



Decoherence in Charge Qubit: Interacting Resonant Levels

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Columbia U., NY

Qubit decoherence

- Qubit is a 2-level system which can be controlled and manipulated – represented as a generalised spin

$$\hat{H} = \frac{1}{2}g_z B_z \hat{\sigma}_z + \frac{1}{2}g_x B_x \hat{\sigma}_x + V_{\text{coupl}} + \hat{H}_{\text{bath}}$$

Generic qubit Hamiltonian:

- Quantum computing: a controlled unitary evolution of a set of entangled qubits
- Major impediment: loss of unitarity – decoherence – due to uncontrolled coupling to the environment
- One of the main theoretical challenges: to understand principal mechanisms and develop a theory of low-T decoherence and relaxation by environment
- Two ways:
 - ✓ Generic for any qubit (Caldeira-Leggett environment)
 - ✓ Microscopic – qubit-specific

Solid-state qubits

- Main challenge – to separate two levels from “zillions” typical for a solid state device:
 - Possible solutions: flux and charge Josephson junctions
 - Double quantum dots
- Main advantages:
 - Potential scalability: ‘many-qubit devices’
 - High-fidelity state preparation
 - Almost non-invasive “readout”
 - Possible coherent evolution at low T
- Main impediment – **decoherence**

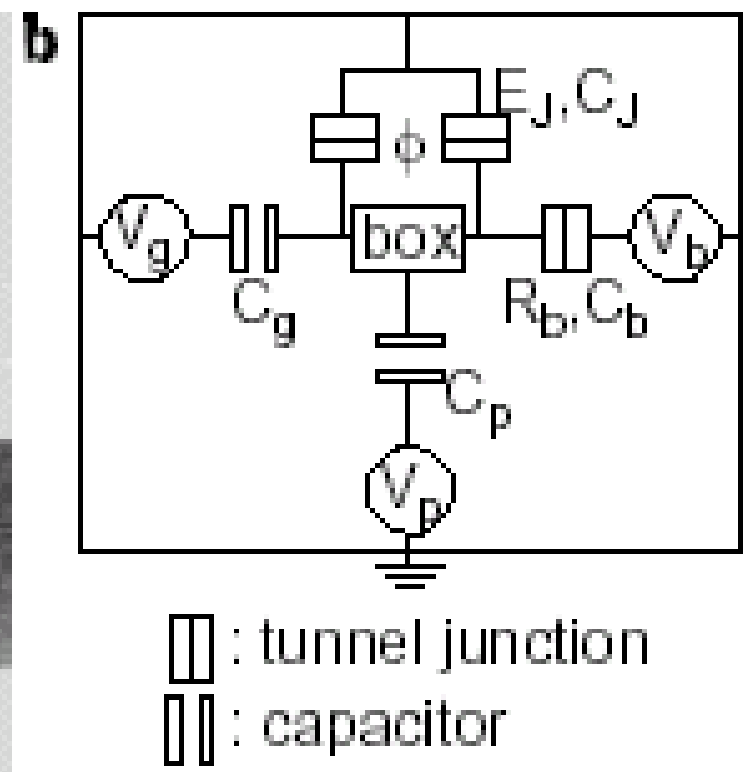
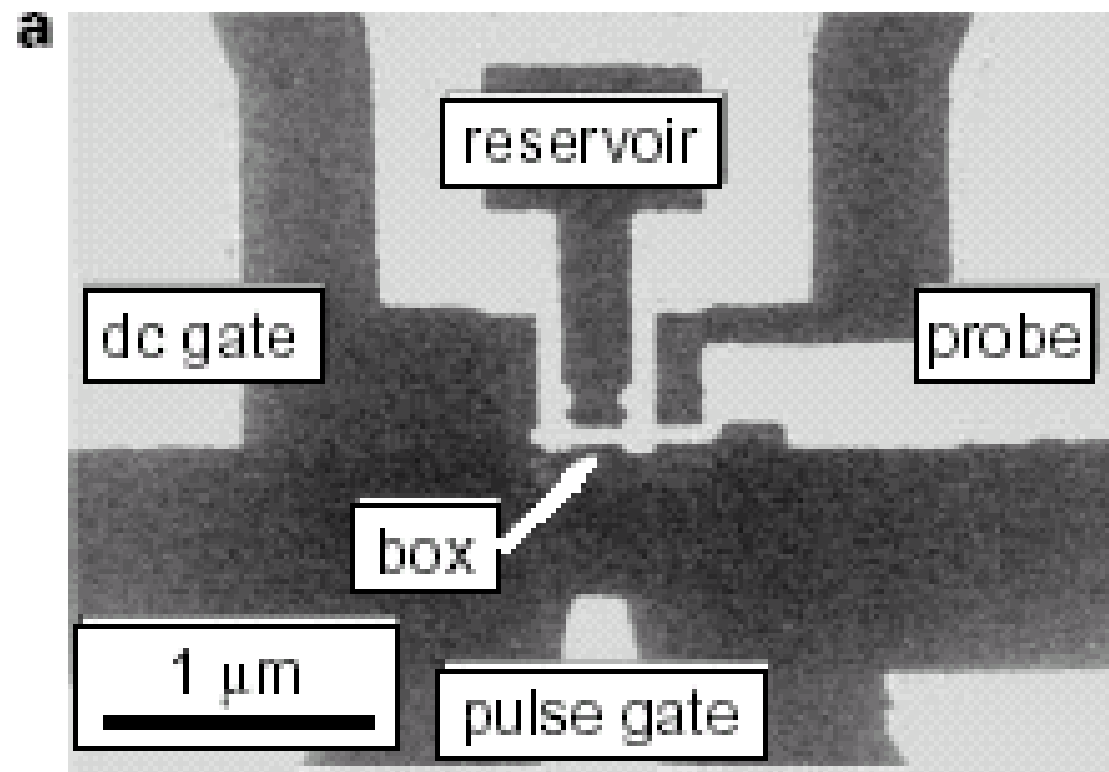


Figure 1 Single-Cooper-pair box with a probe junction. **a**, Micrograph of the sample. The electrodes were fabricated by electron-beam lithography and shadow evaporation of Al on a SiN_x insulating layer (400-nm thick) above a gold ground plane (100-nm thick) on the oxidized Si substrate. The 'box' electrode is a $700 \times 50 \times 15$ nm Al strip containing $\sim 10^8$ conduction electrons. The reservoir electrode was evaporated after a slight oxidation of the surface of the box so that the overlapping area becomes two parallel low-resistive tunnel junctions (~ 10 k Ω in total) with Josephson energy E_J , which can be tuned through magnetic flux ϕ penetrating through the loop. Before the evaporation of the probe electrode we further oxidized the box to create a highly resistive probe junction ($R_b \approx 30$ M Ω).

Two gate electrodes (d.c. and pulse) are capacitively coupled to the box electrode. The sample was placed in a shielded copper case at the base temperature ($T \approx 30$ mK; $k_B T \approx 3$ μ eV) of a dilution refrigerator. The single-electron charging energy of the box electrode $E_C = e^2/2C_2$ was 117 ± 3 μ eV, where C_2 is the total capacitance of the box electrode. The superconducting gap energy Δ was 230 ± 10 μ eV. **b**, Circuit diagram of the device. The C s represent the capacitance of each element and the V s are the voltage applied to each electrode.

$$T_2 \sim 5\text{ns}$$

500 nm

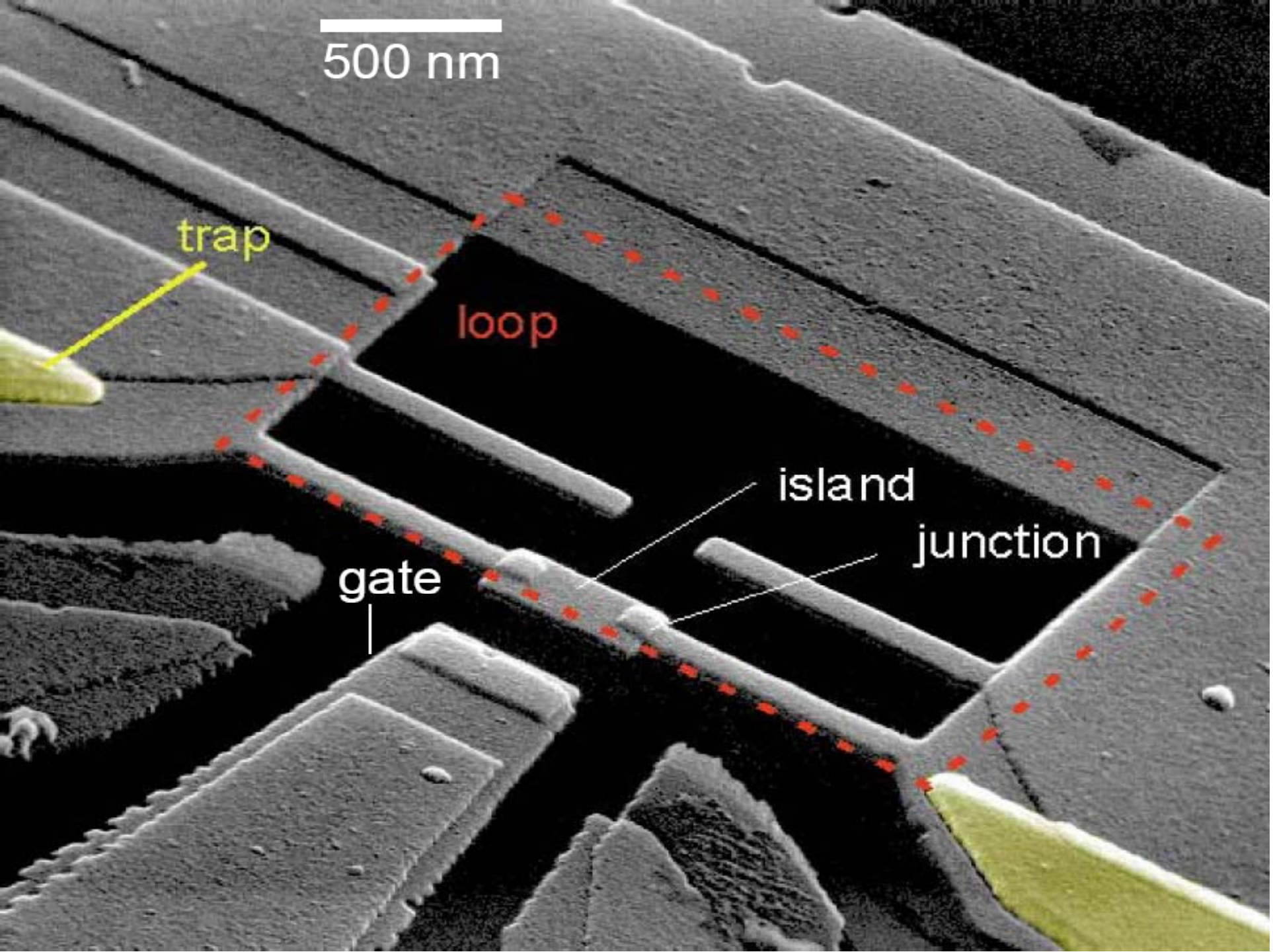
trap

loop

island

junction

gate



Experiment on Double QD qubits

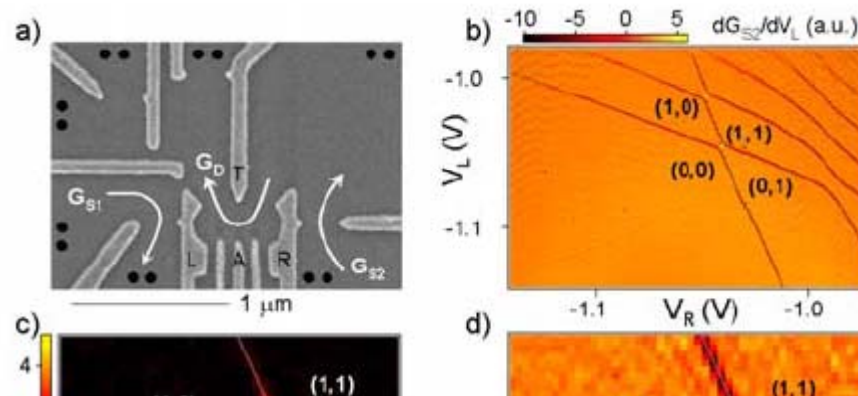
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Manipulation of a Single Charge in a Double Quantum Dot

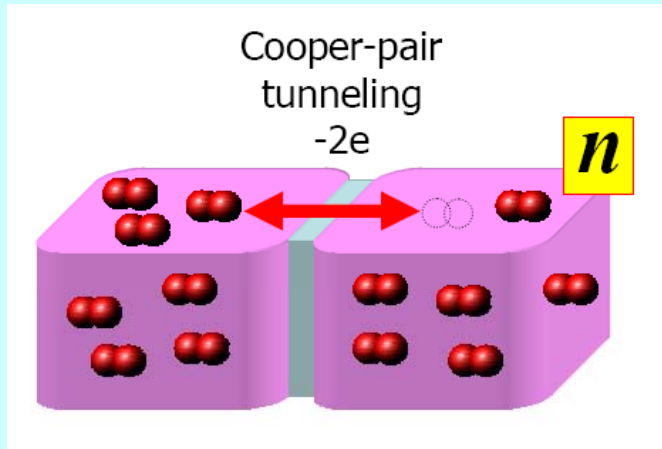
J. R. Petta,¹ A. C. Johnson,¹ C. M. Marcus,¹ M. P. Hanson,² and A. C. Gossard²



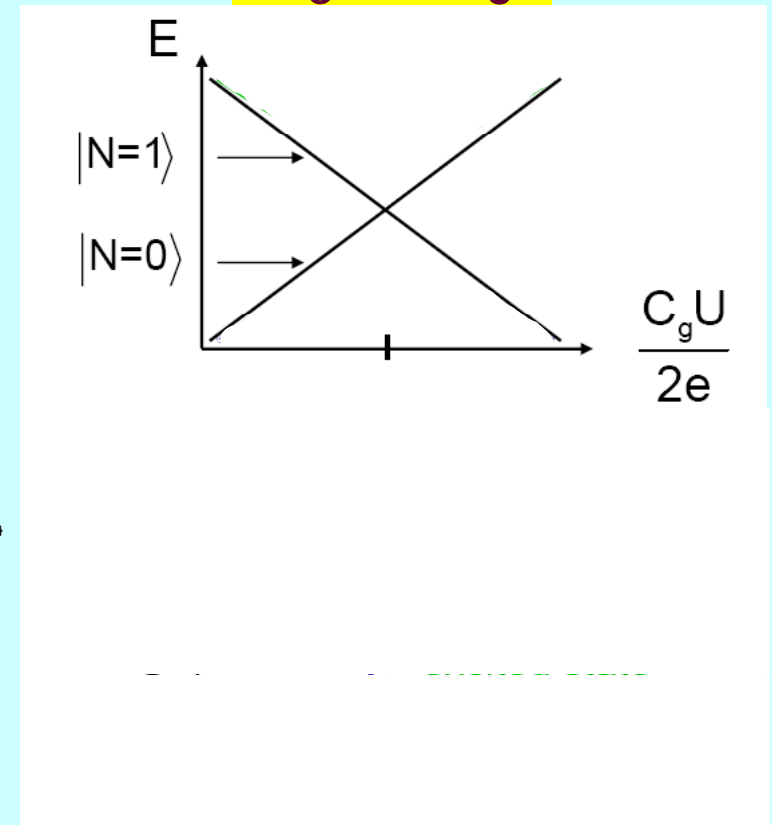
Time-domain experiments and measured linewidths allow us to extract $T_1 = 16$ ns and $T_2^* \geq 400$ ps for the charge two-level system. In addition, we

Fig. 1. Transport and charge sensing of a few-electron double dot. (a) Scanning electron microscope (SEM) image of

Josephson-box charge qubit



$$E_C \gg E_J$$



$$\hat{H} = E_C (\hat{n} - N_g)^2 - E_J \cos \hat{\theta}$$

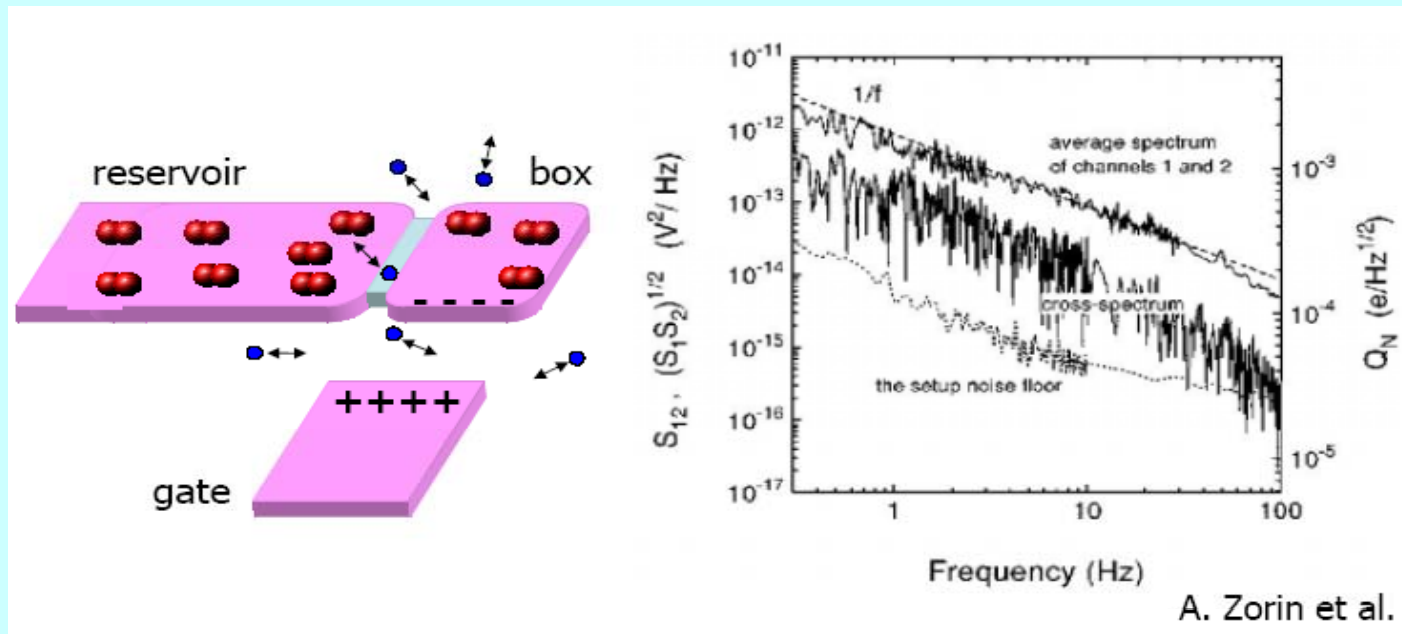
$$[\hat{\theta}, \hat{n}] = i \quad \hat{n} |n\rangle = n |n\rangle$$

$$\hat{H} = \frac{\omega_0}{2} \hat{\sigma}_z + \frac{E_J}{2} \hat{\sigma}_x$$

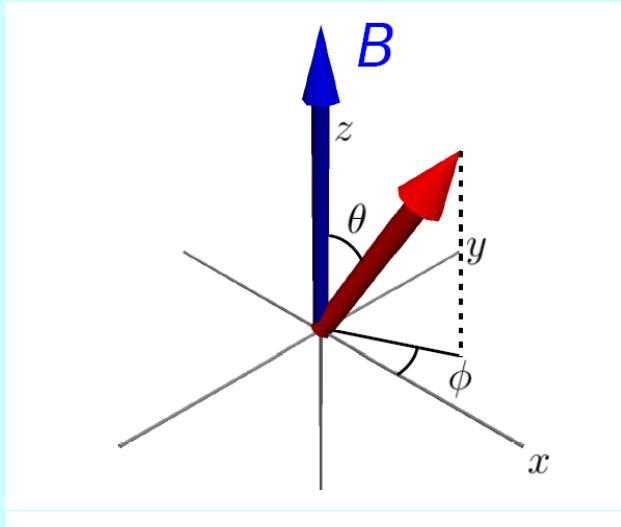
+ $V_{\text{coupl}} + V_{\text{bath}} \Rightarrow$ decoherence

Fluctuating background charges

- charge state of impurities could change due to their coupling to the leads
- their wide spread is responsible for $1/f$ noise



Decoherence & relaxation



Density matrix

$$\rho(t) = \frac{1}{2} (1 + \mathbf{S}(t) \cdot \boldsymbol{\sigma})$$

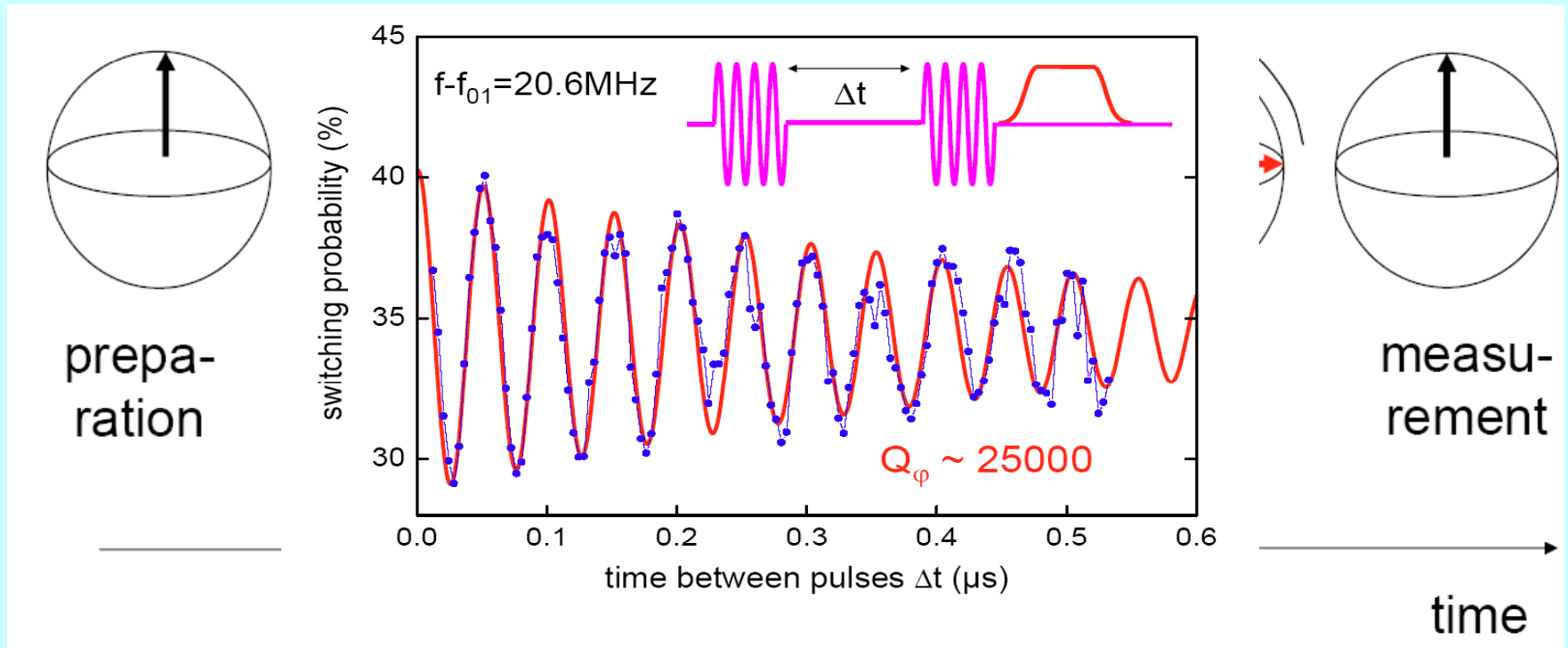
$$\langle \rho_{11}(t) \rangle \propto \rho_{11}(0) e^{-\Gamma_1 t}$$

relaxation

$$\langle \rho_{12}(t) \rangle = \rho_{12}(0) e^{i\omega_0 t} e^{-\Gamma_2 t}$$

decoherence

Measuring decoherence time



Most of the time – free evolution (‘computing’)

Measuring decay time of oscillations gives $1/\Gamma_2$

Possible Sources of Decoherence

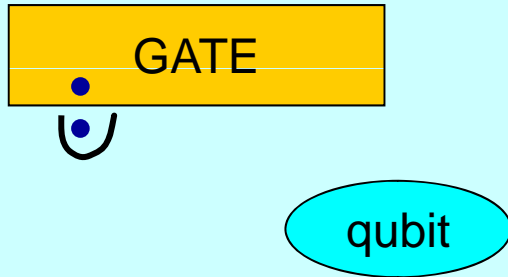
Intrinsic sources

- ❖ Coupling to /emitting phonons –contributes but not sufficient (Ioffe et al, '04; Barranger; Mucciolo, '05)
- ❖ Quasiparticle motion – frozen
- ❖ Electromagnetic radiation –inefficient (Bulaevski and Martin, '05)
- ❖ Fluctuating background charges - believed to be dominant for any charge qubit
- ❖ Something else

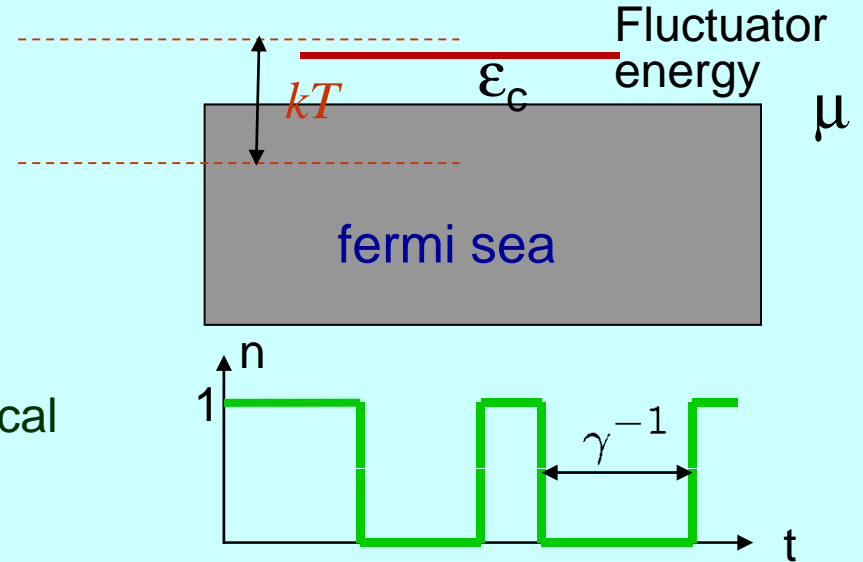
Artificial sources

- ❖ Control circuits
- ❖ Switching on-off measuring devices

Background charges and decoherence



A few charges close to ϵ_F produce classical telegraph noise at $T > \gamma$ (switching rate)



Noise power $S(\omega) \equiv \int \frac{d\omega}{2\pi} e^{i\omega t} \langle \{\hat{n}(t), \hat{n}(0)\} \rangle_B$ gives the decoherence rate:

$$\Gamma_2 \propto S(\omega)|_{\omega \rightarrow 0}$$

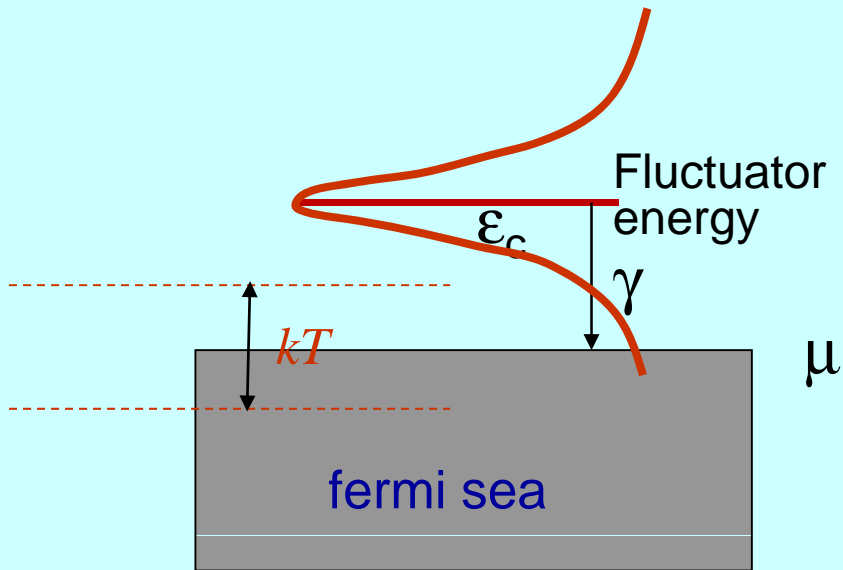
calculated semi-classically for this model (via rate eqs)

E Paladino, L Faoro, G Falci, R Fazio, '02;
Yu Galperin, B Altshuler, D Shantsev, '03

Do the numbers add up?

- ❖ Even a single impurity with $\varepsilon < \gamma$ might be enough to cause a noticeable decoherence – but many requires to explain $1/f$ noise
- ❖ For a DQD qubit, using the experimental geometry and assuming hybridization to be efficient up to 10 atomic distances, we find that $N_{\text{geom}} \sim (10 \div 100)c$, where c is the impurity # in ppm.
- ❖ However, the impurity energies are spread over the bandwidth D , so that **if** all the impurities with $\varepsilon_0 > T$ are frozen, $N_{\text{eff}} \sim (T/D) N_{\text{geom}} \ll 1$

Quantum smearing of the fluctuator



Fluctuators with $\epsilon_c > T$ seem to be exponentially suppressed (frozen out)

NOT REALLY! Hybridization broadens the level

Thus all fluctuators with $\epsilon < \gamma$ contribute to Γ_2 , while the contribution of those with $\epsilon > \gamma$ is power-law suppressed

A. Grishin, I.V.Yurkevich, IVL, '05

Do the numbers add up now?

- ❖ The impurity energies are spread over the bandwidth D , while the effective impurities are spread over γ , so that $N_{\text{eff}} \sim (\gamma/D) N_{\text{geom}}$ which **still** may be <1 , if not $\ll 1$
- ❖ The discrepancy is “worse” for the superconducting bath: effective fluctuators should obey $(\varepsilon_0^2 + \gamma^2)^{1/2} \lesssim T$ but ε_0 is pushed to the gap boundary, $\Delta \sim 2\text{K}$ while $T \sim 0.01\text{K}$.
- ❖ A solution to this “puzzle” is two-fold:
 - ✓ Interaction of the resonant level with the bulk should be included for make the ends meet for a QDQ.
 - ✓ A macroscopic number of two types of “Andreev fluctuators” is required (and present) in case of JCQ.

The model considered

Qubit coupled electrostatically to the FBC(s)

$$\hat{H}_q = \frac{\omega_0}{2} \hat{\sigma}_z + \hat{\sigma}_z \hat{V} + \frac{E}{2} \hat{\sigma}_x, \quad \hat{V} \equiv \frac{1}{2} \sum_i v_i \hat{d}_i^\dagger \hat{d}_i$$

FBCs are hybridized with the conducting electrons bath

$$\hat{H}_B = \sum_i \varepsilon_i \hat{d}_i^\dagger \hat{d}_i + \sum_{i,\mathbf{k}} [t_{\mathbf{k}i} \hat{c}_{\mathbf{k}}^\dagger \hat{d}_i + h.c.] + \sum_{\mathbf{k}} \varepsilon_{\mathbf{k}} \hat{c}_{\mathbf{k}}^\dagger \hat{c}_{\mathbf{k}}$$

The model applicable to the CHARGE QUBIT: $\omega_0 \sim E_C \gg E_J$

Alternatively, the bath is a superconductor and the free-fermion part of H_B is substituted by H_{BCS}

Adding the interaction

$$\hat{H}_{\text{imp}} = \sum_i \left[\varepsilon_i \hat{n}_i + U_i^H \hat{n}_{i\uparrow} \hat{n}_{i\downarrow} \right] ,$$

A huge Hubbard U^H ensures the absence of double-occupied states – and there its role ends. An empty state is quasi degenerate with single-occupied ones but spin degrees of freedom are irrelevant for charge fluctuations.

$$H_{\text{int}} = \sum_i U_i \hat{\rho}_i \hat{n}_i$$

“Interacting Resonant Level” model – can be mapped to the Kondo model (Toulouse, ‘70; Matveev & Larkin ‘97) where two pseudo-spin states correspond to the empty and single-occupied electron impurity states

Reduced density matrix

$$(\rho_S)_{\alpha\beta} = \sum_n \rho_{\alpha\beta}^{nn} \equiv \langle \alpha | \text{tr}_B \hat{\rho} | \beta \rangle$$

since for a measurement on the qubit only

$$\langle \hat{S} \rangle = \text{tr} \hat{\rho} \hat{S} = \sum_n \rho_{\alpha\beta}^{nm} \delta_{mn} S_{\beta\alpha} = \text{tr} \hat{\rho}_S \hat{S}$$

Time evolution:
$$\frac{d}{dt} \hat{\rho}(t) = -i \left[\hat{H}(t), \hat{\rho}(t) \right]$$

Starting with $\rho(0) = \rho_q(0) \otimes \rho_B(0)$, one finds

$$\hat{\rho}(t) = T_{\leftarrow} e^{-i \int_0^t \mathcal{H}_{\text{int}}(t') dt'} \hat{\rho}(0) T_{\rightarrow} e^{i \int_0^t \mathcal{H}_{\text{int}}(t') dt'}$$

The decoherence function

Decoherence matrix in a “qubit” space

$$\hat{\rho}(t) = \begin{pmatrix} n(t) & \rho_{12}(0)e^{-i\omega_0 t} D(t) \\ \rho_{21}(0)e^{i\omega_0 t} D^*(t) & 1 - n(t) \end{pmatrix}$$

Here the decoherence function

$$D(t) = \left\langle T_{\leftarrow} e^{i(\hat{H}_B + \hat{V}(t))t} T_{\rightarrow} e^{-i(\hat{H}_B - \hat{V}(t))t} \right\rangle$$

Define the relaxation rate Γ_1 and decoherence Γ_2 in long- t limit:

$$n(t) \propto e^{-\Gamma_1 t}$$

$$D(t) \propto e^{-\Gamma_2 t}$$

Relations to Noise Power

In the lowest order in the (weak) coupling, Γ_1 and Γ_2 are expressed in terms of $S(\omega)$. Here

$$\hat{H} = \frac{1}{2}\omega_0\hat{\sigma}_z + \frac{1}{2}E\hat{\sigma}_x + V_{\text{coupl}} + \hat{H}_{\text{bath}}$$

and
$$S(\omega) \equiv \int \frac{d\omega}{2\pi} e^{i\omega t} \langle \{\hat{V}(t), \hat{V}(0)\} \rangle.$$

$$\Gamma_1 = \sin^2 \theta S(\omega = \Omega),$$

$$\theta = E/\omega_0$$

$$\Gamma_2 = \frac{1}{2}\Gamma_1 + \tilde{\Gamma}_2, \quad \tilde{\Gamma}_2 = \cos^2 \theta S(\omega = 0)$$

$$\Omega = \sqrt{\omega_0^2 + E^2}$$

Experimentally, $\Gamma_1 \ll \Gamma_2$ so that only low-frequency fluctuations are relevant – which are improbable in the above model of Andreev's BFCs.

Pure decoherence model

Two main features:

- Neglect σ_x term in H_{qubit}
- Choose coupling to the bath $\propto \sigma_z$

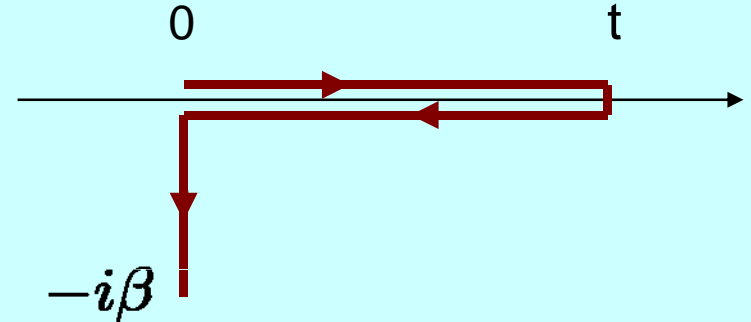
$$\hat{H} = \hat{H}_0 + \hat{H}_{\text{bath}} + \frac{1}{2}\sigma_z V$$

The model for FBCs can be solved asymptotically exactly ($t \rightarrow \infty$) in the absence of the interaction and is good enough for the charge qubit in the “operational mode” (in the absence of the controlled fields and neglecting the Josephson coupling for JCQ).

Formal representation

$$D(t) = \left\langle \hat{T}_K e^{-i \int dt' \hat{V}(t')} \right\rangle_B$$

Keldysh contour ordering



$$D(t) = \int \frac{\mathcal{D}\{\text{all}\}}{\mathcal{Z}} \exp \left[i \int_{c_K} dt' \left(\sum_{ij} S_{ij} + \sum_{\mathbf{k}} S_{\mathbf{k}} \right) \right]$$

$$S_{ij} = \bar{d}_j(t') \left(i\partial_{t'} - \varepsilon_j^0 + \frac{1}{2}v_j(t') \right) d_j(t') \delta_{ij} ,$$

$$S_{\mathbf{k}} = \bar{c}_{\mathbf{k}}(t') (i\partial_{t'} - \varepsilon_{\mathbf{k}}) c_{\mathbf{k}}(t')$$

$$- \left(\bar{c}_{\mathbf{k}} \sum_i t_{\mathbf{k}i}(t') d_i(t') + \text{h.c.} \right) .$$

Integrating out “bulk” electrons:

For a local hybridization, $t_{\mathbf{k}i} = t_i V^{-1/2} e^{i\mathbf{k}\cdot\mathbf{r}_i}$ one finds

$$\Sigma_{ij} = t_i t_j^* g(\mathbf{r}_i - \mathbf{r}_j, t' - t'') \rightarrow |t_i|^2 \delta_{ij} g(0)$$

so that $G_j^{\text{R/A}}(\omega) = \left(\omega - \varepsilon_j \pm i\frac{\gamma_j}{2}\right)^{-1}$, with $\gamma_j \equiv 2\Im\text{m}\Sigma_{jj}^{\text{R}} = 2\pi\nu|t_j|^2$

First expanding and then re-exponentiating

$$\Gamma_2 = -\Re\int_{-\infty}^{+\infty} \frac{d\omega}{2\pi} \text{tr} \ln \left[1 + \frac{v}{2} \hat{G}^{\text{K}} - \frac{v}{2} \hat{G}^{\text{R}} \frac{v}{2} \hat{G}^{\text{A}} \right]$$

At $T=0$

$$\Gamma_2(0) = -\Re\int_{-\infty}^{+\infty} \frac{d\omega}{2\pi} \text{tr} \ln \left[\left(1 + \frac{\hat{v}}{2} \hat{G}^{\text{R}}\right) \left(1 - \frac{\hat{v}}{2} \hat{G}^{\text{A}}\right) \right] = 0$$

Decoherence rate

Performing asymptotically exact calculations, we find

$$\Gamma_2(T) = - \int_{-\infty}^{+\infty} \frac{d\omega}{4\pi} \ln \left\{ 1 - \frac{\cosh^{-2}(\omega/2T)}{1 + 4(\omega - \varepsilon_+)^2 (\omega - \varepsilon_-)^2 / v^2 \gamma^2} \right\}$$
$$\varepsilon_{\pm} \equiv \varepsilon \pm \frac{1}{2}v\sqrt{1 - g^{-2}}, \quad g \equiv \frac{v}{\gamma}$$

Classical (high-T) asymptotics: $T \gg \max |\varepsilon_{\pm}|, \gamma$

$$\Gamma_2 = \frac{\gamma}{2} \left[1 - \Re \sqrt{1 - g^2} \right] = \begin{cases} \frac{1}{2}\gamma, & g \gg 1 \\ \frac{1}{4}\gamma g^2 & g \ll 1 \end{cases}$$

E Paladino, L Faoro, G Falci,
R Fazio, '02

Yu Galperin, B Altshuler,
DShantsev, '03

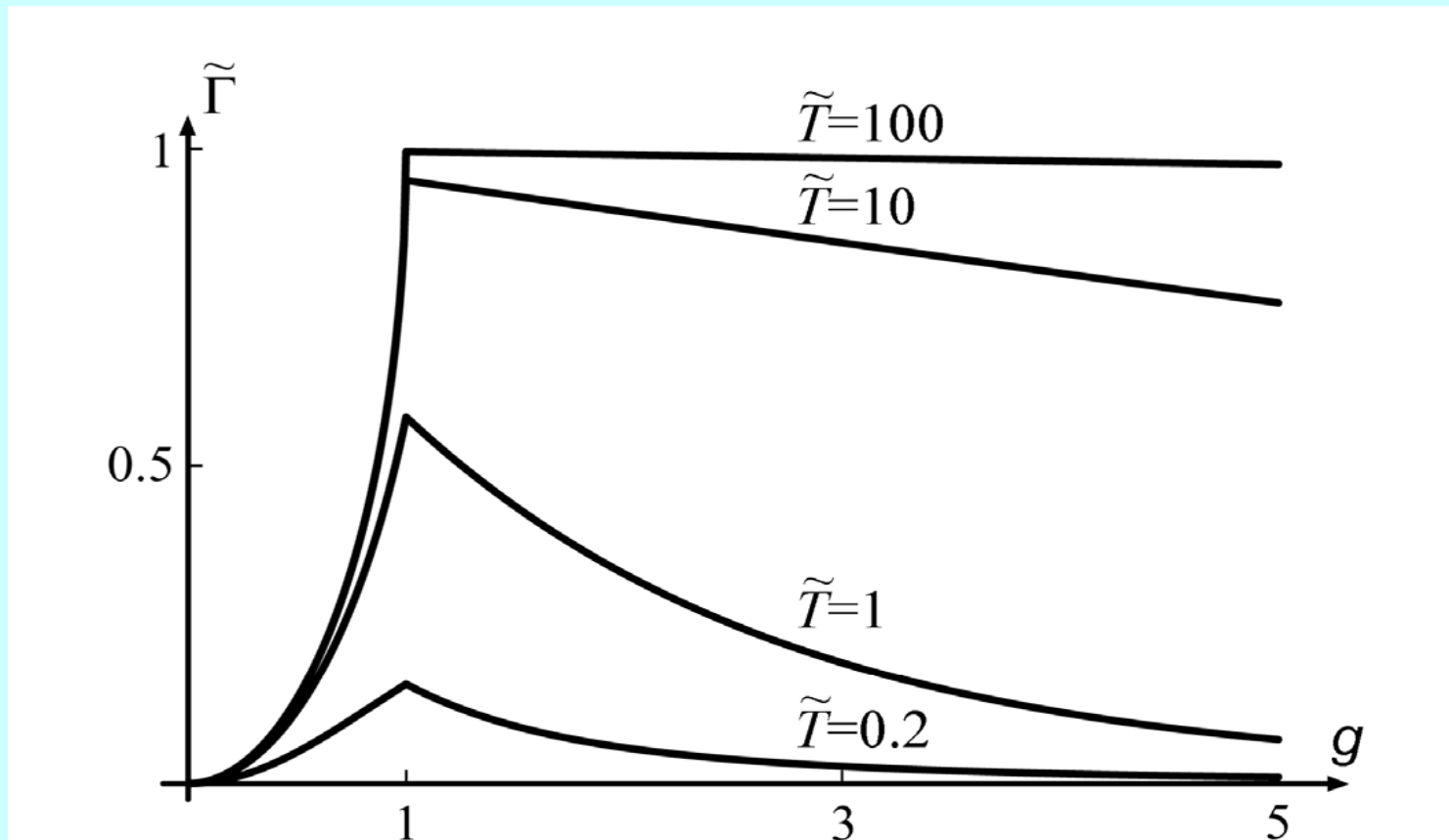
Quantum (low-T) result: $T \ll \min |\varepsilon_{\pm}|$ or $T \ll \gamma$

$$\Gamma_2 = \frac{T}{\pi} \arctan^2 \left(\frac{2g}{4\varepsilon^2/\gamma^2 - g^2 + 1} \right)$$

A Grishin, I Yurkevich, IVL, '05

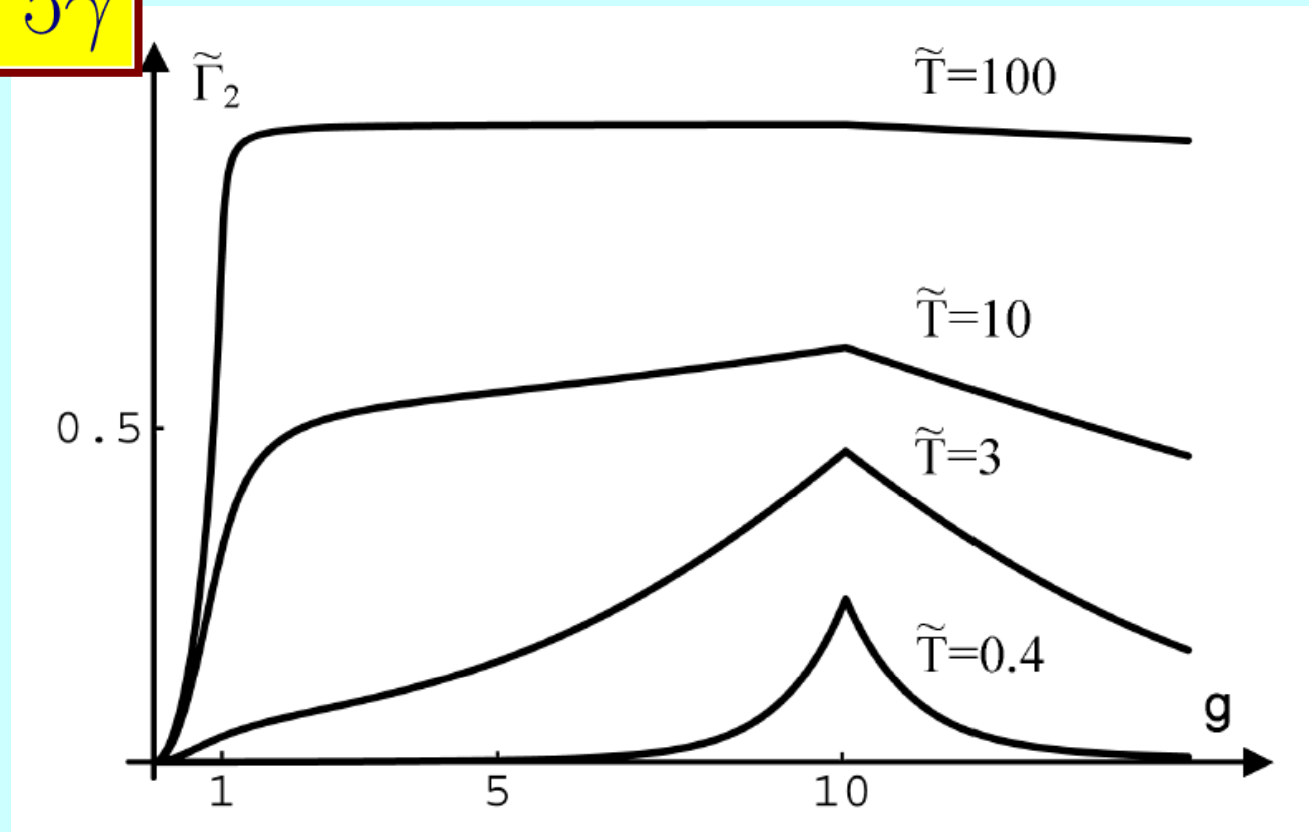
Decoherence rate as function of coupling strength g

$$\varepsilon = 0$$

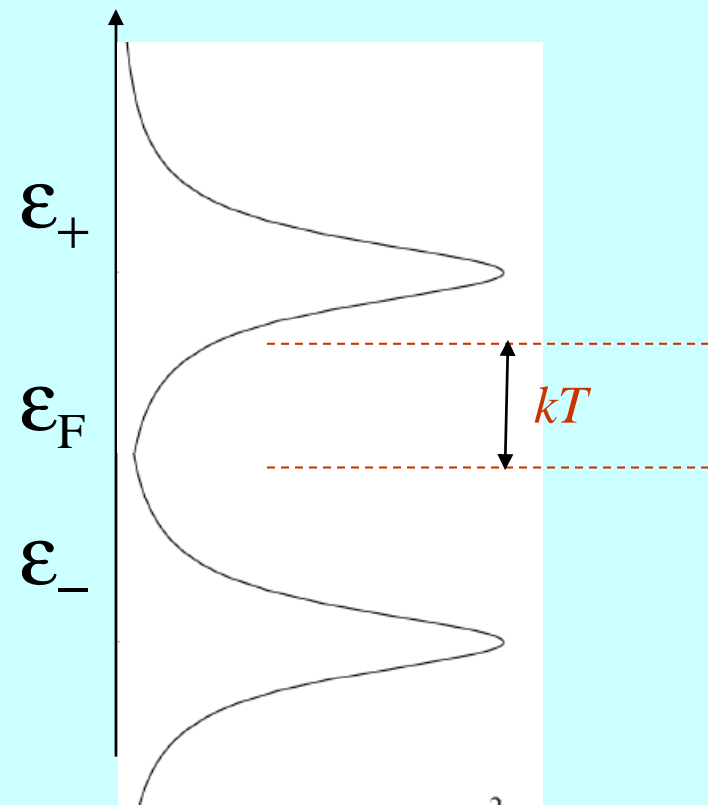
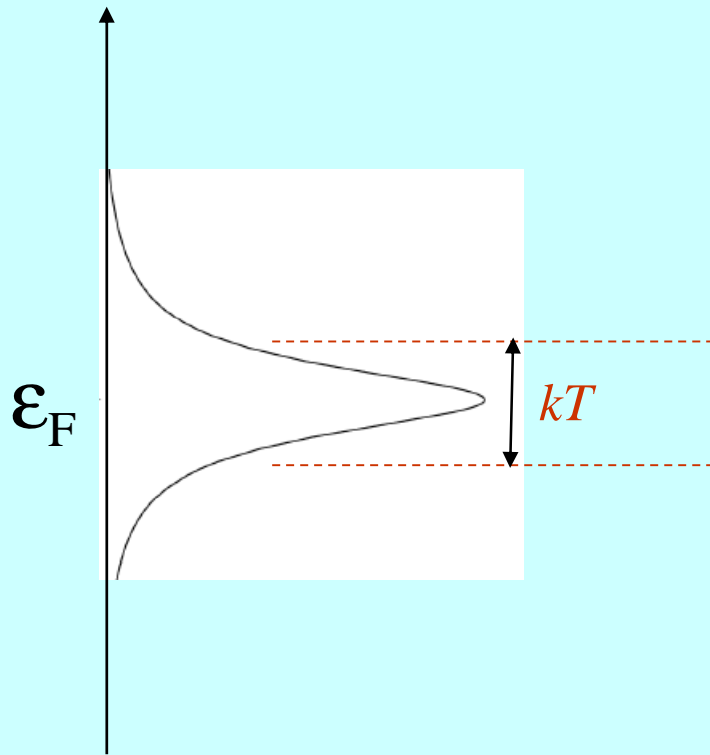


Decoherence rate as function of coupling strength g

$$\varepsilon = 5\gamma$$

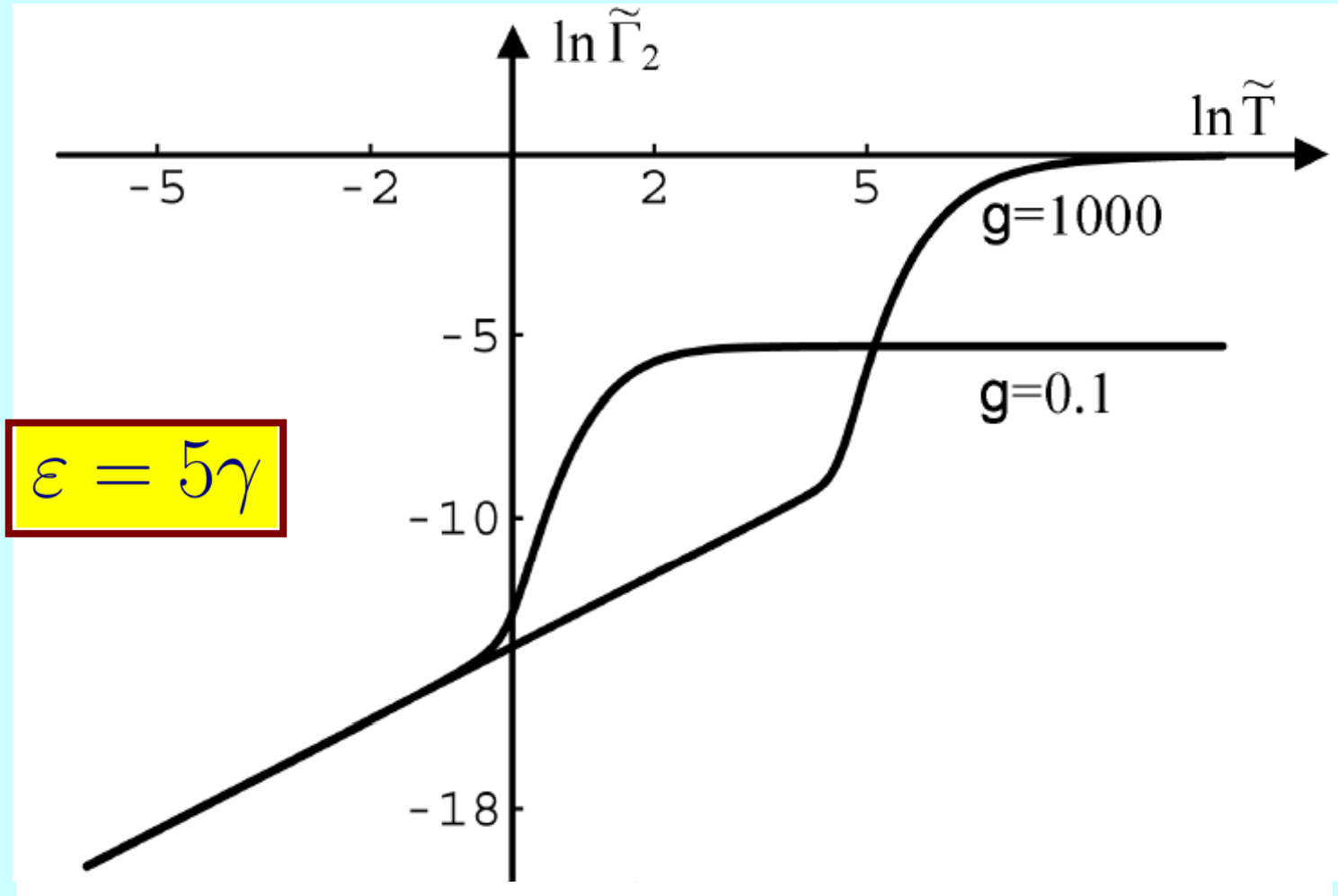


Why non-monotonic?



$$\epsilon_{\pm} \equiv \epsilon \pm \frac{1}{2}v\sqrt{1 - g^{-2}}$$

Decoherence rate as function of T



Adding the interaction

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A huge Hubbard U^H ensures the absence of double-occupied states – and there its role ends. An empty state is quasi degenerate with single-occupied ones but spin degrees of freedom are irrelevant for charge fluctuations.

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“Interacting Resonant Level” model – can be mapped to the Kondo model (Toulouse, ‘70; Matveev & Larkin ‘97) where two pseudo-spin states correspond to the empty and single-occupied electron impurity states

Interacting resonant level matters

Logarithmic divergences for scattering of conducting electrons via impurity state are accounted for by the appropriate RG.

The relevant equation for renormalization of effective switching rate g contains Mahan's and Anderson's terms, competing for $U > 0$

$$\frac{d \ln \gamma}{d \ln \epsilon} = -\alpha, \quad \alpha \equiv 2 \frac{\delta_0}{\pi} - \left(\frac{\delta_0}{\pi} \right)^2, \quad \delta_0 = 2 \arctan(\pi \nu_0 U / 2)$$

This leads to a dramatic increase of effective γ when $\alpha > 0$

$$\gamma \rightarrow \gamma(\epsilon) = \gamma \left[\frac{D}{\epsilon} \right]^\alpha.$$

This results in $N_{\text{eff}} \sim (\gamma/D)^{(1/1+\alpha)} N_{\text{geom}}$ – the probability to find efficient fluctuators strongly increases

Interaction works

(I Yurkevich, J Baldwin,
B Altshuler, IVL, '09)

It strongly enhances
also a contribution
from the Andreev's
fluctuators for JCQ.

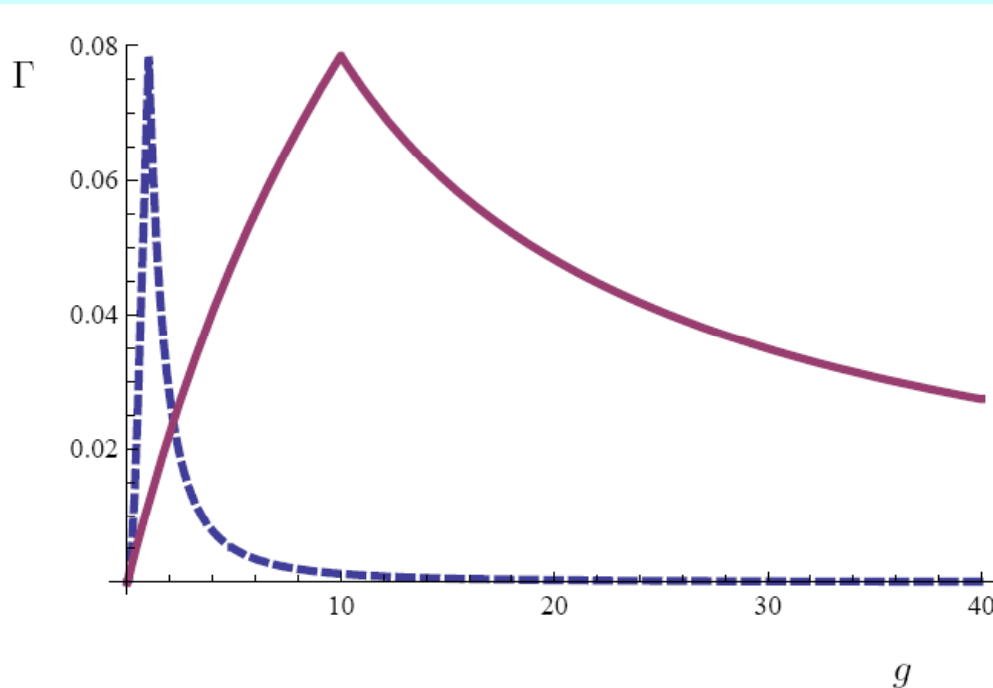
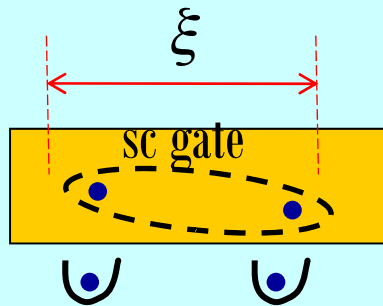


Fig. 1: Decoherence rate as a function of coupling strength, $g \equiv v/\gamma$, for $T = 0.1$, $D=10$. The dashed line shows the non-interacting case ($\alpha = 0$), while the solid line includes the interaction ($\alpha = 0.5$). We see that a high decoherence is achieved for a much wider parametric range with the interaction present.

Superconducting bath: Andreev fluctuators



0-2e Andreev fluctuator

L.Faoro, J.Bergli, B.L. Altshuler, Y.M. Galperin, '05;
L.M.Lutchin, L.Cywinski, C.P.Nave, S.Das Sarma, '08

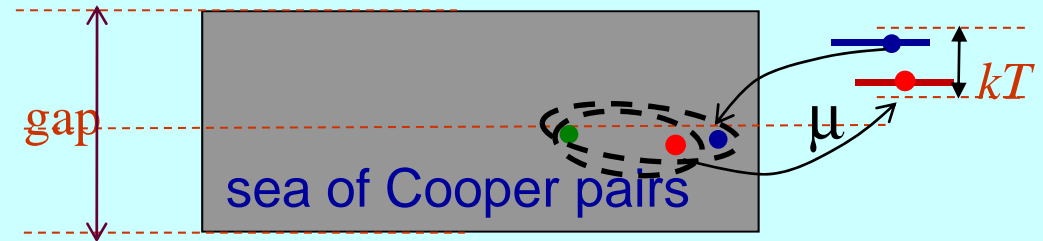
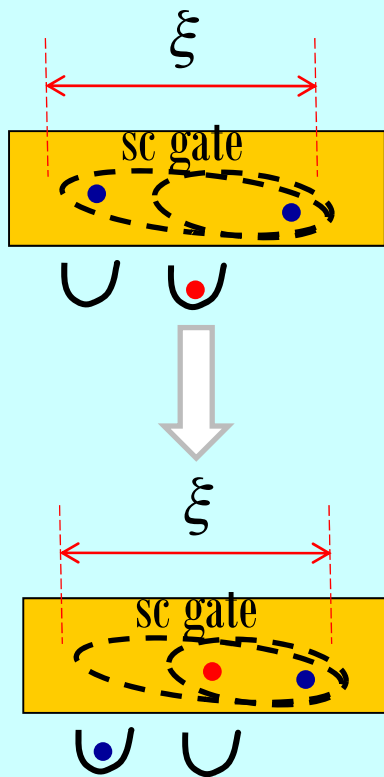
In contrast to fluctuators coupled to a normal metal, one needs **many** Andreev fluctuators to result in decoherence

$$G^{R,A}(\omega) \propto \frac{1}{\epsilon_i - \omega - \Sigma_{ii}^{R,A}(\omega)}, \quad \Sigma_{ii}^{R,A}(\omega) \propto \sum_{j \neq i} \frac{\gamma_{ij}^2}{(\epsilon_j + \omega \mp i0)}$$

However, it doesn't work like that: hybridization changes ϵ_j to

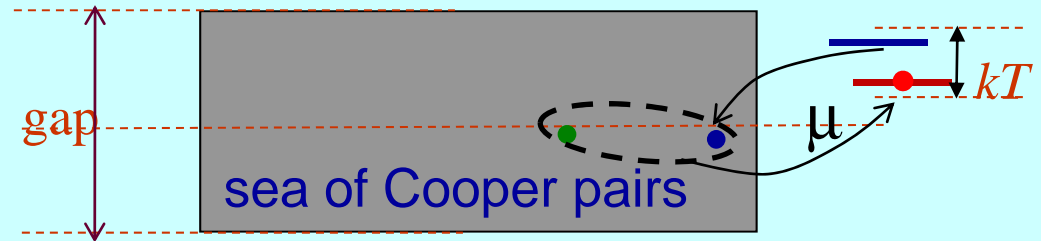
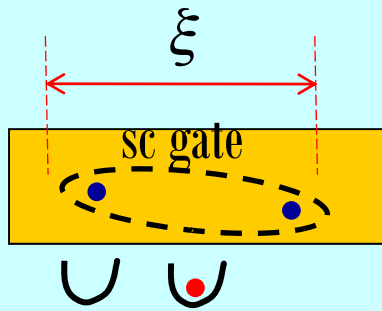
$$\tilde{\epsilon}_{ij} = \pm \frac{\sqrt{(\epsilon_i - \epsilon_j)^2 + \gamma_{ij}^2}}{1 + \gamma_{ij}/\Delta}$$

e - e Andreev fluctuators



In contrast to 0-2e fluctuators, the positions of levels are not changed by the hybridization and the probability to find a **macroscopic number** of e - e Andreev's pairs within the correlation length is substantial. Still, a macroscopic number is required to result in decoherence.

e - e Andreev fluctuators



In contrast to 0-2e fluctuators, the positions of levels are not changed by the hybridization and the probability to find a **macroscopic number** of e - e Andreev's pairs within the correlation length is substantial. Still, a macroscopic number is required to result in decoherence.

The two types of Andreev fluctuators form an impurity band. For large enough concentration, charges delocalise. Stray charges diffusion lead to charge fluctuations which act as a main cause of decoherence

(I Yurkevich, IVL, B Altshuler, 2010)

Conclusions

Most probably, the contribution from background fluctuating charges alone is enough to explain the observed decoherence in DQDQ and JCQ

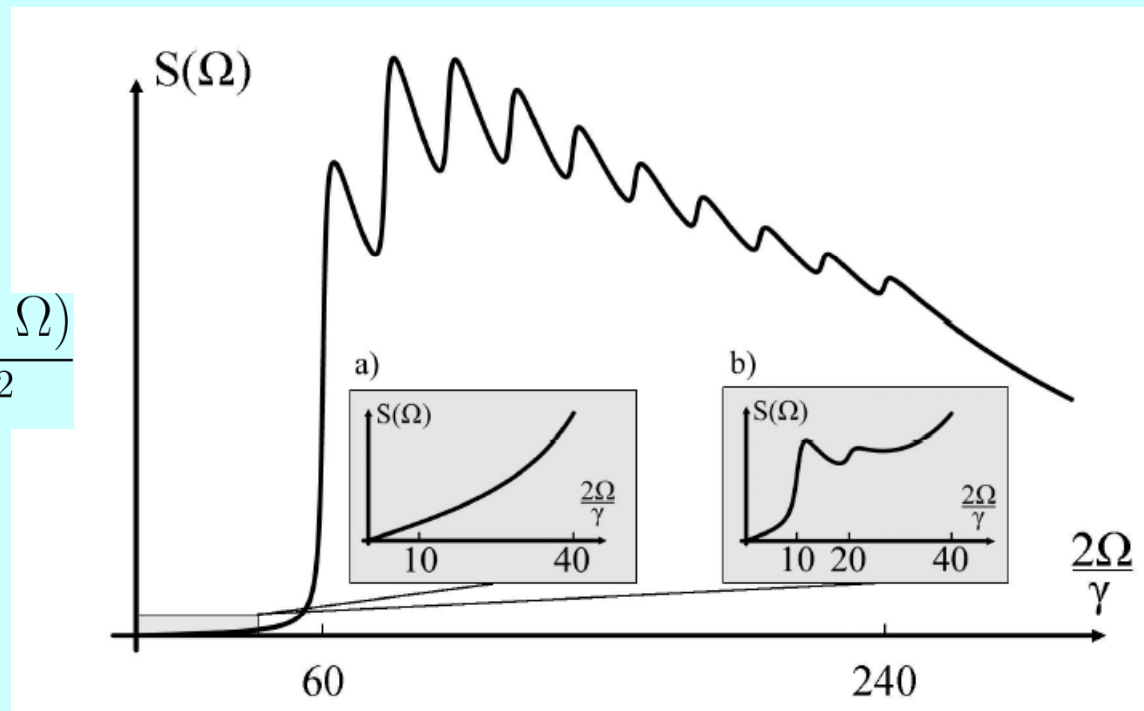
Including the electrostatic interaction of FBS with environment leads to drastic increase of the efficiency and a parametric enhancement of the contribution of the FBCs

For superconducting JCQ, the two types of Andreev fluctuators form an impurity band, with stray charges diffusion as a main cause of decoherence

Relaxation rate

$$\Gamma_1 = \frac{v^2}{4} \left(\frac{E_J}{\Delta E} \right)^2 \langle \{ \hat{X}(t), \hat{X}(0) \} \rangle_{\omega=\Delta E}, \quad \Delta E = \sqrt{(\omega_0)^2 + E_J^2}$$

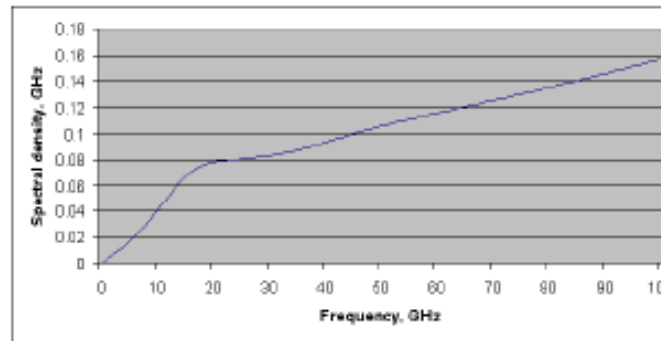
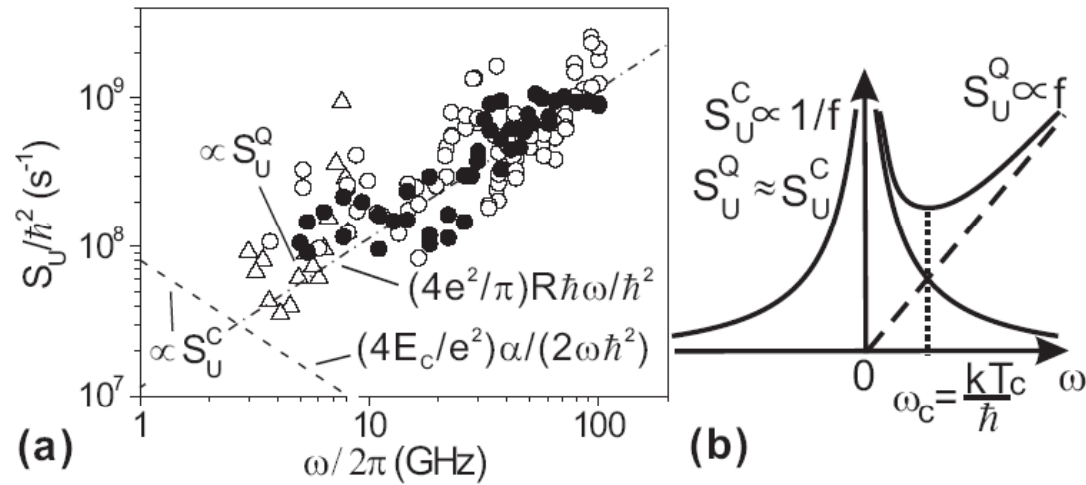
$$S(\Omega) = \frac{2 \Gamma_1 (\Delta E \rightarrow \Omega)}{\pi (E_J/\Omega)^2}$$



Relaxation rate: experiment

Quantum noise in the Josephson charge qubit

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Rabi oscillations and Ramsey fringes

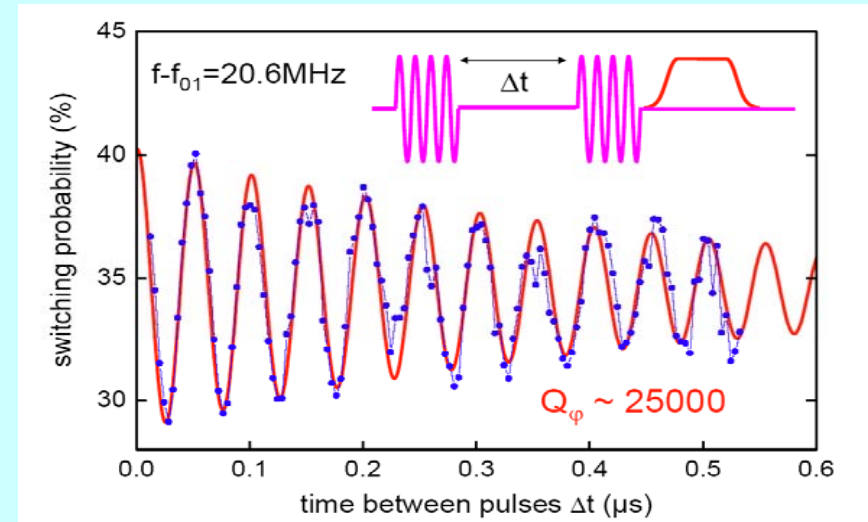
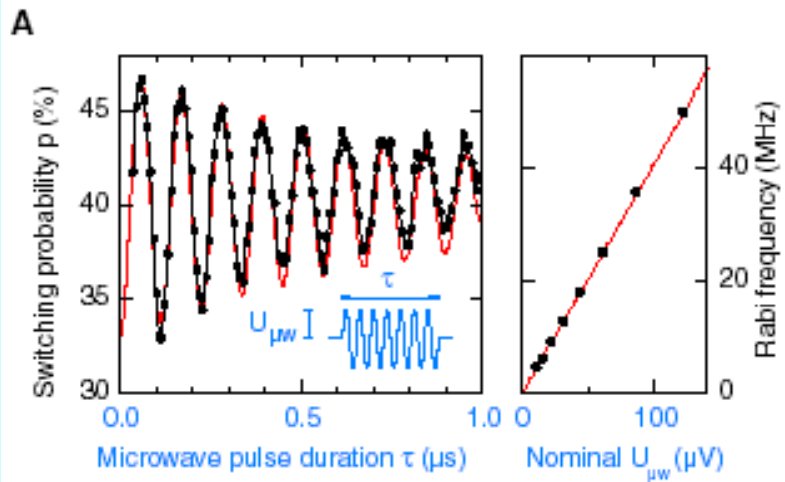


FIG. 3: (A) Left: Rabi oscillations of the switching probability p (5×10^4 events) measured just after a resonant microwave pulse of duration τ . Data were taken at 15 mK for a nominal pulse amplitude $U_{\mu w} = 22 \mu\text{V}$ (joined dots). The Rabi frequency is extracted from an exponentially damped sinusoidal fit (continuous line). Right: Measured Rabi frequency (dots) varies linearly with $U_{\mu w}$, as expected. (B) Ramsey fringes of the switching probability p (5×10^4 events) after two phase-coherent microwave pulses separated by Δt . Joined dots: Data at 15 mK; the total acquisition time was 5 mn. Continuous line: Fit by exponentially damped sinusoid with time constant $T_\varphi = 0.50 \mu\text{s}$. The oscillation corresponds to the “beating” of the free evolution of the spin with the external microwave field. Its period indeed coincides with the inverse of the detuning frequency (here $\nu - \nu_{01} = 20.6 \text{ MHz}$).

Decoherence rate as a function of T

