Enhancement of superconducting Tc near SIT

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Collaboration:

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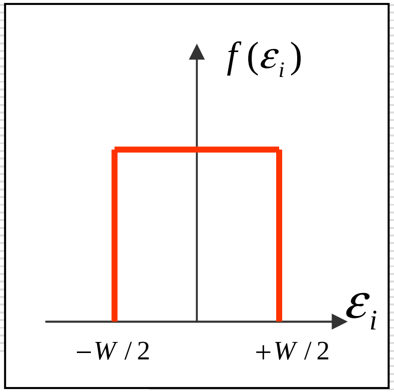
Emilio Cuevas (University of Murcia)

KITP, Santa Barbara, September 13, 2010

The Hamiltonian

- 3d lattice tight-binding model with diagonal disorder
- Local attraction



$$H = \varepsilon_r \delta_{r',r} + V\delta_{r,r+\hat{a}} + H_{\text{int}}$$

$$H_{\text{int}} = -\frac{\lambda}{V_0} \sum_r \psi_\uparrow^+(r)\psi_\downarrow^+(r)\psi_\downarrow(r)\psi_\uparrow(r)$$

NO Coulomb interaction

NO Coulomb interaction?

Where can it happen?

- Cold fermionic atoms in disordered optical traps: atoms are neutral
- In solids with a certain band structure: large background dielectric constant

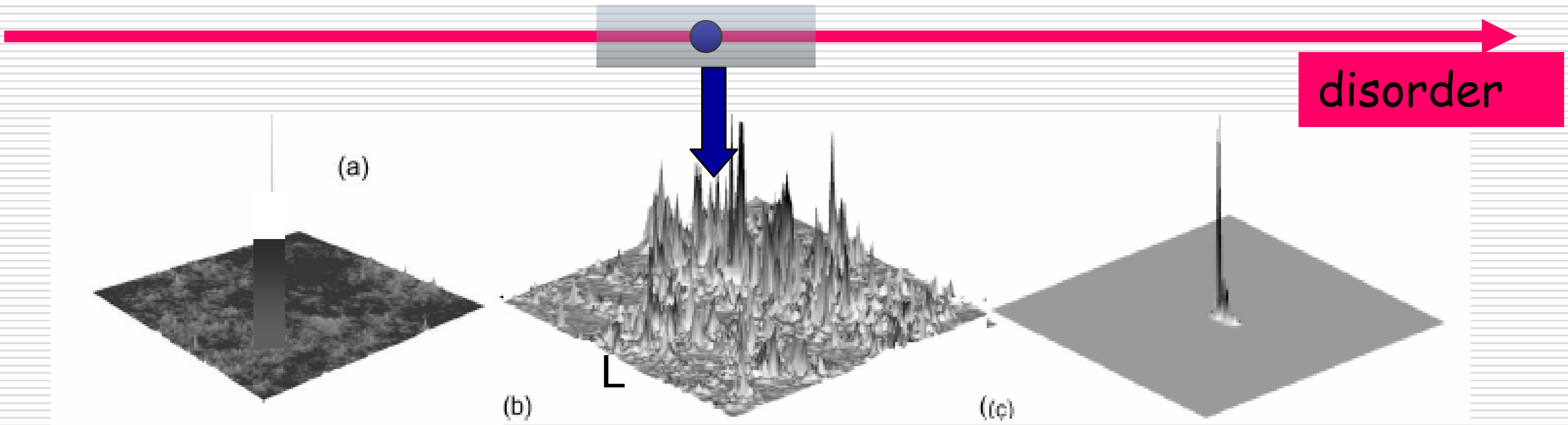
Q: What does the strong disorder do to superconductivity?

A1: weak disorder do not do anything; strong disorder kills superconductivity

A2: eventually disorder kills superconductivity but before killing it enhances it

Weak and strong disorder

Anderson transition



Extended states

Critical states

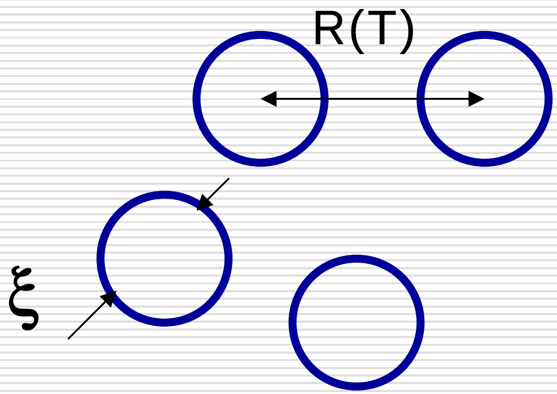
Localized states

$$\ell \sim \lambda_F$$

Direct superconductor to insulator transition

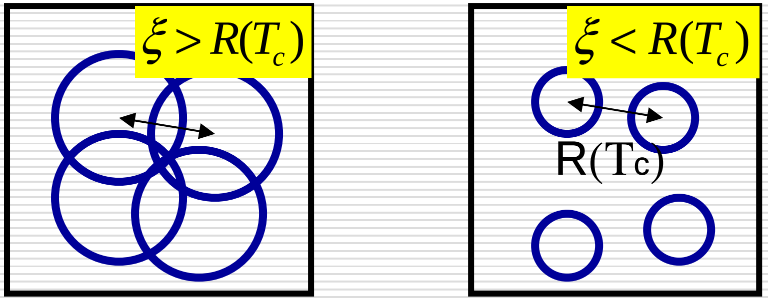
FAQ: Why superconductivity is possible even when single-particle states are localized

Single-particle conductivity: only states in the energy strip ~T near Fermi energy contribute



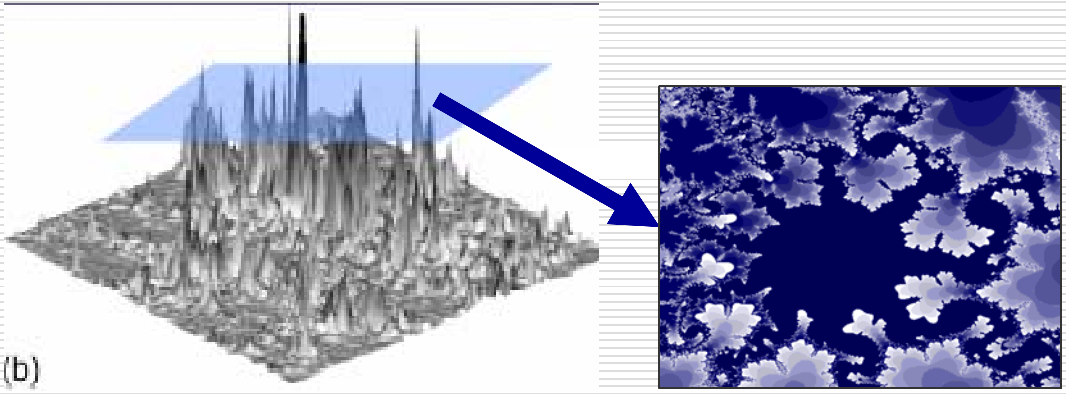
$$R(T) = \left(\frac{1}{\nu_0 T}\right)^{1/3}$$

Superconductivity: states in the energy strip ~Δ~Tc near the Fermi-energy contribute

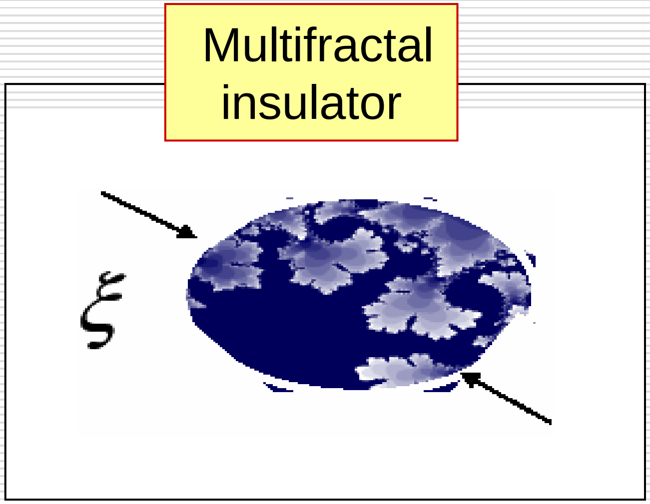


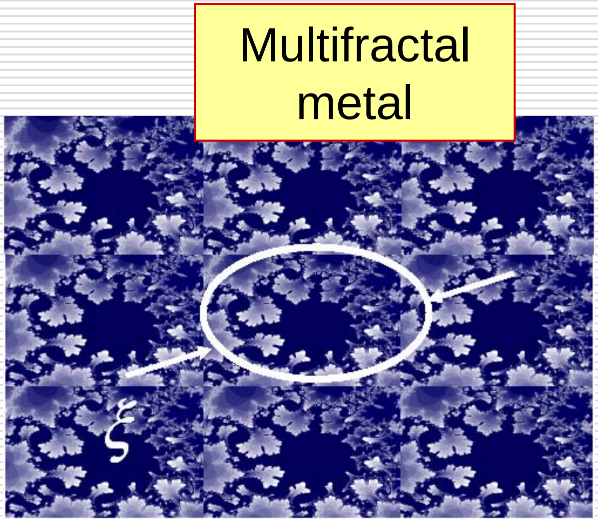
Interaction sets in a new scale Δ~Tc which stays constant as T->0

Multifractality of critical and off-critical states



$$\sum_r |\Psi_i(r)|^{2n} = \frac{1}{L^{d_n(n-1)}}$$





Matrix elements

Interaction comes to play via matrix elements

$$\lambda M_{nm} = \lambda V \int d^d r \quad \Psi_n^2(r)\Psi_m^2(r)$$

Ideal metal and insulator

$$\langle M_{nm} \rangle = \int V d^d r \left\langle \left|\Psi_n(r)\right|^2 \left|\Psi_m(r)\right|^2 \right\rangle$$

Metal:

$$V \quad V \quad \frac{1}{V} \qquad \frac{1}{V} = 1$$

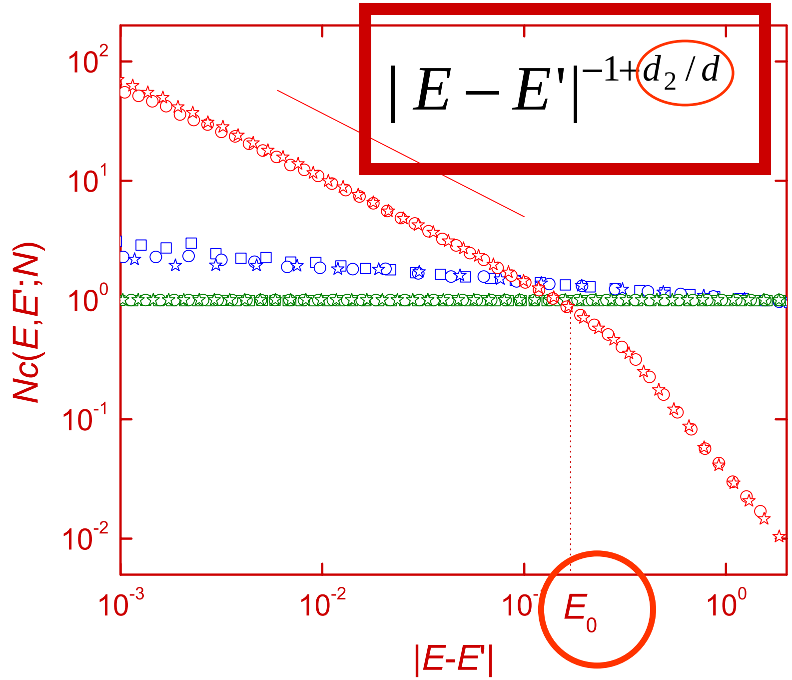
Small amplitude
100% overlap

Insulator:

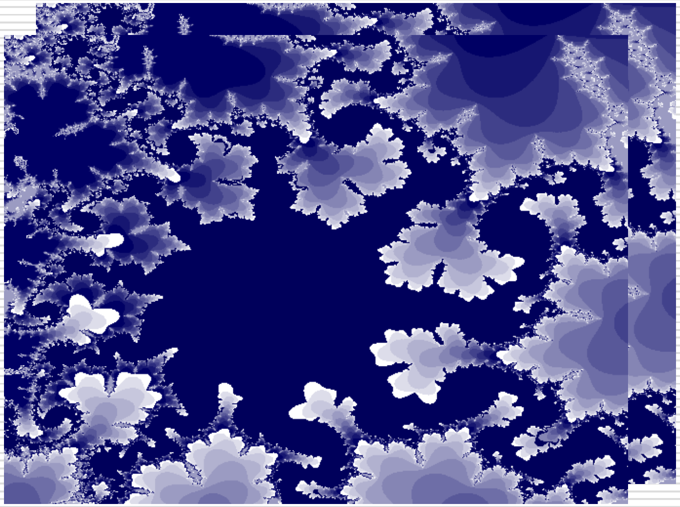
$$V \quad \xi^d \quad \frac{1}{\xi^d} \qquad \frac{1}{\xi^d} \times \left(\frac{\xi^d}{V}\right) = 1$$

Large amplitude
but rare overlap

Critical enhancement of correlations



Amplitude higher than in a metal but almost full overlap



States rather remote (δ<<|E-E'|<E_0) in energy are strongly correlated

Simulations on 3D Anderson model

$$C(E-E') = \frac{\sum_{n,m} V \int d^d r \left\langle |\Psi_n(r)|^2 |\Psi_m(r)|^2 \delta(E_n - E)\delta(E_m - E') \right\rangle}{\sum_{n,m} \left\langle \delta(E_n - E)\delta(E_m - E') \right\rangle}$$

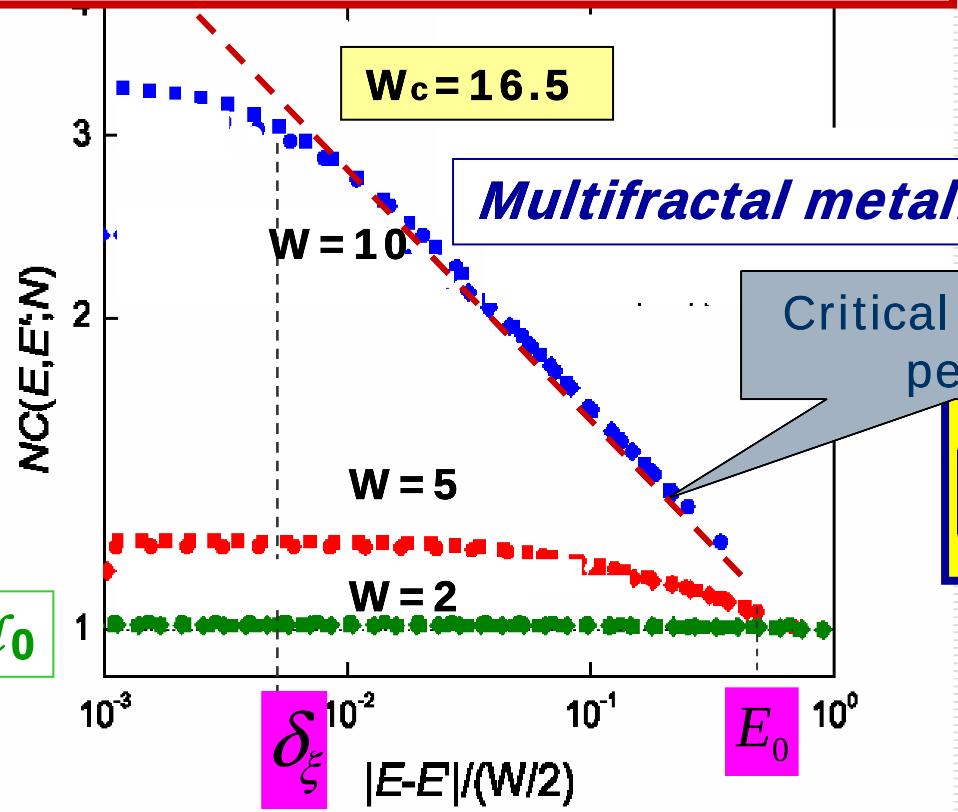
$$E_0 = \left(\nu_0 \ell_0^3\right)^{-1} \sim \frac{D}{W_c}$$

$$\xi = \lambda_F \left| \frac{W_c}{W_c - W} \right|^{\nu}$$

$$\delta_\xi = \left(\nu_0 \xi^d\right)^{-1}$$

$$\ell_0 \sim \lambda_F W_c^{1/3}$$

Ideal metal: $\xi < \ell_0$

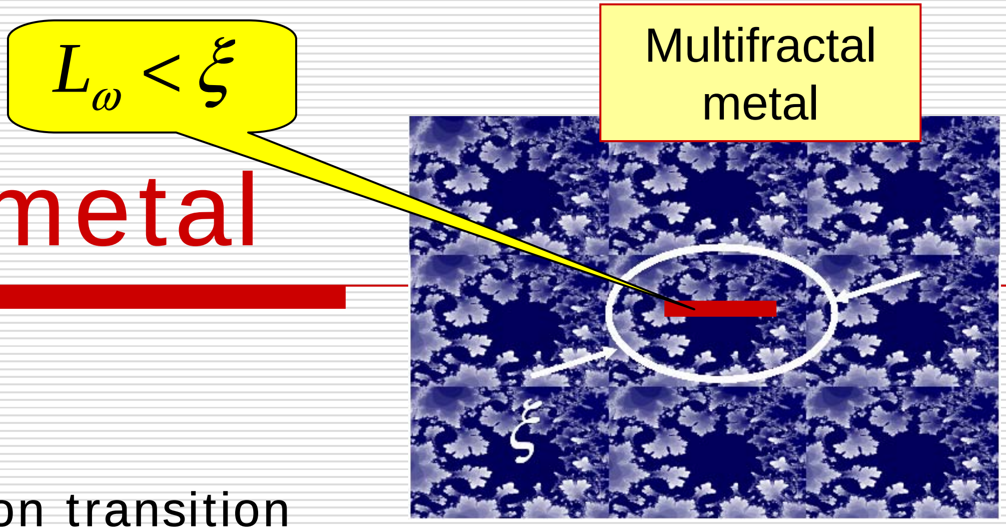


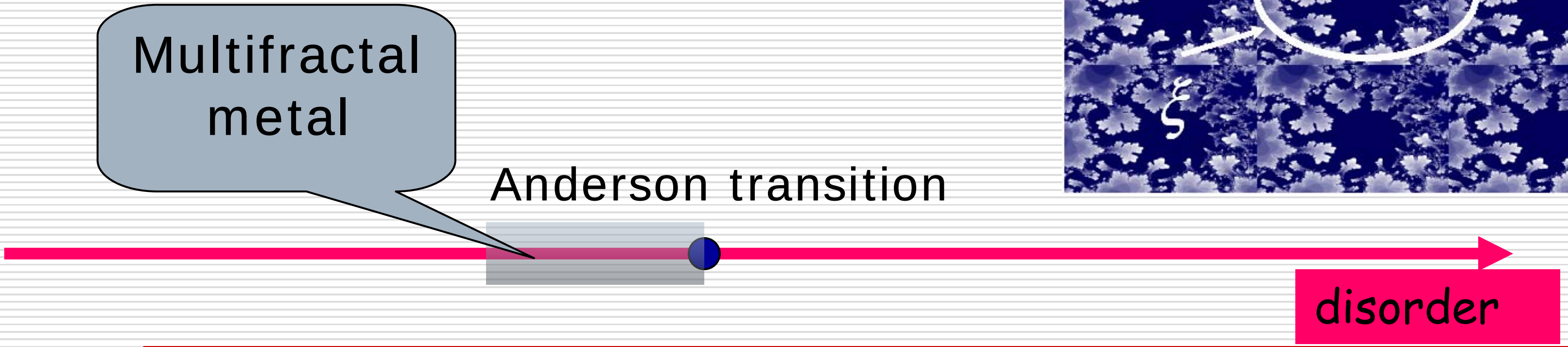
Multifractal metal: $\xi > \ell_0$

Critical power law persists

$$\left(\frac{E_0}{|E - E'|}\right)^{1 - d_2/d}$$

Multifractal metal

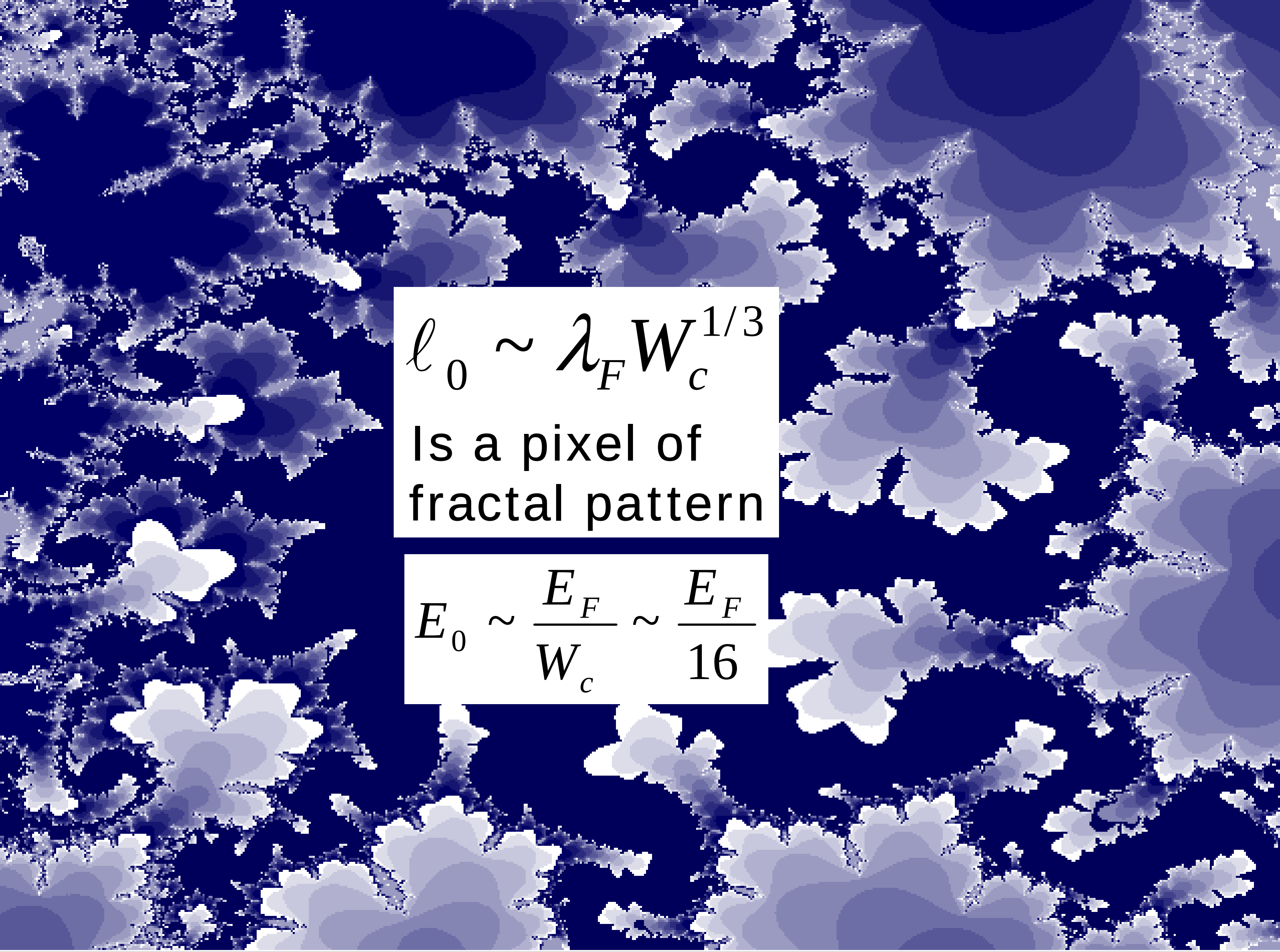




Wave function does not occupy all the available space: enhanced amplitude (normalization)

Regions with enhanced amplitude are strongly correlated for different wave functions as long as

1/3



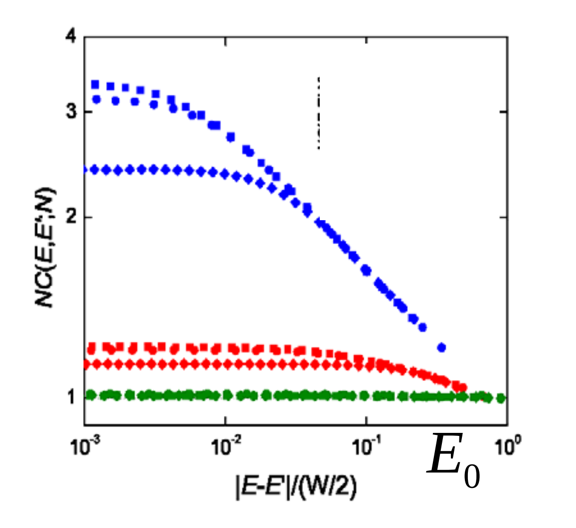
$$\ell_0 \sim \lambda_F W_c^{1/3}$$

Is a pixel of fractal pattern

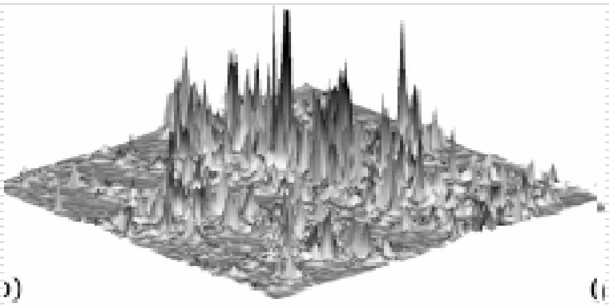
$$E_0 \sim \frac{E_F}{W_c} \sim \frac{E_F}{16}$$

Enhancement of matrix element

$$\lambda M_{nm} = \lambda V \int d^d r \quad \Psi_n^2(r)\Psi_m^2(r)$$



What to do at strong disorder?

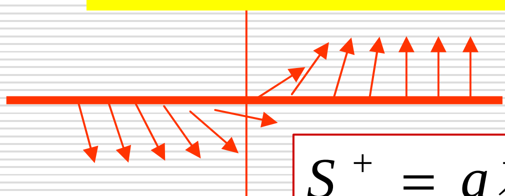


Δ(r) cannot be averaged independently of the MF kernel K(r,r'): No "Anderson theorem"

Single-particle states, strong disorder included

Fock space instead of the real space

$$H_{eff} = -2\sum_i \varepsilon_i S_i^z - U \sum_{i\neq j} M_{ij} (S_i^+ S_j^- + S_i^- S_j^+)$$

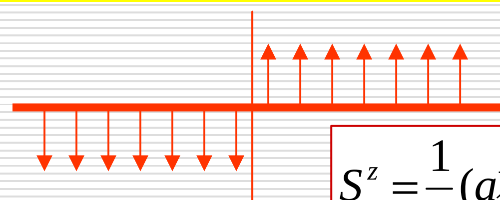


$$S^+ = a_\uparrow^+ a_\downarrow^+$$

E_F

Superconducting phase

$$\langle S_i^{x,y} \rangle \neq 0$$



$$S^z = \frac{1}{2}(a_\uparrow^+ a_\uparrow + a_\downarrow^+ a_\downarrow - 1)$$

E_F

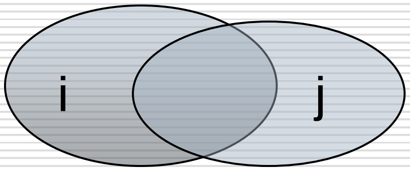
Normal phase

$$\langle S_i^{x,y} \rangle = 0$$

Why the Fock-space mean field is better than the real-space one?

$$H_{eff} = -2\sum_i \varepsilon_i S_i^z - \lambda \sum_{i \neq j} M_{ij}(S_i^x S_j^x + S_i^y S_j^y)$$

$$M_{ij} = \int dr \; \Psi_i^2(r)\Psi_j^2(r)$$



Infinite or large coordination number for extended and weakly localized states: supports mean field approximation

$$\Delta_i = \lambda \sum_j \Delta_j \frac{\tanh(E_j / 2T)}{E_j} M_{ij}$$

M_{ij} is energy dependent

MF critical temperature close to critical disorder

At a small λ parametrically large enhancement of T_c

$$\Delta(E) = \lambda \int dE' \; \Delta(E') \frac{\tanh(E'/2T)}{E'} \; M(E-E')$$

$$M(E-E') = \left(\frac{E_0}{|E-E'|} \right)^{1-d_2/d}$$

Critical enhancement of correlations

$$T_c \sim E_0 \; \lambda^{1/(1-d_2/d)} \sim E_0 \; \lambda^{1.78} \gg \omega_D \; \exp[-1/\lambda]$$

M.V.Feigelman, L.B.Ioffe, V.E.K. and E.Yuzbshyan, Phys.Rev.Lett. v.98, 027001 (2007);

Isotop effect

$$T_c \sim E_0 \quad \lambda^{1.78}$$

$$E_0 \sim E_F / W_c \sim E_F / 16$$

No isotop effect

Annals of Physics 325, 1368 (2010)

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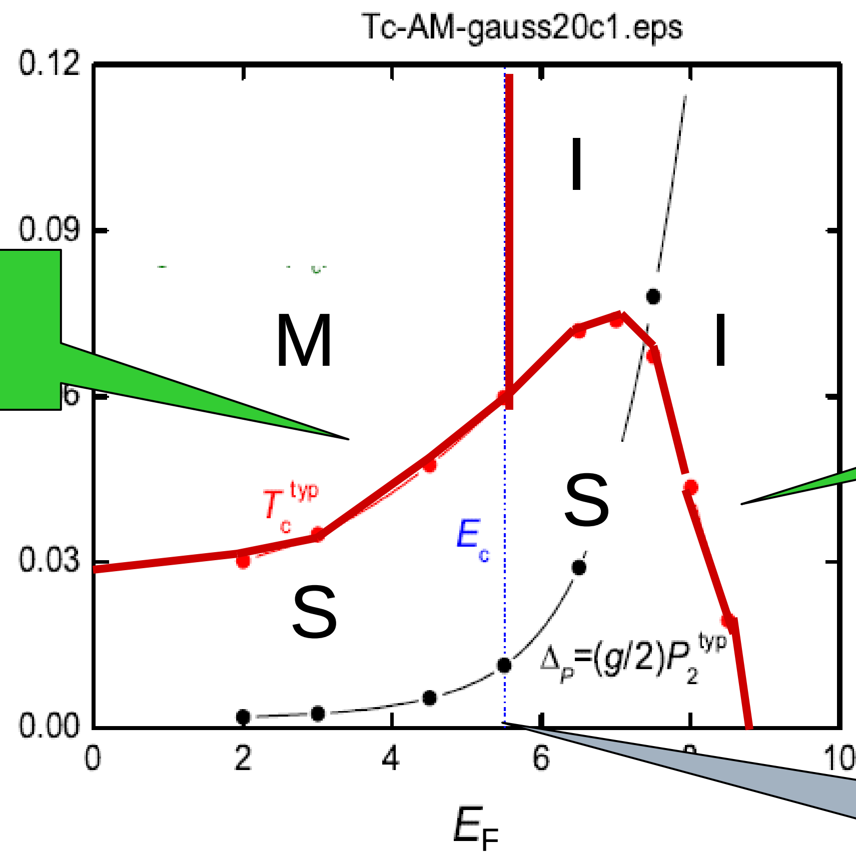
Fractal superconductivity near localization threshold

M.V. Feigel'man [a,b], L.B. Ioffe [a,c,d,*], V.E. Kravtsov [a,e], E. Cuevas [f]

The phase diagram

$\lambda_{BCS} = 0.2$

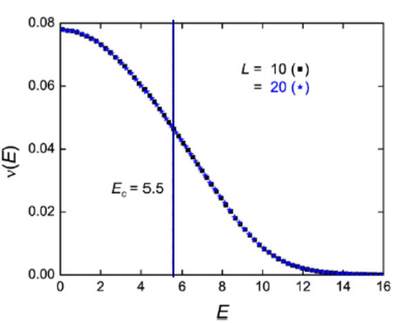
Tc



Extended states

Localized states

Mobility edge

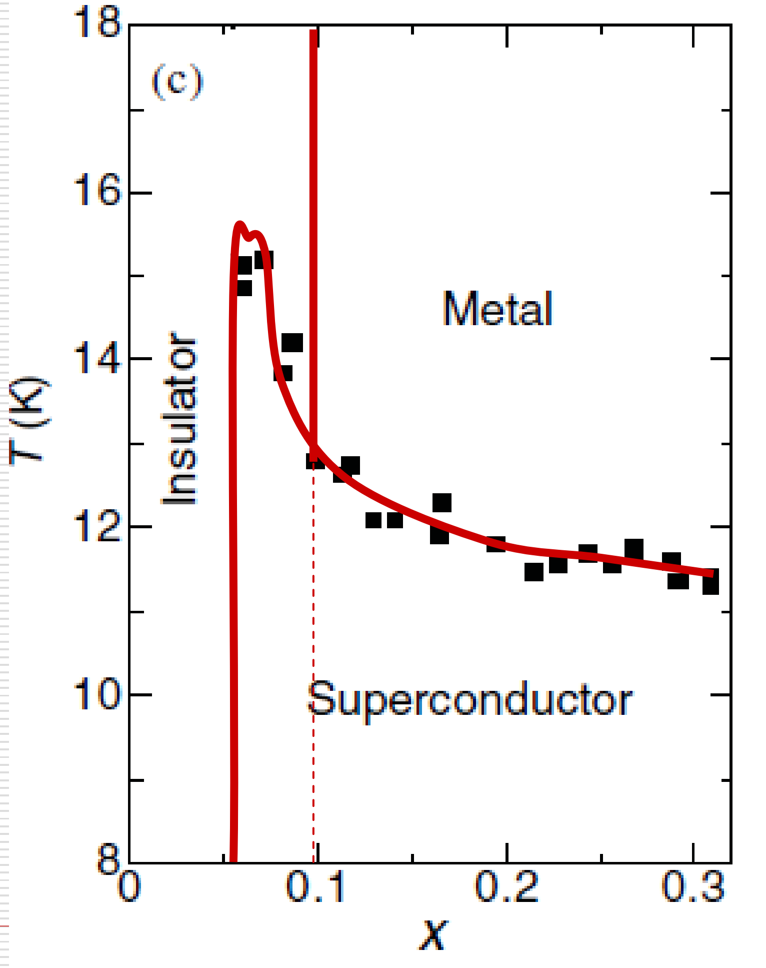


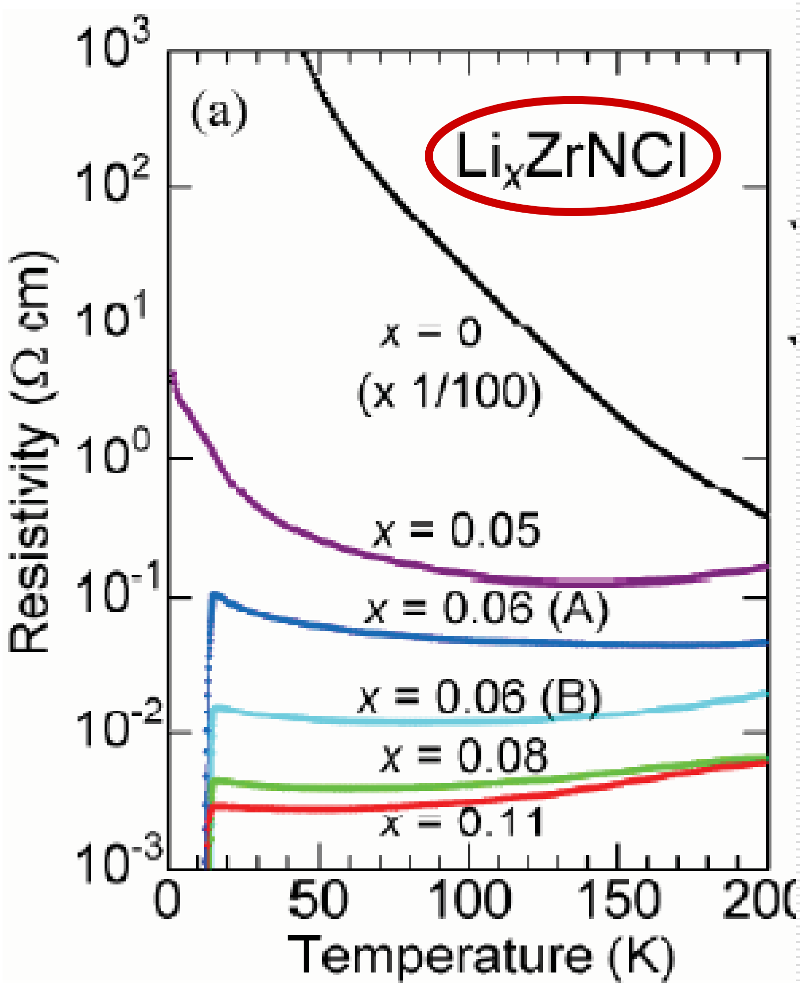
PRL 97, 107001 (2006) PHYSICAL REVIEW LETTERS week ending 8 SEPTEMBER 2006

Increase in T_c upon Reduction of Doping in Li_xZrNCl Superconductors

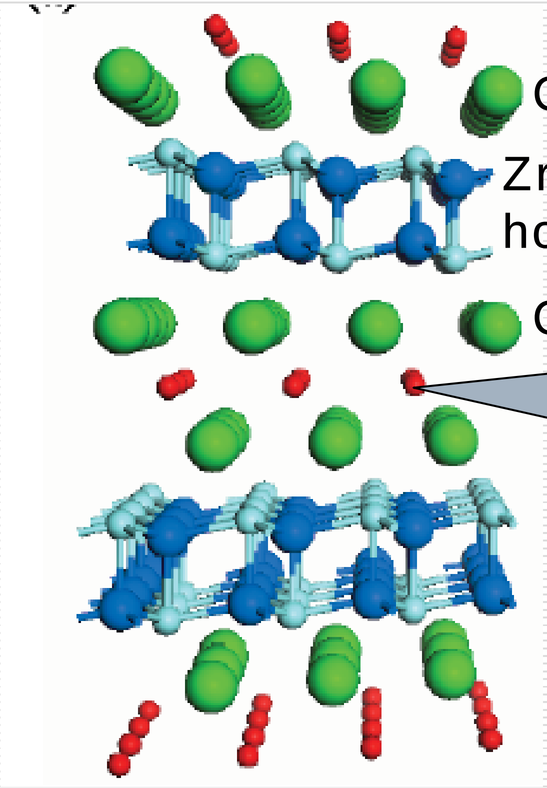
Y. Taguchi,[1,2] A. Kitora,[1] and Y. Iwasa[1,2]

Relevant experiment?





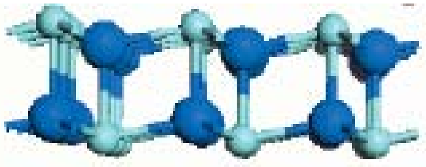
Li-doped ZrNCl



Without Li ZrNCl is a band insulator with a gap of 1.6 eV

Why “natural” in this compound?

Initial motivation for our theory: direct SIT in amorphous films of InO, TiN



ZrN layer

Superconductivity survives until disorder is strong enough to cause Anderson localization. Why Coulomb interaction does not destroy it well before?

Ti 22
Zr 40
Hf 72

In Mendeleev’s periodic Table

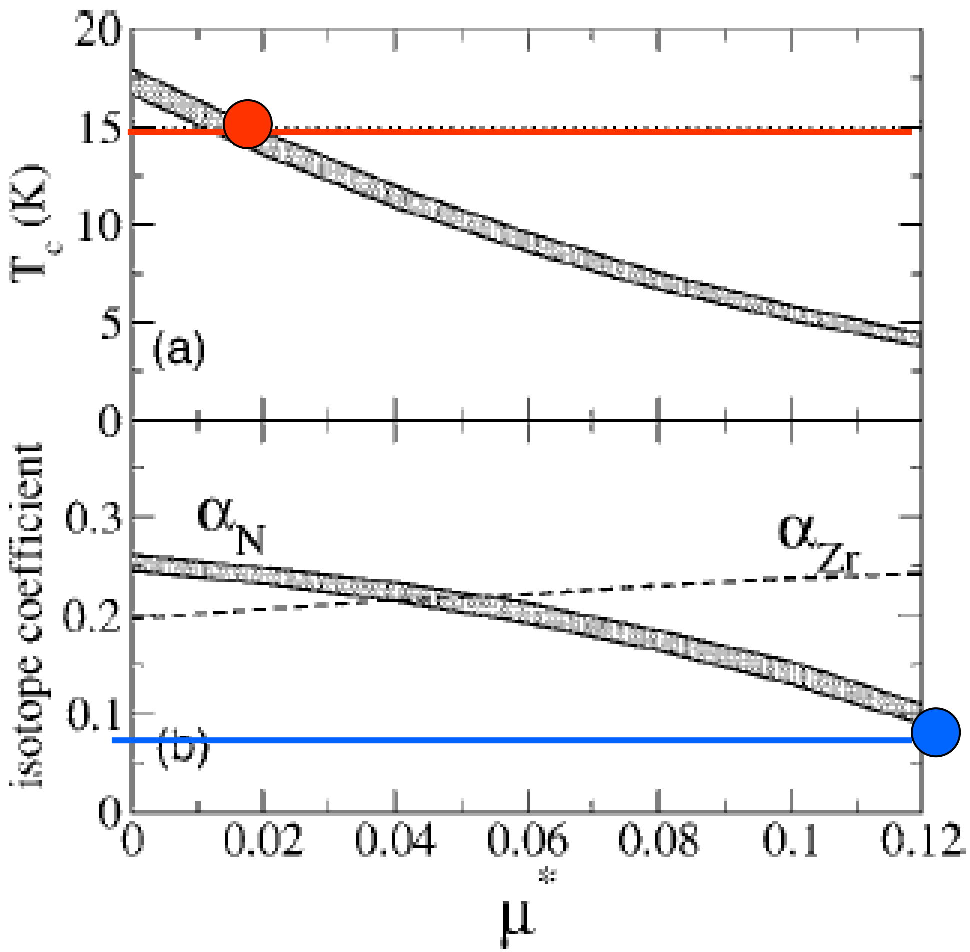
Li-doped HfNCl is a similar SC with higher Tc~25K

Electron-phonon attraction constant ●~0.5 follows from band-structure calculations

Tc is too large

Eliashberg theory
+ band structure
calculations

Isotop effect is
too small



Does the band structure support large background dielectric constant?

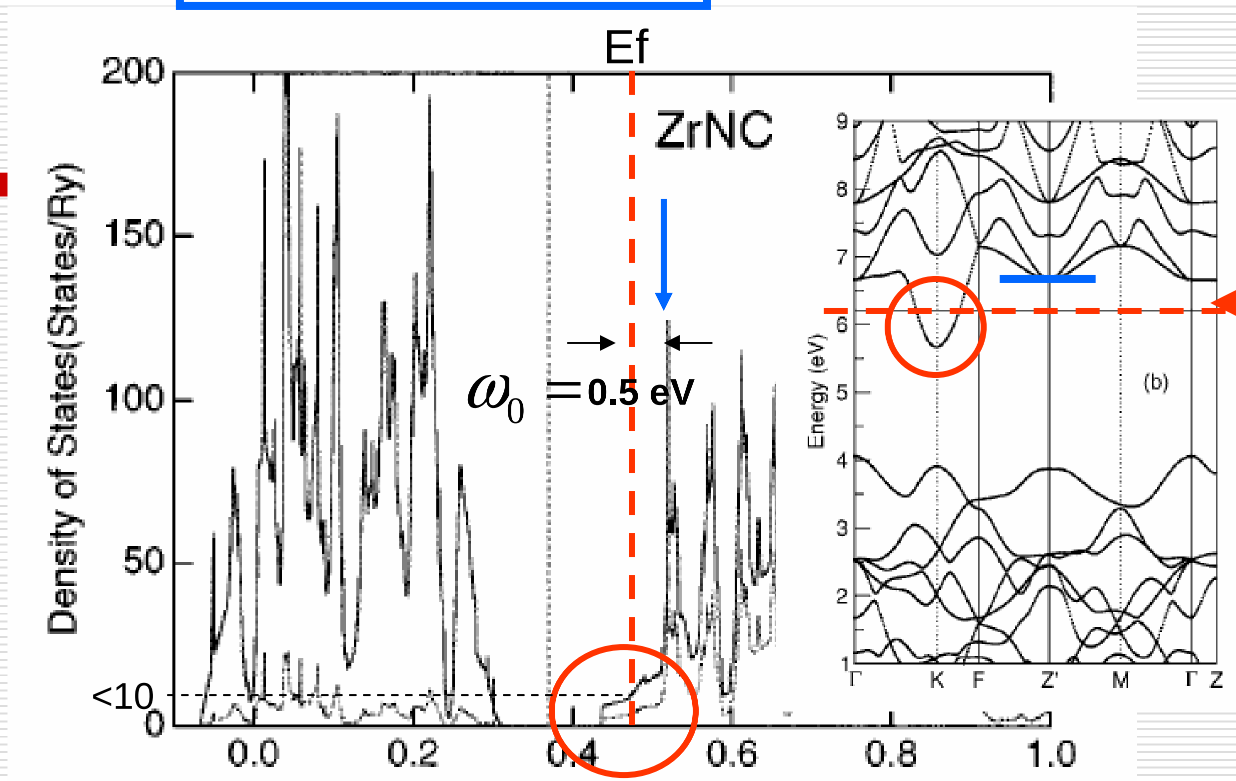
$$\varepsilon \sim \frac{\sigma}{\omega_0 - \omega} \propto \frac{\nu_i \nu_f}{\omega_0}$$

$\nu_i \sim 0.4 \quad states/eV \sim 0.2\nu_{Gold}$

$\nu_f \sim 120\, states/Ry \sim 3\nu_{Gold}$

$\omega_0 \sim 0.5eV \sim 0.1 \quad E_{F_{Gold}}$

DoS in good metals (gold) ~ 2 states/eV (per atom) ~ 30 states/Ry



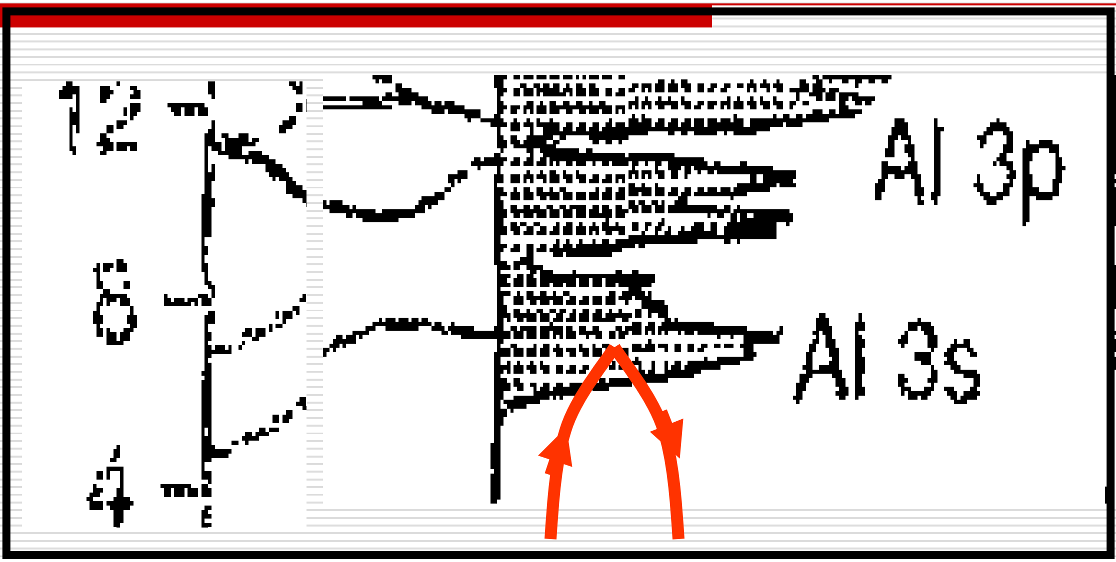
Ry=13.6 eV

Fermi surface occupies a very small part of the Brillouin zone ●F<<a: clear separation between low-energy and “high-energy” physics

High peaks in DoS close to low DoS conduction band

Al N

eV



No high peaks in the DoS close to conduction band

$$\varepsilon_0 \sim 6$$

$$\omega_0 \sim 6\ eV$$

$$\varepsilon_0 \sim 50$$

For large-distance screening

Does not seem to be unreasonable as long as the high DoS peak at ◆ ~0.6 eV persists

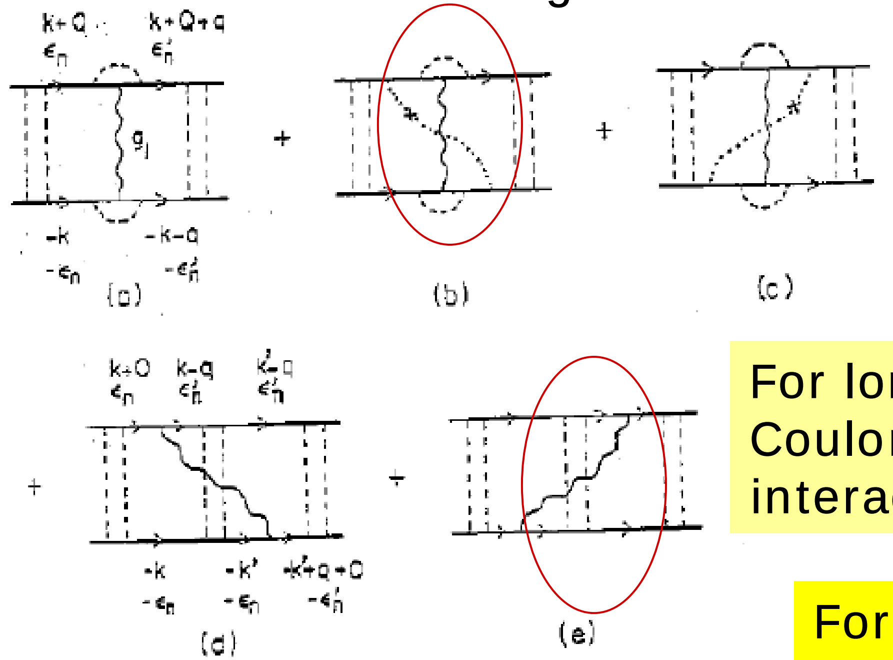
Small-distance (large k~1/a) screening may be much weaker because of the effect of dispersion $\omega_0(k) = \omega_0 + k^2 / 2m$

Conclusions

- Enhancement of superconductive T_c by near-critical disorder: combined action of “holes” in the single-particle wavefunction amplitude and correlation in positions of peaks for different wavefunctions.
- Power-law dependence of T_c on the attractive interaction constant λ at near-critical disorder. The exponent depends on the critical exponent: the fractal dimension d_2.
- Possible experimental system with weak long-range Coulomb interaction

2d weak multifractality: the Ovchinnikov-Mayekawa-Fukuyama-Finkelstein effect

The diffuson diagrams



The cooperon diagrams

$$\frac{\Delta T_c}{T_c} = \frac{\lambda}{g} \ln^3\left(\frac{1}{T_c \tau}\right)$$

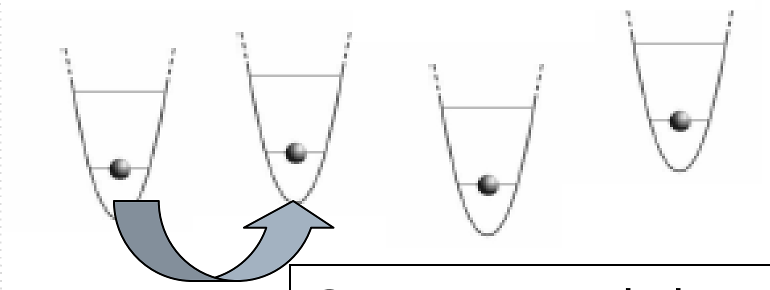
For long-range Coulomb interaction $\lambda = -1$

$$V^{-1} = V_0^{-1}(q) - \Pi \approx -\nu_0$$

For short-range attraction one has effectively ln^2 as $\lambda = 1/\ln$

Cold atoms trapped in an optical lattice

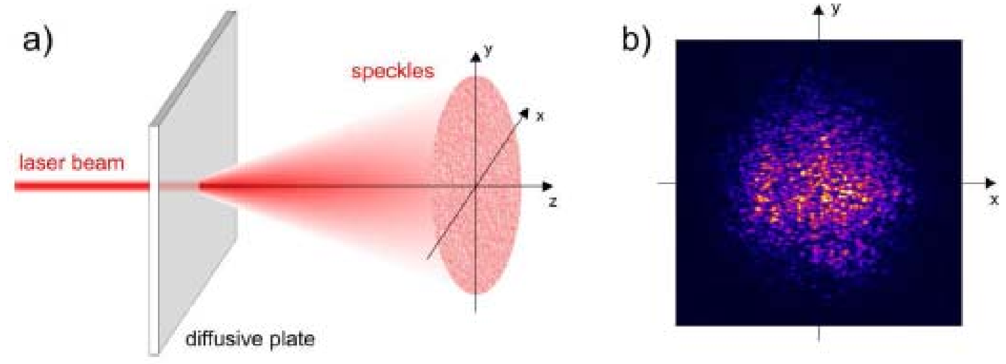
Fermionic atoms trapped in an optical lattice



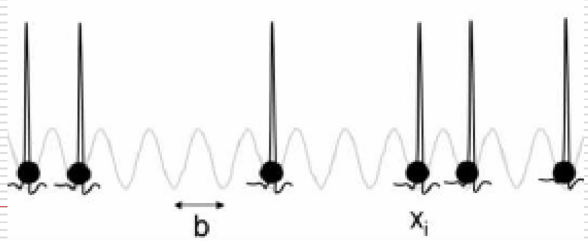
Disorder is produced by:

One, two and three-dimensional tight-binding systems with disorder

speckles



Other trapped atoms (impurities)



One-dimensional Anderson localization in systems of cold atoms

Alain Aspect group

J.Billy et al., Nature **453**, 891 (2008);

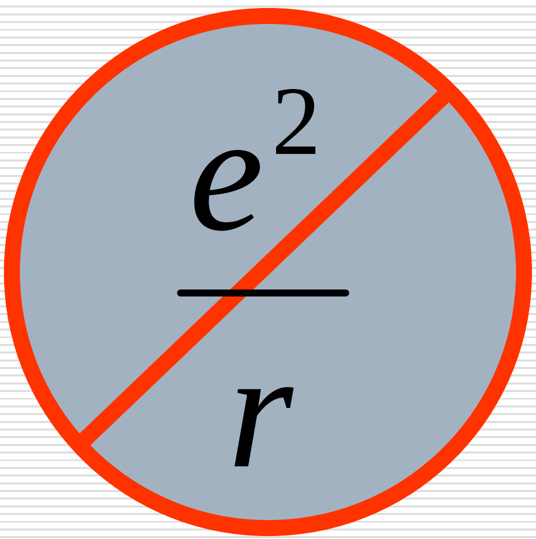
Massimo Inguscio group

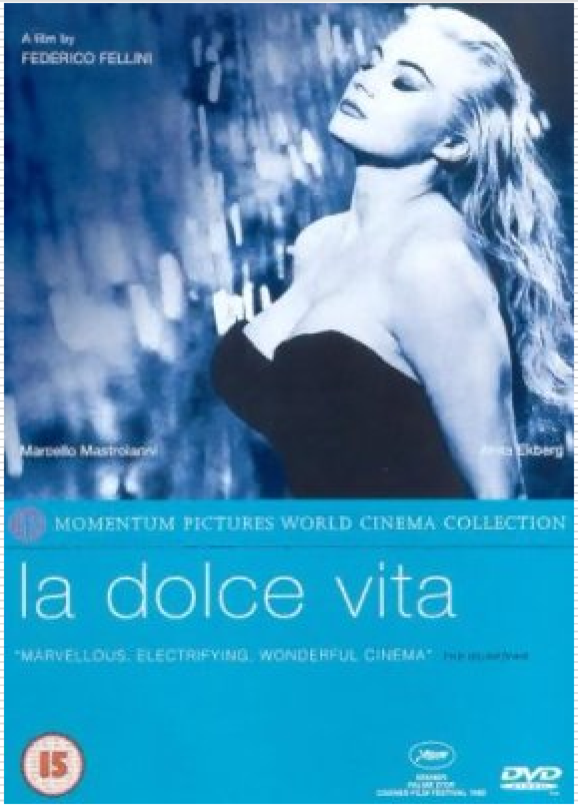
Roati et al., Nature **453**, 895 (2008)

Work on two- and three-dimensional localization is in progress

BUT…

Sweet life is only possible without Coulomb interaction





Mean-field approximation and the Anderson theorem

$$\Delta(r) = \int dr' \langle \Delta(r') \rangle \langle K(r,r';T) \rangle$$

$$K(r,r';T) = U \sum_{ij} \eta_{ij}(T) \; 1/V \qquad \delta_{ij}$$

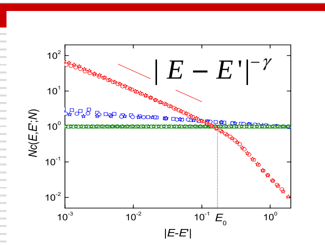
$$\eta_{ij}(T) = \frac{\tanh(E_i/2T) + \tanh(E_j/2T)}{E_i + E_j}$$

Wavefunctions drop out of the equation

Anderson theorem: Tc does not depend on properties of wavefunctions

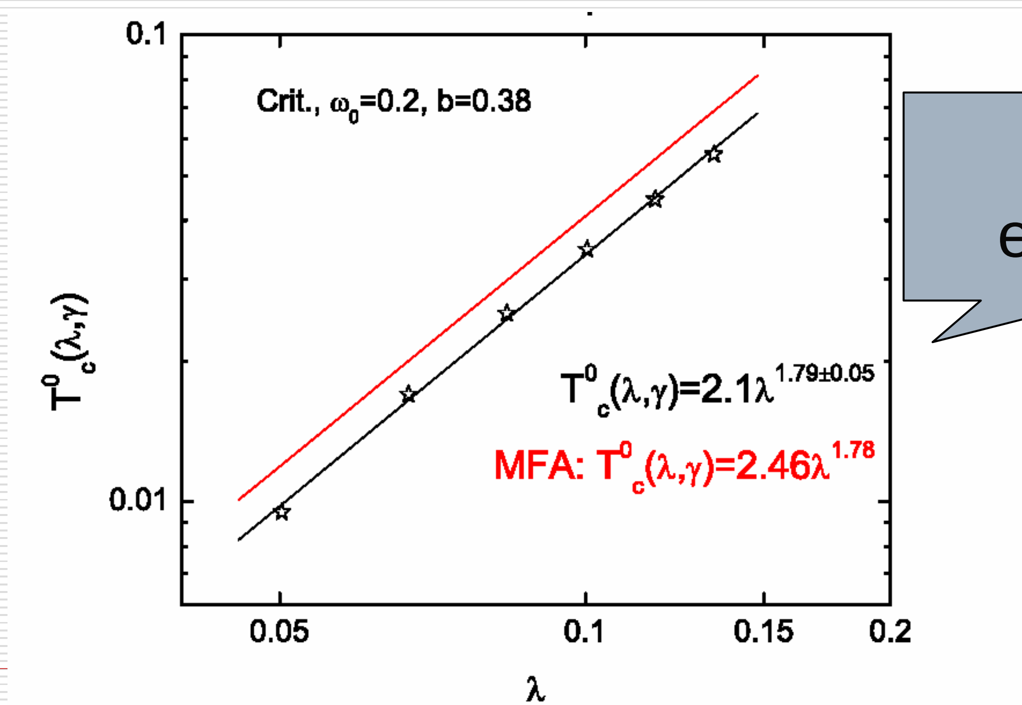
Tc at the Anderson transition: MF vs virial expansion

M.V.Feigelman, L.B.Ioffe, V.E.K. and E.Yuzbshyan,
Phys.Rev.Lett. v.98, 027001 (2007);



$$T_{crit} = c_\gamma E_0 \lambda^{1/\gamma}, \qquad \gamma = 1 - d_2/d$$

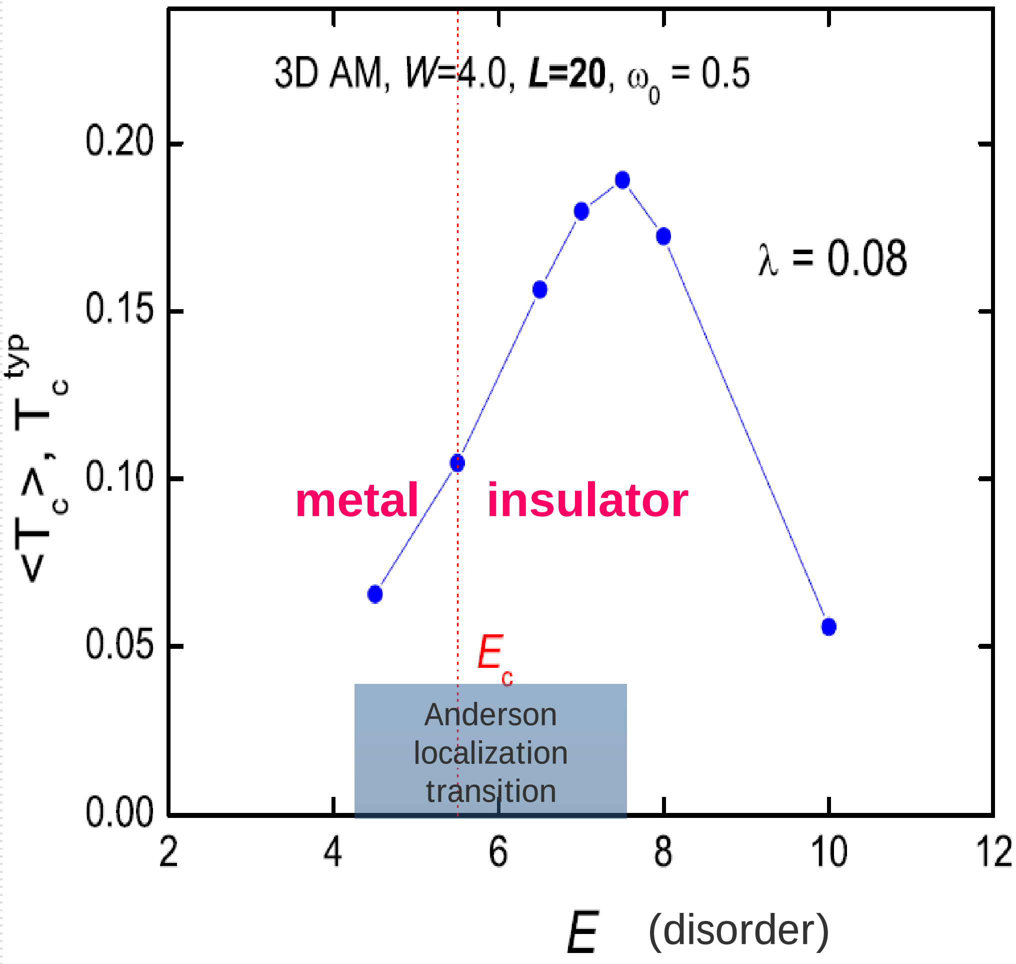
MF result

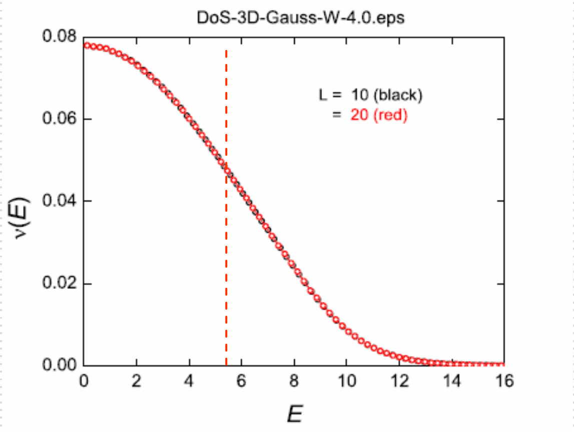


Virial expansion

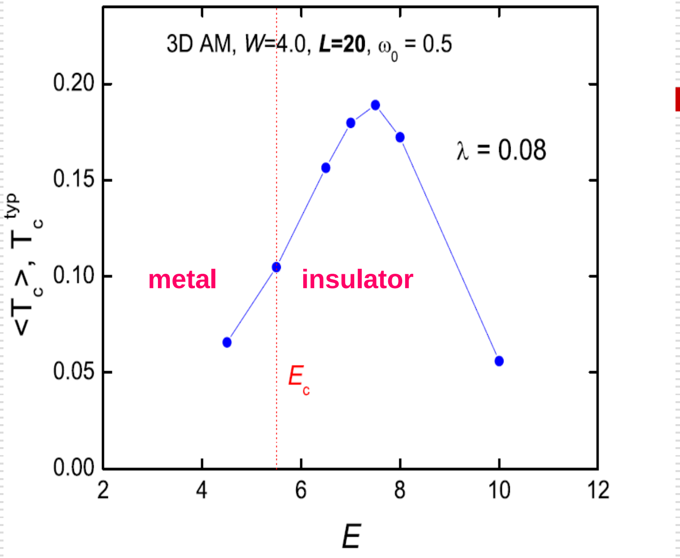
Superconducting transition temperature

Virial expansion on the 3d Anderson model





Conclusion:



Enhancement of Tc by disorder

Maximum of Tc in the insulator

Direct superconductor to insulator transition

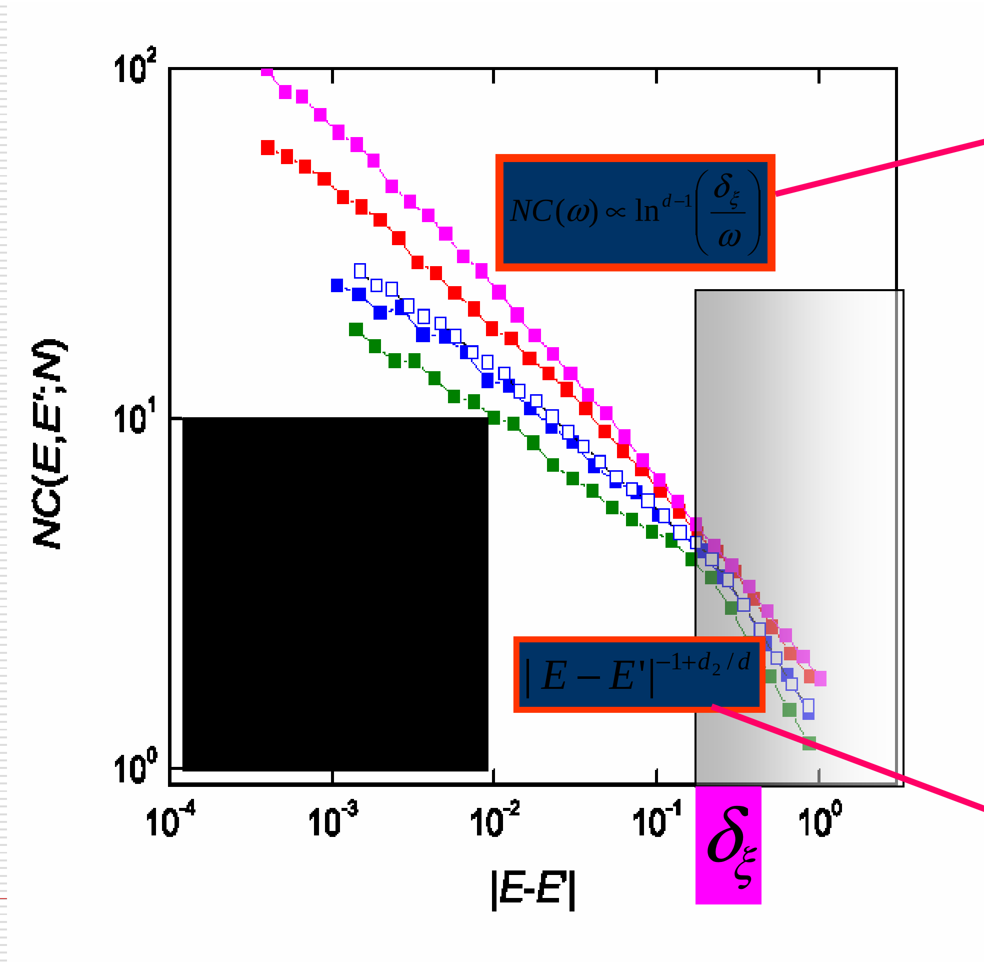
BUT

Fragile superconductivity:

Small fraction of superconducting phase

Critical current decreasing with disorder

Two-eigenfunction correlation in 3D Anderson model (insulator)

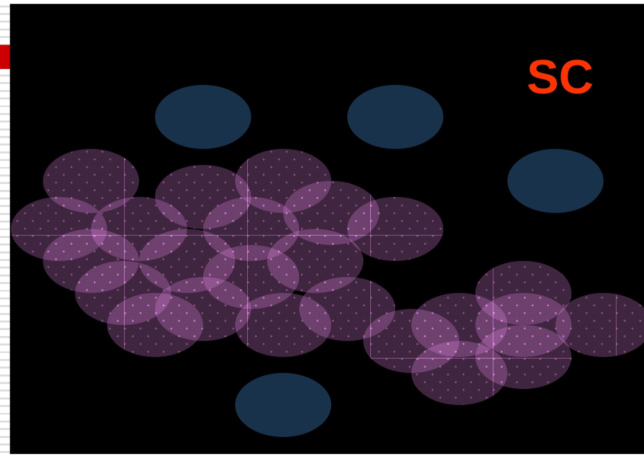


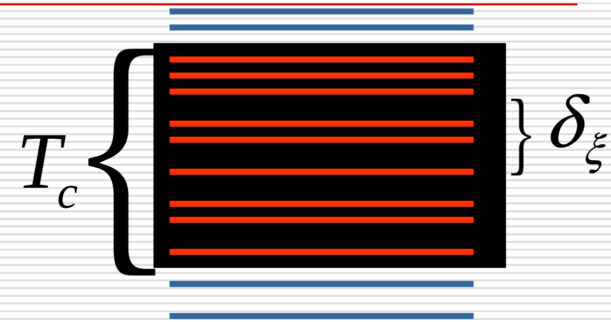
Mott's resonance physics

critical, multifractal physics

Superconductor-Insulator transition: percolation without granulation

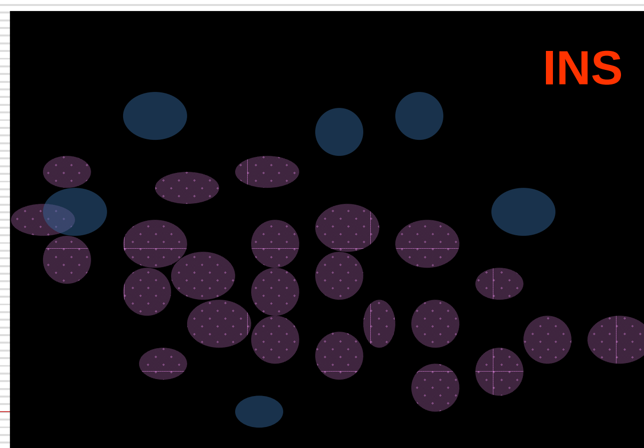
Coordination number K>>1

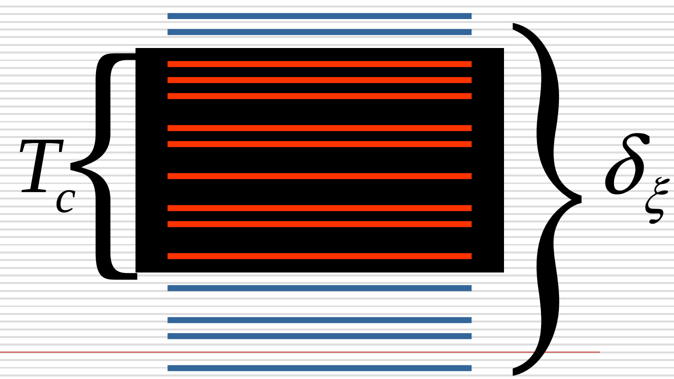




Only states in the strip ~T_c near the Fermi level take part in superconductivity

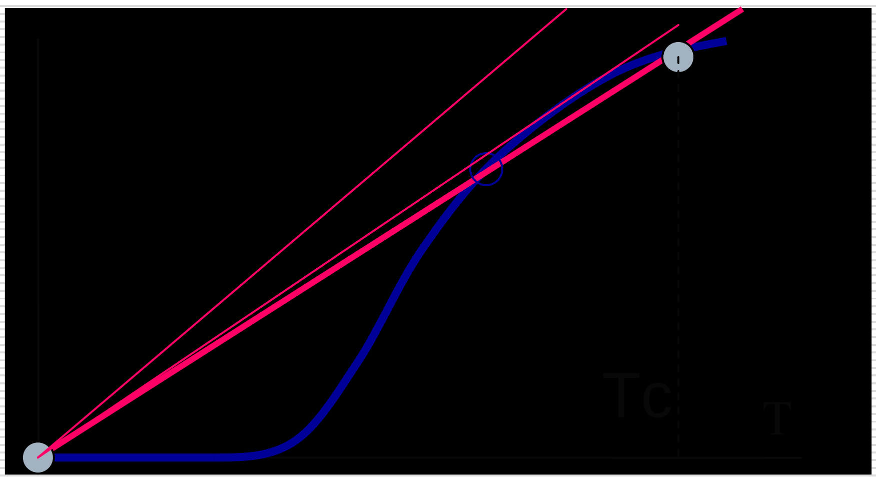
Coordination number K=0





First order transition?

[redacted]



[redacted]

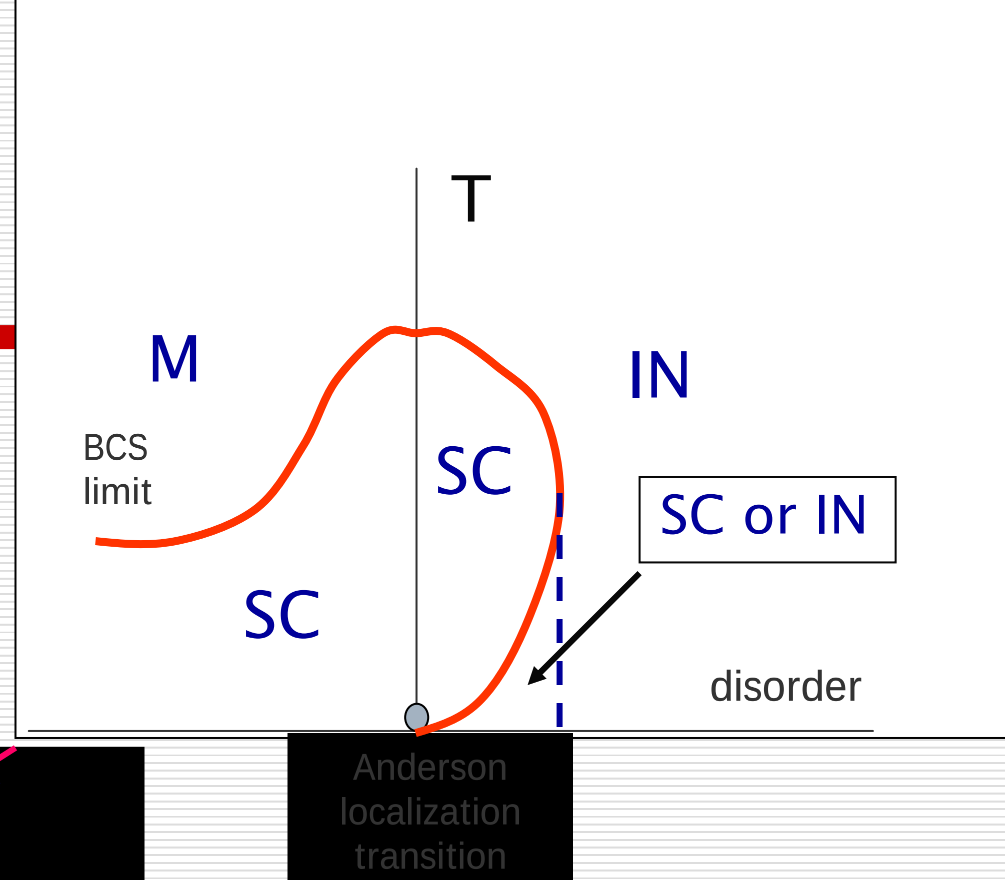
f(x)->0 at x<<1

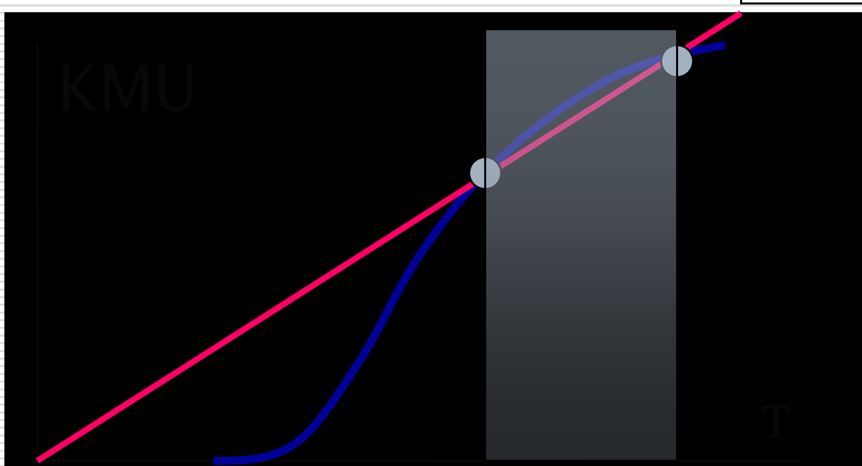
Conclusion

- Fraclal texture of eigenfunctions persists in metal and insulator (multifractal metal and insulator).
- Critical power-law enhancement of eigenfunction correlations persists in a multifractal metal and insulator.
- Enhancement of superconducting transition temperature

  due to critical wavefunction correlations.

[redacted]





[redacted]

Corrections due to off-diagonal terms

[redacted]

[redacted]

Average value of the correction term increases T_c

Average correction is small when [redacted]

Melting of phase by disorder

$$\Delta(r) = \sum_{ij} \Delta_{ij} \eta_{ij} \Psi_i(r) \Psi_j(r)$$

In the diagonal approximation [redacted]

The sign correlation <$\Delta(r)\Delta(r')$> is perfect : solutions $\Delta_i > 0$ do not lead to a global phase destruction

Beyond the diagonal approximation : [redacted]

stochastic term destroys phase correlation

How large is the stochastic term?

Stochastic term: [redacted]

$d_2<d/2$

[redacted]

$d_2>d/2$

$d_2 > d/2$ weak oscillations

$d_2 < d/2$ strong oscillations but still too small to support the glassy solution

More research is needed

Conclusions

- Mean-field theory beyond the Anderson theorem: going into the Fock space
- Diagonal and off-diagonal matrix elements
- Diagonal approximation: enhancement of T_c by disorder.
- Enhancement is due to sparse single-particle wavefunctions and their strong correlation for different energies
- Off-diagonal matrix elements and stochastic term in the MF equation
- The problem of “cold melting” of phase for $d_2 < d/2$