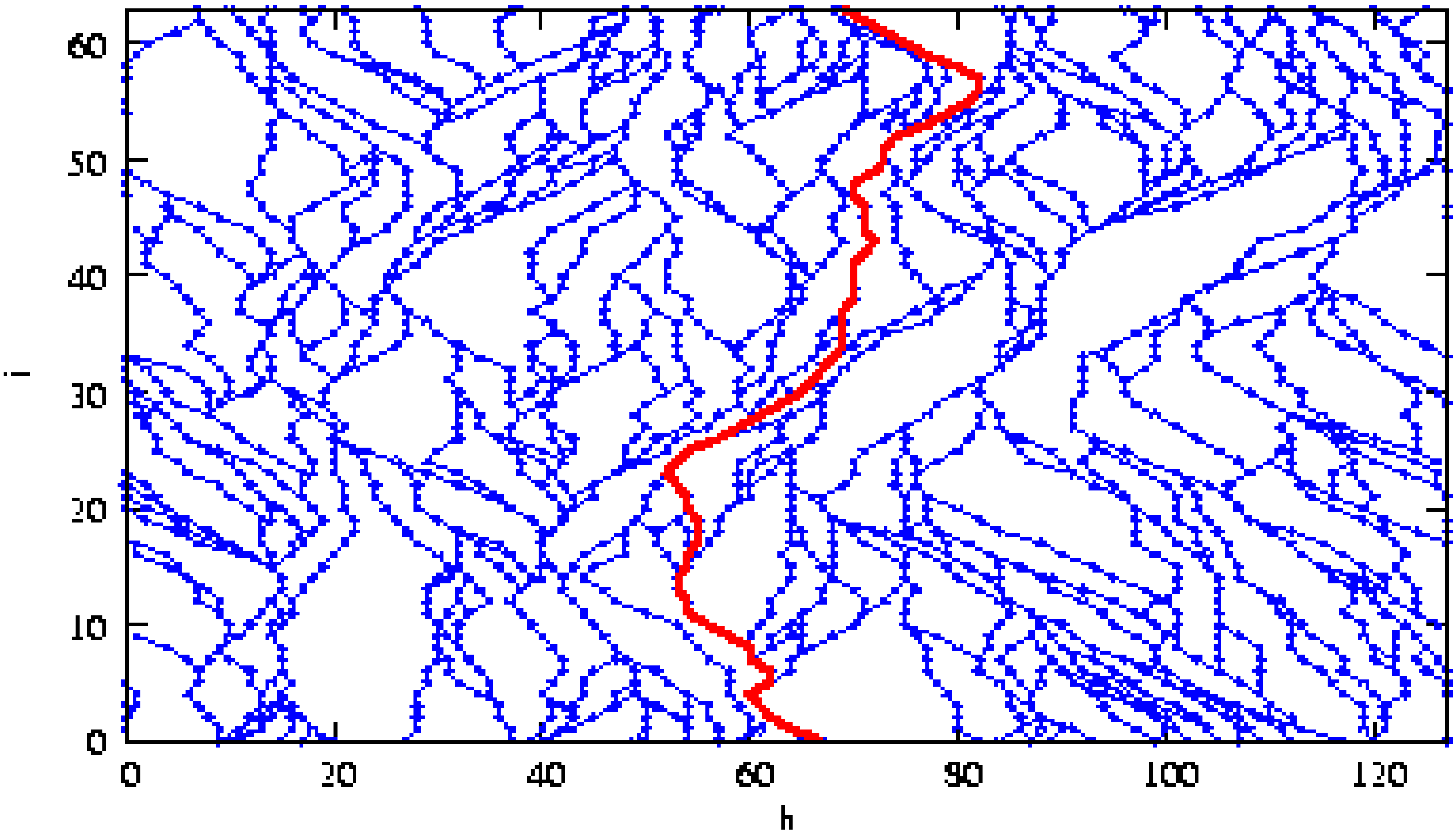


Unique sequence of metastable states

“Optimal” T→ 0 Path vs force

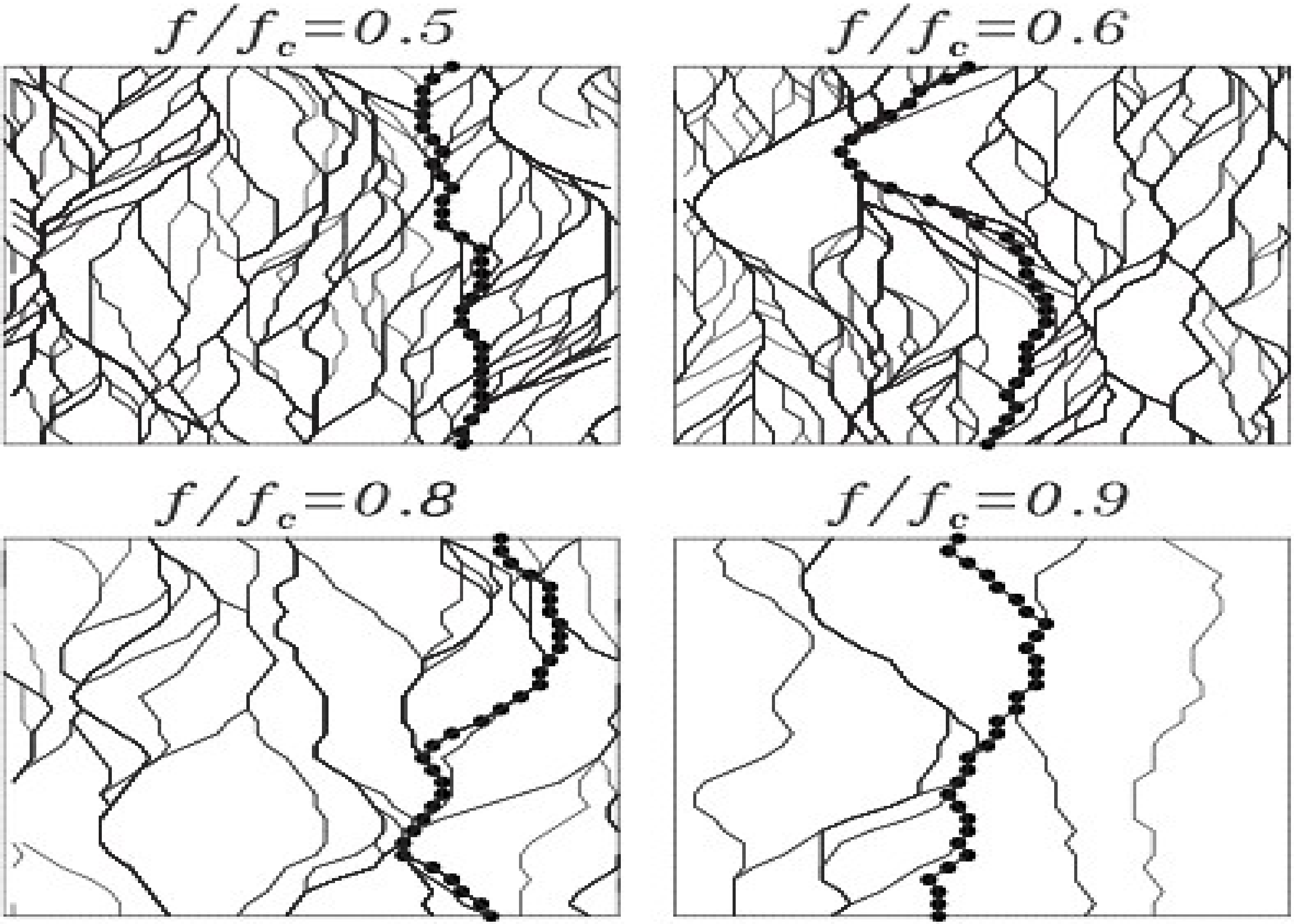


Dominant configuration f<fc

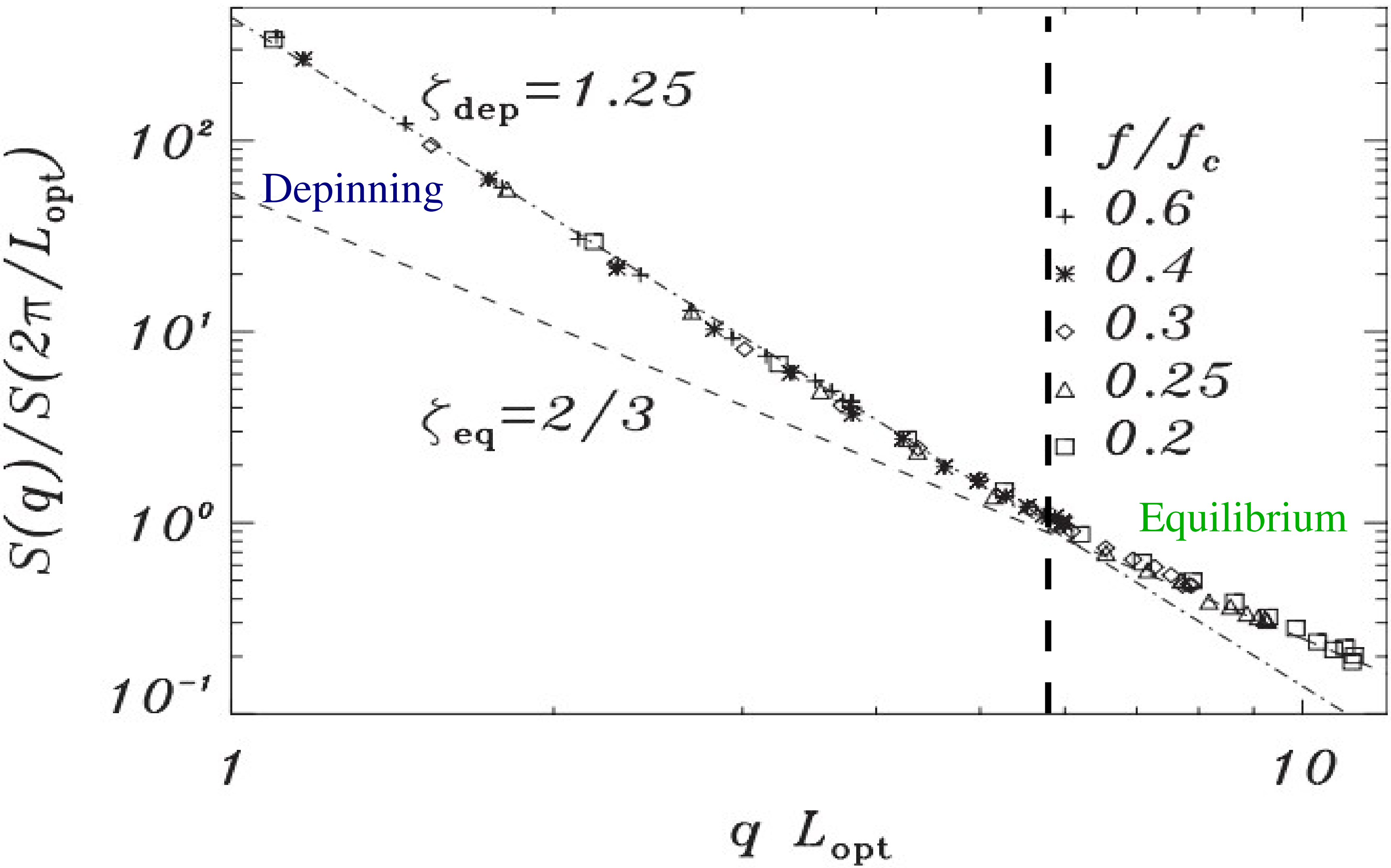


The dominant configuration is the only statistically relevant configuration of the $T \to 0$ steady state dynamics: Under the conditions of Arrhenius activation the system will spend much more time on it than in any other configuration.

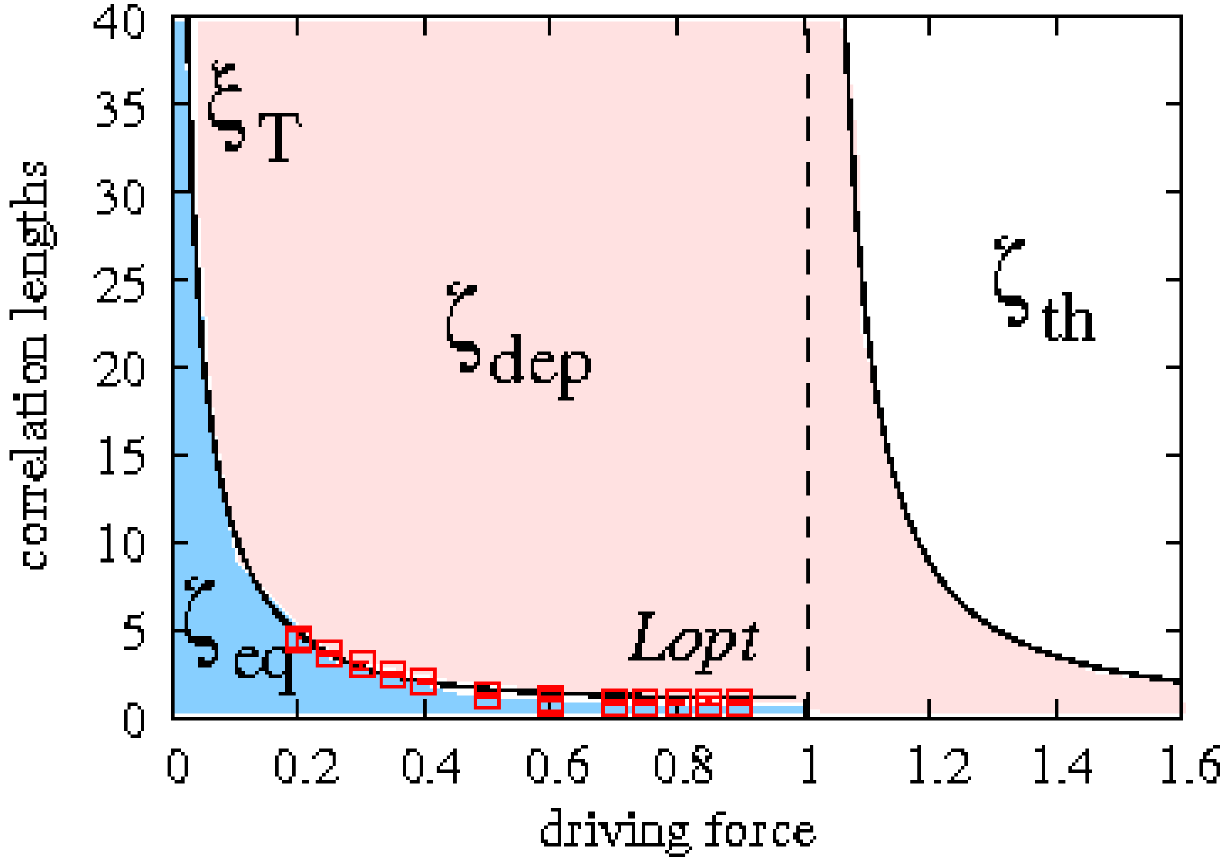
Dominant configurations

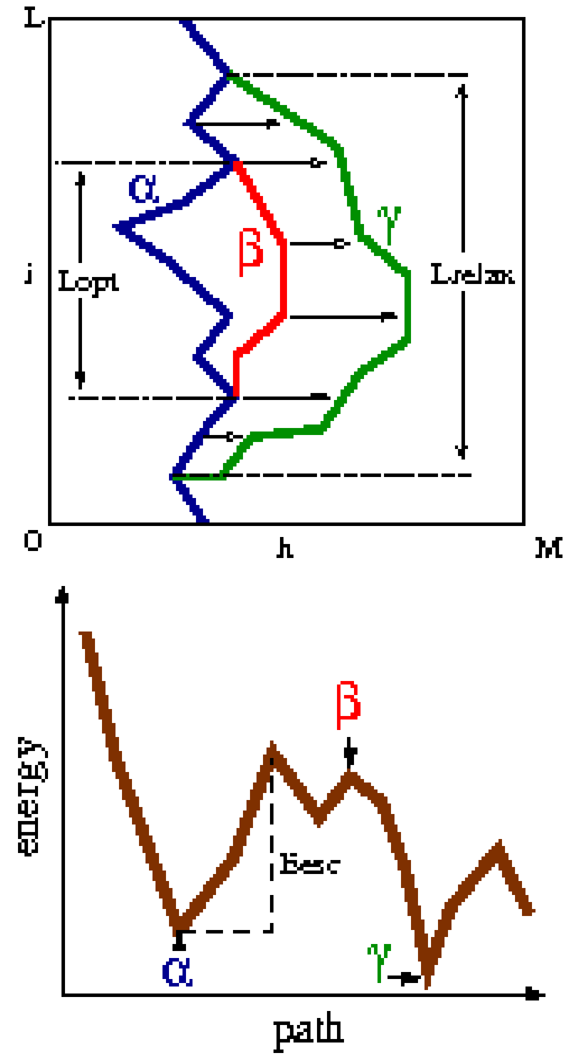


Geometry of the dominant configuration = low temperature steady-state geometry



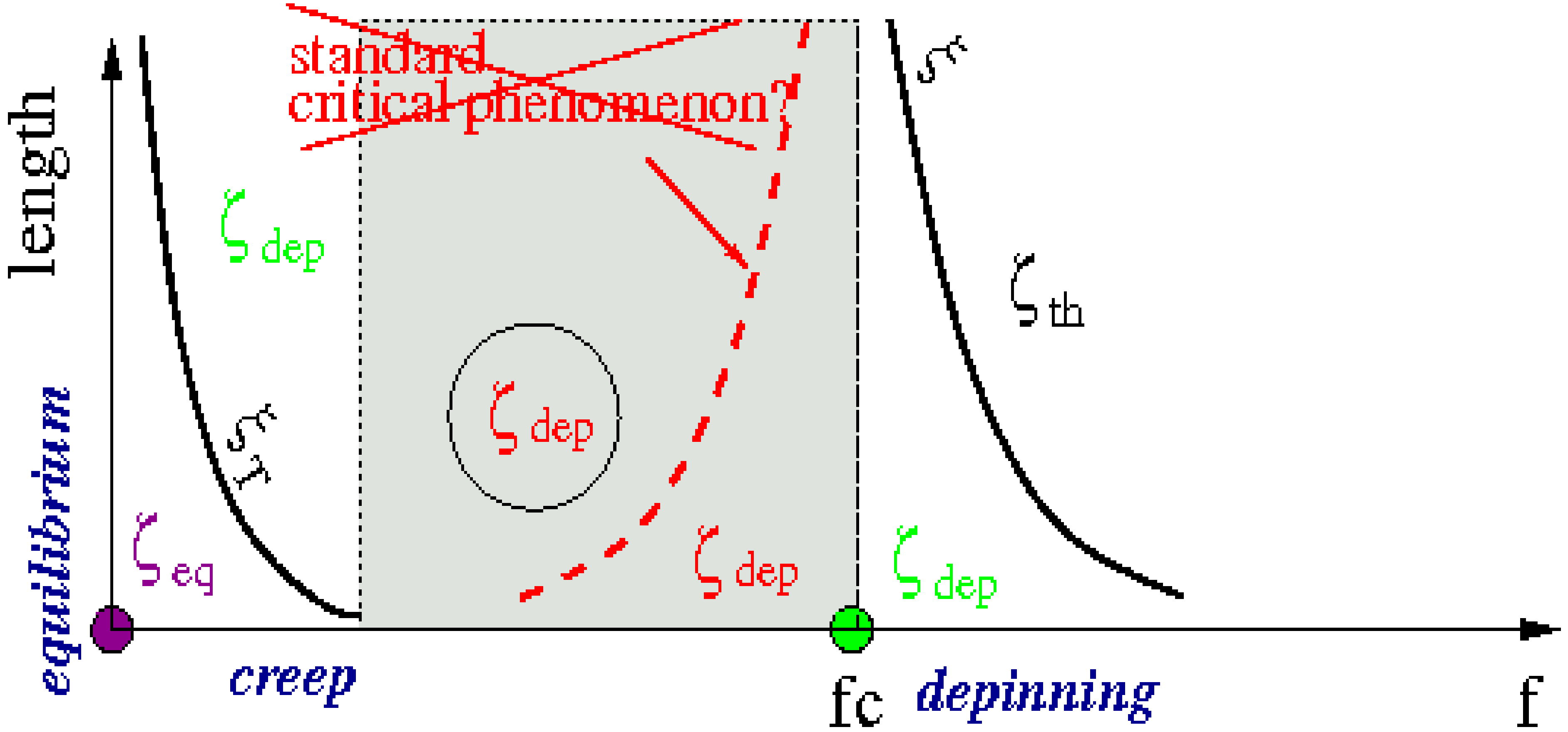
Dynamical Phase diagram
T→ 0



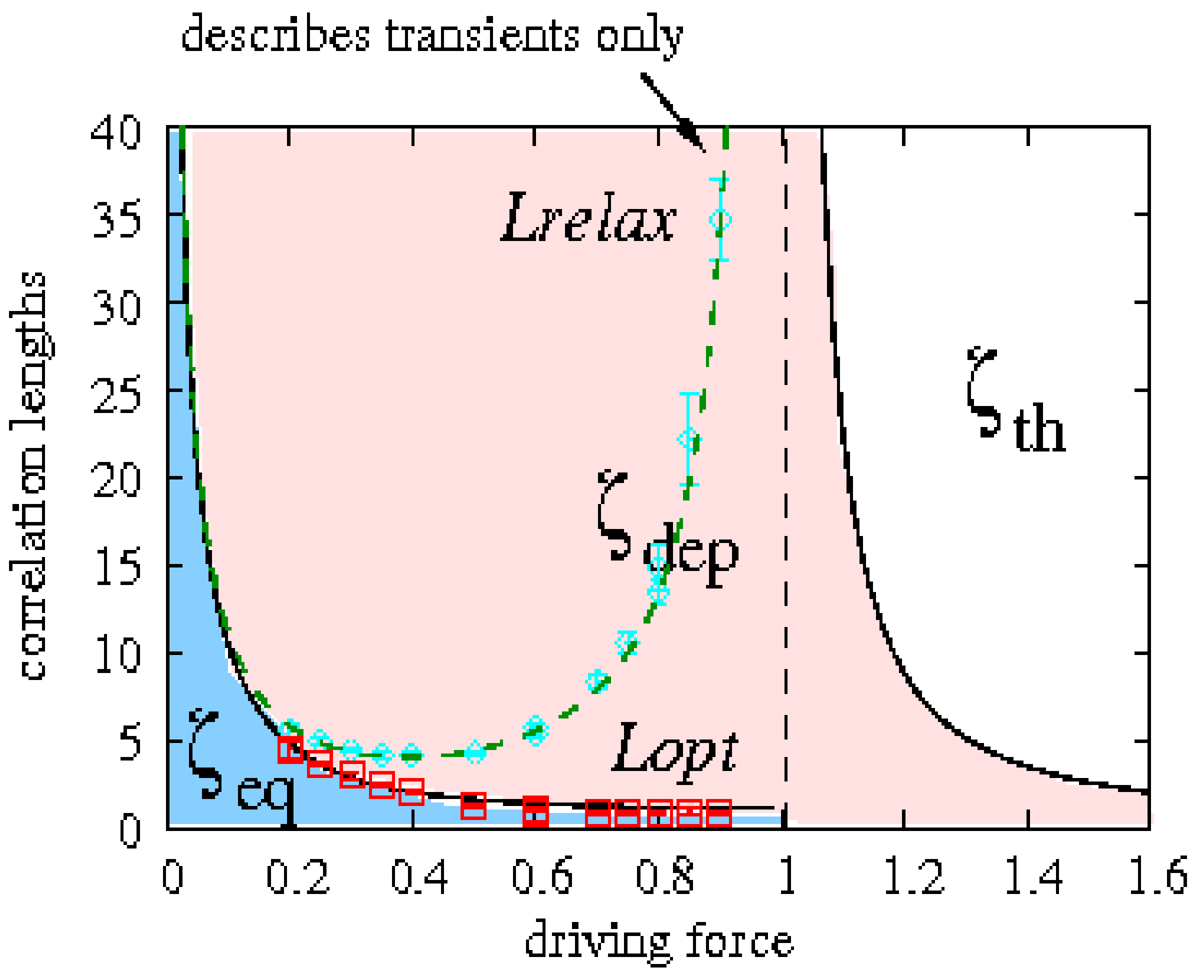


PRL 97, 057001 (2006)

Depinning: non-standard critical phenomenon

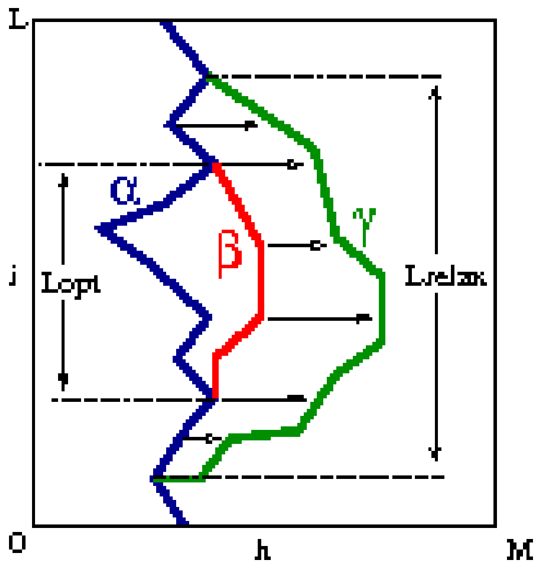


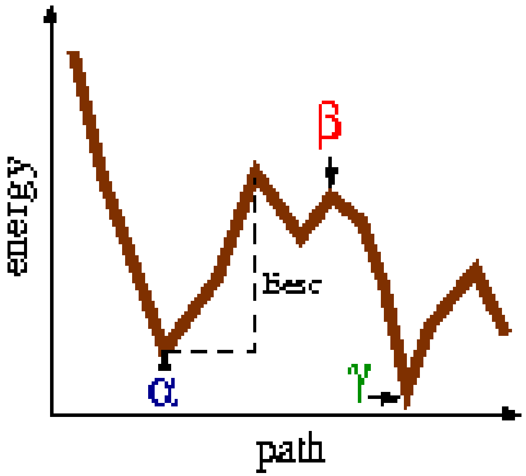
Dynamical Phase diagram T→ 0



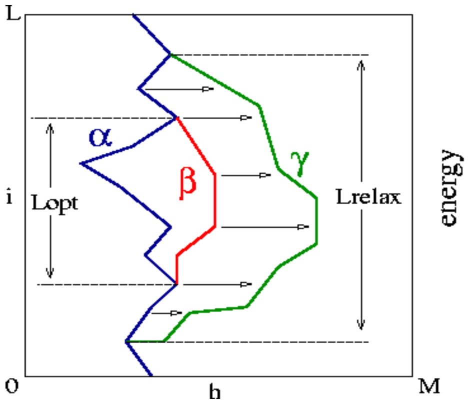
Creep Depinning

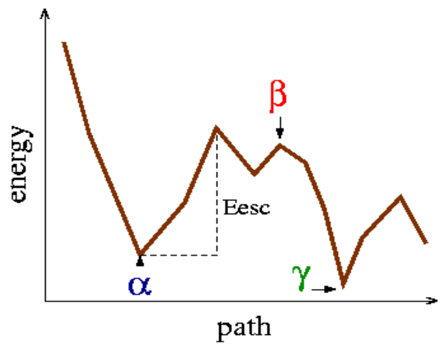
Creep T→0



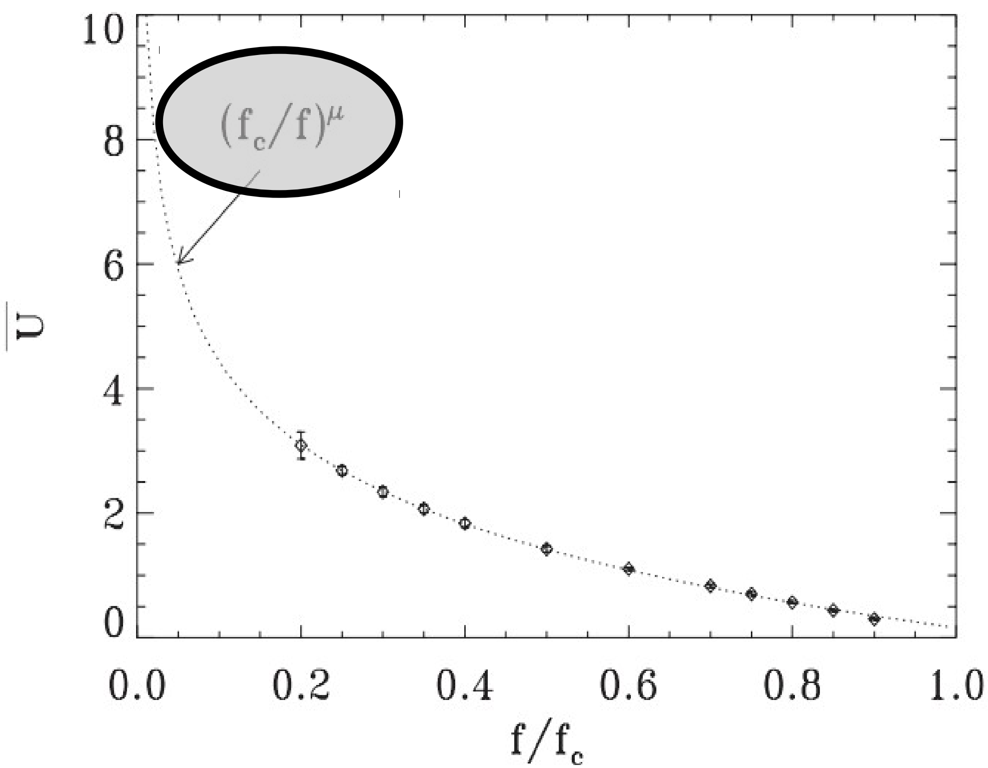


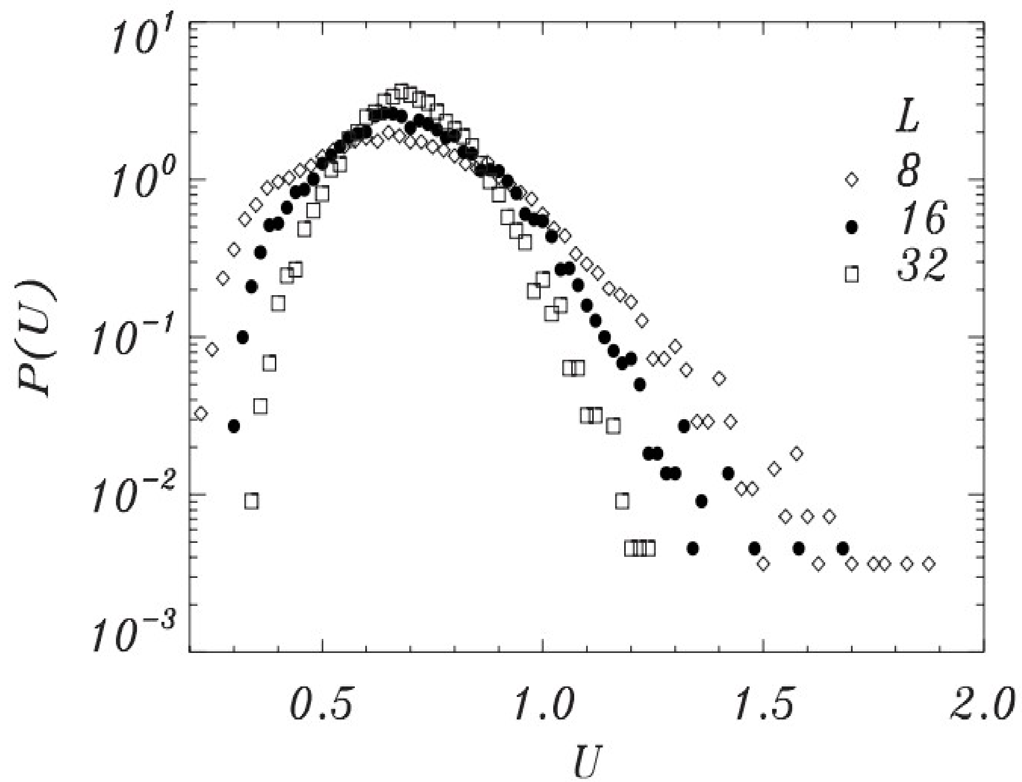
Barriers & transport





$$v(f) = L_{\mathrm{opt}}^{\zeta_{\mathrm{eq}}}\left(\frac{L_{\mathrm{relax}}}{L_{\mathrm{opt}}}\right)^{\zeta_{\mathrm{dep}}} e^{-\beta U(f)}$$





Creep law

Domain Wall Creep in an Ising Ultrathin Magnetic Film

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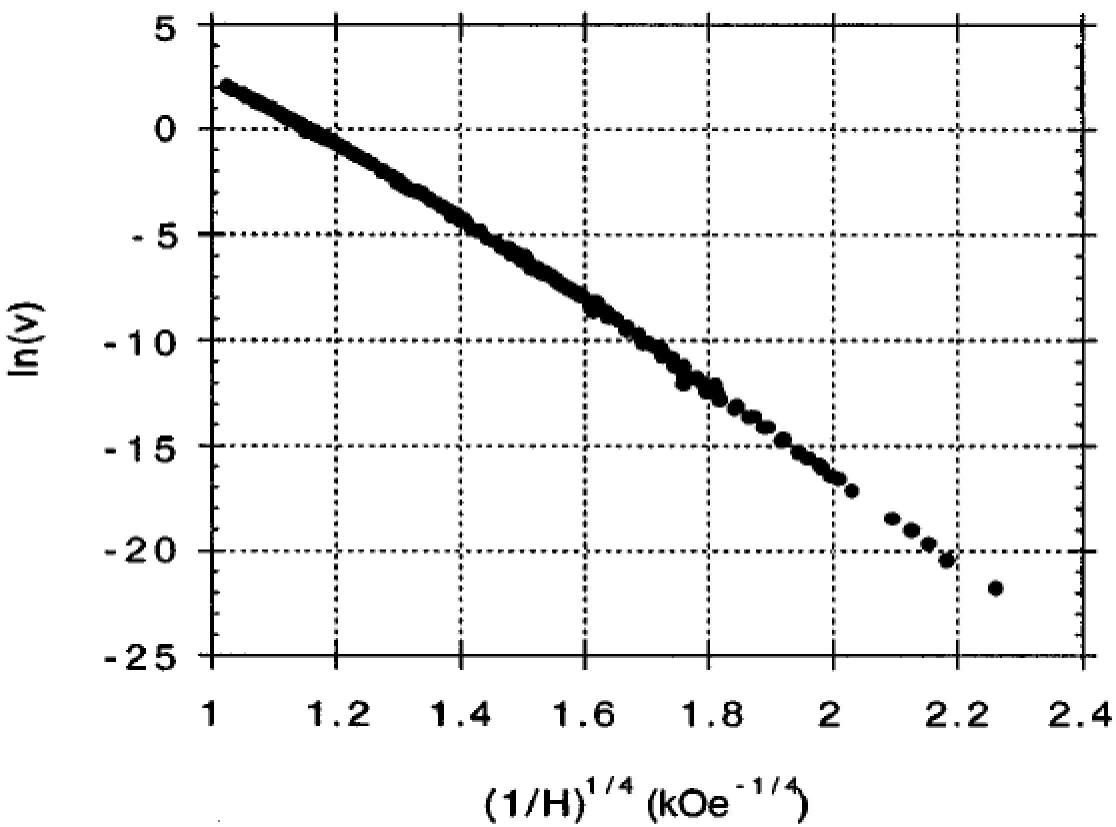
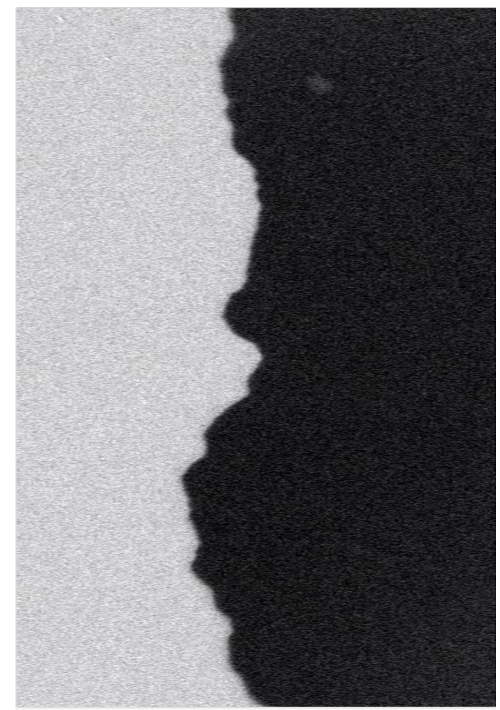
(Received 5 August 1997)





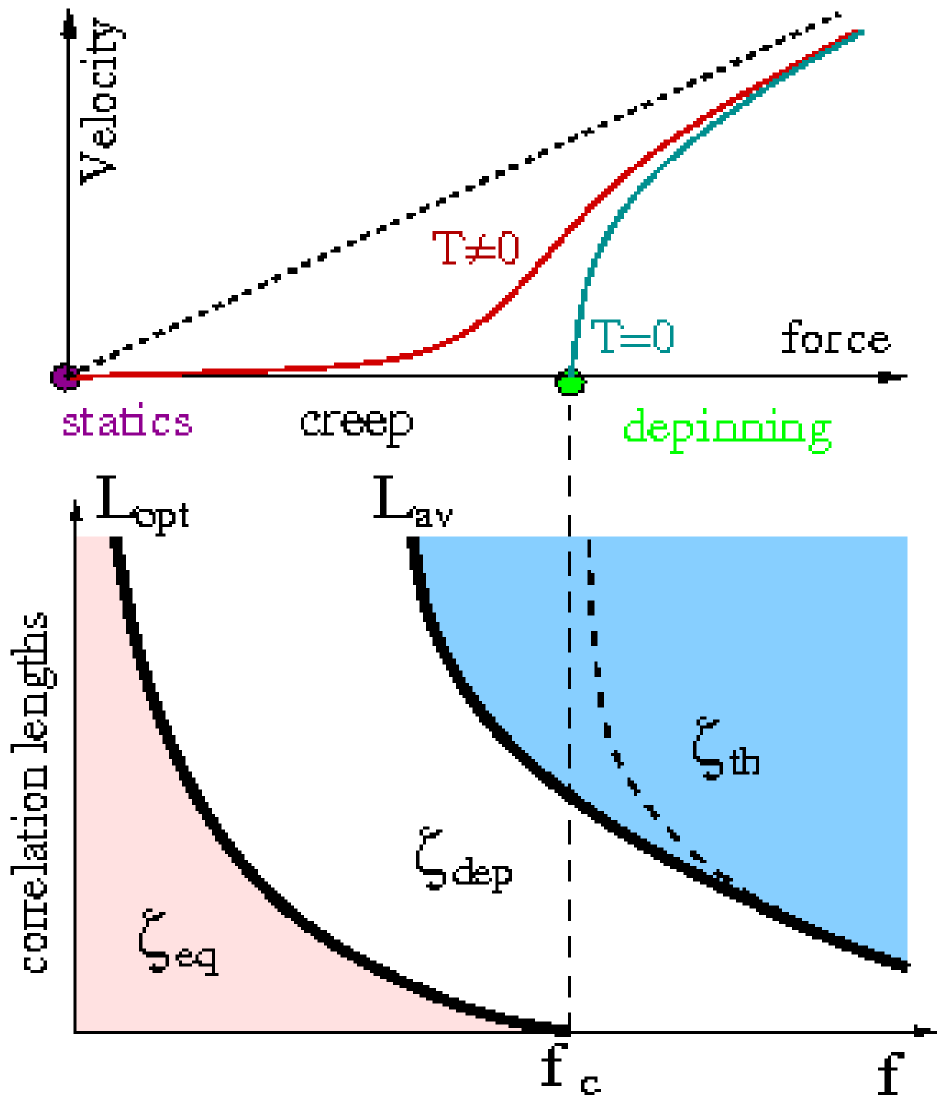
$v \sim nm/s$

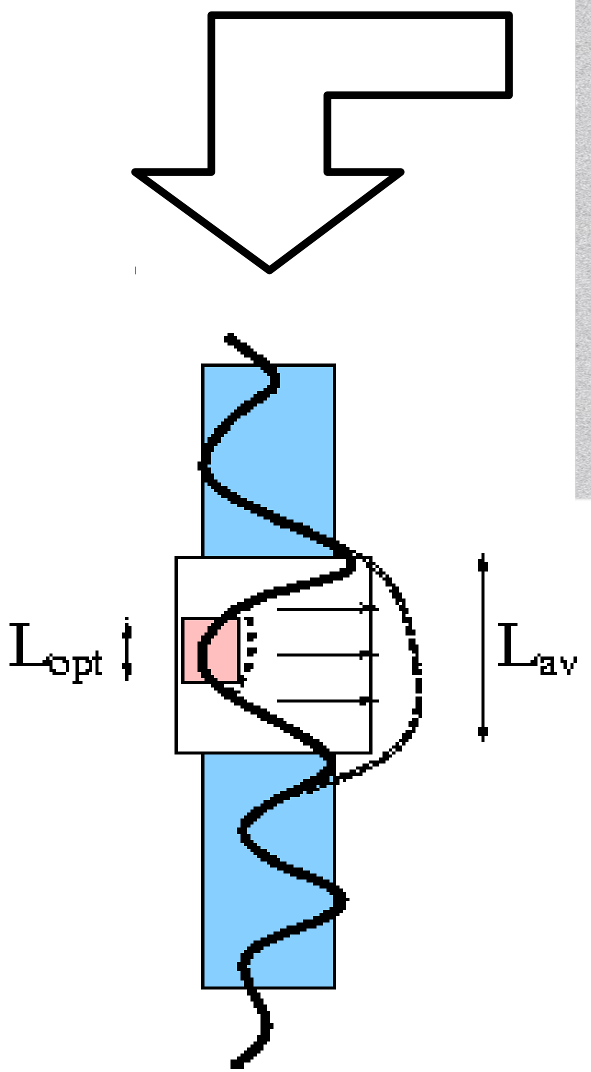
~1 h measurements

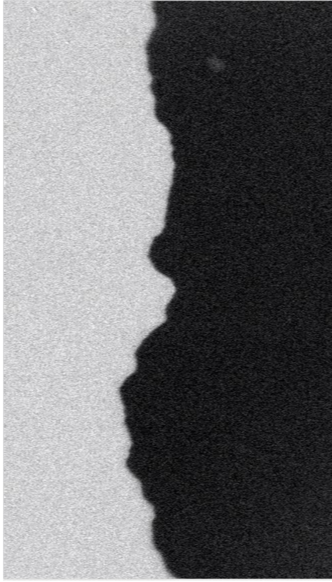
Conclusions

- No length scale diverges in the steady state regime as the depinning threshold is approached. The depinning transition is not a standard critical phenomenon.
- Steady-state low-temperature dynamics of an elastic line in a disordered medium below the depinning threshold using a novel exact algorithm.
- Below threshold, the roughness of the line at large scales is identical to the one at depinning (critical).
- Divergent length scales below threshold are only associated with the transient deterministic relaxation dynamics between metastable states.

Conclusions







REFS:
PRL 94, 047002 (2005); PRL 95, 180604 (2005); **PRL 97, 057001 (2006)**; PRB 75, 140201R (2006); Lecture Notes in Physics 688, 91 (2006); EPL 81, 26005 (2008), **PRB 79, 184207 (2009).**

Theorems: interfaces of dimension d in d+1 with convex elastic energy, not neccesarily harmonic nor local

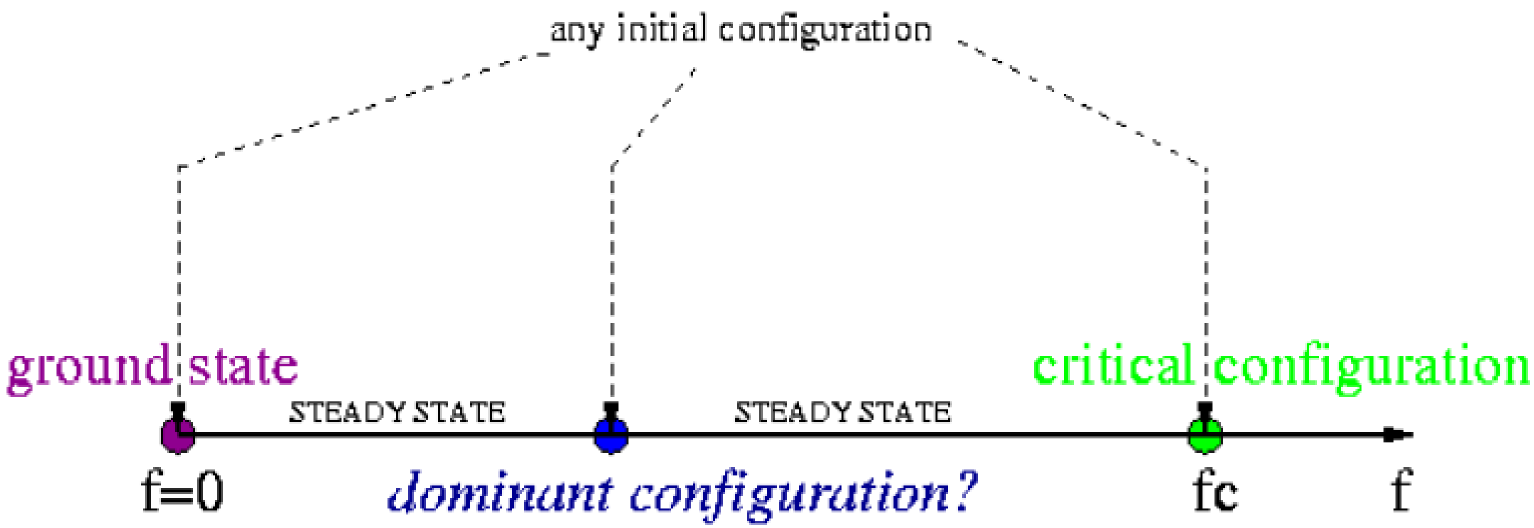
- *Theorem 1*: If there is no configuration which lowers the energy of α in the backward direction, the coarse-grained dynamics starting from α is always forward-directed.

- *Theorem 2*: Let α be any metastable configuration escaping into a configuration γ with $h^{\gamma} \geq h^{\alpha}$ and γ' any configuration such that $h^{\gamma'} \geq h^{\alpha}$ and having an energy barrier $E^{\gamma'}_{esc} > E^{\alpha}_{esc}$: all γ' then satisfy $h^{\gamma'} \geq h^{\gamma}$.

- Practical Consequences:
  - The dynamics is *periodic* after a single pass of the line around the system, and we only have to consider forward motion.
  - The metastable configuration with the largest barrier (dominant) is *always* encountered, independently of the initial condition.

T→ 0 Steady State

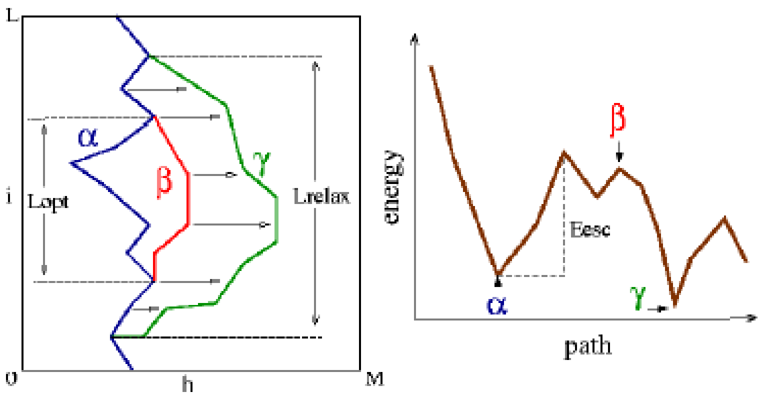
- $\lim_{T\to 0} \lim_{t\to\infty}$



- How to target efficiently these configurations?
  - ⋆ *Ground State*: transfer Matrix approach [Huse and Henley; Kardar (1985)]
  - ⋆ *Critical Configuration*: variant Monte Carlo technique [Rosso, Krauth (2001)]
  - ⋆ *Dominant configuration* $0 < f < f_c$?

Model

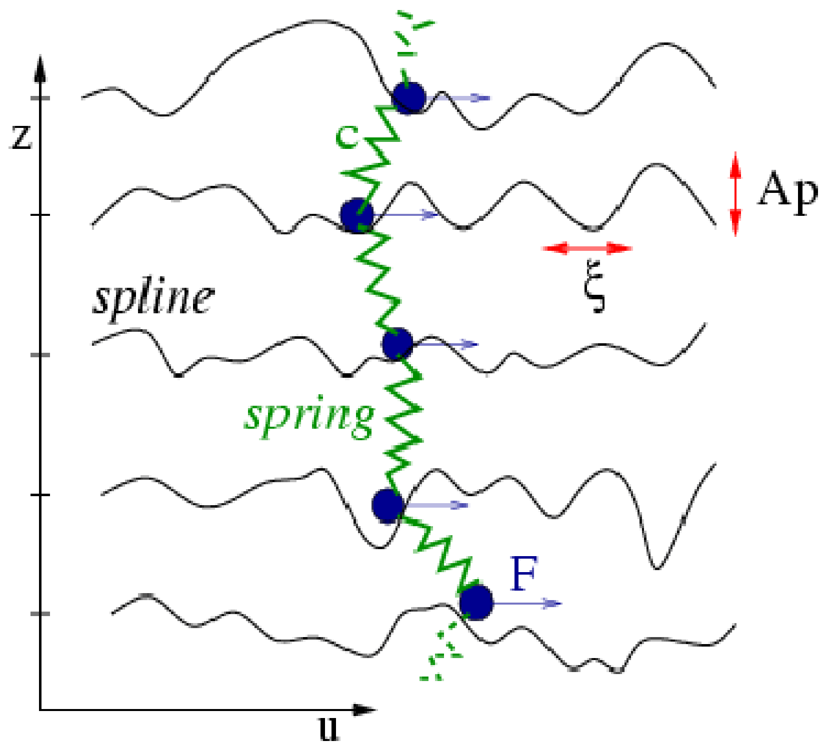
- $E = \sum_i \frac{1}{2}(h_{i+1} - h_i)^2 - f h_i + V_{disorder}(i, h_i)$
- Elementary (global) moves:
  $h_i \rightarrow h_i + \delta_i$, $\delta_i = 0, \pm 1$.
- Optimal path & characteristic lengths



the path with minimal barriers, without avoiding valleys, which connects two metastable states α *and* γ*, such that* $E_\alpha > E_\gamma$. [ask for more details]

Numerical Simulation

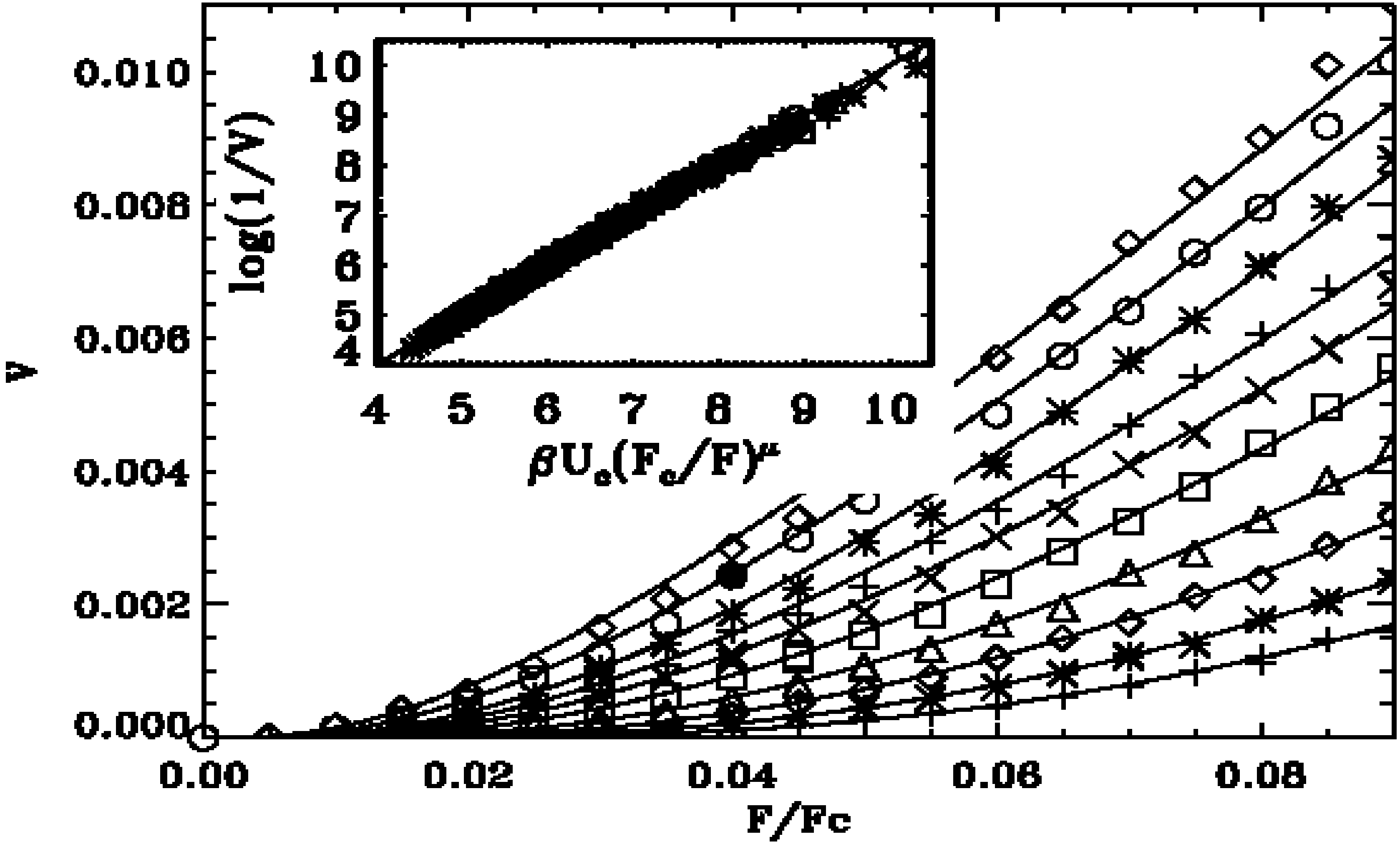
- Discretization in z, Disorder = $spline\ potential$



- *Integration*: stochastic Runge Kutta 2^{nd} order (R-K-Helfand).

- *Exact* F_c calculated by a high precision fastly convergent algorithm.

High temperature creep



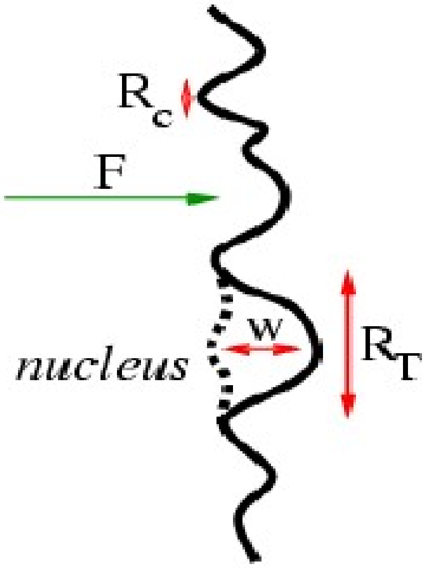
Divergent Barriers

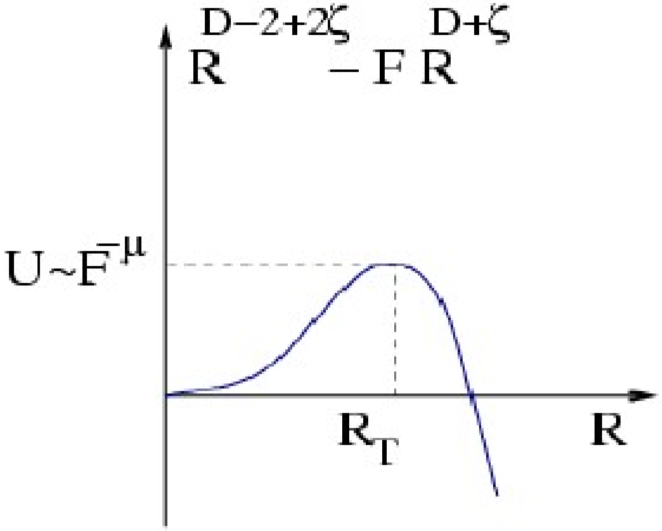
$$V(F) \sim \exp\left[-\frac{U_c}{T}\left(\frac{F_c}{F}\right)^{\mu} \right],$$

PRL 95, 180604 (2005)

Creep motion: scaling theory

Ioffe and Vinokur, Nattermann (1987)



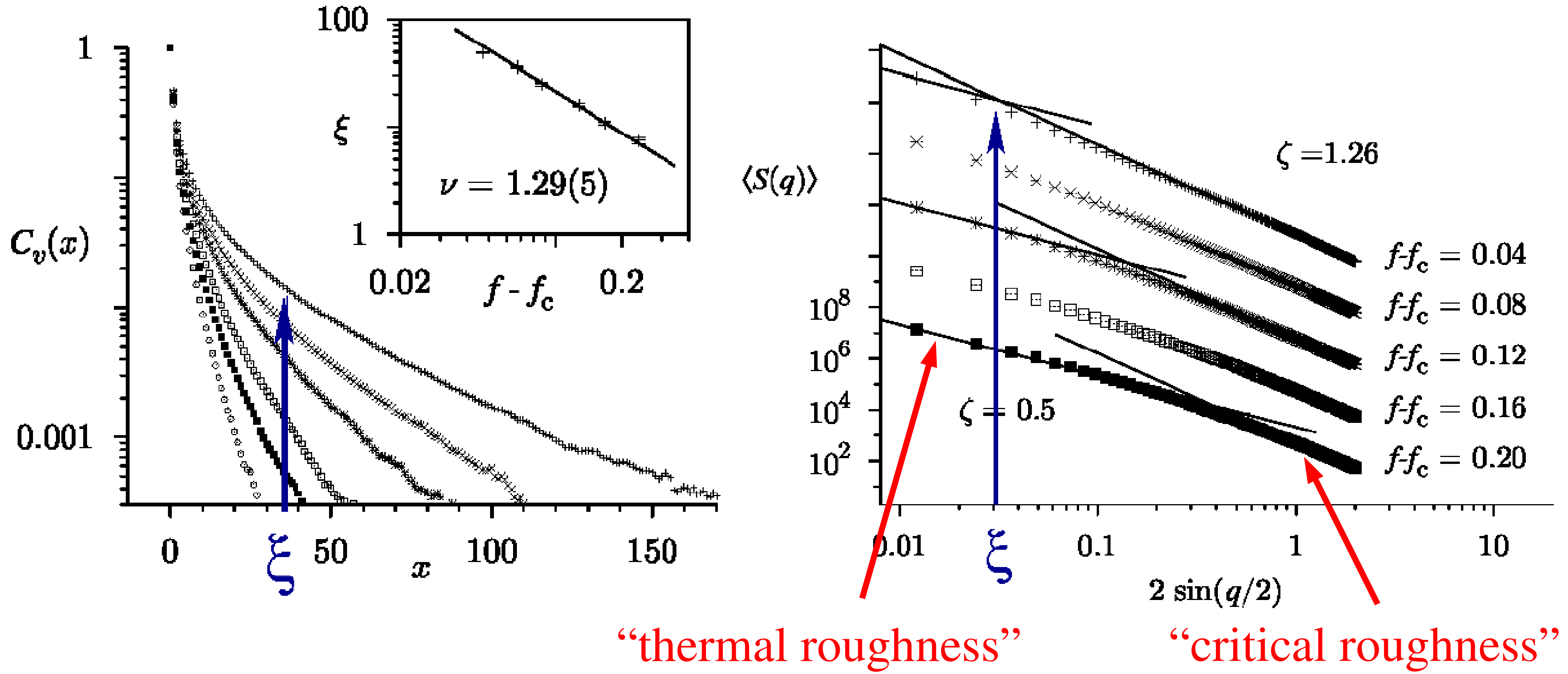


- roughness (statics): $w(R) \sim \xi(\frac{R}{R_c})^{\zeta_{eq}}$, $R > R_c$
- effective barrier: $U_c(\frac{R}{R_c})^{D-2+2\zeta_{eq}} - FR^D\xi(\frac{R}{R_c})^{\zeta_{eq}}$
- optimal nucleus: $R_T \sim R_c(F_c/F)^{1/(2-\zeta_{eq})}$
- Arrhenius: $V \sim e^{-\beta U(R_T)} = e^{-\beta U_c\left(\frac{F_c}{F}\right)^{\mu}}$; $\mu = \frac{D-2+2\zeta_{eq}}{2-\zeta_eq}$

Interfaces in disordered media

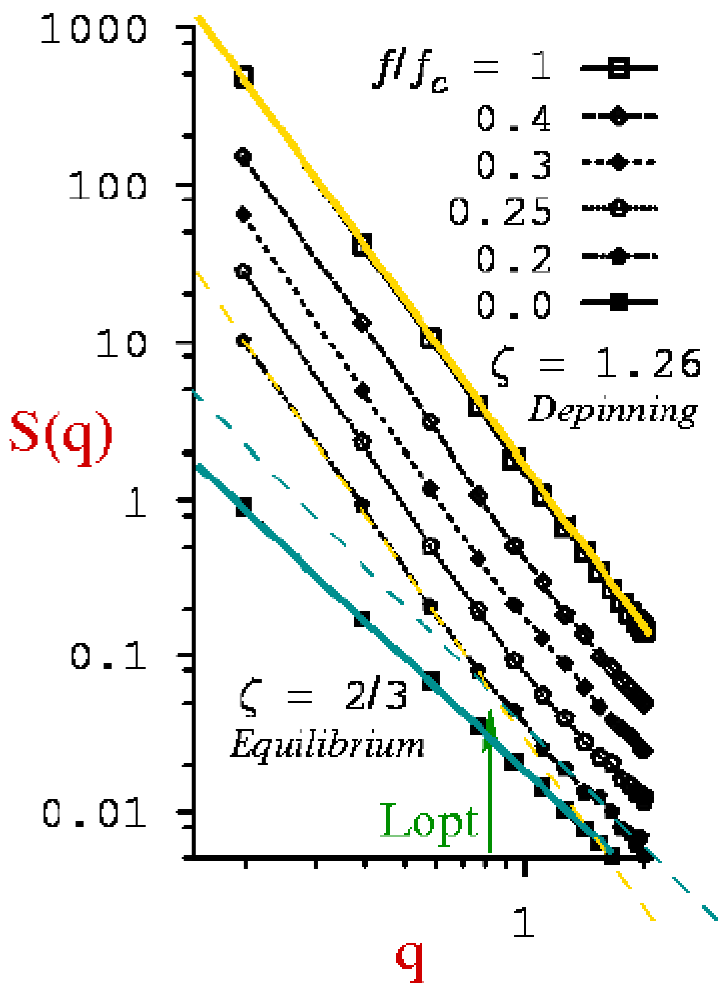
Depinning

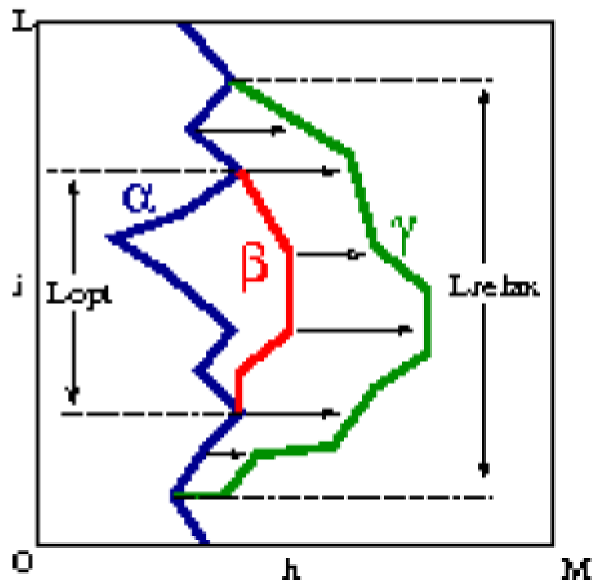
O. Duemmer and W. Krauth (2005)



Average size of avalanches = crossover length in roughness

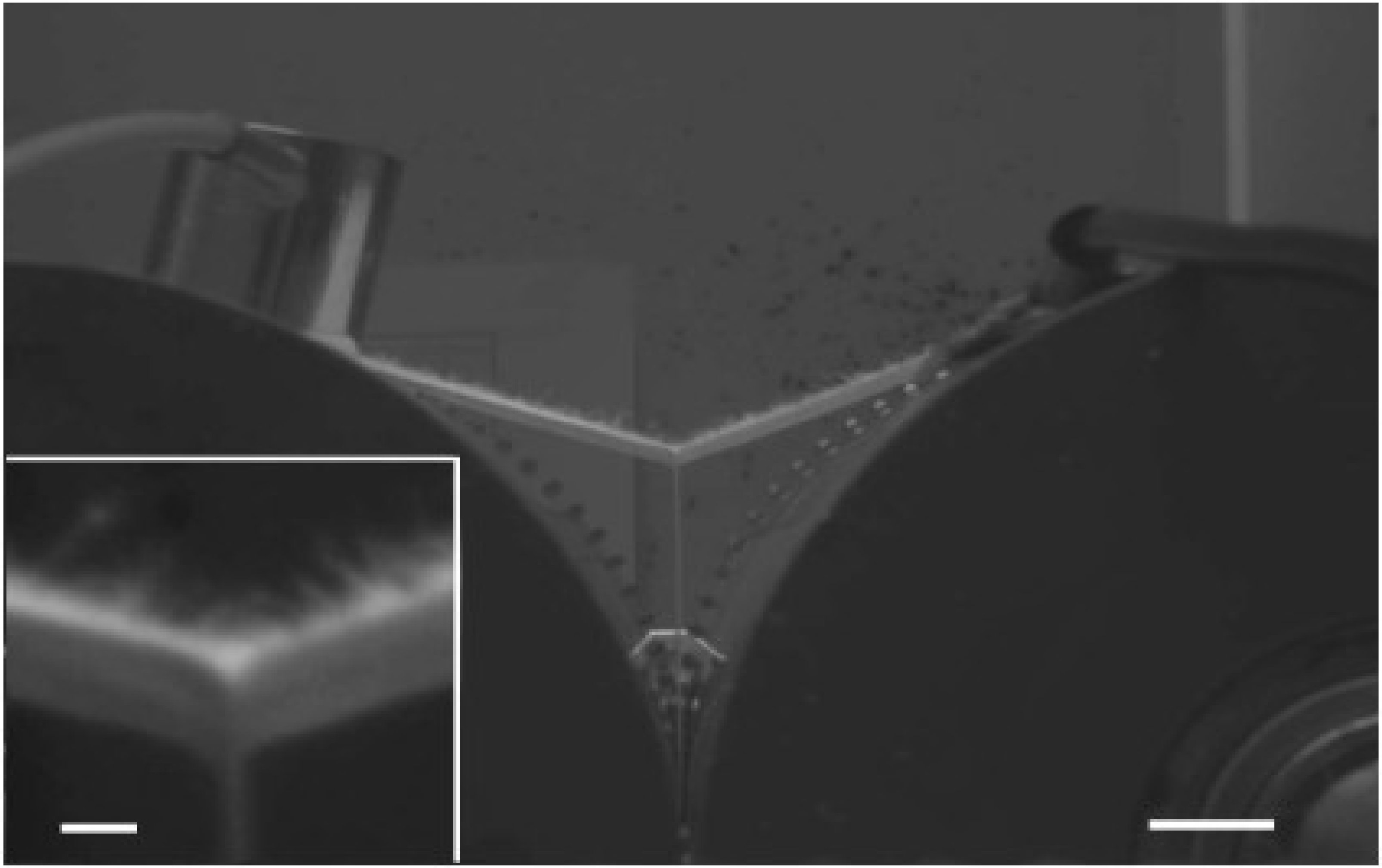
Geometry





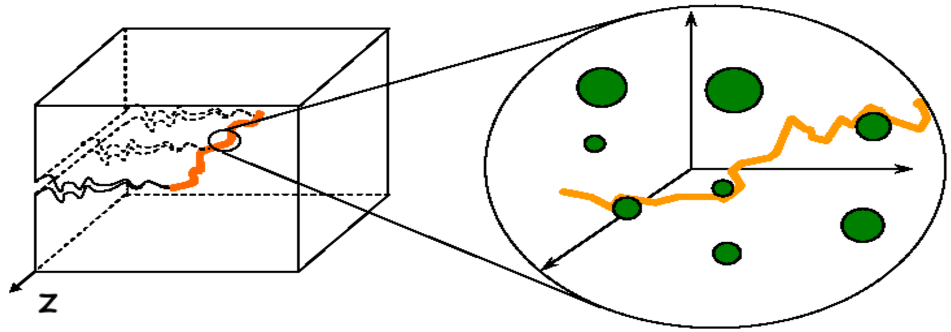
- $\zeta = \zeta_{dep} = 1.26$ at large scales
- $\zeta = \zeta_{eq} = 2/3$ at small scales
- Single crossover length *decreases* with $f \Longrightarrow L_{opt}$

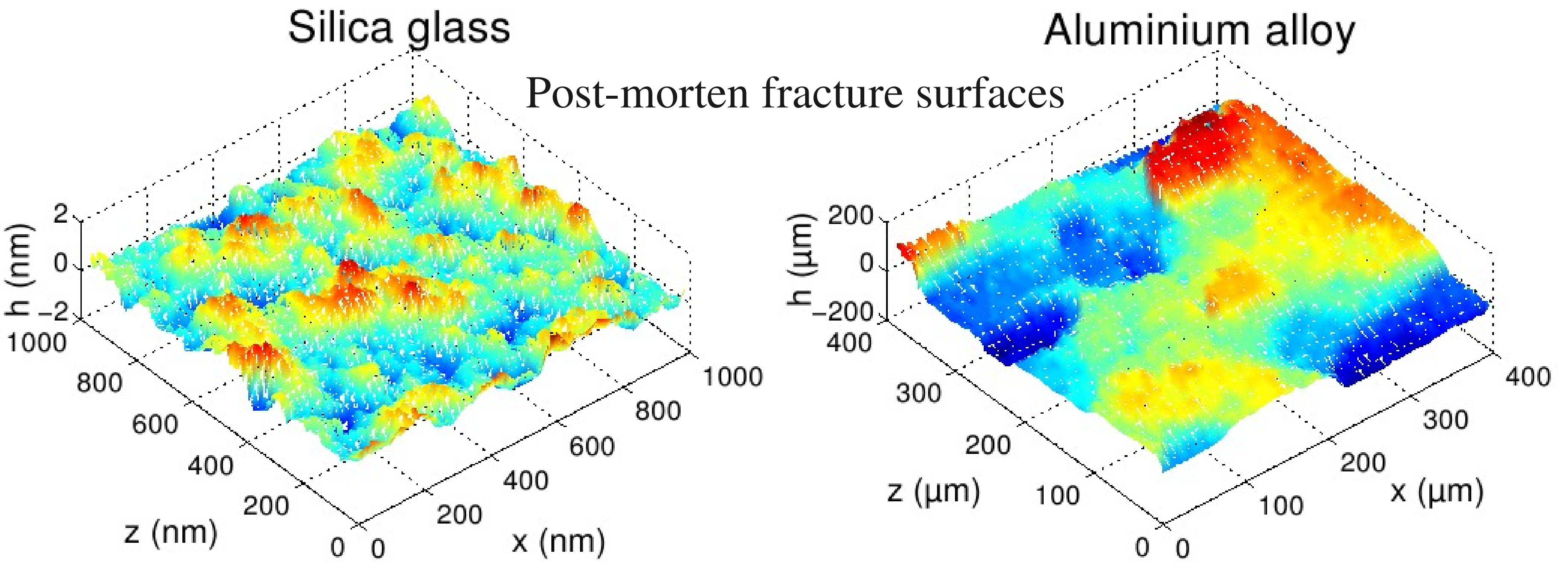
Paper peeling



J. Koivisto, J. Rosti, and M.J. Alava (2007)

Crack propagation





Post-morten fracture surfaces

L. Ponson, D. Bonamy, E. Bouchaud, SPCSI (2006)